“Super” Dilatation Symmetry of the Top-Higgs System

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This work is preliminary:

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$$m_{Higgs}^2 \approx \frac{1}{2} m_{top}^2 \qquad m_{top} \approx v_{weak}$$

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Our main interest is in mechanisms that can relate the quantities m_H, m_t and v_{weak}.

Higgs as a $\bar{t}t$ Boundstate:

V. A. Miransky, M. Tanabashi, and K. Yamawaki, Phys. Lett. **B221**, 177 (1989); Mod. Phys. Lett. **A4**, 1043 (1989);

W. J. Marciano, Phys. Rev. Lett. **62**, 2793 (1989);

Y. Nambu, Enrico Fermi Institute Report No. 89-08, 1989 (unpublished); in *Proceedings of the 1989 Workshop on Dynamical Symmetry Breaking*, edited by T. Muta and K. Yamawaki (Nagoya University, Nagoya, Japan, 1990).

W. A. Bardeen, C. T. Hill, and M. Lindner, Phys. Rev. **D41**, 1647 (1990).

C. T. Hill, Phys. Lett. B **266**, 419 (1991); C. T. Hill, Phys. Lett. B **345**, 483 (1995).

Pure NJL: $m_H = 2m_t$

RG improved with GUT hierarchy: $m_H \sim 260$ GeV $\quad m_t \sim 220$ GeV

Top Seesaw: $m_H \sim 1$ TeV.

Hence $\bar{t}t$ composite Higgs models do succeed in relating m_H, m_t and v_{weak},

but generally predict a Higgs boson that is significantly heavier than the top quark

Quick Review (1): Nambu Jona-Lasinio Model

"high energy physics" scale Λ

$$\mathcal{L}_\Lambda = \bar{\psi}_L^a i\partial\!\!\!/ \, \psi_L^a + \bar{\psi}_R^a i\partial\!\!\!/ \, \psi_R^a + \frac{g^2}{\Lambda^2}(\bar{\psi}_L^a \psi_{Ra})(\bar{\psi}_R^b \psi_{Lb})$$

Fierz rearrangement of a single–coloron (massive gluon) exchange potential:

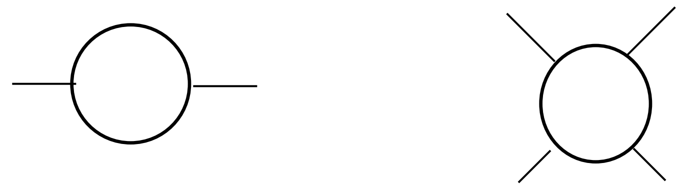
$$\tfrac{1}{2}g^2 \left(\bar{\psi}\gamma_\mu \frac{\lambda^A}{2}\psi\right) \frac{g^{\mu\nu}}{q^2 - \Lambda^2} \left(\bar{\psi}\gamma_\nu \frac{\lambda^A}{2}\psi\right) \quad \text{thus } G = g^2/\Lambda^2 \text{ at } q^2 = 0.$$

introduce auxiliary field:

$$\mathcal{L} = \mathcal{L}_{kinetic} + (g\overline{\psi}_L \psi_R H + h.c.) - \Lambda^2 H^\dagger H$$

"factorized interaction"

integrate out the fermion field components on scales $\mu \leftrightarrow \Lambda$



$$\begin{aligned}
\mathcal{L}_\mu &= \mathcal{L}_{kinetic} + g\overline{\psi}_L\psi_R H + h.c. \\
&\quad + Z_H|\partial_\nu H|^2 - m_H^2 H^\dagger H - \frac{\lambda_0}{2}(H^\dagger H)^2 - \xi_0 R H^\dagger H
\end{aligned}$$

$$\begin{aligned}
Z_H &= \frac{g^2 N_c}{(4\pi)^2}\log(\Lambda^2/\mu^2); \qquad m_H^2 = \Lambda^2 - \frac{2g^2N_c}{(4\pi)^2}(\Lambda^2-\mu^2) \\
\lambda_0 &= \frac{2g^4N_c}{(4\pi)^2}\log(\Lambda^2/\mu^2); \qquad \xi_0 = \frac{1}{6}\frac{g^2N_c}{(4\pi)^2}\log(\Lambda^2/\mu^2).
\end{aligned}$$

$$m_H^2 = \Lambda^2 - \frac{2g^2N_c}{(4\pi)^2}(\Lambda^2-\mu^2)$$

The broken phase is selected by demanding that $m_H^2 < 0$ $\Rightarrow$ $\Lambda^2(1 - g^2N_c/8\pi^2) < 0$

$$\Rightarrow \quad g^2 > 8\pi^2/N_c = g_c^2$$

Renormalize $H \to H/\sqrt{Z_H}$:

Renormalized Low Energy Effective Lagrangian:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{kinetic} + \tilde{g}\overline{\psi}_L \psi_R H + h.c. \\ &+|\partial_\nu H|^2 - \widetilde{m}_H^2 H^\dagger H - \frac{\tilde{\lambda}}{2}(H^\dagger H)^2 - \xi R H^\dagger H\end{aligned}$$

(equivalent to renormalization group running)

$$\begin{aligned}\tilde{g}^2 &= g^2/Z_H = \frac{16\pi^2}{N_c \log(\Lambda^2/\mu^2)} \\ \widetilde{m}_H^2 &= m_H^2/Z_H \\ \tilde{\lambda} &= \lambda_0/Z_H^2 = \frac{32\pi^2}{N_c \log(\Lambda^2/\mu^2)} \\ \xi &= \xi_0/Z_H = 1/6\end{aligned}$$

Applied to Higgs = top--anti-top boundstate models:

$m_{top} = \tilde{g}v/\sqrt{2}$; $\quad m_h^2/m_{top}^2 = 2\tilde{\lambda}/\tilde{g}^2 = 4 \quad$ or: $\quad m_h = 2m_{top}$

Improved RG Generally predicts heavier top quark and/or heavier Higgs boson.

