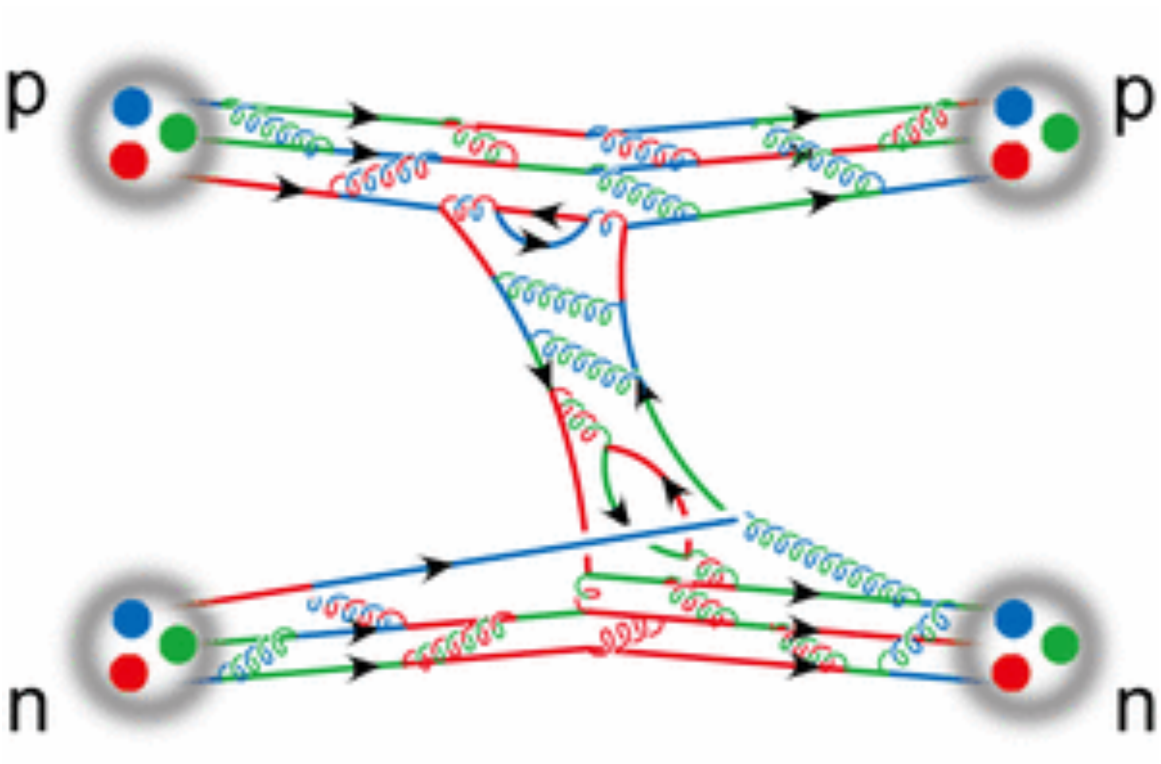
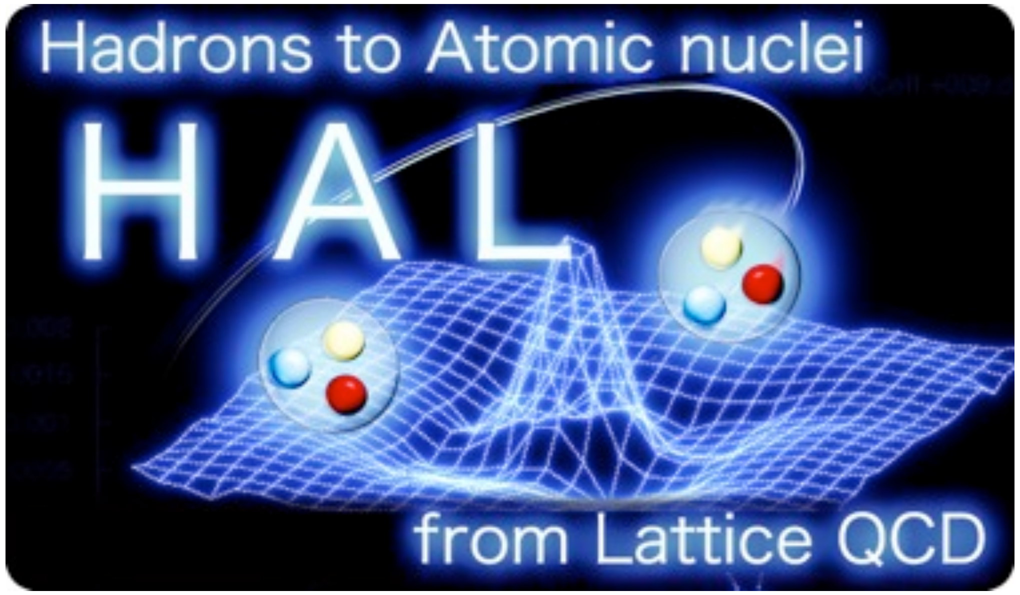
Hadron interactions from lattice QCD

Sinya Aoki
University of Tsukuba



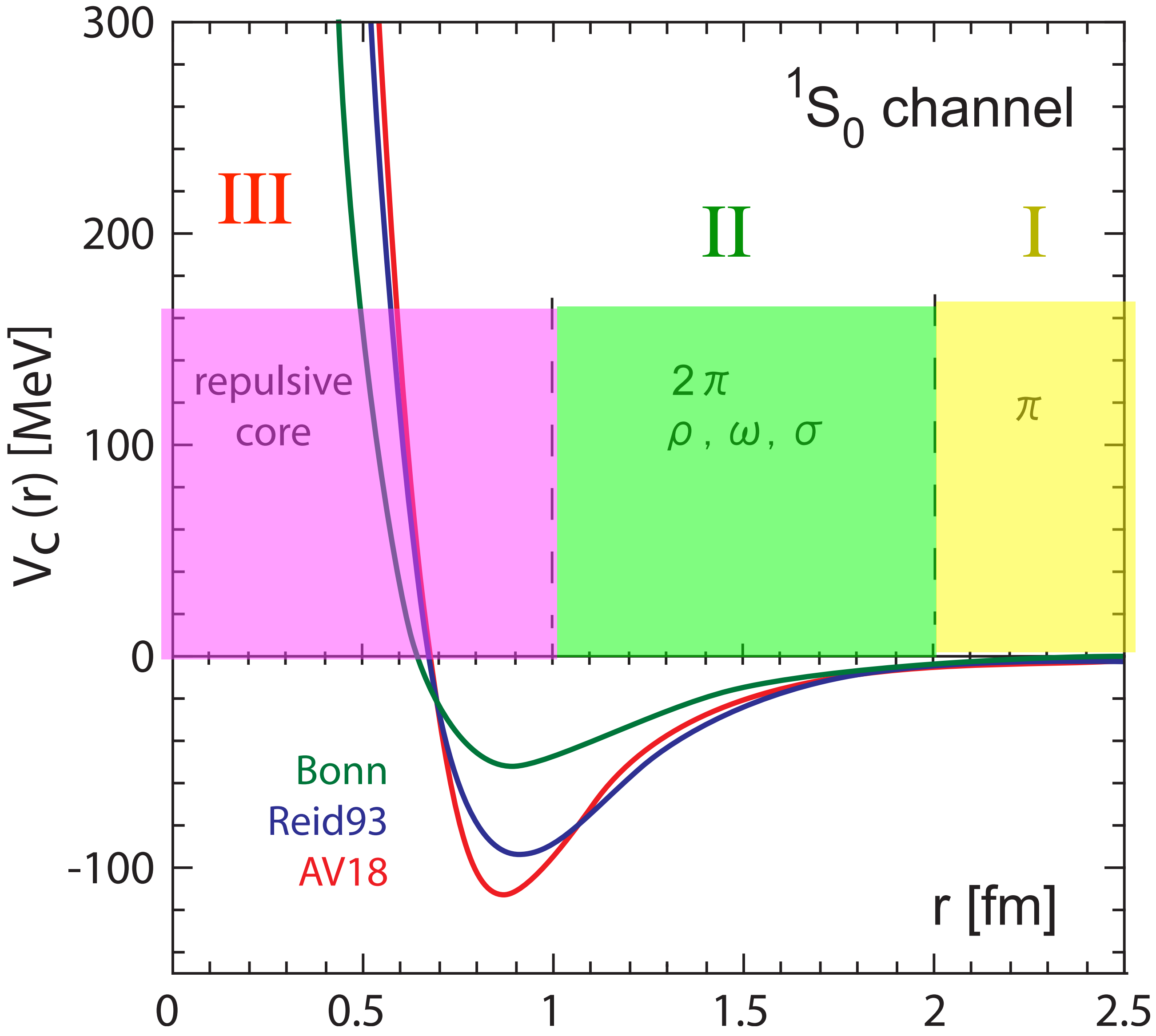


KMI workshop on “Strong Coupling Gauge Theories in the LHC Perspective”
Sakata-Hirata Hall, Nagoya University, December 4-7, 2012

1. Introduction

How can we extract hadronic interaction from lattice QCD ?

