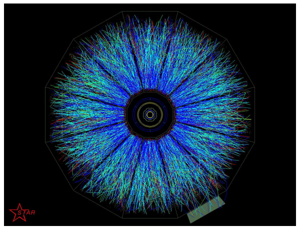
Relativistic Hydrodynamics in High-Energy Heavy Ion Collisions

Kobayashi-Maskawa Institute
for the Origin of Particles and the Universe

Kobayashi Maskawa Institute
Department of Physics, Nagoya University
Chiho NONAKA

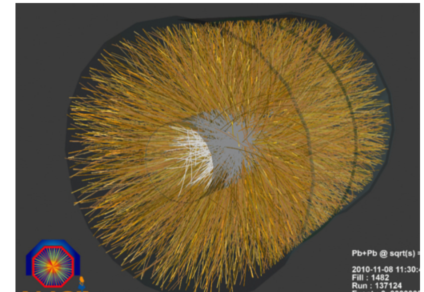
December 13, 2013@KMI 2013, Nagoya

Relativistic Heavy Ion Collisions



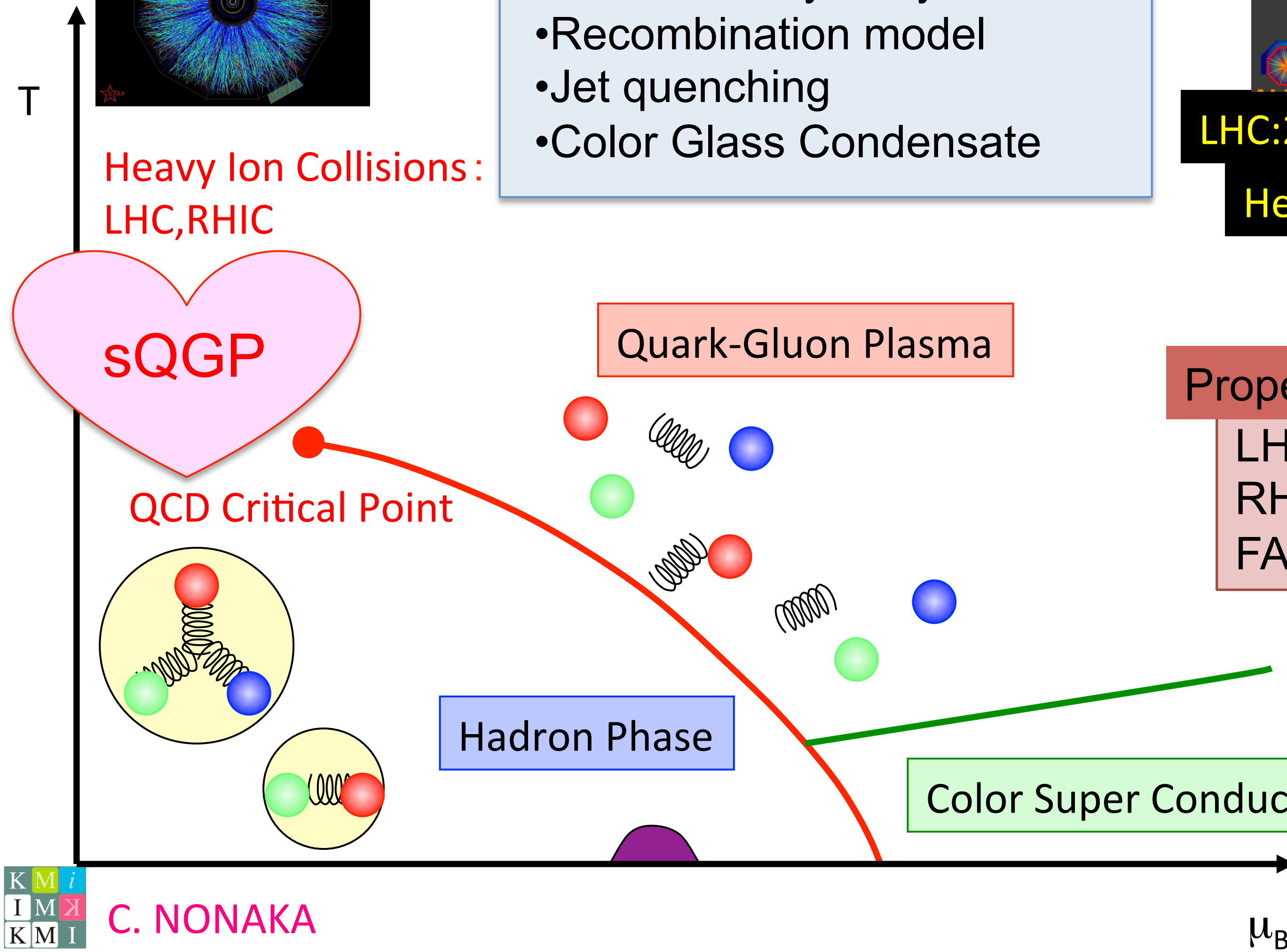
RHIC:2000

Strongly interacting QGP
- Relativistic hydrodynamics
- Recombination model
- Jet quenching
- Color Glass Condensate



LHC:2010

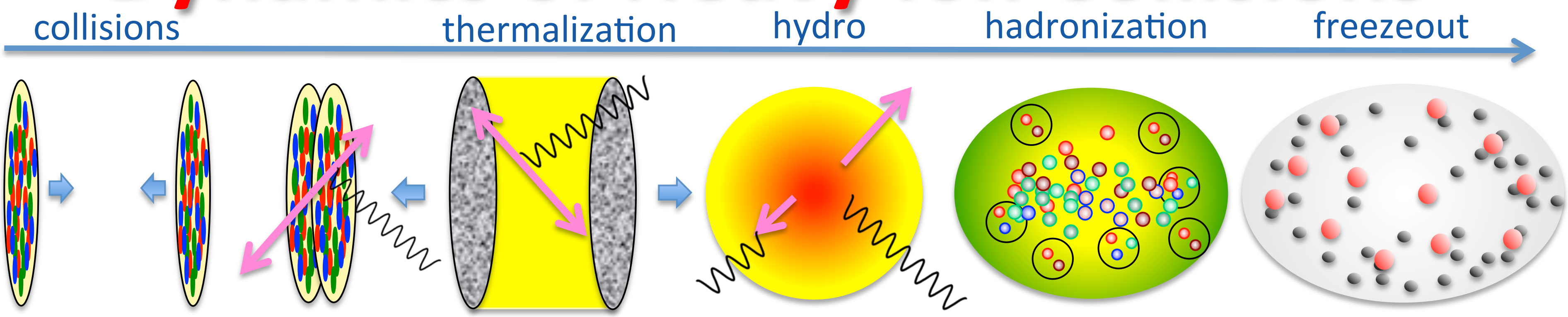
Heavy Ion collisions start!



Property of QGP

LHC: Energy frontier
RHIC: energy scan
FAIR, NICA : high density

Dynamics of Heavy Ion Collisions



Observables: a lot of experimental data at RHIC and LHC

- photons/leptons
- bulk property
- Jets
- heavy quarkonia

Phenomenological model

sQGP

Initial condition → **Hydrodynamic model** → **Freezeout process**

?

Experimental data

higher harmonics

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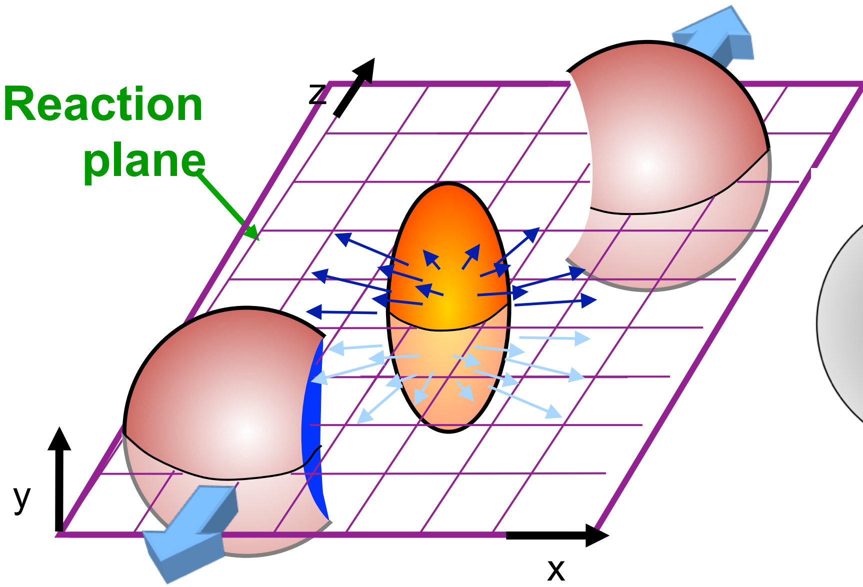
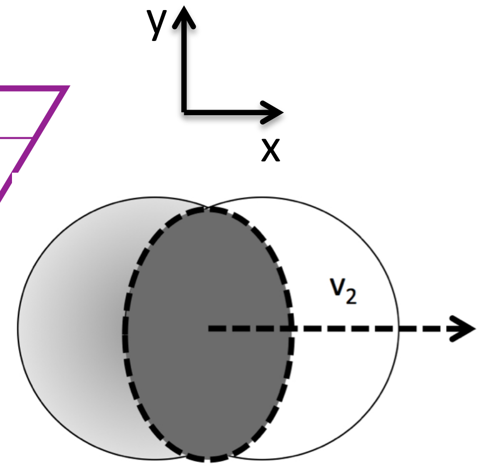
Higher Harmonics

