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Conformal Window and Correlation Functions in Lattice Conformal QCD

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In Collaboration with

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Plan of Talk

- Briefly review our previous works on the phase structure of the lattice SU(3) QCD. Thereby clarify the reason why we conjecture that the conformal window is $7 \leq N_f \leq 16$
- Introduce the concept of "conformal theory with IR cutoff".
  Propose new predictions about the propagator of a meson.
  We verify that new numerical results satisfy our proposals for Nf=7 and Nf=16.

(to be continued)

Plan of Talk (Cont.)

- Point out and verify that the propagator of a meson at T/Tc >1 shows the characteristics of “conformal theory with IR cutoff”.

STAGES and TOOLS

- Lattice gauge theory
  - one-plaqutte gauge action
    - Improved RG action: future plan
  - Wilson fermion action
- Lattice size: Nx=Ny=Nz=N; Nt=r N
- Lattice spacing: a
- PCAC quark mass: m_q
- G(t): propagators of mesons

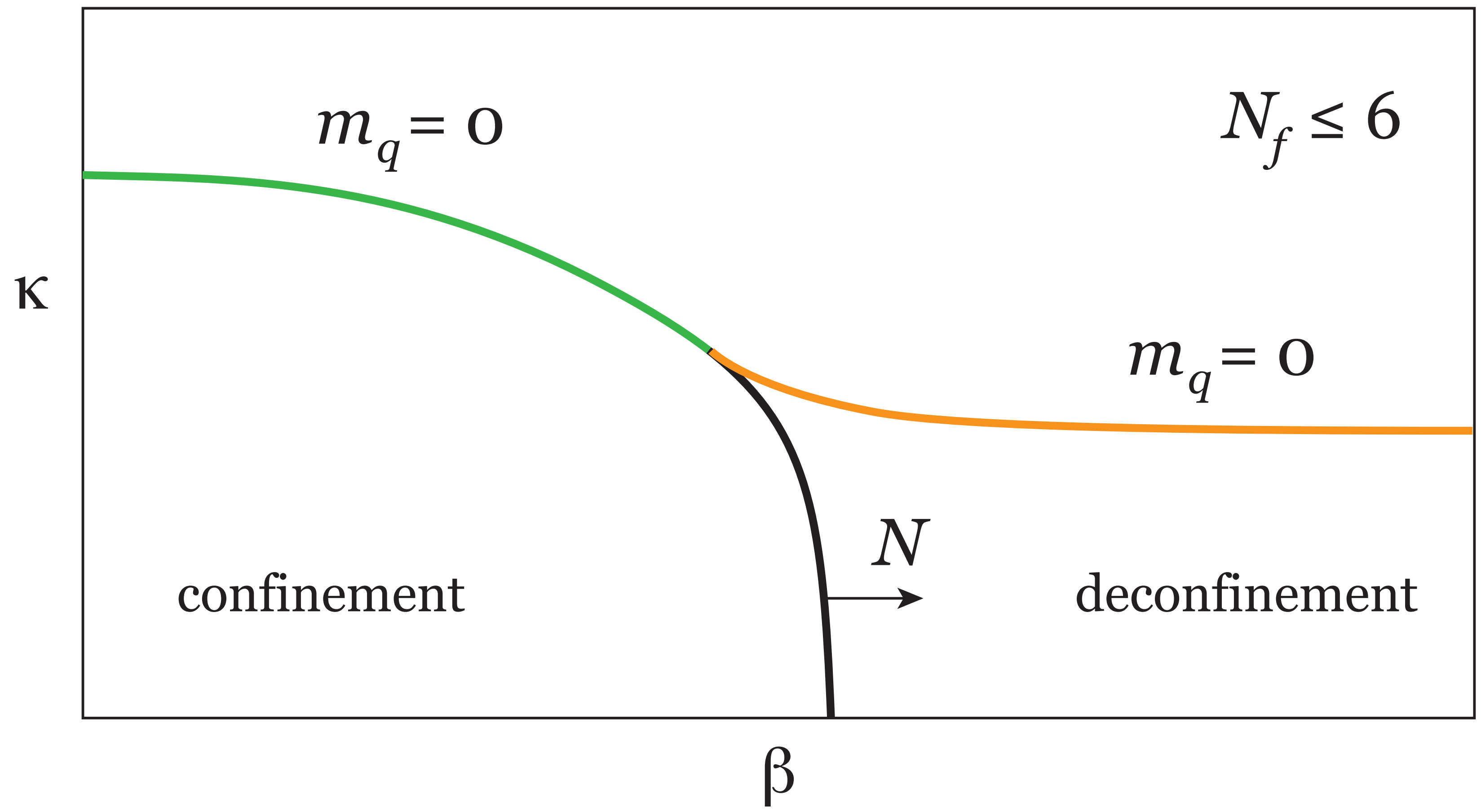
Strategy for Part 1

- Investigate the phase structure in terms of g_0, m_{q0} for $N_f \leq 16$
- UV fixed point at $g_0 = 0, m_{q0} = 0$
- Find critical Nf for the Banks-Zaks IRFP on the massless line starting from UVFP
- Construct the field theory toward UVFP, taking the limit $a \to 0$ and $N \to \infty$ with $L = a\,N$ constant

Phase Diagram: $N_f \leq 6$

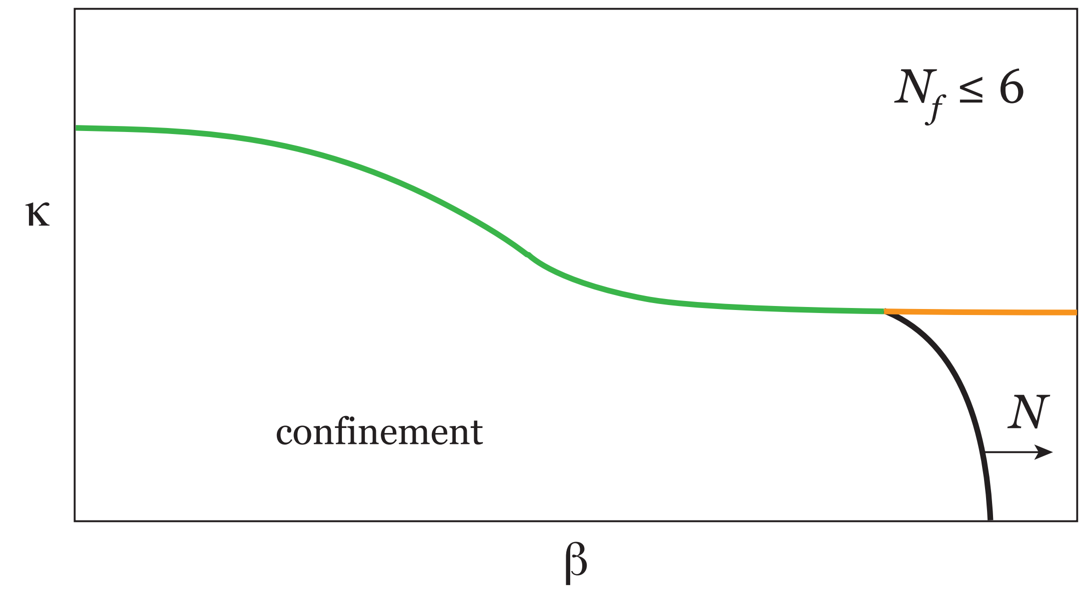
Chiral transition on the massless line starting from the UVFP

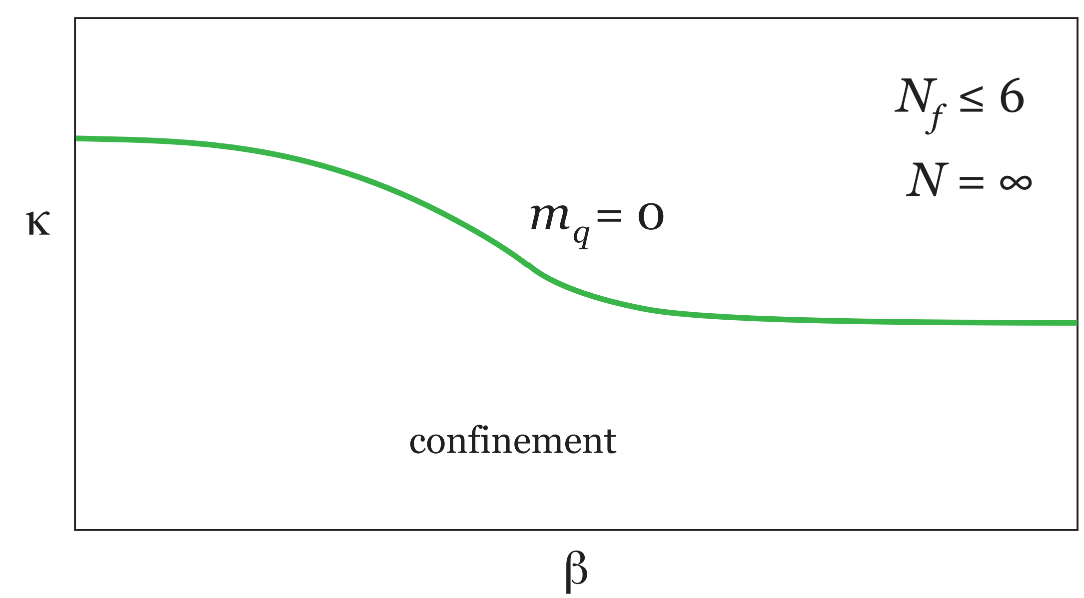
The finite temperature phase transition in the quenched QCD transition and the chiral transition move toward larger beta, as N increases.



Phase Diagram: $N_f \leq 6$; N larger

As N increases, the green line
becomes longer and
in the limit N => infinity
only the green part survives.



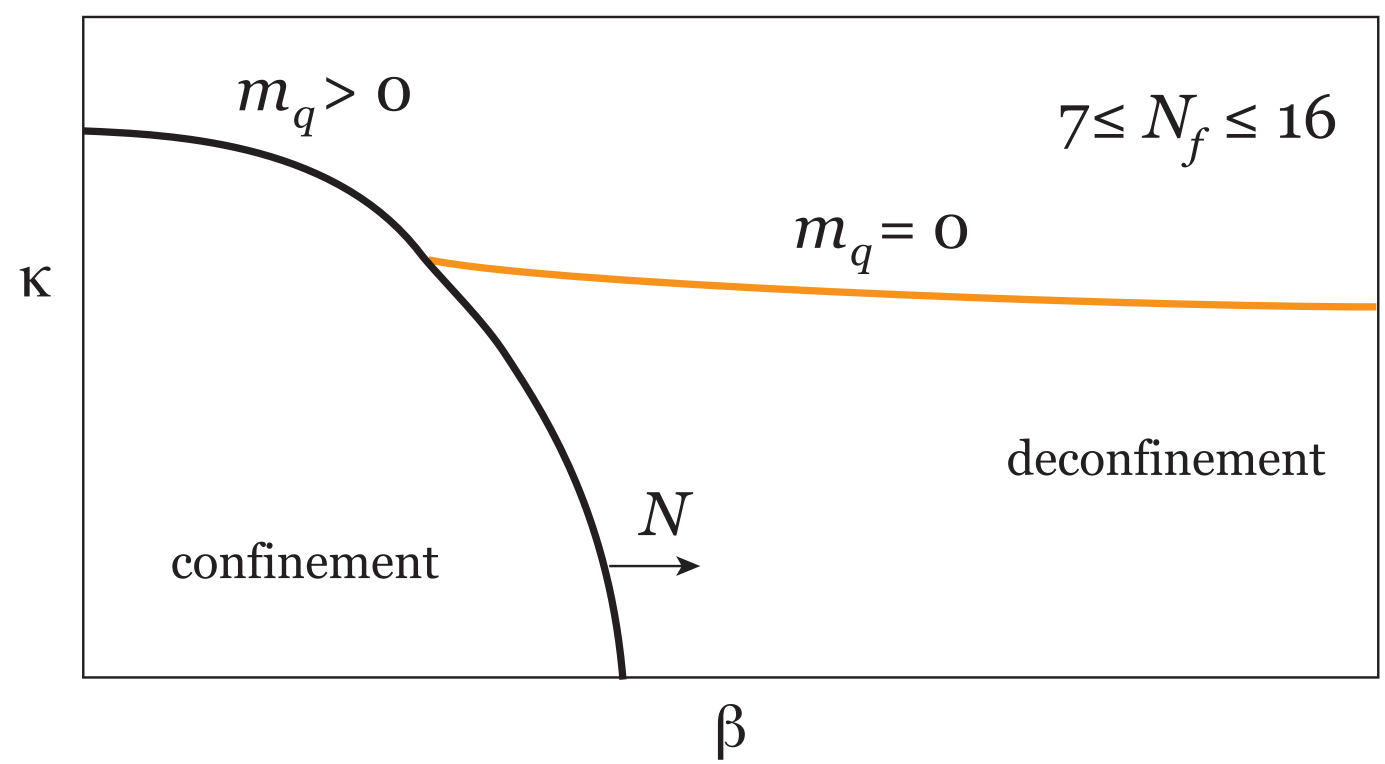


Phase Diagram: $7 \leq N_f \leq 16$

Complicated due to lack of chiral symmetry

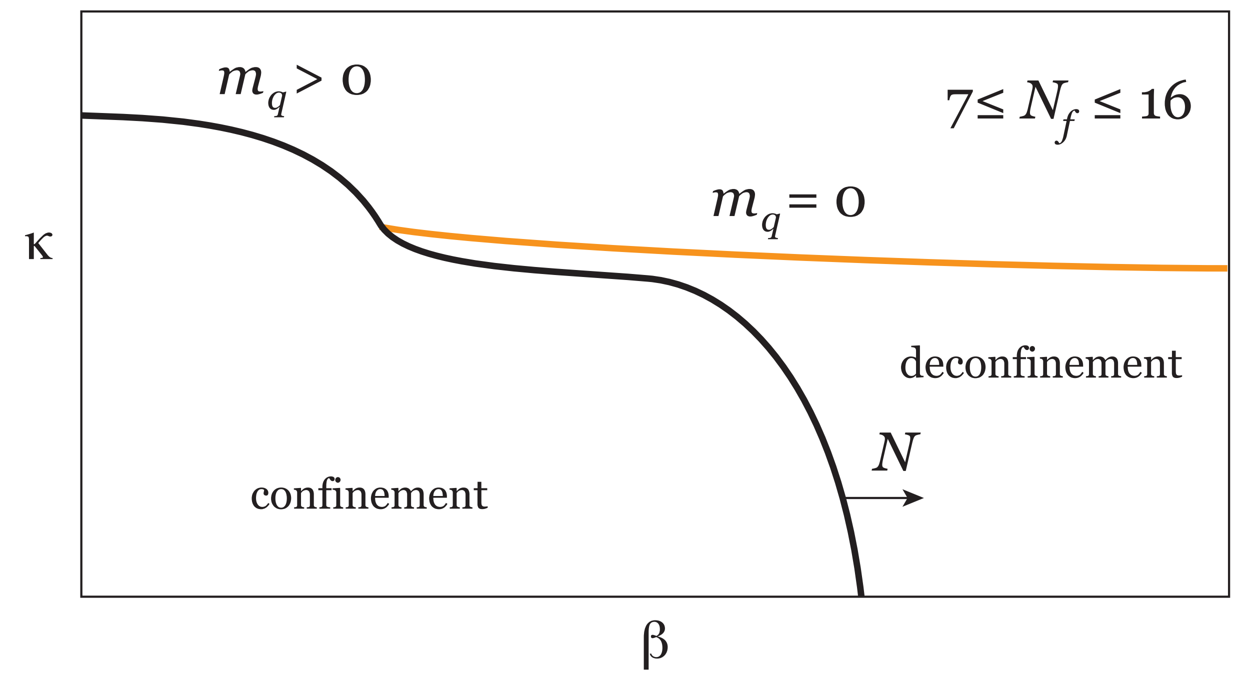
1. the massless line from the UVFP hits the bulk transition
2. no massless line in the confining phase at strong coupling region

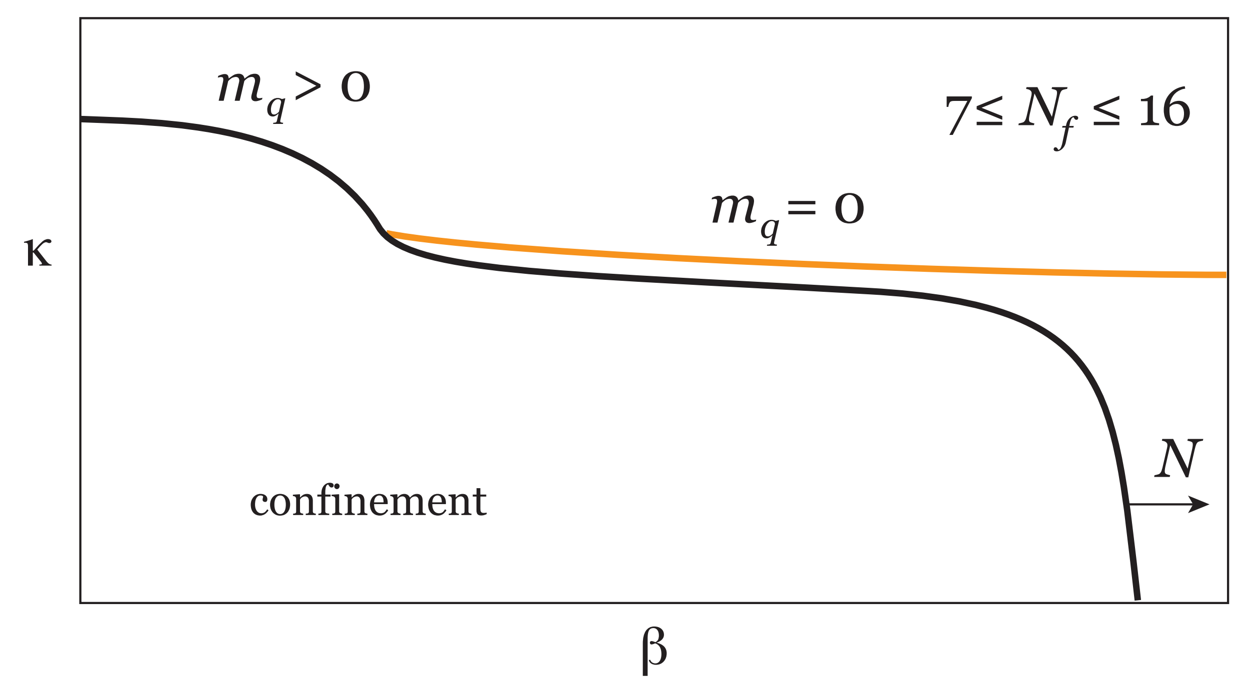
massless quark line only in the deconfining phase



Phase Diagram: $7 \leq N_f \leq 16$

As N increases,
massless quark line is, still,
only in the deconfining phase





What found

- There are no green lines
  (massless line in the confining phase)
  for $7 \leq N_f \leq 16$
- Conformal window is $7 \leq N_f \leq 16$
- Indirect way to conclude this

More direct way

- Identify the IR fixed point

  For small Nf, g* is in strong coupling region

  Only upper limit for g* ?

- Find out characteristics of Conformal theories <= this work

(End of part 1)

Strategy for part 2

- Define a continuous theory by continuum limit of lattice theory, keeping L =finite or infinity
- Introduce the concept “Conformal theories with IR cutoff” in continuous theories
- Then propose Conformal theories on a lattice

Continuum limit

- $a \to 0$ and $N \to \infty$, keeping $L = N a$ constant
- case 1: L=infinity IR cut-off $\Lambda_{IR} = 0$; R^4
- case 2: L=finite IR cut-off $\Lambda_{IR} = 1/L$; T^4
- A huge difference between case1 and case2

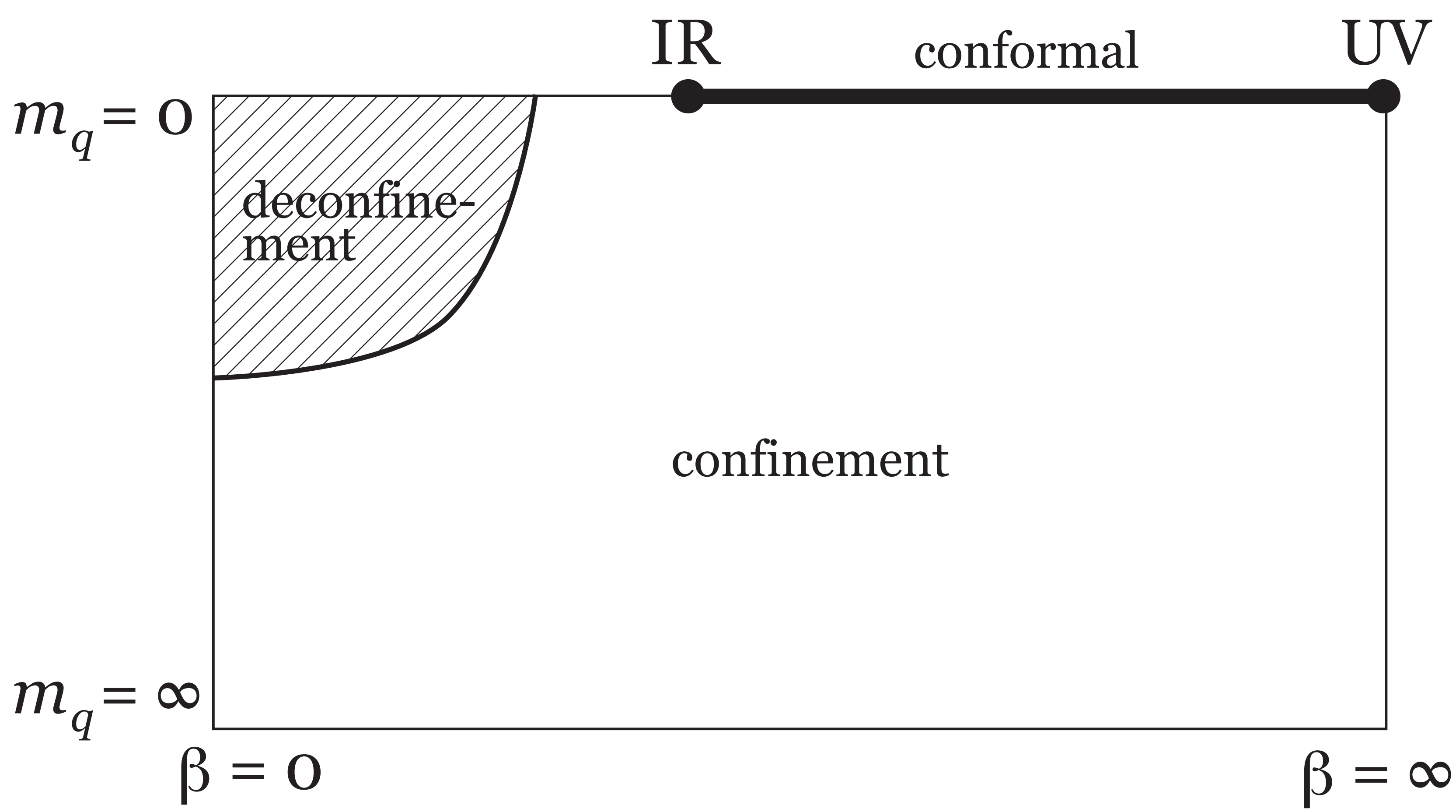
Case 1: $\Lambda_{IR} = 0$

No physical quantities with physical dimensions

conformal region exists only on the massless line

massive region is confining phase

deconfining phase in strong coupling region is conjectured based on numerical simulations



