Conformal dynamics in Nf=12 lattice QCD

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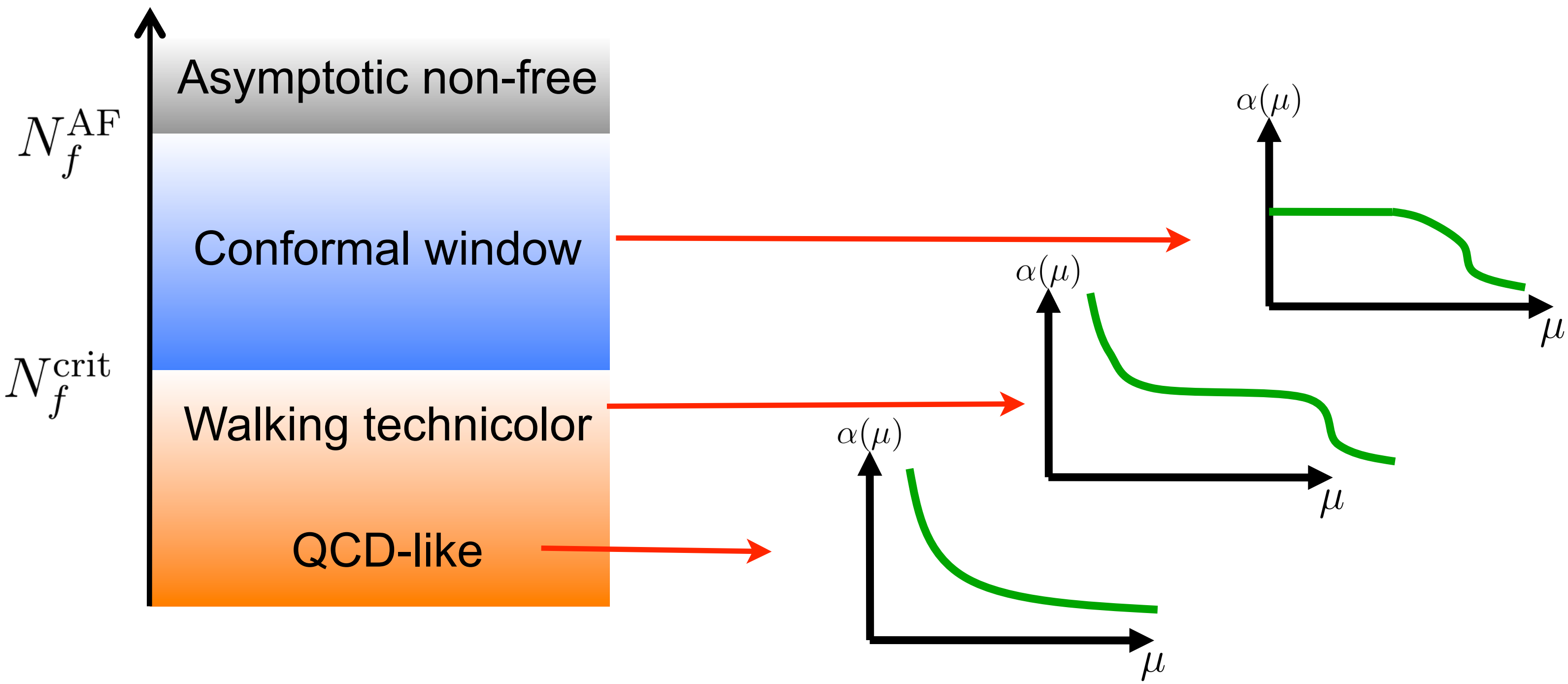
Introduction

Walking and conformal behavior -> non-perturbative dynamics

Many flavor QCD: benchmark test of walking dynamics

N_f : Number of flavor $\alpha(\mu)$: running gauge coupling



- Understanding of the conformal dynamics is important (e.g. critical phenomena)
- Walking technicolor (WTC) could be realized just below conformal window.
- What the value of the anomalous dimensions γ? (γ : critical exponent)
- Rich hadron structures may be observed in LHC.

LatKMI-Nagoya project (since 2011)

Systematic study of flavor dependence in Large Nf QCD using single setup of the lattice simulation

Our goals:

- Understand the flavor dependence of the theory
- Find the conformal window
- Find the walking regime and investigate the anomalous dimension

Status (lattice):

- Nf=16: likely conformal
- Nf=12: **controversial** → This talk
- Nf=8: controversial, our study suggests walking behavior? → talk by K.-i. Nagai (next)
- Nf=4: chiral broken and enhancement of chiral condensate

Observables:

- pseudoscalar, vector meson -> chiral behavior
- Glueball (O++) and/or flavor-singlet scalar

Is this lighter compared with others? If so, Good candidate of “Higgs” (techni-dilaton).

→ talk by T. Yamazaki

Our work

- use of improved staggered action

  Highly improved staggered quark action [HISQ]
- use MILC version of HISQ action

  use tree level Symanzik gauge action

  no $(ma)^2$ improvement (no interest to heavy quarks)= HISQ/tree

Simulation setup

- **SU(3), Nf=12 flavor**

simulation parameters

two bare gauge couplings (β) & four volumes & various fermion masses

- $\beta=6/g^2=3.7$ and 4.0
- $V=L^3 \times T$: L/T=3/4; L=18, 24, 30, 36
- $0.03 \leqq m_f \leqq 0.2$ for β=3.7, $0.04 \leqq m_f \leqq 0.2$ for β=4.0

Statistics ~ 2000 trajectory

- Measurement of meson spectrum

  in particular pseudoscalar ("NG-pion") mass (M_π), decay constant (F_π)

  vector meson mass (M_ρ), flavor-singlet scalar mass (M_σ)

Machine: φ @ KMI, CX400 @ Kyushu Univ.

