Holographic Mean-Field Theory for Baryon Many-Body Systems

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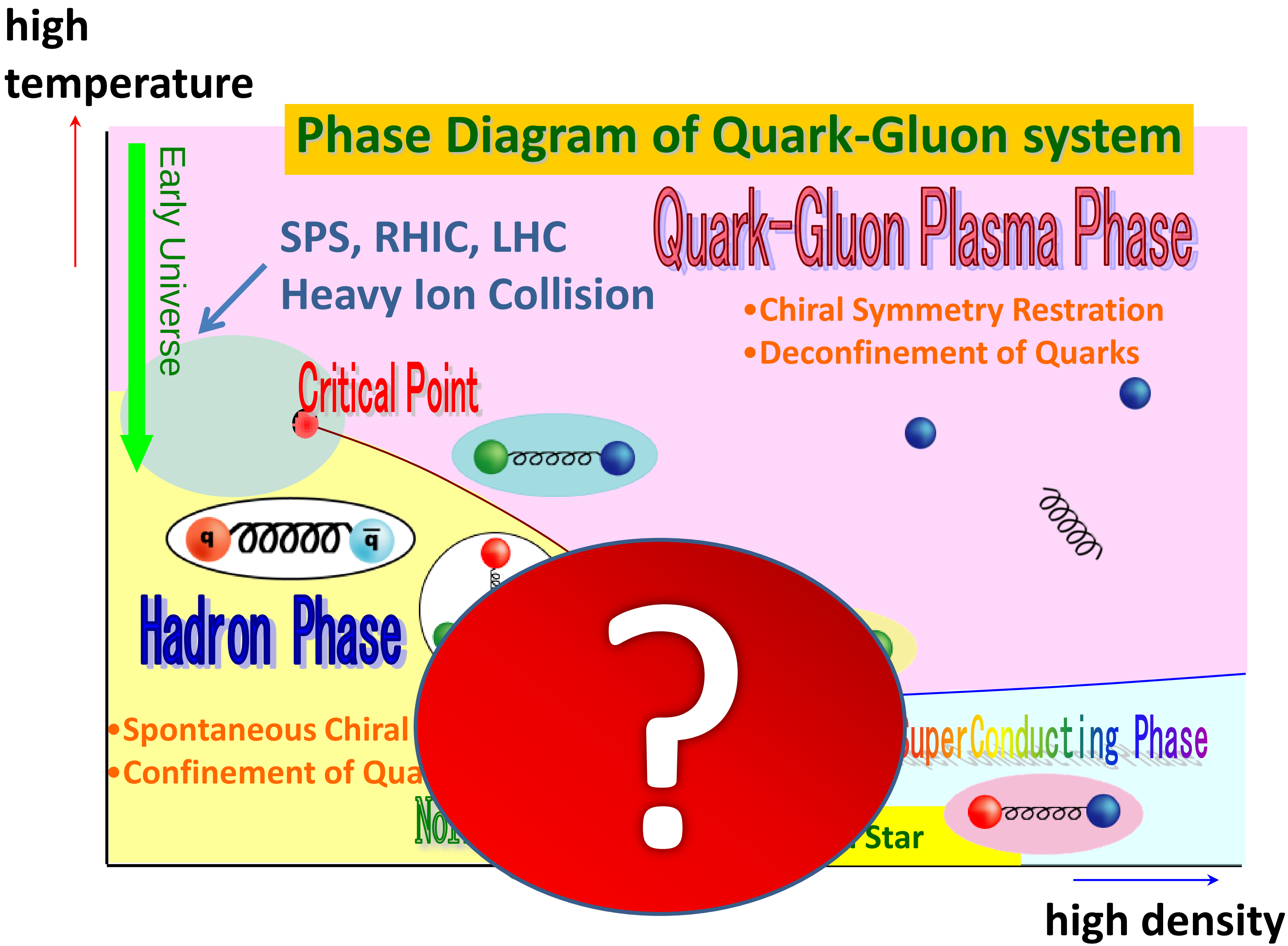
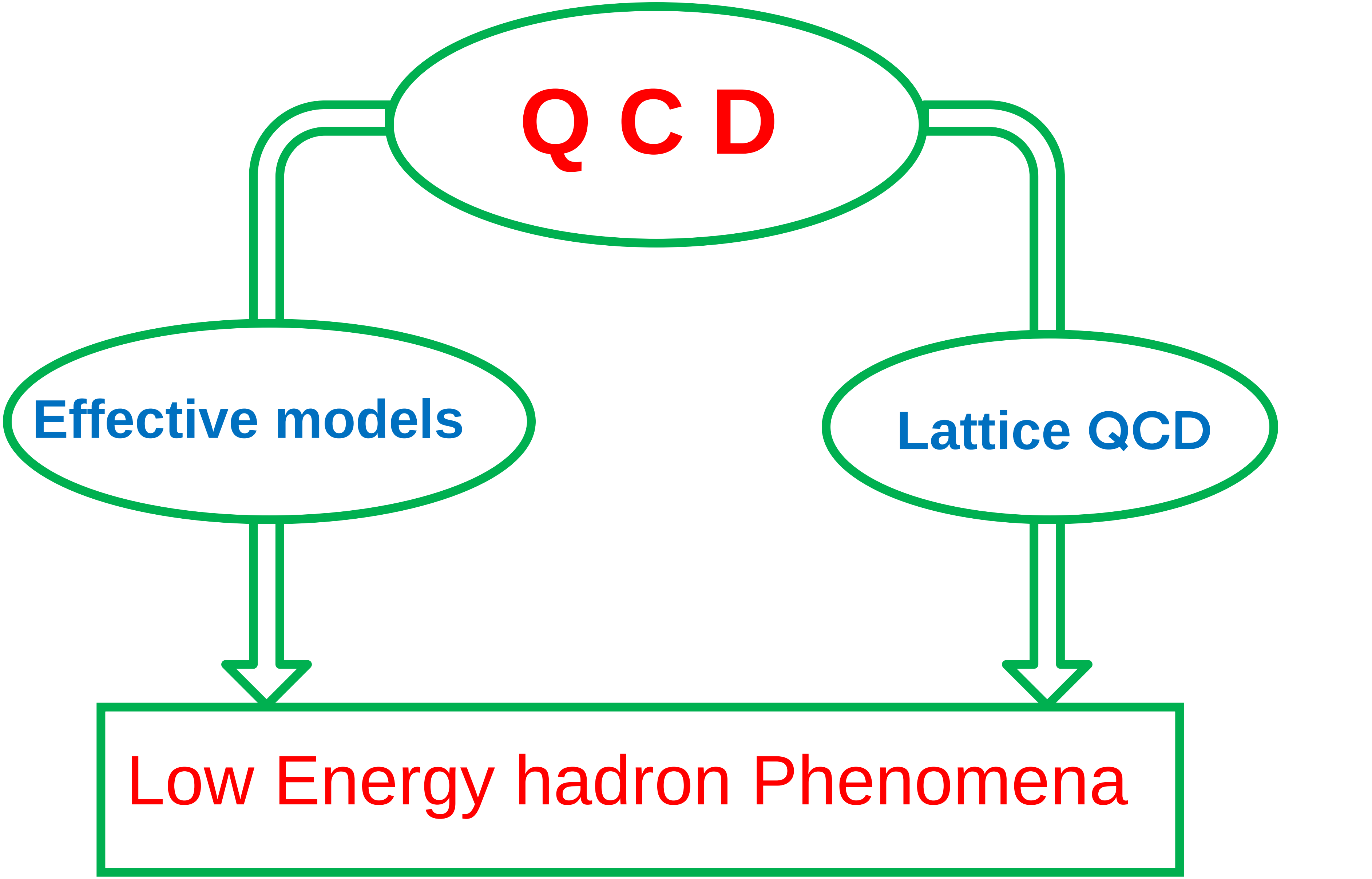
based on