Ex. **Phenomenological NN potential**
(~40 parameters to fit 5000 phase shift data)



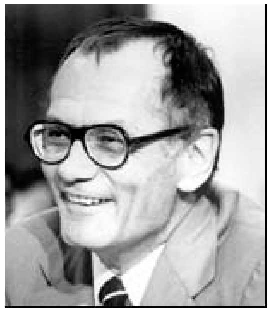
I One-pion exchange

Yukawa(1935)

II Multi-pions

Taketani et al.(1951)

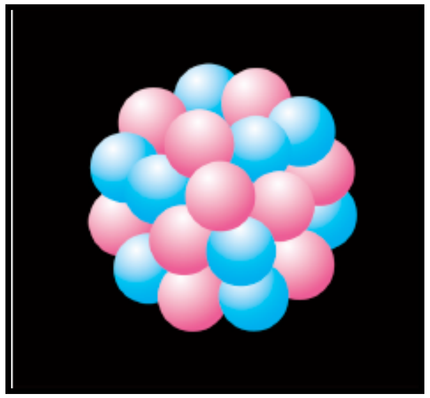
III Repulsive core

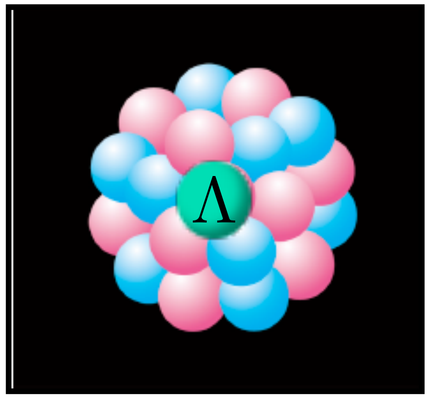


Jastrow(1951)

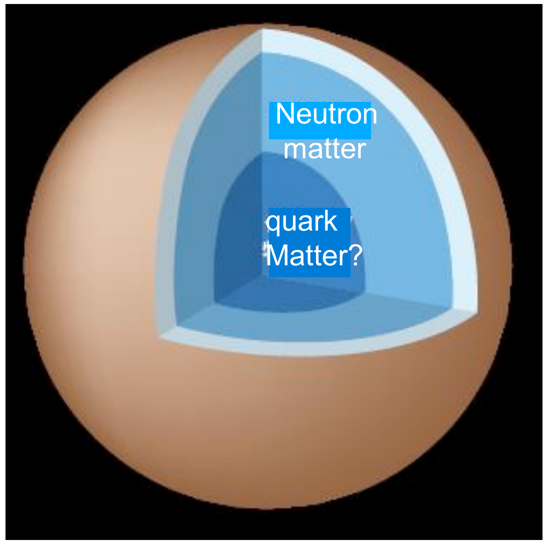
Nuclear force is a basis for understanding ...

- Structure of ordinary and hyper nuclei



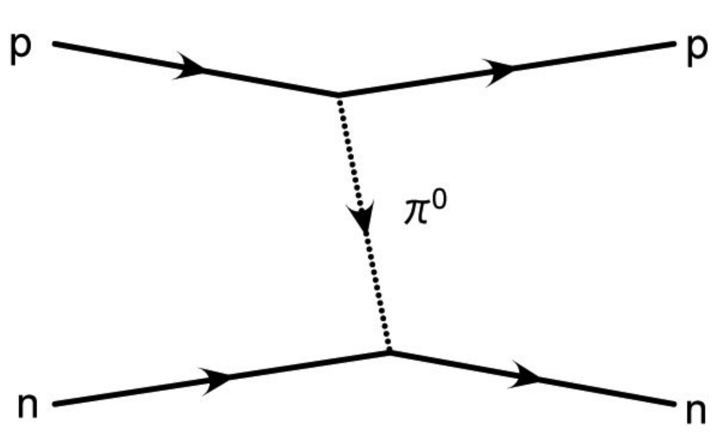


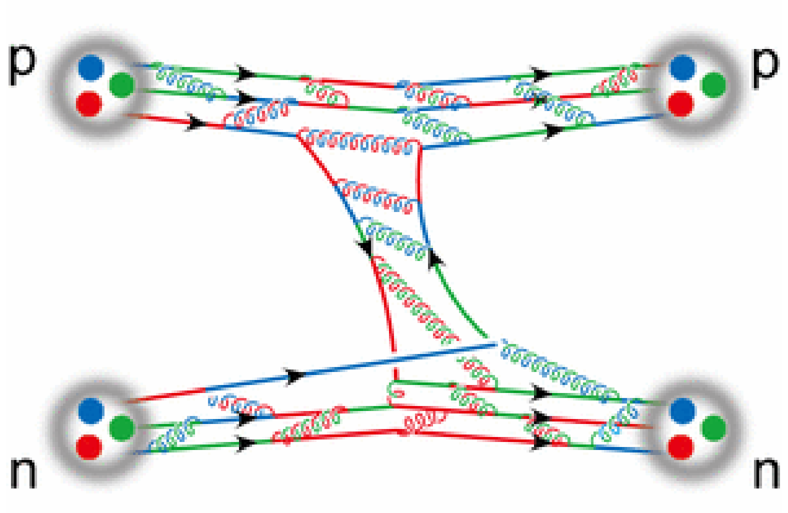
- Structure of neutron star



- Ignition of Type II SuperNova

Can we extract a nuclear force in (lattice) QCD ?





Plan of my talk

1. Introduction
2. Strategy
3. Nuclear potential
4. H-dibaryon
5. Conclusion

2. Strategy

Potentials in QCD ?

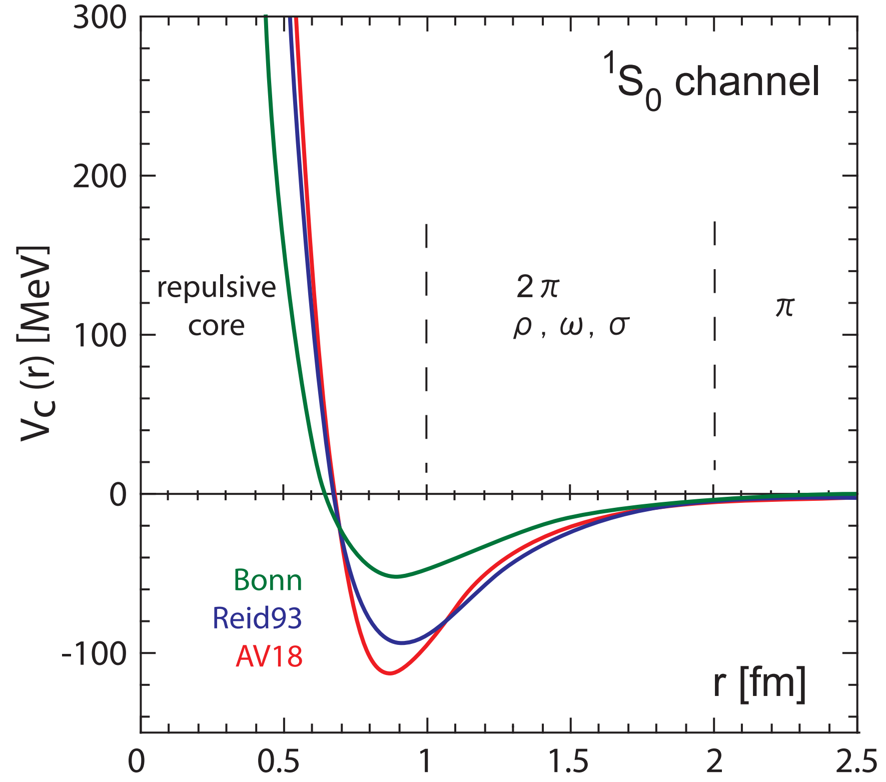
What are “potentials” (quantum mechanical objects) in quantum field theories such as QCD ?

“Potentials” themselves can NOT be directly measured.

cf. running coupling in QCD

scheme dependent, Unitary transformation

experimental data of scattering phase shifts → potentials, but not unique



“Potentials” are useful tools to extract observables such as scattering phase shift.

One may adopt a convenient definition of potentials as long as they reproduce correct physics of QCD.

Our strategy in lattice QCD

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89.

Step 1 define (Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

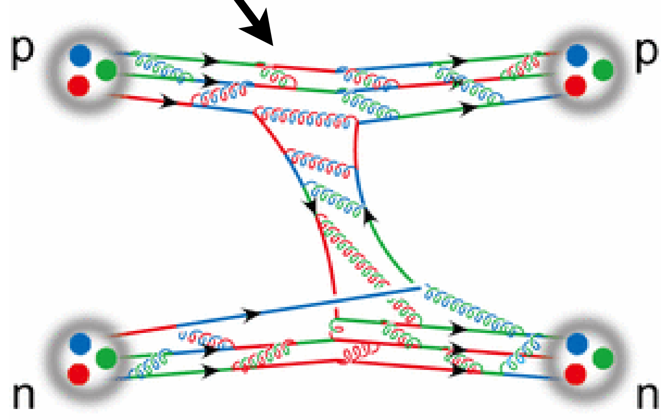
Spin model: Balog et al., 1999/2001

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

$$W_k = 2\sqrt{\mathbf{k}^2 + m_N^2}$$

energy

$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator



Important property

partial wave $\varphi_{\mathbf{k}}^l \to A_l \dfrac{\sin(kr - l\pi/2 + \delta_l(k))}{kr}$ $r = |\mathbf{r}| \to \infty$

Lin et al., 2001; CP-PACS, 2004/2005

$\delta_l(k)$ scattering phase shift (phase of the S-matrix by unitarity) in QCD !

How can we extract it ?

cf. Luescher's finite volume method

Step 2 define non-local but energy-independent "potential" as

$$\mu = m_N/2$$

reduced mass

$$\left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, \underbrace{U(\mathbf{x},\mathbf{y})}_{\text{non-local potential}} \varphi_{\mathbf{k}}(\mathbf{y})$$

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

Properties & Remarks

1. Potential itself is NOT an observable. Using this freedom, we can construct a non-local but energy-independent potential as

inner product

$$U(\mathbf{x},\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'}^{W_k, W_{k'} \le W_{\text{th}}} \left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(\mathbf{x}) \eta^{-1}_{\mathbf{k},\mathbf{k}'} \varphi^\dagger_{\mathbf{k}'}(\mathbf{y})$$

$\eta^{-1}_{\mathbf{k},\mathbf{k}'}$: inverse of $\eta_{\mathbf{k},\mathbf{k}'} = (\varphi_{\mathbf{k}}, \varphi_{\mathbf{k}'})$

$\varphi_{\mathbf{k}}$ is linearly independent.

For ${}^\forall W_{\mathbf{p}} < W_{\text{th}} = 2m_N + m_\pi$ (threshold energy)

$$\int d^3y\, U(\mathbf{x},\mathbf{y}) \phi_{\mathbf{p}}(\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'} \left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(x) \eta^{-1}_{\mathbf{k},\mathbf{k}'} \eta_{\mathbf{k}',\mathbf{p}} = \left[\epsilon_p - H_0\right] \varphi_{\mathbf{p}}(x)$$

Proof of existence (cf. Density Functional Theory)

Of course, potential satisfying this is not unique. (Scheme dependence. cf. running coupling)

2. Non-relativistic approximation is NOT used. We just take the specific (equal-time) flame.

Step 3 expand the non-local potential in terms of derivative as

$$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$$

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\sigma_{\mathbf{1}}\cdot\sigma_{\mathbf{2}}) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

spins

tensor operator $S_{12} = \frac{3}{r^2}(\sigma_{\mathbf{1}}\cdot\mathbf{x})(\sigma_{\mathbf{2}}\cdot\mathbf{x}) - (\sigma_{\mathbf{1}}\cdot\sigma_{\mathbf{2}})$

$V_A(\mathbf{x})$ local and energy independent coefficient function
(cf. Low Energy Constants(LOC) in Chiral Perturbation Theory)

Step 4 extract the local potential at LO as

$$V_{\mathrm{LO}}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$$

Step 5 solve the Schroedinger Eq. in the infinite volume with this potential.

phase shifts and binding energy below inelastic threshold

(We can calculate the phase shift at all angular momentum.)