(2) FYI: NJL Model of Composite Dirac Fermion:

Dobrescu & CTH (2000) unpublished

$$\mathcal{L}_0 = i\overline{\psi_L}\partial\!\!\!/\,\psi_L + \partial_\mu\phi^\dagger\partial^\mu\phi - m_\phi^2\phi^\dagger\phi$$

$$\mathcal{L}_{int} = -\frac{g^2}{\Lambda^2}\overline{\psi_L}\gamma_\mu\frac{\lambda^A}{2}\psi_L\ \phi^\dagger(i\ \overset{\leftrightarrow}{\partial}{}^{\mu})\frac{\lambda^A}{2}\phi$$

color Fierz identity to leading order in $1/N$

$$\mathcal{L}_{int} = -\frac{ig^2}{2\Lambda^2}\left([\overline{\psi_L}\gamma_\mu\ \overset{\rightarrow}{\partial}{}^{\mu}\ \phi][\phi^\dagger\psi_L] - [\overline{\psi_L}\phi][\gamma_\mu\psi_L\ \overset{\rightarrow}{\partial}{}^{\mu}\ \phi^\dagger]\right)$$

NJL Model of Composite Dirac Fermion:

$$\mathcal{L}_{int} = -\frac{ig^2}{2\Lambda^2}\left([\overline{\psi_L}\gamma_\mu \overleftrightarrow{\partial}^\mu \phi][\phi^\dagger\psi_L] - [\overline{\psi_L}\phi][\gamma_\mu\psi_L \overleftrightarrow{\partial}^\mu \phi^\dagger]\right)$$

Factorize with Dirac auxilliary field:

$$\mathcal{L}_{int} = \frac{ig}{\Lambda\sqrt{2}}\overline{\psi_L}\gamma_\mu(\overleftrightarrow{\partial}^\mu \phi)\chi_L - \frac{g}{\sqrt{2}}\overline{\chi_R}\phi^\dagger\psi_L - \Lambda\overline{\chi_R}\chi_L + h.c.$$

RG Running:

$$\mathcal{L}_\mu = \mathcal{L}_0 + \mathcal{L}_{int} + Z_L\overline{\chi_L}i\partial\!\!\!/\,\chi_L + Z_R\overline{\chi_R}i\partial\!\!\!/\,\chi_R - \overline{\chi_L}\widetilde{M}\chi_R + h.c.$$

where:

$$Z_R = \frac{g^2N}{32\pi^2}\ln\left(\frac{\Lambda}{\mu}\right) \qquad Z_L = \frac{g^2N\eta}{32\pi^2}\left(\frac{\Lambda^2-\mu^2}{\Lambda^2}\right)$$

and:

$$\widetilde{M} = \Lambda - \frac{g^2N}{32\pi^2}\left(\frac{\Lambda^2-\mu^2}{\Lambda}\right)$$

Quick Review (3): Nonlinearly Realized Symmetries: The "bottoms-up derivation"

$$\mathcal{L}_K = \overline{\psi}_L i\partial\!\!\!/ \,\psi_L + \overline{\psi}_R i\partial\!\!\!/ \,\psi_R + \frac{1}{2}\partial_\mu \pi \partial^\mu \pi$$

The RH-chiral symmetry:
$$\begin{cases} \delta\psi_L = 0 & \\ \delta\psi_R = i\theta\psi_R & \delta\overline{\psi}_R = -i\theta\overline{\psi}_R \\ \delta\pi = f_\pi \theta & \end{cases}$$

Global transformation: $\delta\mathcal{L}_K = 0$

RH-chiral current from local transformation:

$$-\frac{\delta\mathcal{L}_K}{\delta\,\partial^\mu\theta} = \overline{\psi}_R\gamma_\mu\psi_R - f_\pi\partial_\mu\pi$$

f_π determined from experiment, $\pi \to \mu\nu$.

Nonlinearly Realized Symmetry “bottoms-up derivation” beyond the kinetic term:

Massive “nucleon” coupled to pion:

$$\mathcal{L}' = M\overline{\psi}\psi - ig\pi\overline{\psi}\gamma^5\psi = M\overline{\psi}_L\psi_R - ig\pi\overline{\psi}_L\psi_R + h.c.$$

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$$\mathcal{L}' = M\overline{\psi}\psi - ig\pi\overline{\psi}\gamma^5\psi = M\overline{\psi}_L\psi_R - ig\pi\overline{\psi}_L\psi_R + h.c.$$

Perform transformation:

$$\delta\mathcal{L}' = (i\theta M - igf_\pi\theta + g\pi f_\pi\theta)\,\overline{\psi}_L\psi_R + h.c.$$

$$\delta\mathcal{L}' = 0 \qquad \longrightarrow \qquad g = \frac{M}{f_\pi}$$

Goldberger-Treiman

Nonlinearly Realized Symmetry "bottoms-up derivation" beyond the kinetic term:

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Perform transformation:

$$\delta\mathcal{L}' = (i\theta M - igf_\pi\theta + g\pi f_\pi\theta)\,\overline{\psi}_L\psi_R + h.c.$$

But, we still must cancel "higher order term" $\propto \pi\theta\overline{\psi}_L\psi_R$. Include an $\mathcal{O}\pi^2$ term:

$$\mathcal{L}' \to M\left(1 - \frac{i\pi}{f_\pi} + c\frac{\pi^2}{f_\pi^2}\right)\overline{\psi}_L\psi_R + h.c.$$

$$\delta\mathcal{L}' \to M\left(i\theta - i\frac{f_\pi\theta}{f_\pi} + \frac{\pi\theta}{f_\pi} + 2c\frac{\pi}{f_\pi^2}f_\pi\theta + ic\frac{\pi^2}{f_\pi^2}f_\pi\theta\right)\overline{\psi}_L\psi_R + h.c.$$

$$\delta\mathcal{L}' = 0 \qquad \longrightarrow \qquad g = \frac{M}{f_\pi}, \qquad c = -\frac{1}{2}$$

Nonlinearly Realized Symmetry "bottoms-up derivation" beyond the kinetic term:

But, must now cancel higher order term $\propto \pi^2\theta\overline{\psi}_L\psi_R$:

$$\mathcal{L}' \rightarrow M\left(1 - \frac{i\pi}{f_\pi} - \frac{\pi^2}{2f_\pi^2} + ic'\frac{\pi^3}{f_\pi^3}\right)\overline{\psi}_L\psi_R + h.c. + ...$$

We find, iteratively, the solution:

$$\mathcal{L}' = M\overline{\psi}_L U\psi_R + h.c. \qquad\qquad U = \exp(i\pi/f_\pi)$$

$$\mathcal{L}_K = \overline{\psi}_L i\partial\!\!\!/\, \psi_L + \overline{\psi}_R i\partial\!\!\!/\, \psi_R + \frac{f_\pi^2}{2}\partial_\mu U^\dagger \partial^\mu U$$

"nonlinear σ-model lagrangian."