MH, S. Nakamura and S. Takemoto, Phys. Rev. D 84, 036010 (2012)

1. Introduction



high
temperature
Phase Diagram of Quark-Gluon system
Early Universe
SPS, RHIC, LHC
Heavy Ion Collision
Quark-Gluon Plasma Phase
•Chiral Symmetry Restration
•Deconfinement of Quarks
Critical Point
q
q̄
Hadron Phase
•Spontaneous Chiral
•Confinement of Qua
SuperConducting Phase
Star
?
high density



QCD
Effective models
Lattice QCD
Low Energy hadron Phenomena

☆ Holographic QCD Models

Effective models of QCD

・Gauge/Gravity correspondence

4-dimension
Strong coupling gauge theory

⇔ equivalent

10-dimension
Gravity

Physical quantities with full quantum effects ⇐ Solving classical equations of motion

・Large Nc limit

QCD ⇒ weakly interacting theory of mesons
Baryons are given as solitons.

・large $\lambda = Nc\, g^2$ ('t Hooft coupling) limit

Correspondence in real-life QCD ?

☆ How do we include density effects into hQCD models ?

There are many proposals.

- K. Y. Kim, S. J. Sin and I. Zahed, arXiv:hep-th/0608046; JHEP 0801, 002 (2008);
- N. Horigome and Y. Tanii, JHEP 0701, 072 (2007);
- S. Kobayashi, D. Mateos, S. Matsuura, R. C. Myers and R. M. Thomson, JHEP 0702, 016 (2007).
- O. Bergman, G. Lifschytz and M. Lippert, JHEP 0711, 056 (2007);
- Y. Seo and S. J. Sin, JHEP 0804, 010 (2008).
- S. Nakamura, Y. Seo, S. J. Sin and K. P. Yogendran, J. Korean Phys. Soc. 52, 1734 (2008);
- S. Nakamura, Y. Seo, S. J. Sin and K. P. Yogendran, Prog. Theor. Phys. 120, 51 (2008).
- M. Rozali, H. H. Shieh, M. Van Raamsdonk and J. Wu, JHEP 0801, 053 (2008)

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In this talk, I will introduce our work in
MH, S. Nakamura and S. Takemoto, PRD84, 036010 (2012)
where we proposed "holographic mean field theory"
to study density effects in hQCD models.

