In-medium QCD forces at high temperature

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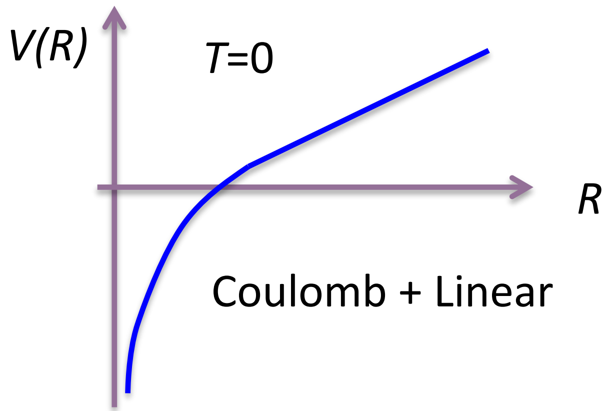
Y.Akamatsu, A.Rothkopf, PRD85(2012),105011 (arXiv:1110.1203[hep-ph])
Y.Akamatsu, arXiv:1209.5068[hep-ph]

Contents

1. INTRODUCTION

Confinement & Deconfinement

- Vacuum & In-Medium Potentials

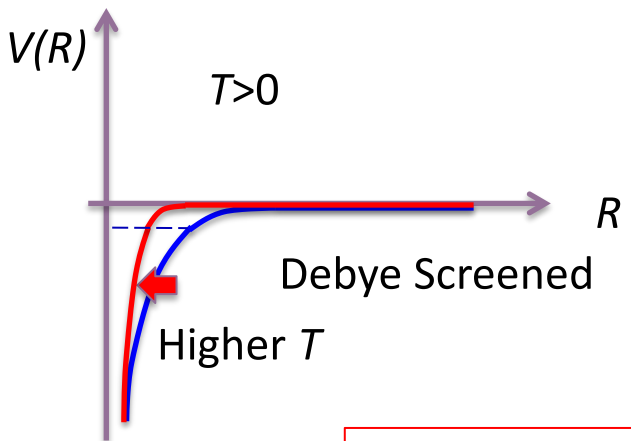


Singlet channel

$$V(R) = KR - \frac{4}{3}\frac{\alpha_s}{R} + O\left(1/M^2\right)$$

String tension K ~ 0.9GeVfm^{-1}

Mass spectra ($c\bar{c}$, $b\bar{b}$)



Debye screened potential

$$V(R) = -\frac{4}{3}\frac{\alpha_s}{R}\exp\left(-\omega_D R\right) + O\left(1/M^2\right)$$

Debye mass ω_D ~ gT (HTL)

The Schrödinger equation

Existence of bound states ($c\bar{c}$, $b\bar{b}$)

→ J/Ψ suppression in heavy-ion collisions

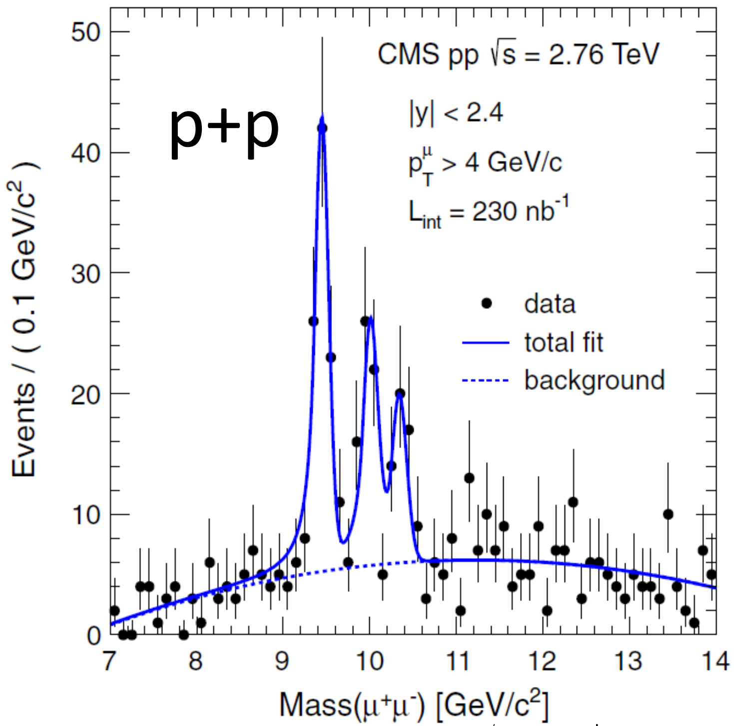
Matsui & Satz (86)

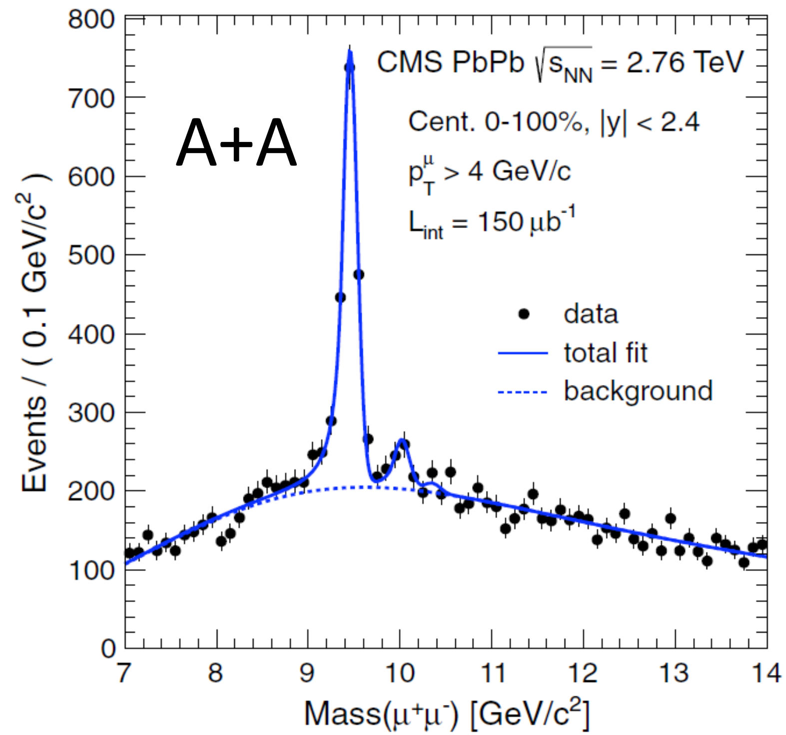
What is the in-medium potential?