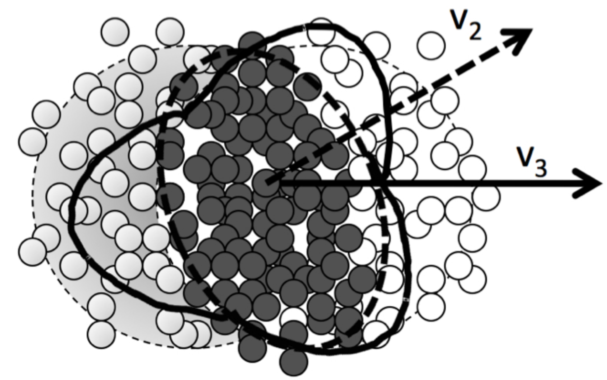




elliptic flow

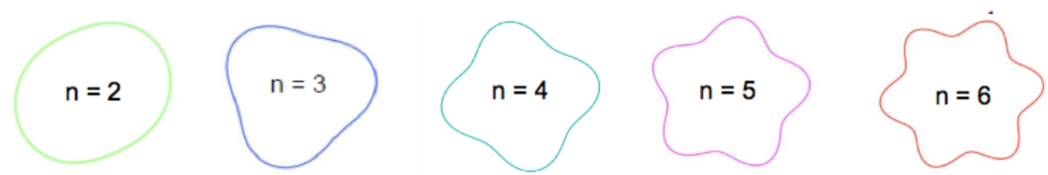
more realistic

event by event
fluctuations



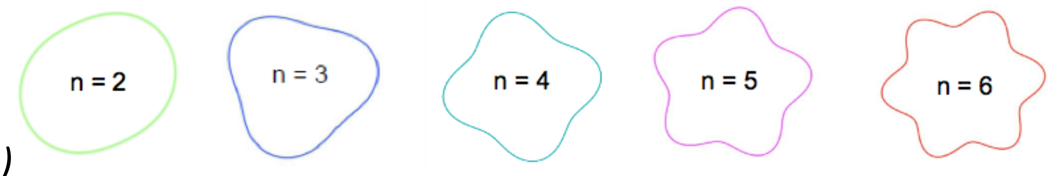
higher harmonics

$$\frac{dN}{dyd\phi} \propto 1+2v_1\cos(\phi-\Theta_1)+2v_2\cos 2(\phi-\Theta_2)+2v_3\cos 3(\phi-\Theta_3)+2v_4\cos 4(\phi-\Theta_4)+\cdots$$

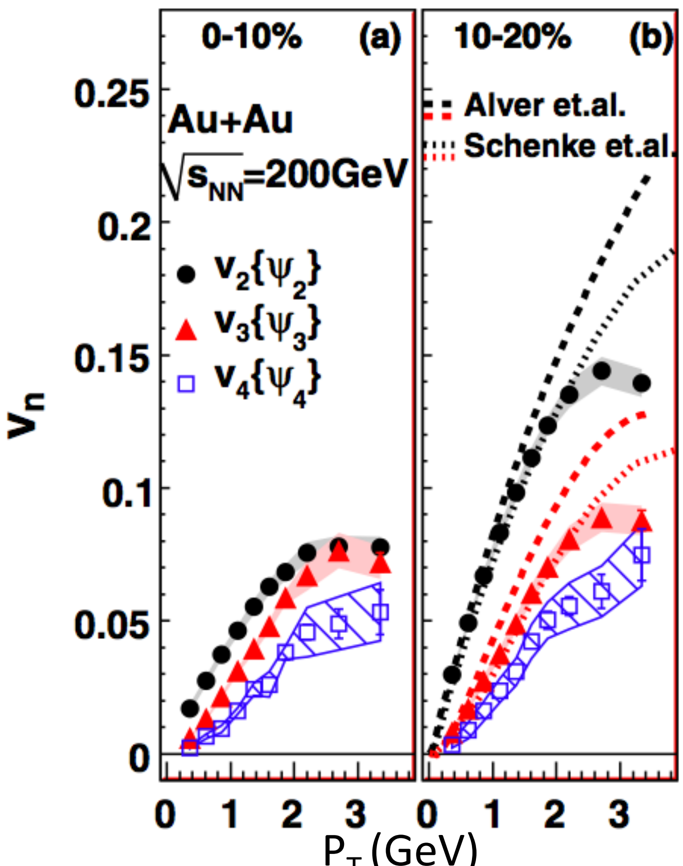


Higher Harmonics @ RHIC & LHC

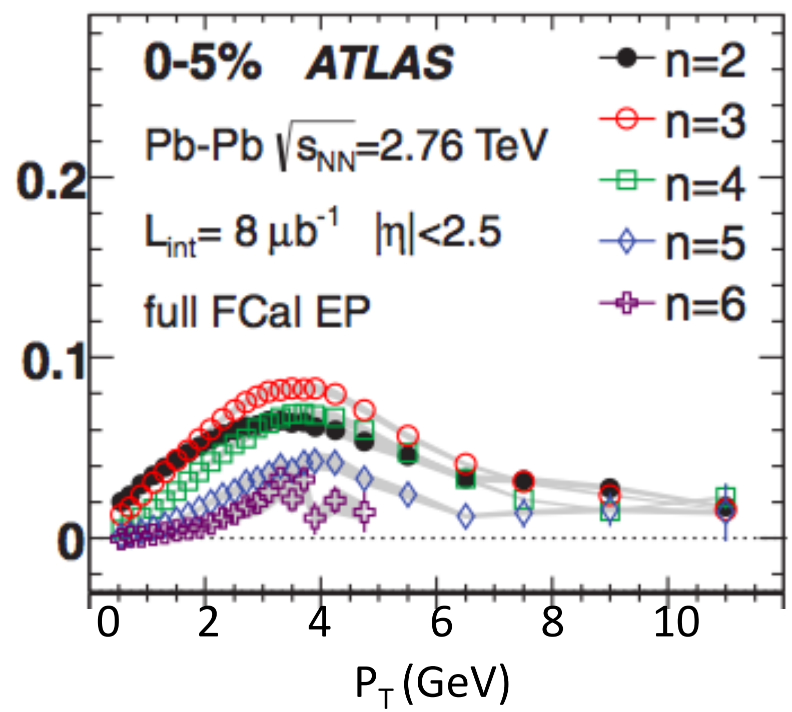
$$\frac{dN}{dyd\phi} \propto 1+2v_1\cos(\phi-\Theta_1)+2v_2\cos 2(\phi-\Theta_2)+2v_3\cos 3(\phi-\Theta_3)+2v_4\cos 4(\phi-\Theta_4)+\cdots$$



PHENIX@RHIC, PRL107,252301(2011)

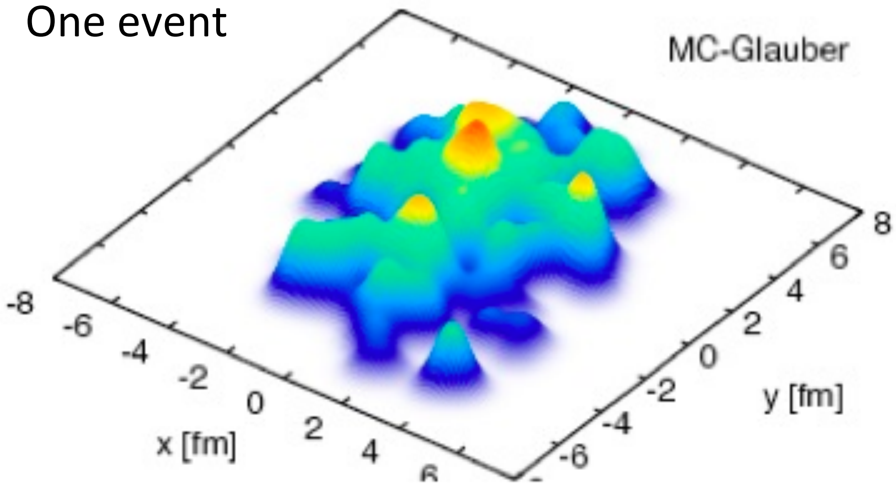


ATLAS@LHC, PRC86,014907(2012)