Case 1: Propagators of mesons

When $g(\mu) = g^*$

$G(t) = c\frac{1}{t^\alpha}$ $\alpha = 3 - 2\gamma_H^*$ scale invariant

When $0 < g(\mu) < g^*$

$$G(t) = c\,\frac{1}{t^{\alpha(t)}}$$

$\alpha(t) = 3$ $t \ll \Lambda_{CFT}$ UV fixed point

$\alpha(t) = 3 - 2\gamma_H^*$ $t \gg \Lambda_{CFT}$ IR fixed point

Λ_{CFT} is a scale parameter for the transition region from UV to IR

Case 2: Conformal theories with IR cutoff

Physical quantities: Λ_{CFT} Λ_{IR} m_H

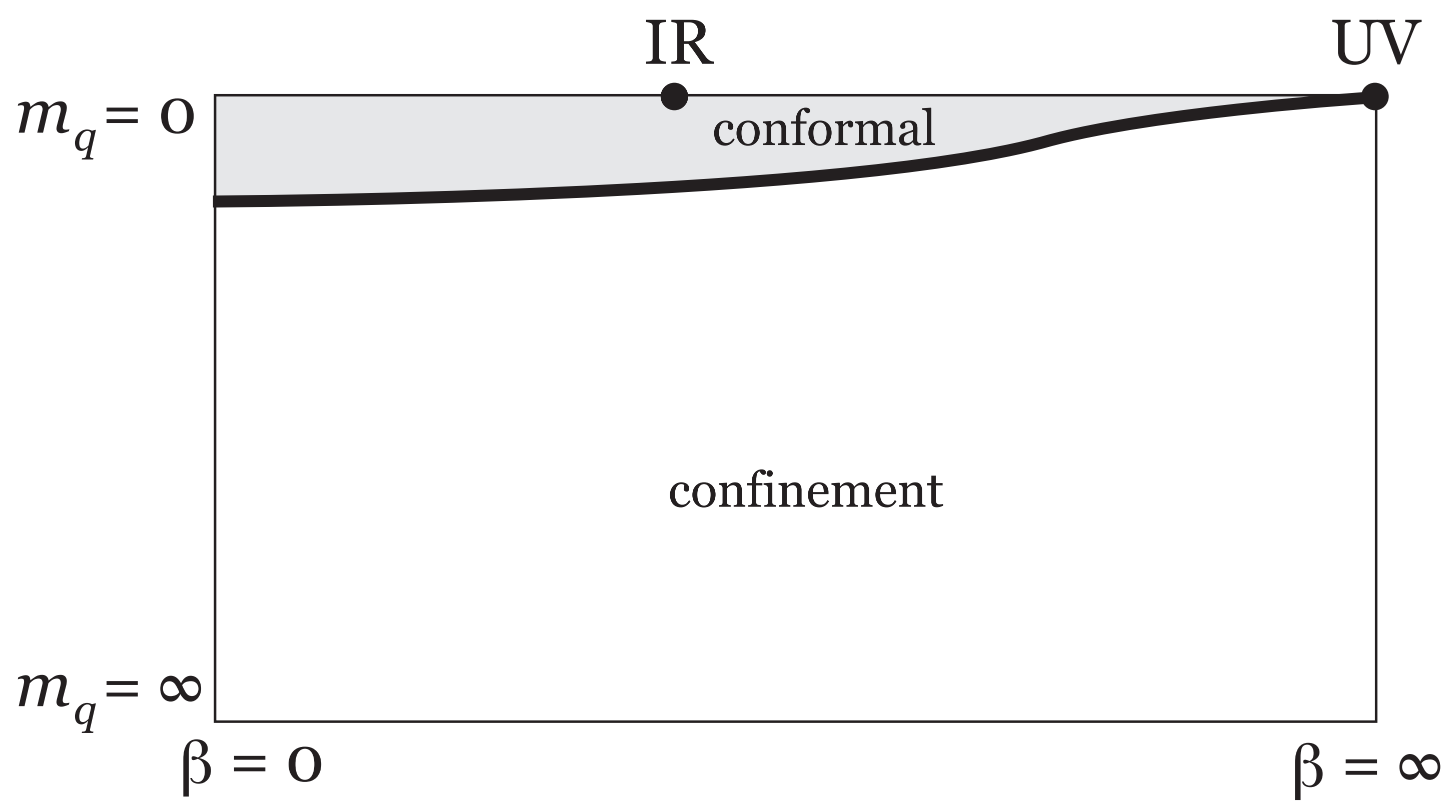
"Conformal theories with IR cutoff" region: see figure

Boundary of the "conformal" region is given by

$$m_H \leq c\Lambda_{IR}$$

Propagators :

$$G(t) = c\frac{\exp(-mt)}{t^{\alpha}}$$



Case 2: Conformal theories with IR cutoff (Cont.)

$$G(t) = c\,\frac{\exp(-mt)}{t^{\alpha}}$$

α and m are t-dependent: $\alpha(t)$ $m(t)$

evolve with RG transformation

$t \ll \Lambda_{CFT}$ $\alpha(t) = 3$ $m(t) = 2\,mq$

$t \gg \Lambda_{CFT}$ $\alpha(t) = 3 - 2\gamma_H^*$ $m(t) = m_H$

Conformal theories on the Lattice

- Note: IR cutoff is inherent in numerical simulations on a lattice: $\Lambda_{IR} = 1/(a\,N)$
- Primary target of part 2 is to verify the transition of meson propagators from an exponential damping form to a modified Yukawa-type, that is, an exponential form with power correction

Size Dependence of Critical mass

When the lattice size is increased

$$N_2 = s\, N_1$$

The critical mass is decreased

$$m_q^{critical}(\beta, N_2) = 1/s\, m_q^{critical}(\beta, N_1)$$

If we keep the quark mass in the region

$$m_q^{critical}(\beta, N_2) \leq m_q \leq m_q^{critical}(\beta, N_1)$$

Yukawa-type disappears

Have to carefully choose the parameters to find the “Conformal region”

Numerical Simulations

- Algorithm: Blocked HMC for 2N and RHMC for 1 : Nf=2N + 1
- Computers:
  U. Tsukuba: CCS HAPACS;
  KEK: HITAC 16000
- Nf=7, 16 and (Nf=2, 6)
- Lattice size: $8^3 \times 32, 16^3 \times 64, 24^3 \times 96$
- Statistics: 1,000 +1000 trajectories

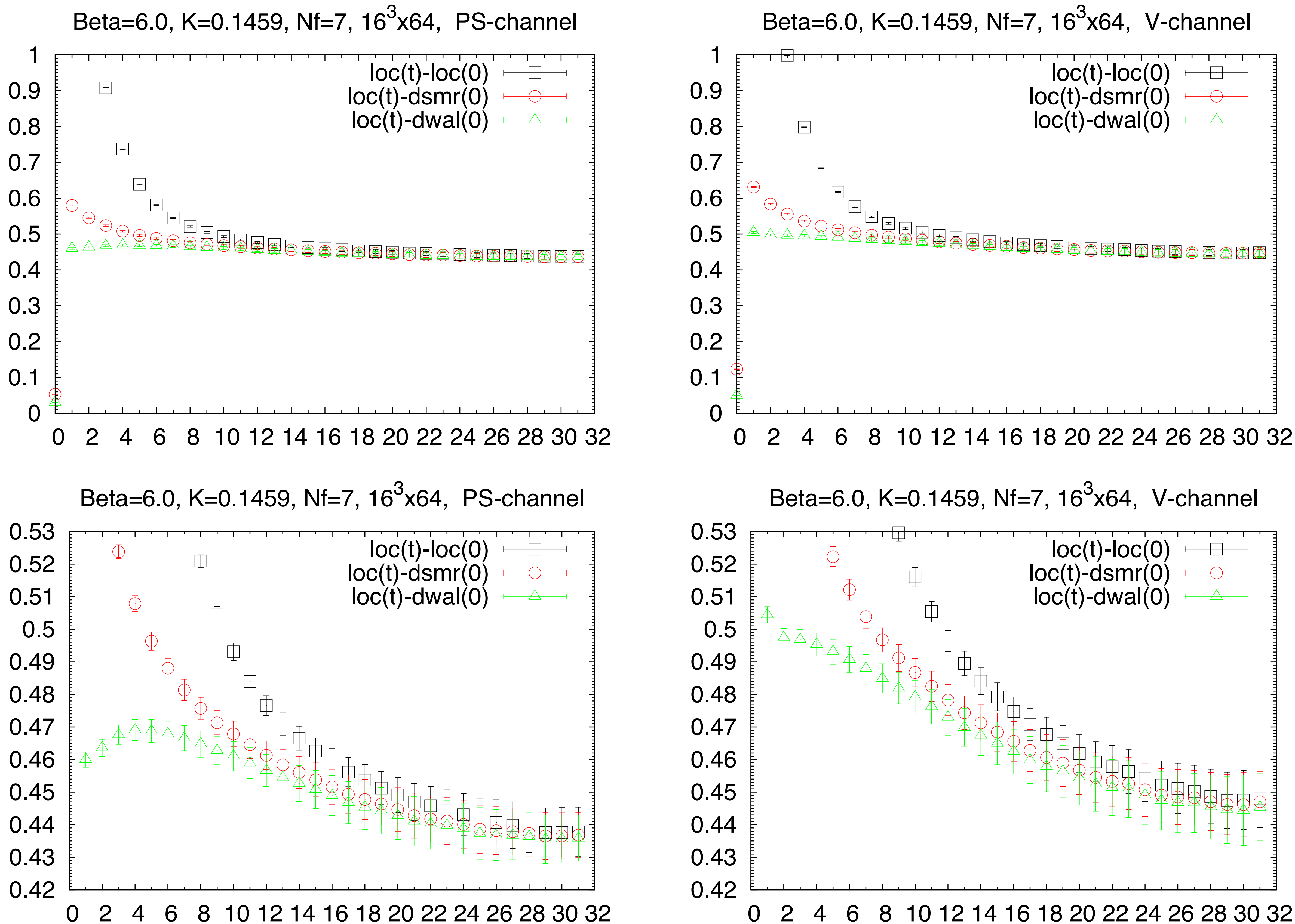
Parameters of Simulations

Masses are preliminary !

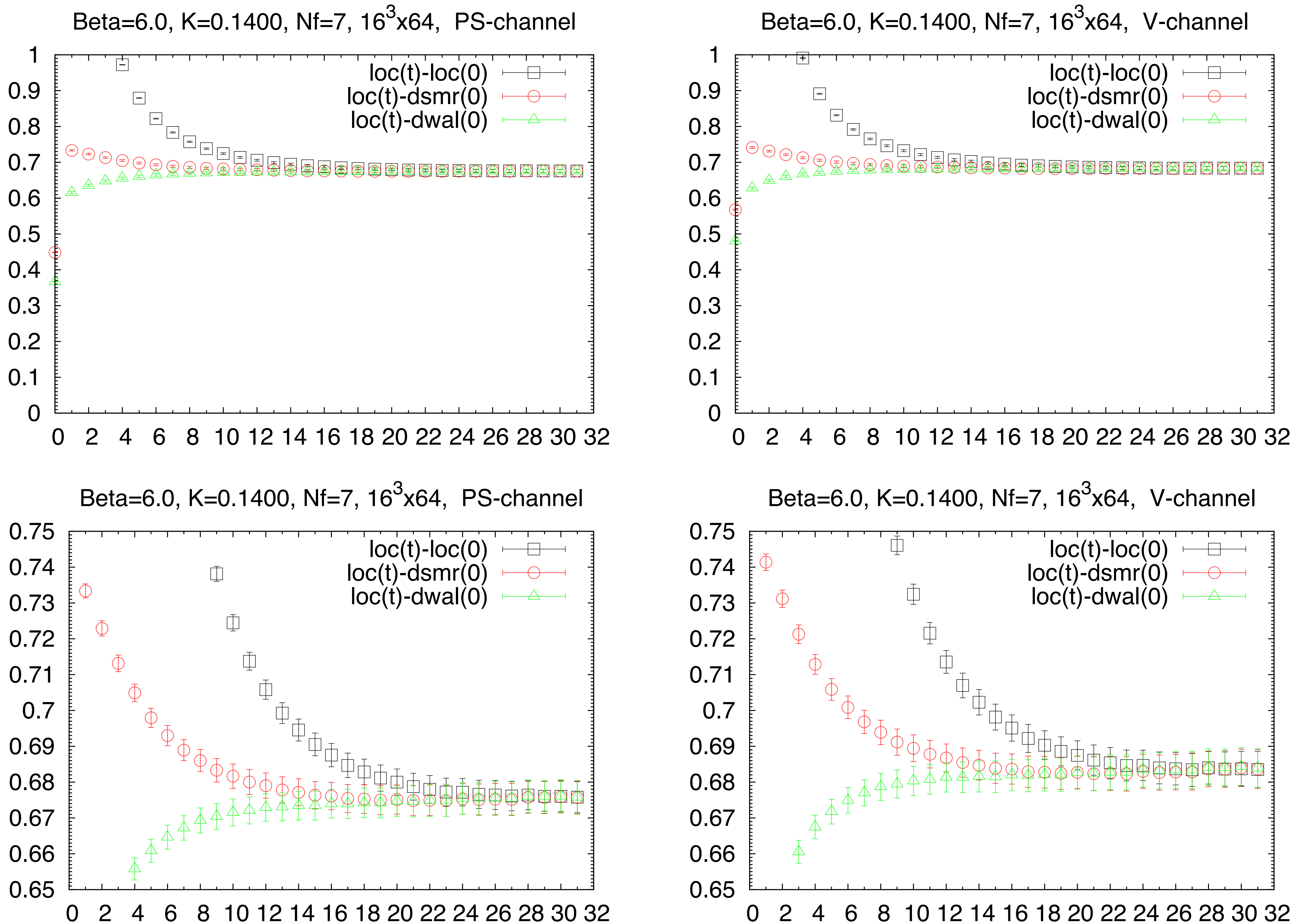
Nf7						
K	0.1400	0.1446	0.1452	0.1459	0.1472	
mq	0.22	0.084	0.062	0.045	0.006	
mH(96)	0.66	0.33	0.33	0.20		
mH(64)	0.68	0.46	0.42	0.41	0.41	
mH(32)	0.74		0.74	0.74		
Nf16						
K	0.125	0.126	0.127	0.13	0.1315	0.13322
mq	0.25	0.22	0.19	0.1	0.055	0.003
mH(96)				0.30	0.27	0.32
mH(64)	0.54	0.54	0.49	0.43	0.38	0.38
mH(32)						

From now on, let me show you
examples of Yukawa-type propagators
for Nf=7 at beta=6.0 and Nf=16 at beta=11.5
detailed analyses will come later

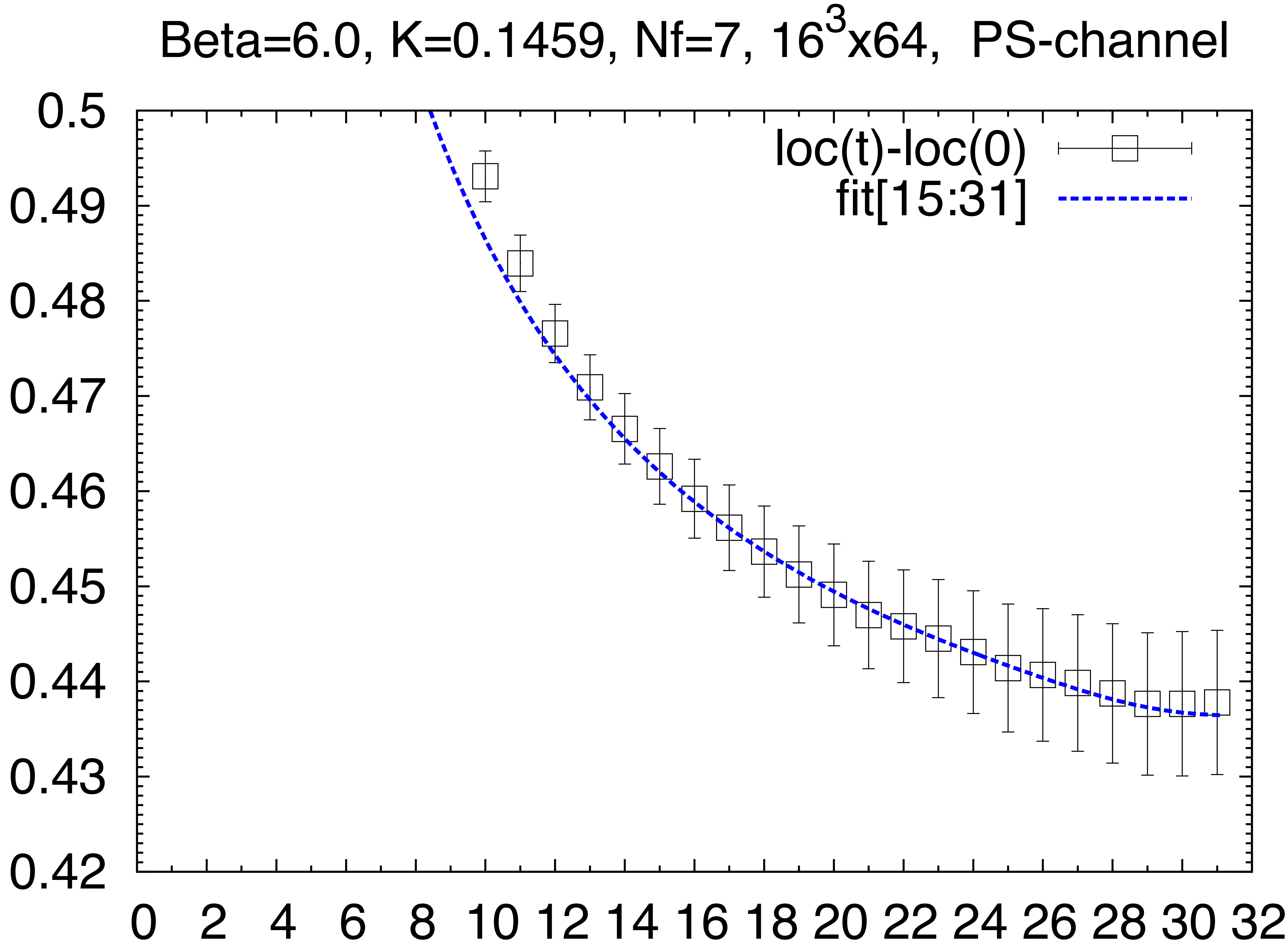
Nf7: mq=0.045: example of Yukawa-type



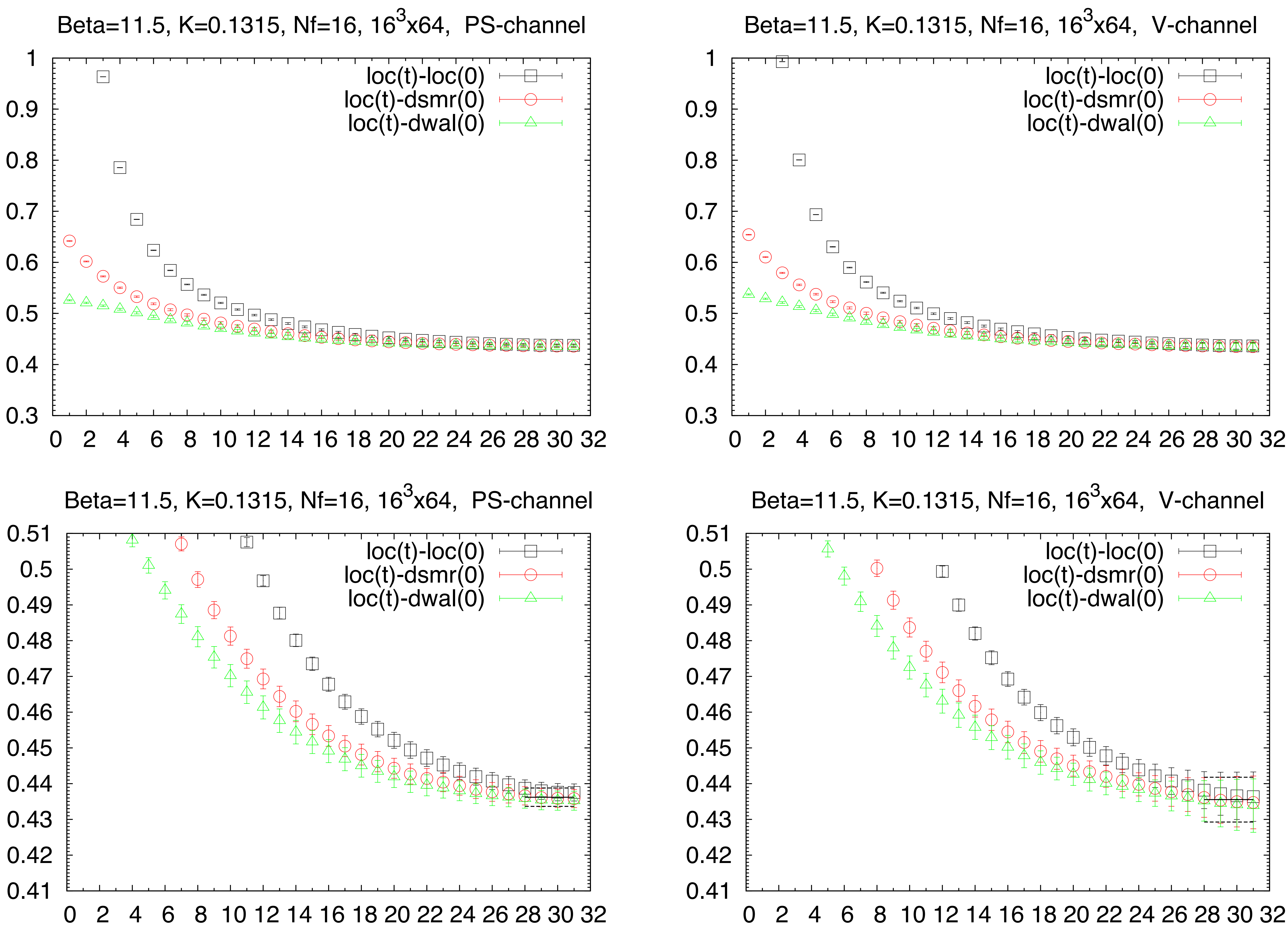
Nf=7: mq=0.22; example of exp!!-damp



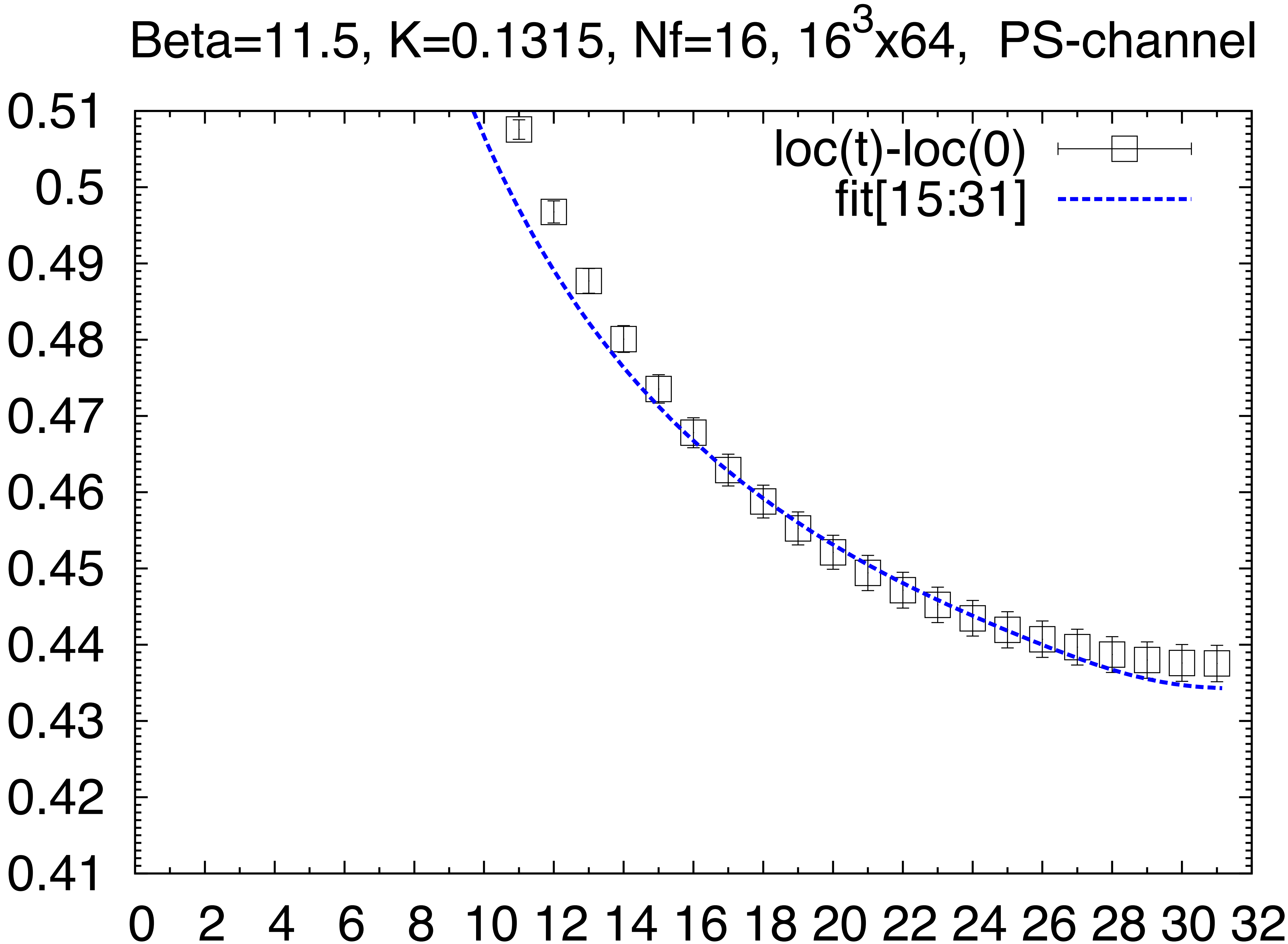
Nf=7: mq=0.045 Yukawa-type fit[15:31]