(4) A popular approach to understanding Higgs: Higgs as Dilaton

Consider the pure Higgs Lagrangian (no gauge fields or couplings to fermions):

$$\mathcal{L}_0 = \partial_\mu H^\dagger \partial^\mu H - \frac{\lambda}{2}(H^\dagger H - v^2)^2$$

As usual, the groundstate has a minimum for:

$$\langle H^i \rangle = \theta^i \qquad \text{where} \qquad \theta^i = (v, 0)$$

In the present case it pulls H^i to the minimum VEV. But once we take the limit $\lambda \rightarrow 0$ the Lagrangian acquires a "shift symmetry,"

$$\delta H^i = \theta^i \epsilon \qquad \longrightarrow \qquad \delta\, \partial_\mu H^\dagger \partial^\mu H = 0$$

The Noether current is: $$J_\mu = \frac{\delta \mathcal{L}_0}{\delta \partial^\mu \epsilon} = \theta^\dagger \partial_\mu H + H^\dagger \partial_\mu \theta$$

(4) A popular approach to understanding Higgs: Higgs as Dilaton

We see that θ is a defining part of the current. Since θ rotates with H under the global $SU(2) \times U(1)$ transformations, the charge $\int d^3x J_0$ commutes with the gauge group.

$$J_\mu = \frac{\delta \mathcal{L}_0}{\delta \partial^\mu \epsilon} = \theta^\dagger \partial_\mu H + H^\dagger \partial_\mu \theta$$

In the broken phase of the theory

$$\frac{1}{2v}(\theta^\dagger H + H^\dagger \theta) = v + \frac{h}{\sqrt{2}}$$

$$J_\mu \to \sqrt{2} v \partial_\mu h$$

This becomes the "dilatonic current" of the standard model Higgs boson in the broken phase. The Higgs h in the limit $\lambda \to 0$ is now a Nambu-Goldstone boson of spontaneous scale symmetry breaking, inherited from the shift symmetry of the flat potential, *i.e.*, h has become a "dilaton."

To see this, consider the top quark mass term:

$$g\overline{\psi_L}t_R H + h.c. \;\longrightarrow\; m_t\bar{t}t\left(1+\frac{h}{\sqrt{2}v}\right)$$

Under an infinitesimal scale transformation

$$t(x) \rightarrow (1-\epsilon)^{3/2}t(x') \qquad h(x) - (1-\epsilon)h(x') \qquad x_\mu = (1+\epsilon){x'}_\mu$$

Hence the action transforms as:

$$\begin{aligned}
S_0 &= \int d^4x\, m_t\bar{t}t(x)\left(1+\frac{h(x)}{\sqrt{2}v}\right) \\
&\rightarrow \int d^4x\,(1-\epsilon)^3 m_t\bar{t}t(x')\left(1+\frac{h(x')(1-\epsilon)}{\sqrt{2}v}\right) \\
&= \int d^4x'(1+\epsilon)^4(1-\epsilon)^3 m_t\bar{t}t(x')\left(1+\frac{h(x')(1-\epsilon)}{\sqrt{2}v}\right) \\
&= \int d^4x'\left((1+\epsilon)m_t\bar{t}t(x') + m_t\bar{t}t(x')\frac{h(x')}{\sqrt{2}v}\right)
\end{aligned}$$

However, with the dilaton we can compensate the rescaled mass term by a shift

$$h(x') \to h(x') - \sqrt{2}v\epsilon$$

we see that:

$$\int d^4x' \left((1+\epsilon) m_t \bar{t}t(x') + m_t \bar{t}t(x') \frac{h(x')}{\sqrt{2}v} \right) \to \int d^4x' \left(m_t \bar{t}t(x') + m_t \bar{t}t(x') \frac{h(x')}{\sqrt{2}v} \right) = S_0$$

Hence, the top quark mass is ultaimately invariant and the scale symmetry is broken spontaneously. The same conclusion applies to the masses of all fermions, and of the gauge fields, W and Z. The Higgs self-interactions that involve nonzero λ would not be invariant under scale transformations with dilatonic shifts in h.

Remarks on Higgs Boson Interactions with Nucleons.
Mikhail A. Shifman, **A.I. Vainshtein**, **Valentin I. Zakharov** (**Moscow, ITEP**). 1978.
Phys.Lett. B78 (1978) 443

Low Energy Theorems for Higgs Boson Couplings to Photons.
Mikhail A. Shifman, **A.I. Vainshtein**, **M.B. Voloshin**, **Valentin I. Zakharov** (Moscow, ITEP). 1979
Sov.J.Nucl.Phys. 30 (1979) 711-716, Yad.Fiz. 30 (1979) 1368-1378

Our mission:

Generalize dilatational shift to a nonlinear “super”-symmetry:

Generalize dilatational shift to a nonlinear "super"-symmetry:

$$\delta\psi_L = \theta_L \epsilon + ... \text{ and } \delta t_R = \theta_R \epsilon + ...,$$

"spurions" $\theta_{L,R}$ carry the same global color, isospin, weak hypercharge and baryon number quantum numbers as the corresponding ψ_L and t_R fermion fields, ie, $\theta_L \sim (N_c = 3, I = \frac{1}{2}, Y = \frac{1}{3}, B = \frac{1}{3})$ and $\theta_R \sim (N_c = 3, I = 0, Y = \frac{4}{3}, B = \frac{1}{3})$.

Generalize dilatational shift to a nonlinear "super"-symmetry:

$$\mathcal{L}_K = \overline{\psi}_L i\partial\!\!\!/\, \psi_L + \overline{t}_R i\partial\!\!\!/\, t_R + \partial H^\dagger \partial H$$

We define the infinitesimal transformation :

$$\delta\psi_L^{ia} = \theta_L^{ia}\epsilon_0 - i\frac{\partial\!\!\!/\, H^i \theta_R^a}{\Lambda^2}\epsilon; \qquad \delta\overline{\psi}_{L\ ia} = \overline{\theta}_{L\ ia}\epsilon_0 + i\frac{\overline{\theta}_{Ra}\partial\!\!\!/\, H_i^\dagger}{\Lambda^2}\epsilon;$$

$$\delta t_R^a = \theta_R^a\epsilon_0 - i\frac{\partial\!\!\!/\, H_i^\dagger \theta_L^{ia}}{\Lambda^2}\epsilon; \qquad \delta\overline{t}_{Ra} = \overline{\theta}_{Ra}\epsilon_0 + i\frac{\overline{\theta}_{Lia}\partial\!\!\!/\, H^i}{\Lambda^2}\epsilon;$$

$$\delta H^i = \frac{\overline{\theta}_{Ra}\psi_L^{ia} + \overline{t}_{Ra}\theta_L^i}{\Lambda^2}\epsilon; \qquad \delta H_i^\dagger = \frac{\overline{\psi}_{Lai}\theta_R^a + \overline{\theta}_{Lai}t_R^a}{\Lambda^2}\epsilon.$$

$$\partial_\mu \theta_{L,R} = 0$$

Generalize dilatational shift to a nonlinear “super”-symmetry:

$$\mathcal{L}_K = \overline{\psi}_L i\partial\!\!\!/ \psi_L + \bar{t}_R i\partial\!\!\!/ t_R + \partial H^\dagger \partial H$$