$\delta_L(k)$ exact by construction

$\delta_L(p \neq k)$ approximated one by the derivative expansion

expansion parameter

$$\frac{W_p - W_k}{W_{\mathrm{th}} - 2m_N} \simeq \frac{\Delta E_p}{m_\pi}$$

We can check a size of errors at LO of the expansion. (See later).
We can improve results by extracting higher order terms in the expansion.

3. Nuclear potential

Extraction of NBS wave function

NBS wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0|N(\mathbf{x}+\mathbf{r},0)N(\mathbf{x},0)|NN, W_k\rangle$$

Potential

$$[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

4-pt Correlation function

source for NN

$$F(\mathbf{r}, t-t_0) = \langle 0|T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\}\overline{\mathcal{J}}(t_0)|0\rangle$$

complete set for NN

$$\begin{aligned} F(\mathbf{r}, t-t_0) &= \langle 0|T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\}\sum_{n,s_1,s_2}|2N, W_n, s_1, s_2\rangle\langle 2N, W_n, s_1, s_2|\overline{\mathcal{J}}(t_0)|0\rangle + \cdots \\ &= \sum_{n,s_1,s_2} A_{n,s_1,s_2}\varphi^{W_n}(\mathbf{r})e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N, W_n, s_1, s_2|\overline{\mathcal{J}}(0)|0\rangle. \end{aligned}$$

ground state saturation at large t

$$\lim_{(t-t_0)\to\infty} F(\mathbf{r}, t-t_0) = A_0\varphi^{W_0}(\mathbf{r})e^{-W_0(t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})$$

NBS wave function

This is a standard method in lattice QCD and was employed for our first calculation.

Improved method

Ishii *et al.* (HALQCD), PLB712(2012) 437

normalized 4-pt Correlation function $R(\mathbf{r},t) \equiv F(\mathbf{r},t)/(e^{-m_N t})^2 = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-\Delta W_n t}$

$$\Delta W_n = W_n - 2m_N = \frac{\mathbf{k}_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N}$$

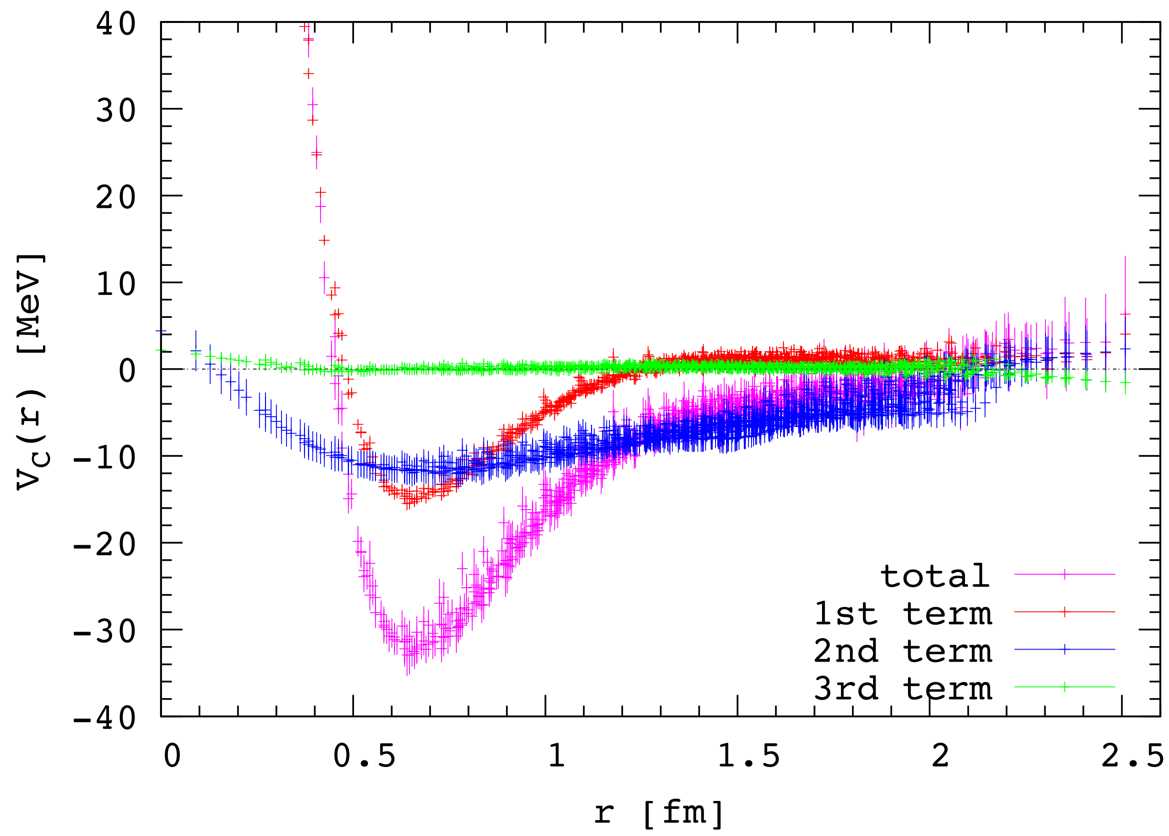
$$-\frac{\partial}{\partial t} R(\mathbf{r},t) = \left\{ H_0 + U - \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t)$$

potential

Leading Order

$$\left\{ \underbrace{-H_0}_{\text{1st}} \underbrace{- \frac{\partial}{\partial t}}_{\text{2nd}} + \underbrace{\frac{1}{4m_N}\frac{\partial^2}{\partial t^2}}_{\text{3rd}} \right\} R(\mathbf{r},t) = \int d^3r'\, U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t) = \underbrace{V_C(\mathbf{r}) R(\mathbf{r},t)}_{\text{total}} + \cdots$$

3rd term(relativistic correction)
is negligible.

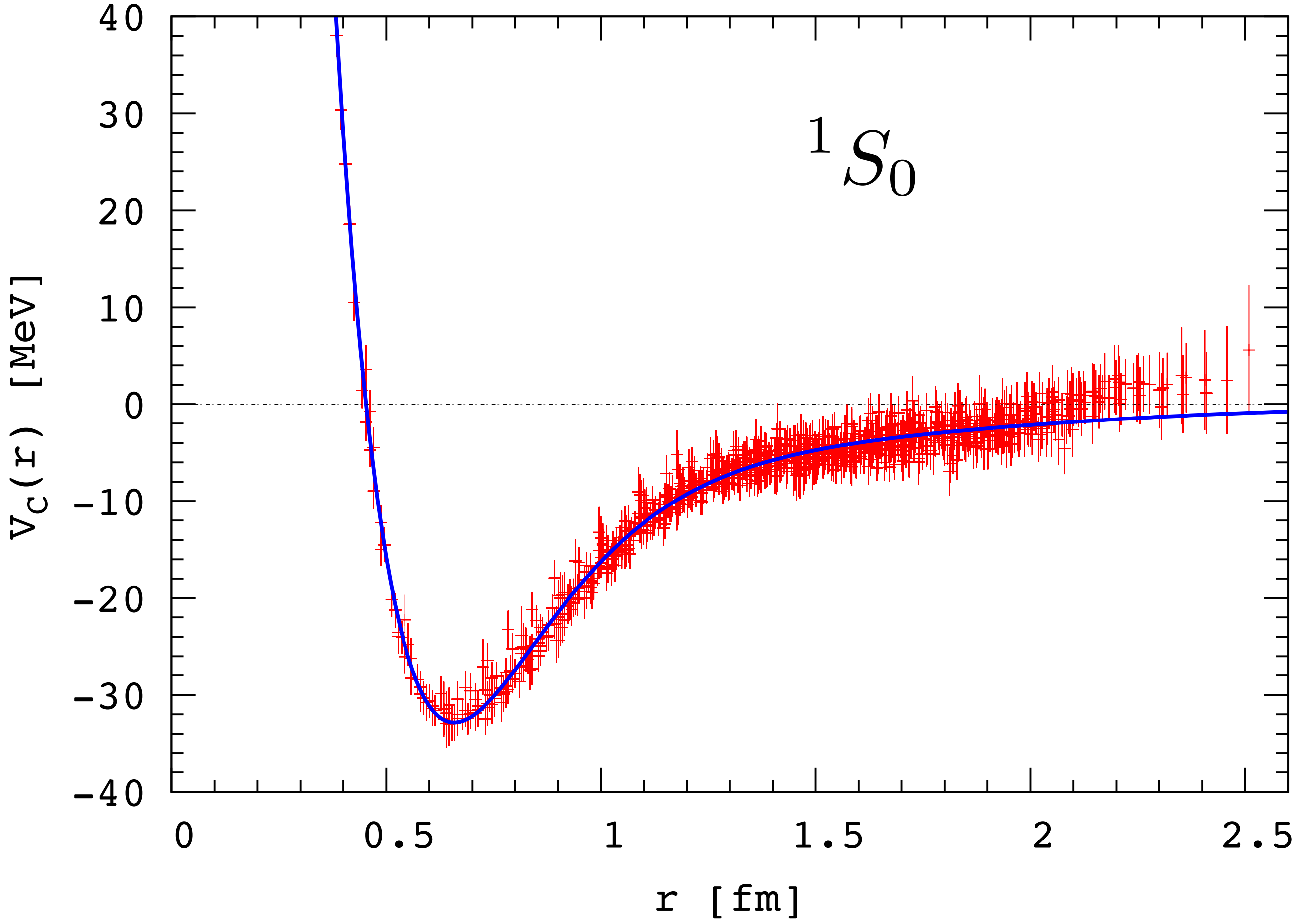


Ground state saturation is no more required ! (advantage over finite volume method.)