Nf=12 theory: Conformal phase v.s. Chiral broken phase

From the fermion mass (mf) dependence of the hadron mass, we study the phase structure of the theory.

Conformal hypothesis: critical phenomena near the fixed point

hyper-scaling, γ : mass anomalous dimension at the fixed point

- $M_H \propto mf^{1/(1+\gamma)}$
- $F_\pi \propto mf^{1/(1+\gamma)} + \ldots$ (for small mf)

$\Rightarrow$ $F_\pi/M_\pi \rightarrow$ constant (mf$\rightarrow$0)

$M_\rho/M_\pi \rightarrow$ constant

Chiral symmetry breaking hypothesis: π is NG-boson.

Chiral perturbation theory (ChPT) works.

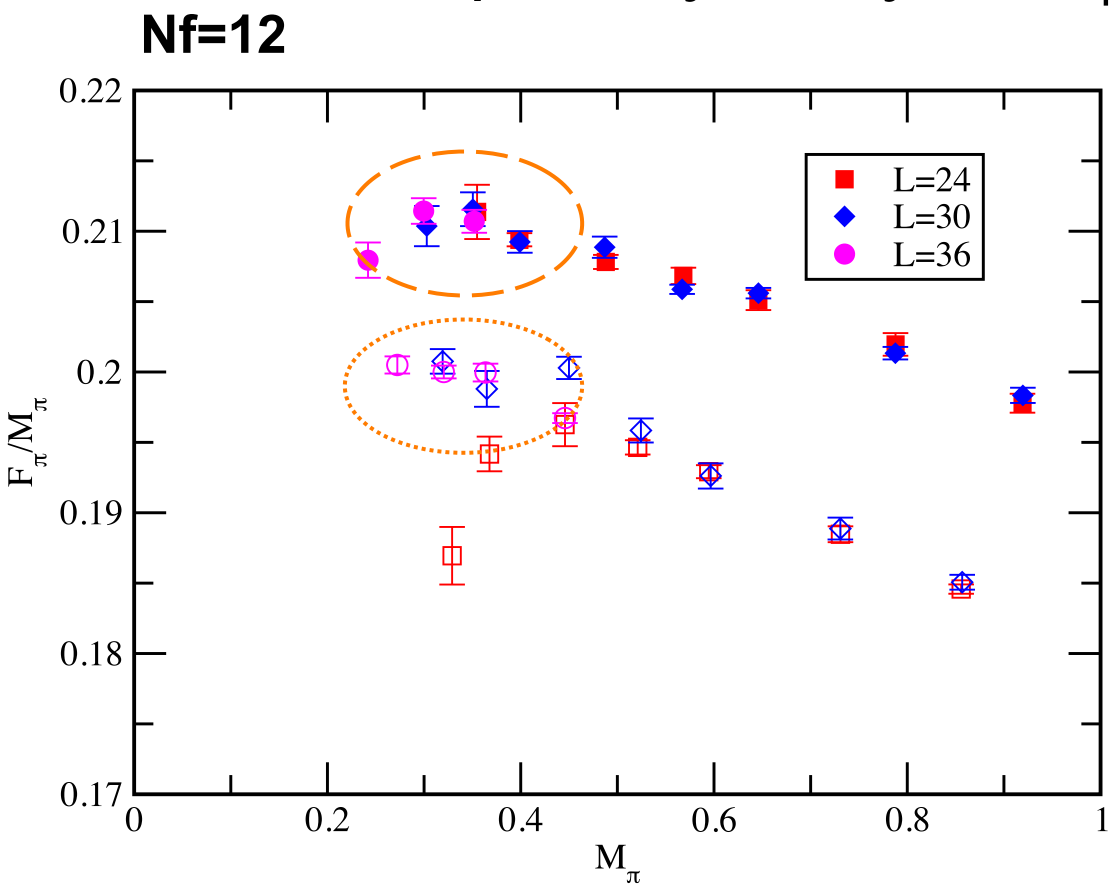
- $M_\pi^2 \propto mf$ (PCAC relation)
- $F_\pi = F + c\, M_\pi^2 + \ldots$ (for small mf)

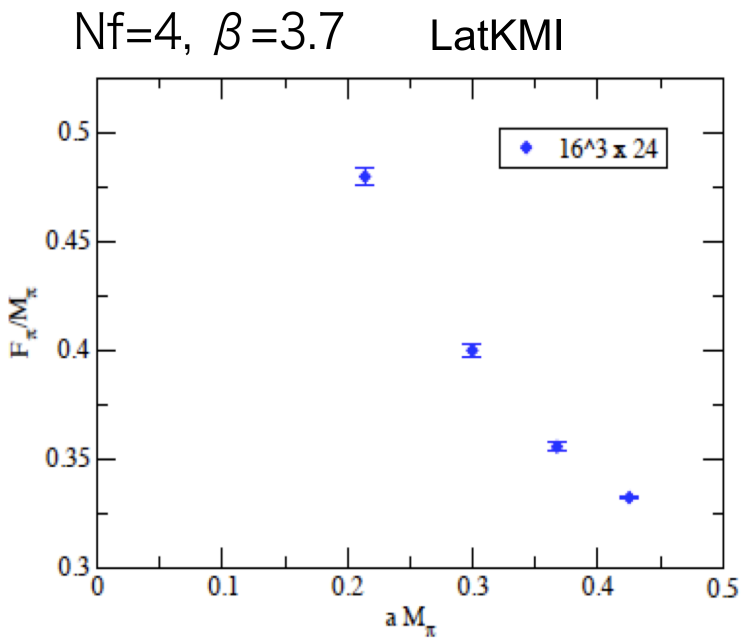
$\Rightarrow$ $F_\pi/M_\pi \rightarrow \infty$ (mf $\rightarrow$ 0)