$$\begin{aligned}
\delta(\overline{\psi}_L i\partial\!\!\!/ \psi_L) &= \frac{(\overline{\psi}_L \theta_R)\cdot \partial^2 H}{\Lambda^2}\epsilon + h.c. + t.d. \\
\delta(\bar{t}_R i\partial\!\!\!/ t_R) &= \frac{(\overline{\theta}_L t_R)\cdot \partial^2 H}{\Lambda^2}\epsilon + h.c. + t.d. \\
\delta(\partial H^\dagger \partial H) &= -\frac{(\overline{\psi}_L \theta_R + \overline{\theta}_L t_R)\cdot \partial^2 H}{\Lambda^2}\epsilon + h.c. + t.d.
\end{aligned}$$

$$\text{hence,} \qquad \delta\mathcal{L}_K = 0 + t.d.$$

A "super"-dilatation symmetry :

$$\delta\psi_L^{ia} = \theta_L^{ia}\epsilon_0 - i\frac{\partial\!\!\!/ H^i\theta_R^a}{\Lambda^2}\epsilon; \qquad \delta\overline{\psi}_{L\,ia} = \overline{\theta}_{L\,ia}\epsilon_0 + i\frac{\overline{\theta}_{Ra}\partial\!\!\!/ H_i^\dagger}{\Lambda^2}\epsilon;$$

$$\delta t_R^a = \theta_R^a\epsilon_0 - i\frac{\partial\!\!\!/ H_i^\dagger\theta_L^{ia}}{\Lambda^2}\epsilon; \qquad \delta\overline{t}_{Ra} = \overline{\theta}_{Ra}\epsilon_0 + i\frac{\overline{\theta}_{Lia}\partial\!\!\!/ H^i}{\Lambda^2}\epsilon;$$

$$\delta H^i = \frac{\overline{\theta}_{Ra}\psi_L^{ia} + \overline{t}_{Ra}\theta_L^i}{\Lambda^2}\epsilon; \qquad \delta H_i^\dagger = \frac{\overline{\psi}_{Lai}\theta_R^a + \overline{\theta}_{Lai}t_R^a}{\Lambda^2}\epsilon.$$

this transformation is a bona-fide invariance
of the Top-Higgs gaugeless kinetic terms:

The Noether current

$$J_\mu = \frac{\delta \mathcal{L}_0}{\delta \partial^\mu \epsilon} = \eta \theta_R^\dagger \gamma_\mu t_R + \eta \theta_L^\dagger \gamma_\mu \psi_L + \frac{1}{\Lambda^2} \theta_R^\dagger \sigma_{\mu\nu} \partial^\nu (H^\dagger \psi_L) + \frac{1}{\Lambda^2} \theta_L^\dagger \sigma_{\mu\nu} \partial^\nu (H\ t_R)$$

where $\eta = \epsilon_0/\epsilon$.

Note that the parameters, $\theta_{L,R}$, are structurally part of the current,

We emphasize that this is therefore *not a representation of the supersymmetry algebra.*

$$[\delta_{\epsilon'}, \delta_\epsilon](\psi, H, t_R) = 0$$

For example,

$$\delta_\epsilon \psi_L = \theta_L \epsilon_0 - i\frac{\partial\!\!\!/ H\theta_R}{\Lambda^2}\epsilon$$

$$\delta_{\epsilon'}\delta_\epsilon \psi_L = i\frac{\partial\!\!\!/ ((\overline{\theta}_R\psi)\theta_R + (\bar{t}_R\theta_L)\theta_R)}{\Lambda^4}\epsilon\epsilon'$$

$$[\delta_\epsilon, \delta_\epsilon']\psi_L = i\frac{\partial\!\!\!/ ((\overline{\theta}_R\psi)\theta_R + (\bar{t}_R\theta_L)\theta_R)}{\Lambda^4}[\epsilon', \epsilon] = 0$$

“super”-dilatation symmetry of the standard model beyond the kinetic terms (preliminary):

We presently turn to the full Lagrangian of the top-Higgs system in the standard model with gauge fields turned off:

$$\begin{aligned}\mathcal{L}_H &= \overline{\psi}_L i\partial\!\!\!/ \psi_L + i\bar{t}_R \partial\!\!\!/ t_R + \partial H^\dagger \partial H \\ &+ g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2\end{aligned}$$

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$$\delta(-M_H^2 H^\dagger H) = -\frac{\epsilon}{\Lambda^2} M_H^2 (\overline{\psi}_L \theta_R + \overline{\theta}_L t_R) \cdot H + h.c$$

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$$\begin{aligned}\delta(-M_H^2 H^\dagger H) &= -\frac{\epsilon}{\Lambda^2} M_H^2 (\overline{\psi}_L \theta_R + \overline{\theta}_L t_R) \cdot H + h.c \\ \delta(-\frac{\lambda}{2}(H^\dagger H)^2) &= -\frac{\epsilon}{\Lambda^2} \lambda(\overline{\psi}_L \theta_R + \overline{\theta}_L t_R) \cdot H H^\dagger H + h.c.\end{aligned}$$

A "super"-dilatation symmetry:

We presently turn to the full Lagrangian of the top-Higgs system in the standard model with gauge fields turned off:

$$\mathcal{L}_H = \overline{\psi}_L i\partial\!\!\!/ \psi_L + i\overline{t}_R \partial\!\!\!/ t_R + \partial H^\dagger \partial H + g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2$$

$$\delta(-M_H^2 H^\dagger H) = -\frac{\epsilon}{\Lambda^2} M_H^2 (\overline{\psi}_L \theta_R + \overline{\theta}_L t_R) \cdot H + h.c$$

$$\delta(-\frac{\lambda}{2}(H^\dagger H)^2) = -\frac{\epsilon}{\Lambda^2} \lambda (\overline{\psi}_L \theta_R + \overline{\theta}_L t_R) \cdot H H^\dagger H + h.c.$$

$$\begin{aligned}\delta(g\overline{\psi}_L t_R H + h.c.) = \; & g\epsilon_0(\overline{\psi}_L \theta_R + \overline{\theta}_L t_R) H + g^2 \frac{\epsilon}{2\Lambda^2} (\overline{\theta}_R \psi_L + \overline{t}_R \theta_L) \cdot (H^\dagger H^\dagger H) \\ & + g\frac{\epsilon}{\Lambda^2} \overline{\psi}_L t_R (\overline{\theta}_R \psi_L + \overline{t}_R \theta_L) \\ & + ig\frac{2\epsilon}{\Lambda^2} \overline{\psi}_L \gamma_\mu \frac{\tau^A}{2} \theta_L \left(H^\dagger \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2} H \right) + ig\frac{\epsilon}{2\Lambda^2} \overline{\psi}_L \gamma_\mu \theta_L \left(H^\dagger \overleftrightarrow{\partial^\mu} H \right) \\ & - ig\frac{\epsilon}{\Lambda^2} \overline{\theta}_R \gamma_\mu t_R \left(H^\dagger \overleftrightarrow{\partial^\mu} H \right) + h.c. + t.d.\end{aligned}$$