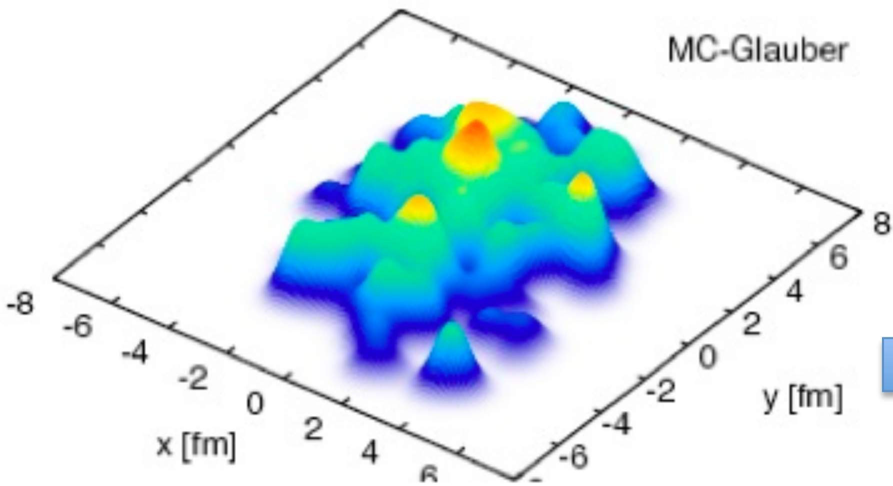


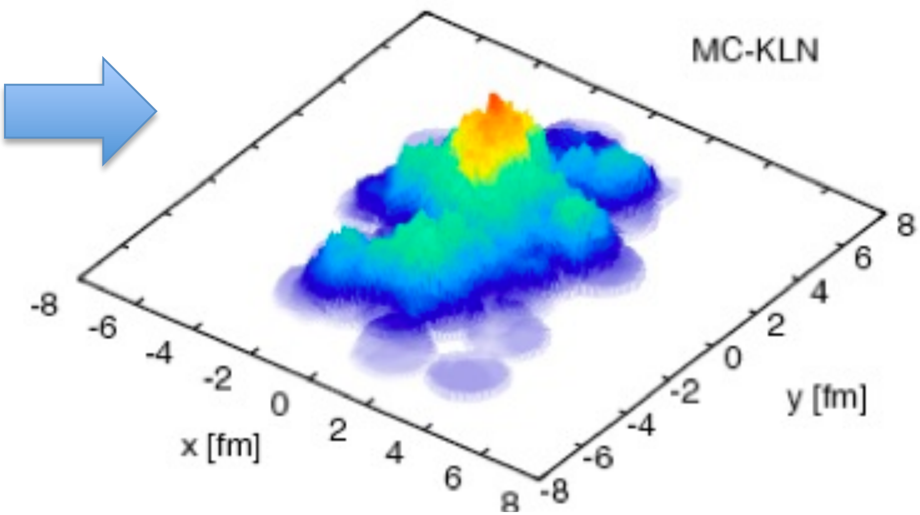
Initial Conditions

One event



Initial Conditions





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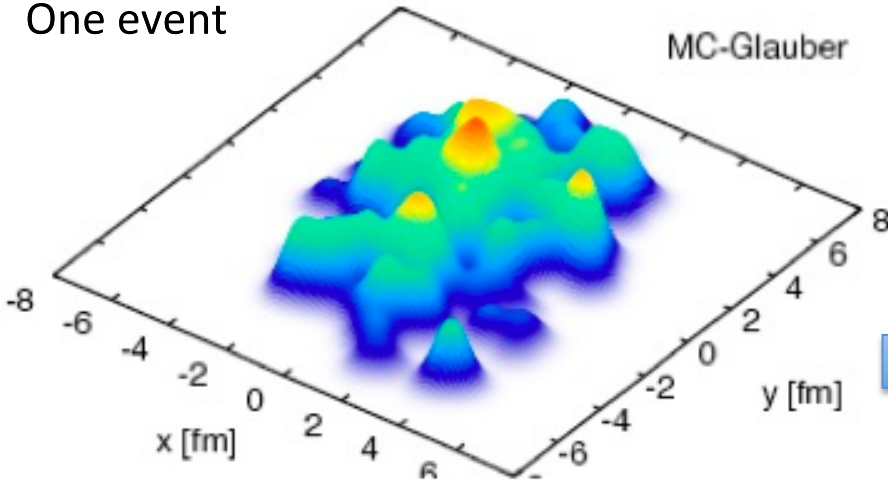
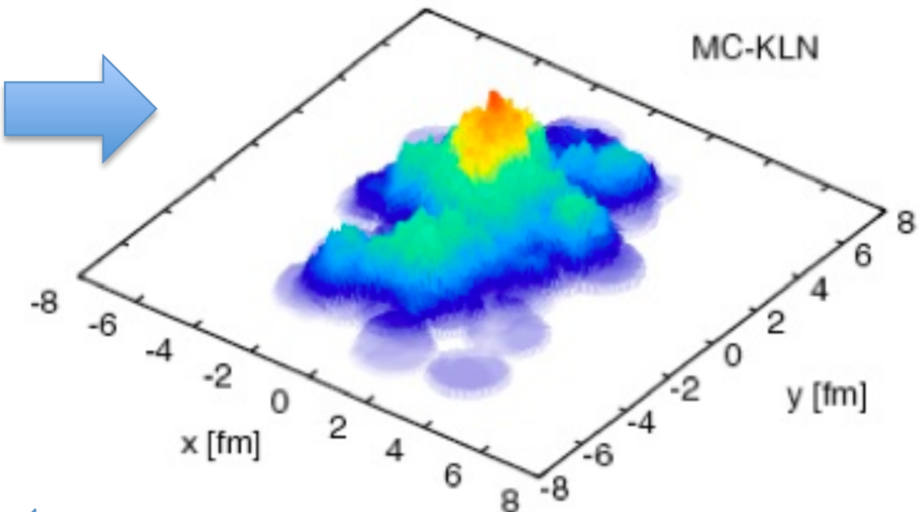
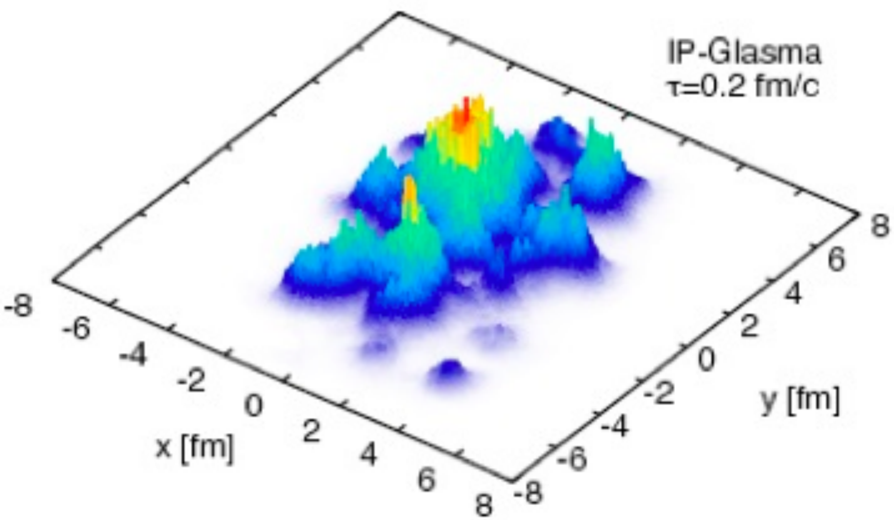
Initial Conditions