Quarkonium Suppression

- Sequential melting of bottomonia

CMS





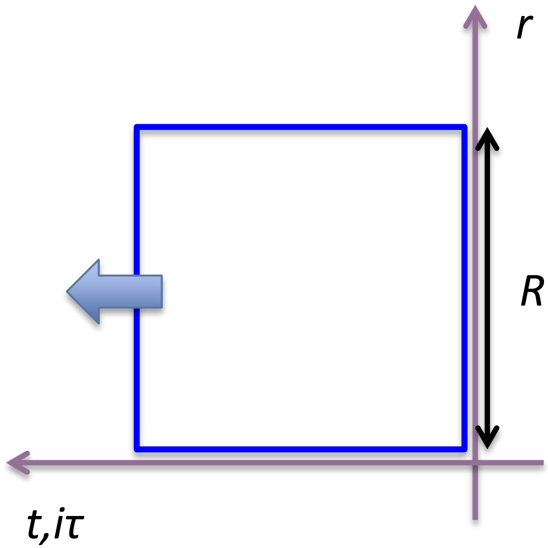
$$\frac{\Upsilon(2S)/\Upsilon(1S)\big|_{PbPb}}{\Upsilon(2S)/\Upsilon(1S)\big|_{pp}} = 0.21 \pm 0.07(\text{stat}) \pm 0.02(\text{syst})$$

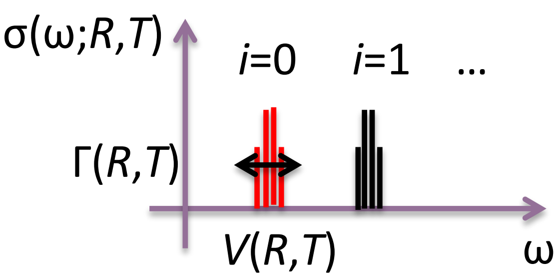
$$\frac{\Upsilon(3S)/\Upsilon(1S)\big|_{PbPb}}{\Upsilon(3S)/\Upsilon(1S)\big|_{pp}} = 0.06 \pm 0.06(\text{stat}) \pm 0.06(\text{syst}) < 0.17(95\%\text{CL})$$

2. IN-MEDIUM QCD FORCES

In-Medium Potential

- Definition





T>0, M=∞

In analogy to vacuum case

$$\left\langle \Psi(t;R) \right\rangle_T \propto \left\langle J(t;R) J^\dagger(0;R) \right\rangle_T$$

$$\propto \sum_{n,m} \left| \left\langle m \right| J^\dagger(0;R) \left| n \right\rangle \right|^2 e^{-\beta E_n(R)} \exp\left[i\{E_n(R) - E_m(R)\} t \right]$$

$$\sim \sum_{\substack{\alpha=(n,m) \\ \in \text{lowest peak}}} W_{i=0}(\alpha;T) \exp\left[-iE_\alpha(R) t \right]$$

Long time dynamics

$$\sim \exp\left[-i\{V(R,T) - i\Gamma(R,T)/2\} t \right]$$

Lorentzian fit of σ(ω;*R*,*T*)

$$G(\tau;R,T) = \left\langle J(-i\tau;R) J^\dagger(0;R) \right\rangle_T \quad \sim \int D[A] e^{-S(A;T)}$$

$$= \sum_i \sum_{\alpha \in \text{peak}(i)} W_i(\alpha;T) \exp\left[-E_\alpha(R) \tau \right]$$

($0<\tau<\beta$)

Spectral decomposition

In-Medium Potential

- Complex Potential

Laine et al (07), Beraudo et al (08), Bramilla et al (10), Rothkopf et al (12).

$$\langle \Psi(t;R) \rangle_T \sim \sum_{\alpha \in \text{lowest peak}} W_{i=0}(\alpha;T) \exp\left[-iE_\alpha(R)t\right]$$

Long time dynamics

$$\sim \exp\left[-i\{V(R,T) - i\Gamma(R,T)/2\}t\right]$$

Lorentzian fit of σ(ω;*R*,*T*)

$$\sim \int D\Theta(s) \exp\left[-\int_0^t ds\, \Theta(s)^2 / \Gamma(R,T)\right] \exp\left[-i\int_0^t ds \{V(R,T) + \Theta(s)\}\right]$$

Suggests stochastic & unitary description

In-Medium Potential

- Stochastic Potential

Akamatsu & Rothkopf ('12)

$$\Psi(t+\Delta t, R) = \exp\left[-i\Delta t\{V(R) + \Theta(t,R)\}\right]\Psi(t,R),$$

