Large-Nc gauge theory and Chiral Random matrix theory

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M.H., J.-W. Lee and N. Yamada, 1212.****[hep-lat]

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SU(∞), V=∞ gauge theory
with Nf=2 adjoint fermions
(※ other repsensations are also possible)

conformal? confining?
chiral symmetry breaking?

Large-Nc equivalence
(Eguchi-Kawai equivalence)

in the 't Hooft limit

SU(∞), finite-V gauge theory
(Eguchi-Kawai model)

Study this theory
instead of V=∞

(V=2^4 in our simulation)

Earlier work:
Narayanan-Neuberger,
Hietanen-Narayanan,
Gonzalez-Arroyo-Okawa, etc

※ To establish the method, we numerically study
Nf=0 case, for which we know the answer.

Chiral Random Matrix Theory (chi-RMT)

SU(3) QCD

$\downarrow$ $L \gg 1/\Lambda_{QCD}$

Chiral Perturbation Theory

$\downarrow$ ε-regime ($L \ll 1/m_\pi$), $m_q V \Sigma$: fixed, $V \to \infty$

chi-RMT

<u>QCD and chi-RMT</u>

<u>give the same Dirac spectrum</u>

N×N complex matrix

$V \Leftrightarrow N$

$$\mathcal{D}_f = \begin{pmatrix} m_f \mathbf{1} & \Phi \\ -\Phi^\dagger & m_f \mathbf{1} \end{pmatrix}$$

$$Z = \int d\Phi \left(\prod_{f=1}^{N_f} \det \mathcal{D}_f \right) e^{-N \mathrm{tr} \Phi^\dagger \Phi}$$

$m_q N$: fixed, $N \to \infty$

Chiral Random Matrix Theory (chi-RMT)

QCD-like theory (YM + fermion)

if the chiral sym. breaking is broken

$L \gg 1/\Lambda_{QCD}$

Chiral Perturbation Theory

ε-regime ($L \ll 1/m_\pi$),

$m_q V \Sigma$: fixed, $V \to \infty$

chi-RMT

(3 classes depending on the chiral symmetry breaking pattern)

<u>The Dirac spectrum coincide</u>

<u>if the chiral symmetry is spontaneously broken.</u>

Large-Nc vs chi-RMT

- In QCD, thermodynamic limit is $V \to \infty$.
- In the SU(Nc) case, it is $V \to \infty$ and/or $Nc \to \infty$.

So, when we compare it with chi-RMT,

$$\boxed{\begin{array}{c} V \times (Nc)^{\alpha} \Leftrightarrow N \\ m_q V \times (Nc)^{\alpha} : \text{fixed.} \end{array}} \qquad \begin{array}{c} \Sigma \sim (Nc)^{\alpha} \\ (\alpha > 0) \end{array}$$

Let us call it as 'chi-RMT limit.'

Large-Nc vs chi-RMT

The large-Nc 't Hooft limit and chi-RMT limit are different!

't Hooft limit (planar limit) : m_q, V : fix, $Nc \to \infty$

chi-RMT limit : $m_q V \times (Nc)^{\alpha}$ fixed, $Nc \to \infty$

The Eguchi-Kawai equivalence does not hold in the chi-RMT limit!

(※ $m_q=0$ should be regarded as the chi-RMT limit.)

Large-Nc vs chi-RMT

The large-Nc 't Hooft limit and chi-RMT limit are different!

$$f(m, V, N_c) = \sum_{g=0}^{\infty} \frac{f_g(m, V)}{N_c^{2g}}$$

't Hooft counting holds when this coefficient is Nc-independent

Large-Nc vs chi-RMT

QCD (SU(3))