One event







Gale Jeon Schenke 1301.5893

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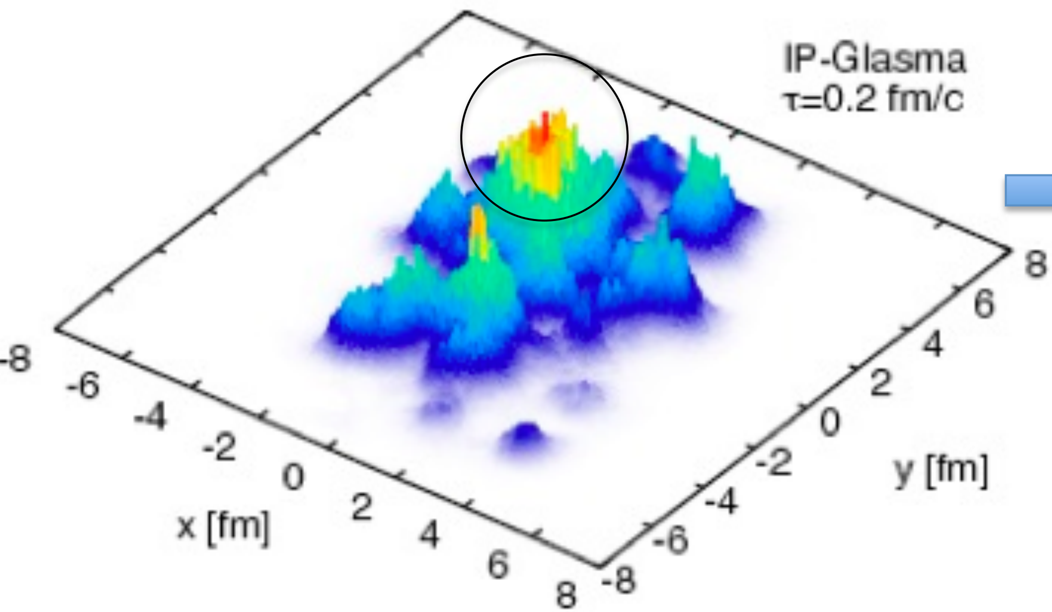
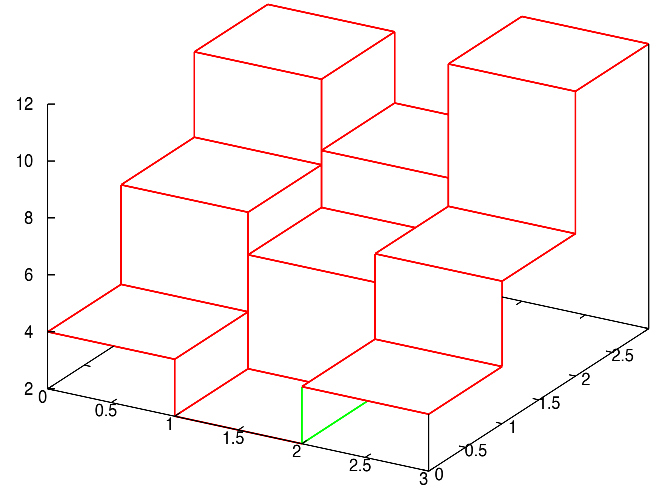
Numerical Scheme





Superposition of shock waves

Numerical algorithm for hydrodynamic evolution

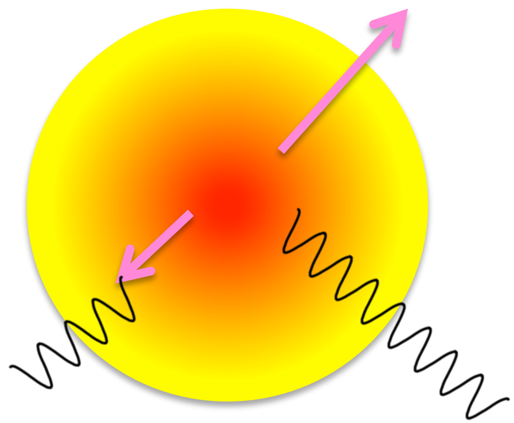
→ ✔shock-wave capturing scheme
✔stable
✔less numerical viscosity

Hydrodynamic Expansion

sQGP

Initial condition → **Hydrodynamic model** → **Freezeout process**

fluctuating
initial conditions



V_n

Bulk property

transport coefficients .....

importance of numerical algorithm !

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Akamatsu, Inutsuka, CN, Takamoto:

arXiv:1302.1665、J. Comp. Phys. (2014) 34

HYDRODYNAMIC MODEL

Viscous Hydrodynamic Model

- Relativistic viscous hydrodynamic equation

$$\partial_\mu T^{\mu\nu} = 0$$

  - First order in gradient: acausality
  - Second order in gradient:
    - Israel-Stewart, Ottinger and Grmela, AdS/CFT, Grad's 14-momentum expansion, Renomarization group
- Numerical scheme
  - Shock-wave capturing schemes: Riemann problem
    - Godunov scheme: analytical solution of Riemann problem
    - SHASTA: the first version of Flux Corrected Transport algorithm, Song, Heinz, Pang, Victor…
    - Kurganov-Tadmor (KT) scheme, McGill

Our Approach

Takamoto and Inutsuka, arXiv:1106.1732
Akamatsu, Inutsuka, CN, Takamoto, arXiv:1302.1665

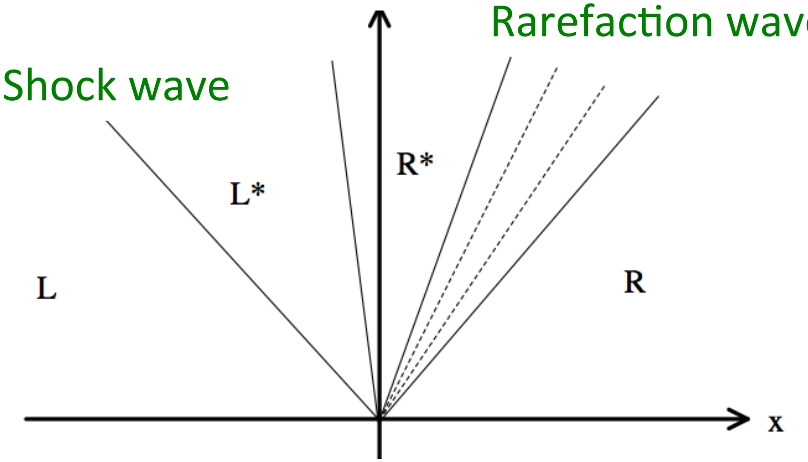
- Israel-Stewart Theory

(ideal hydro)

1. dissipative fluid dynamics = advection + dissipation

Riemann solver: Godunov method

exact solution



Two shock approximation

Mignone, Plewa and Bodo, Astrophys. J. S160, 199 (2005)

Rarefaction wave ⟶ shock wave

2. relaxation equation = advection + stiff equation

Numerical Scheme

- Israel-Stewart Theory

Takamoto and Inutsuka, arXiv:1106.1732

1. Dissipative fluid equation

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p g^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \tau^{\mu\nu}$$
$$= T_{\rm ideal} + T_{\rm dissip}$$

$$\partial_t U + \nabla \cdot F(U) = 0 \qquad U = U_{\rm ideal} + U_{\rm dissip}$$

2. Relaxation equation

$$\hat{D}\Pi = \frac{1}{\tau_\Pi}(\Pi_{NS} - \Pi) - I_\Pi,$$

$$\hat{D}\pi^{\mu\nu} = \frac{1}{\tau_\pi}(\pi^{\mu\nu}_{NS} - \pi^{\mu\nu}) - I^{\mu\nu}_\pi,$$