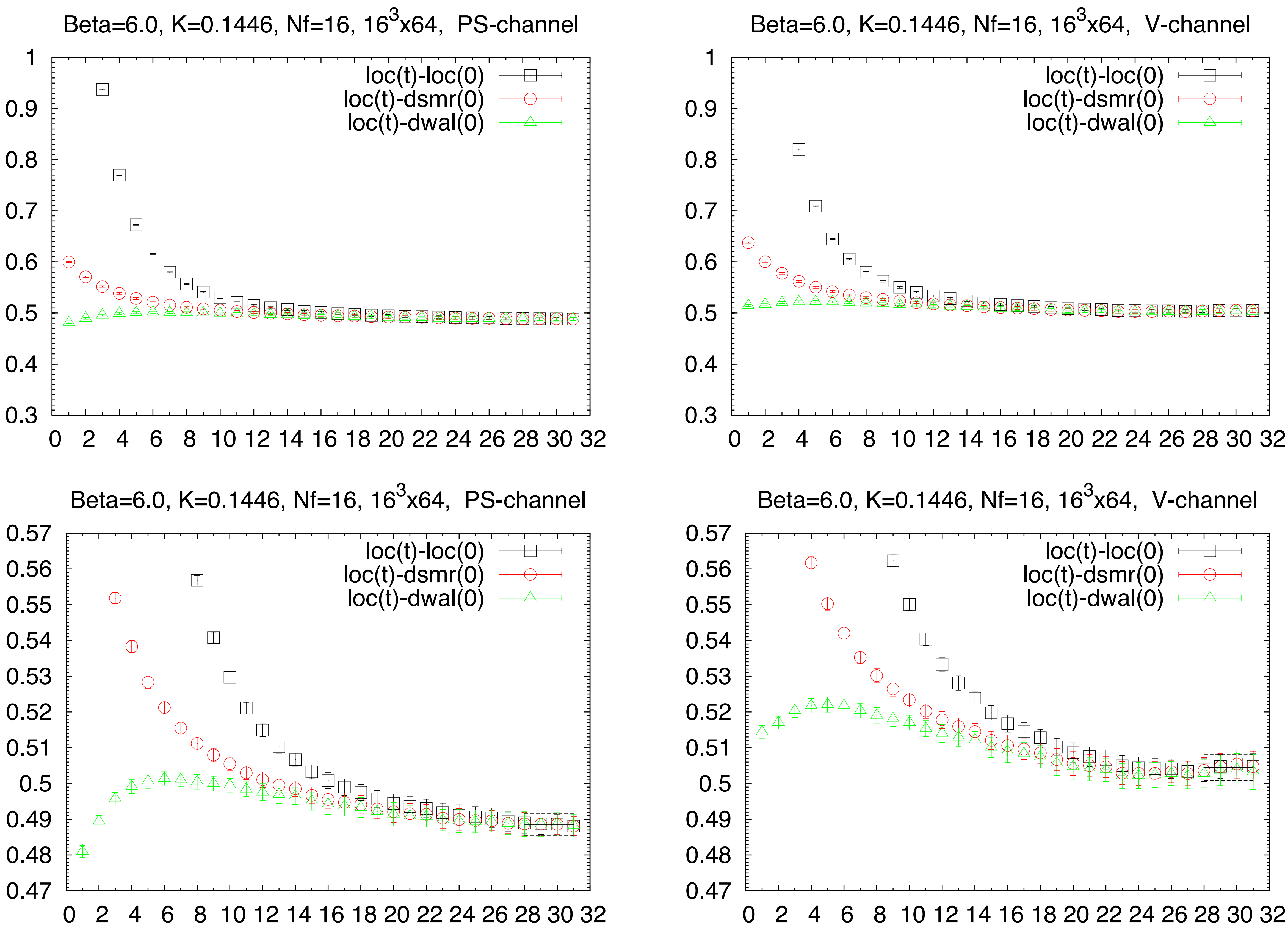
Nf16: mq=0.055; example of Yukawa-type



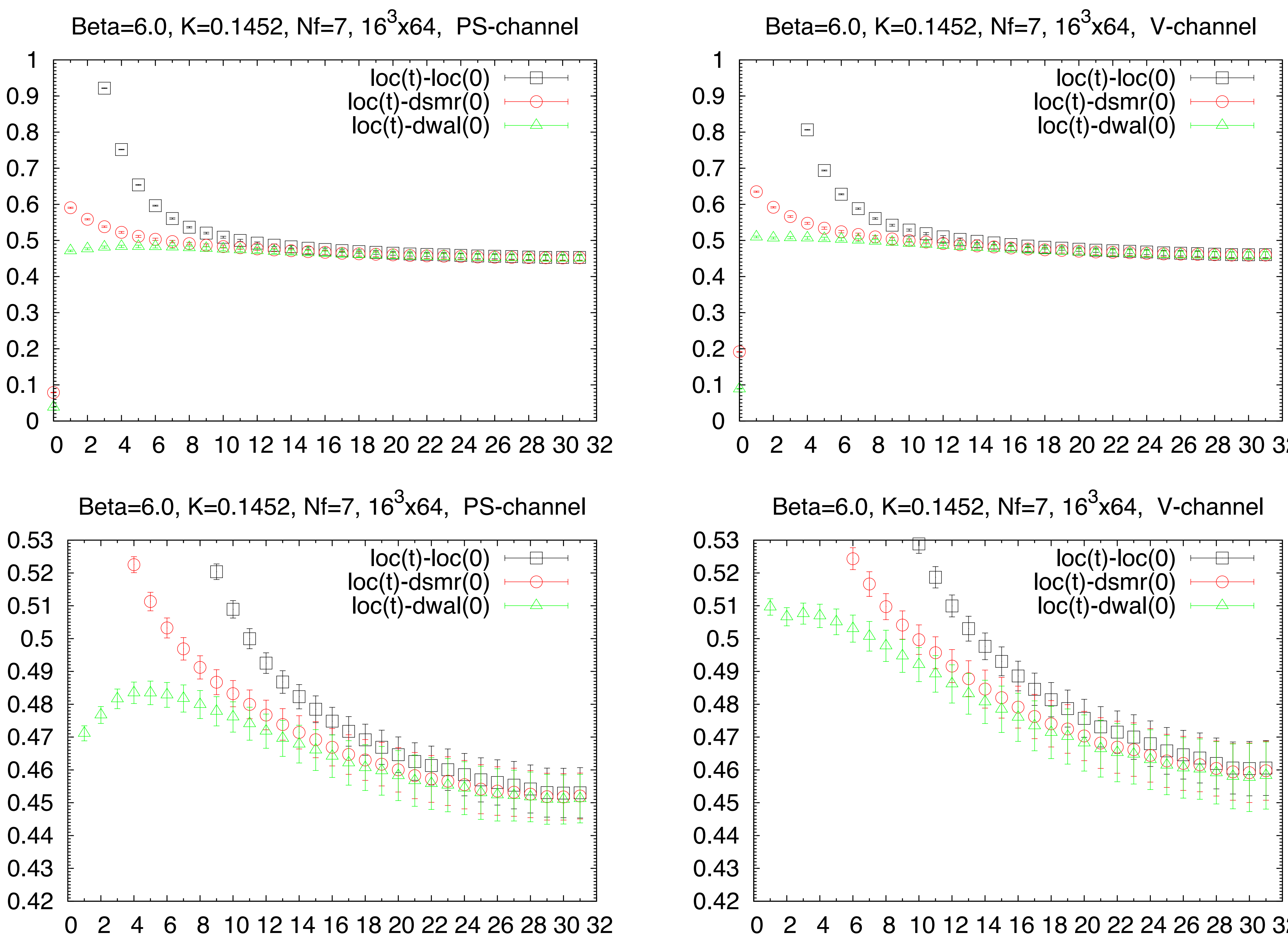
Nf16: mq=0.055:   Yukawa-type fit[15:31]