N_f=12 Result

[LatKMI, PRD86 (2012) 054506]
and
Some updates

A primary analysis, F_π/M_π vs M_π

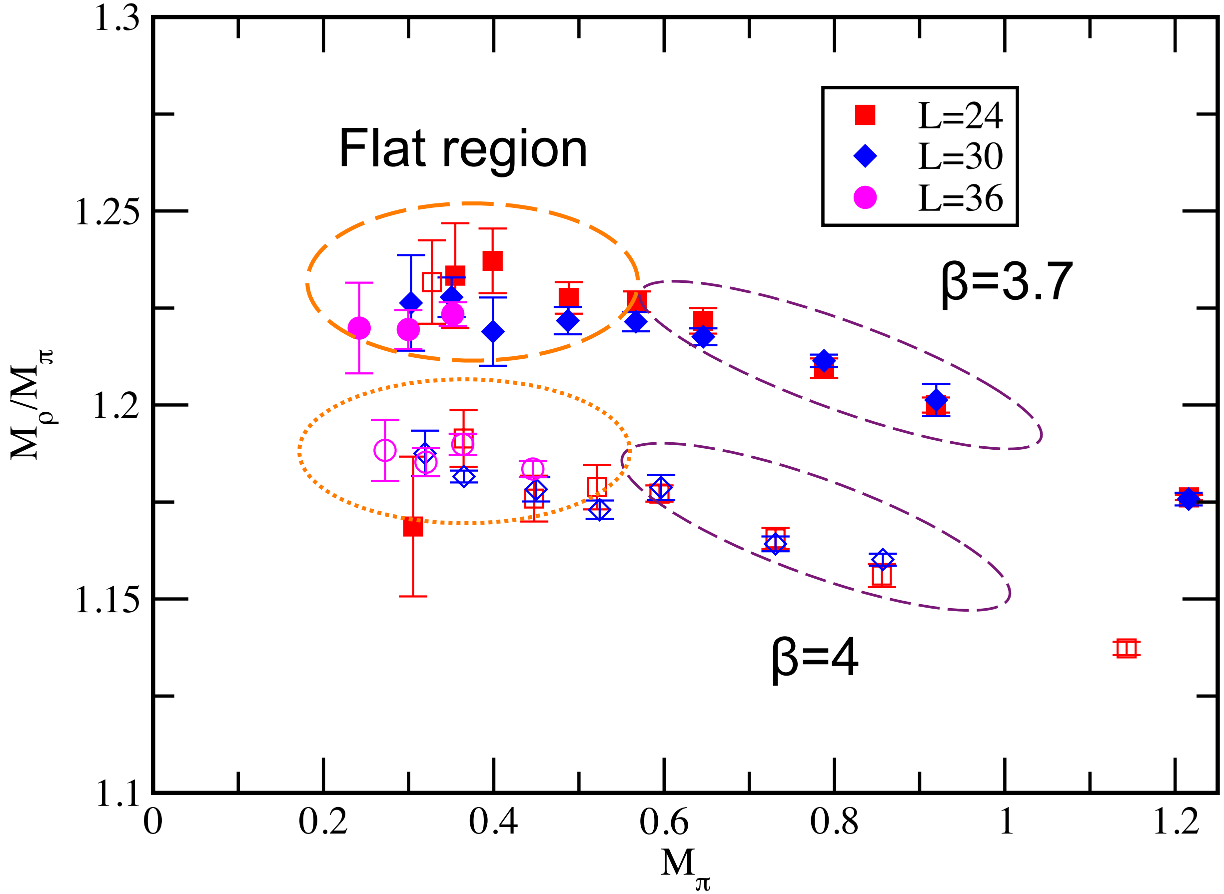




In both of β=3.7 and 4.0, both ratios at L=30 and L=36 seem to be flat in the small mass region, but small volume data (L≦24) shows large finite volume effect.
This behavior is contrast to the result in ordinary QCD system

Mρ/M_π vs M_π

Nf=12



From naïve scale matching, one can obtain the relation

- a(β=3.7) > a(β=4.0)

Our result suggests asymptotically free region for beta=3.7-4.

Ratio is almost flat in small mass region (wider than Fπ/Mπ)
-> consistent with hyper scaling
Volume dependence is smaller than Fπ/Mπ.
In the large mass region, large mass effects show up.
Mρ/M_π should be 1, as mf -> infinity.

Conformal hypothesis in infinite volume & finite volume

- Universal behavior for all hadron masses (hyper-scaling)
- Mass dependence is determined by scaling dimension (mass-deformed CFT.)

$$M_H \propto m_f^{1/(1+\gamma)}, \quad F_\pi \propto m_f^{1/(1+\gamma)}$$ **(infinite volume result)**

Our interest : the same low-energy physics with the one obtained in infinite volume limit

But all the numerical simulations can be done only in finite size system (L).

we use **Finite size scaling hypothesis**

-> Finite size hyper-scaling for hadron mass in L^4 theory
[DeGrand et al. ; Del debbio et. al., '09]

Note: In order to avoid dominant finite volume effect and
to connect with infinite volume limit result,
we focus on the region of L >> ξ (correlation length), (LMπ >>1).

Finite size hyper-scaling

- Universal behavior for all hadron masses
- From RG argument the scaling variable x is determined as a combination of mass and size

$$x = Lm^{1/(1+\gamma_*)}$$

- The universal description for hadron masses are given by the following forms as,

$$L \cdot M_H = f_H(x) \qquad L \cdot F_H = f_F(x)$$

Ref [DeGrand et al. ; Del debbio et. al., '09]

c.f. Finite Size Scaling (FSS) of 2nd order phase transition

$$\xi_L(T) = L f_\xi\left(\frac{L}{\xi_\infty}\right). \qquad \xi_\infty \propto \left|\frac{T_c - T}{T_c}\right|^{-\nu}$$

Test of Finite size hyper-scaling

$$L \cdot M_H = f_H(x) \qquad L \cdot F_H = f_F(x)$$

We test the finite hyper-scaling for our data at L=18, 24, 30, 36.