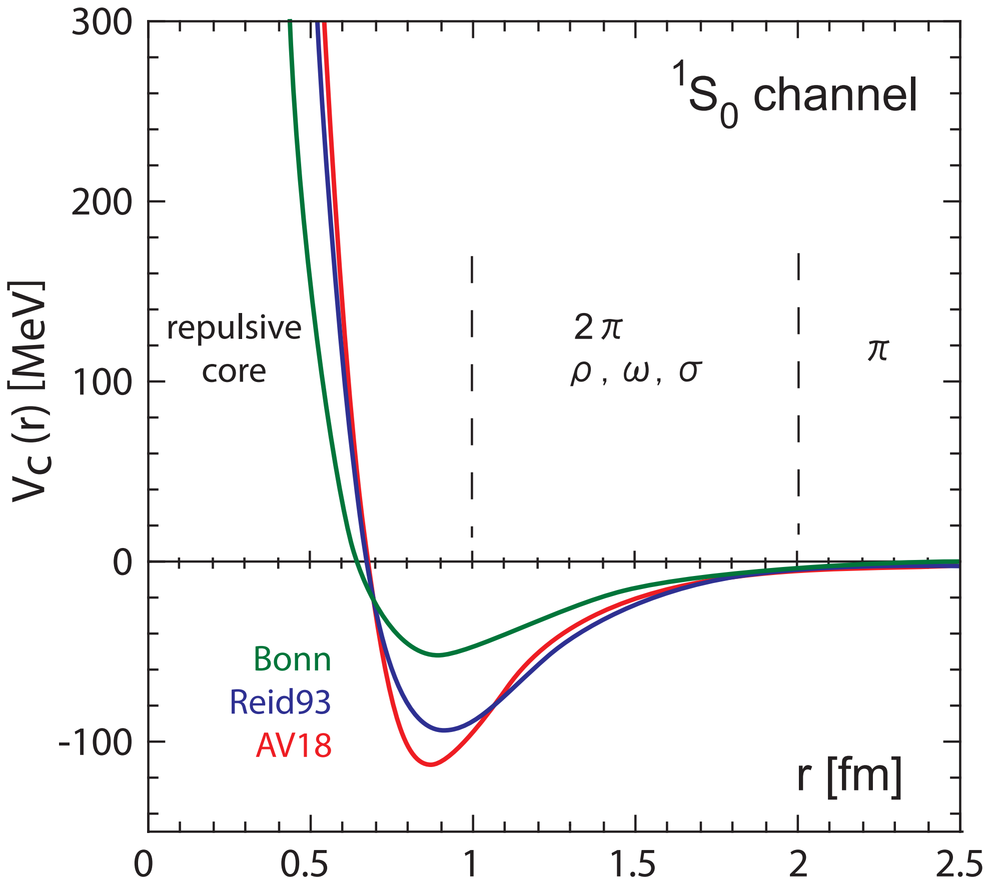
NN potential

2+1 flavor QCD, spin-singlet potential (PLB712(2012)437)

a=0.09fm, L=2.9fm $m_\pi \simeq 700$ MeV



phenomenological potential



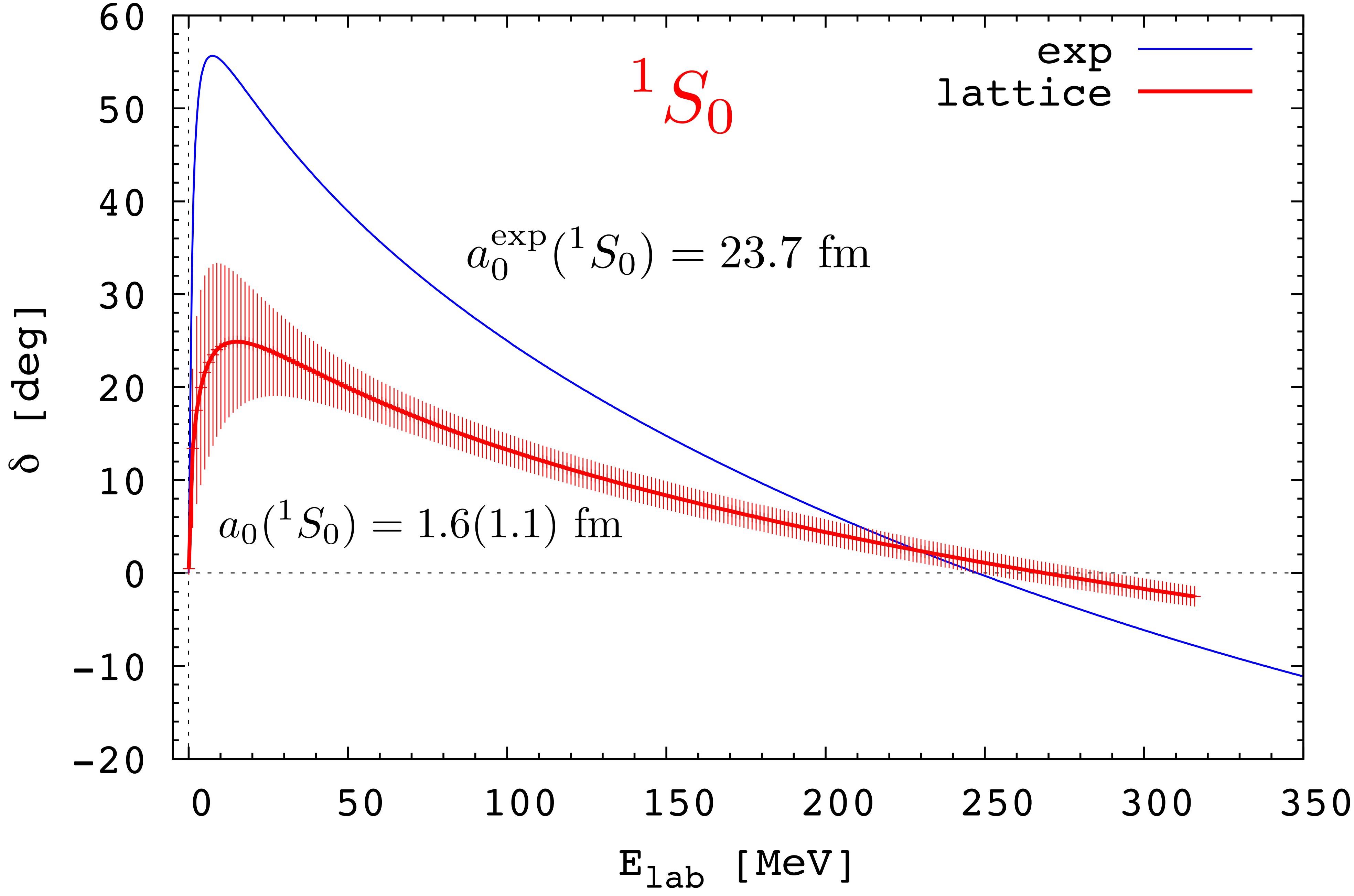
Qualitative features of NN potential are reproduced !

(1)attractions at medium and long distances
(2)repulsion at short distance(repulsive core)

1st paper(quenched QCD): Ishii-Aoki-Hatsuda, PRL90(2007)0022001

This paper has been selected as one of 21 papers in Nature Research Highlights 2007.
(One from Physics, Two from Japan, the other is on “iPS” by Sinya Yamanaka et al.)

NN potential → phase shift



It has a reasonable shape. The strength is weaker due to the heavier quark mass.

Need calculations at physical quark mass.

Convergence of velocity expansion

If the higher order terms are large, LO potentials determined from NBS wave functions at different energy become different.(cf. LOC of ChPT).