(*T* omitted)

$$\langle \Theta(t,R) \rangle = 0, \quad \langle \Theta(t,R)\Theta(t',R') \rangle = \Gamma(R,R')\delta_{tt'} / \Delta t,$$

Introduce noise field Θ(*t*,*R*)

Density matrix: Non-local correlation relevant

$$\rho(t, R_1, R_2) \equiv \langle \Psi^*(t, R_1)\Psi(t, R_2) \rangle$$

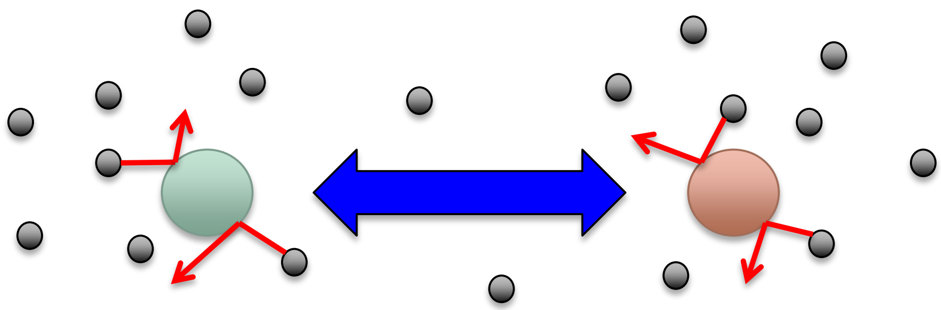
$$i\frac{\partial}{\partial t}\Psi(t,R) = \left\{V(R) - \frac{i}{2}\Gamma(R,R) + \Xi(t,R)\right\}\Psi(t,R),$$

Imaginary potential = Local correlation

$$\Xi(t,R) \equiv \Theta(t,R) - \frac{i\Delta t}{2}\left\{\Theta(t,R)^2 - \langle \Theta(t,R)^2 \rangle\right\}, \quad \langle \Xi(t,R) \rangle = 0$$

In-Medium Forces

- $M<\infty$



Debye screened force

$M=\infty$ +

Fluctuating force

(Stochastic) Potential force
→Hamiltonian dynamics

$M<\infty$ Drag force

Non-potential force
→Not Hamiltonian dynamics

Langevin dynamics

How to describe in-medium QCD forces?

3. INFLUENCE FUNCTIONAL OF QCD

Open Quantum System

- Basics

$$\begin{cases} \text{sys = heavy quarks} \\ \text{env = gluon, light quarks} \end{cases}$$

Hilbert space $\mathrm{H}_{\mathrm{tot}} = \mathrm{H}_{\mathrm{sys}} \otimes \mathrm{H}_{\mathrm{env}}$

von Neumann equation $i\dfrac{d}{dt}\hat{\rho}_{\mathrm{tot}}(t) = \left[\hat{H}_{\mathrm{tot}}, \hat{\rho}_{\mathrm{tot}}(t)\right]$

⬇ Trace out the environment

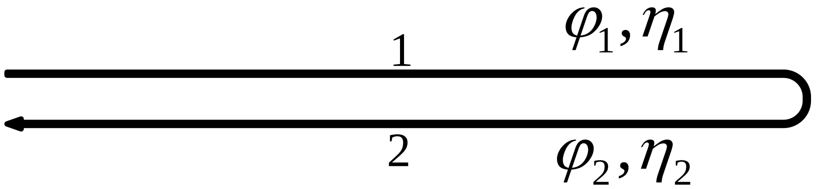
Reduced density matrix $\hat{\rho}_{\mathrm{red}}(t) \equiv \mathrm{Tr}_{\mathrm{env}}\left[\hat{\rho}_{\mathrm{tot}}(t)\right]$

Master equation $i\dfrac{d}{dt}\hat{\rho}_{\mathrm{red}}(t) = ?$

(Markovian limit)

Closed-Time Path

$$\rho[\varphi_1^{\rm ini},\varphi_2^{\rm ini}]$$



- QCD

$$Z[\eta_1,\eta_2] \sim \int D[\psi_{1,2} q_{1,2} A_{1,2}]\rho[\psi_1^* q_1^* A_1|^{\rm ini}, \psi_2 q_2 A_2|^{\rm ini}]$$

$$\times \exp\left[iS[\psi_1] - iS[\psi_2] + i\int \psi_1\eta_1 - i\int \psi_2\eta_2\right]$$

$$\times \exp\left[iS[q_1 A_1] - iS[q_2 A_2] + ig\int j_1 A_1 - ig\int j_2 A_2\right]$$

$$\rho_{\rm tot} = \rho_{\rm env}^{\rm eq} \otimes \rho_{\rm sys}$$ Factorized initial density matrix

$$\to \rho_{\rm tot}[\psi_1^* q_1^* A_1|^{\rm ini}, \psi_2 q_2 A_2|^{\rm ini}] = \rho_{\rm env}^{\rm eq}[q_1^* A_1|^{\rm ini}, q_2 A_2|^{\rm ini}] \cdot \rho_{\rm sys}[\psi_1^{*\rm ini}, \psi_2^{\rm ini}]$$