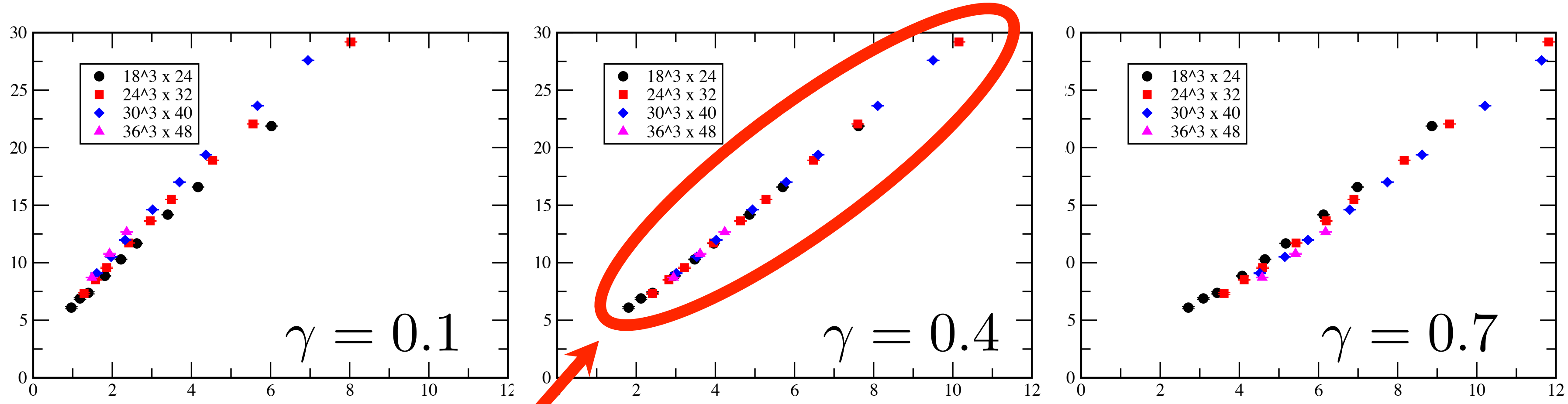
The scaling function f(x) is unknown in general,

But if the theory is inside the conformal window,

the data should be described by one scaling parameter x.

Data alignment at a certain γ

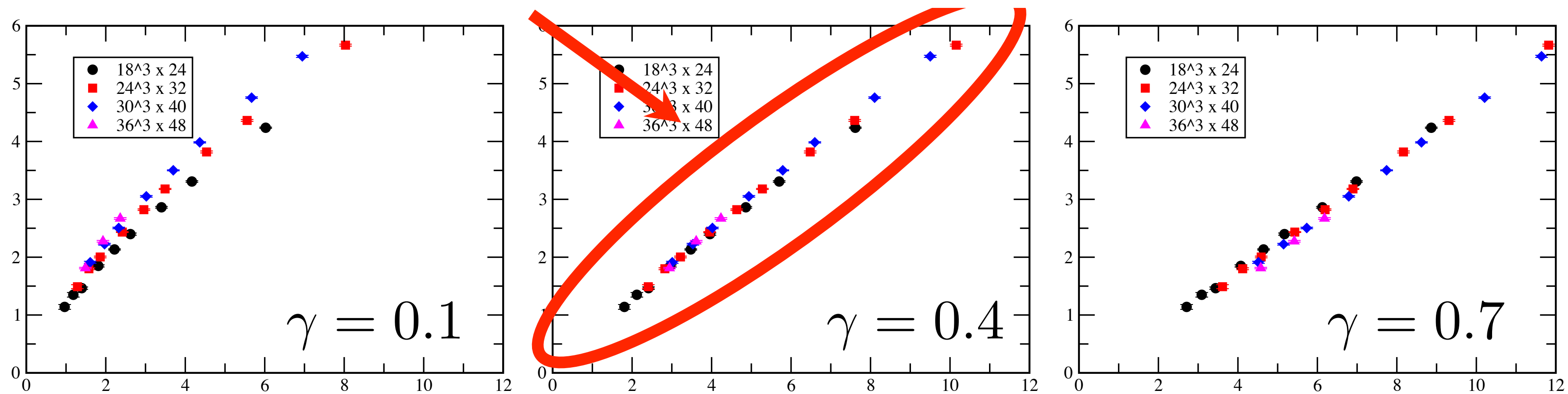
$$\xi = LM_\pi$$



x

LF_π

good alignment! **How to quantify this situation?**



$$x = L \cdot m^{1/(1+\gamma)}$$

■To quantify the alignment and obtain the optimal γ

We define a function P(γ) to quantify how much the data “align” as a function of x.

$$P(\gamma) = \frac{1}{\mathcal{N}} \sum_{L} \sum_{j \notin K_L} \frac{|y^j - f(K_L)(x_j)|^2}{|\delta y^j|^2}$$

[LatKMI, PRD86 (2012) 054506]

Optimal value of γ for alignment will minimize P(γ).

our analysis: three observables of $y_p=LM_p$ for p=π, ρ; $y_F=LF_\pi$.