Numerical check in quenched QCD

$m_\pi \simeq 0.53$ GeV

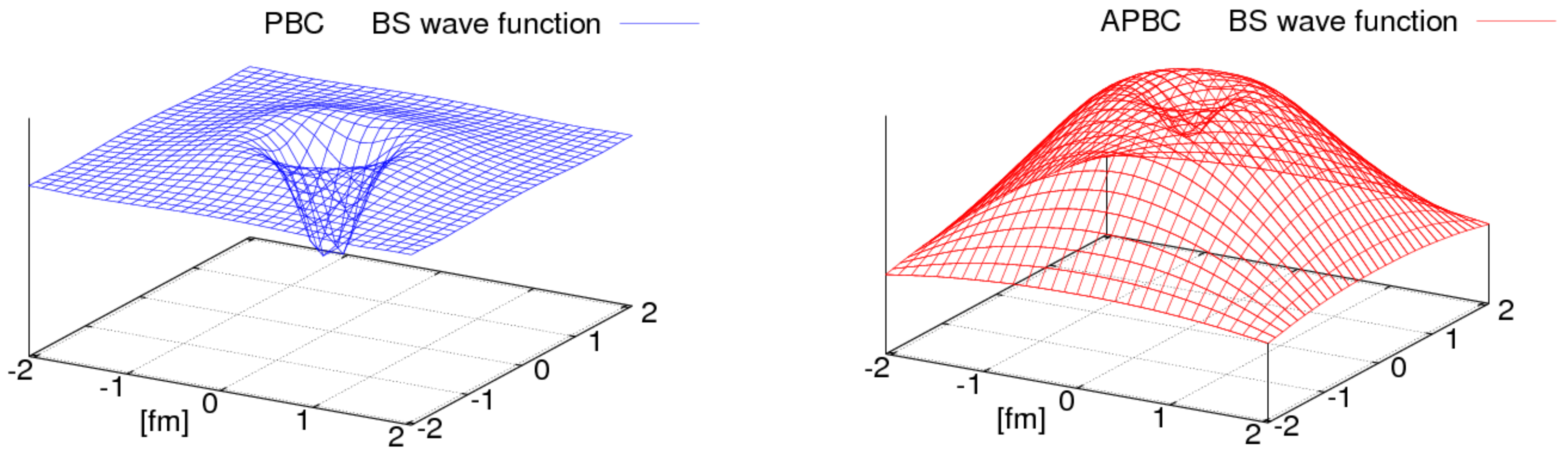
a=0.137fm, L=4.0 fm

K. Murano, N. Ishii, S. Aoki, T. Hatsuda

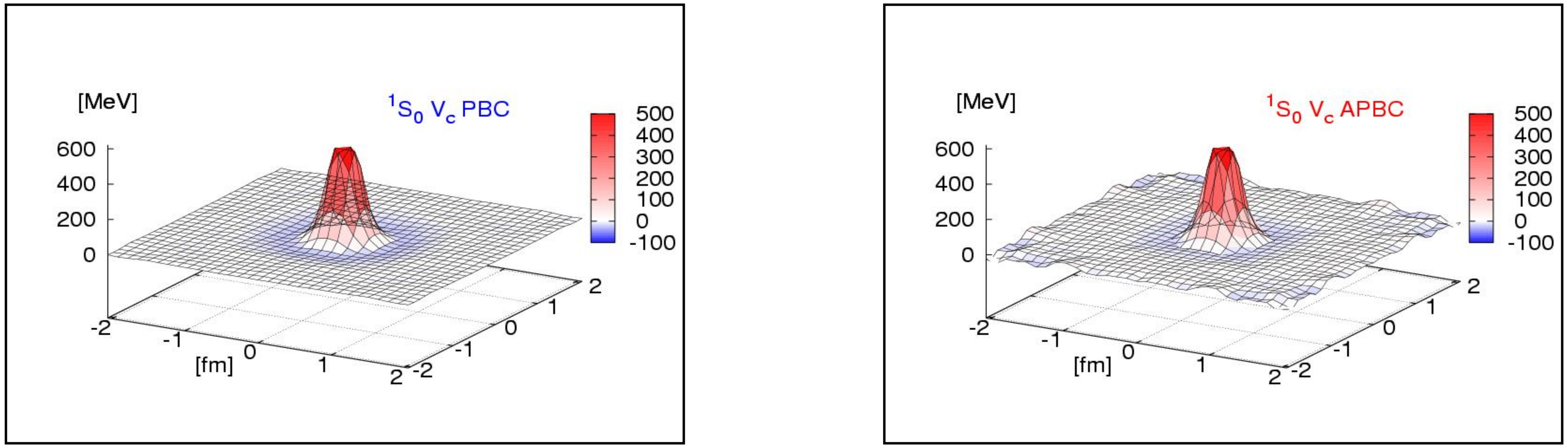
PTP 125 (2011)1225.

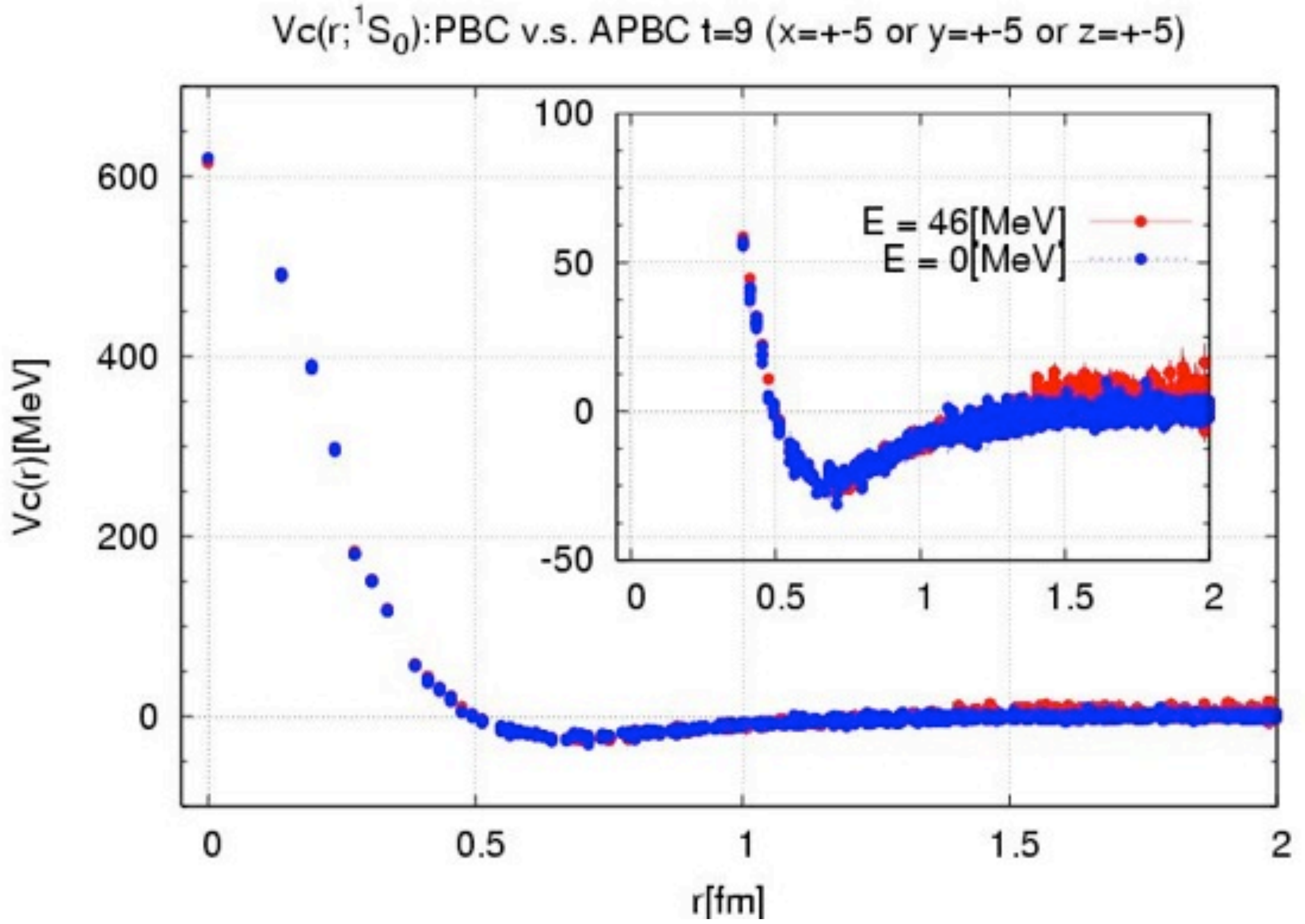
● PBC (E∼0 MeV)	● APBC (E∼46 MeV)

NBS wave functions

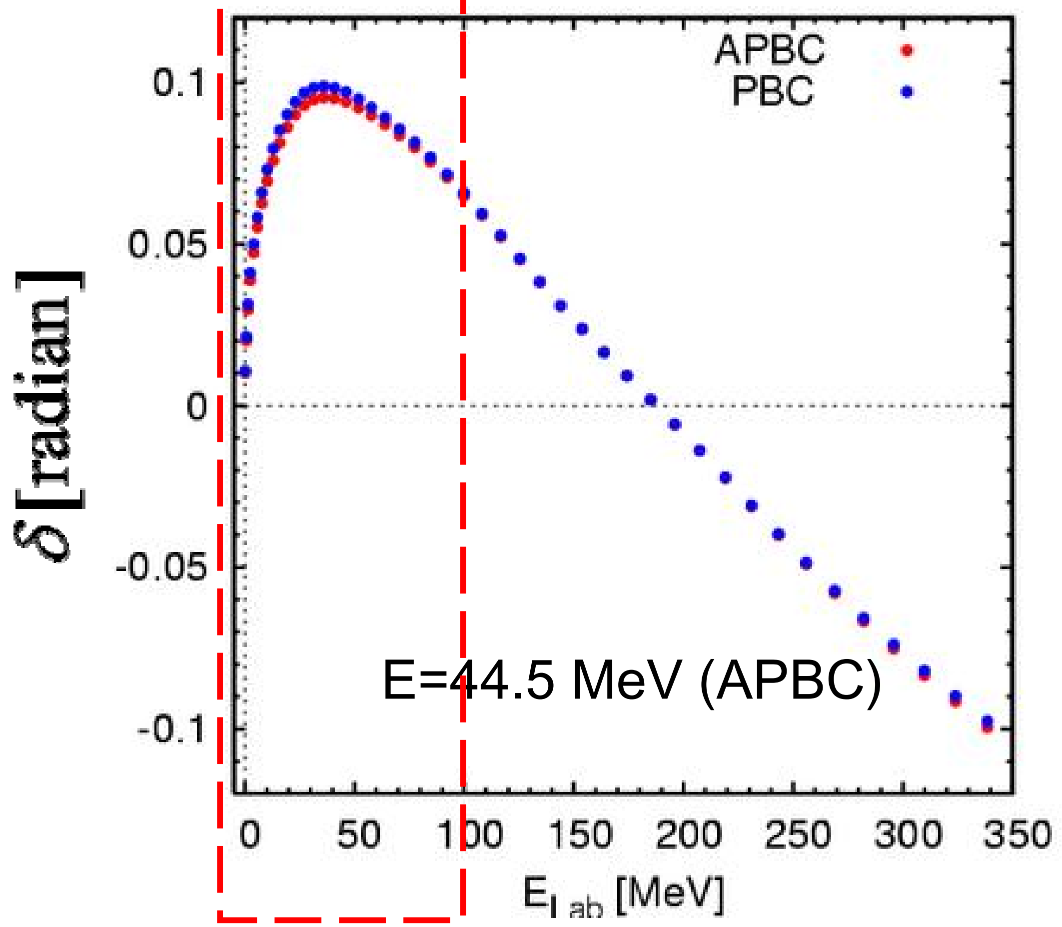


potentials





phase shifts from potentials



Higher order terms turn out to be very small at low energy in HAL scheme.

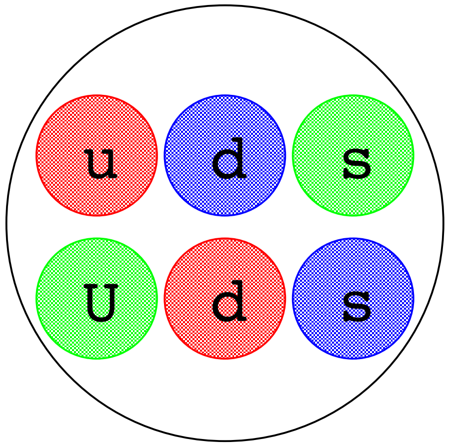
Need to be checked at lighter pion mass in 2+1 flavor QCD.

Note: convergence of the velocity expansion can be checked within this method.

