IR fixed points from lattice simulations

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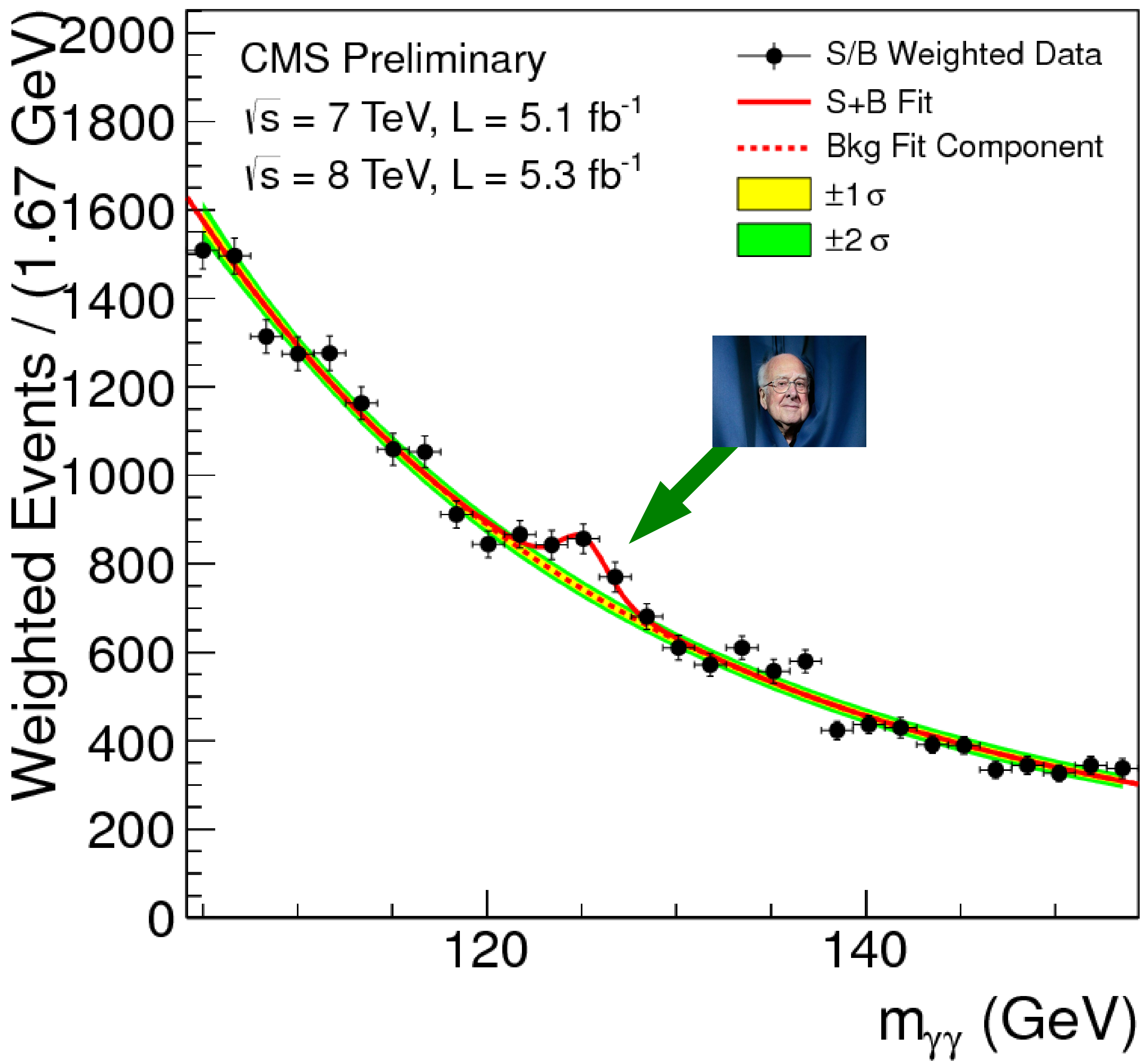
2012: a first glimpse of Higgs



Is Technicolor dead?

Motivations

"Strong EW symmetry breaking is very appealing. I believe news of its demise are premature, driven mostly by our frustration with our inability to calculate. What we do know is that it is not a scaled up version of QCD. These comments apply equally to all alternatives to TC. So the strategy for progress is to (i) search and discover, (ii) study in detail and (iii) build a model and learn about strong dynamics. In that order!"

[Grinstein 11]

- Calculate on the lattice
- What are we searching for?
- Results and perspectives

Hierarchies and scaling dimensions

Linearized RG flow in a neighbourhood of a fixed point

$$\mu \frac{d}{d\mu} g = (\Delta - D)\, g + O(g^2) \qquad\qquad g(\mu) = \left(\frac{\mu}{\Lambda_{\rm UV}}\right)^{\Delta - D} g(\Lambda_{\rm UV})$$

Associated IR scale:

$$\Lambda_{\rm IR} \sim g_0^{1/(D-\Delta)} \Lambda_{\rm UV} \qquad\qquad \begin{cases} D - \Delta = O(1) & g_0 \text{ must be tuned} \\ D - \Delta \ll 1 & \text{natural hierarchy} \end{cases}$$

Stable hierarchy related to *weakly* relevant operators. [Strassler 03, Sannino 04,Luty&Okui 04]

YM theory at the GFP is a limiting case:

$$\Lambda_{\rm IR} \sim \Lambda_{\rm UV} \exp\{-\frac{1}{\beta_0 g^2}\}$$

Global-singlet relevant operators (GSRO) require fine-tuning.

Flavor sector

In the SM:

$$\boxed{\dim(H^\dagger H) \simeq 2}$$

$$\mathcal{L}_Y = y^u \, H \bar{L} u_R + y^d \, H^\dagger \bar{L} d_R$$

dimension = 1+3 = 4

In DEWSB: scalar is composite [Dimopoulos et al 79, Eichten et al 1980]

$$\mathcal{L}_Y = \frac{y}{\Lambda_{\mathrm{UV}}^2} \bar{Q} Q \, \bar{q} q$$

dimension = 3+3 = 6

Tension with suppressing FCNC

$$\frac{f}{\Lambda_{\mathrm{UV}}^2} \bar{q} q \bar{q} q$$

dimension = 6

Walking TC

Alleviate the problem due to the **large** dimension of the composite scalar

Theory at the EW scale is **near** a non-trivial fixed point

Scaling dimension of the fermion bilinear is smaller

$$\dim(\bar{Q}Q) = 3 - \gamma$$

[Holdom,Yamawaki,Appelquist,Eichten,Lane]