agreement with chi-RMT @ mV fixed, $V \to \infty$

$\uparrow$

nonzero chiral condensate @ $m \to 0$ *after* $V \to \infty$

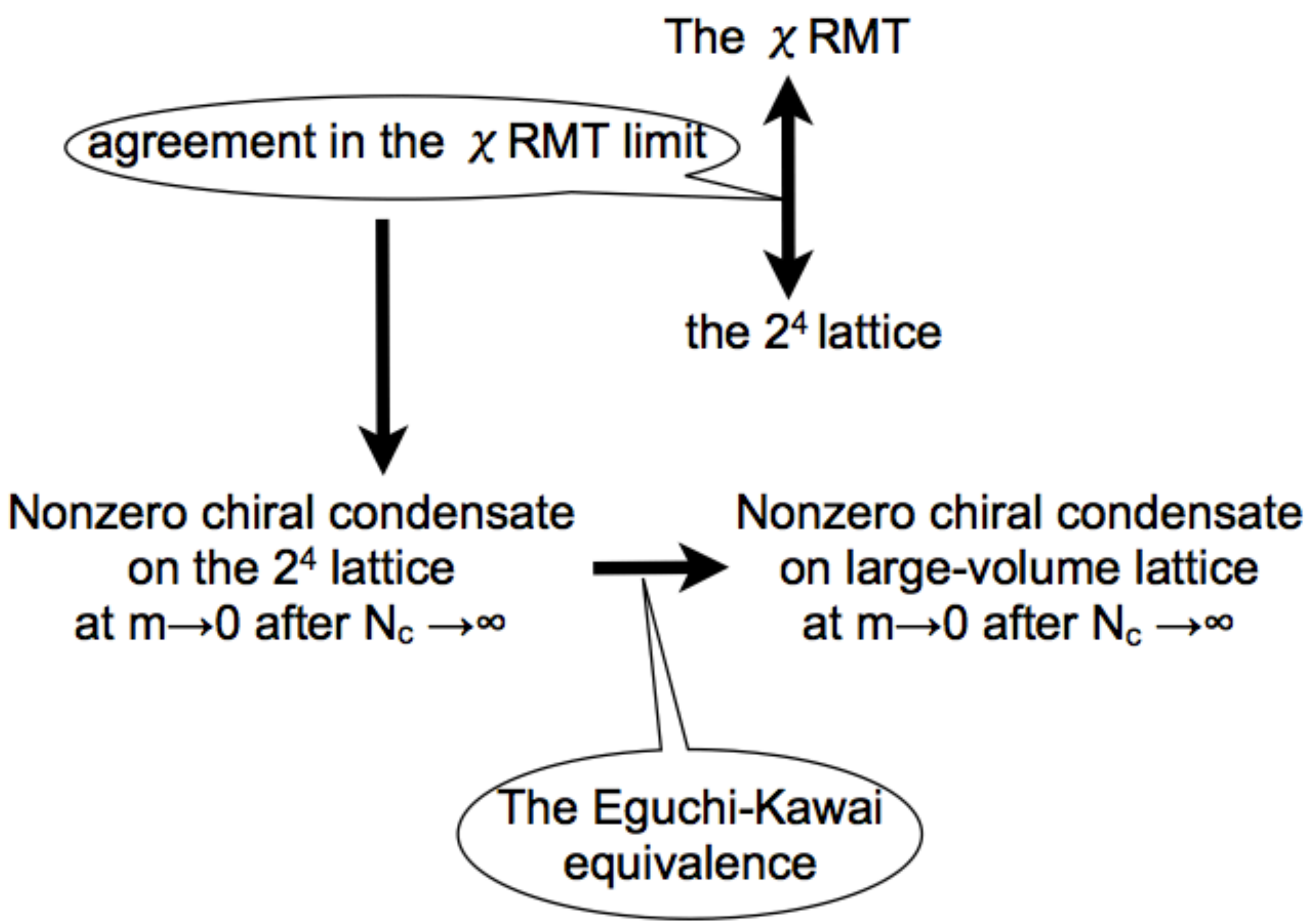
large-Nc YM

agreement with chi-RMT @ $mV \times (Nc)^{\alpha}$ fixed, $Nc \to \infty$

$\uparrow$

nonzero chiral condensate @ $m \to 0$ *after* $Nc \to \infty$

't Hooft limit



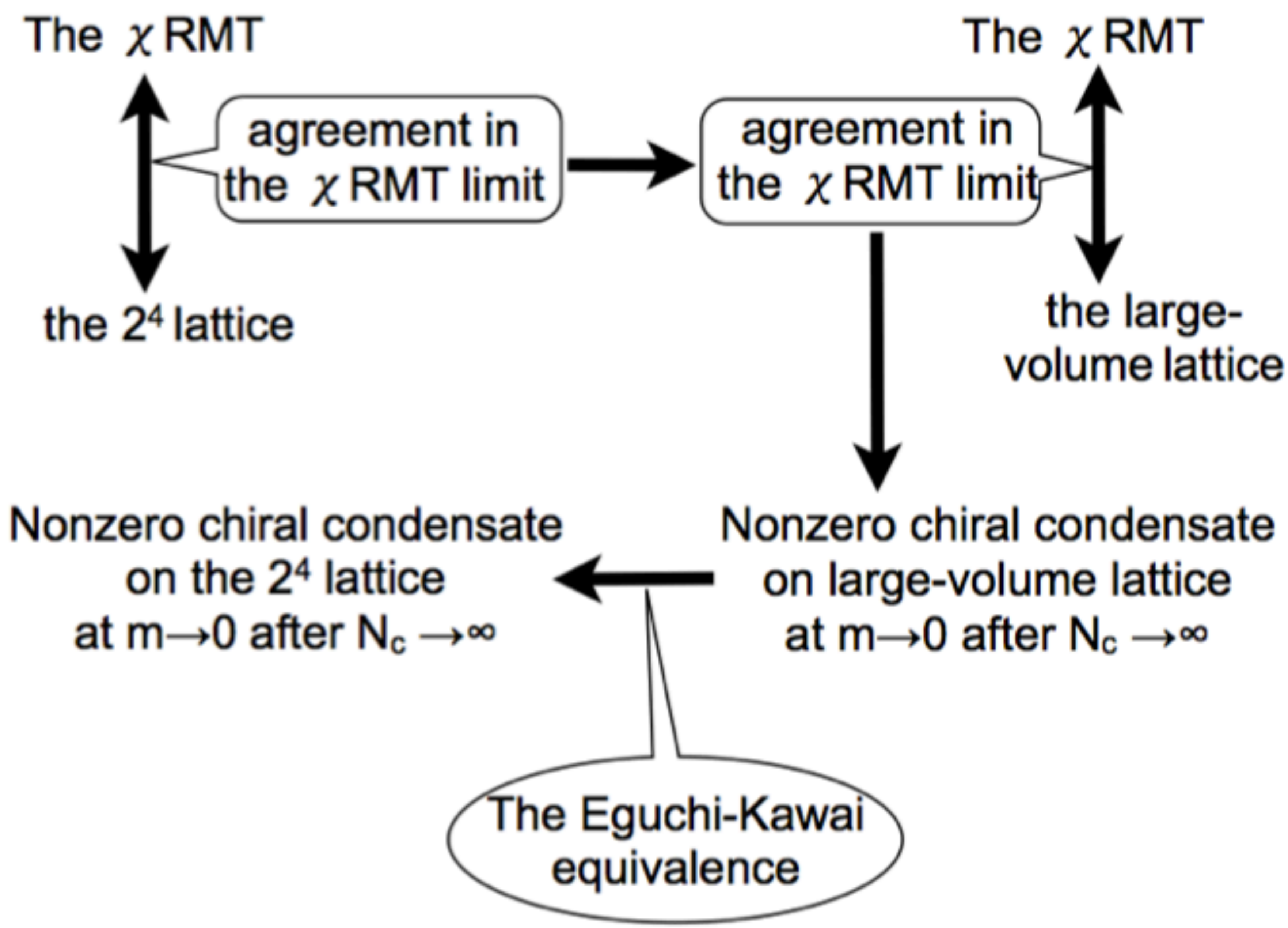
The χ RMT
agreement in the χ RMT limit
the 2^4 lattice
Nonzero chiral condensate on the 2^4 lattice at $m \to 0$ after $N_c \to \infty$
Nonzero chiral condensate on large-volume lattice at $m \to 0$ after $N_c \to \infty$
The Eguchi-Kawai equivalence

Large-Nc vs chi-RMT

This argument might be too naive for the Eguchi-Kawai model, because the chiral perturbation might not be applicable to 2^4 lattice straightforwardly.

<u>Still, however</u>:

- For sufficiently large lattice, there is no problem. There, the eigenvalue distribution depends only on $mV\times(Nc)^{\alpha}$.
- If there is no phase transition (center symmetry breaking), the same expression should hold even at small V.