Influence functional — Feynman & Vernon (63)

$$= Z_{qA}[j_1, j_2] \equiv \exp\left[iS^{\rm FV}[j_1, j_2]\right]$$

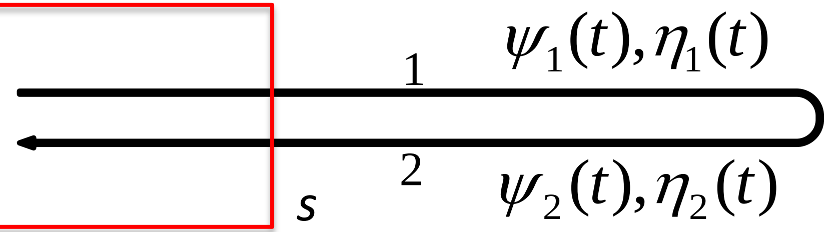
$$= \exp\left[-g^2/2\int j_1 G_A^{\rm F} j_1 + j_2 G_A^{\tilde{\rm F}} j_2 - j_1 G_A^{>} j_2 - j_2 G_A^{<} j_1 + \int g^3 G_A^{(3)} jjj + g^4 G_A^{(4)} jjjj + \cdots\right]$$

Influence Functional

- Open Quantum System

$$Z[\eta_1,\eta_2] \sim \int D[\psi_{1,2}]\rho_{\rm sys}[\psi_1^{*\rm ini},\psi_2^{\rm ini}]$$
$$\times \exp\left[iS[\psi_1]-iS[\psi_2]+iS^{\rm FV}[j_1,j_2]+i\int\psi_1\eta_1-i\int\psi_2\eta_2\right]$$

$$\rho_{\rm sys}[\psi_1^{*\rm ini},\psi_2^{\rm ini}]$$



Path integrate until *s*, with boundary condition $\psi_1(s)=\psi_1,\psi_2(s)=\psi_2$

$$=\rho_{\rm red}[s,\psi_1^*,\psi_2]=\langle\psi_1^*|\hat{\rho}_{\rm red}(s)|\psi_2\rangle$$

Influence Functional

- Functional Master Equation

Effective initial wave function

$$\rho_{\rm red}[t,\psi_1^*,\psi_2] \sim \int_{-\infty}^{t,\psi_1^*,\psi_2} D[\psi_{1,2}]\,\boxed{\rho_{\rm sys}[\psi_1^{*\rm ini},\psi_2^{\rm ini}]}$$

$$\times\exp\boxed{\left[iS[\psi_1]-iS[\psi_2]+\boxed{iS^{\rm FV}[j_1,j_2]}\right]}$$

Effective action S_{1+2} → Single time integral

Long-time behavior (Markovian limit)

Analogy to the Schrödinger wave equation

Functional differential equation

$$i\frac{\partial}{\partial t}\rho_{\rm red}[t,\psi_1^*,\psi_2] = H_{1+2}^{\rm func}[\psi_1^*,\psi_2]\rho_{\rm red}[t,\psi_1^*,\psi_2]$$

How does this formalism work in perturbation theory in non-relativistic limit?

4. DYNAMICAL EQUATIONS

Density Matrix

- Coherent State

$$\psi_1 \sim (Q_1, Q_{1c}^*)$$
$$\psi_2 (\equiv \tilde{\psi}_2^*) \sim (Q_2, Q_{2c}^*)$$

$$\rho_{\rm red}\left[t, Q_{1(c)}^*, \tilde{Q}_{2(c)}^*\right] = \left\langle Q_{1(c)}^* \middle| \hat{\rho}_{\rm red}(t) \middle| \tilde{Q}_{2(c)}^* \right\rangle$$

$$\left\langle Q_{1(c)}^* \right| = \langle \Omega | \exp\left[-\int_{\vec{x}} \left\{ \hat{Q} Q_1^* + \hat{Q}_c Q_{1(c)}^* \right\} \right]$$

$$\left| \tilde{Q}_{2(c)}^* \right\rangle = \exp\left[-\int_{\vec{x}} \left\{ \tilde{Q}_2^* \hat{\tilde{Q}}^\dagger + \tilde{Q}_{2c}^* \hat{\tilde{Q}}_c^\dagger \right\} \right] |\Omega\rangle$$

Source for HQs

$$\frac{\delta}{\delta Q_1^*(\vec{x})} \left\langle Q_{1(c)}^* \right| \Bigg|_{Q_{1(c)}^*=0} = \langle \Omega | \hat{Q}(\vec{x})$$

$$\frac{\delta}{\delta \tilde{Q}_2^*(\vec{x})} \left| \tilde{Q}_{2(c)}^* \right\rangle \Bigg|_{\tilde{Q}_{2(c)}^*=0} = -\hat{Q}^\dagger(\vec{x}) |\Omega\rangle$$

Density Matrix

- A few HQs

One HQ

$$\rho_Q(t,\vec{x},\vec{y}) = \left\langle \vec{x} \middle| \hat{\rho}_Q(t) \middle| \vec{y} \right\rangle \propto \left\langle \Omega \middle| \hat{Q}(\vec{x}) \hat{\rho}_{\rm red}(t) \hat{Q}^\dagger(\vec{y}) \middle| \Omega \right\rangle$$

$$= -\frac{\delta}{\delta Q_1^*(\vec{x})} \frac{\delta}{\delta \tilde{Q}_2^*(\vec{y})} \rho_{\rm red}\left[t, Q_{1(c)}^*, \tilde{Q}_{2(c)}^*\right]_{Q_{1(c)}^* = \tilde{Q}_{2(c)}^* = 0}$$

Similar for two HQs, …

$$\rho_{QQ_c}(t,\vec{x}_1,\vec{x}_2,\vec{y}_1,\vec{y}_2),\cdots$$

Master Equation

- Functional Master Equation

$$i\frac{\partial}{\partial t}\rho_{\mathrm{red}}[t, Q^*_{1(c)}, \tilde{Q}^*_{2(c)}] = H^{\mathrm{func}}_{1+2}[Q^*_{1(c)}, \tilde{Q}^*_{2(c)}]\rho_{\mathrm{red}}[t, Q^*_{1(c)}, \tilde{Q}^*_{2(c)}]$$