Small dimension allows a better description of the flavor sector, BUT

$$\dim(H) \simeq 1 \quad \Longrightarrow \quad \dim(H^\dagger H) \simeq 2$$

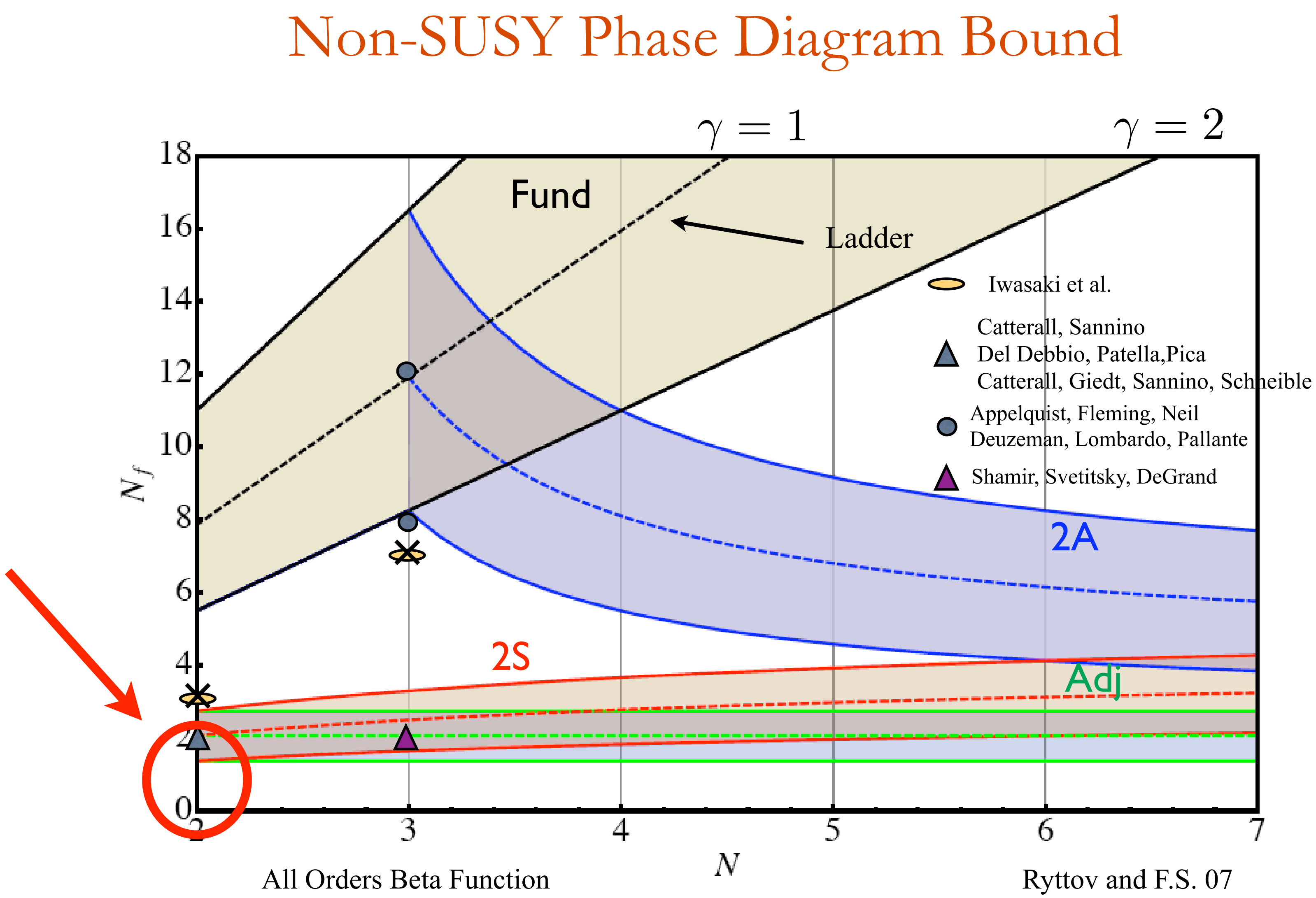
At a strongly coupled IRFP we could have:

$$\dim(H) \text{ small, } \textbf{but} \;\; \dim(H^\dagger H) > 2 \dim(H)$$

[Sannino 04, Luty 04, Rattazzi et al 08]

Phase diagram of SU(N) gauge theories

Use lattice tools to **search** for IRFPs in 4D SU(N) gauge theories



[Yamawaki, Appelquist, Miransky, Schrock, Nunez, Piai, Hong, Braun, Gies]

Lattice tools

- Scaling of the spectrum
  - dependence on the fermion mass
  - finite-size scaling
  - eigenvalue spectrum
- RG flows
  - Schroedinger functional
  - Monte Carlo Renormalization Group

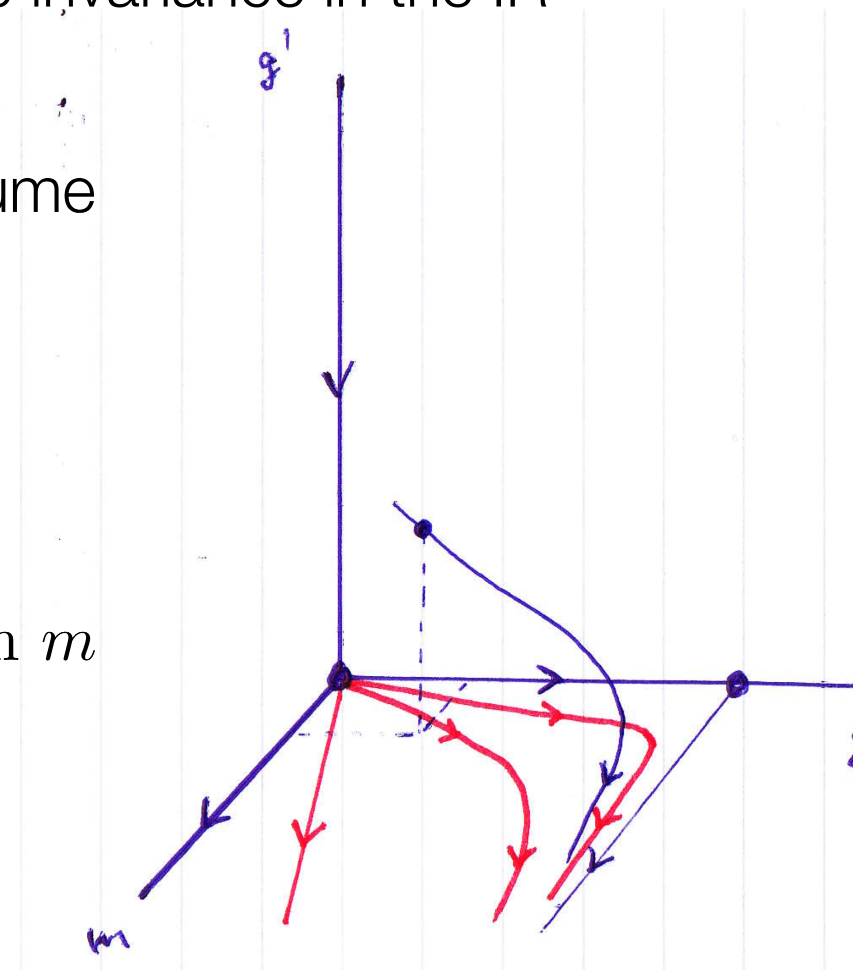
Mass-deformed CFT on the lattice

- The identification of a CFT by numerical simulations is a difficult task
- No massive spectrum; power-law behaviour of correlators at large distances
- Numerical simulations are performed at *finite fermion mass*, and/or in a *finite-volume* box; both the mass and the finite volume break scale invariance in the IR
- Consider a CGT deformed by a mass term/finite volume
- Determine the scaling of physical observables

$$\mathcal{O} \sim m^{\eta_{\mathcal{O}}} + \text{higher order in } m + \text{terms analytic in } m$$