The χRMT
agreement in the χRMT limit
agreement in the χRMT limit
The χRMT
the 2⁴ lattice
the large-volume lattice
Nonzero chiral condensate on the 2⁴ lattice at m→0 after Nc →∞
Nonzero chiral condensate on large-volume lattice at m→0 after Nc →∞
The Eguchi-Kawai equivalence

Numerical results (Nf=0)

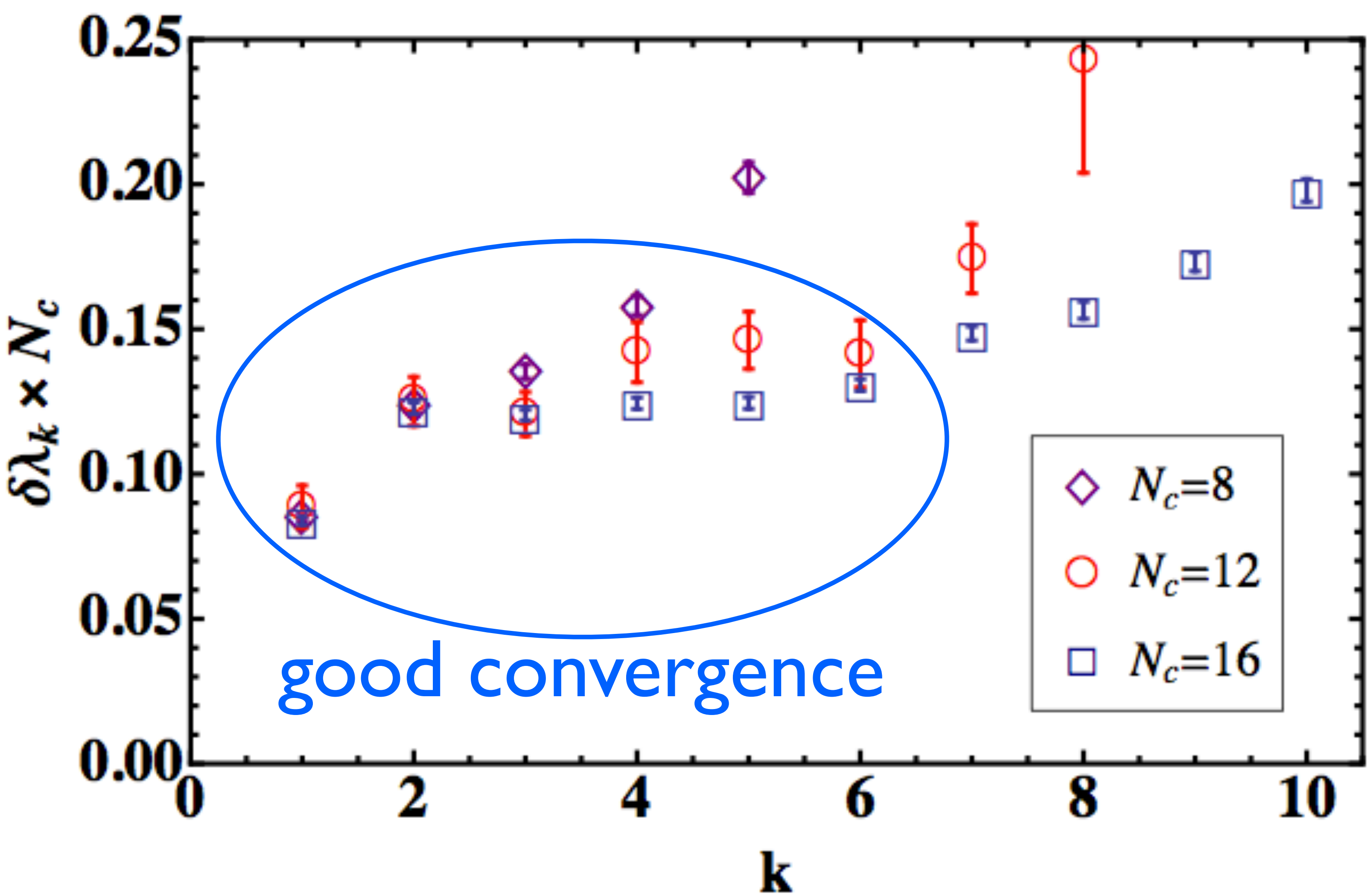
•2^4 plaquette action + heavy Dirac adjoint fermion
→ unbroken center symmetry

• Probe massless overlap fermion
in the adjoint representation

• Low-lying Dirac eigenvalues scales as 1/Nc→ α=1
(Naive expectation from the 't Hooft counting is α=2)

• Chiral symmetry must be broken.
Can we detect it by comparing the simulation data with the chi-RMT prediction?