Functional differentiation $\frac{\delta}{\delta Q^*_1(\vec{x})}\frac{\delta}{\delta \tilde{Q}^*_2(\vec{y})}$

Color traced

Master equation

$$i\partial_t \rho_Q(t,\vec{x},\vec{y}) = \left\{ \left(a - a^*\right)M + \left(-\frac{\nabla_x^2 - \nabla_y^2}{2M} \right) \right\} \rho_Q(t,\vec{x},\vec{y})$$

$$+ C_F \left\{ -iD(\vec{x}-\vec{y}) + \frac{\vec{\nabla}_x D(\vec{x}-\vec{y})}{4T} \cdot \frac{\vec{\nabla}_x - \vec{\nabla}_y}{iM} \right\} \rho_Q(t,\vec{x},\vec{y})$$

Reduces to Caldeira-Leggett master equation at x~y

Master Equation

- HQ Number Conservation

$$\mathrm{Tr}\hat{\rho}_Q(t) = \int_{\vec{x}} \rho_Q(t,\vec{x},\vec{x})$$

$$i\frac{d}{dt}\mathrm{Tr}\hat{\rho}_Q(t) = \int_{\vec{x},\vec{y}} \delta(\vec{x}-\vec{y})\Big(i\partial_t \rho_Q(t,\vec{x},\vec{y})\Big) = 0$$

- Ehrenfest Equation

$$\frac{d}{dt}\langle\vec{x}\rangle = \frac{\langle\vec{p}\rangle}{M},$$

$$\frac{d}{dt}\langle\vec{p}\rangle = -\frac{\gamma}{2MT}\langle\vec{p}\rangle, \qquad \gamma = \frac{C_F}{3}\nabla^2 D(x)\Big|_{x=0} = -\frac{g(T)^2 C_F}{9}\nabla^2 \tilde{G}^{>}_{00,aa}(\omega=0,x)\Big|_{x=0}$$

$$\frac{d}{dt}\langle E\rangle = -\frac{\gamma}{MT}\left(\langle E\rangle - \frac{3T}{2}\right). \qquad = \frac{g(T)^2 C_F}{9}\int\frac{d^3k}{(2\pi)^3}k^2\tilde{G}^{>}_{00,aa}(\omega=0,k)$$

Moore et al (05,08,09)

Other Results

- Complex Potential

$$\left\langle \Psi(t;\vec{x},\vec{y}) \right\rangle_T \propto \left\langle J(t;\vec{x},\vec{y}) J^{\dagger}(0;\vec{x},\vec{y}) \right\rangle_T$$

$$\propto \frac{\delta^2}{\delta Q_1^*(\vec{x}) \delta Q_1^*(\vec{y})} \rho_{\text{red}} \left[Q_{1(c)}^*, \tilde{Q}_{2(c)}^*, t \right]_{Q_{1(c)}^* = \tilde{Q}_{2(c)}^* = 0}$$

⬇ Time-evolution equation + Project on singlet state

$$V_{\text{singlet}}(R) = 2(a-1)M - C_F V(R) = -\frac{g(T)^2 C_F}{4\pi} \left(\omega_D + \frac{e^{-\omega_D R}}{R} + iT\phi(\omega_D R) \right)$$

Laine et al (07), Beraudo et al (08), Brambilla et al (10)

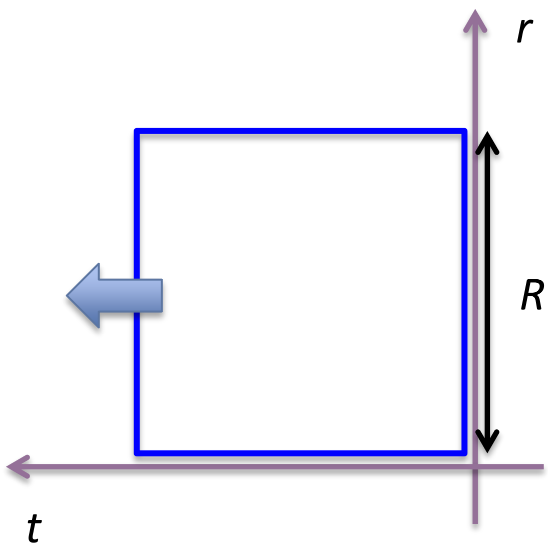
5. SUMMARY & OUTLOOK

- Quantum Dynamics of HQs in Medium
  - Stochastic potential, drag force
- Non-Equilibrium Quantum Field Theory
  - Open quantum system, closed-time path, influence functional
  - Functional master equation, master equation, etc.
- Non-Perturbative Region
  - Model the renormalized effective Hamiltonian
  - Higher-order perturbative analyses (process involving real gluons)
  - Application to phenomenology

BACKUP

In-Medium Potential

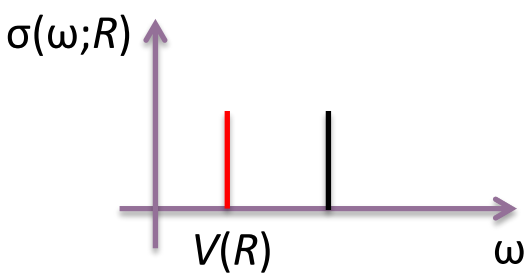
- Definition



T=0, *M*=∞

$$\Psi(t;R) \propto \langle \mathrm{vac} | J(t;R) J^\dagger(0;R) | \mathrm{vac} \rangle$$
$$= \sum_m \left| \langle m | J^\dagger(0;R) | \mathrm{vac} \rangle \right|^2 \exp\left[-iE_m(R)t \right]$$
$$\sim \exp\left[-iE_{\min}(R)t \right] = \mathrm{e}^{-iV(R)t}$$