(in contrast to convergence of ChPT, convergence of perturbative QCD)

4. H-dibaryon

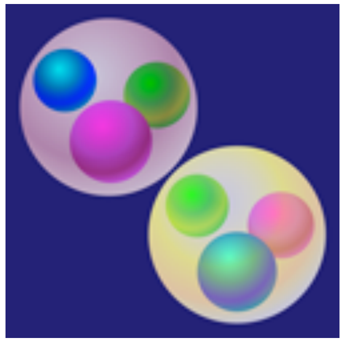
H-dibaryon:
a possible six quark state(uuddss)
predicted by the model but not observed yet.



http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.106.162001

Binding baryons on the lattice

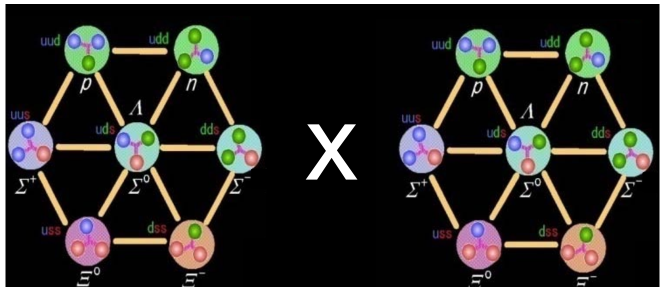
April 26, 2011



Baryon Potentials in the flavor SU(3) symmetric limit

$m_u = m_d = m_s$

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underbrace{27 + 8s + 1}_{\text{Symmetric}} + \underbrace{10^* + 10 + 8a}_{\text{Anti-symmetric}}$$

6 independent potentials in flavor-basis

$$V^{(27)}(r),\ V^{(8s)}(r),\ V^{(1)}(r) \longleftarrow {}^1S_0$$

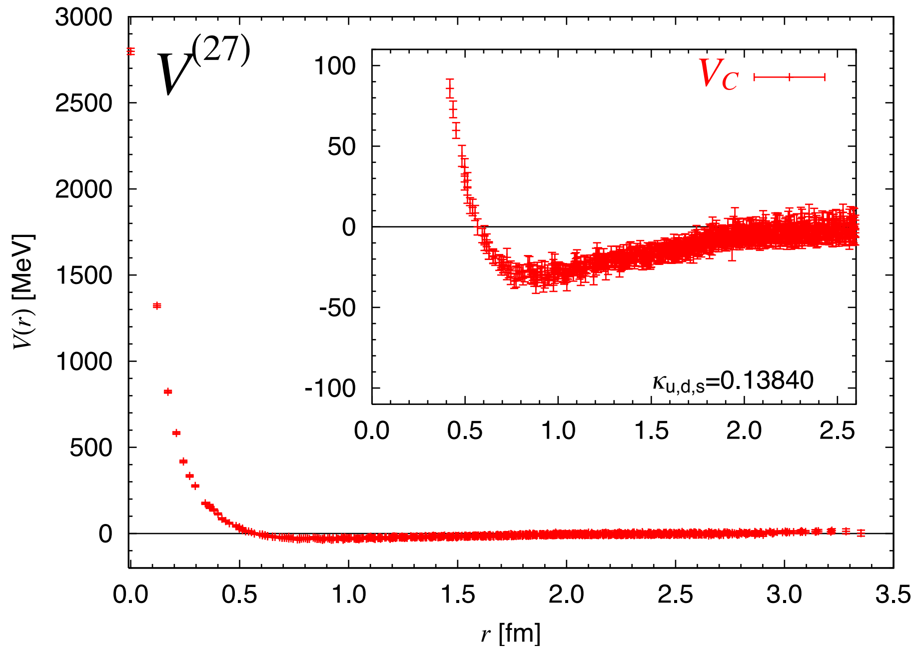
$$V^{(10^*)}(r),\ V^{(10)}(r),\ V^{(8a)}(r) \longleftarrow {}^3S_1$$

3-flavor QCD a=0.12 fm

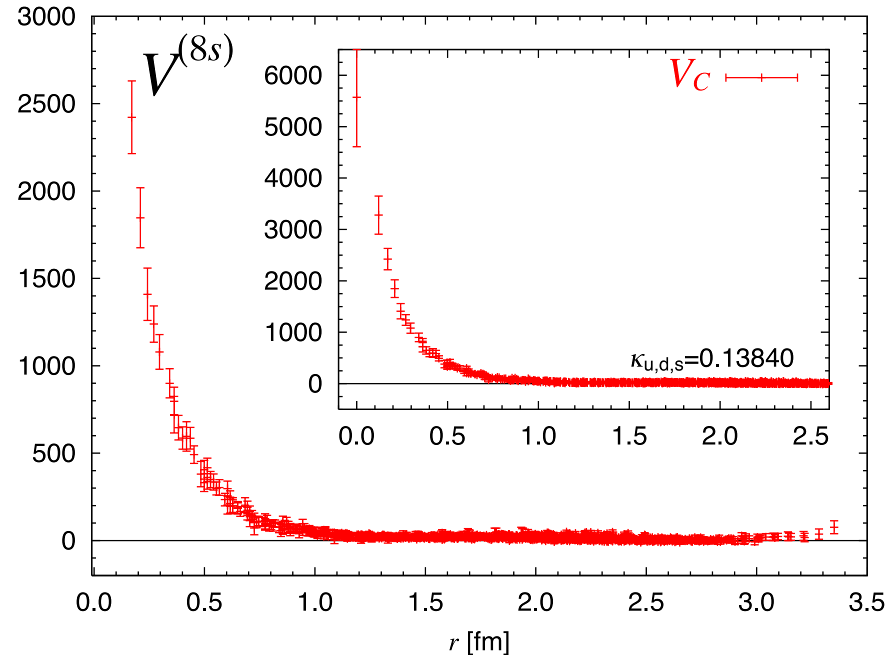
Inoue *et al.* (HAL QCD Coll.), PTP124(2010)591 L=2 fm

Inoue *et al.* (HAL QCD Coll.), NPA881(2012)28 L=2-4 fm

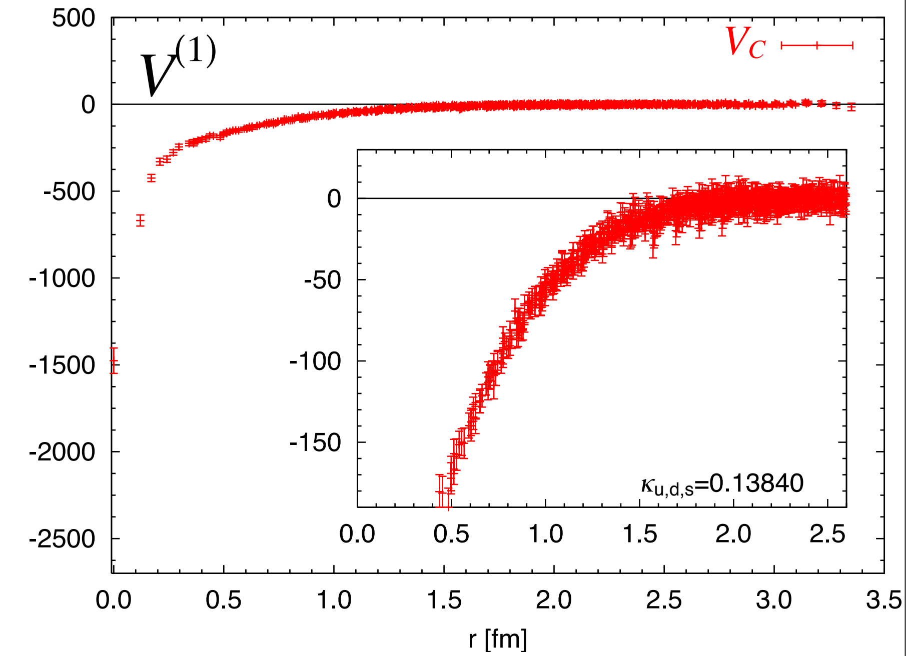
$L \simeq 4$ fm, $\quad m_\pi \simeq 470$ MeV