Outline

1. Introduction
2. Holographic Mean field Theory
3. A Prediction
4. Summary

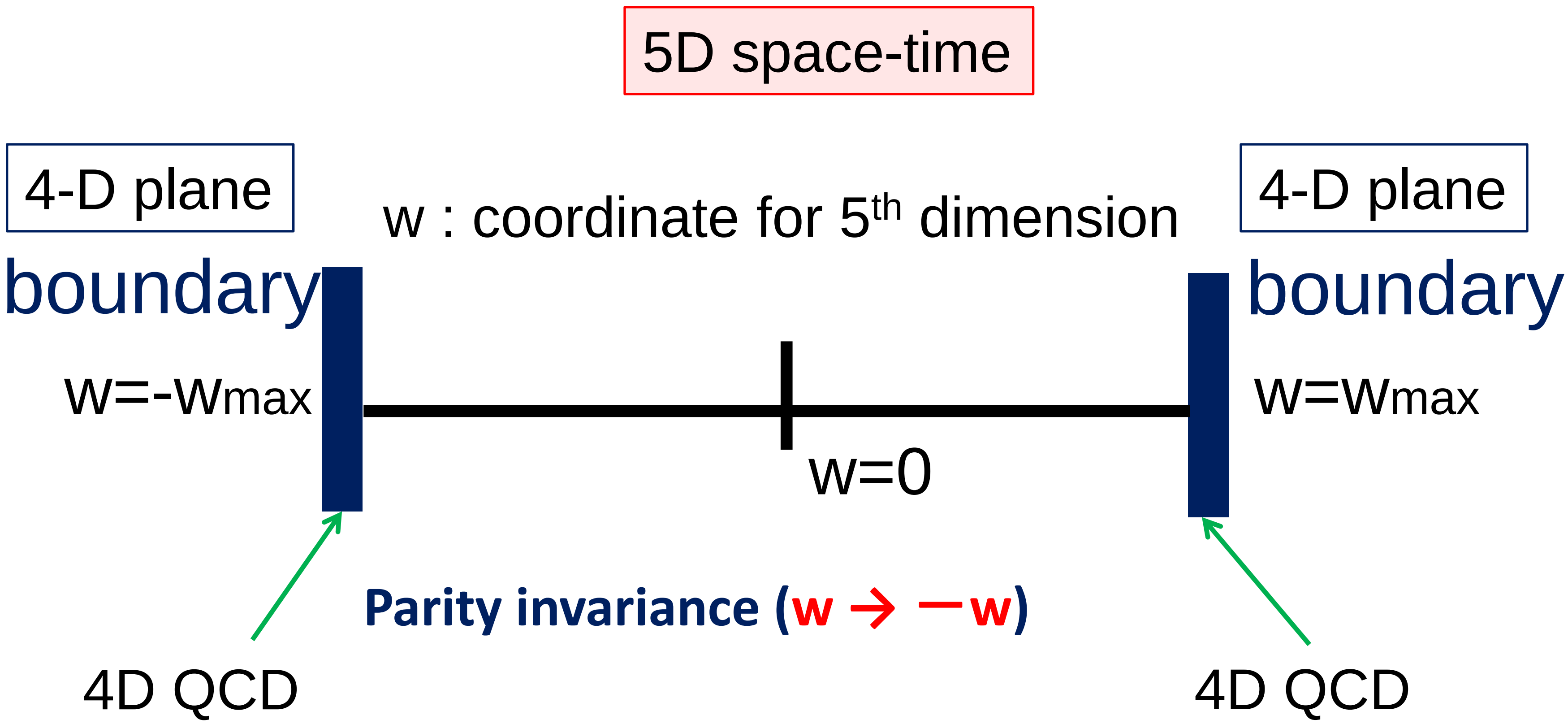
2. Holographic Mean Field Theory

- HRYY model = Sakai-Sugimoto model + Baryon

D.K.Hong, M.Rho, H.-U.Yee and P.Yi,
Phys. Rev. D **76**, 061901 (2007); JHEP **0709**, 063 (2007)

5-dimensional Baryon field $\Psi(x, w)$ ← Dirac spinor

U(1) gauge field $A_\mu(x, w)$



・HRYY model

5-dimensional Baryon field $\Psi(x,w)$

U(1) gauge field $A_\mu(x,w)$

$$S = S_\Psi + S_A$$

Baryonic action

$$S_\Psi = \int d^4x dw \left[i\bar{\Psi}\Gamma^M(\partial_M - iqA_M)\Psi - m_5(w)\bar{\Psi}\Psi \right]$$

Gauge action

$$S_A = \int d^4x dw \mathcal{L}_A$$

$\mathcal{L}_A$: DBI action

In our study we study density effects at zero temperature assuming that baryons are uniformly distributed.

Holographic Mean Field for Baryon

$$\Psi(x,w) = \boxed{\Psi(w)} + \psi(x,w)$$

mean field **Fluctuation field**

In the large Nc limit at non-zero density, the baryon becomes a classical objet, so that non-zero value of the mean field does not imply that the fermion field has a VEV.
The mean field shows the distribution of the baryon number density in 5th direction.

This exhibits the fluctuation (quasi-particle) in the matter. This is not a field corresponding to the effective nucleon which makes the matter.

Holographic Mean Field for the Gauge FIeld

$$A_\mu(x,w) = \boxed{A_\mu(w)} + a_\mu(x,w)$$

(μ = 0,1,2,3)

mean fields

fluctuation

These mean fields are generated in the 5th direction depending on the distribution of the baryonic mean field.