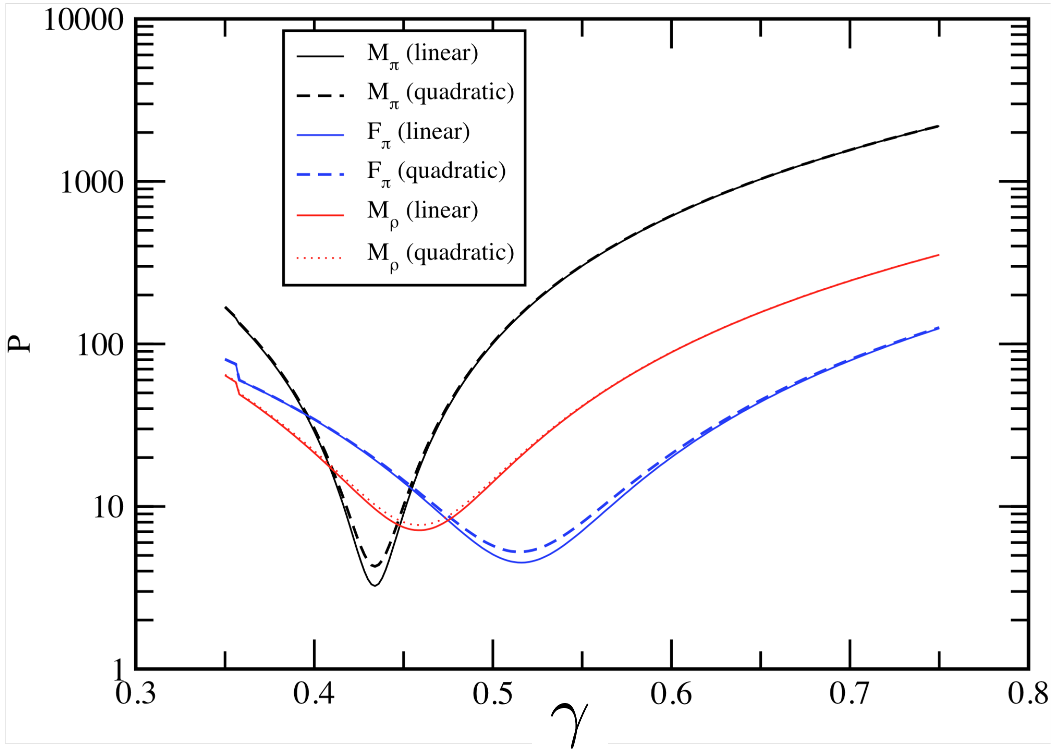
A scaling function f(x) is unknown,

→ f(xj) is obtained by interpolation (spline) with linear ansatz (quadratic for a systematic error).

If ξ^j is away from $f(x_i)$ by $\delta\,\xi^j$ as average → P=1.

P(γ) analysis

- P(γ) has minimum at a certain value of γ, from which we evaluate the optimal value of γ.
- At minimum, P(γ) is close to 1.



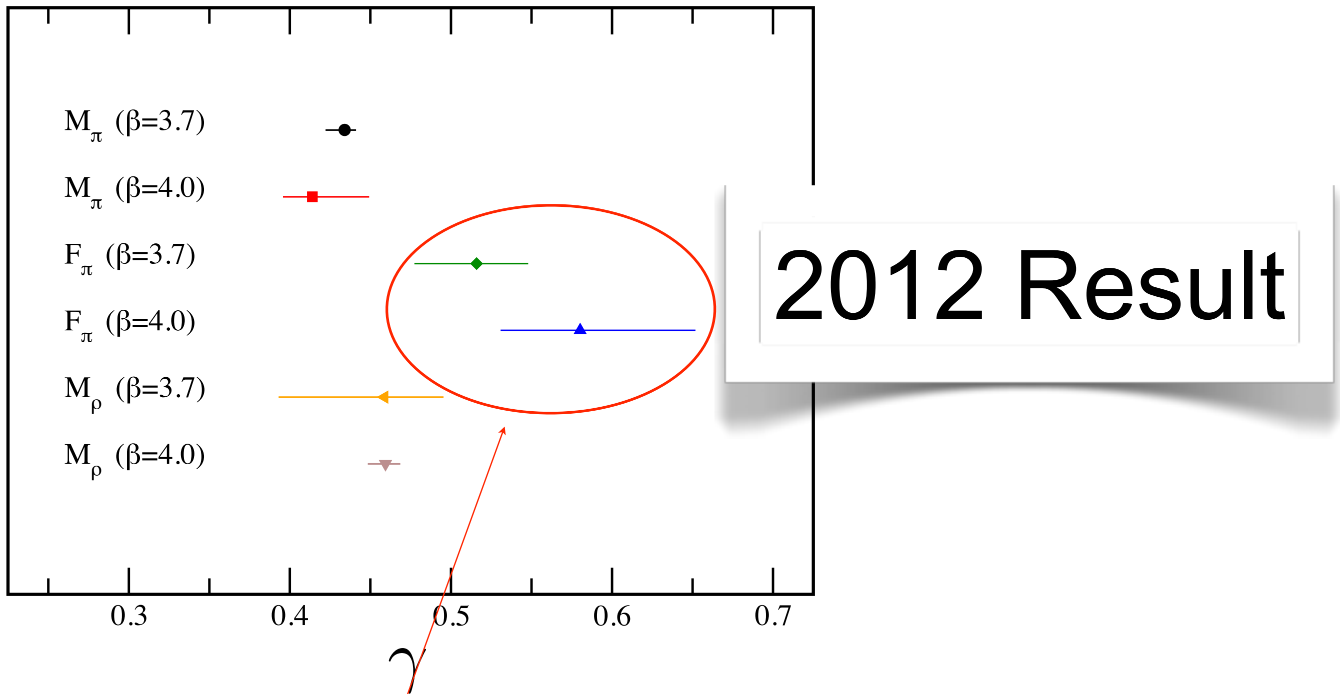
Results for data for L=18, 24, 30 at β=3.7

L > ξ is satisfied in our analysis.