The last transformation emerges as follows:

1

$$\begin{aligned}
\delta(g\overline{\psi}_L t_R H + h.c.) &= g\left(\overline{\psi}_L(\theta_R\epsilon_0 - i\epsilon\frac{\not{\partial} H^\dagger\theta_L}{\Lambda^2})H + (\overline{\theta}_L\epsilon_0 + i\epsilon\frac{\overline{\theta}_R\not{\partial} H^\dagger}{\Lambda^2})t_R H\right) \\
&\quad + g\epsilon\left(\overline{\psi}_L t_R(\overline{\theta}_R\psi_L + \overline{t}_R\theta_L)\frac{1}{\Lambda^2}\right) + h.c. \\
&= g\epsilon_0(\overline{\psi}_L\theta_R + \overline{\theta}_L t_R)H + \frac{g\epsilon}{\Lambda^2}\overline{\psi}_L t_R(\overline{\theta}_R\psi_L + \overline{t}_R\theta_L) \\
&\quad + i\frac{g\epsilon}{2\Lambda^2}(\partial^\mu\overline{\psi}_L\gamma_\mu)\cdot H(H^\dagger\cdot\theta_L) - i\frac{g\epsilon}{2\Lambda^2}\overline{\theta}_R(\gamma_\mu\partial^\mu t_R)(H^\dagger\cdot H) \\
&\quad + i\frac{2g\epsilon}{\Lambda^2}\overline{\psi}_L\gamma_\mu\frac{\tau^A}{2}\theta_L\left(H^\dagger\overleftrightarrow{\partial^\mu}\frac{\tau^A}{2}H\right) + i\frac{g\epsilon}{2\Lambda^2}\overline{\psi}_L\gamma_\mu\theta_L\left(H^\dagger\overleftrightarrow{\partial^\mu}H\right) \\
&\quad - i\frac{g\epsilon}{\Lambda^2}\overline{\theta}_R\gamma_\mu t_R\left(H^\dagger\overleftrightarrow{\partial^\mu}H\right) + h.c. + t.d.
\end{aligned}$$

2 use the isospin Fierz identity, $[\tau^A]_{ij}[\tau^A]_{kl} = 2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl}$, and: $\overleftrightarrow{\partial^\mu} = \frac{1}{2}(\overrightarrow{\partial^\mu} - \overleftarrow{\partial^\mu})$.

3 apply the fermionic equations of motion

$$i\not{\partial}\, t_R + g\psi_L\cdot H^\dagger = 0 \qquad\qquad i\not{\partial}\,\psi_L + g t_R H = 0$$

Summary:

$$\mathcal{L}_H = \overline{\psi}_L i\partial\!\!\!/\, \psi_L + i\overline{t}_R \partial\!\!\!/\, t_R + \partial H^\dagger \partial H$$
$$+ g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2$$

$$\left\{\begin{aligned}
\delta(-M_H^2 H^\dagger H) &= -\frac{\epsilon}{\Lambda^2}\, M_H^2(\overline{\psi}_L \theta_R + \overline{\theta}_L t_R)\cdot H + h.c \\
\delta(-\frac{\lambda}{2}(H^\dagger H)^2) &= -\frac{\epsilon}{\Lambda^2}\, \lambda(\overline{\psi}_L \theta_R + \overline{\theta}_L t_R)\cdot H H^\dagger H + h.c. \\
\delta(g\overline{\psi}_L t_R H + h.c.) &= g\epsilon_0(\overline{\psi}_L \theta_R + \overline{\theta}_L t_R) H + g^2 \frac{\epsilon}{2\Lambda^2}(\overline{\theta}_R \psi_L + \overline{t}_R \theta_L)\cdot (H^\dagger H^\dagger H) \\
&\quad + g\frac{\epsilon}{\Lambda^2}\overline{\psi}_L t_R (\overline{\theta}_R \psi_L + \overline{t}_R \theta_L) \\
&\quad + ig\frac{2\epsilon}{\Lambda^2}\overline{\psi}_L \gamma_\mu \frac{\tau^A}{2}\theta_L \left(H^\dagger \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2} H\right) + ig\frac{\epsilon}{2\Lambda^2}\overline{\psi}_L \gamma_\mu \theta_L \left(H^\dagger \overleftrightarrow{\partial^\mu} H\right) \\
&\quad - ig\frac{\epsilon}{\Lambda^2}\overline{\theta}_R \gamma_\mu t_R \left(H^\dagger \overleftrightarrow{\partial^\mu} H\right) + h.c. + t.d.
\end{aligned}\right.$$

arrange cancellations amongst the d=4 terms

Note D=6 ops: we have generated higher dimension operator terms of the form:

$$\frac{g\epsilon}{\Lambda^2}\overline{\psi}_L t_R(\overline{\theta}_R\psi_L + \overline{t}_R\theta_L) + i\frac{2g\epsilon}{\Lambda^2}\overline{\psi}_L\gamma_\mu\frac{\tau^A}{2}\theta_L(H^\dagger \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2}H)$$
$$+i\frac{g\epsilon}{2\Lambda^2}\overline{\psi}_L\gamma_\mu\theta_L(H^\dagger \overleftrightarrow{\partial^\mu} H) - i\frac{g\epsilon}{\Lambda^2}\overline{\theta}_R\gamma_\mu t_R(H^\dagger \overleftrightarrow{\partial^\mu} H) + h.c. + t.d.$$

These terms can be cancelled by adding higher dimension operators to the Lagrangian of the form:

$$\mathcal{L}_{d=6} = \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R\overline{t}_R\psi_L) + \frac{2\kappa}{\Lambda^2}(\overline{\psi}_L\gamma_\mu\frac{\tau^A}{2}\psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2}H)$$
$$+\frac{\kappa}{2\Lambda^2}(\overline{\psi}_L\gamma_\mu\psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} H) - \frac{\kappa}{\Lambda^2}(\overline{t}_R\gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H)$$