Long time dynamics



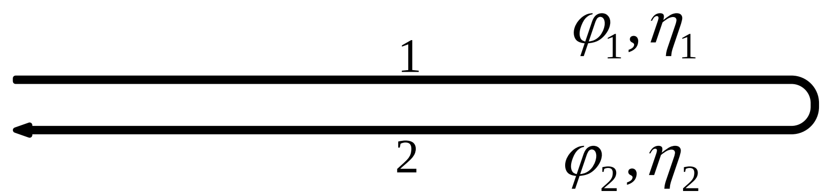
$$G(\tau;R) = \langle \mathrm{vac} | J(-i\tau;R) J^\dagger(0;R) | \mathrm{vac} \rangle \sim \int D[A] \mathrm{e}^{-S(A)} \square$$
$$\sim \exp\left[-E_{\min}(R)\tau \right] = \mathrm{e}^{-\tau V(R)}$$

V(*R*) from large τ behavior

Closed-Time Path

- Basics

$\rho[\varphi_1^{\mathrm{ini}}, \varphi_2^{\mathrm{ini}}]$



Partition function

$$
\begin{aligned}
Z[\eta_1, \eta_2] &= \mathrm{Tr}\left(\hat{U}(\infty, -\infty; \eta_1)\hat{\rho}\hat{U}(\infty, -\infty; \eta_2)^\dagger\right) \\
&= \mathrm{Tr}\left(\hat{U}(\infty, -\infty; \eta_2)^\dagger \hat{U}(\infty, -\infty; \eta_1)\hat{\rho}\right) \\
&\sim \int D\varphi_{1,2}\,\rho[\varphi_1^{\mathrm{ini}}, \varphi_2^{\mathrm{ini}}]\exp\left[iS[\varphi_1] - iS[\varphi_2] + i\int \eta_1\varphi_1 - i\int \eta_2\varphi_2\right]
\end{aligned}
$$

$$
\prod_i \frac{\delta}{\delta\eta_i(x_i)} \ln Z[\eta_1, \eta_2]\Bigg|_{j_{1,2}=0} \propto \left\langle \mathrm{T_C} \prod_i \hat{\varphi}_i(x_i) \right\rangle_{T,\mathrm{conn}}
$$

$$
\frac{\delta^2}{\delta\eta_1(x_1)\delta\eta_1(x_2)} \ln Z[\eta_1, \eta_2]\Bigg|_{j_{1,2}=0} \propto \left\langle \mathrm{T}\hat{\varphi}(x_1)\hat{\varphi}(x_2) \right\rangle_{T,\mathrm{conn}} = G_\varphi^{\mathrm{F}}(x_1, x_2)
$$

$$
\frac{\delta^2}{\delta\eta_1(x_1)\delta\eta_2(x_2)} \ln Z[\eta_1, \eta_2]\Bigg|_{j_{1,2}=0} \propto \left\langle \hat{\varphi}(x_2)\hat{\varphi}(x_1) \right\rangle_{T,\mathrm{conn}} = G_\varphi^{<}(x_1, x_2)
$$

…

Approximations

- Leading-Order Perturbation

Influence Functional

$$\exp\left[iS^{\mathrm{FV}}[j_1, j_2]\right]$$
$$\approx \exp\left[-g^2/2\int j_1 G_A^{\mathrm{F}} j_1 + j_2 G_A^{\tilde{\mathrm{F}}} j_2 - j_1 G_A^{>} j_2 - j_2 G_A^{<} j_1\right]$$

Expansion up to 4-Fermi interactions

$$G_A^{\mathrm{F}}(x_1 - x_2) = \left\langle \mathrm{T}\hat{A}(x_1)\hat{A}(x_2)\right\rangle_T, \quad G_A^{<}(x_1 - x_2) = \left\langle \hat{A}(x_2)\hat{A}(x_1)\right\rangle_T$$
$$G_A^{>}(x_1 - x_2) = \left\langle \hat{A}(x_1)\hat{A}(x_2)\right\rangle_T, \quad G_A^{\tilde{\mathrm{F}}}(x_1 - x_2) = \left\langle \tilde{\mathrm{T}}\hat{A}(x_1)\hat{A}(x_2)\right\rangle_T$$

Leading-order result by HTL resummed perturbation theory

Approximations

- Heavy Mass Limit

Non-relativistic kinetic term

$$S[\psi] \approx S_{\mathrm{kin}}^{\mathrm{NR}}[Q, Q_c] \quad \psi \sim (Q, Q_c^\dagger)$$

$$S_{\mathrm{kin}}^{\mathrm{NR}}[Q, Q_c] = Q^\dagger [i\partial_0 - M + \nabla^2 / 2M] Q + Q_c [i\partial_0 + M + \nabla^2 / 2M] Q_c^\dagger$$

$$\nabla Q \sim \sqrt{MT} \cdot Q$$

$$(\Leftrightarrow \nabla G \sim (g) T \cdot G)$$

Expansion up to

$$\sim \sqrt{\frac{T}{M}}$$

Non-relativistic 4-current (density, current)

$$j_a^0 = Q^\dagger t^a Q + Q_c t^a Q_c^\dagger \equiv \rho_a$$

$$\vec{j}_a \approx Q^\dagger \left(\frac{\overleftrightarrow{\nabla}}{2iM} \right) t^a Q - Q_c \left(\frac{\overleftrightarrow{\nabla}}{2iM} \right) t^a Q_c^\dagger$$