$A_0(w)$: time-component
$A_1(w)$, $A_2(w)$, $A_3(w)$: spatial components

Boundary Conditions for Holographic Mean Fields

Boundary conditions are determined, corresponding to the physical situation at the boundary where 4D QCD exists.

$\Psi(\pm w_{\mathrm{max}}) = 0$ $\longleftarrow$ No baryonic source is included.

$A_0(\pm w_{\mathrm{max}}) = \mu$ $\longleftarrow$ chemical potential μ at the boundary

$A_3(\pm w_{\mathrm{max}}) = 0$ $\longleftarrow$ No baryonic current at the boundary

Ansatz for Holographic Mean Fields

We assume the uniform distribution of baryons:

A) Using the rotational invariance of the system, we can take $A_1(w) = A_2(w) = 0$.

B) We take $\Psi^T = (\Psi_+, 0, \Psi_-, 0)$.

We can show that A) and B) are consistent with the Equations of motions.
In other words, A) and B) give a set of solutions for the EoM.

→ In our calculation we keep $A_0(w)$ and $A_3(w)$ only.

→ We can show the existence of 5D parity invariance.

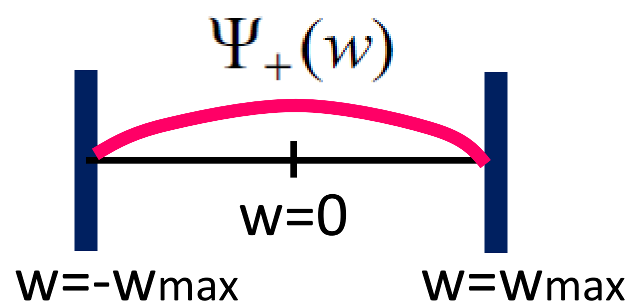
Regularity Conditions for Holographic Mean Fields

We have more conditions
(regularity conditions or IR boundary conditions),
by requiring

- **Parity invariance of the system (invariance under w → −w)**
- **Smoothness of the distribution of baryons in 5th direction**

$$\Psi'_{+}(0) = 0$$

(Even parity baryons)

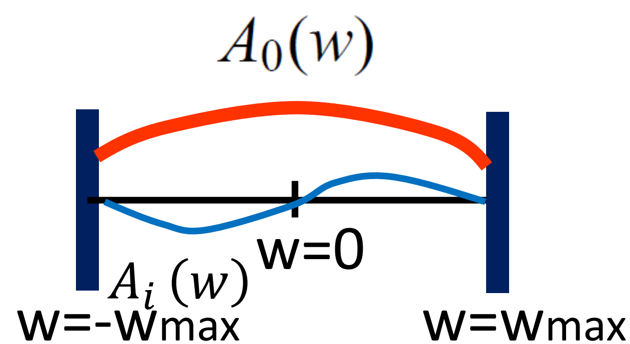


$$A'_{0}(0) = 0$$

(Even parity)

$$\boldsymbol{A_i(0) = 0}$$

(Odd parity)



☆ Equations of Motion for Holographic Mean Fields

Dirac equation — $\Psi(\pm w_{\max}) = 0 \quad \Psi'_+(0) = 0$

$$(\partial_w - qA_3(w))\Psi_+ - (m_5(w) + qA_0(w))\Psi_- = 0$$
$$(\partial_w + qA_3(w))\Psi_- - (m_5(w) - qA_0(w))\Psi_+ = 0$$

$$\Psi = \begin{pmatrix} \Psi_+ \\ 0 \\ \Psi_- \\ 0 \end{pmatrix}$$

"Maxwell equation" — $A_0(\pm w_{\max}) = \mu, \ A'_0(0) = 0$

$$\partial_w \frac{\partial \mathcal{L}_A}{\partial A'_0(w)} = \rho(w) = q(\Psi_+^\dagger \Psi_+ + \Psi_-^\dagger \Psi_-)$$
$$\partial_w \frac{\partial \mathcal{L}_A}{\partial A'_3(w)} = J_3(w) = q(\Psi_+^\dagger \Psi_- + \Psi_-^\dagger \Psi_+)$$