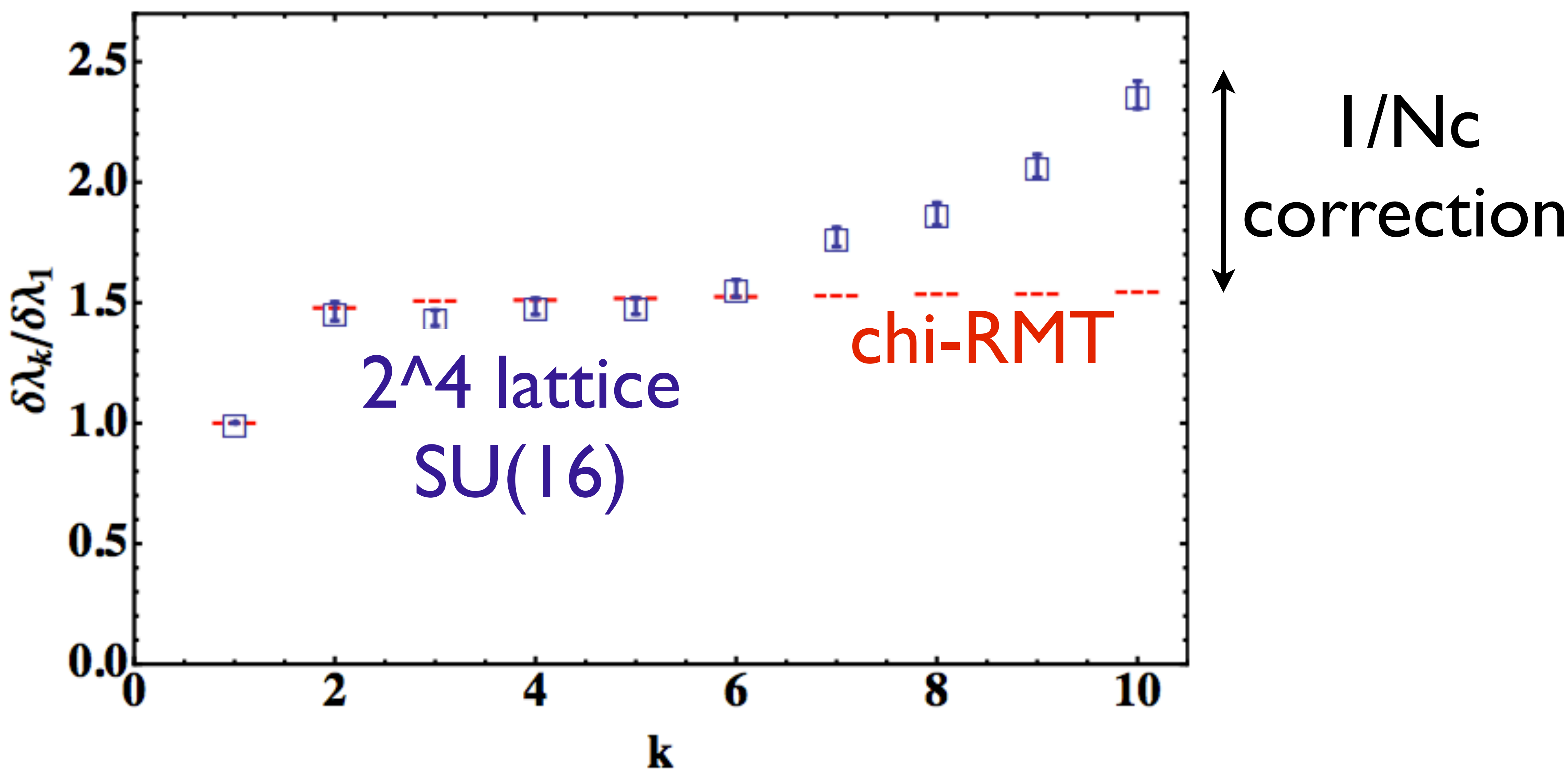
Numerical results (Nf=0)



2^4 lattice

$\delta\lambda_k$ = < Im[λ_k-λ_{k-1}] > , $\delta\lambda_1$ = <λ_1>

Numerical results (Nf=0)



perfect agreement with chi-RMT!

Conclusion & Outlook

- Chiral symmetry breaking at large-Nc can be detected by comparing small-size lattice and chi-RMT.
- 2^4, SU(8) (or SU(16)) is good enough.
- Simulaton of Nf=2 theory is ongoing.
- Be careful about the difference between the 't Hooft limit and chi-RMT limit when you use them.
- Twisted boundary condition (→ M. Okawa's talk)

Thanks!