(quenched)

Approximations

- Long-Time Behavior

Time-retardation in interaction

$$\tilde{G}(\vec{x}-\vec{y},\omega) \approx \tilde{G}(\vec{x}-\vec{y},0) + \omega \tilde{G}'(\vec{x}-\vec{y},0) \equiv \overline{G}(\vec{x}-\vec{y}) + \omega \overline{G}'(\vec{x}-\vec{y})$$

$$\Leftrightarrow \quad G(x-y) \approx \overline{G}(\vec{x}-\vec{y})\delta(x^0-y^0) + i\overline{G}'(\vec{x}-\vec{y}) \frac{\partial}{\partial(x^0-y^0)} \delta(x^0-y^0)$$

Low frequency expansion

$$\int_{xy} j(x)G(x-y)j(y) \approx \int_t \int_{\vec{x}\,\vec{y}} \begin{bmatrix} \overline{G}(\vec{x}-\vec{y})\, j(t,\vec{x})\, j(t,\vec{y}) \\ -\frac{i}{2}\overline{G}'(\vec{x}-\vec{y})\{\partial_0 j(t,\vec{x})\, j(t,\vec{y}) - j(t,\vec{x})\partial_0 j(t,\vec{y})\} \end{bmatrix}$$

Using free equation of motion

Effective Action

- LO pQCD, NR Limit, Slow Dynamics

$$S_{1+2} \approx S_{\rm kin}^{\rm NR}[Q_1, Q_{1c}] - S_{\rm kin}^{\rm NR}[Q_2, Q_{2c}] + S_{\rm FV}^{\rm LONR}[j_1, j_2]$$

$$S_{\rm FV}^{\rm LONR}[j_1, j_2] = -1/2 \int_{t,\vec{x},\vec{y}} \begin{Bmatrix} V(\vec{x}-\vec{y})\rho_{1a}(t,\vec{x})\rho_{1a}(t,\vec{y}) \\ -V^*(\vec{x}-\vec{y})\rho_{2a}(t,\vec{x})\rho_{2a}(t,\vec{y}) \end{Bmatrix} + \int_{t,\vec{x},\vec{y}} \begin{Bmatrix} iD(\vec{x}-\vec{y})\rho_{1a}(t,\vec{x})\rho_{2a}(t,\vec{y}) \\ -1/4T\, \vec{\nabla} D(\vec{x}-\vec{y}) \cdot \begin{pmatrix} \vec{j}_{1a}(t,\vec{x})\rho_{2a}(t,\vec{y}) \\ +\rho_{1a}(t,\vec{x})\vec{j}_{2a}(t,\vec{y}) \end{pmatrix} \end{Bmatrix}$$

Stochastic potential (finite in M→∞)

Drag force (vanishes in M→∞)

$$-g^2 \left\{ \overline{G}^R_{00,ab}(\vec{x}-\vec{y}) + i\overline{G}^>_{00,ab}(\vec{x}-\vec{y}) \right\} \equiv V(\vec{x}-\vec{y})\delta_{ab}$$

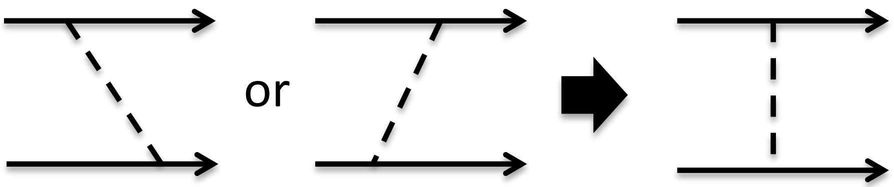
$$-g^2 \overline{G}^>_{00,ab}(\vec{x}-\vec{y}) \equiv D(\vec{x}-\vec{y})\delta_{ab} = {\rm Im}V(\vec{x}-\vec{y})\delta_{ab}$$

Hamiltonian Formalism (technical)

- Order of Operators = Time Ordered

Kinetic term $\psi_1^*(t_+,\vec{x}) \Leftrightarrow \psi_1(t_-,\vec{x}),\ \psi_2^*(t_-,\vec{x}) \Leftrightarrow \psi_2(t_+,\vec{x})$

Instantaneous interaction



Remember the original order

- Change of Variables (canonical transformation)

Make 1 & 2 symmetric

$$\tilde{\psi}_2 = (\tilde{Q}_2, \tilde{Q}_{2c}^*) = \psi_2^* = (Q_2^*, Q_{2c})$$

⇒ Determines $H_{1+2}^{\rm func}[\psi_1^*, \tilde{\psi}_2^*]$ without ambiguity

Hamiltonian Formalism (technical)

- Variables of Reduced Density Matrix

$$\rho_{\mathrm{red}}[t,\psi_1^*,\tilde{\psi}_2^*]=\left\langle \psi_1^* \middle| \hat{\rho}_{\mathrm{red}}(t) \middle| \tilde{\psi}_2^* \right\rangle$$ Latter is better (explained later)

$$\Leftrightarrow \rho_{\mathrm{red}}[t,Q_{1(c)}^*,\tilde{Q}_{2(c)}^*]=\left\langle Q_{1(c)}^* \middle| \hat{\rho}_{\mathrm{red}}(t) \middle| \tilde{Q}_{2(c)}^* \right\rangle$$