$(LM_\pi > 8.5 \text{ for our simulation parameter region})$

■Result of gamma (data L=18,24,30)

[LatKMI, PRD86 (2012) 054506]

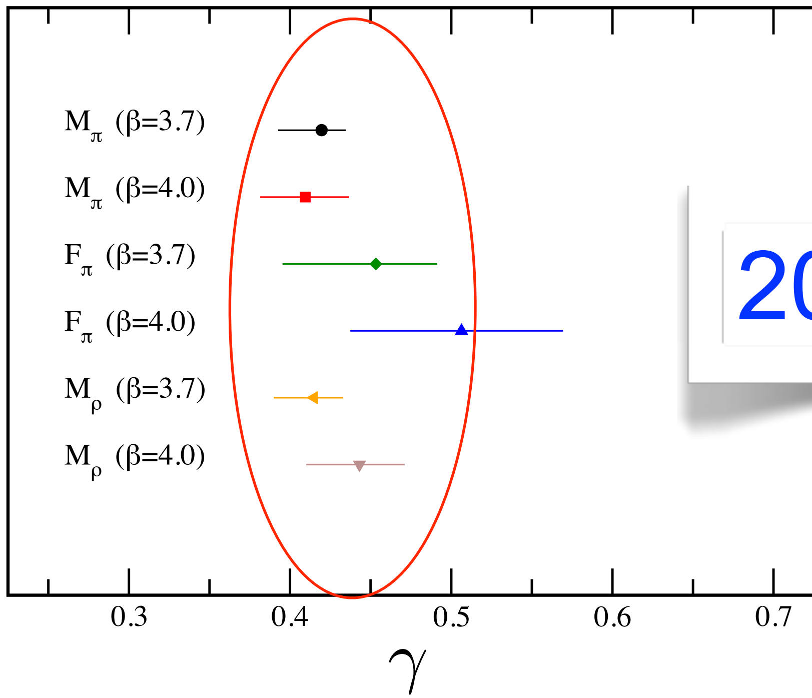


- The error -> both statistical & systematic errors
  <- estimation by changing x range of the analysis

•Remember: <u>Fπ data seems to be out of scaling region due to finite mass & volume corrections. Flat range is smaller than Mρ/Mπ.</u>

■Result of gamma (data L=24,30,36 with lighter mass region)

[LatKMI, 2013]



2013 Update

- γ(Mπ) is stable against the change of the mass (x) and β .
- smaller mass with larger volume (18,24,30 ->24,30,36)

→closer value to γ(Mπ)

The universal scaling is obtained for both values of β =3.7 & 4.0
γ=0.4-0.5.

Short summary

- β=3.7-4.0: Mπ, Fπ, Mρ show conformal hyper scaling
- Fπ : large mass corrections in our whole mass parameters, likely too heavy mf to be neglect. → Approaching small mass region, we obtain hyper-scaling behavior.
- We find that the hyper-scaling is realized in larger volume region together with smaller mass region.
- In such a region, the universal γ can be obtained for Mπ, Fπ, Mρ.

Scalar mass in Nf=12

[LatKMI, PRL(2013)]

Scalar in conformal phase

motivation

- The scalar in mass-deformed CFT could be lighter due to the dilatonic nature [Bando-Matumoto-Yamawaki, '86].
  However, it has never been showed in many flavor QCD system from the first principle lattice calculation.

  This is the first result for the scalar measurement in Nf=12 QCD.
- Information of the scalar could be a hint for the composite Higgs boson in the walking technicolor model, emerging as the techni-dilaton from the (near-) conformal dynamics.

method

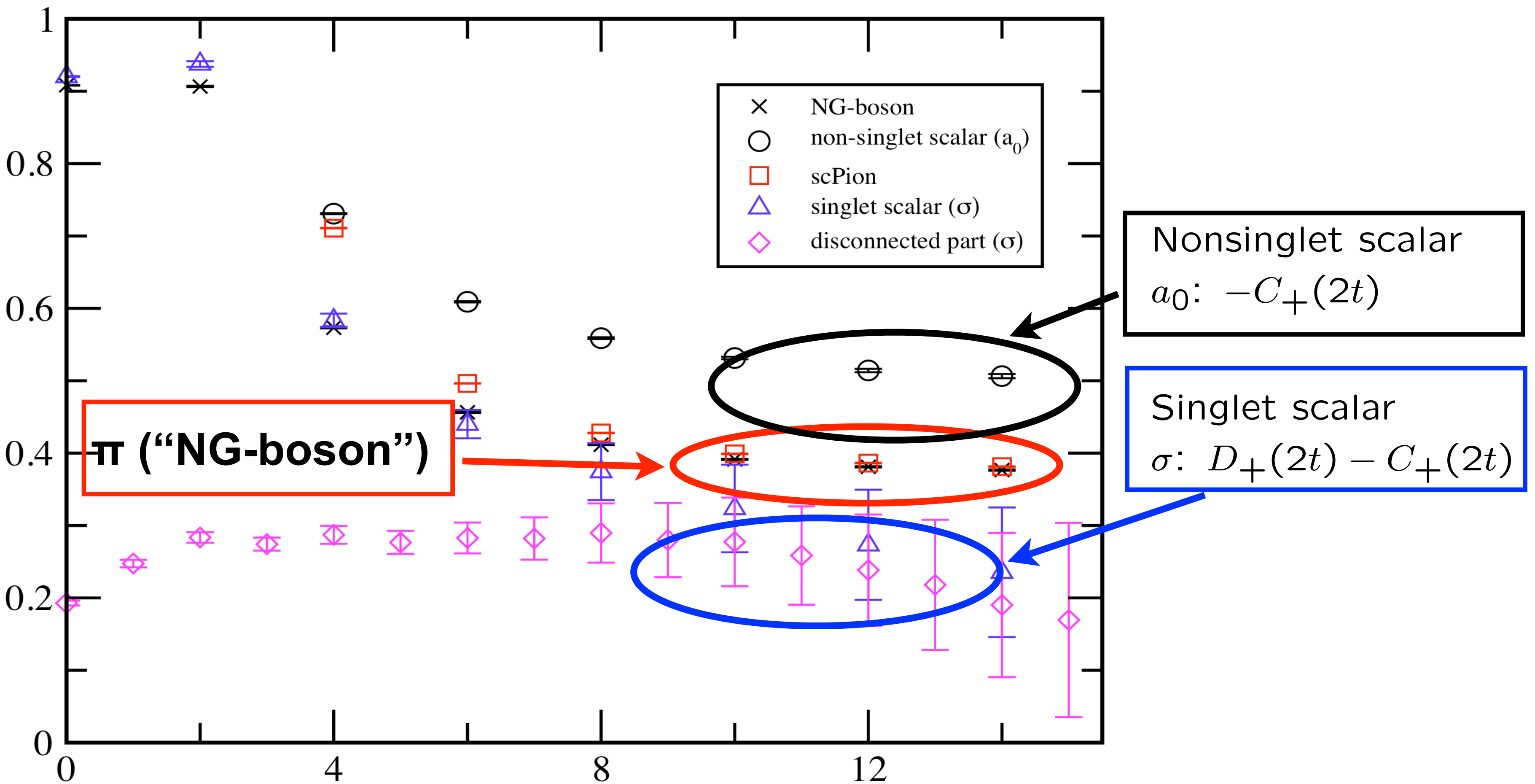
- Flavor-singlet scalar from fermion bilinear
- very noisy in general for disconnected diagram
- we use high statistics: a few 1000 ~14000 configurations
- Details of the calculation -> talk by T. Yamazaki.