Define the full Lagrangian with D=6 ops

$$
\begin{aligned}
\mathcal{L}_H \; &= \overline{\psi}_L i\partial\!\!\!/\, \psi_L + i\overline{t}_R \partial\!\!\!/\, t_R + \partial H^\dagger \partial H \\
&+ g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\
&+ \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \overline{t}_R \psi_L) + \frac{2\kappa}{\Lambda^2}(\overline{\psi}_L \gamma_\mu \frac{\tau^A}{2} \psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2} H) \\
&+ \frac{\kappa}{2\Lambda^2}(\overline{\psi}_L \gamma_\mu \psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} H) - \frac{\kappa}{\Lambda^2}(\overline{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H)
\end{aligned}
$$

require:

$$
\delta \mathcal{L}_H = \mathcal{O}\left(\frac{1}{\Lambda^4}\right)
$$

Full Lagrangian

$$\begin{aligned}\mathcal{L}_H &= \overline{\psi}_L i\partial\!\!\!/\, \psi_L + i\bar{t}_R \partial\!\!\!/\, t_R + \partial H^\dagger \partial H \\ &+ g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\ &+ \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \bar{t}_R \psi_L) + \frac{2\kappa}{\Lambda^2}(\overline{\psi}_L \gamma_\mu \frac{\tau^A}{2}\psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2} H) \\ &+ \frac{\kappa}{2\Lambda^2}(\overline{\psi}_L \gamma_\mu \psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} H) - \frac{\kappa}{\Lambda^2}(\bar{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H)\end{aligned}$$

$$g\epsilon_0 = \frac{M_H^2}{\Lambda^2}\epsilon \qquad g\epsilon_0(\overline{\psi}_L \theta_R + \overline{\theta}_L t_R)H \qquad -\frac{\epsilon}{\Lambda^2} M_H^2 (\overline{\psi}_L \theta_R + \overline{\theta}_L t_R) \cdot H$$

This is just a calibration of ϵ_0 and ϵ

Full Lagrangian

$$\begin{aligned}\mathcal{L}_H &= \overline{\psi}_L i\partial\!\!\!/ \psi_L + i\bar{t}_R \partial\!\!\!/ t_R + \partial H^\dagger \partial H \\ &\quad + g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\ &\quad + \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \bar{t}_R \psi_L) + \frac{2\kappa}{\Lambda^2}(\overline{\psi}_L \gamma_\mu \frac{\tau^A}{2}\psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2} H) \\ &\quad + \frac{\kappa}{2\Lambda^2}(\overline{\psi}_L \gamma_\mu \psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} H) - \frac{\kappa}{\Lambda^2}(\bar{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H)\end{aligned}$$

$$g\epsilon_0 = \frac{M_H^2}{\Lambda^2}\epsilon$$

$$\epsilon_0 \kappa = -g\epsilon,$$

Full Lagrangian

$$\begin{aligned}\mathcal{L}_H &= \overline{\psi}_L i\not{\partial}\,\psi_L + i\bar{t}_R\not{\partial}\,t_R + \partial H^\dagger \partial H \\ &+ g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\ &+ \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \bar{t}_R \psi_L) + \frac{2\kappa}{\Lambda^2}(\overline{\psi}_L \gamma_\mu \frac{\tau^A}{2}\psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2} H) \\ &+ \frac{\kappa}{2\Lambda^2}(\overline{\psi}_L \gamma_\mu \psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} H) - \frac{\kappa}{\Lambda^2}(\bar{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H)\end{aligned}$$

$$g\epsilon_0 = \frac{M_H^2}{\Lambda^2}\epsilon$$

$\epsilon_0 \kappa = -g\epsilon,$ or,

$$\boxed{\frac{\kappa}{\Lambda^2} = -\frac{g^2}{M_H^2}}$$

relationship (1)

Full Lagrangian

$$\begin{aligned}\mathcal{L}_H \;=\; & \overline{\psi}_L i\partial\!\!\!/\, \psi_L + i\overline{t}_R \partial\!\!\!/\, t_R + \partial H^\dagger \partial H \\ & + g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\ & + \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \overline{t}_R \psi_L) + \frac{2\kappa}{\Lambda^2}(\overline{\psi}_L \gamma_\mu \frac{\tau^A}{2}\psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2} H) \\ & + \frac{\kappa}{2\Lambda^2}(\overline{\psi}_L \gamma_\mu \psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} H) - \frac{\kappa}{\Lambda^2}(\overline{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H)\end{aligned}$$

$$0 = (\lambda - \frac{1}{2}g^2)\frac{\epsilon}{\Lambda^2}(\overline{\psi}_L \theta_R + \overline{\theta}_L t_R) \cdot H H^\dagger H + h.c$$

$$\boxed{\lambda = \frac{1}{2}g^2}$$

relationship (2)

Full Lagrangian

$$
\begin{aligned}
\mathcal{L}_H \; = \; & \overline{\psi}_L i\partial\!\!\!/ \psi_L + i\overline{t}_R \partial\!\!\!/ t_R + \partial H^\dagger \partial H \\
& + g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\
& + \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \overline{t}_R \psi_L) + \frac{2\kappa}{\Lambda^2}(\overline{\psi}_L \gamma_\mu \frac{\tau^A}{2}\psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2} H) \\
& + \frac{\kappa}{2\Lambda^2}(\overline{\psi}_L \gamma_\mu \psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} H) - \frac{\kappa}{\Lambda^2}(\overline{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H)
\end{aligned}
$$

require:

$$\delta \mathcal{L}_H = \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

Finally, the most interesting relationship

Analogue of Goldberger-Treiman

$$0 = (\lambda - \frac{1}{2}g^2)\frac{\epsilon}{\Lambda^2}(\overline{\psi}_L \theta_R + \overline{\theta}_L t_R) \cdot H H^\dagger H + h.c$$

$$\boxed{\lambda = \frac{1}{2}g^2}$$

relationship (2)

$$\Rightarrow \quad m_H^2 \; = \; 2\lambda v_{weak}^2 \; = \; m_t^2 \text{ in the broken phase}$$

Full Lagrangian

$$\begin{aligned}\mathcal{L}_H &= \overline{\psi}_L i\partial\!\!\!/\, \psi_L + i\bar{t}_R \partial\!\!\!/\, t_R + \partial H^\dagger \partial H \\ &+ g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\ &+ \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \bar{t}_R \psi_L) + \frac{2\kappa}{\Lambda^2}(\overline{\psi}_L \gamma_\mu \frac{\tau^A}{2}\psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2} H) \\ &+ \frac{\kappa}{2\Lambda^2}(\overline{\psi}_L \gamma_\mu \psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} H) - \frac{\kappa}{\Lambda^2}(\bar{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H)\end{aligned}$$

$$\boxed{\lambda = \frac{1}{2} g^2}$$

$$\boxed{\frac{\kappa}{\Lambda^2} = -\frac{g^2}{M_H^2}}$$

$$m_H^2 = 2\lambda v_{weak}^2 = m_t^2 \text{ in the broken phase}$$

Invariant under "super" dilatation up to D=8 operators

$$\delta\mathcal{L}_H = \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

RG improvement of Higgs Mass?