- Renormalization

Convenient to move all the functional differential operators to the right in

$$i\frac{\partial}{\partial t}\rho_{\mathrm{red}}[t,Q_{1(c)}^*,\tilde{Q}_{2(c)}^*]=H_{1+2}^{\mathrm{func}}[Q_{1(c)}^*,\tilde{Q}_{2(c)}^*]\rho_{\mathrm{red}}[t,Q_{1(c)}^*,\tilde{Q}_{2(c)}^*]$$

In this procedure, divergent contribution from Coulomb potential at the origin appears → needs to be renormalized

Functional Master Equation

- Renormalized Effective Hamiltonian

$$\hat{H}_{1+2} \approx \int_{\vec{x}} \left[aM\hat{Q}_1^\dagger \hat{Q}_1 + \hat{Q}_1^\dagger \left(-\nabla^2/2M \right) \hat{Q}_1 \right] + (\text{same for } \hat{Q}_{1c})$$

$$+ \int_{\vec{x}} \left[a^* M\hat{\tilde{Q}}_2^\dagger \hat{\tilde{Q}}_2 + \hat{\tilde{Q}}_2^\dagger \left(-\nabla^2/2M \right) \hat{\tilde{Q}}_2 \right] + (\text{same for } \hat{\tilde{Q}}_{2c})$$

$$+ \frac{1}{2} \int_{\vec{x}\vec{y}} \begin{bmatrix} V(\vec{x}-\vec{y}) N\left\{ \hat{j}_1^{a0}(\vec{x})\, \hat{j}_1^{a0}(\vec{y}) \right\} - V^*(\vec{x}-\vec{y}) N\left\{ \hat{j}_2^{a0}(\vec{x})\, \hat{j}_2^{a0}(\vec{y}) \right\} \\ -2iD(\vec{x}-\vec{y}) N\left\{ \hat{j}_1^{a0}(\vec{x})\, \hat{j}_2^{a0}(\vec{y}) \right\} \\ + \frac{1}{2T} \vec{\nabla} D(\vec{x}-\vec{y}) \cdot N\left\{ \hat{\vec{j}}_{1,NR}^{a}(\vec{x})\, \hat{j}_2^{a0}(\vec{y}) + \hat{j}_1^{a0}(\vec{x})\, \hat{\vec{j}}_2^{a}(\vec{y}) \right\} \end{bmatrix}$$

$$a \equiv 1 + \frac{C_{\mathrm{F}}}{2M} \lim_{r\to 0} V^{(T>0)}(r), \quad V^{(T>0)}(r) \equiv V(r) - V^{(T=0)}(r)$$

Functional Master Equation

- Schrödinger wave equation

Anti-commutator in functional space

$$\left\{\hat{Q}_1(\vec{x}),\hat{Q}_1^{\dagger}(\vec{y})\right\}=\left\{\hat{Q}_{1c}(\vec{x}),\hat{Q}_{1c}^{\dagger}(\vec{y})\right\}=\delta(\vec{x}-\vec{y})\Leftrightarrow Q_{1(c)}=\frac{\delta}{\delta Q_{1(c)}^{*}}$$

$$\left\{\hat{\tilde{Q}}_2(\vec{x}),\hat{\tilde{Q}}_2^{\dagger}(\vec{y})\right\}=\left\{\hat{\tilde{Q}}_{2c}(\vec{x}),\hat{\tilde{Q}}_{2c}^{\dagger}(\vec{y})\right\}=-\delta(\vec{x}-\vec{y})\Leftrightarrow \tilde{Q}_{2(c)}=-\frac{\delta}{\delta \tilde{Q}_{2(c)}^{*}}$$

➡ $\hat{H}_{1+2}\Leftrightarrow H_{1+2}^{\mathrm{func}}$

$$i\frac{\partial}{\partial t}\rho_{\mathrm{red}}[t,Q_{1(c)}^{*},\tilde{Q}_{2(c)}^{*}]=H_{1+2}^{\mathrm{func}}[Q_{1(c)}^{*},\tilde{Q}_{2(c)}^{*}]\rho_{\mathrm{red}}[t,Q_{1(c)}^{*},\tilde{Q}_{2(c)}^{*}]$$

Other Results

- Stochastic Dynamics

M=∞ : Stochastic potential

$$\exp\left[iS_{\mathrm{FV}}^{\mathrm{LONR}}[j_1, j_2]\right]$$

Debye screened potential

$$= \exp\left[-i/2\int_{t,\vec{x},\vec{y}} \mathrm{Re}\,V(\vec{x}-\vec{y})\left\{\rho_{1a}(t,\vec{x})\rho_{1a}(t,\vec{y})-\rho_{2a}(t,\vec{x})\rho_{2a}(t,\vec{y})\right\}\right]$$

$$\times\left\langle \exp\left[-i\int_{t,\vec{x},\vec{y}} \xi_a(t,\vec{x})\left\{\rho_{1a}(t,\vec{x})-\rho_{2a}(t,\vec{x})\right\}\right]\right\rangle_{\xi}$$

Fluctuation

$$\left\langle \xi_a(t,\vec{x})\xi_b(s,\vec{y})\right\rangle = -\delta_{ab}\delta(t-s)D(\vec{x}-\vec{y})$$

D(*x-y*): Negative definite

M<∞ : Drag force

Two complex noises c_1,c_2

→ Non-hermitian evolution

$$\rho_Q(t,\vec{x},\vec{y}) = \left\langle \Psi(t,\vec{x})\tilde{\Psi}(t,\vec{y})\right\rangle_{\xi, c_1, c_2}$$

$$\tilde{\Psi}(t,\vec{x}) \neq \Psi^*(t,\vec{x})$$