Result

LatKMI,
Phys. Rev. Lett. 111 (2013)162001

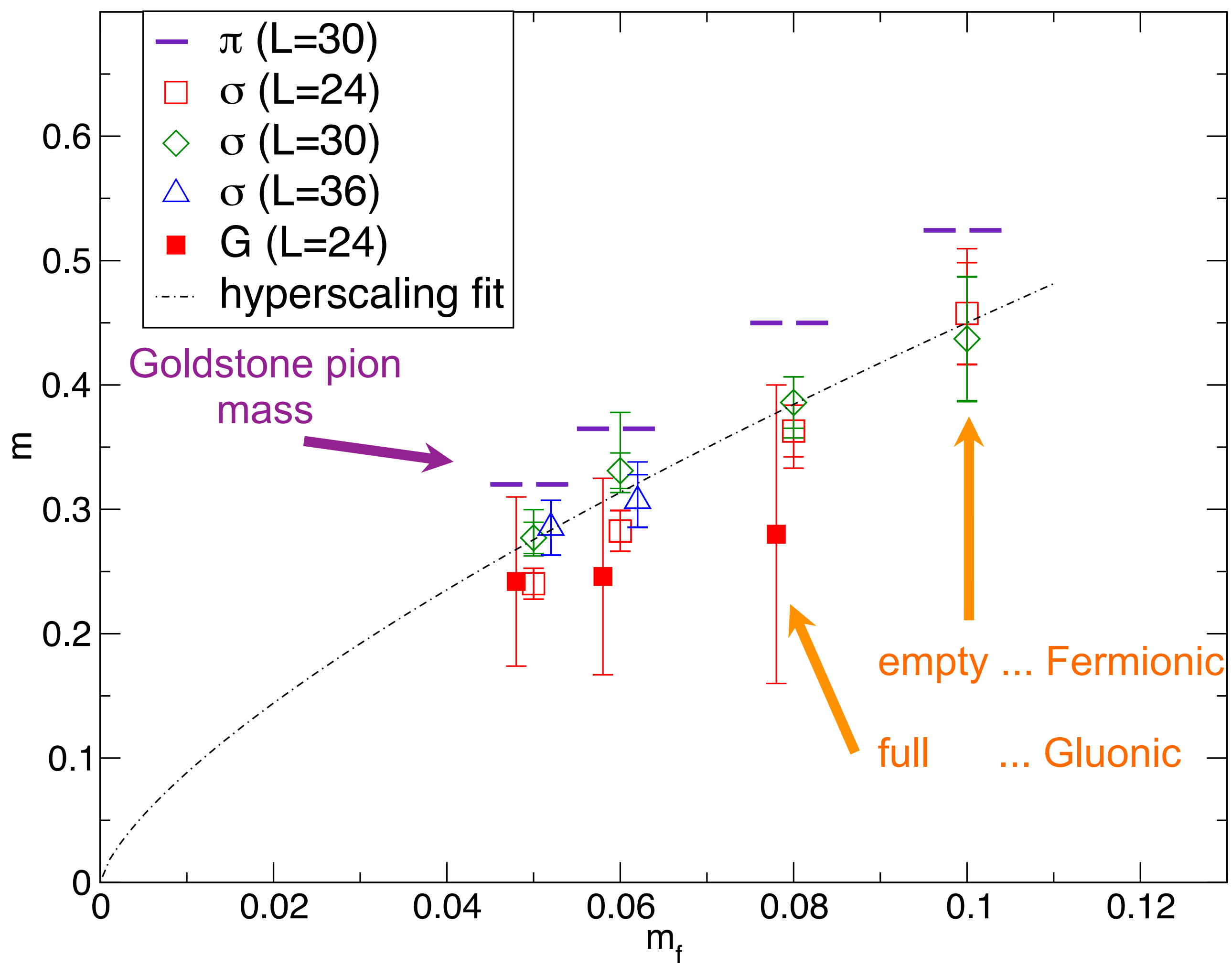
Effective mass (mf=0.06, L=24, 14000config.)

$$m_{\text{eff}}(t) = \log(C_H(t)/C_H(t+1)) \xrightarrow{t\gg 1} m_H$$



good signal !!