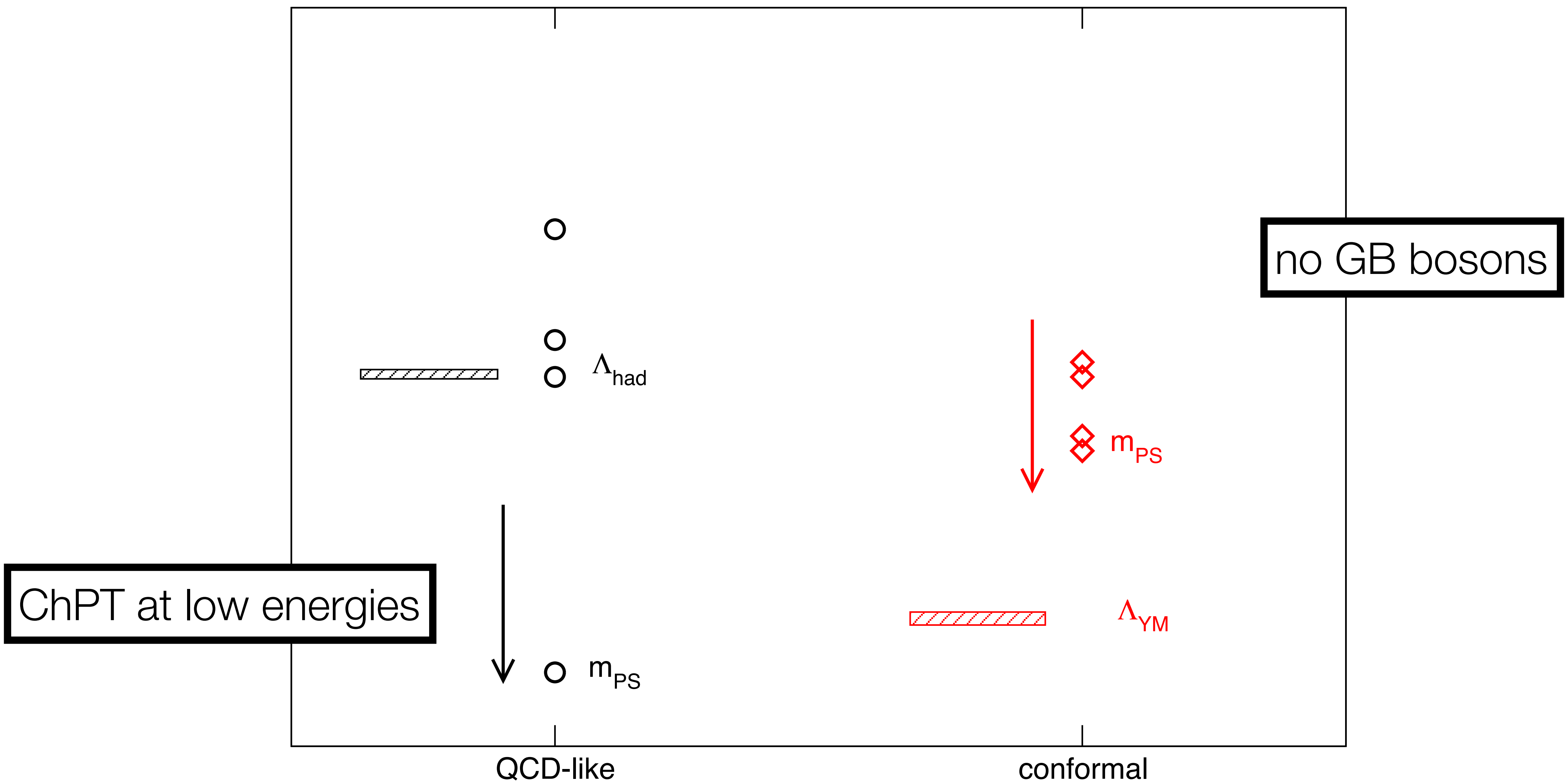
$$\boxed{M_H \propto \mu\, m^{\frac{1}{1+\gamma_*}}}$$

[LDD, Zwicky 10]