We can estimate the radiative effects on the Higgs mass using the renormalization group (RG) equations for λ and g. We include the QCD effects, and integrate in the approximation of constant rh sides of the RG equations. This yields the leading log effect:

$$16\pi^2 \frac{\partial \lambda}{\partial \ln(\mu)} = 12\lambda^2 + 4N_c\lambda g^2 - 4N_c g^4 \approx (3-2N_c)g^4 \approx -3,$$
$$16\pi^2 \frac{\partial g^2}{\partial \ln(\mu)} = (2N_c+3)\, g^4 - 2(N_c^2-1)g^2 g_{QCD}^2 \approx 9g^4 - 16\times(4\pi\alpha_{QCD})g^2 \approx -13$$

In the last expressions we've substituted $\lambda = g^2/2$, $\alpha_{QCD} = 0.11$ and $g = 1$. This yields, upon imposing the boundary condition $\lambda(\Lambda) - g^2(\Lambda)/2 = 0$:

$$\lambda(v_{weak}) - \frac{1}{2}g^2(v_{weak}) \approx -\frac{3.5}{16\pi^2}\ln\left(\frac{\Lambda}{v_{weak}}\right) \approx -0.04$$

This implies $m_H = \sqrt{2\lambda}v_{weak} \approx 167$ GeV.

Naively, with $\Lambda \sim 10^7$ GeV we bring $m_H \sim 125$ GeV.

A more minimal "super"symmetry: shift only t_R

$$
\begin{cases}
\delta\psi_L^{ia} = -i\dfrac{\not\partial H^i \theta_R^a}{\Lambda^2}\epsilon; & \delta\overline{\psi}_{L\,ia} = i\dfrac{\overline{\theta}_{Ra}\not\partial H_i^\dagger}{\Lambda^2}\epsilon; \\
\delta t_R^a = \theta_R^a \epsilon_0; & \delta\overline{t}_{Ra} = \overline{\theta}_{Ra}\epsilon_0; \\
\delta H^i = \dfrac{\overline{\theta}_{Ra}\psi_L^{ia}}{\Lambda^2}\epsilon; & \delta H_i^\dagger = \dfrac{\overline{\psi}_{Lai}\theta_R^a}{\Lambda^2}\epsilon.
\end{cases}
$$

$$
\begin{aligned}
\Rightarrow \quad \mathcal{L}_H = & \ \overline{\psi}_L i\not\partial\psi_L + i\overline{t}_R\not\partial t_R + \partial H^\dagger\partial H \\
& + g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\
& + \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \overline{t}_R \psi_L) - \frac{\kappa}{\Lambda^2}(\overline{t}_R\gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H) \ + \ \mathcal{O}\left(\frac{1}{\Lambda^4}\right)
\end{aligned}
$$

$$
\Rightarrow \quad \lambda = \frac{1}{2}g^2 \qquad\qquad \frac{\kappa}{\Lambda^2} = -\frac{g^2}{M_H^2}
$$

A more minimal "super"symmetry: shift only t_R

$$\left\{ \begin{aligned} \delta\psi_L^{ia} &= -i\frac{\partial\!\!\!/ H^i \theta_R^a}{\Lambda^2}\epsilon; & \delta\overline{\psi}_{L\,ia} &= i\frac{\overline{\theta}_{Ra}\partial\!\!\!/ H_i^\dagger}{\Lambda^2}\epsilon; \\ \delta t_R^a &= \theta_R^a \epsilon_0; & \delta\overline{t}_{Ra} &= \overline{\theta}_{Ra}\epsilon_0; \\ \delta H^i &= \frac{\overline{\theta}_{Ra}\psi_L^{ia}}{\Lambda^2}\epsilon; & \delta H_i^\dagger &= \frac{\overline{\psi}_{Lai}\theta_R^a}{\Lambda^2}\epsilon. \end{aligned} \right.$$

$$\Rightarrow$$

$$\begin{aligned} \mathcal{L}_H =\; & \overline{\psi}_L i\partial\!\!\!/ \psi_L + i\overline{t}_R \partial\!\!\!/ t_R + \partial H^\dagger \partial H \\ & + g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\ & + \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \overline{t}_R \psi_L) - \frac{\kappa}{\Lambda^2}(\overline{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right) \end{aligned}$$

NJL-like fermionic NJL-like

Factorize into NJL-like "Auxilliary Fields"

$$\mathcal{L}_H = \overline{\psi}_L i\partial\!\!\!/ \psi_L + i\bar{t}_R \partial\!\!\!/ t_R + \partial H^\dagger \partial H$$
$$+g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2$$
$$+\frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \bar{t}_R \psi_L) - \frac{\kappa}{\Lambda^2}(\bar{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H)$$

$$\mathcal{L}_H = \overline{\psi}_L i\partial\!\!\!/ \psi_L + i\bar{t}_R \partial\!\!\!/ t_R + \partial H^\dagger \partial H$$
$$+g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2$$
$$+k\overline{\psi}_L t_R \Phi - \Lambda^2 \Phi^\dagger \Phi + h.c.$$
$$+k\bar{t}_R H^\dagger \chi_L + \frac{k}{\Lambda}\bar{t}_R \partial\!\!\!/ H^\dagger \chi_R - \Lambda \overline{\chi}_L \chi_R + h.c.$$

Usual NJL

$k^2 = \kappa$

fermionic NJL

Auxillary scalar Φ and Dirac fermion χ

Renormalized Factorized Lagrangian

$$\begin{aligned}
\mathcal{L}_H \;=\; & \overline{\psi}_L i\partial\!\!\!/\, \psi_L + i\bar{t}_R \partial\!\!\!/\, t_R + \partial H^\dagger \partial H \\
& + g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 \\
& + \tilde{k}\overline{\psi}_L t_R \Phi - M^2 \Phi^\dagger \Phi + h.c. + \partial \Phi^\dagger \partial \Phi \\
& + \tilde{k}\bar{t}_R H^\dagger \chi_L + \frac{\tilde{k}}{\Lambda}\bar{t}_R \partial\!\!\!/\, H^\dagger \chi_R - M\overline{\chi}_L \chi_R + h.c. + \overline{\chi}\partial\!\!\!/\, \chi
\end{aligned}$$

“predicts” Higgs boson recurrence Φ

and heavy Dirac fermion χ

Mass scale Λ

Analogues of KK-modes, composite t_R models

Conclusions

$$\mathcal{L}_H = \overline{\psi}_L i\partial\!\!\!/ \psi_L + i\bar{t}_R \partial\!\!\!/ t_R + \partial H^\dagger \partial H$$
$$+ g(\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2$$
$$+ \frac{\kappa}{\Lambda^2}(\overline{\psi}_L t_R \bar{t}_R \psi_L) + \frac{2\kappa}{\Lambda^2}(\overline{\psi}_L \gamma_\mu \frac{\tau^A}{2}\psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} \frac{\tau^A}{2} H)$$
$$+ \frac{\kappa}{2\Lambda^2}(\overline{\psi}_L \gamma_\mu \psi_L)(H^\dagger i \overleftrightarrow{\partial^\mu} H) - \frac{\kappa}{\Lambda^2}(\bar{t}_R \gamma_\mu t_R)(H^\dagger i \overleftrightarrow{\partial^\mu} H) \quad + \; 1/\Lambda^4$$

where: $\lambda = \frac{1}{2} g^2 \quad$ and $\quad g^2 \Lambda^2 = -\kappa M_H^2.$