$A_3(\pm w_{\max}) = 0, \ A_3(0) = 0$

The chemical potential μ is determined as an eigenvalue for given value of the baryon number density

$$n = \int_{-w_{\max}}^{w_{\max}} dw\, \rho(w)$$

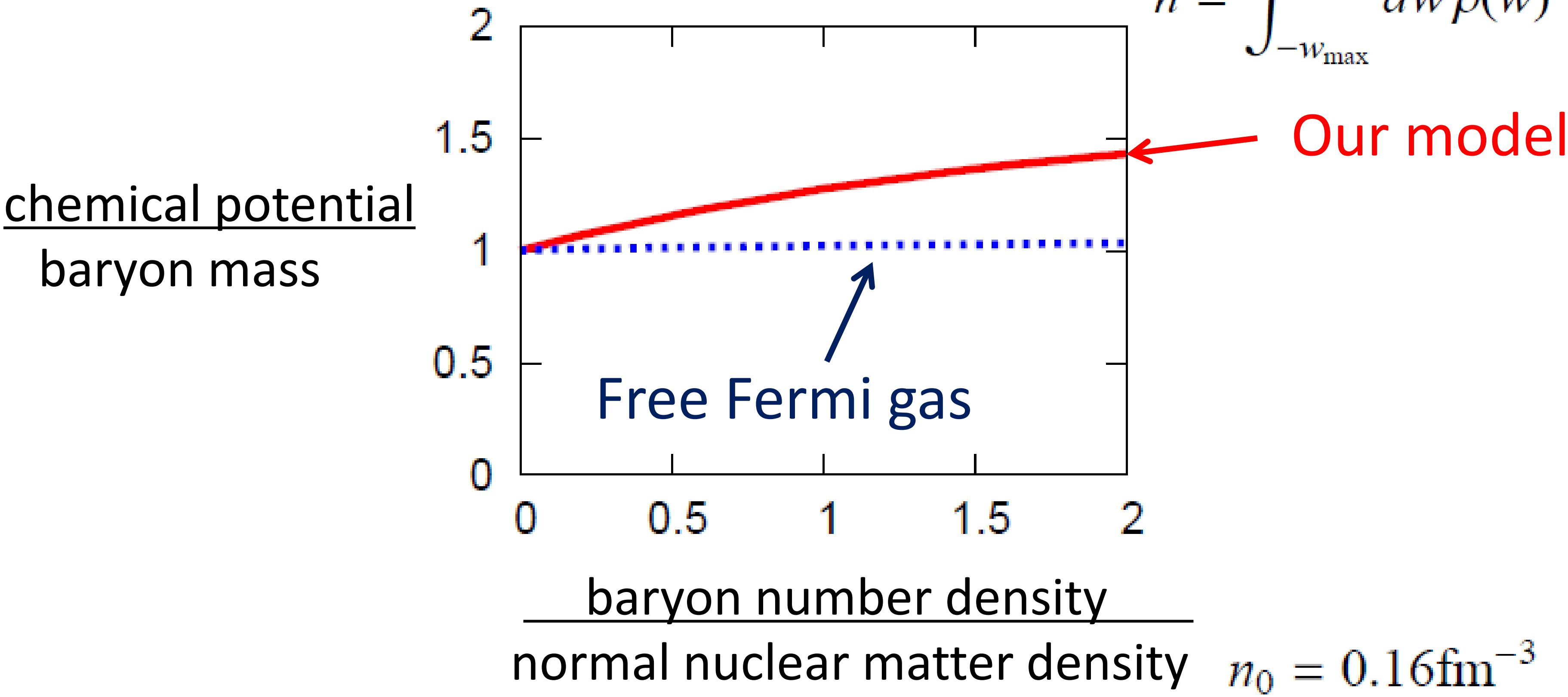
Equation of state : $\mu \Leftrightarrow n$