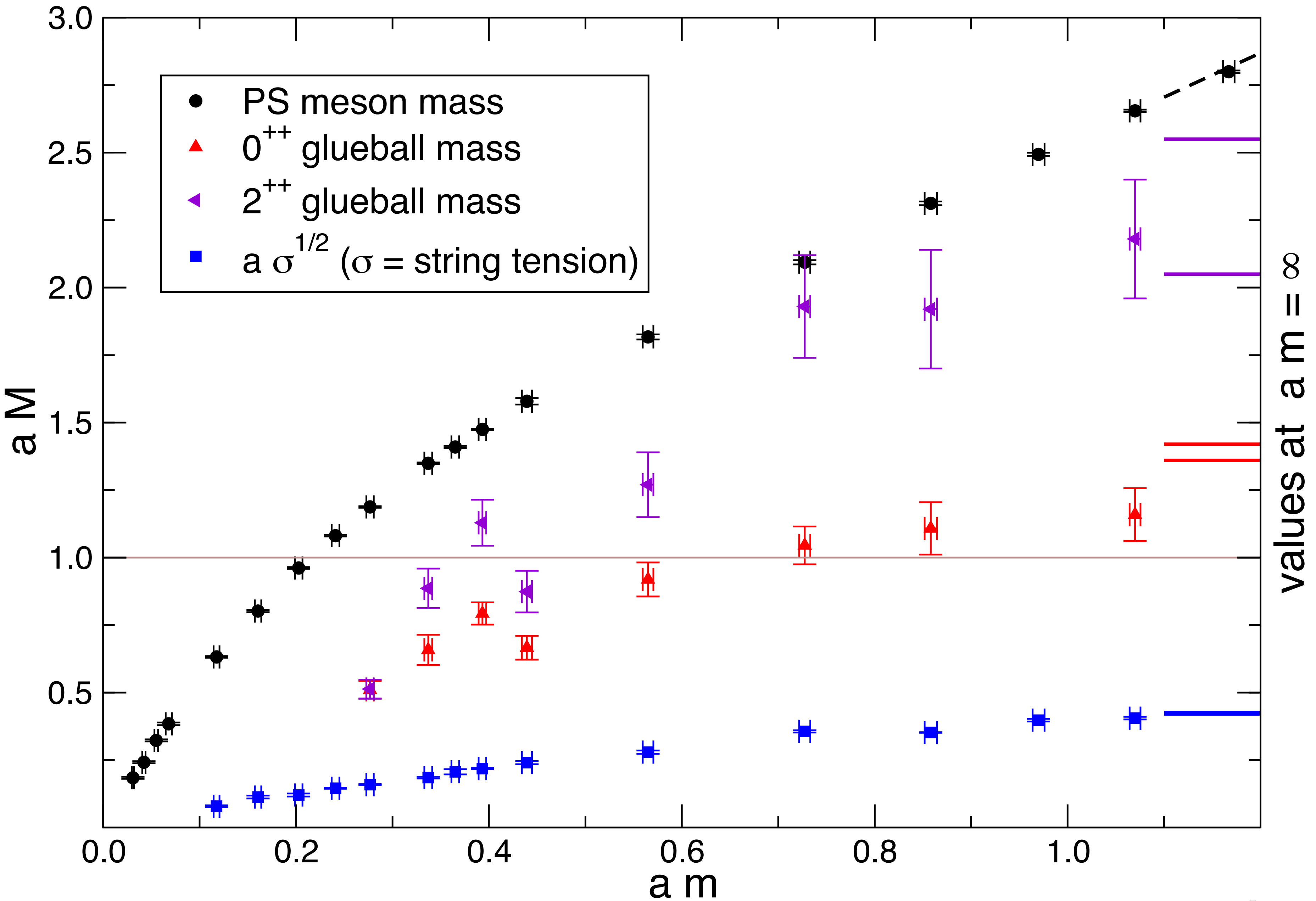
Conformal spectrum

- Different qualitative behaviours in the chiral limit



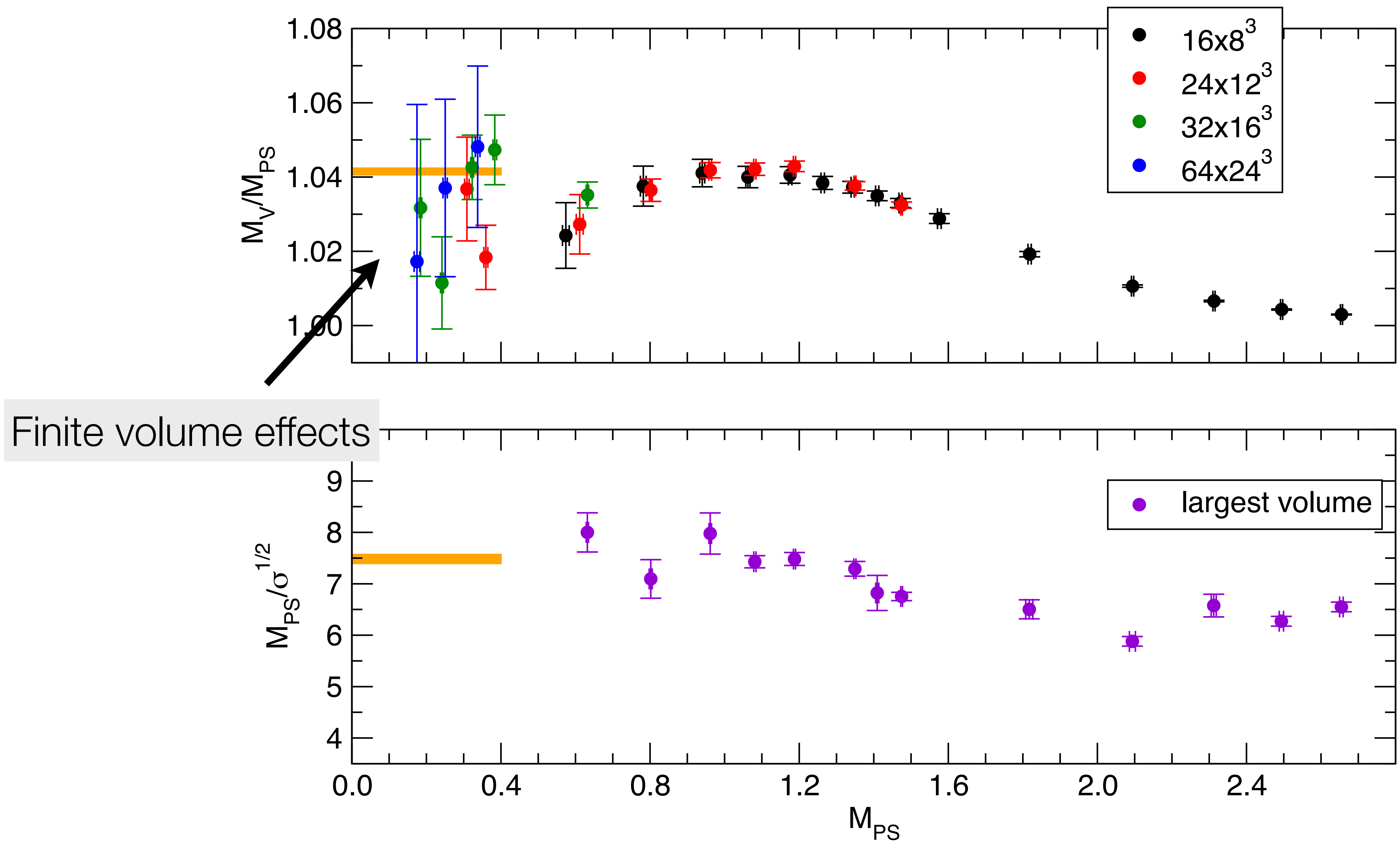
Spectrum for SU(2) + 2 adjoint fermions

- Overall picture



[LDD et al 09]