$$\hat{D}q^\mu = \frac{1}{\tau_q}(q^\mu_{NS} - q^\mu) - I^\mu_q,$$

$$\hat{D} = u^\mu \partial_u$$

I: second order terms

$$\tau^{\mu\nu} = \Pi\Delta^{\mu\nu} + \pi^{\mu\nu}$$

→

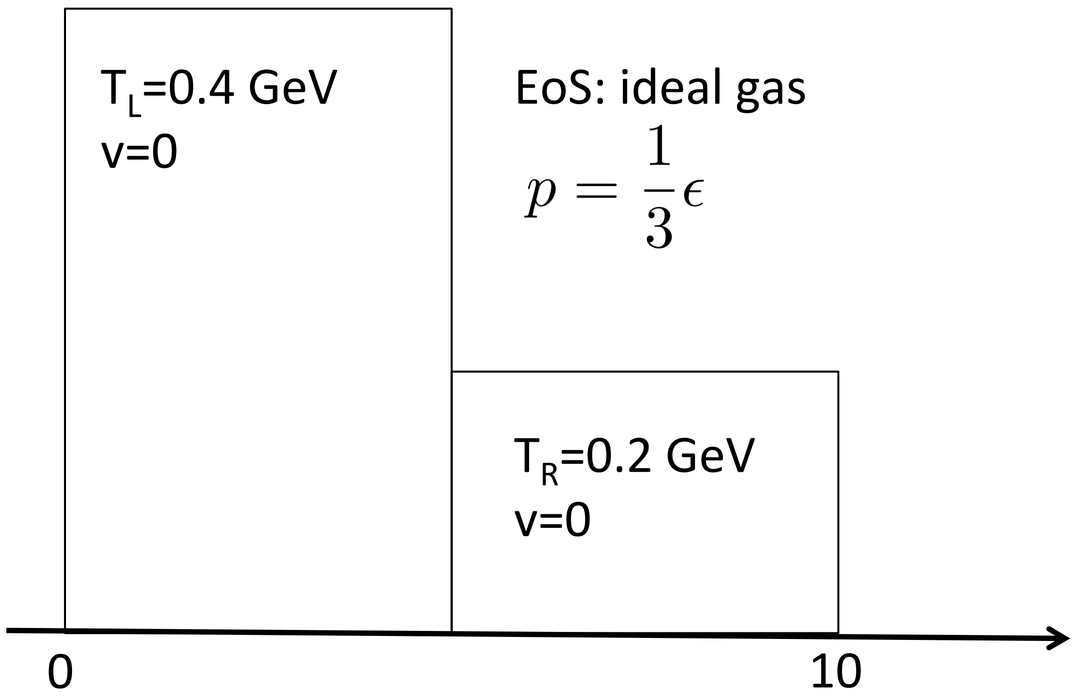
$$\left(\frac{\partial}{\partial t} + v^j \frac{\partial}{\partial x^j}\right)\Pi = -\frac{I_\Pi}{\gamma}, \quad + \quad \frac{\partial}{\partial t}\Pi = \frac{1}{\gamma\tau_\Pi}(\Pi_{NS} - \Pi),$$

advection + stiff equation

$$\Delta t < \tau_{\rm relax} << \tau_{\rm fluid}$$

Comparison

- Shock Tube Test : *Molnar, Niemi, Rischke*, Eur.Phys.J.C65,615(2010)

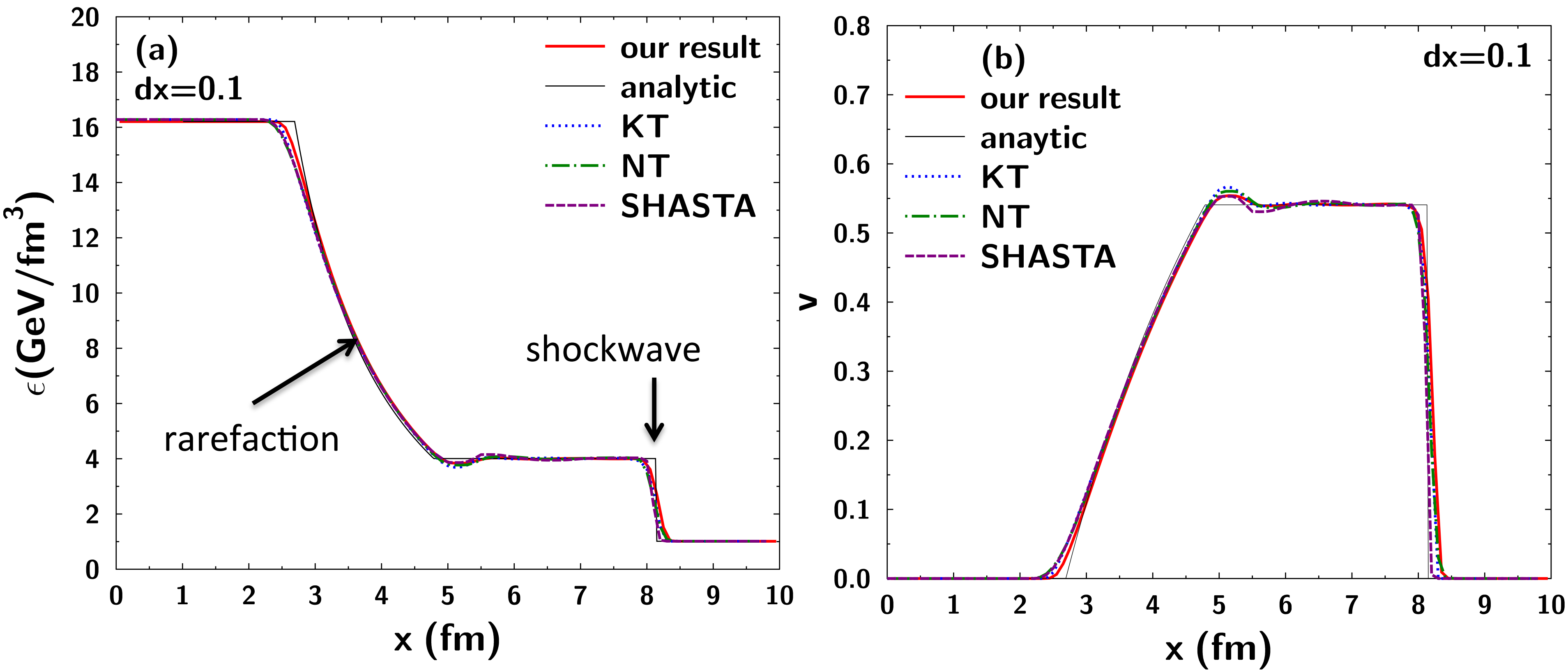


Nx=100, dx=0.1, dt=0.04

- Analytical solution
- Numerical schemes
  SHASTA, KT, NT
  Our scheme

Shocktube problem

- Ideal case



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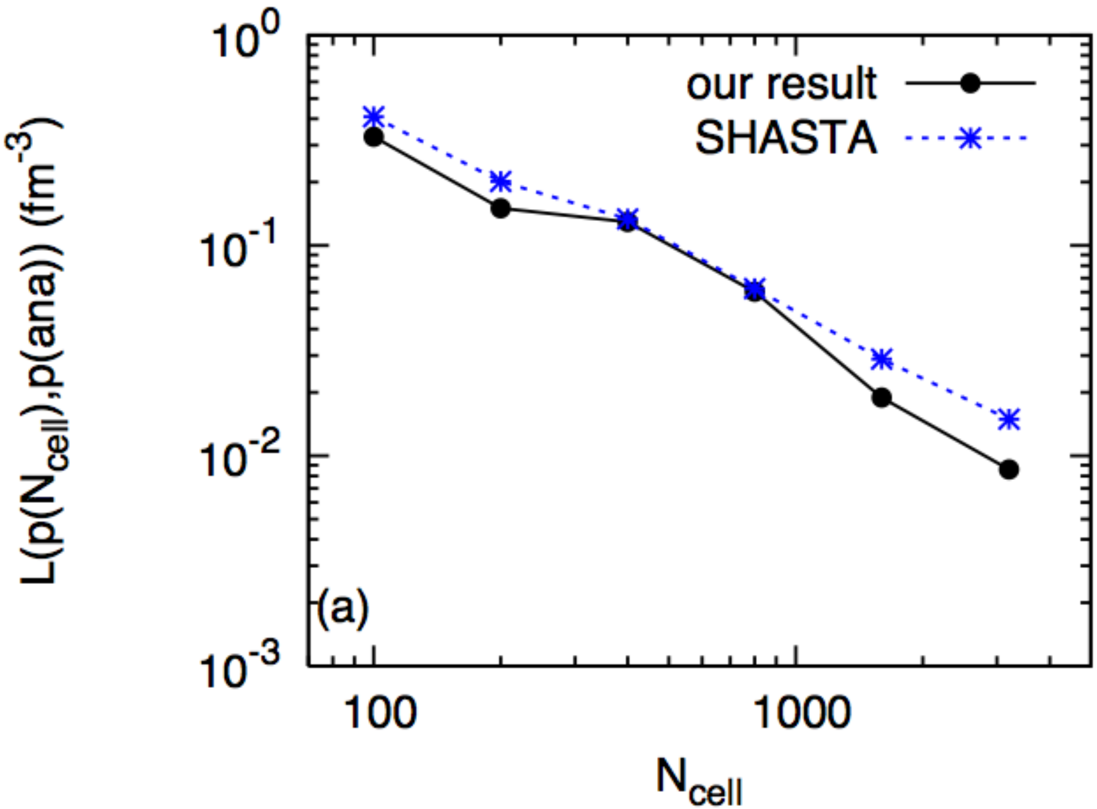
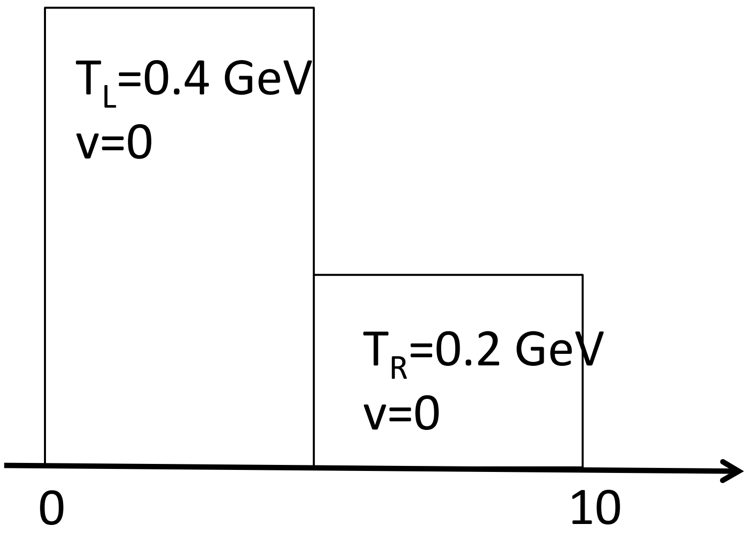
L1 Norm