3. Predictions

☆ Equation of State

The chemical potential μ is determined as an eigenvalue for given value of the baryon number density

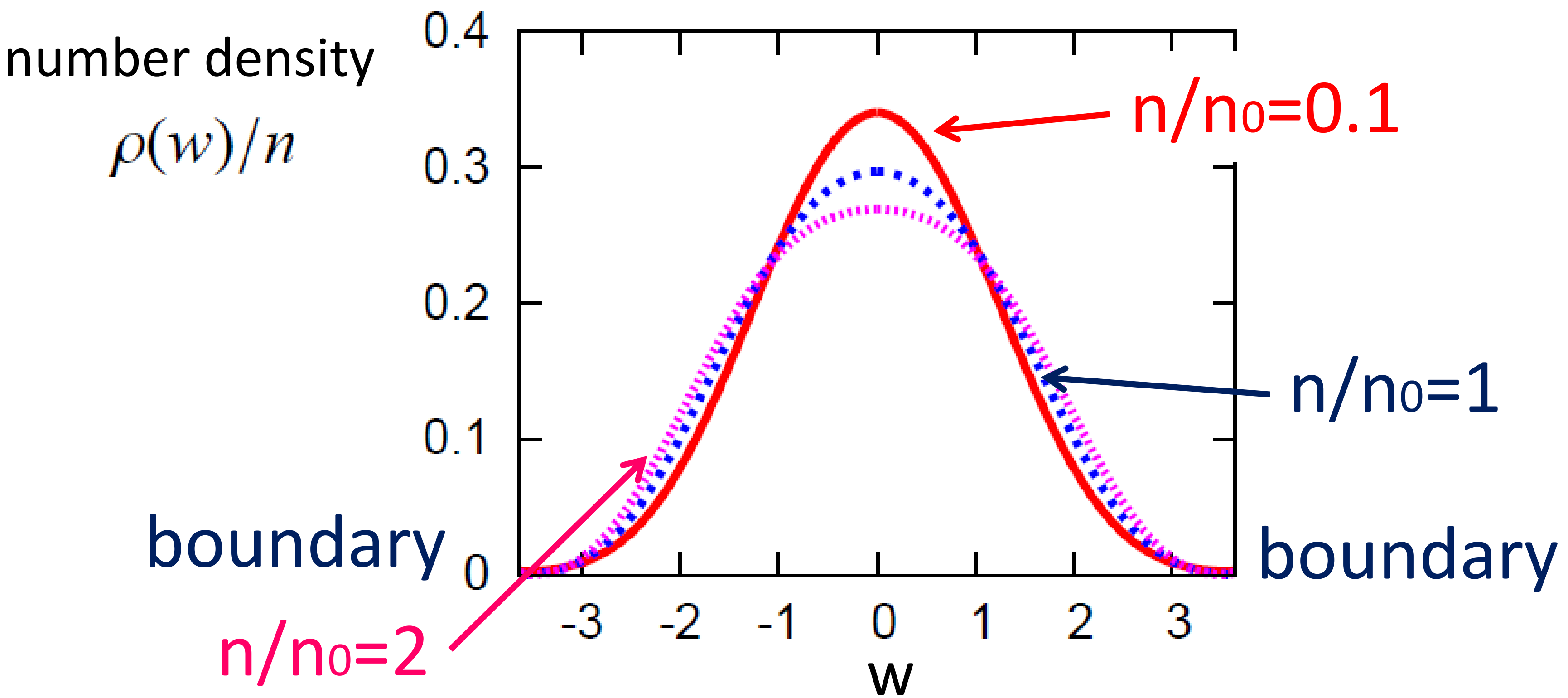
$$n = \int_{-w_{\max}}^{w_{\max}} dw\, \rho(w)$$



$n_0 = 0.16\mathrm{fm}^{-3}$

μ increases rapidly with the density, reflecting the existence of the repulsive force mediated by ω, ω', ω'', … mesons.