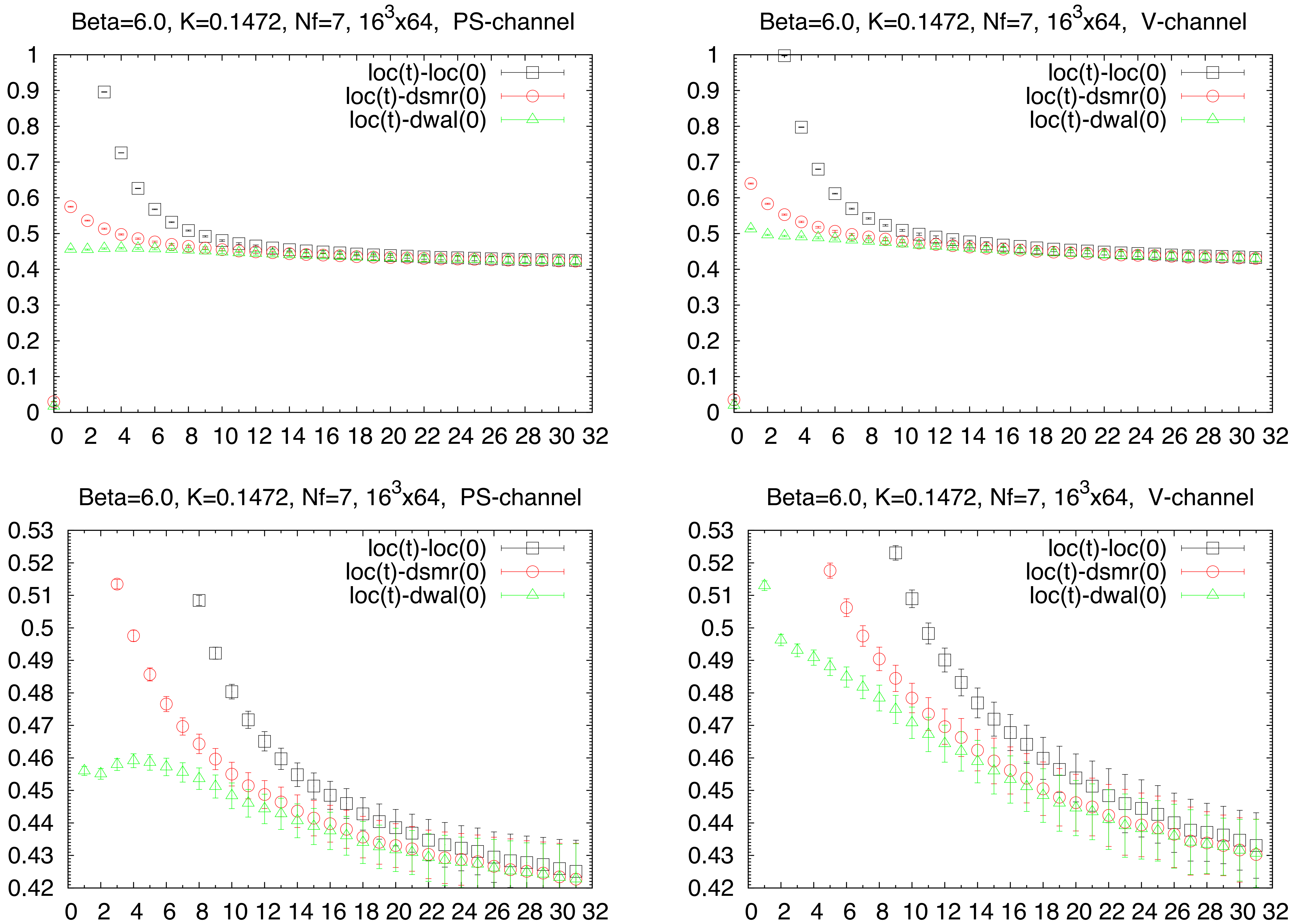
More example :Nf=7; mq=0.084



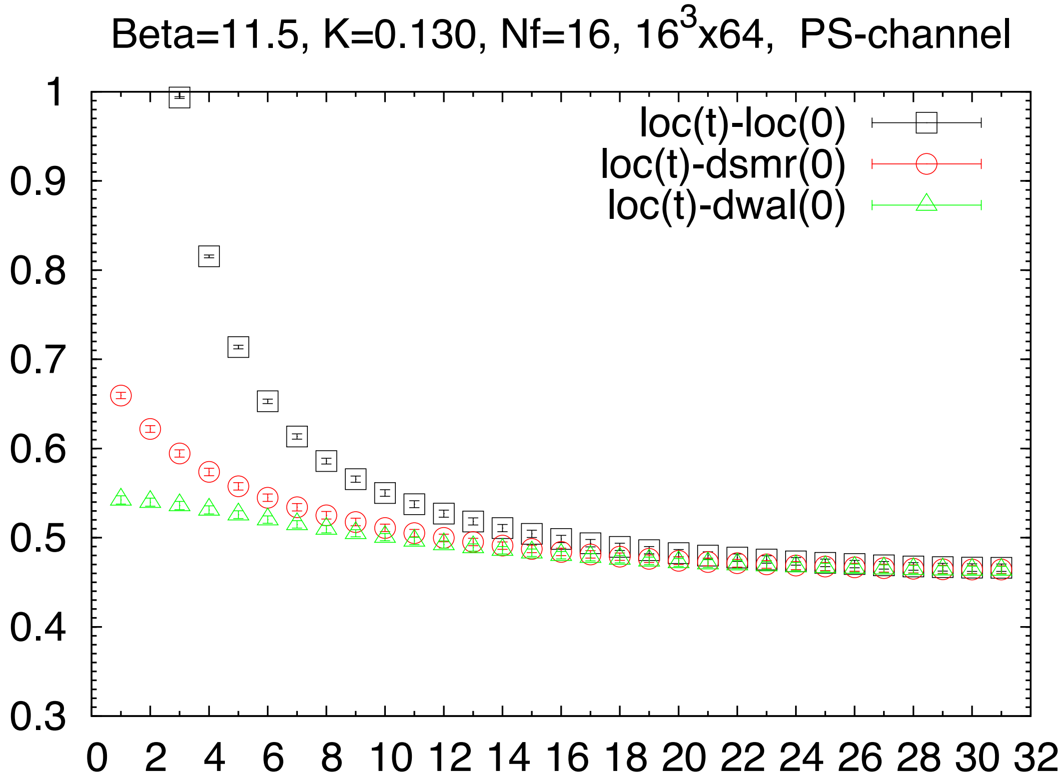
More Nf=7; mq=0.062

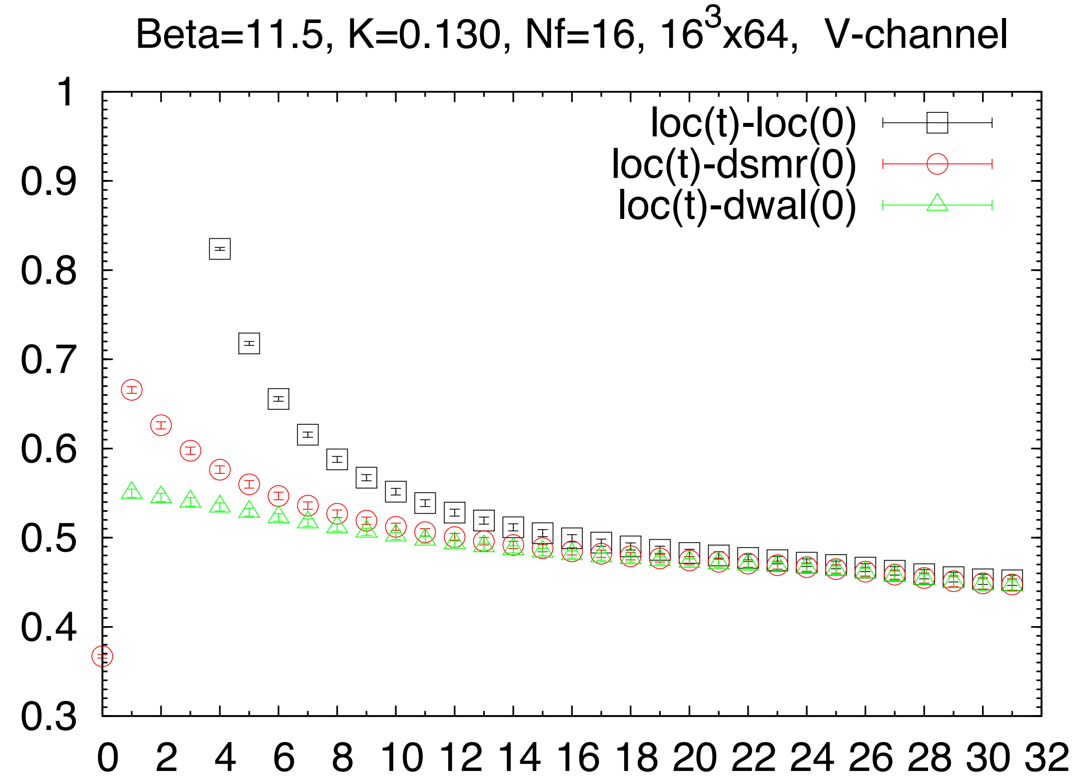


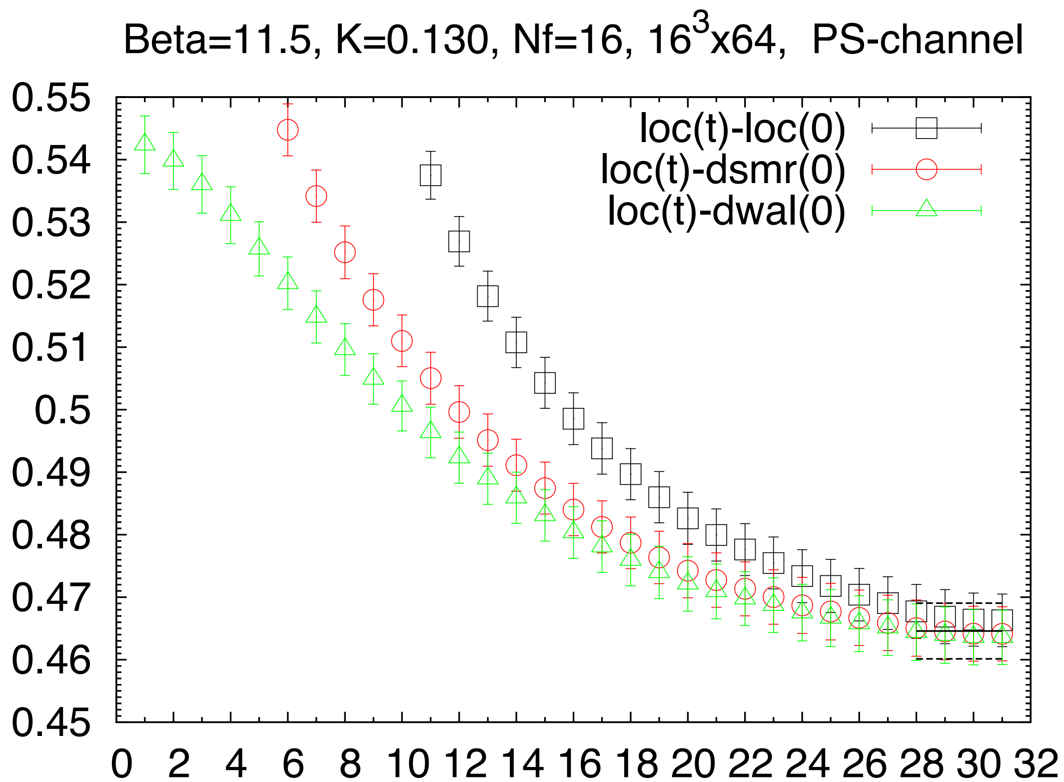
More Nf=7; mq=0.0006

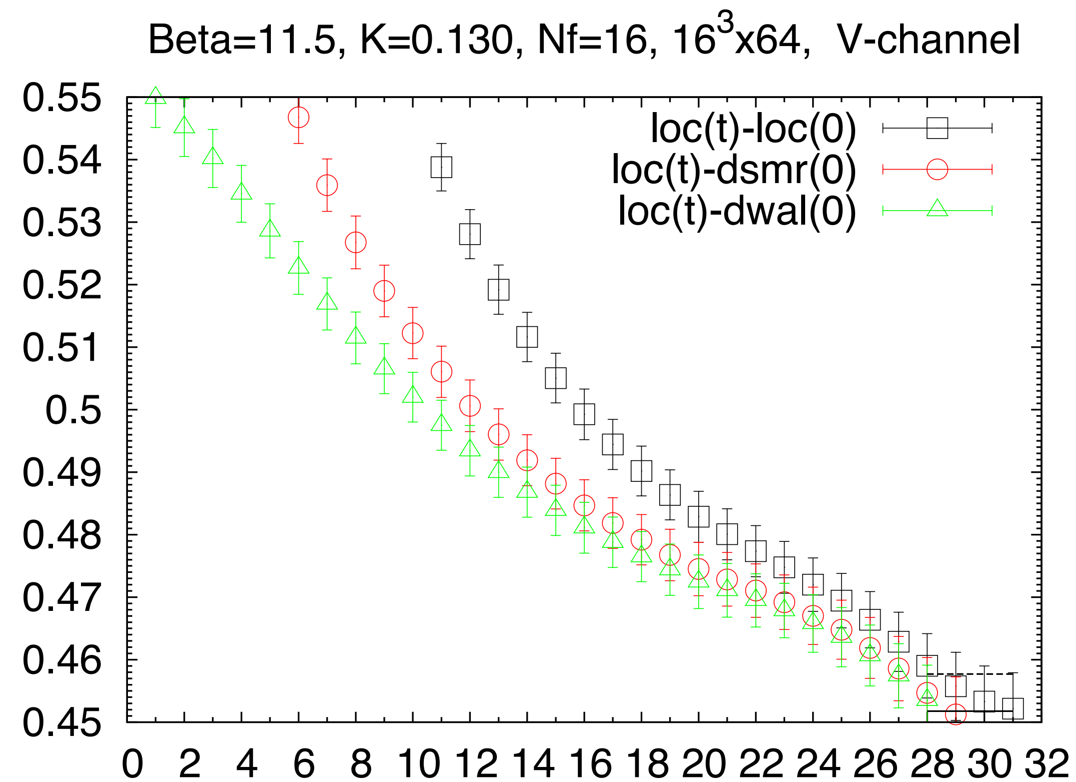


Now Nf=16; mq=0.1

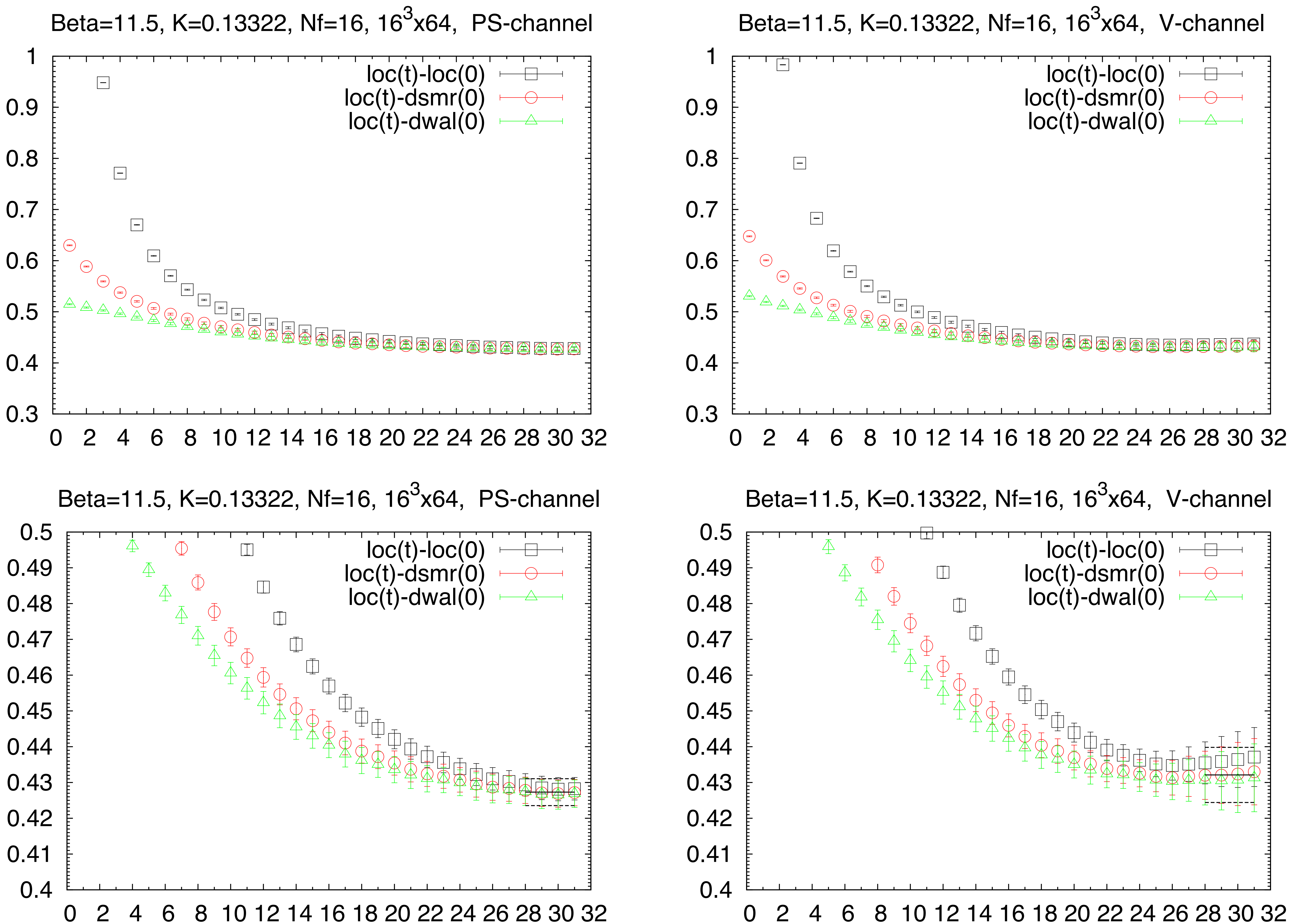








More NF=16; mq=0.003



Verified the existence of
"Conformal theories with IR cutoff"
for Nf=7 and 16

$$m_H \leq c\,\Lambda_{IR} \qquad \Lambda_{IR} = 1/(N^3 \times N_t)^{1/4}$$

$c \sim 11.0$ Nf=7 for all lattice sizes at beta=6.0

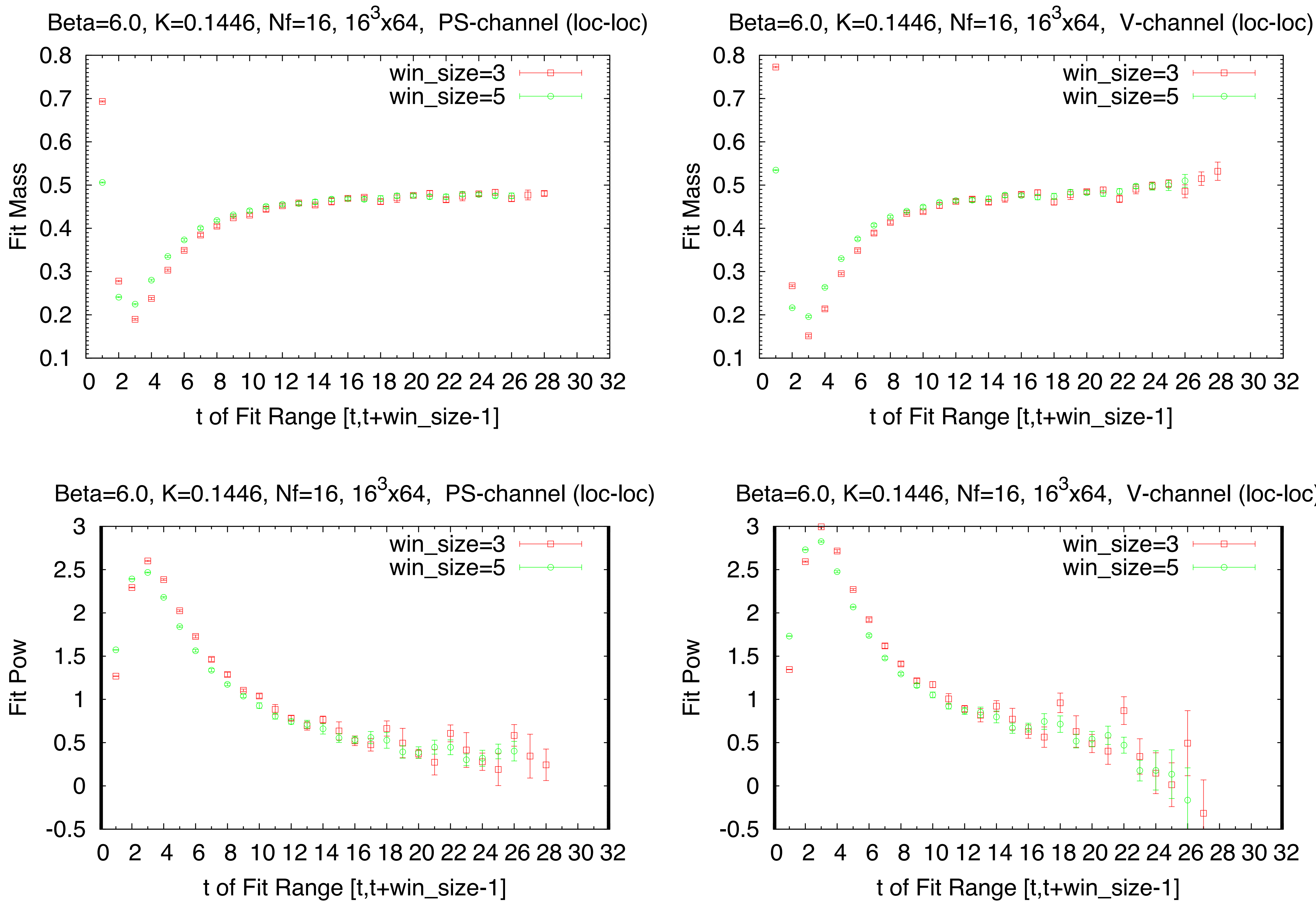
$c \sim 12.4$ Nf=16 for all lattice sizes at beta=11.5

Achieved the first target in part 2

Second primary target in part 2

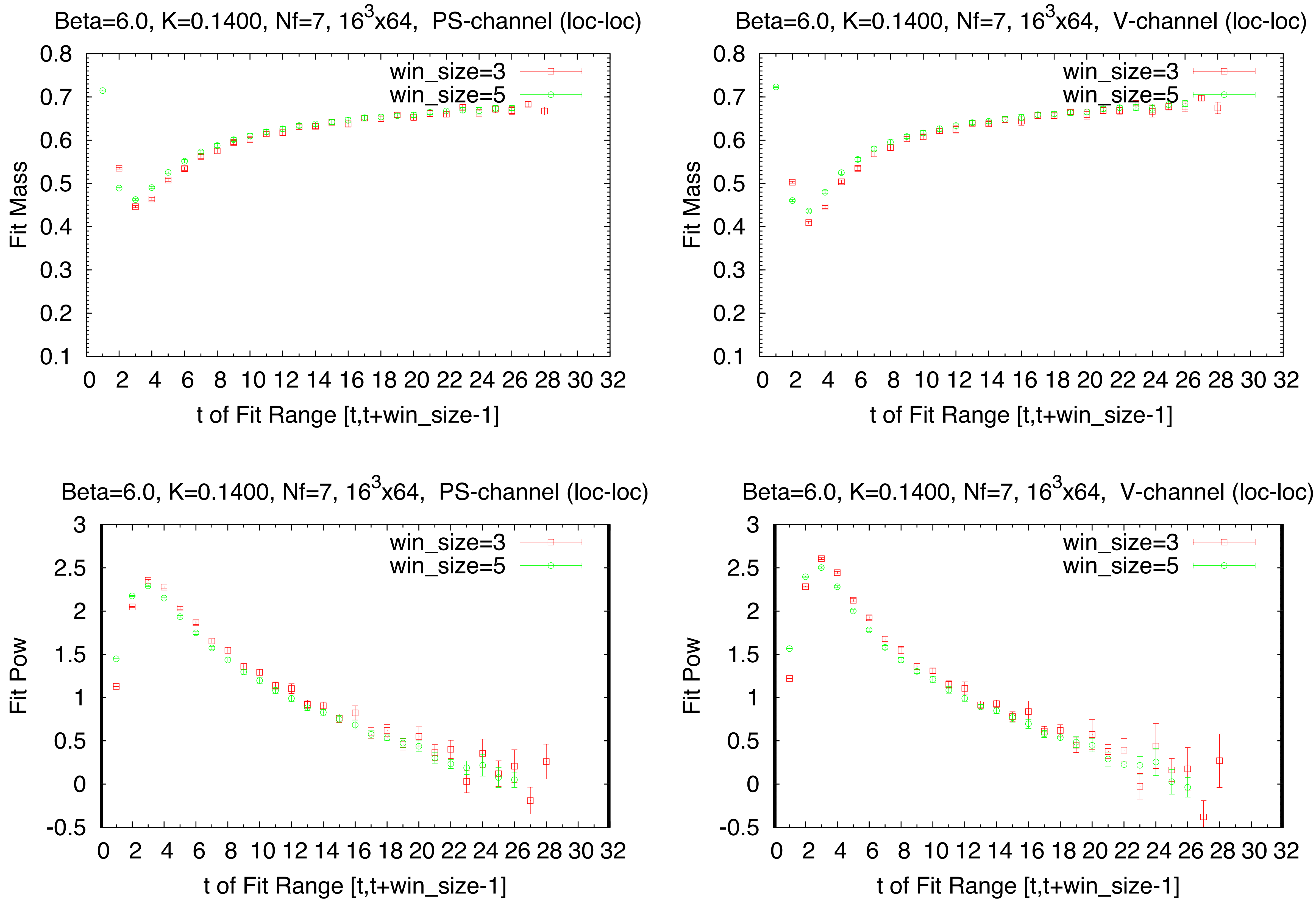
- What kind of theory is defined ?
- $\alpha(t)$ and $m(t)$ reflect the dynamics
- Investigate t-dependence of $\alpha(t)$ and $m(t)$

$m(t)$, $\alpha(t)$: Nf=7; mq=0.084

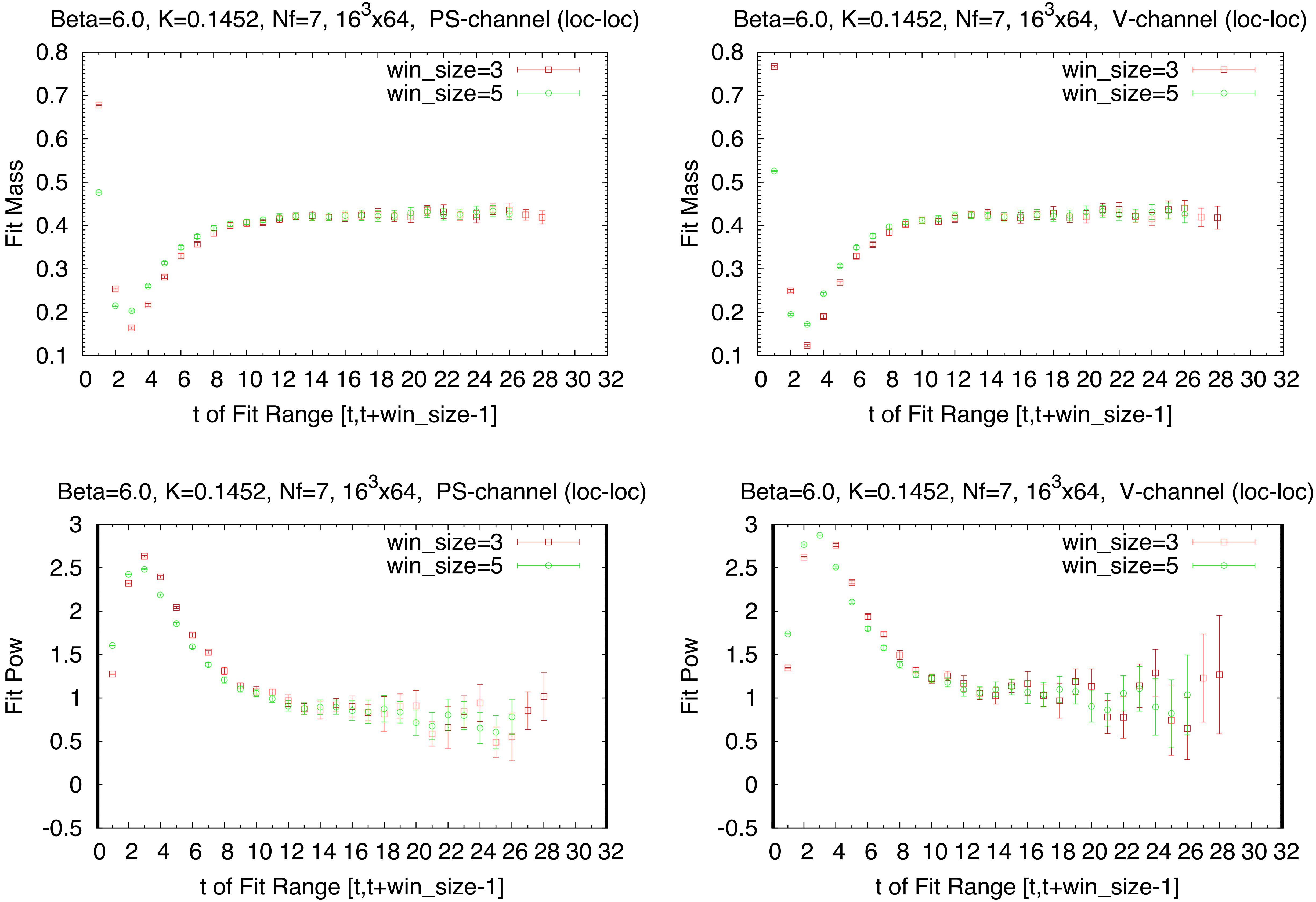


Compare with exp. damping

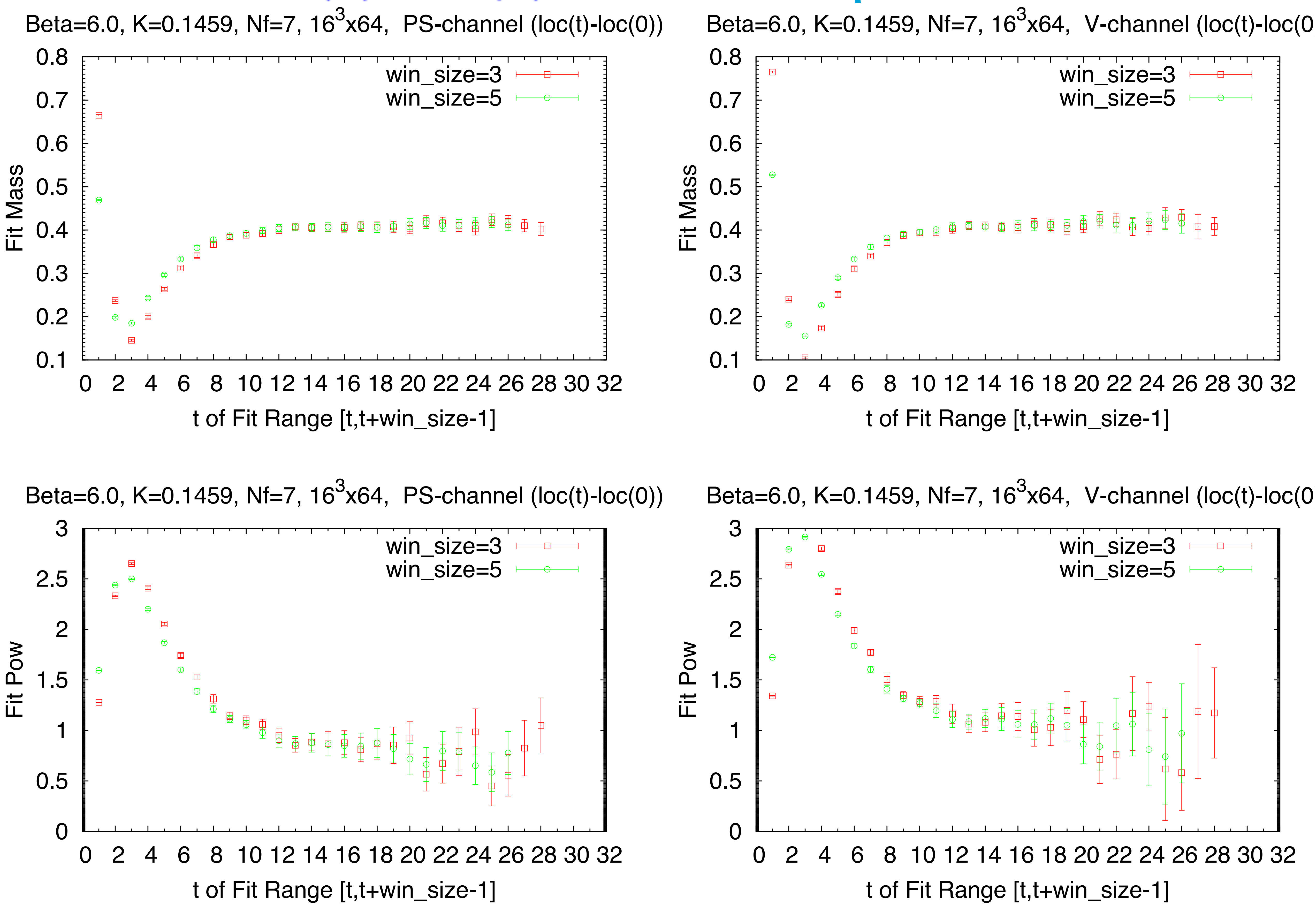
Nf=7; K=0.1400(mq=0.22)