- Numerical dissipation: deviation from analytical solution





For analysis of heavy ion collisions

N_{cell}=100: dx=0.1 fm

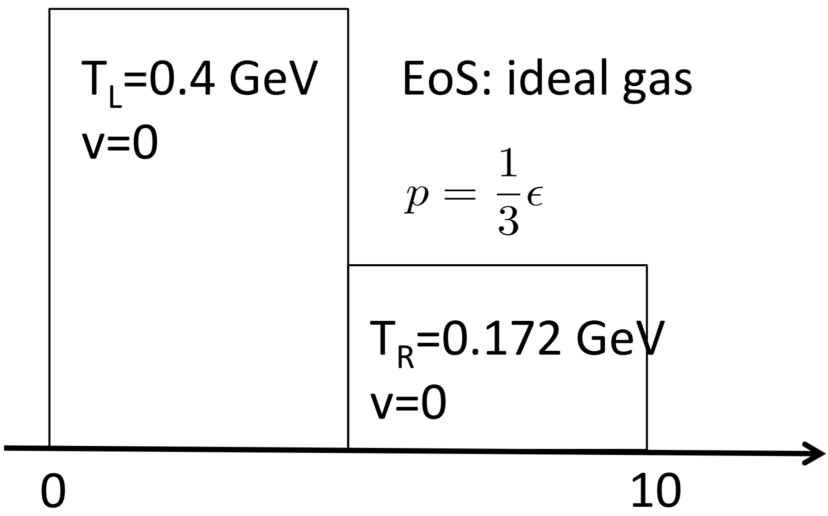
$$L(p(N_{\rm cell}), p({\rm anaytic})) = \sum_{i=1}^{N_{\rm cell}} |p(N_{\rm cell}) - p({\rm analytic})| \frac{\lambda}{N_{\rm cell}}$$

λ=10 fm

Large ΔT difference

- T_L=0.4 GeV, T_R=0.172 GeV
  - SHASTA becomes unstable.
  - Our algorithm is stable.
- SHASTA: anti diffusion term, A_{ad}
  - A_{ad} = 1 : default value, unstable
  - A_{ad} =0.99: stable,

    more numerical dissipation



Nx=100, dx=0.1, dt=0.04

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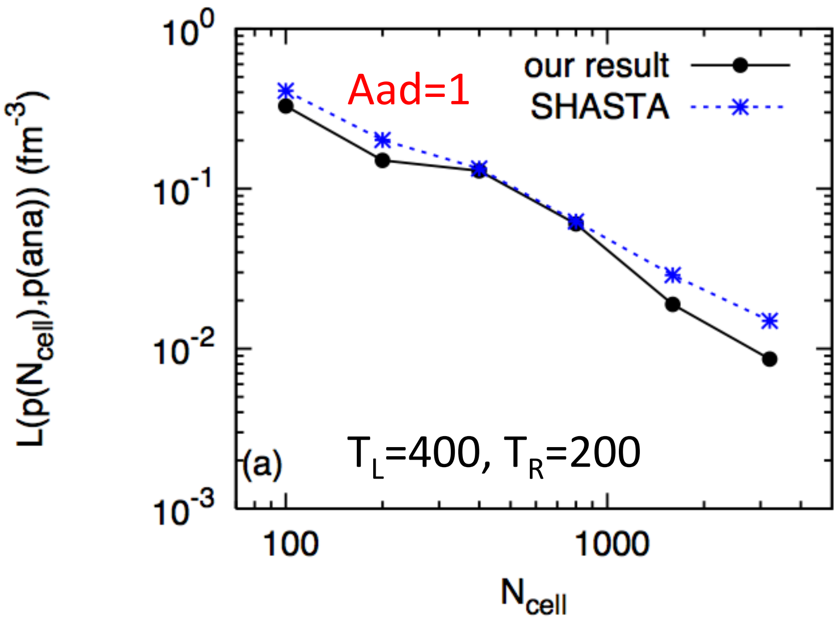
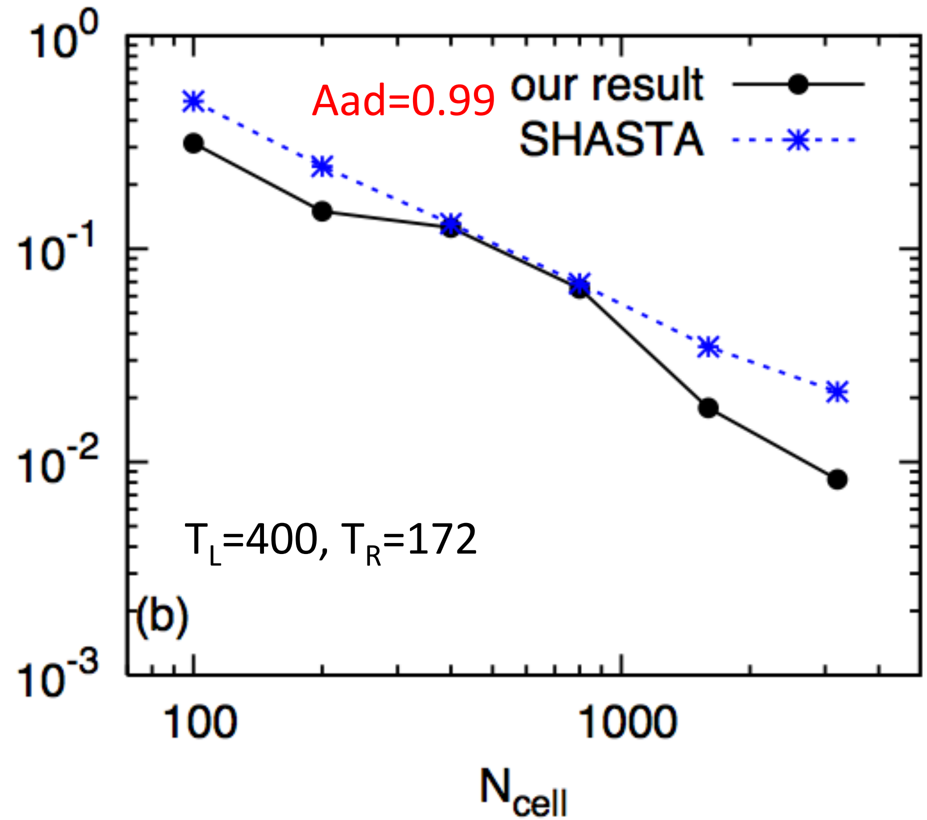
L1 norm