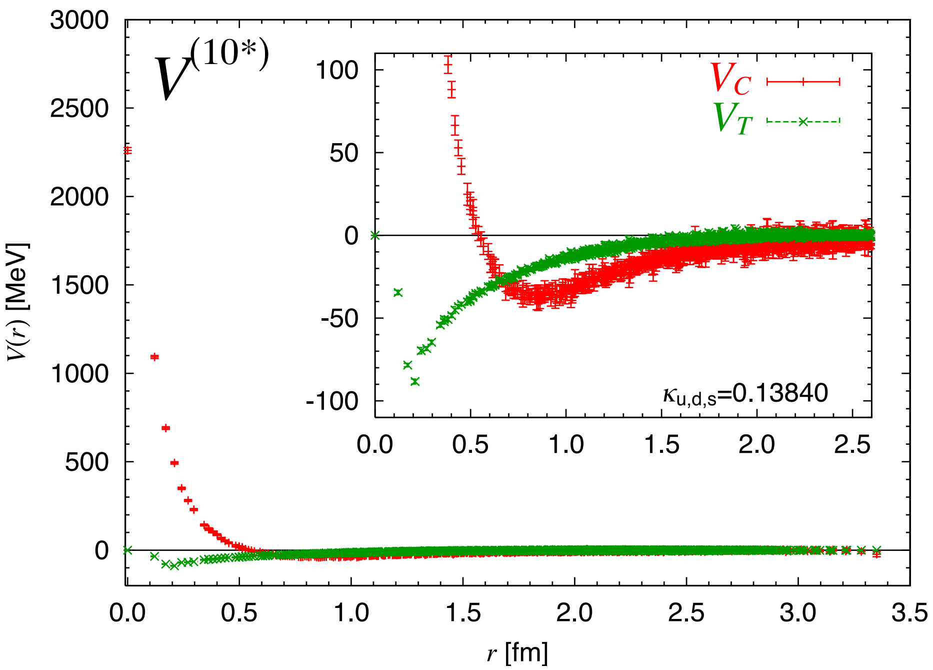
same as NN



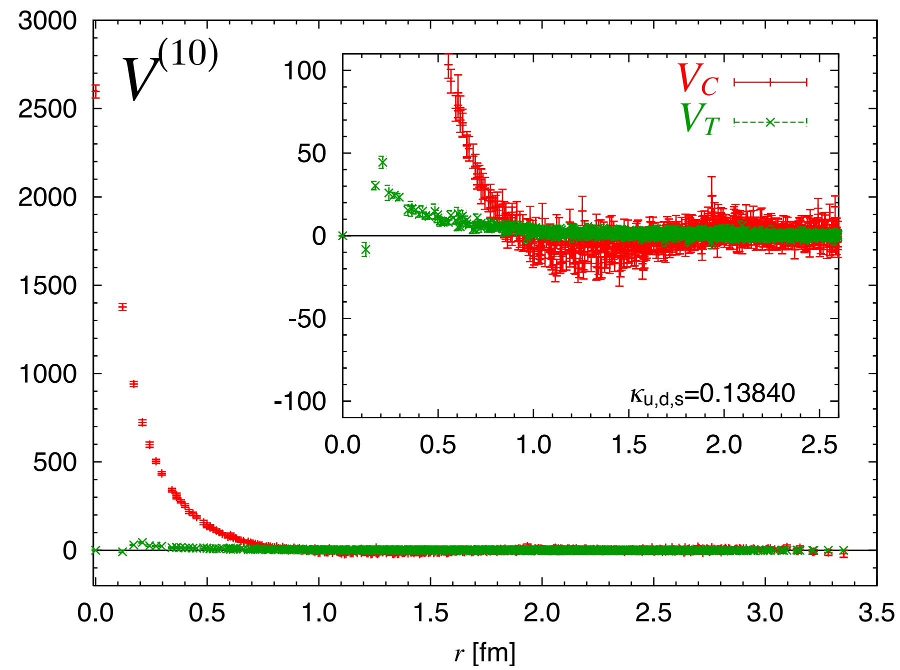
8s: strong repulsive core. repulsion only.



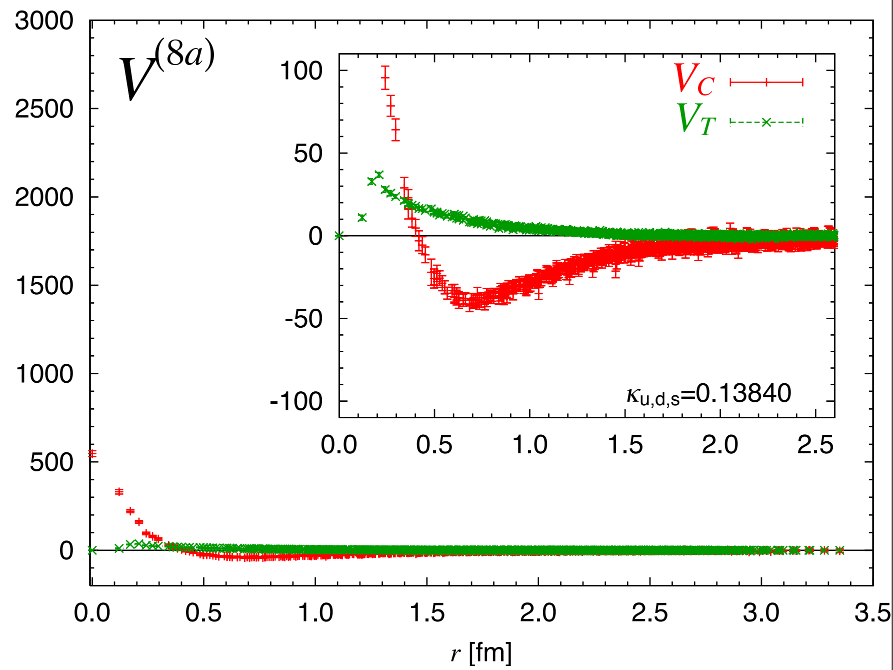
1: attractive instead of repulsive core ! attraction only .



same as NN



10: strong repulsive core. weak attraction.



8a: weak repulsive core.
strong attraction.

Flavor dependences of BB interactions become manifest in SU(3) limit !

H-dibaryon in the flavor SU(3) symmetric limit

a=0.12 fm

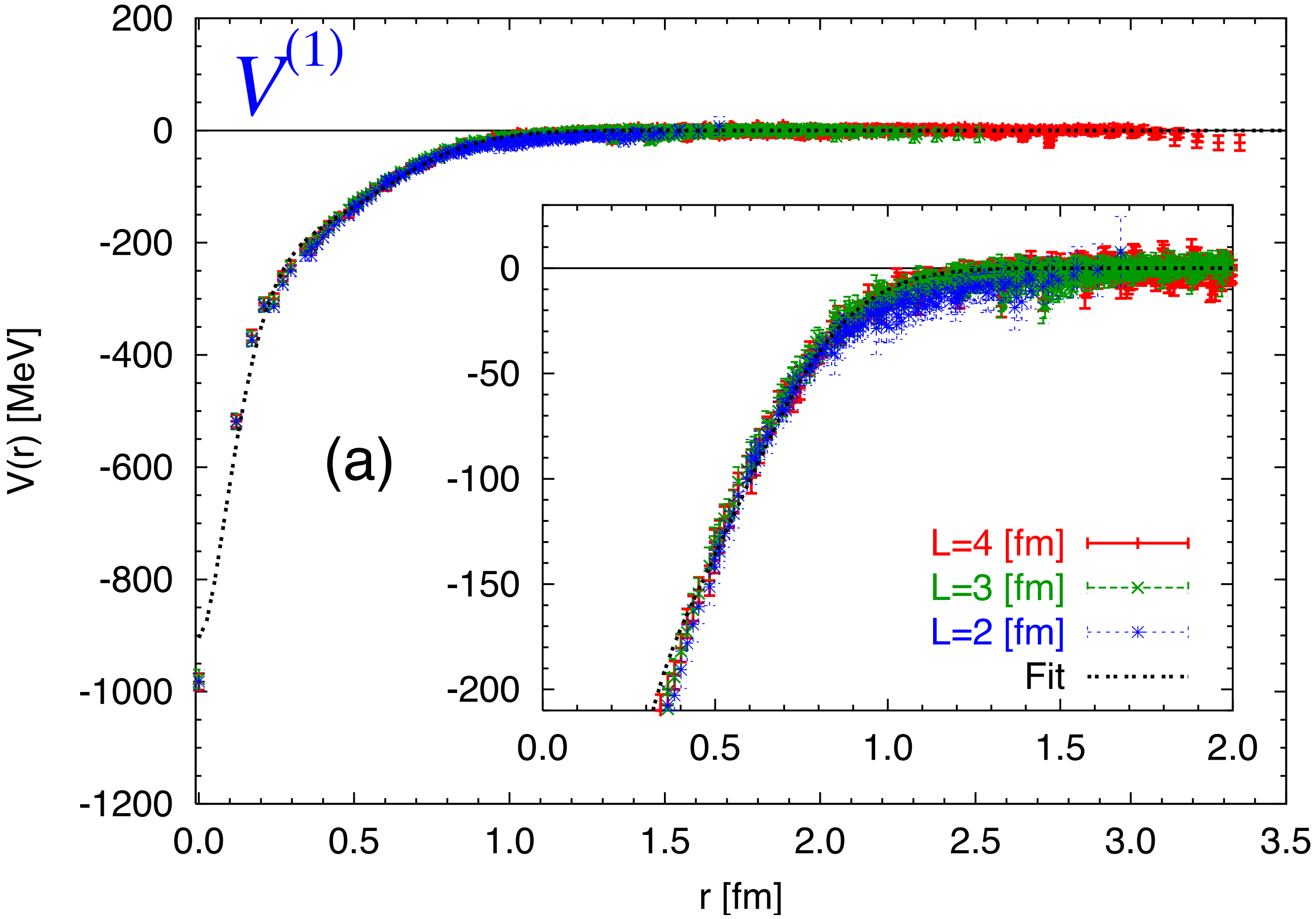
Inoue *et al.* (HAL QCD Coll.), PRL106(2011)162002

Attractive potential
in the flavor singlet channel

possibility of a bound state (H-dibaryon)

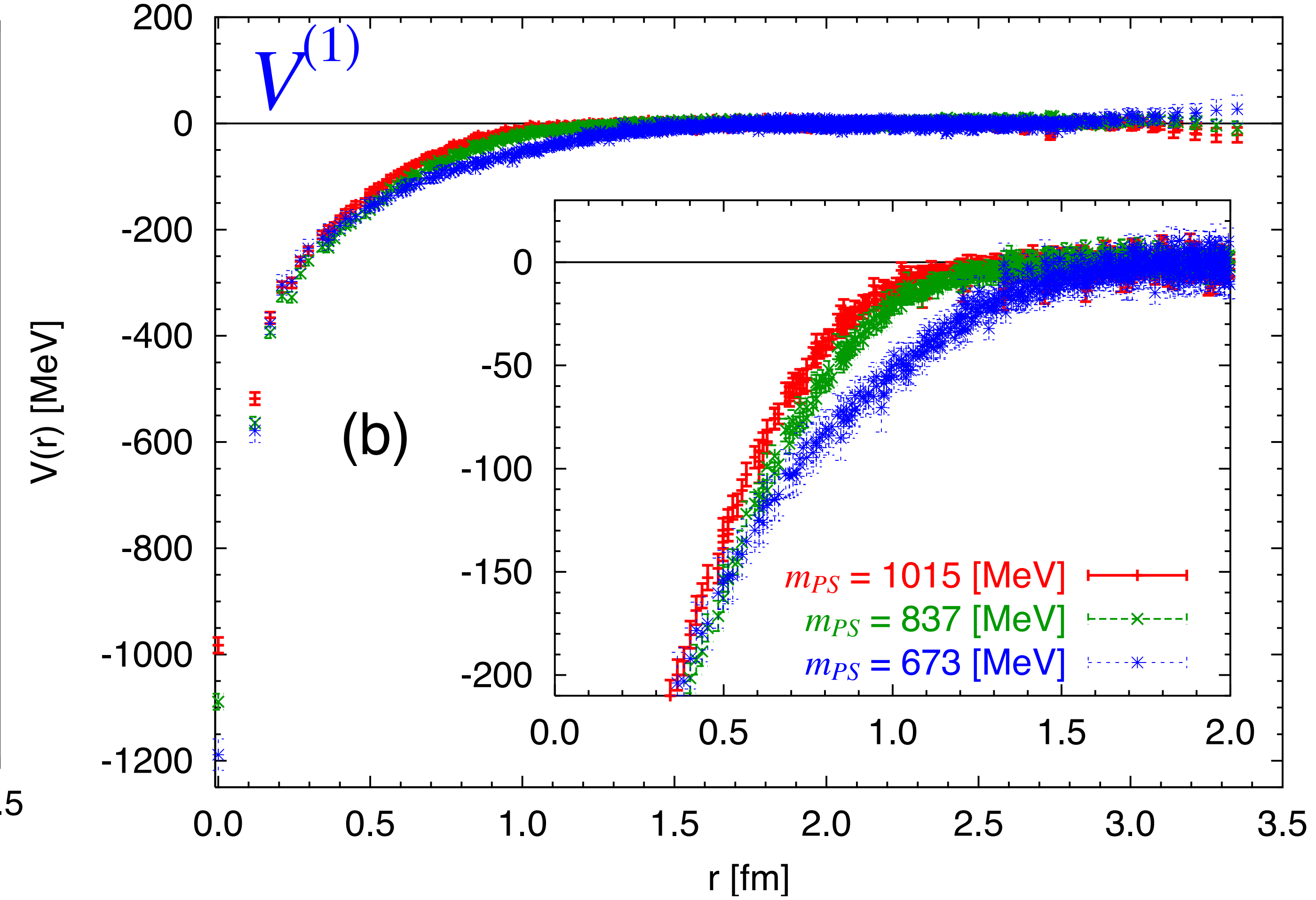
$$\Lambda\Lambda - N\Xi - \Sigma\Sigma$$