Spectrum for SU(2) + 2 adjoint fermions

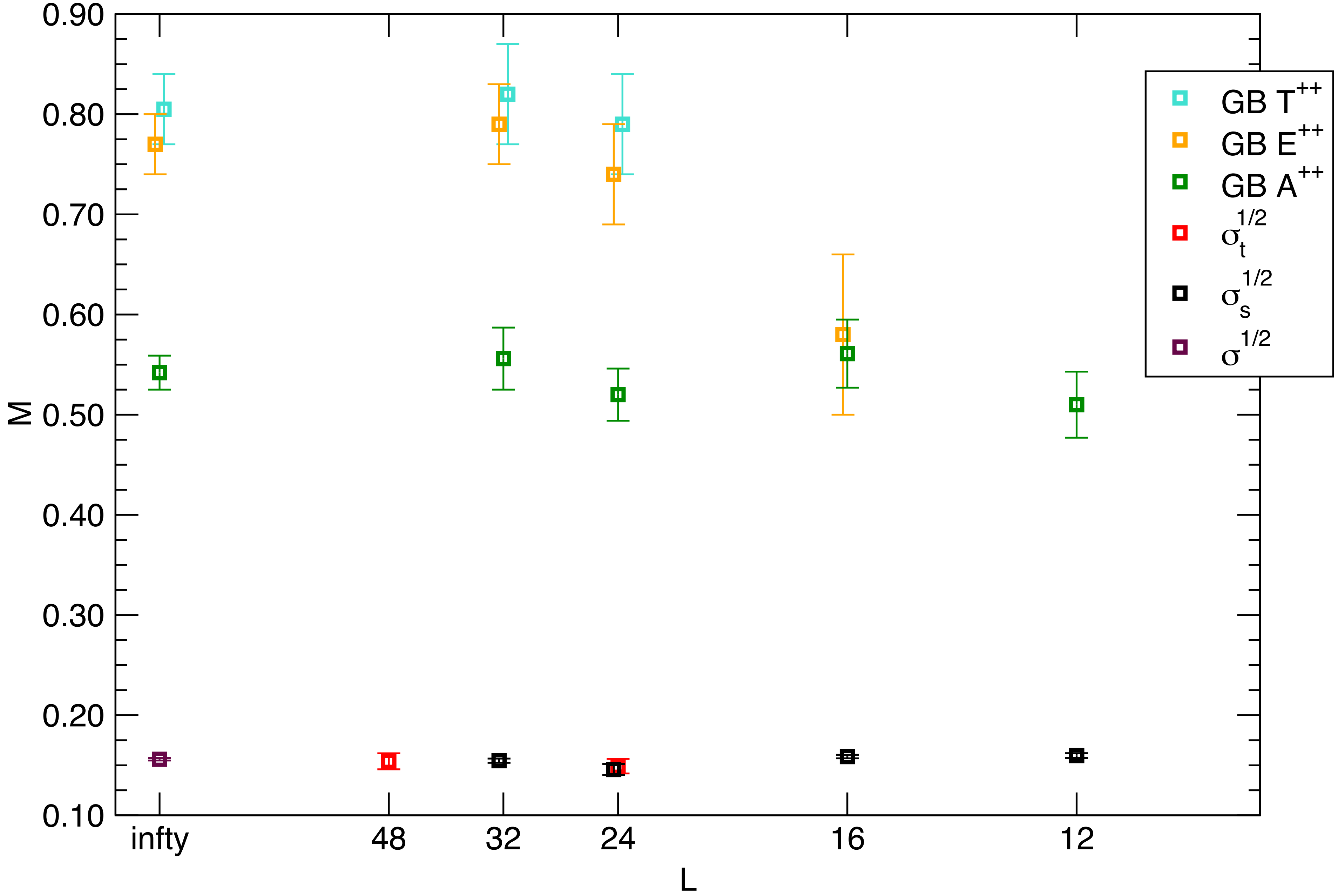


Qualitative evidence for a conformal spectrum

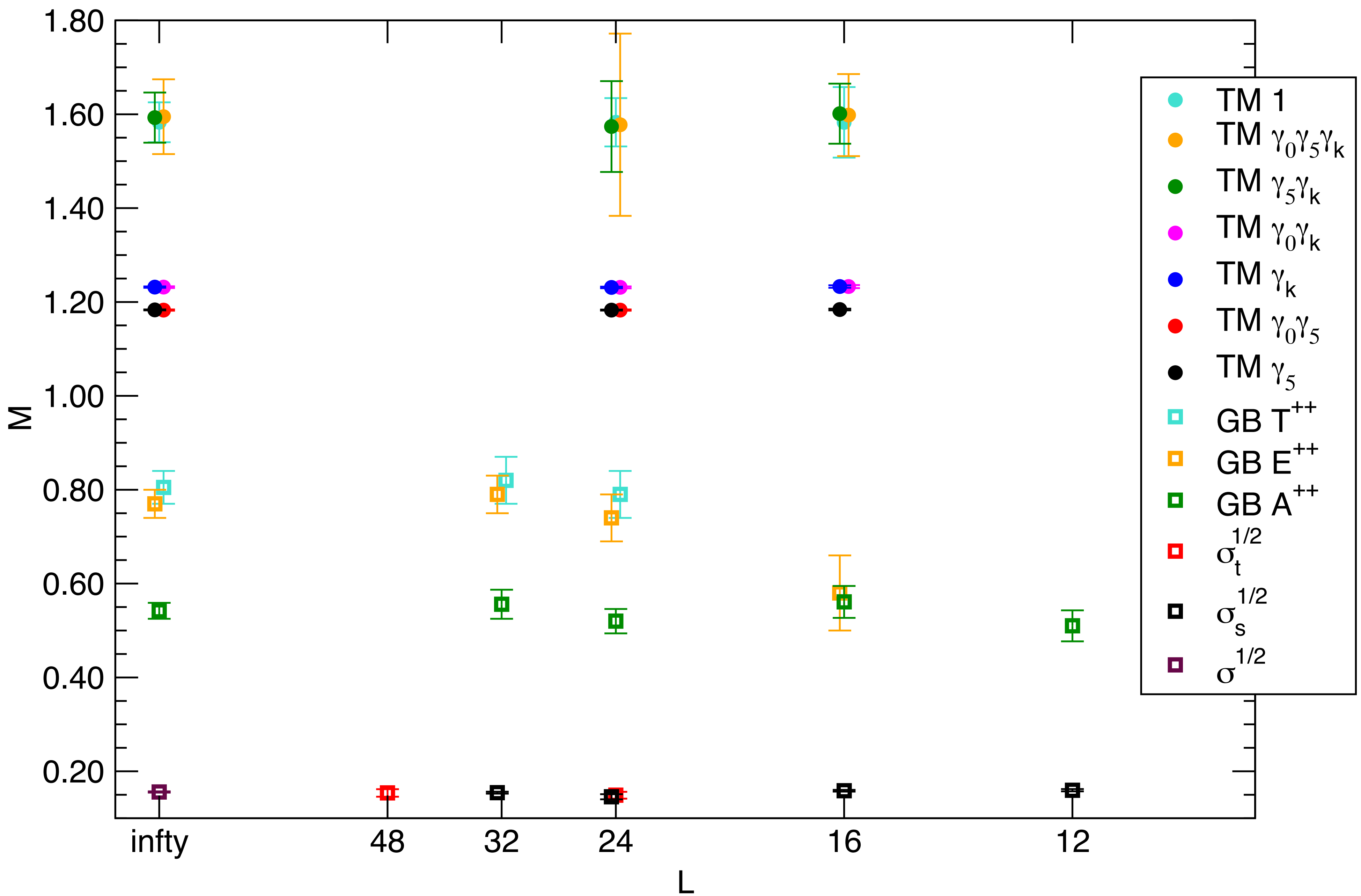
Need large lattices and small masses to extract the scaling exponent

[LDD et al 11]