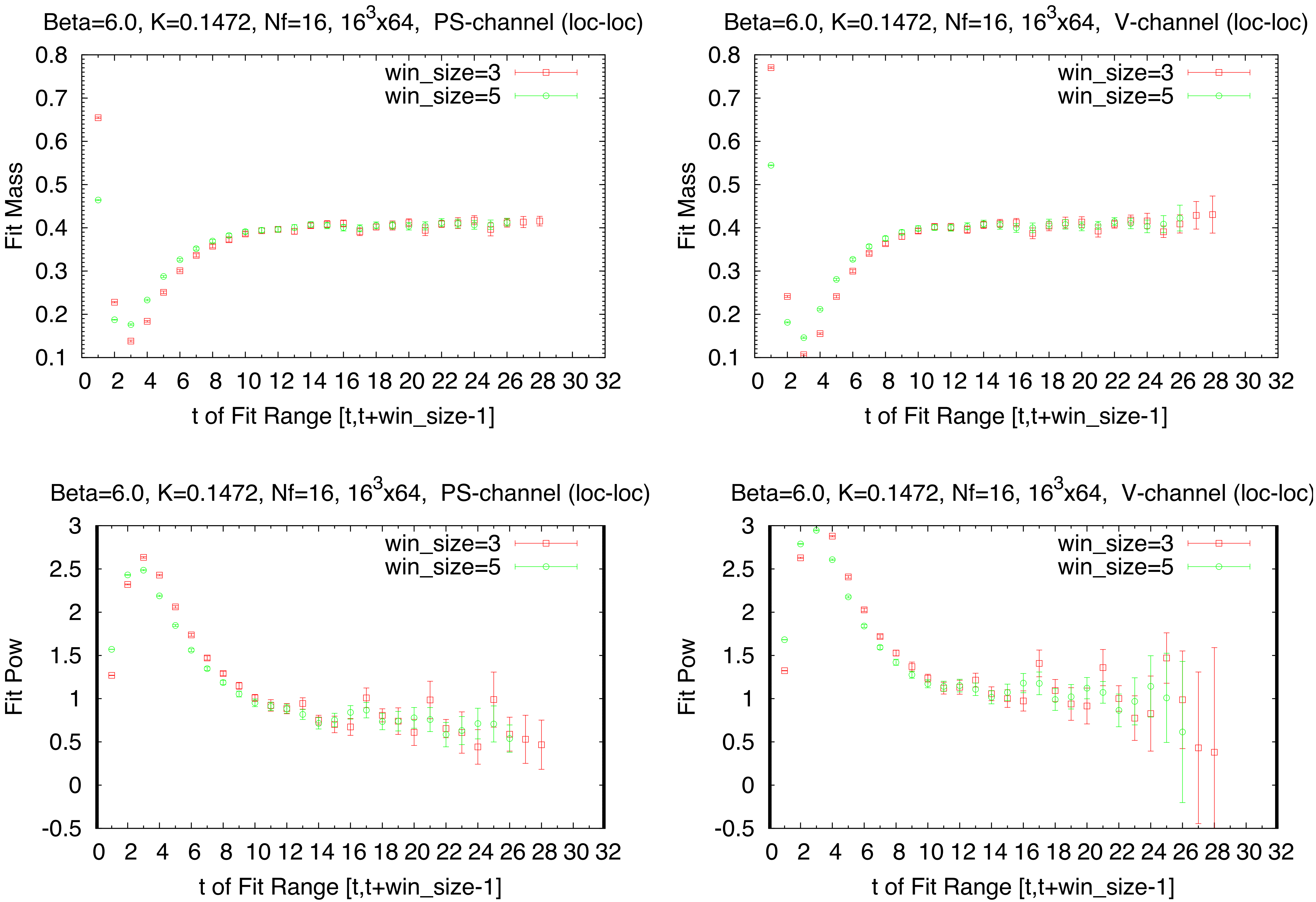
$m(t)$, $\alpha(t)$: Nf7; mq=0.062



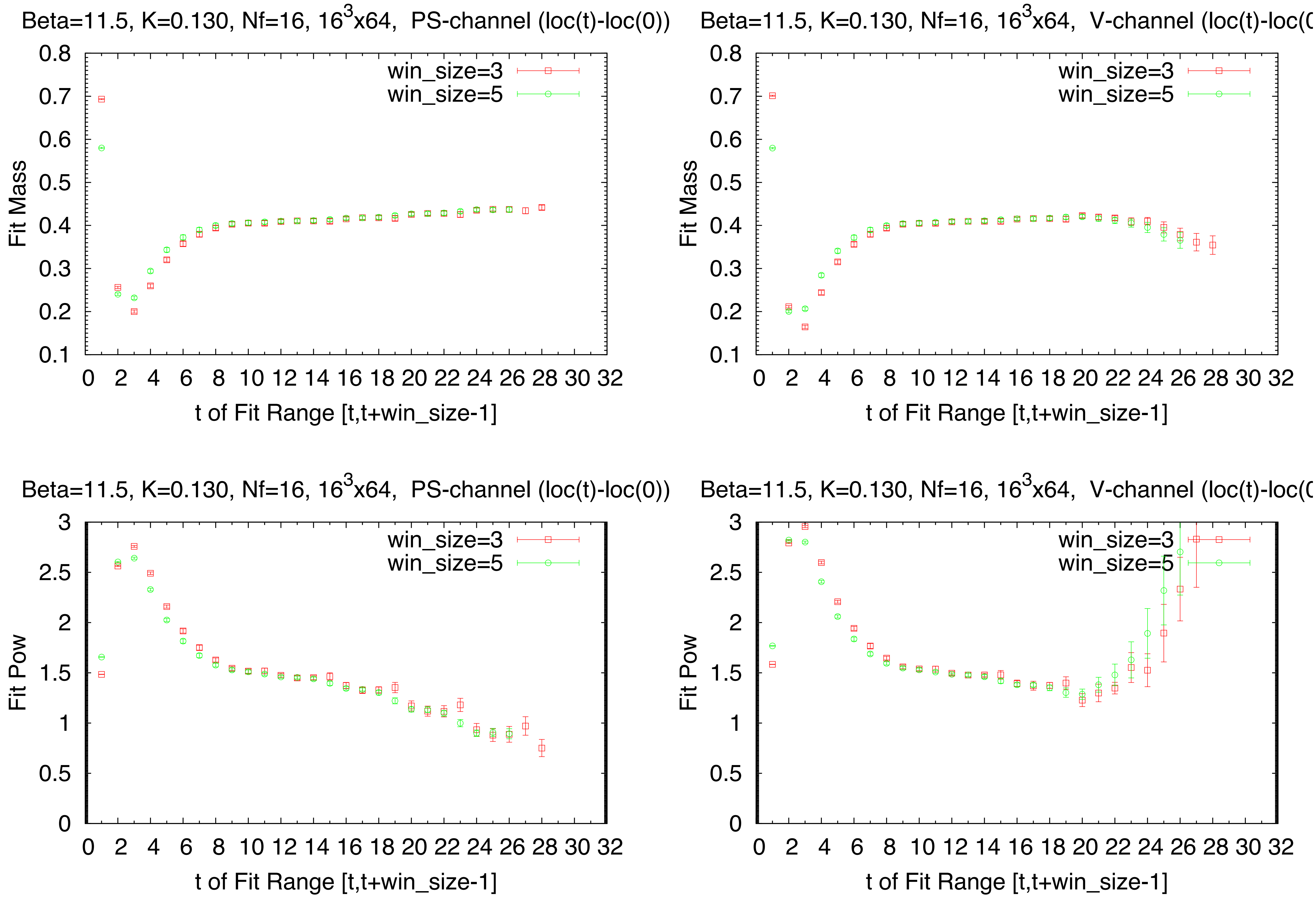
$m(t)$, $\alpha(t)$: Nf7; mq=0.045



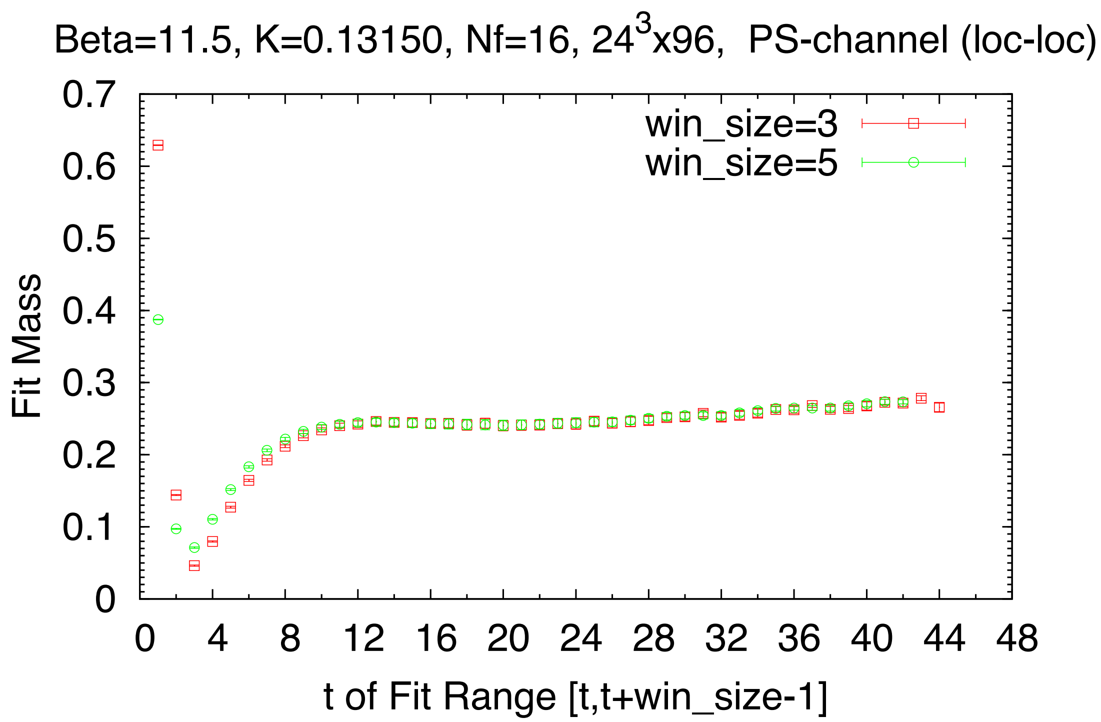
$m(t)$, $\alpha(t)$: Nf7; mq=0.0006

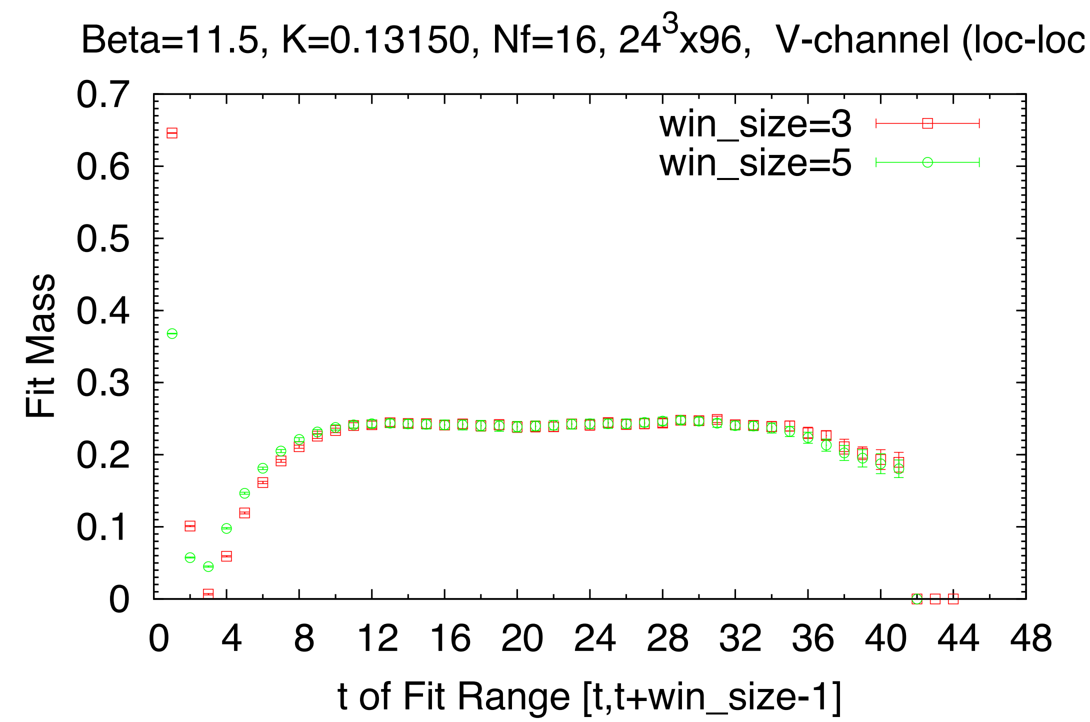


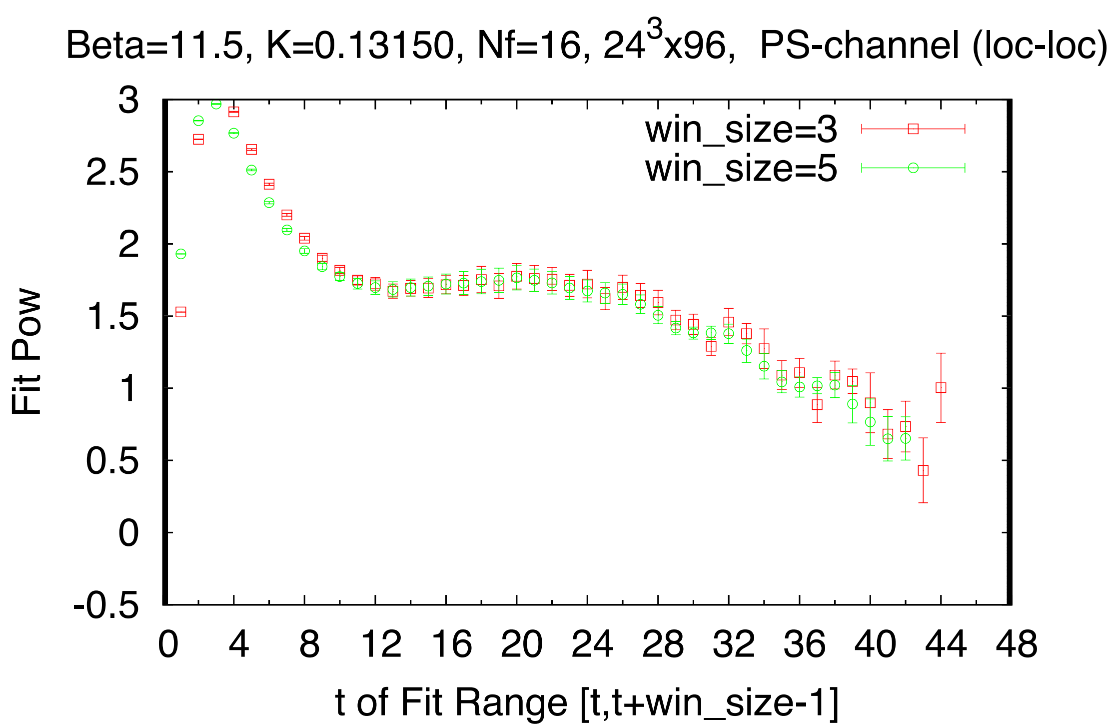
$m(t)$, $\alpha(t)$: Nf16; mq=0.084; N=16

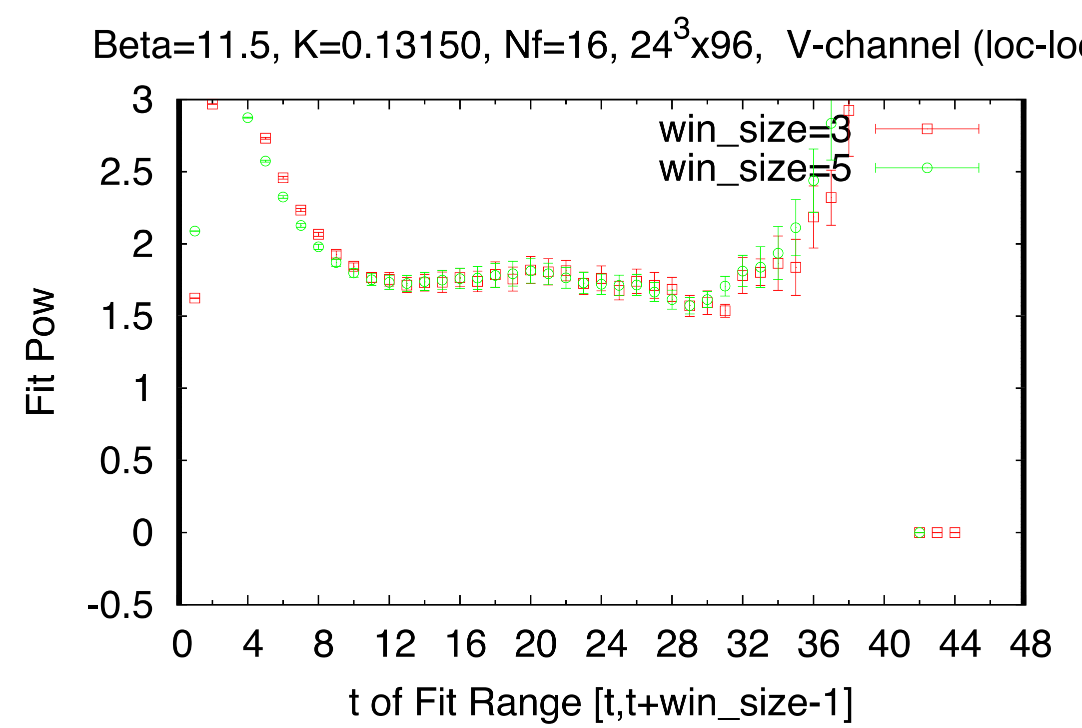


$m(t)$, $\alpha(t)$; Nf16; mq=0.062; N=16

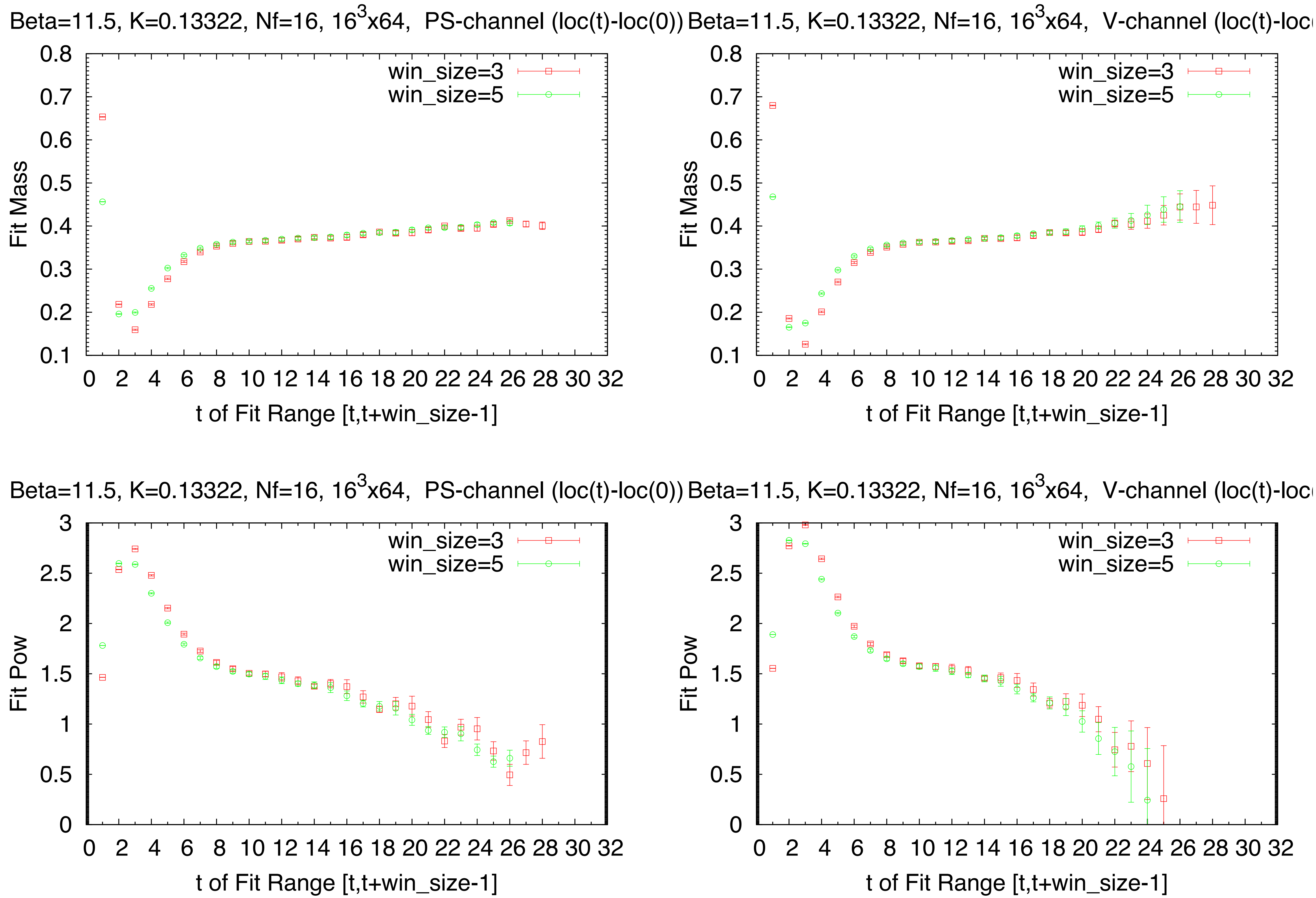




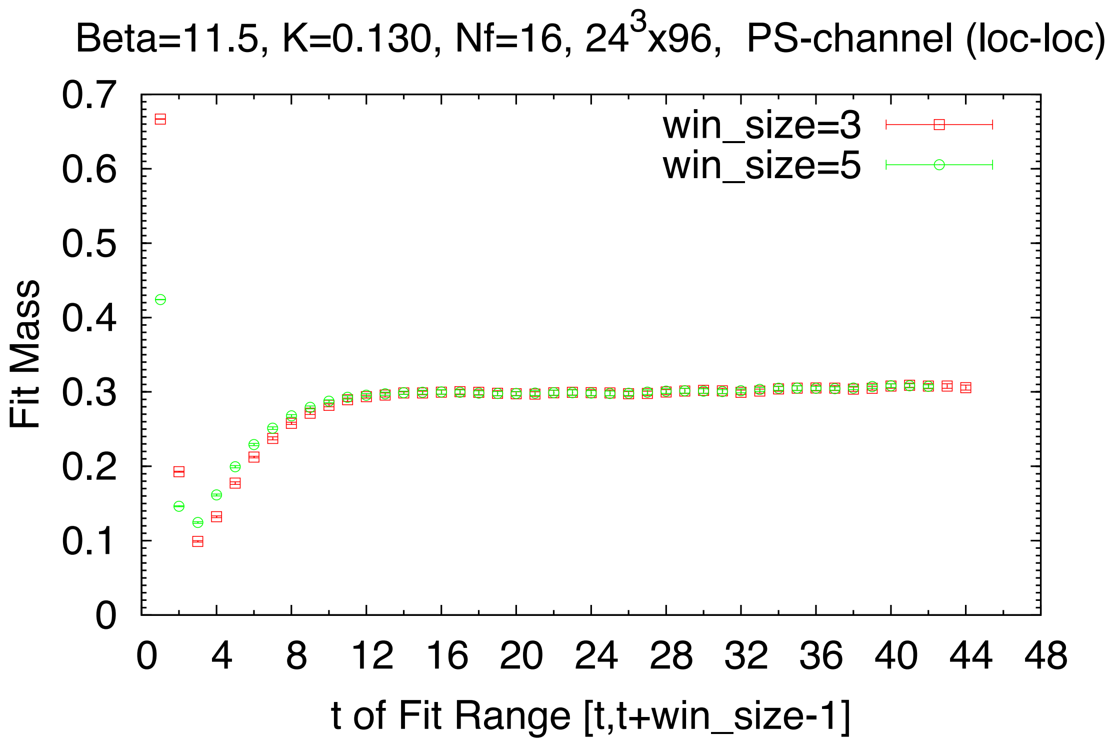


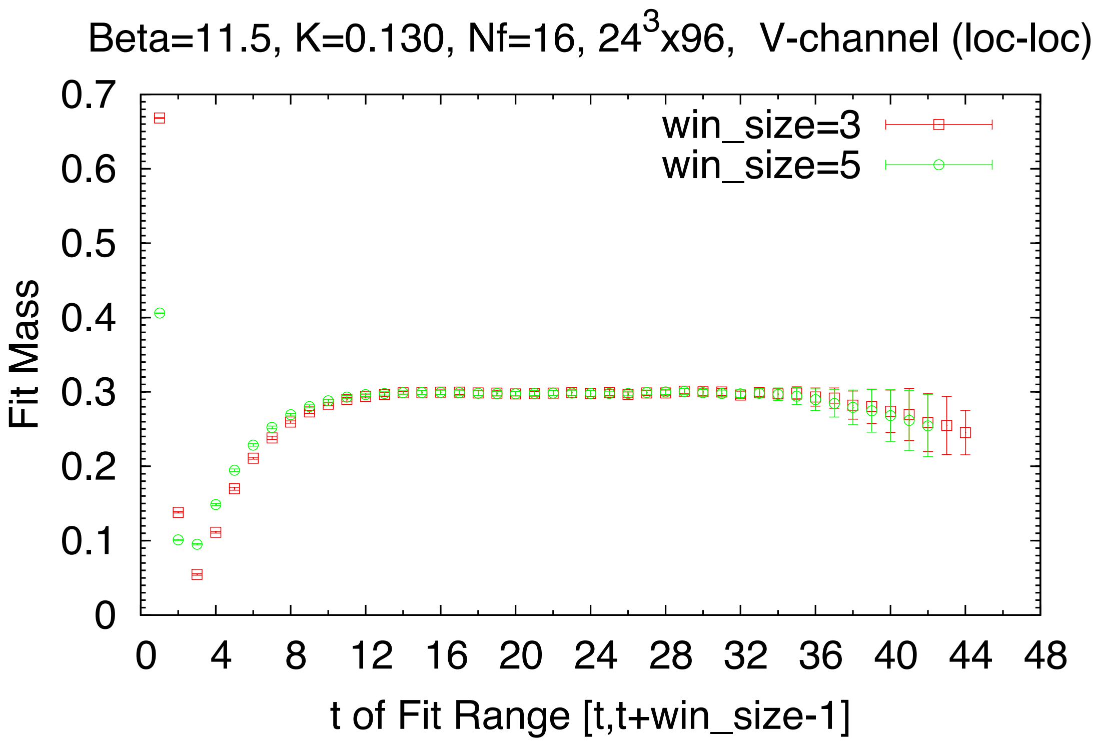


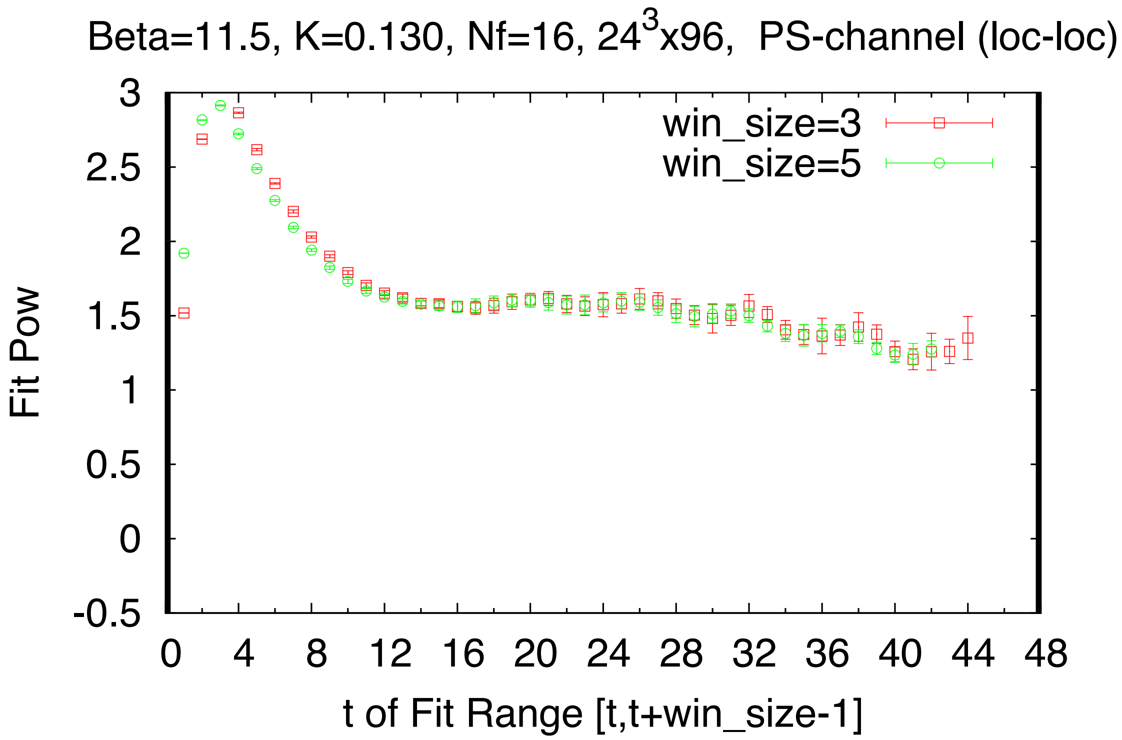
$m(t)$, $\alpha(t)$: Nf16; mq=0.0006; N=16