Is invariant under:

$$\delta \psi_L^{ia} = \theta_L^{ia} \epsilon_0 - i \frac{\partial\!\!\!/ H^i \theta_R^a}{\Lambda^2}\epsilon; \qquad \delta \overline{\psi}_{L\,ia} = \overline{\theta}_{L\,ia}\epsilon_0 + i \frac{\overline{\theta}_{Ra} \partial\!\!\!/ H_i^\dagger}{\Lambda^2}\epsilon;$$
$$\delta t_R^a = \theta_R^a \epsilon_0 - i \frac{\partial\!\!\!/ H_i^\dagger \theta_L^{ia}}{\Lambda^2}\epsilon; \qquad \delta \bar{t}_{Ra} = \overline{\theta}_{Ra}\epsilon_0 + i \frac{\overline{\theta}_{Lia} \partial\!\!\!/ H^i}{\Lambda^2}\epsilon;$$
$$\delta H^i = \frac{\overline{\theta}_{Ra}\psi_L^{ia} + \bar{t}_{Ra}\theta_L^i}{\Lambda^2}\epsilon; \qquad \delta H_i^\dagger = \frac{\overline{\psi}_{Lai}\theta_R^a + \overline{\theta}_{Lai} t_R^a}{\Lambda^2}\epsilon.$$

What is origin/meaning of the D=6 operators?
UV completion?

Possible new boundstates:
recurrences
tower of resonances

Possible new gauge bosons:
Weak coupling
Strong coupling

Possible VanderWaal interactions?

Suggests a possible dynamical origin of:

$$m_{Higgs}^2 \approx \frac{1}{2} m_{top}^2 \qquad m_{top} \approx v_{weak}$$

$$v_{weak} \approx 175 \text{ GeV}$$

Evidence of a (new kind of) supersymmetry in physics?

Suggests a new physics scale of order 1 TeV.

More work

end

Higher Dimension operator effects

$$\mathcal{L}_H = \mathcal{L}_{KT} + g(\overline{\psi}_L t_R H + h.c.)P(H^\dagger H) + M_H^2 H^\dagger H - \frac{\lambda}{2} Q(H^\dagger H)$$

$$P(H^\dagger H) = \sum_{n=0} c_n \left(\frac{H^\dagger H}{\Lambda^2}\right)^n \qquad c_0 = 1$$

$$Q(H^\dagger H) = \sum_{n=0} d_n (H^\dagger H)^2 \left(\frac{H^\dagger H}{\Lambda^2}\right)^n \qquad d_0 = 1$$

The Higgs potential minimum is now modified by Q, given by:

$$0 = -M_H^2 + \frac{\lambda}{2} \frac{\partial Q(v)}{\partial (v)^2}\bigg|_{v=v_{weak}}$$

At the potential minimum, the top quark mass is:

$$m_t = g v_{weak} P(v_{weak}^2)$$

M_H^2 is, of course, the curvature of the potential at the origin, $v \approx 0$, but the physical real Higgs boson mass is determined by the curvature at the potential minimum, $v = v_{weak}$:

$$m_H^2 = -M_H^2 + \frac{\lambda}{4}\frac{\partial^2 Q(v)}{(\partial v)^2}\bigg|_{v=v_[weak}$$

For the quartic potential we have $Q(v) = v^4$, and the usual results obtain:

Thus, to maintain the super-dilatation symmetry in the full lagrangian for all values of Higgs fields we must demand the symmetry condition on the operator towers:

perform the super-dilatation on the Yukawa and potential terms

$$0 = \frac{\epsilon}{\Lambda^2}(\overline{\psi}_L\theta_R + \overline{\theta}_L t_R)\cdot H\left(-\frac{1}{2}g^2 H^\dagger H(P(H^\dagger H))^2 + \frac{\lambda}{2}\frac{\partial Q(H^\dagger H)}{\partial(H^\dagger H)}\right) + h.c.$$

Thus, to maintain the super-dilatation symmetry in the full lagrangian for all values of Higgs fields we must demand the symmetry condition on the operator towers:

$$\frac{1}{2}g^2v^2(P(v^2))^2 = \frac{\lambda}{2}\frac{\partial Q(v^2)}{\partial v^2}$$

At the potential minimum, $v = v_{weak}$ we also have

$$m_t^2 = g^2 v_{weak}^2 (P(v_{weak}^2))^2 = \lambda \frac{\partial Q(v^2)}{\partial v^2}\bigg|_{v=v_{weak}} = 2M_H^2.$$

hence, even in the presence of the tower of operators we get the result $m_t^2 = 2M_H^2$

we emphasize that this is the *curvature at the origin of the potential*, and does not give the physical Higgs mass, which is *the curvature at the minimum.* in general, $2M_H^2 \neq m_H^2$

To estimate the size of the effect, consider:

$$\frac{\lambda}{2}Q(v^2) = \frac{\lambda}{2}\left(v^4 + \eta\kappa\frac{v^6}{\Lambda^2}\right)$$

The minimum $v = v_{weak}$ and physical Higgs mass now satisfy

$$M_H^2 = \lambda v_{weak}^2 + \eta\kappa\lambda\frac{3v_{weak}^4}{2\Lambda^2}, \qquad m_H^2 = -M_H^2 + 3\lambda v_{weak}^2 + \eta\kappa\lambda\frac{15v_{weak}^4}{2\Lambda^2}.$$

Hence:

$$m_t^2 = 2M_H^2 = 2\lambda v_{weak}^2 + 3\eta\kappa\lambda\frac{v_{weak}^4}{\Lambda^2}, \qquad m_H^2 = 2\lambda v_{weak}^2 + 6\eta\kappa\lambda\frac{v_{weak}^4}{\Lambda^2},$$

and we thus predict:

$$m_H^2 \approx m_t^2 + \frac{3}{2}\eta\kappa m_t^2\frac{v_{weak}^2}{\Lambda^2}$$

Thus, we determine η by demanding $m_H^2 \approx 0.5m_t^2$,

$$\eta \approx -\frac{1}{3}\frac{\Lambda^2}{\kappa v_{weak}^2} \approx -6.5/\kappa \approx -0.04\kappa_c/\kappa$$

“Shift”

“Super” ~~Dilatation~~ Symmetry of the Top-Higgs System

Christopher T. Hill

Fermilab