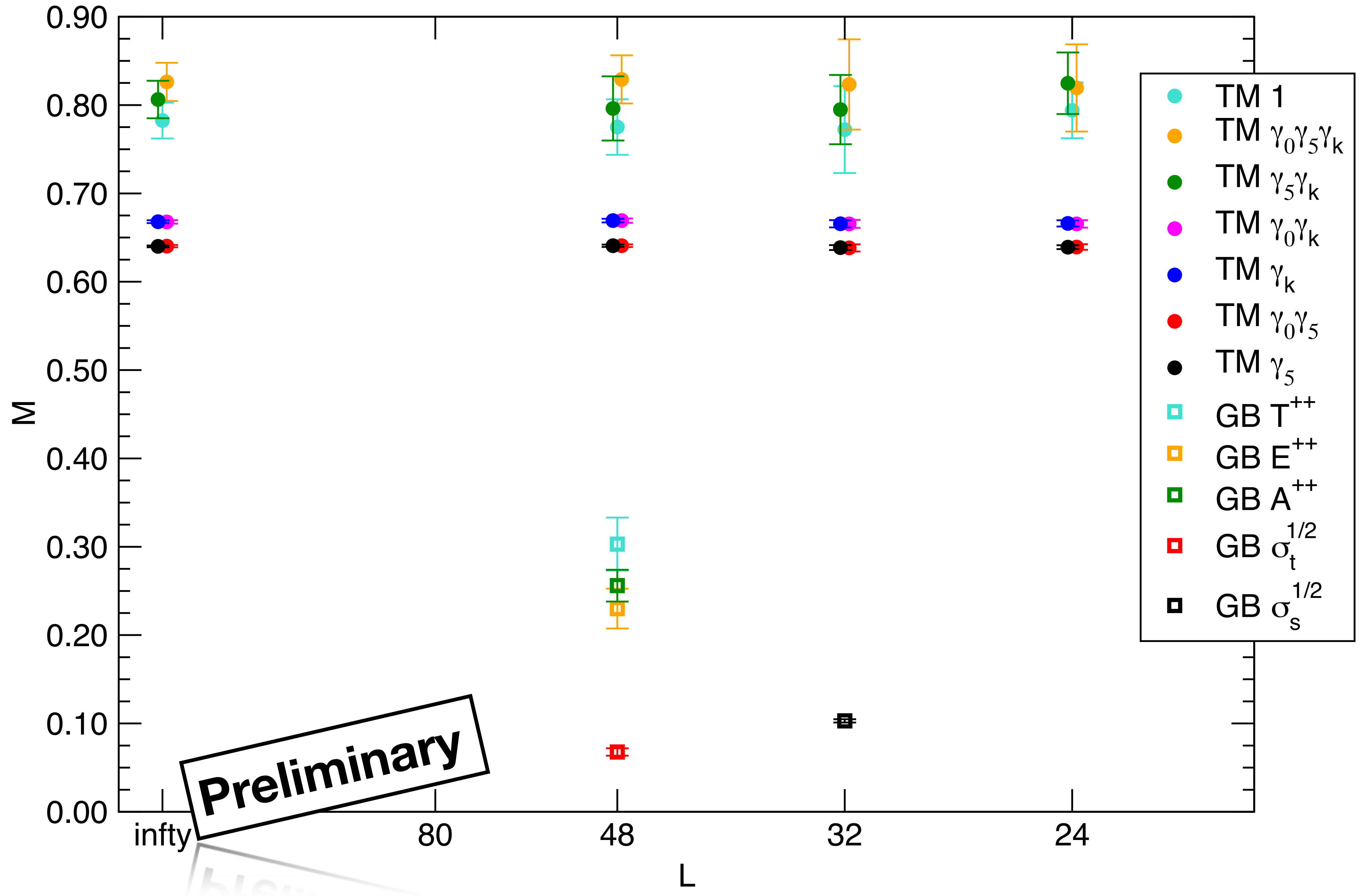
Larger volumes - heavier mass



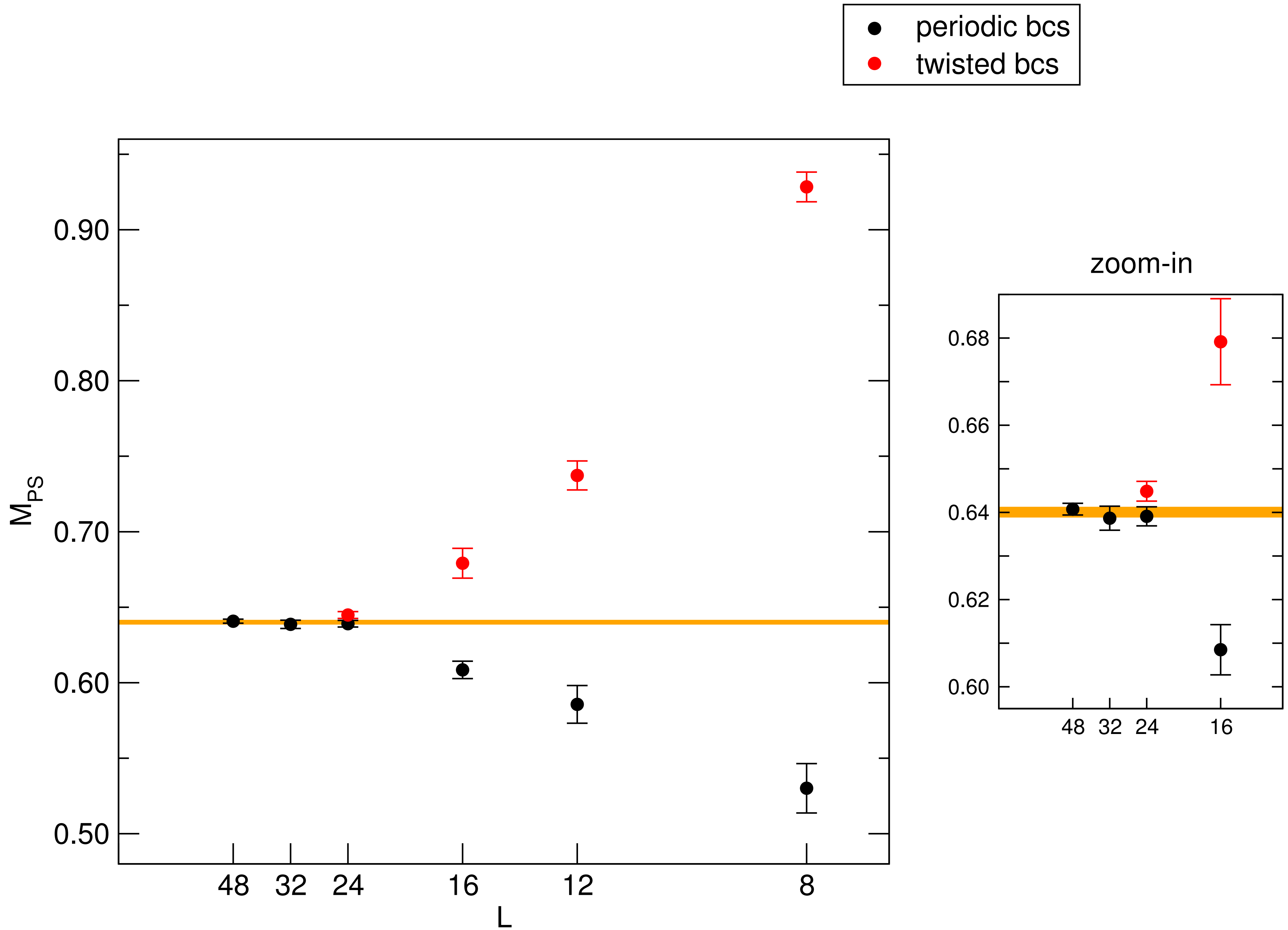
Larger volumes - heavier mass



Larger volumes - lighter mass



Larger volumes - twist



Dirac Eigenvalues

Scaling of the eigenvalue density:

$$\langle \bar{q}q \rangle \overset{m\to 0}{\sim} m^{\eta_{\bar{q}q}} \quad \Leftrightarrow \quad \rho(\lambda) \overset{\lambda\to 0}{\sim} \lambda^{\eta_{\bar{q}q}} \ .$$

[DeGrand 09, LDD & Zwicky 10, Patella 12]

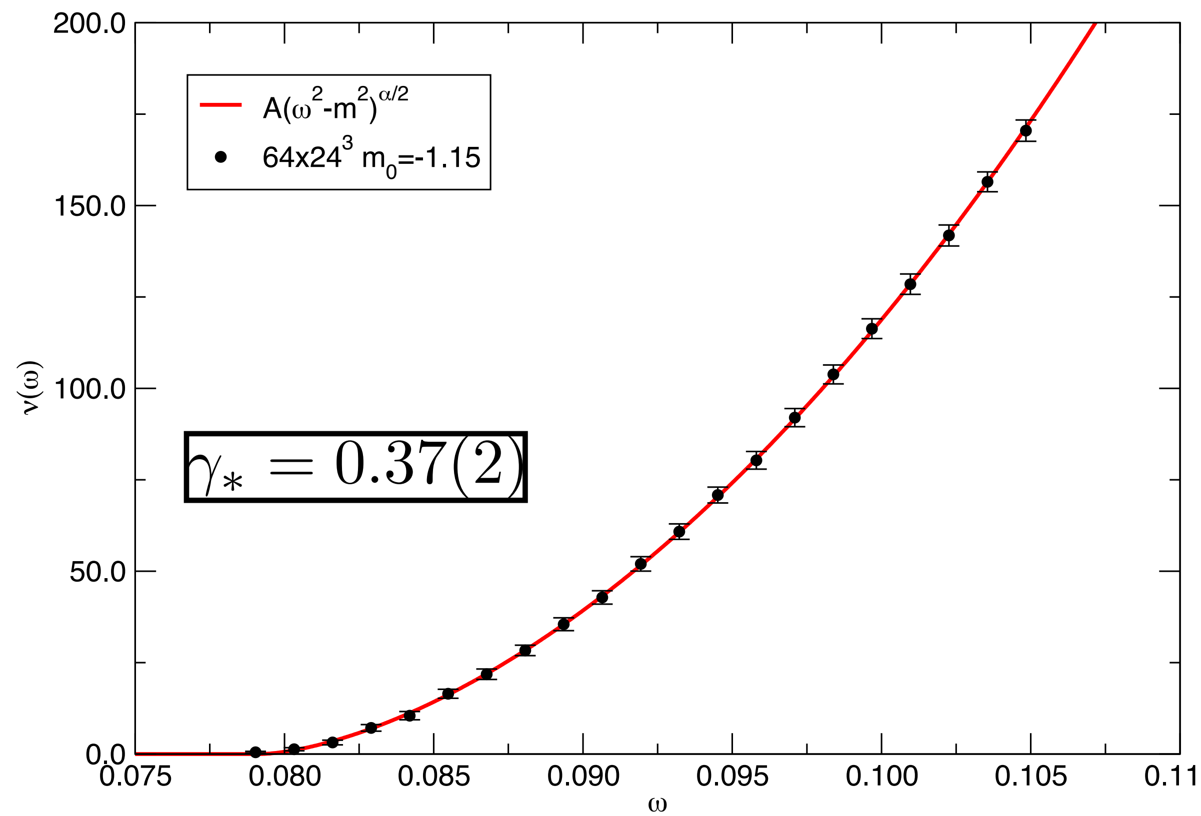
Measure the mode number of $D^\dagger D + m^2$

$$\nu(M,m) = \int_{-\Lambda}^{+\Lambda} d\lambda \rho(\lambda, m),$$

$$\Lambda = \sqrt{M^2 - m^2}$$

$$\boxed{\nu(M) \propto \Lambda^{\eta_{\bar{q}q}+1}}$$

$$\eta_{\bar{q}q} + 1 = \frac{4}{1+\gamma_*}$$



[Patella 12]

Finite-size scaling

Numerical simulations are performed in a finite volume. The finite-volume effects can be incorporated in the RG analysis:

$$C_H(t; m, \mu, L) = b^{-2\gamma_H} C_H(t; m', \mu', L)$$

Using the power-law scaling of the couplings, and dimensional analysis:

$$C_H(t; m, \mu, L) = b^{-2(d_H+\gamma_H)} C_H(b^{-1}t; b^{y_m} m, \mu, b^{-1}L)$$

Choose: $b^{-1}L = L_0$

$$C_H(t; m, \mu, L) = \left(\frac{L}{L_0}\right)^{-2\Delta_H} C_H\left(\frac{t}{L/L_0}; x\frac{1}{\mu L_0^{y_m}}, \mu, L_0\right)$$

$$x = L^{y_m} m$$

$$y_m = 1 + \gamma_*$$

Finite-size scaling 2

Comparing with the asymptotic large-time behaviour:

$$\boxed{M_H = L^{-1} f(x)}$$

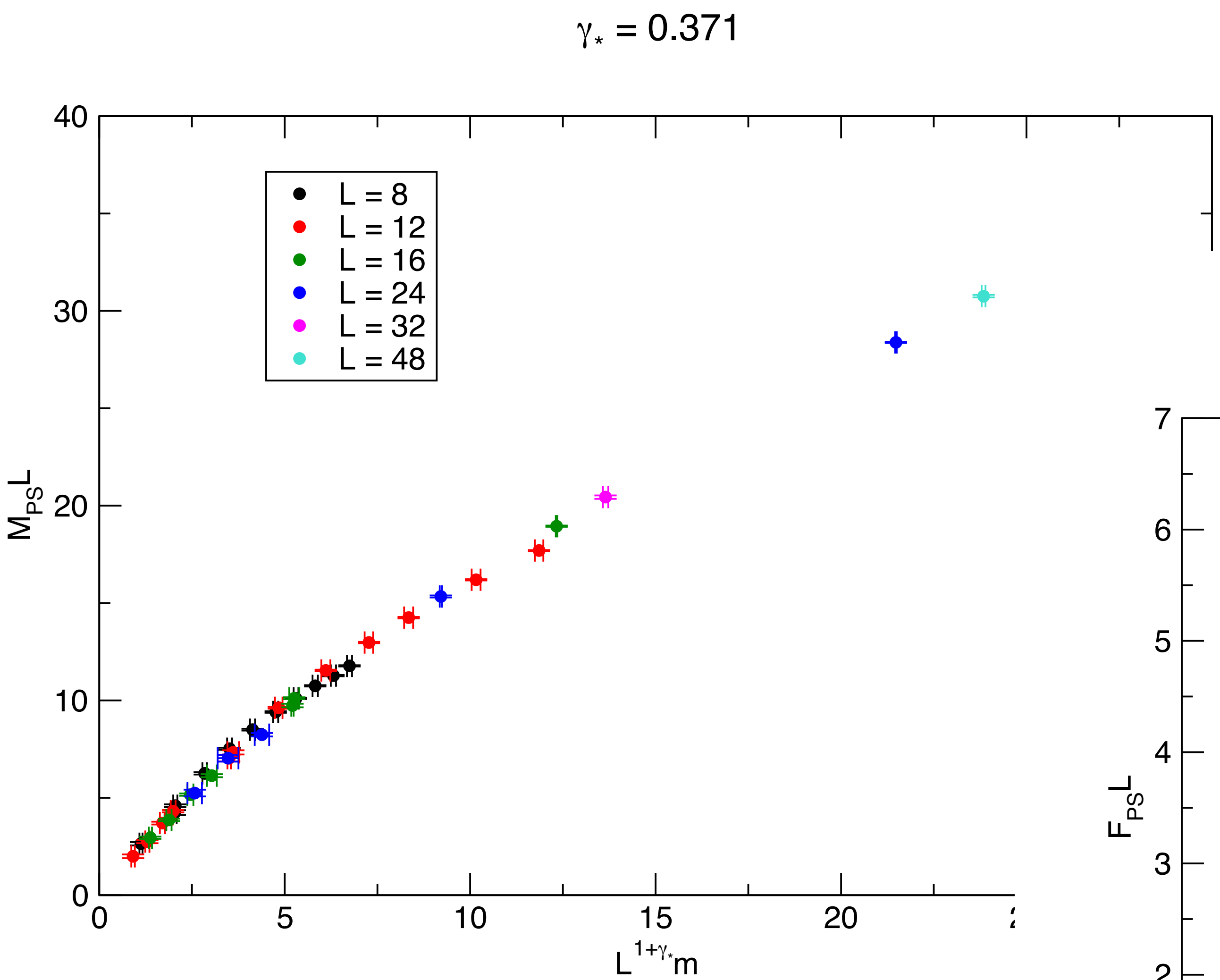
In order to recover the correct behaviour at **infinite volume**:

$$f(x) \sim x^{1/y_m}\,, \quad \text{as } x \to \infty$$

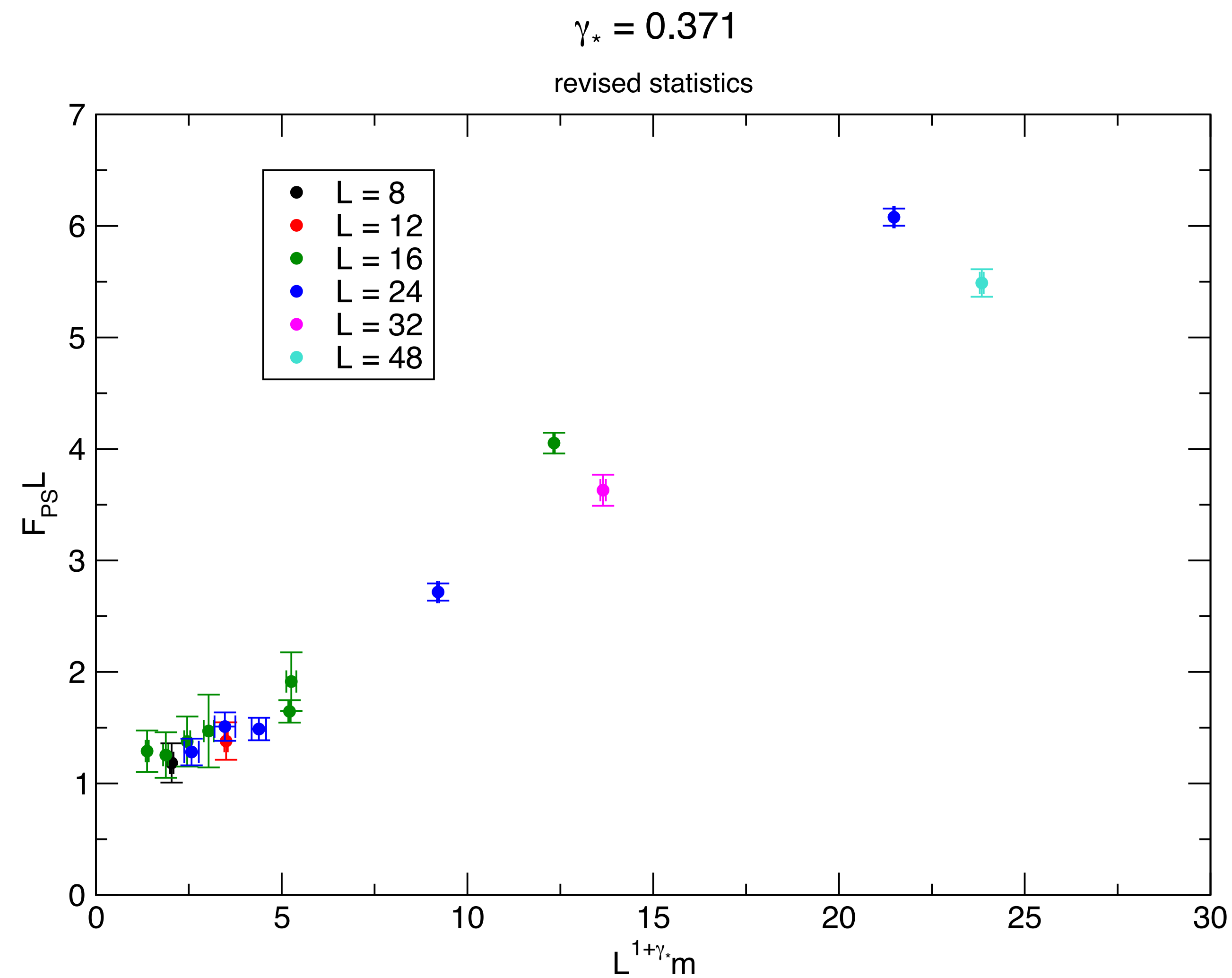
If we go to the massless limit, at **fixed** volume and cut-off, the masses of the states in the spectrum of the theory saturate and scale as:

$$M_H \propto L^{-1}$$

FSS - example



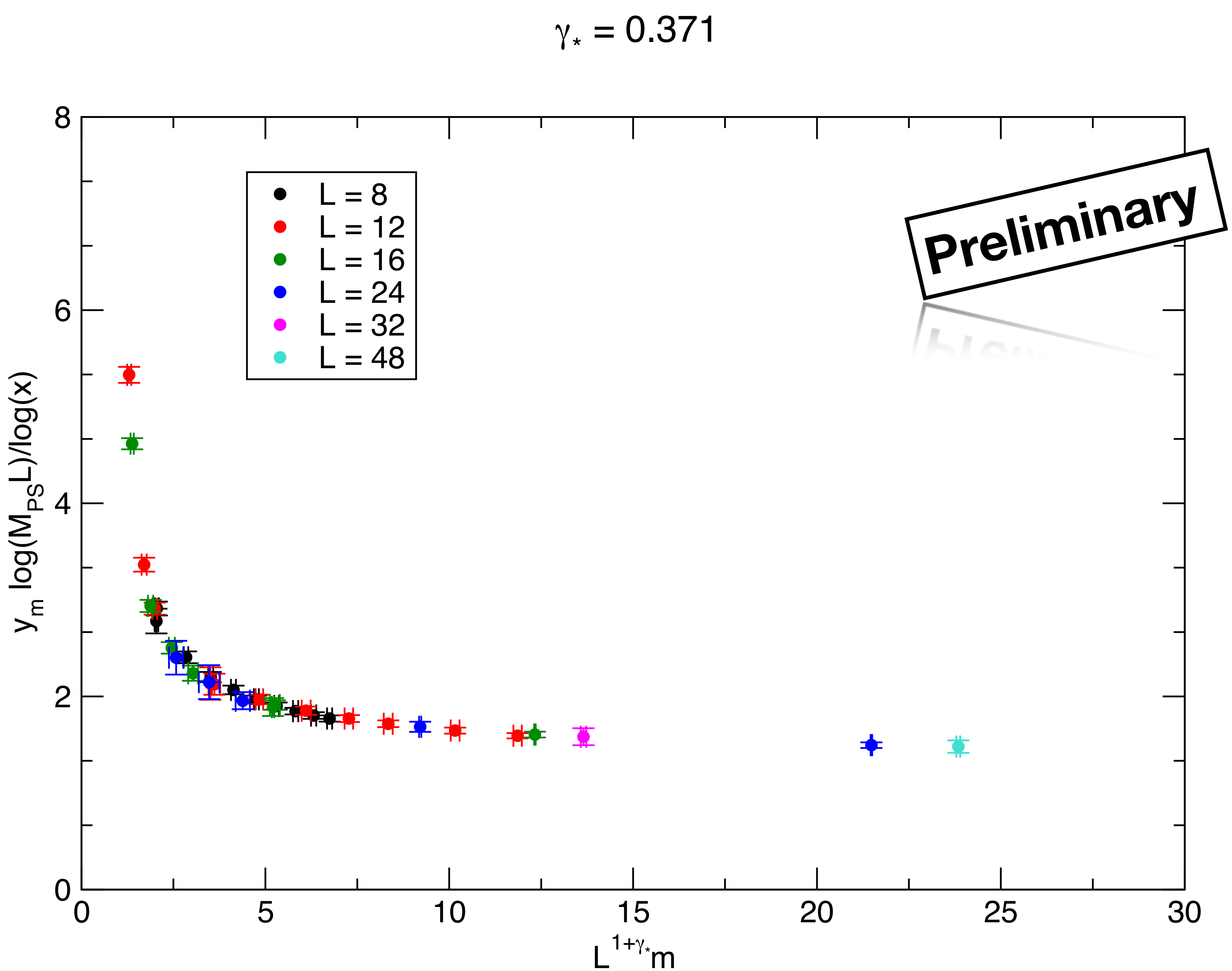
$\gamma_* = 0.371$
$M_{PS}L$
$L^{1+\gamma_*}m$
L = 8
L = 12
L = 16
L = 24
L = 32
L = 48



$\gamma_* = 0.371$
revised statistics
$F_{PS}L$
$L^{1+\gamma_*}m$
L = 8
L = 12
L = 16
L = 24
L = 32
L = 48

Preliminary

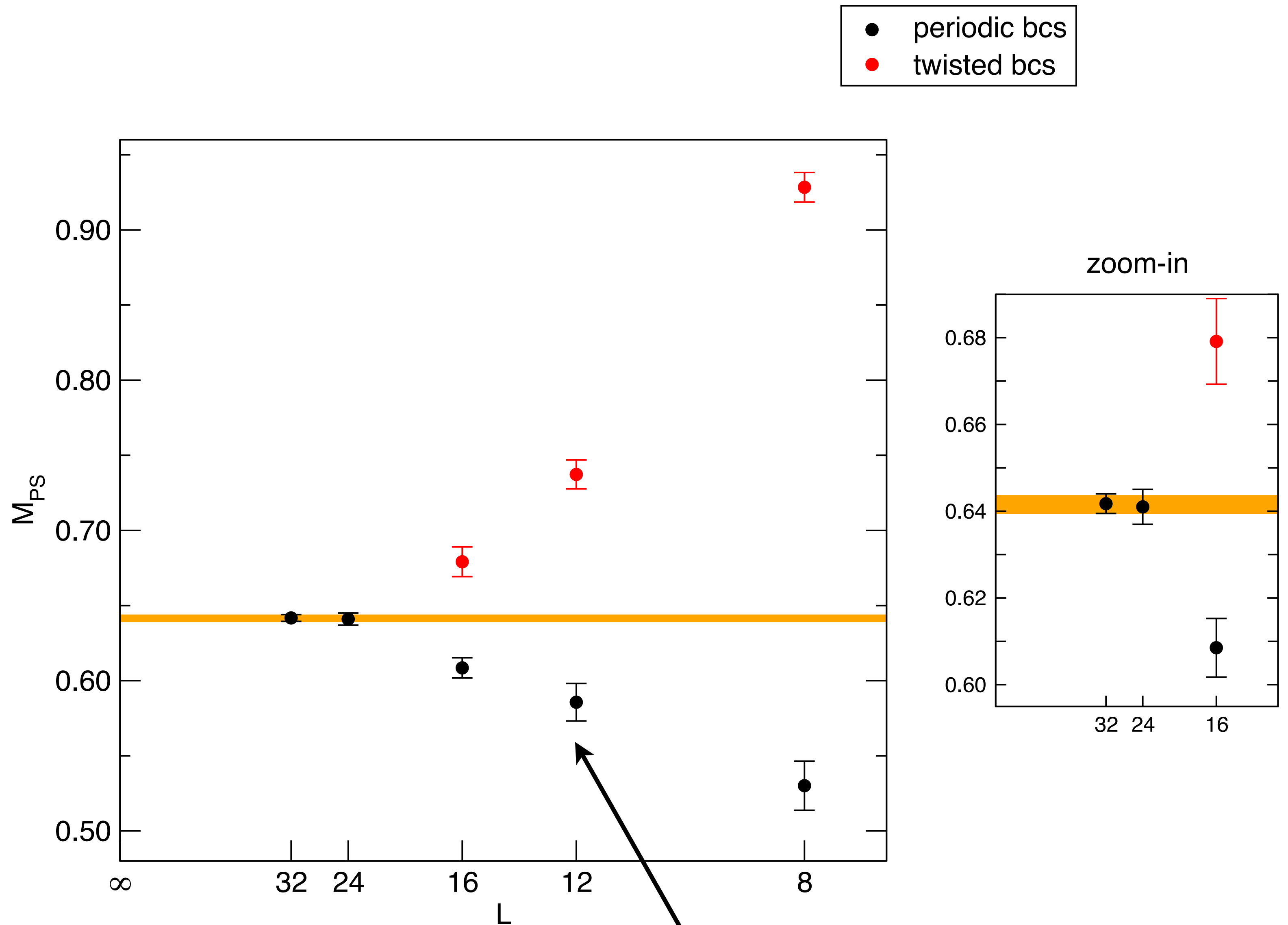
FSS - asymptotic behaviour



Conclusions

- Lattice simulations yield first principle results about the NP dynamics of strongly interacting theories; tools need to be adapted to the new dynamics.
- Studies so far have focused on understanding the phase diagram (IRFP)
- Quantitative results on the spectrum, beta function and the anomalous dimensions
- Searches for new physics rely on effective theory descriptions of the low-energy dynamics: NP effects are encoded in the LECs.
- Signatures of DEWSB have been studied: compare with results from the LHC
- Lattice input to phenomenology: LECs, S parameter. Feasible & expensive!
- **We need to ask the right questions!!!**

Spectrum - systematic errors



finite volume effects

[LDD et al 11]