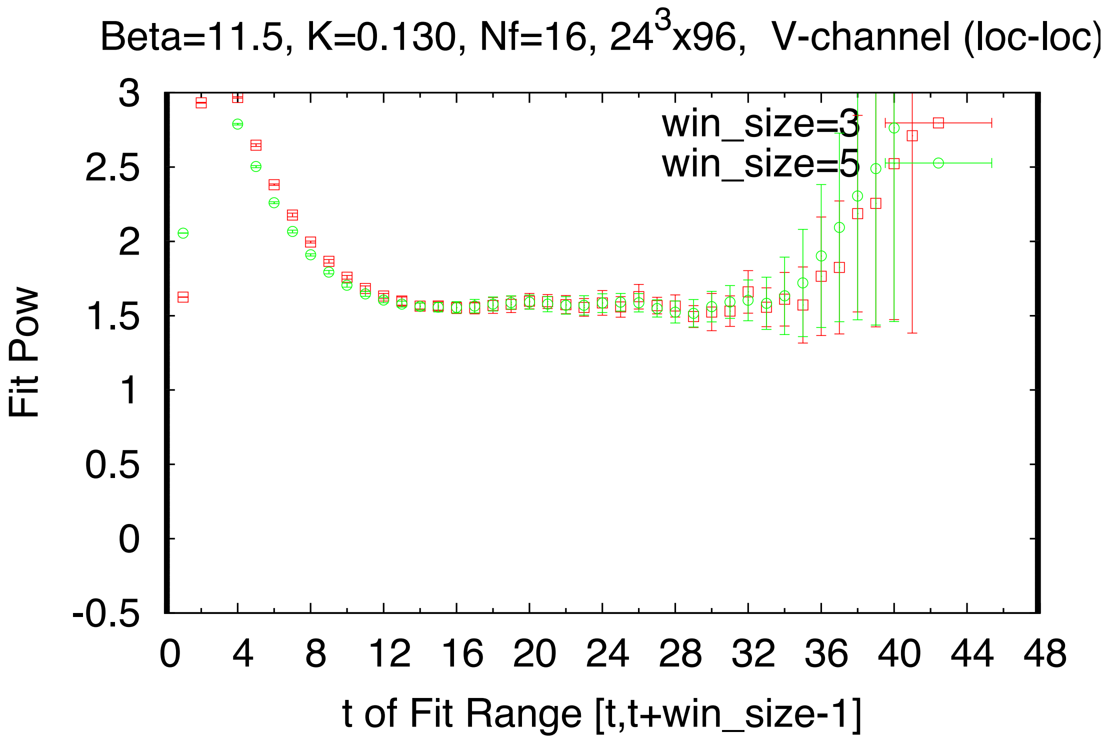


$m(t), \alpha(t)$: Nf16; mq=0.084; N24

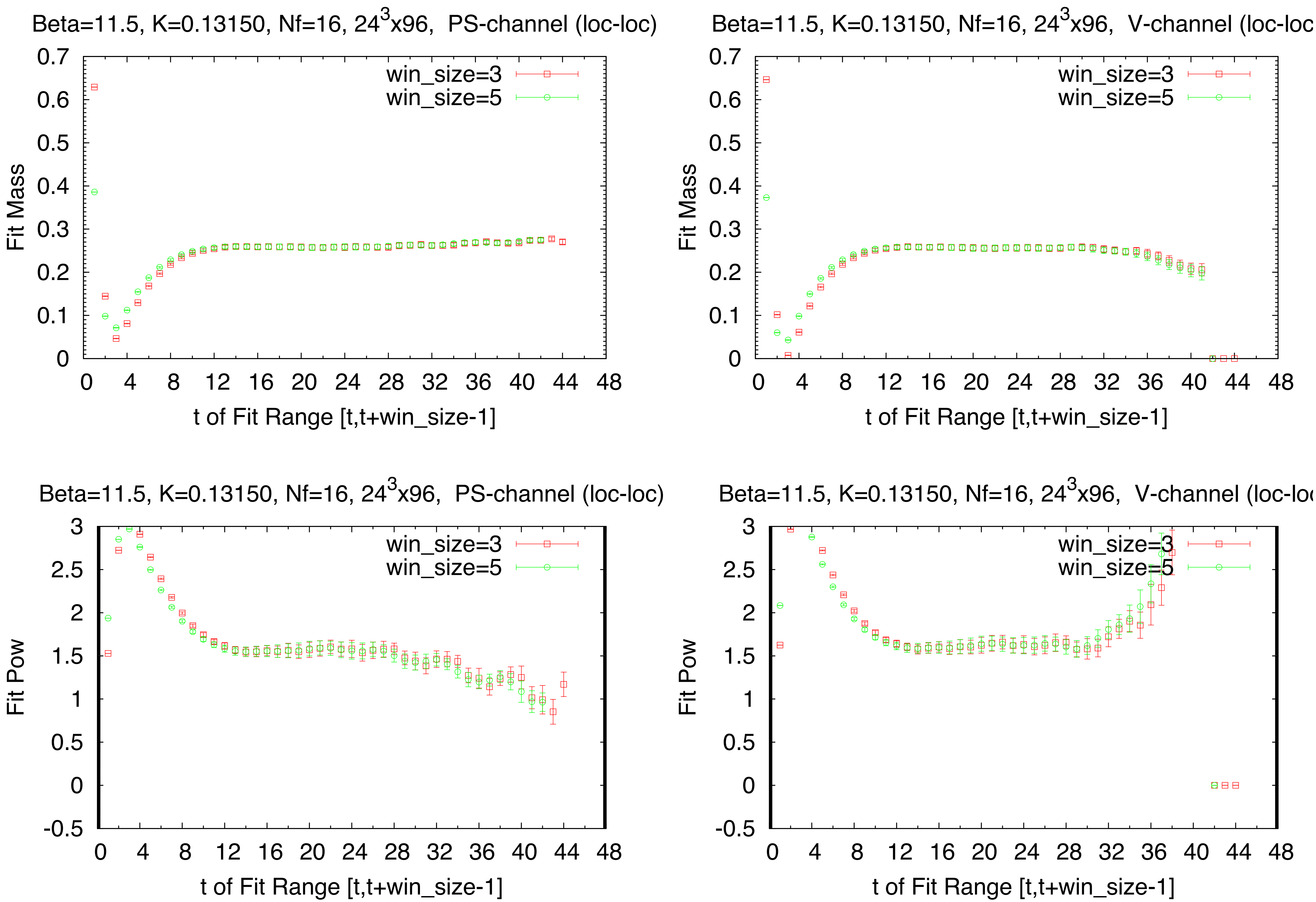




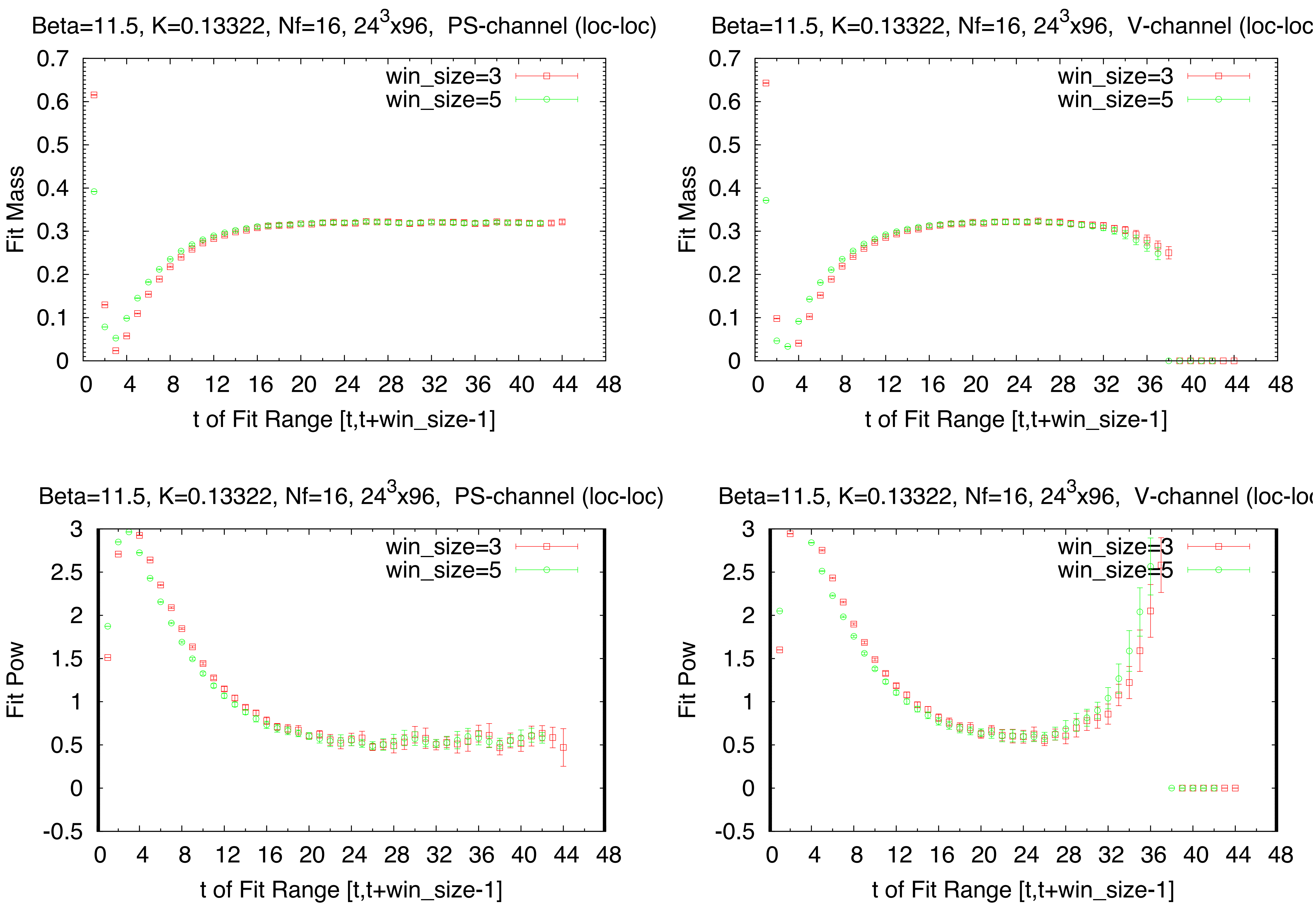




$m(t)$, $\alpha(t)$: Nf16; mq=0.062; N24



$m(t)$, $\alpha(t)$: Nf16; mq=0.0006; N24



Observation of the results

- Finite size effects are severe

  $m_H = 0.2 \sim 0.4$ for $m_q \sim 0.0$

- Clear difference between Nf=7 and Nf=16
- Nf 7 : plateau at t= 15 ~ 31 (16^3x64)
- Nf16: shoulder at t= 12 ~ 24 (both sizes)
- Compare the results with some models

t-dependence of $\alpha(t)$

- In general, in the continuum limit

$$\alpha(t) = 3 - 2\gamma^* \quad \text{for} \quad t \gg \Lambda_{CFT}$$

- In the above derivation, assumed

$$m_H t \ll 1$$

- In simulation results

$$m_H t \geq 1 \quad m_H = 0.3 \sim 0.4,\ t = 30 \sim 45$$

- To estimate $\alpha(t)$ in this case, we need a model

Some models

- a free Wilson quark and an anti-quark
- meson unparticle model*

$$\langle O(p)O(-p)\rangle = \frac{1}{(p^2+m^2)^{2-\Delta}}$$

$O(p)$: meson operator

- fermion unparticle model*

$$\langle \psi(p)\bar{\psi}(-p)\rangle = (p^\mu\gamma_\mu + m)\frac{1}{(p^2+m^2)^{\frac{5}{2}-\Delta_f}}$$

*: motivated by the soft-wall model in AdS/CFT correspondence

Model calculations

In case $m_H t \gg 1$, for $t \gg \Lambda_{CFT}$

Free case: $\alpha(t) = 3/2$

Meson unparticle case:

$$\alpha(t) = 2 - \gamma^*$$

Fermion unparticle case:

$$\alpha(t) = 3/2 - \gamma^*$$

Interpretation of Results

- Nf=7 is close to the meson unparticle model

  plateau at $t = 16 \sim 24$

$$2 - \gamma^* \sim 0.8$$

$$\gamma^* \sim 1.2$$

- Nf=16 is close to the fermion unparticle model

  shoulder at $t = 20 \sim 24$

$$1.5 - \gamma^* = \alpha$$

$$1.5 - \gamma^* \sim 1.5$$

consistent with 2-loop results:

$$\gamma^* \sim 0.025$$

(End of part 2)

Strategy for Part 3

- Note that QCD at high temperature is the theory with IR cutoff
- Apply a similar idea of Part 2 to this case
- Derive physical implications

Running Coupling Constant at T

Define a running coupling constant $g(\mu;T)$

on the line $m_q = 0$

by any method such as Wilson method

cf: Kaczmarek(2004) et. al

$V(r,T) = c\,\alpha(r,T)/r$

In UV region, the theory is asymptotic free,

therefore perturbative RG is applicable

the coupling constant is universal

$$g^2(\mu;T) = g^2(\mu;T=0) + cg^4(\mu;T=0)$$

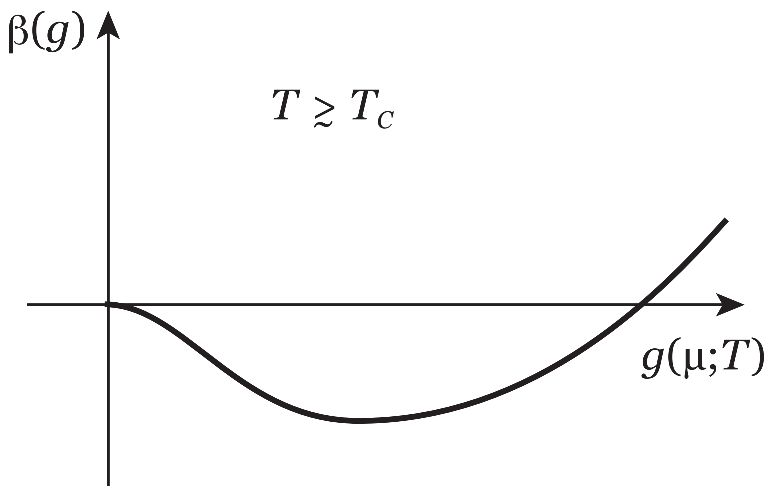
Running Coupling Constant at T (Cont.)

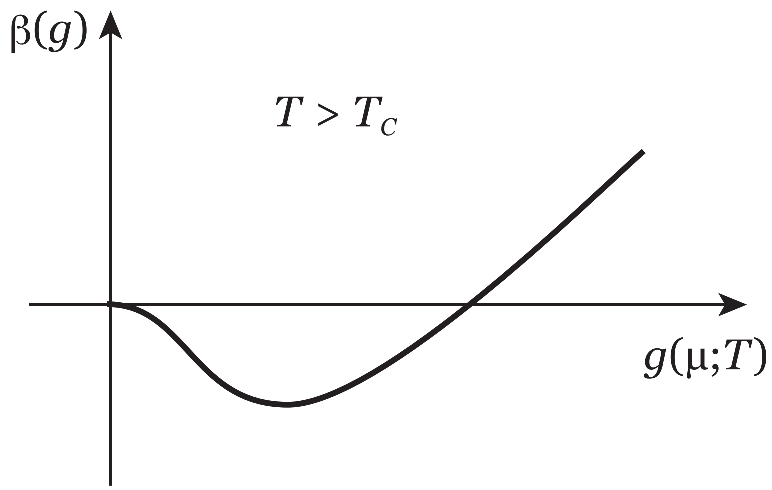
- In IR region, the running constant $g(\mu, T)$ may be quite different from $g(\mu, T = 0)$ since the IR cutoff is finite; 1/T
- When T/Tc >1, $g(\mu, T)$ cannot be arbitrarily large, since the quark is not confined

Beta function for T > Tc

As far as $T < T_c$,
the beta-function is negative all through g

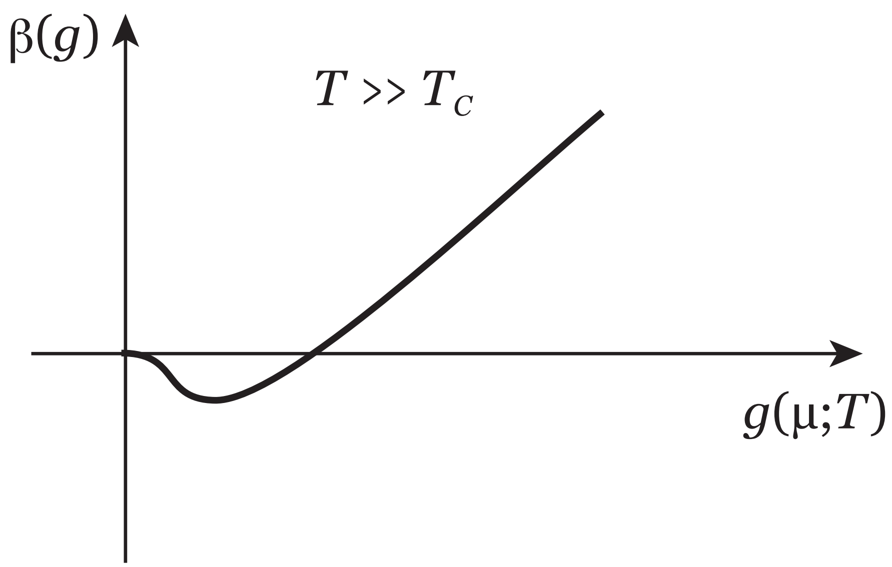
When $T > T_c$, but $T \sim T_c$
the beta function changes the sign
from negative to positive at large g