- SHASTA with small A_{ad} has large numerical dissipation



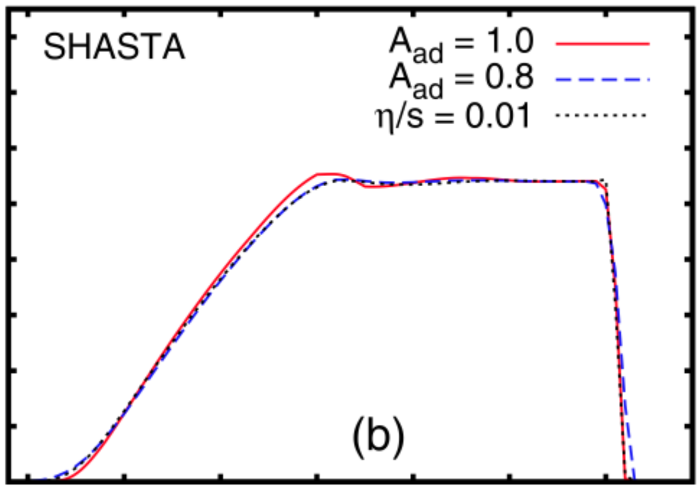


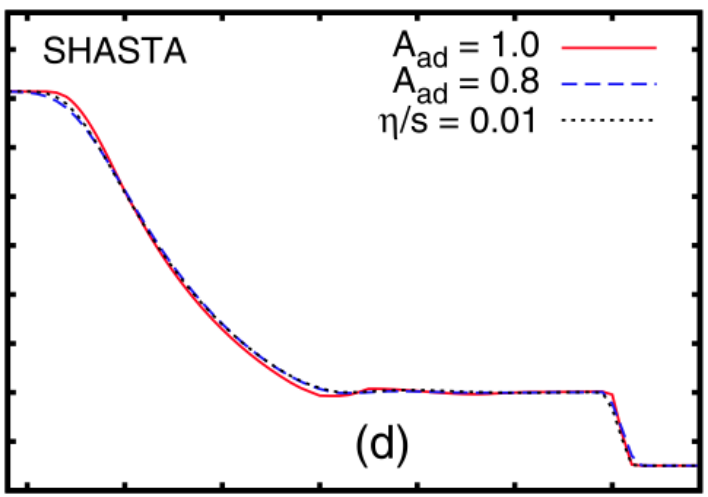
$$L(p(N_{\rm cell}), p(\text{anaytic})) = \sum_{i=1}^{N_{\rm cell}} |p(N_{\rm cell}) - p(\text{analytic})| \frac{\lambda}{N_{\rm cell}}$$

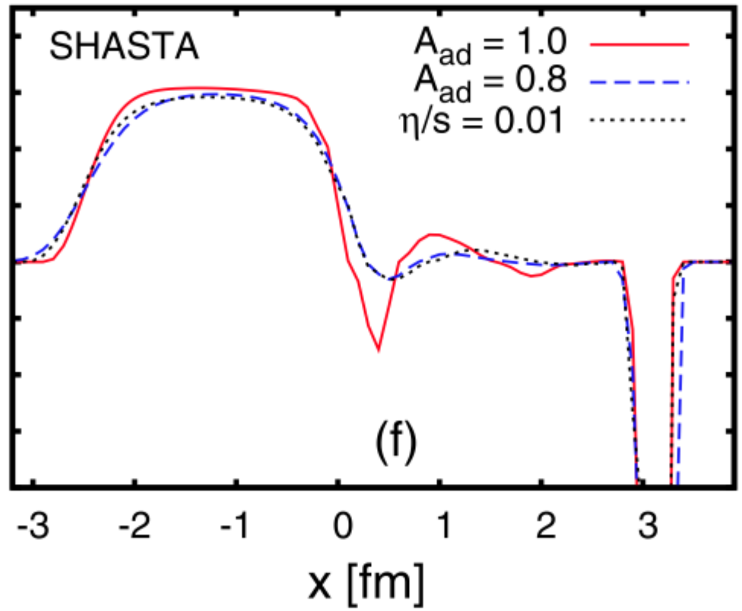
λ=10 fm

Artificial and Physical Viscosities

Molnar, Niemi, Rischke, Eur.Phys.J.C65,615(2010)







Antidiffusion terms : artificial viscosity stability

$$U_i^{n+1} = \tilde{U}_i - \tilde{A}_i + \tilde{A_{i-1}}$$

$$A_i = A\mathrm{ad}\tilde{\Delta}_i/8$$

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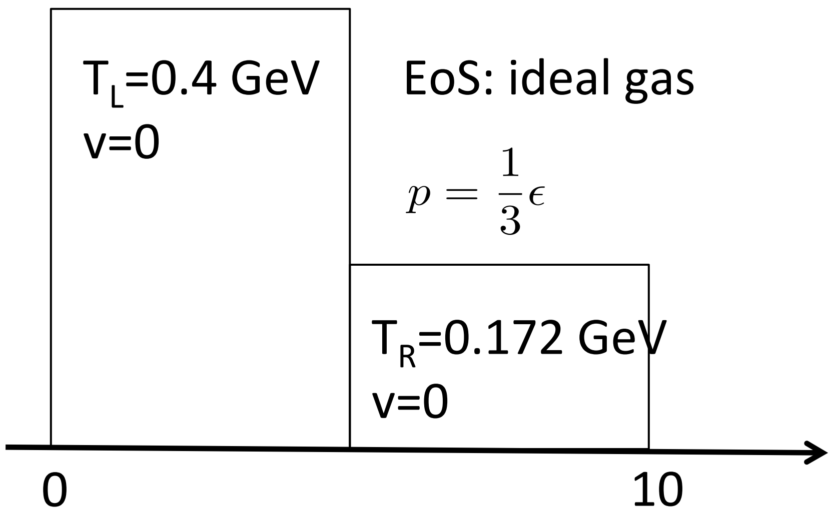

Nx=100, dx=0.1, dt=0.04

- Large fluctuation (ex initial conditions)
  - Our algorithm is stable even with small numerical dissipation.

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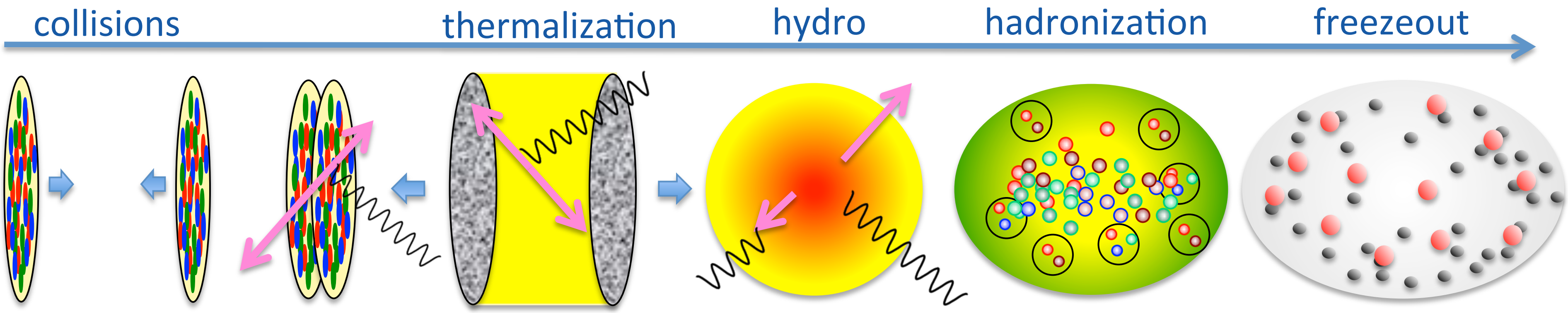
DYNAMICAL MODEL

Our Dynamical Model



Fluctuating Initial conditions

Hydrodynamic expansion

Freezeout process
- From Hydro to particle
- Final state interactions

Akamatsu, Inutsuka, CN, Takamoto, arXiv:1302.1665、J. Comp. Phys. (2014) 34

MC-KLN → hydrodynamic model → Cornelius → Oscar sampler → UrQMD

MC-KLN: *Nara*
http://www.aiu.ac.jp/~ynara/mckln/

Cornelius: Freezeout hypersurface finder
Huovinen, Petersen

Oscar sampler: *Ohio group*

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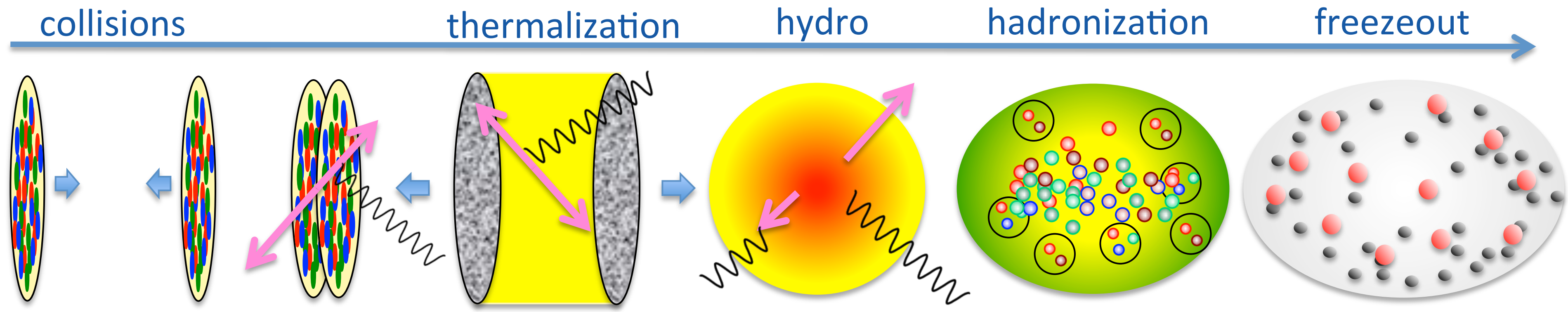
Our Dynamical Model