Results: Nf=12 summary

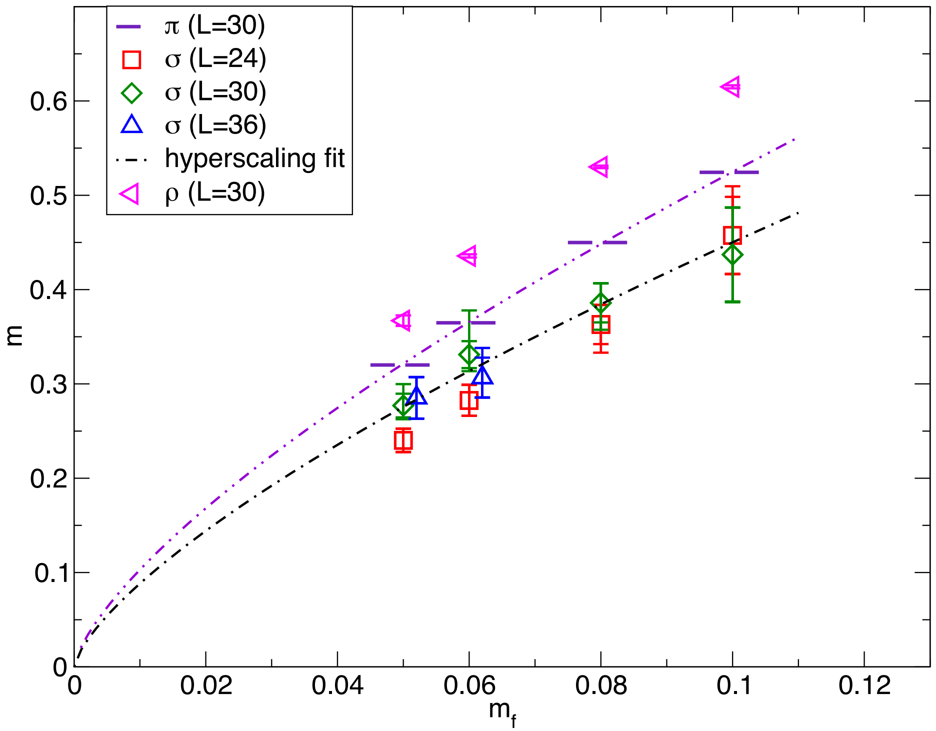


m_σ / m_π
$0.73(4)\binom{0}{0}$
$0.78(4)\binom{1}{0}$
$0.81(5)\binom{0}{5}$
$0.88(8)\binom{6}{1}$
$0.87(4)\binom{6}{2}$
$0.91(4)\binom{12}{3}$
$0.86(5)\binom{0}{4}$
$0.83(9)\binom{1}{2}$
$0.89(7)\binom{0}{1}$
$0.84(6)\binom{6}{1}$

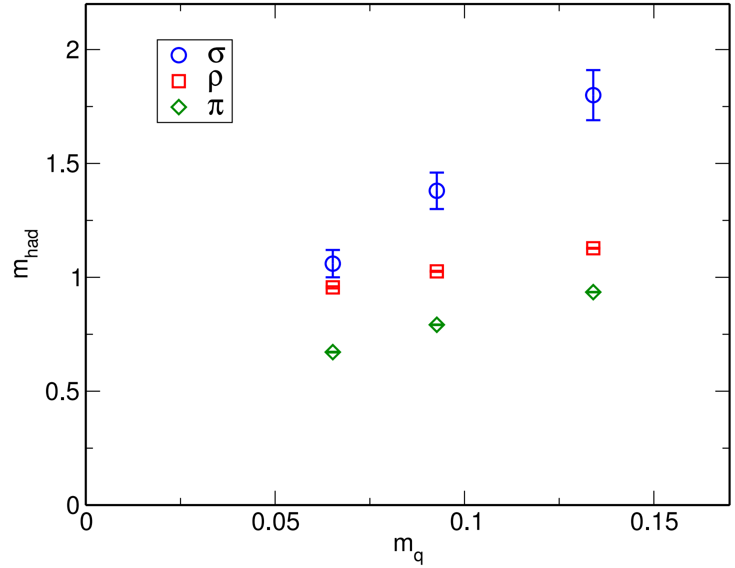
O++ scalar is lighter than π.

It is consistent with hyper-scaling (γ~0.4)

Results: Nf=12 summary



c.f. Nf=2 lattice QCD result
[T. Kunihiro,et al.,
SCALAR Collaboration, 2003]



Mρ > Mπ > Mσ
Nf=12 QCD is in sharp contrast to the real-life QCD
(right figure: Nf=2 lattice QCD result)

Summary

- **Large Nf SU(3) gauge theory is being investigated in LatKMI project.**
- **We focus on the Nf=12 case.**

[LatKMI, PRD 2012].

- **Finite size hyper scaling is observed for the π (“NG-boson”) mass, decay constant and rho meson mass.**
- **Nf=12 is consistent with conformal gauge theory.**
- **The resulting universal γ ~0.4-0.5 (not favored as Walking Technicolor)**
- ChPT expansion is not valid, expansion parameter is much larger than 1.

(Not yet exclude chiral broken scenario (very small Fπ))

[LatKMI, PRL 2013]

- **We measured Flavor-singlet meson (& 0++ glueball) spectrum.**
- Scalar is lighter than π, which is in sharp contrast to the real-life QCD.

How about other # of fermions??

-> e.g. 8 flavor case, talk by K.-i. Nagai (next!)

END

Thank you