When T increases further,
it will change the sign at medium strong g

When $T \gg T_c$
it will change the sign at small g



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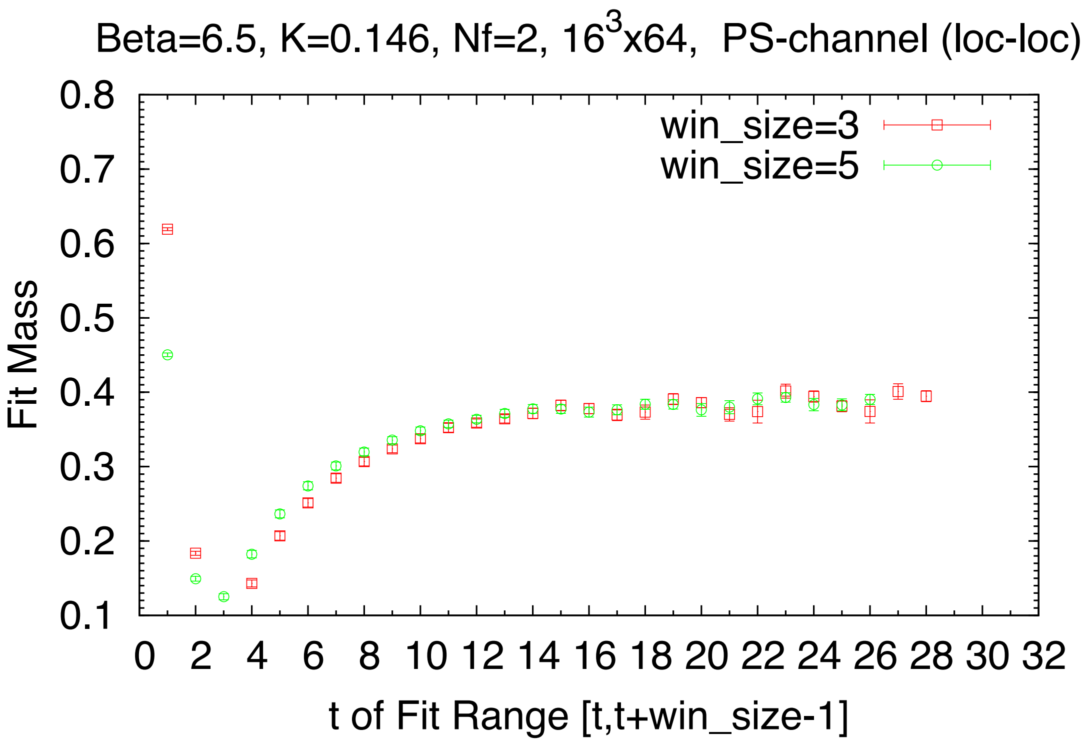
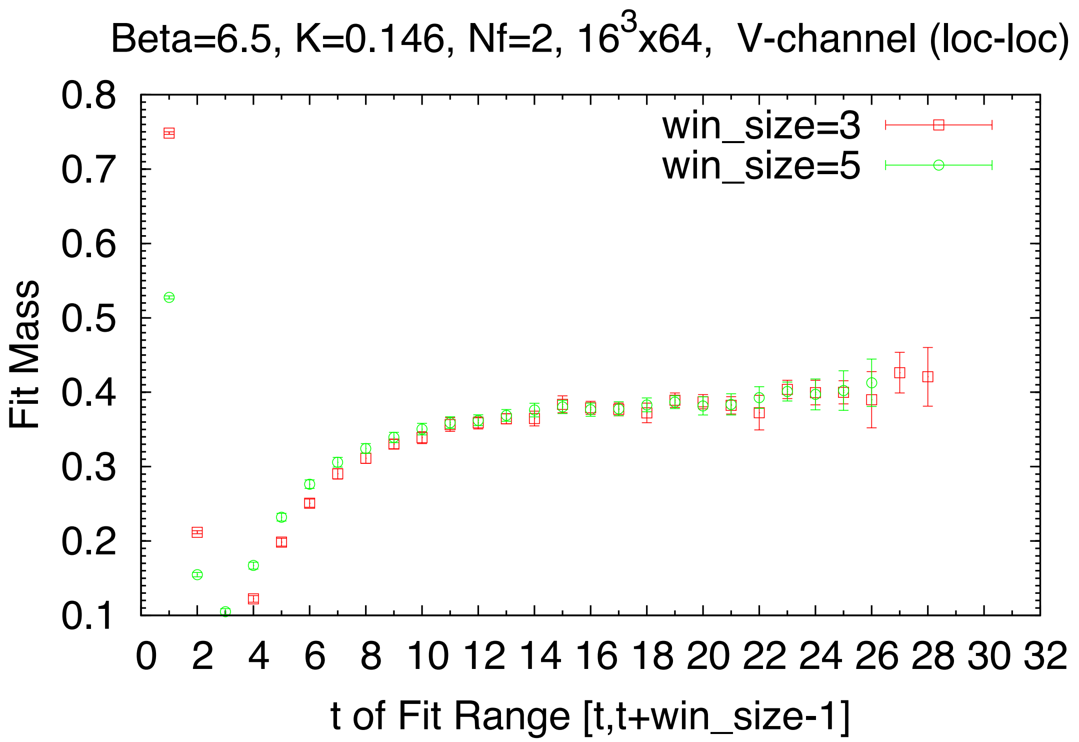
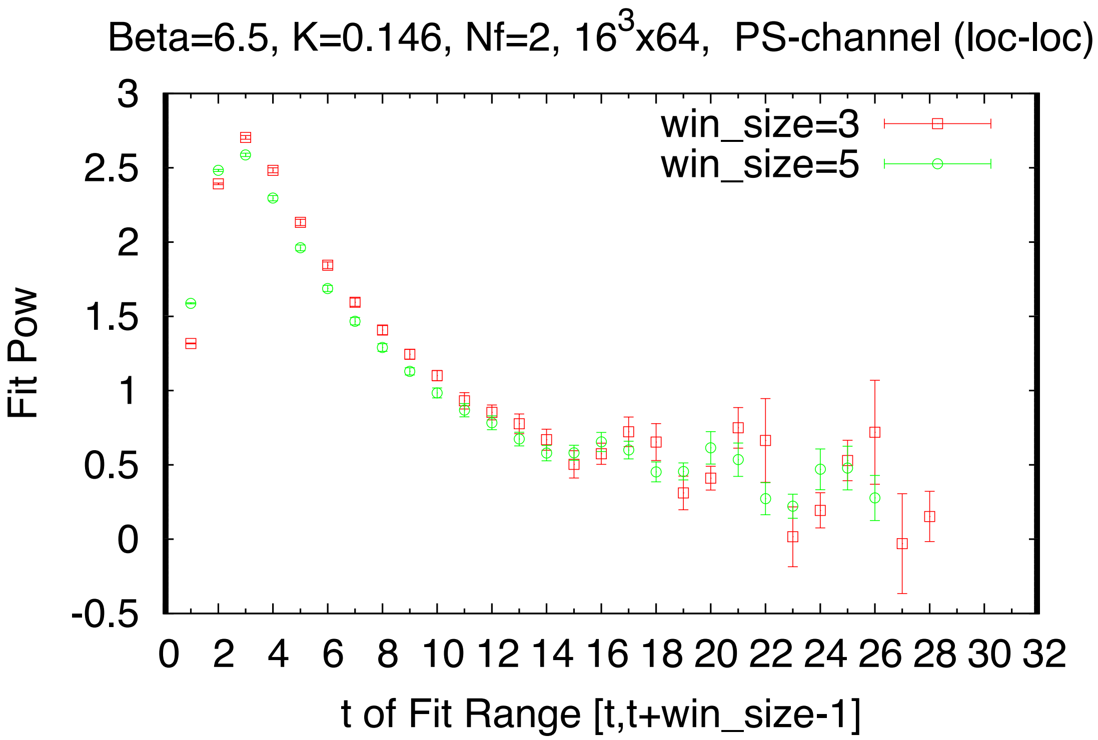
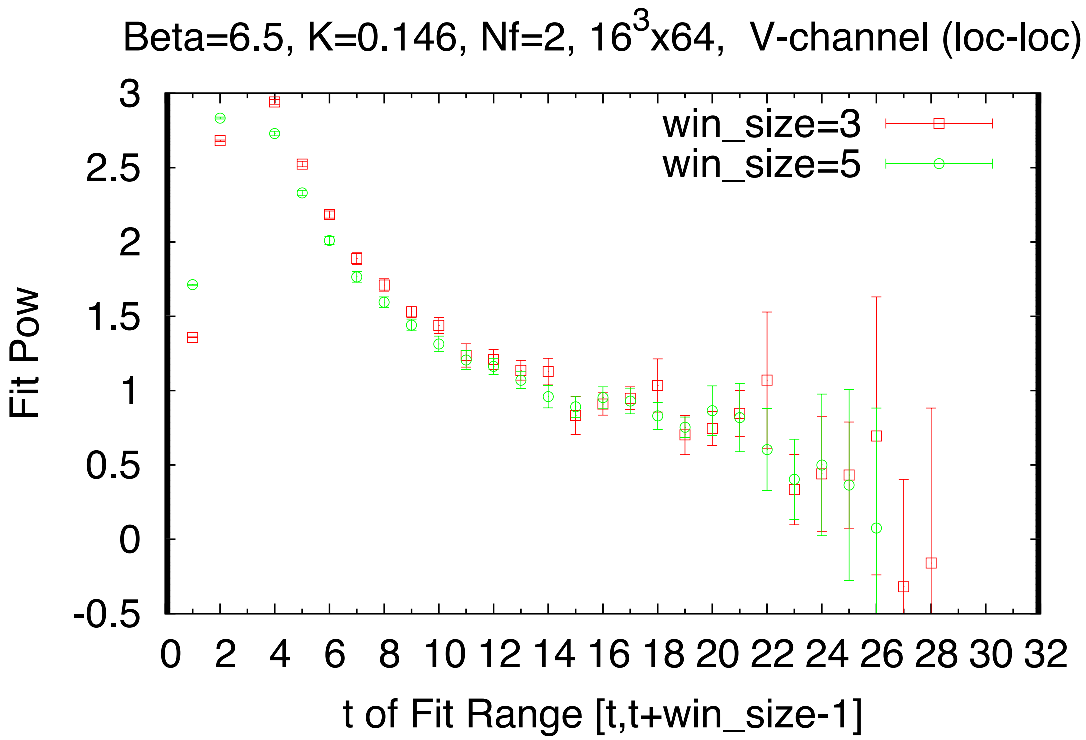
Nf=2; T~ 2 Tc

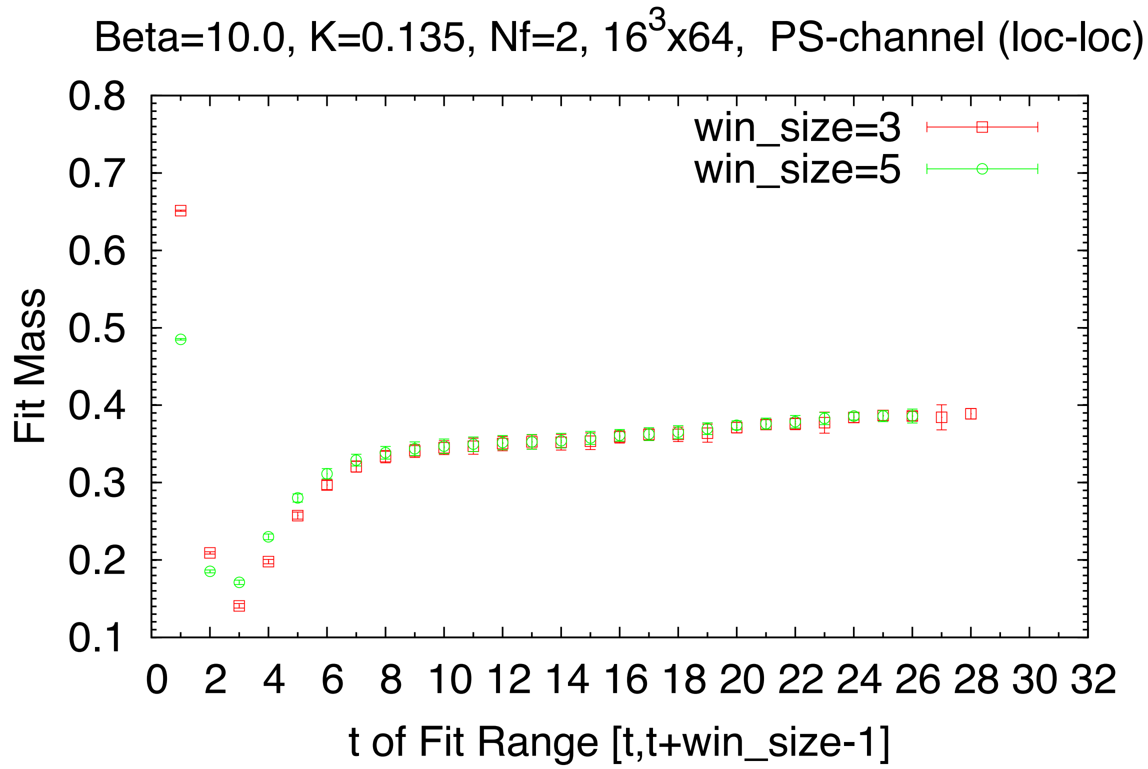

Beta=6.5, K=0.146, Nf=2, 16^3x64, PS-channel (loc-loc)
Fit Mass
t of Fit Range [t,t+win_size-1]
win_size=3
win_size=5

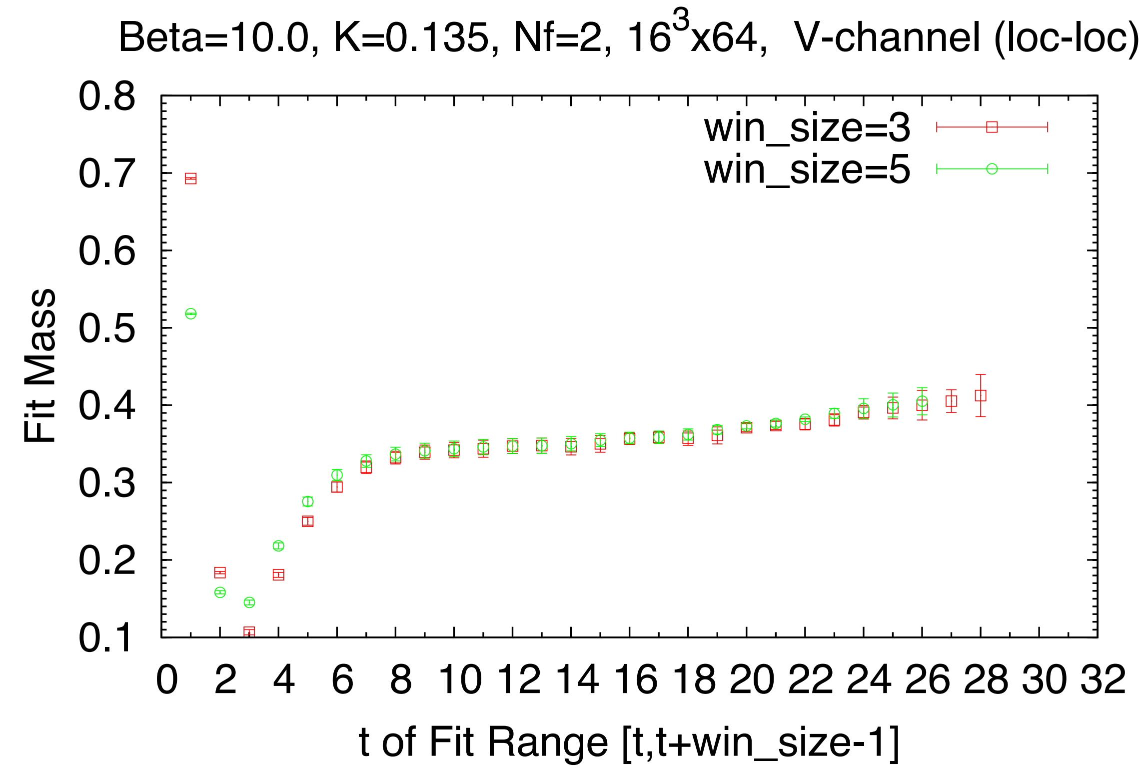

Beta=6.5, K=0.146, Nf=2, 16^3x64, V-channel (loc-loc)
Fit Mass
t of Fit Range [t,t+win_size-1]
win_size=3
win_size=5

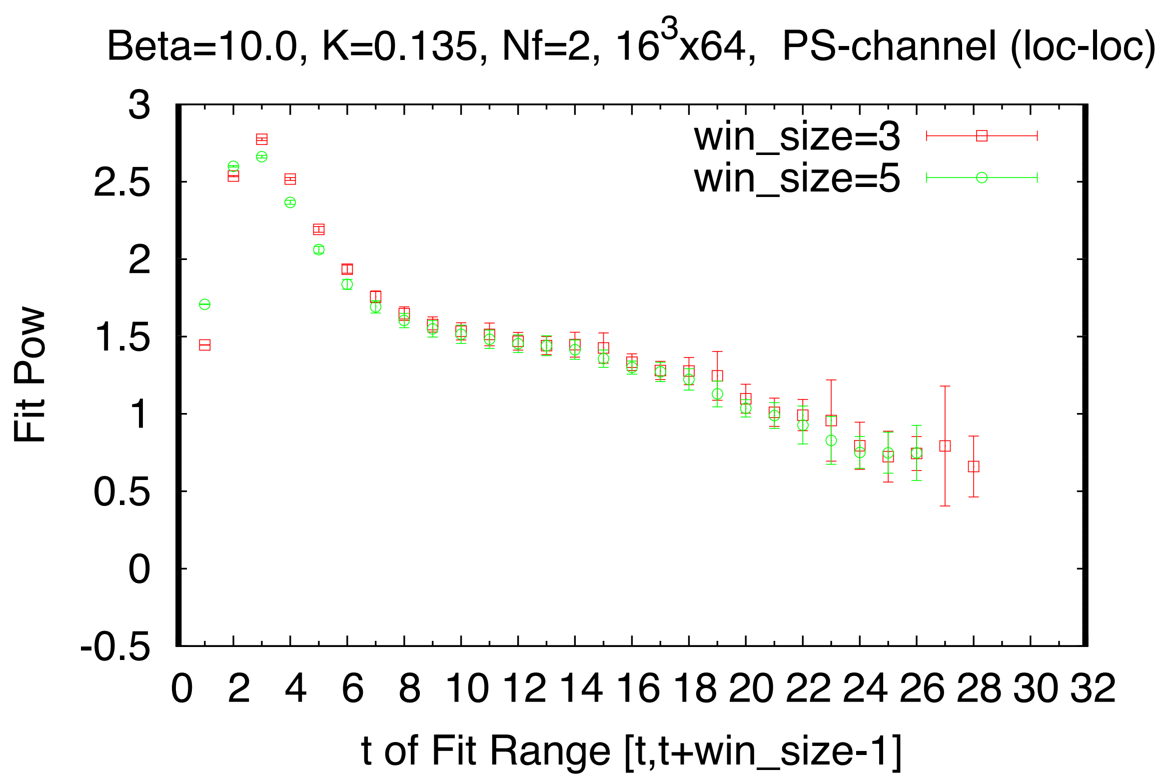

Beta=6.5, K=0.146, Nf=2, 16^3x64, PS-channel (loc-loc)
Fit Pow
t of Fit Range [t,t+win_size-1]
win_size=3
win_size=5

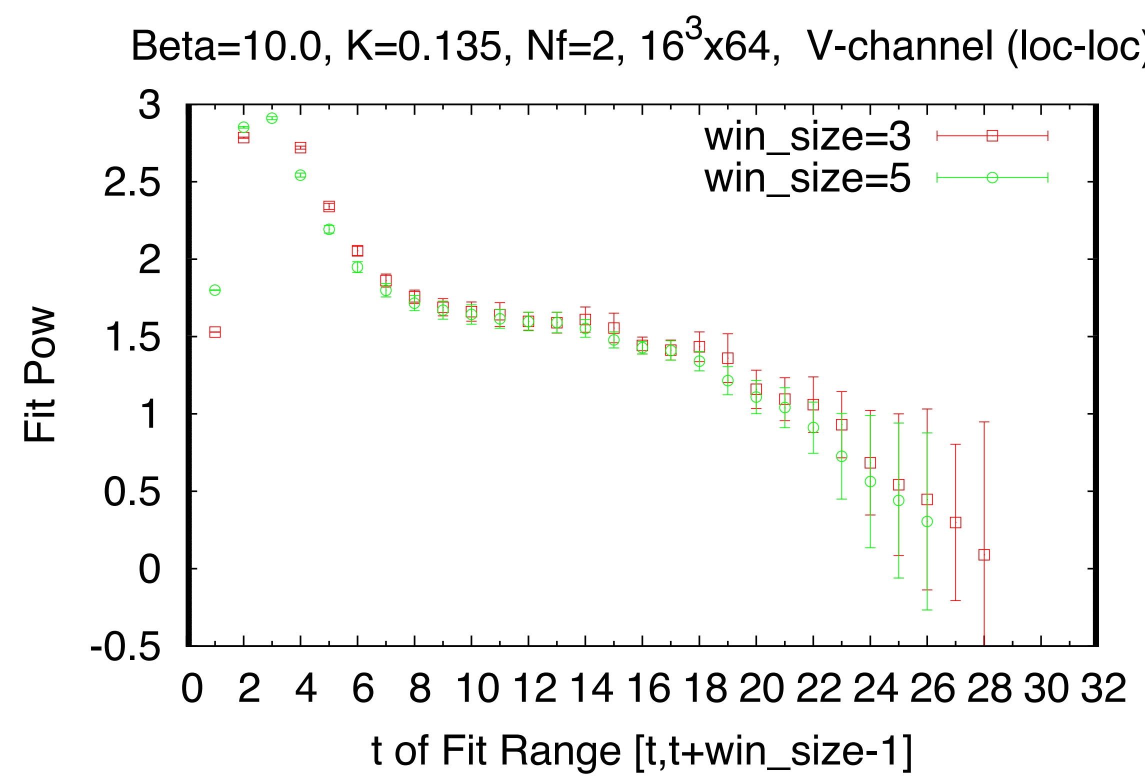

Beta=6.5, K=0.146, Nf=2, 16^3x64, V-channel (loc-loc)
Fit Pow
t of Fit Range [t,t+win_size-1]
win_size=3
win_size=5

T~100Tc

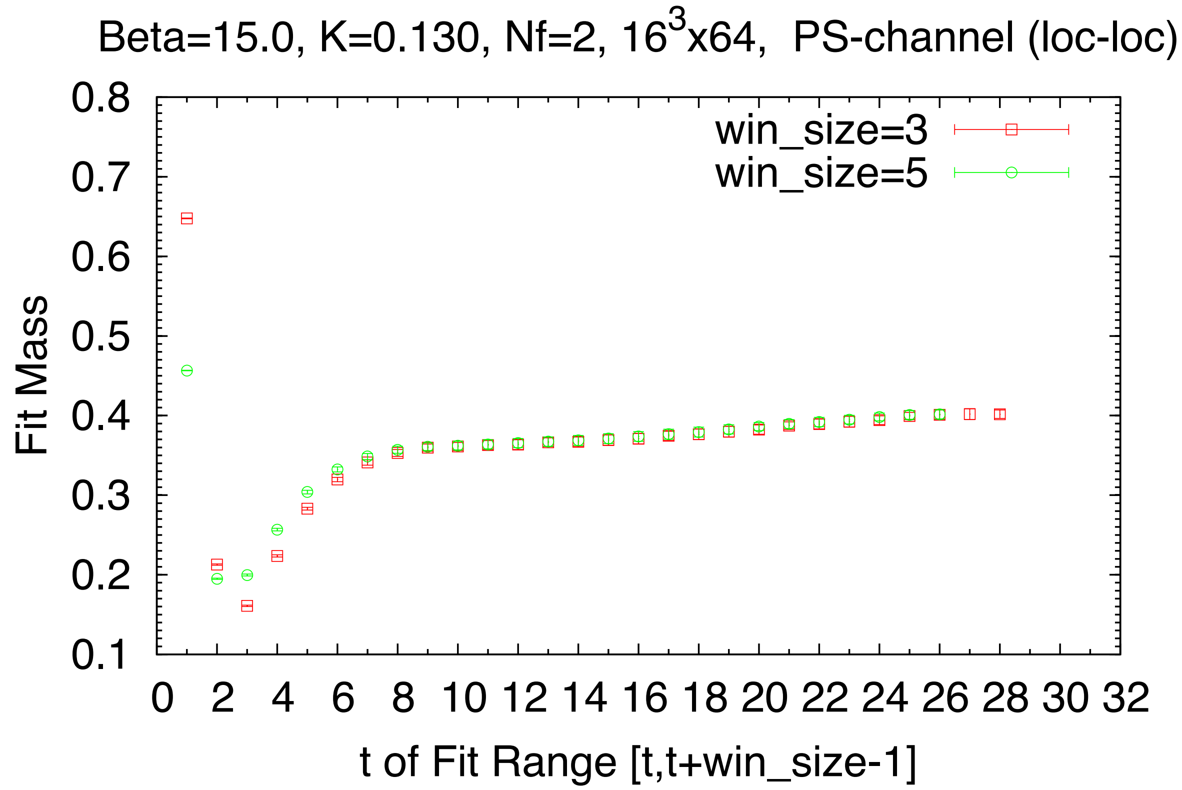

Beta=10.0, K=0.135, Nf=2, 16^3x64, PS-channel (loc-loc)
Fit Mass
t of Fit Range [t,t+win_size-1]
win_size=3
win_size=5

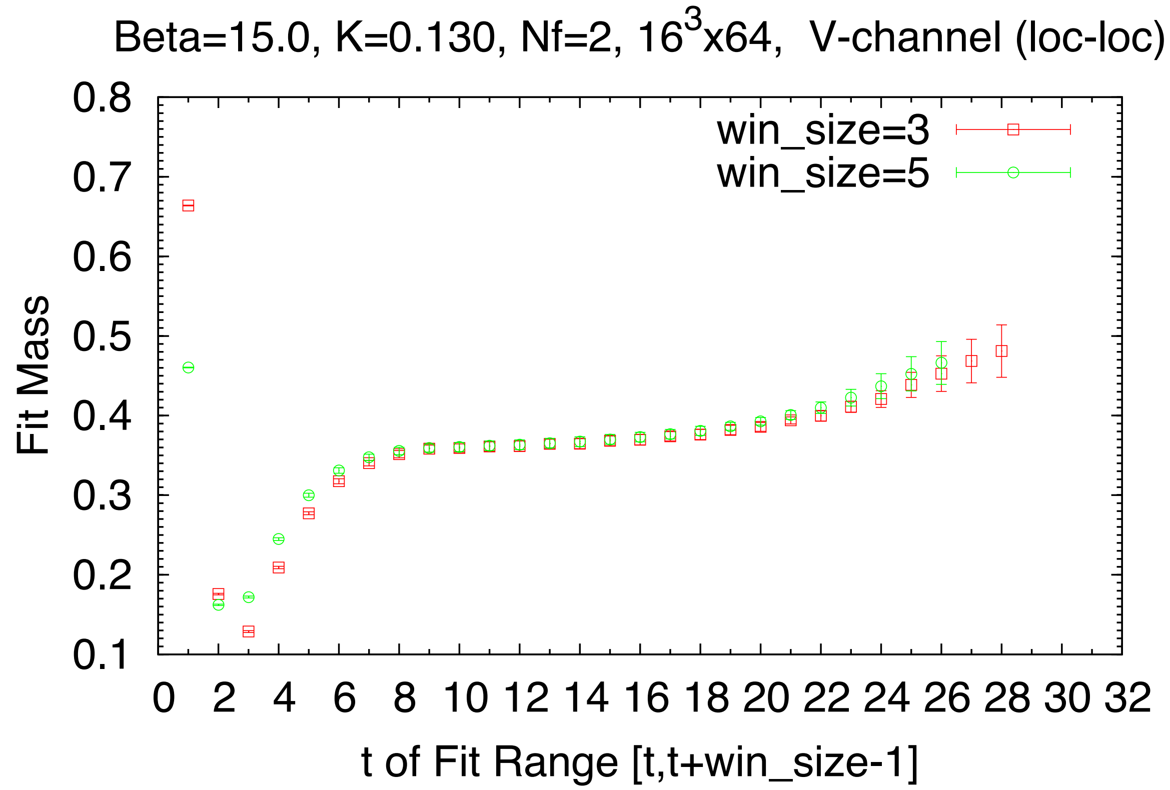

Beta=10.0, K=0.135, Nf=2, 16^3x64, V-channel (loc-loc)
Fit Mass
t of Fit Range [t,t+win_size-1]
win_size=3
win_size=5