☆ Distribution of baryon number density



The distribution shifts to the boundary side as the density grows. This implies that taking account of the distribution of the baryon charge along the 5th direction becomes more important in the higher density region.

4. Summary

1. We proposed the "Holographic Mean Field Theory" to study baryon many-body system in a holographic QCD models.
2. We obtain the equation of state which determines the chemical potential for given baryon number density: In HRYY-SS model, the chemical potential increases monotonically with density, which seems a reflection of the repulsion.
3. We showed the distribution of the baryons in the 5^{th} direction: The resultant distribution is broader for larger density.

Discussions

There are several things to be studied.

1. Spin of the system ?

$$J^i = \varepsilon^{ijk} \int dx \; \bar{\Psi} \left[\gamma^j , \gamma^k \right] \Psi \neq 0$$

Instead of taking $\Psi^T = (\Psi_+, 0, \Psi_-, 0)$, we should take $\Psi^T = (\Psi_1, \Psi_2, \Psi_3, \Psi_4)$ and require $J^i = 0$?

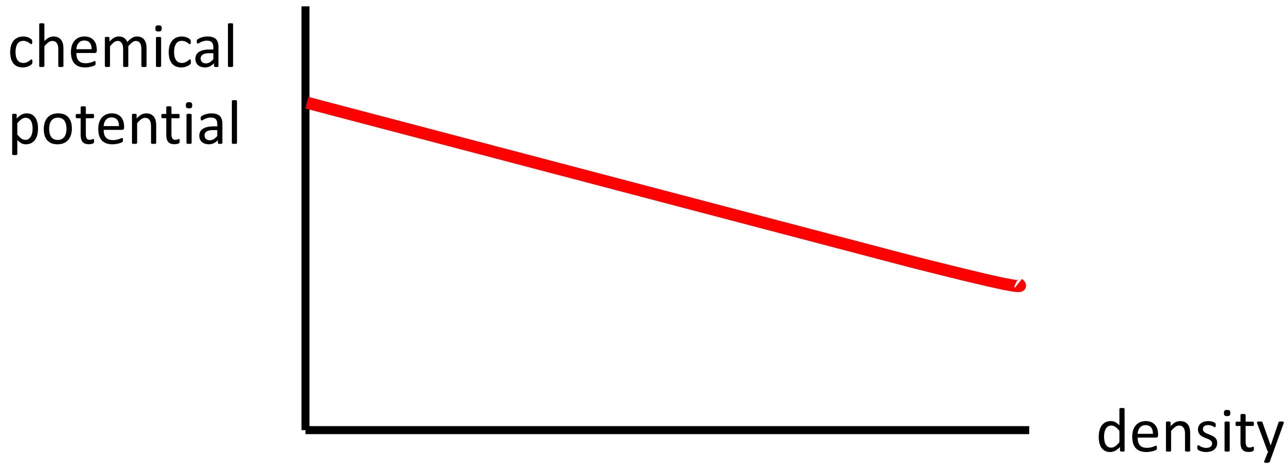
MH, H.Hoshino, S.Nakamura, work in progress.

Discussions

2. Effect of scalar attraction ?

A preliminary analysis in HIY model [D.K.Hong, T.Inami, H.-U.Yi, PLB646] (= EKSS + Baryon) shows that the chemical potential decreases against increasing density.

MH, B.-R. He, work in progress.



Discussions

3. Isospin chemical potential - isospin density relation?

This has a relevance to the symmetry energy.

4. Dispersion relation of fluctuations

$$
\begin{aligned}
&[i\Gamma^w \partial_w + \Gamma^0 (p_0 + qA_0(w)) + \Gamma^3 qA_3(w) + \vec{\Gamma}\cdot\vec{p} \\
&- m_5(w)]\psi(p_0, \vec{p}, w) + \Gamma^\nu \Psi(w) q a_\nu(p_0, \vec{p}, w) = 0
\end{aligned}
$$

coupled with the EoM for *a* and Ψ from “Maxwell equation”.

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The End