volume dependence



L=3 fm is enough for the potential.

pion mass dependence

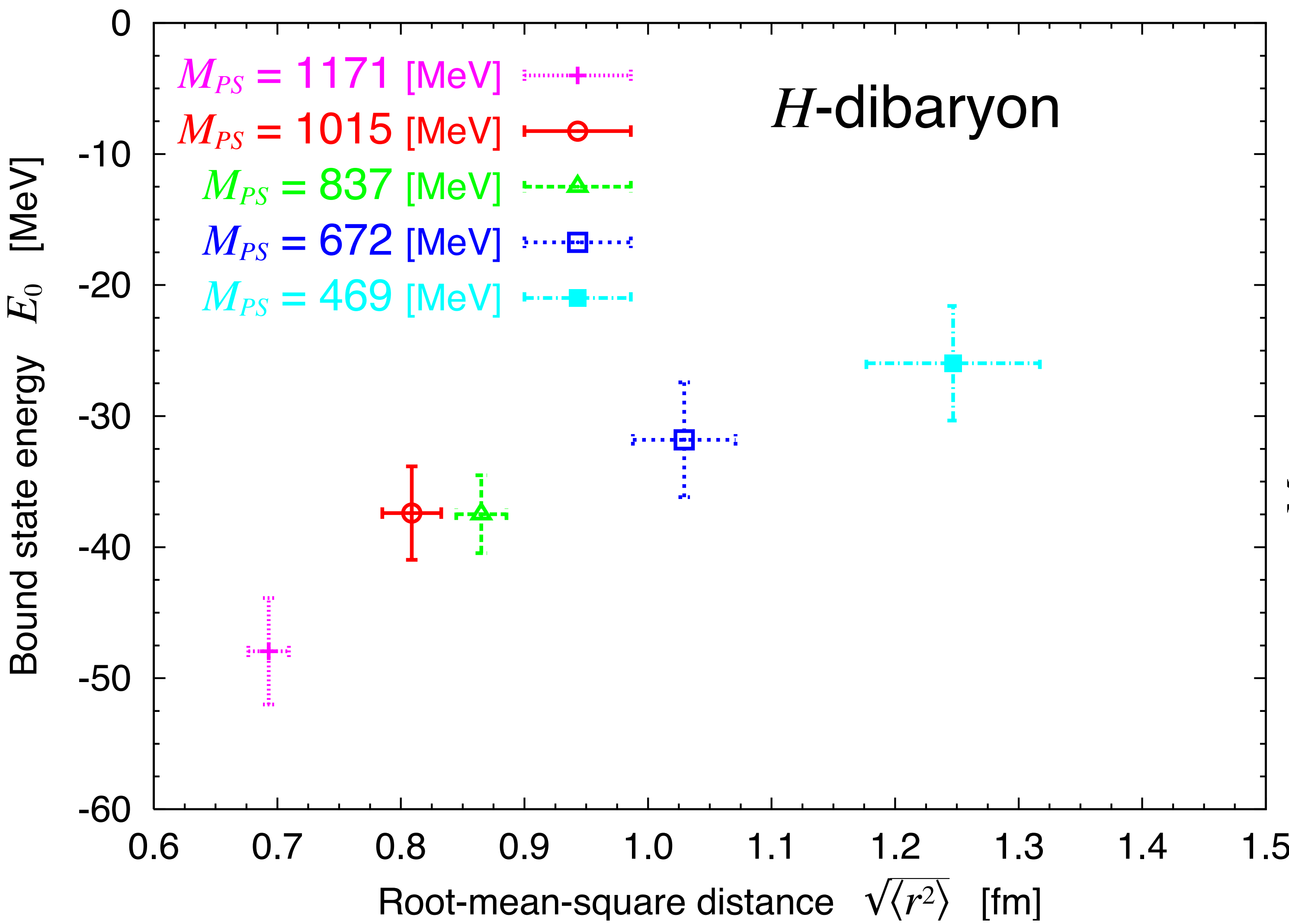


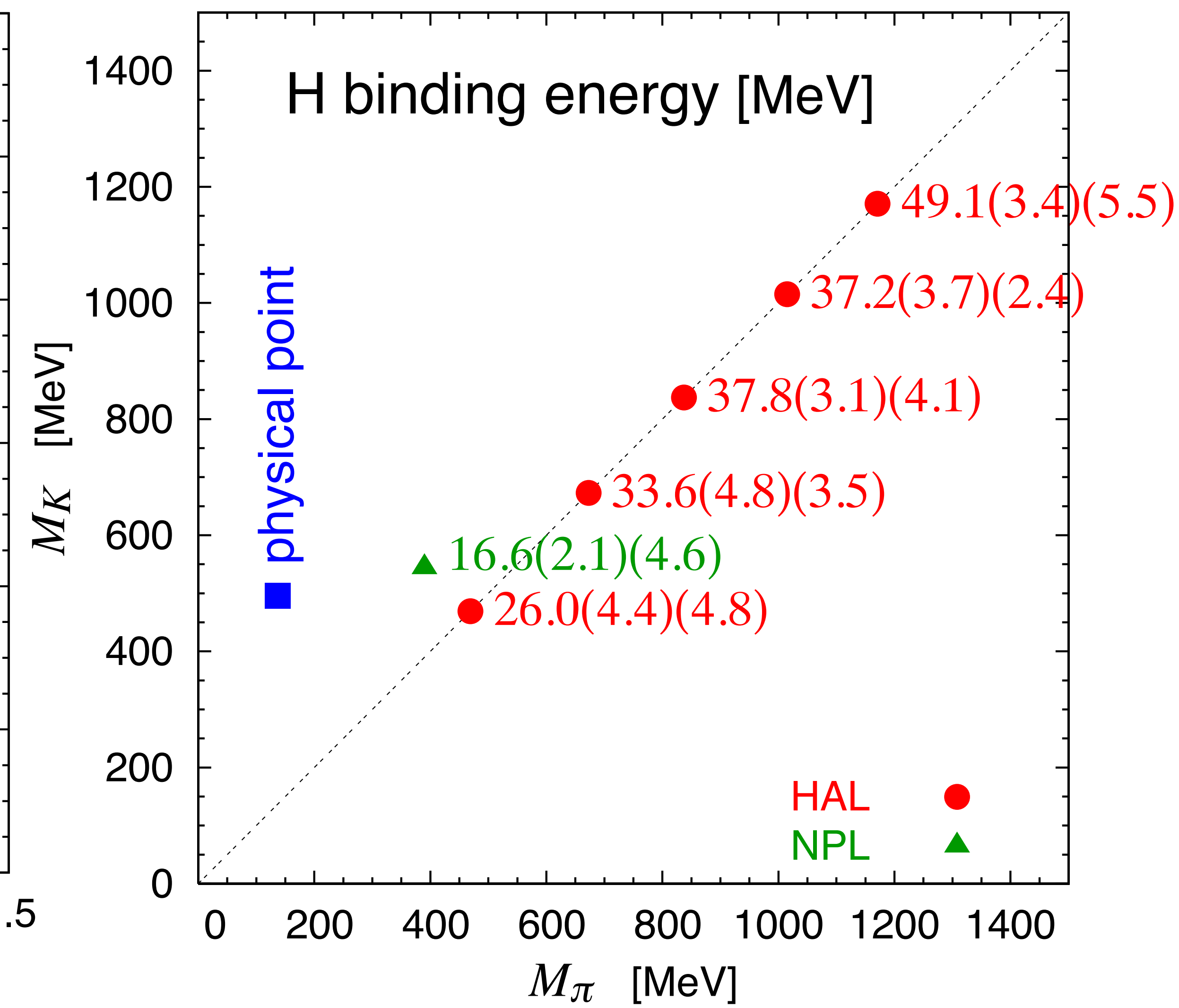
lighter the pion mass, stronger the attraction

fit potentials at L=4 fm by $V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$

Solve Schroedinger equation
in the infinite volume

One bound state (H-dibaryon) exists.





An H-dibaryon exists in the flavor SU(3) limit.
Binding energy = 25-50 MeV at this range of quark mass.
A mild quark mass dependence.

Real world ?

5. Conclusion

- HAL QCD scheme is shown to be a promising method to extract hadronic interactions in lattice QCD.
  - ground state saturation is not required.
  - Calculate potential (matrix) in lattice QCD on a finite box.
  - Calculate phase shift by solving (coupled channel) Shroedinger equation in infinite volume.
  - bound/resonance/scattering
- Future directions
  - calculations at the physical pion mass on “K-computer”
  - hyperon interactions with the SU(3) breaking
  - Baryon-Meson, Meson-Meson
  - Exotic other than H such as penta-quark, X, Y etc.
  - 3 Nucleon forces
  - Other applications ? (weak interaction ?)

Thank you !