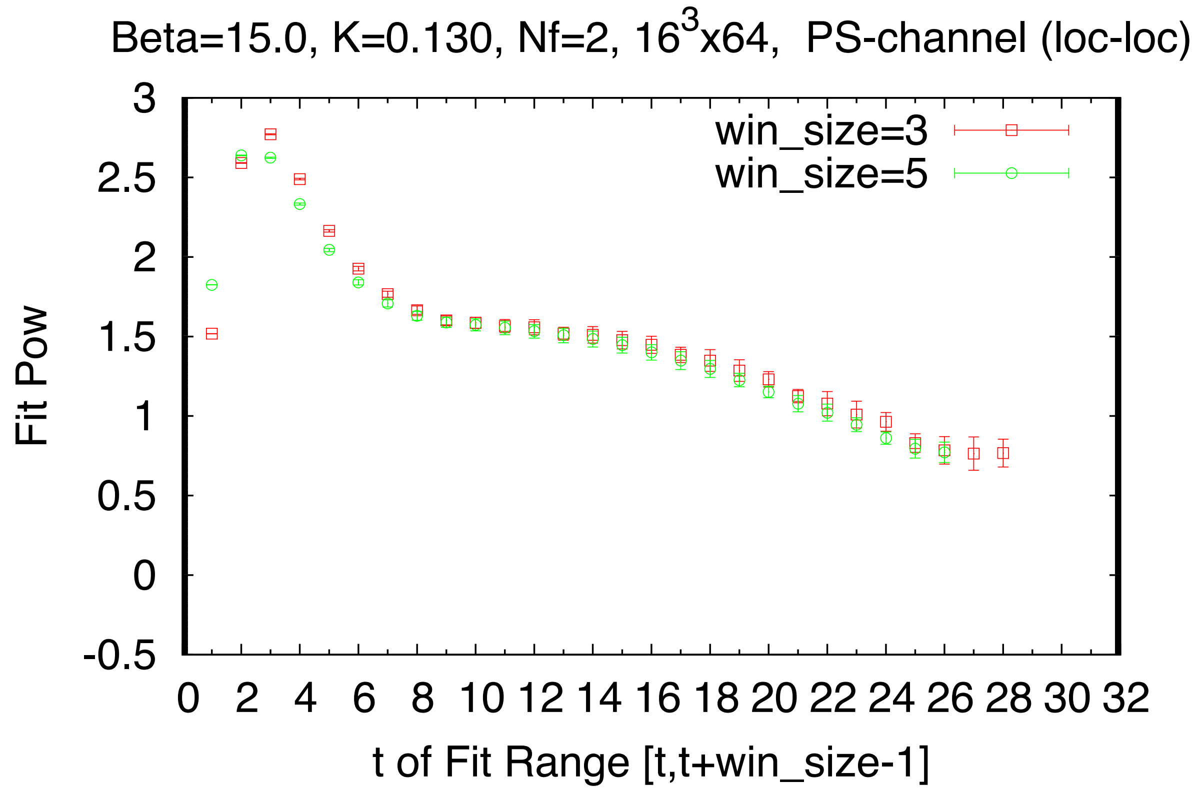

Beta=10.0, K=0.135, Nf=2, 16^3x64, PS-channel (loc-loc)
Fit Pow
t of Fit Range [t,t+win_size-1]
win_size=3
win_size=5

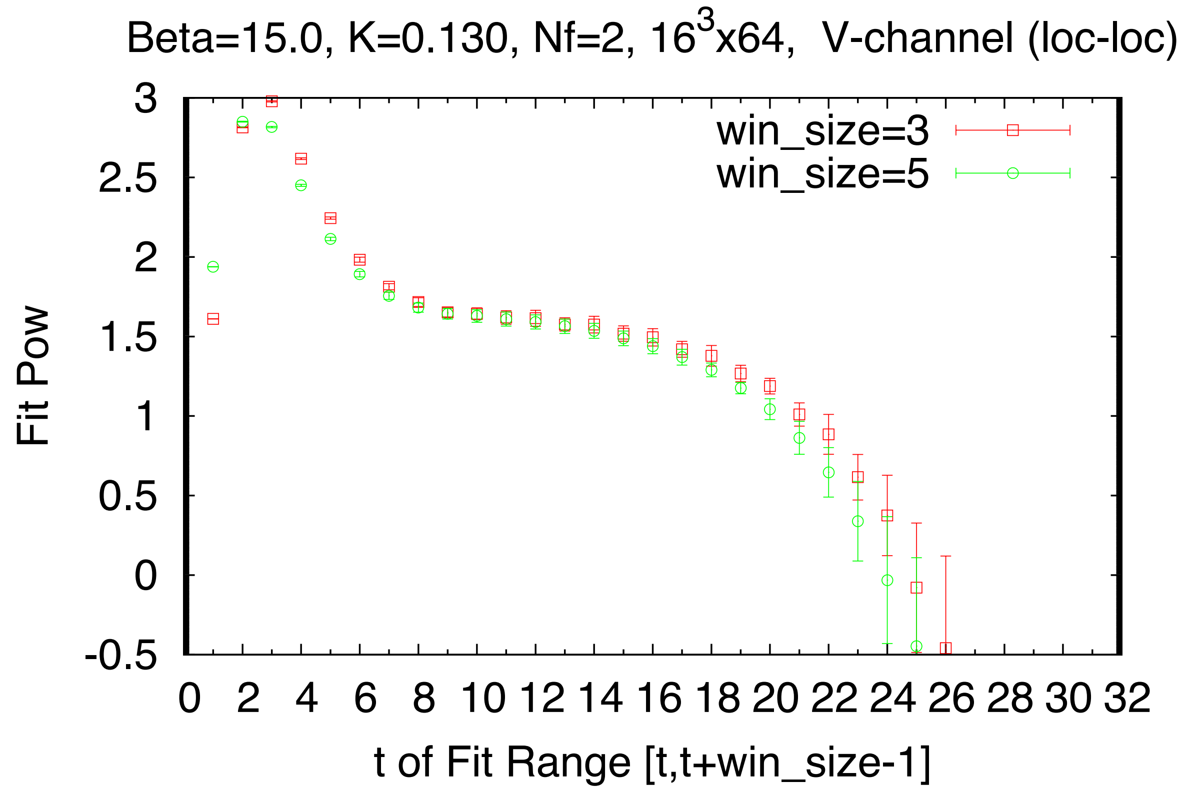

Beta=10.0, K=0.135, Nf=2, 16^3x64, V-channel (loc-loc)
Fit Pow
t of Fit Range [t,t+win_size-1]
win_size=3
win_size=5

T ~ 10^5Tc


Beta=15.0, K=0.130, Nf=2, 16^3x64, PS-channel (loc-loc)
Fit Mass
t of Fit Range [t,t+win_size-1]
win_size=3
win_size=5


Beta=15.0, K=0.130, Nf=2, 16^3x64, V-channel (loc-loc)
Fit Mass
t of Fit Range [t,t+win_size-1]
win_size=3
win_size=5

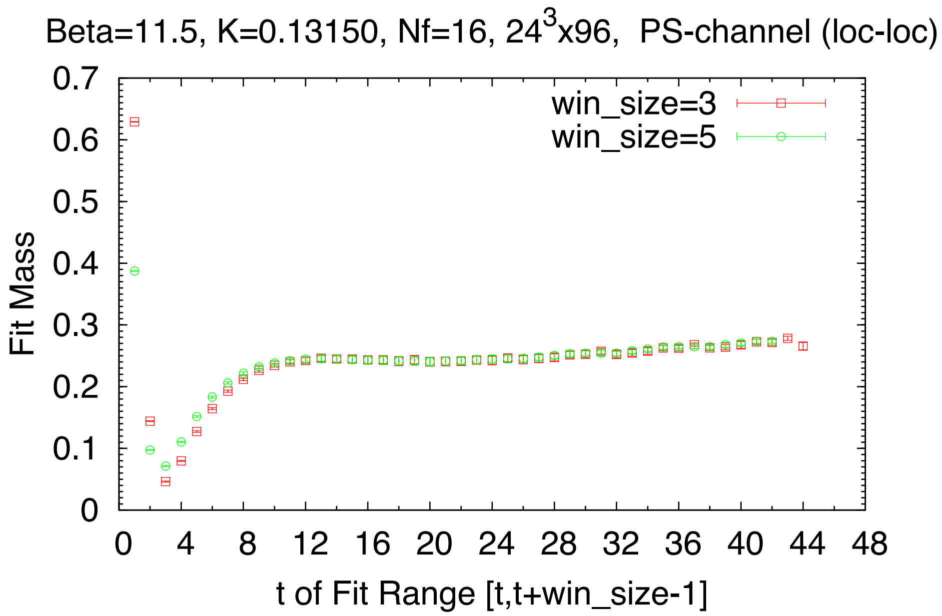

Beta=15.0, K=0.130, Nf=2, 16^3x64, PS-channel (loc-loc)
Fit Pow
t of Fit Range [t,t+win_size-1]
win_size=3
win_size=5

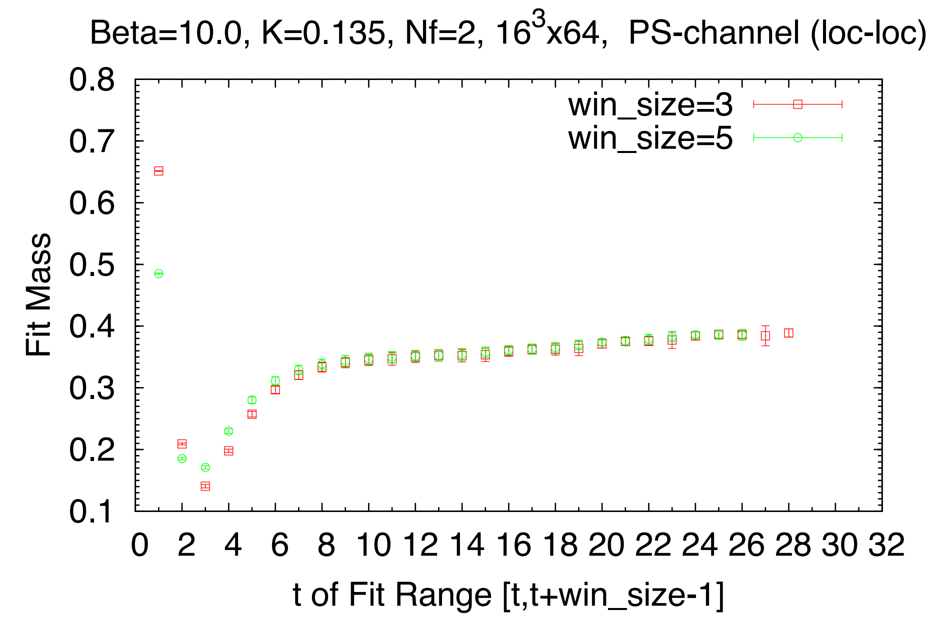

Beta=15.0, K=0.130, Nf=2, 16^3x64, V-channel (loc-loc)
Fit Pow
t of Fit Range [t,t+win_size-1]
win_size=3
win_size=5

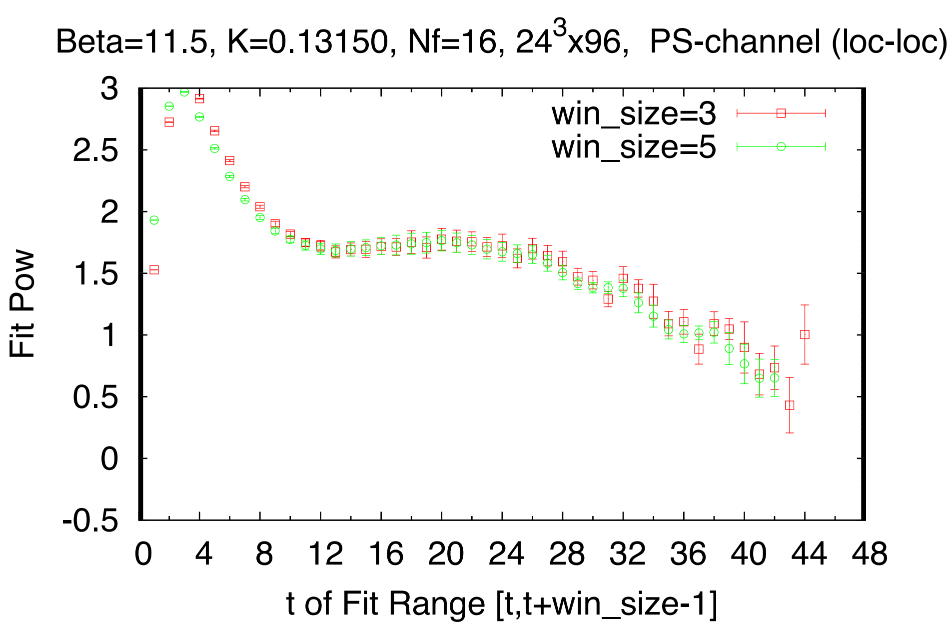
Similarity between large Nf with Λ_{IR} and small Nf at T/Tc >1

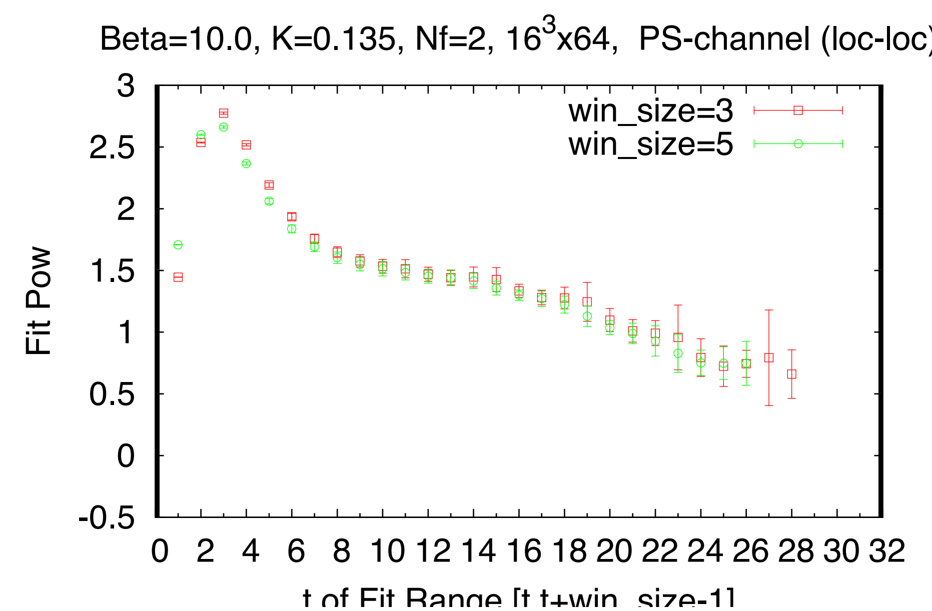
Nf16; with Λ_{IR}

Nf2; T= 100Tc









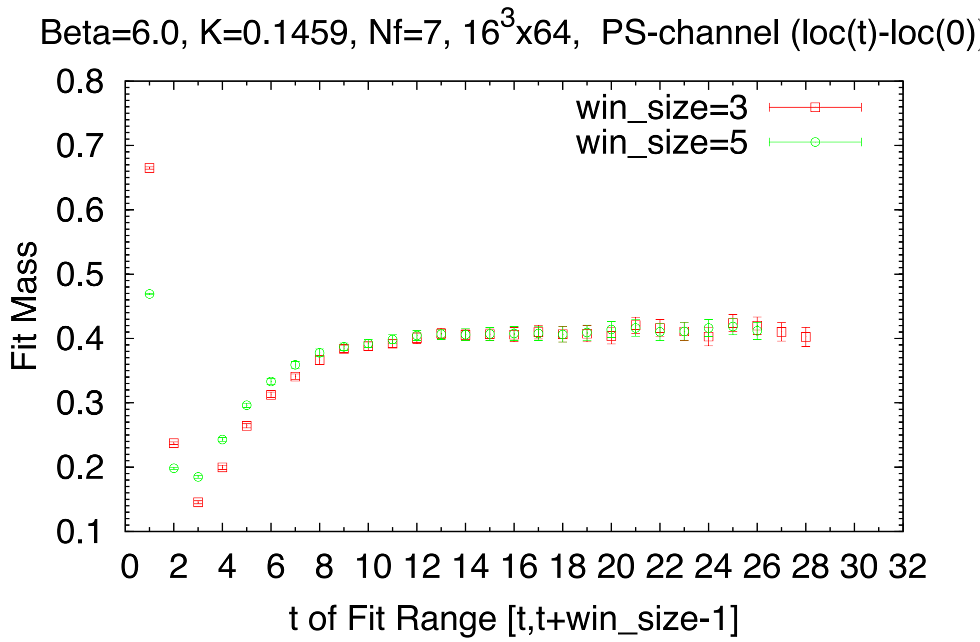
2012年12月5日水曜日

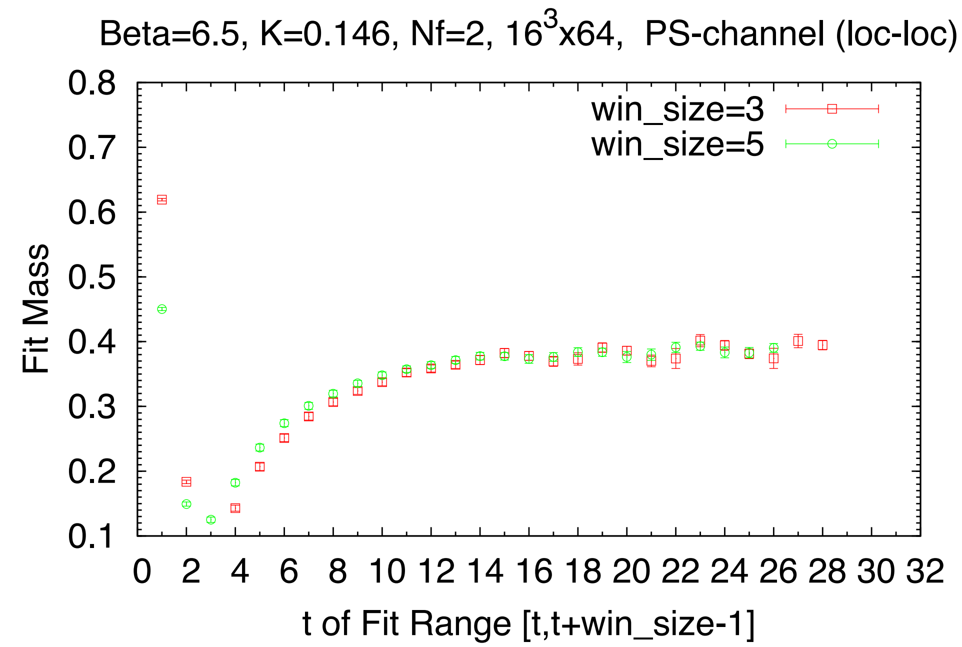
Similarity between

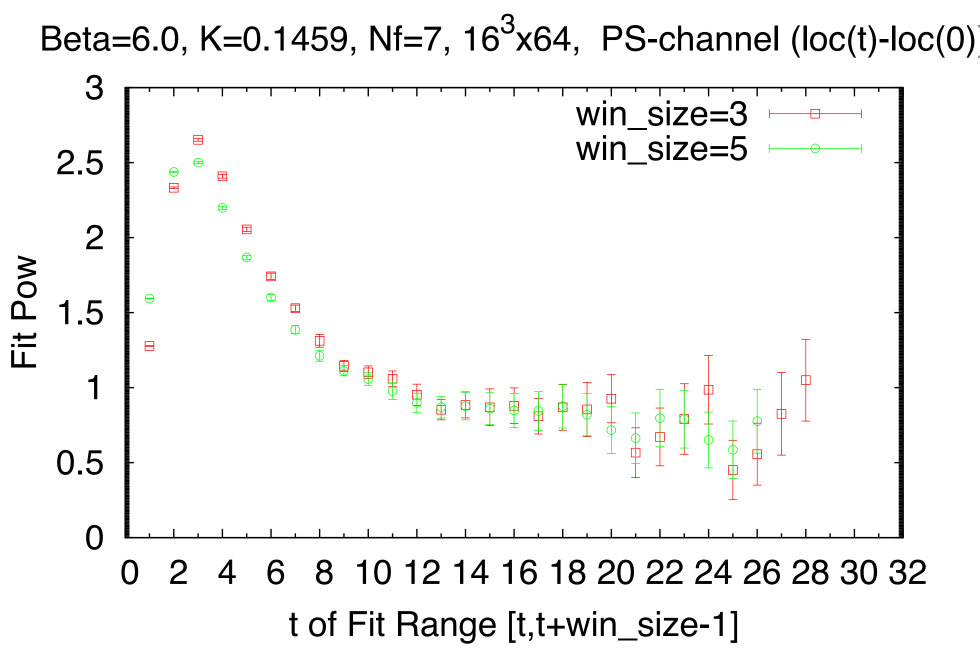
large Nf with Λ_{IR} and small Nf at T/Tc >1

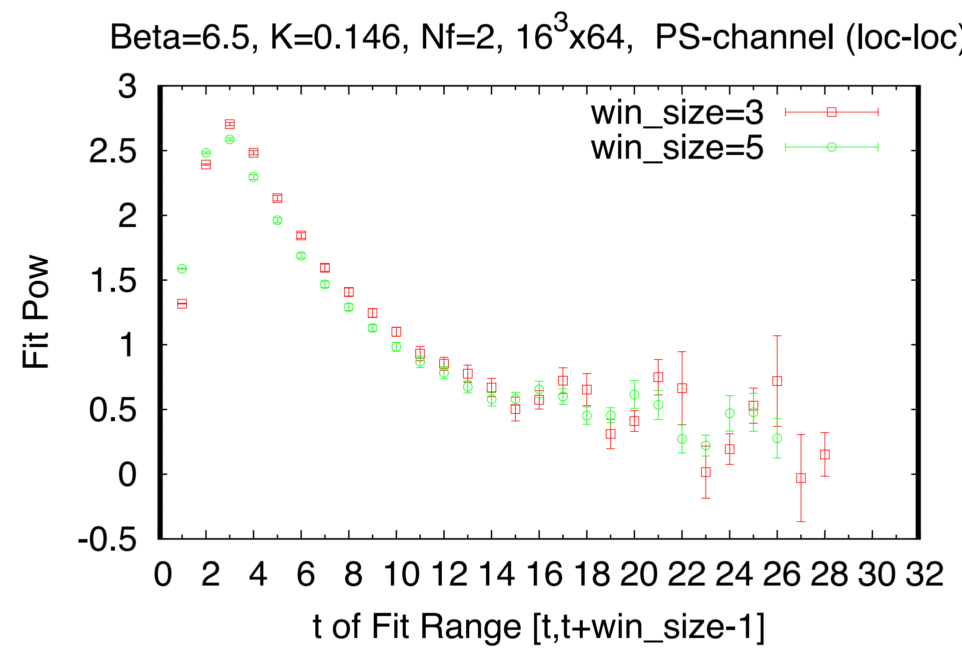
Nf7; with Λ_{IR}

Nf2; T = 2 Tc









Long standing important issues

- Free energy of quark-gluon plasma state does not reach that of the Stefan-Boltzman idea gas state even at T/Tc=100
- Wave function of “meson” just above Tc can be obtained, although quarks are deconfined
- The order of chiral phase transition in Nf=2 case: 1st or 2nd ? : UA(1) symmetry ?

Pisarski and Wilczek(1984), iwasaki et. al(1997);
S. Aoki et. al(2012)

Solutions

- quarks and gluons are not free particles
  When T ~ Tc, meson unparticles
  When T >> Tc, fermion unparticles
- meson unparticles are similar to meson particles in some aspects
- G(t) is not analytic in terms of mq and mH

(End of part 3)

Conclusions

- "Conformal Theories with IR cutoff" are satisfactorily verified in the cases of Nf=7 and Nf=16
- The assertion that the Conformal Window is $7 \leq N_f \leq 16$, is thereby strengthened
- "Conformal Theories with IR cutoff" are also verified in the case of T/Tc >1 in Nf=2 and Nf=6
- IR cutoff is inherent with simulations on a lattice and QCD at high temperatures

(to be continued)

Conclusions (Cont.)

- “Nf=7” and “T~Tc” are similar to each other, and are consistent with meson unparticle model
- “Nf=16” and “T>>Tc” are similar to each other, and are consistent with fermion unparticle model
- Physics implications should be deepened
- A lot of things should be done