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Oscar sampler: *Ohio group*

Simulation setups:
- Free gluon EoS
- Hydro in 2D boost invariant simulation
- UrQMD with |y|<0.5

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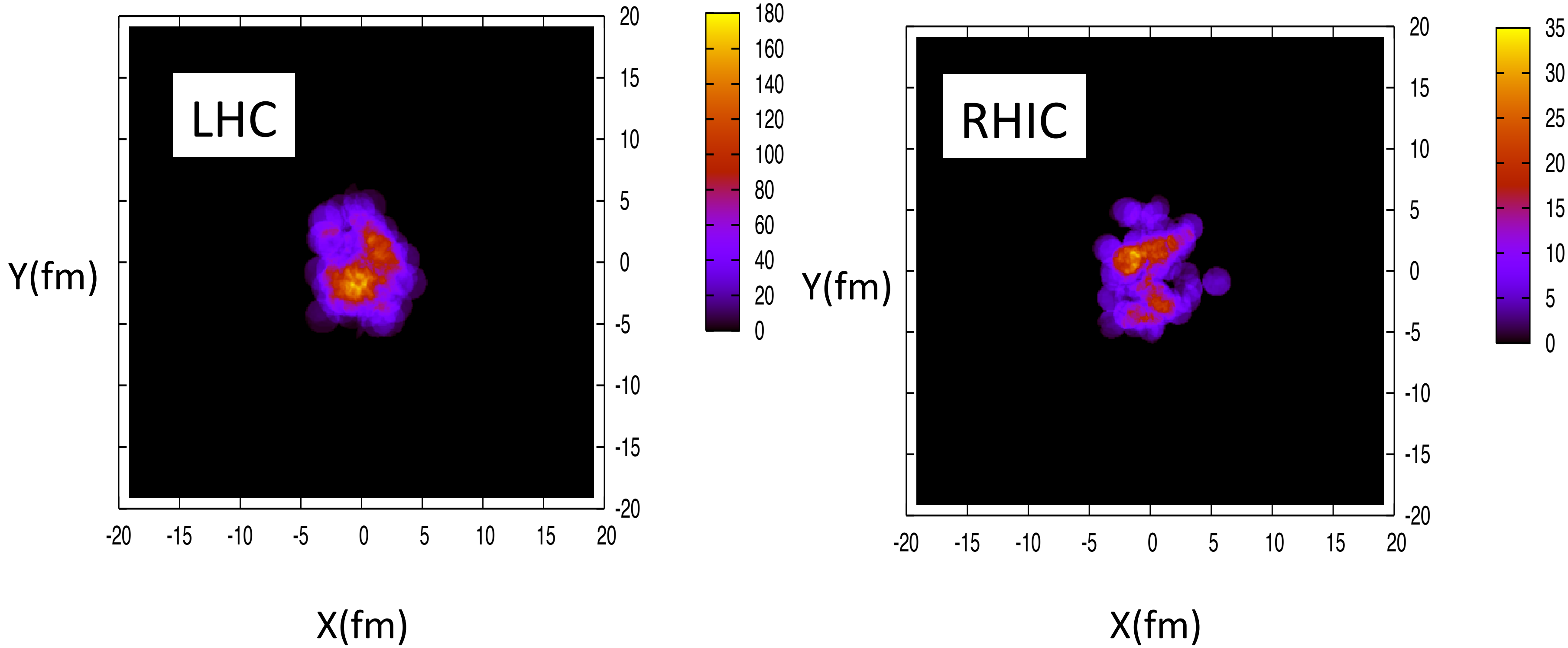
Initial Pressure Distribution

- MC-KLN (centrality 15-20%)

Pressure (fm^{-4})



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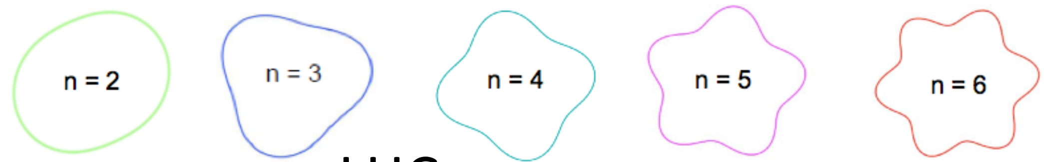
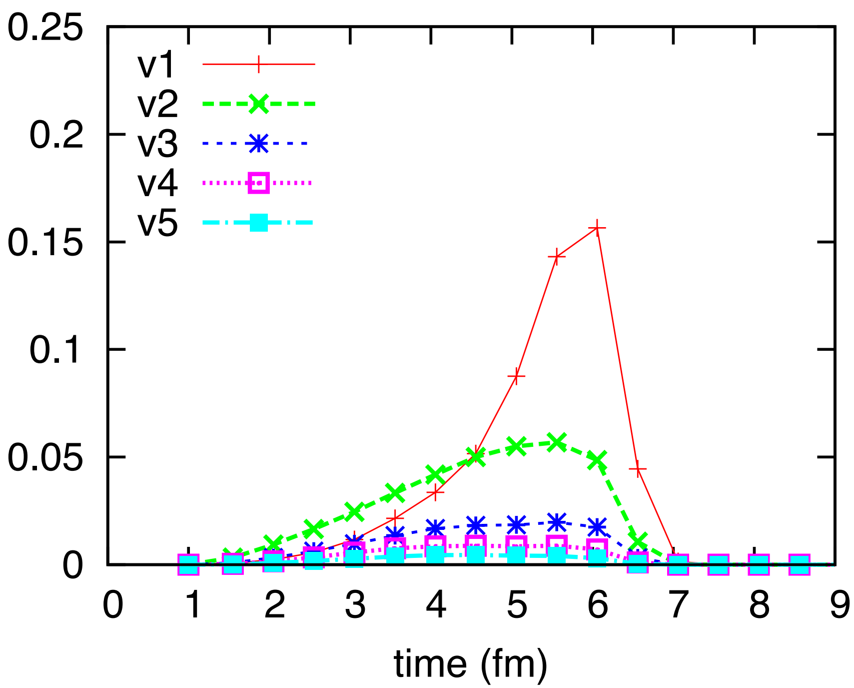
Time Evolution of v_n

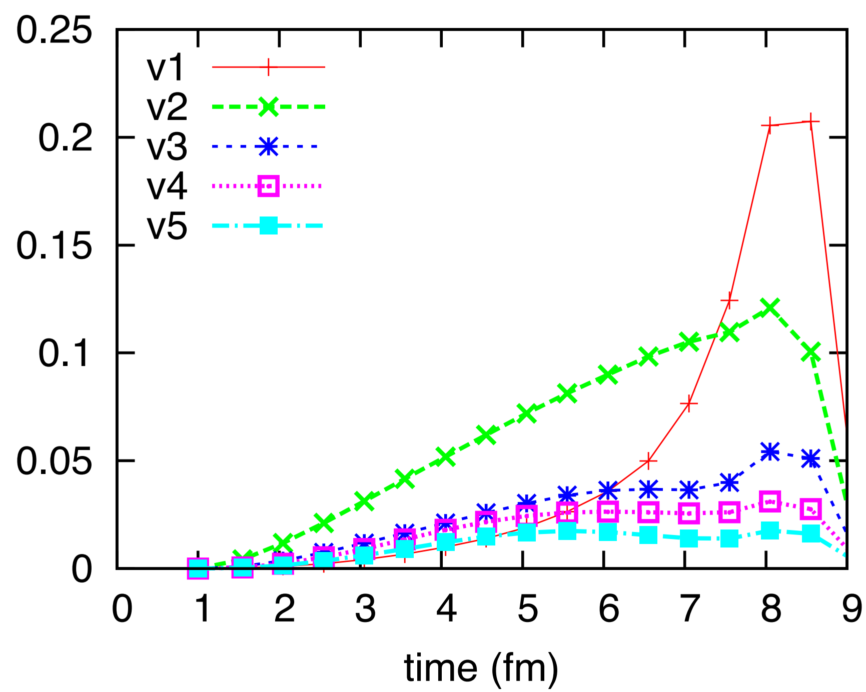
$$\frac{dN}{dy\,d\phi} \propto 1+2v_1\cos(\phi-\Theta_1)+2v_2\cos 2(\phi-\Theta_2)+2v_3\cos 3(\phi-\Theta_3)+2v_4\cos 4(\phi-\Theta_4)+\cdots$$



RHIC



LHC



- Qualitatively RHIC ~ LHC
- v_2 is dominant
- $v_2 > v_3 > v_4 > v_5$

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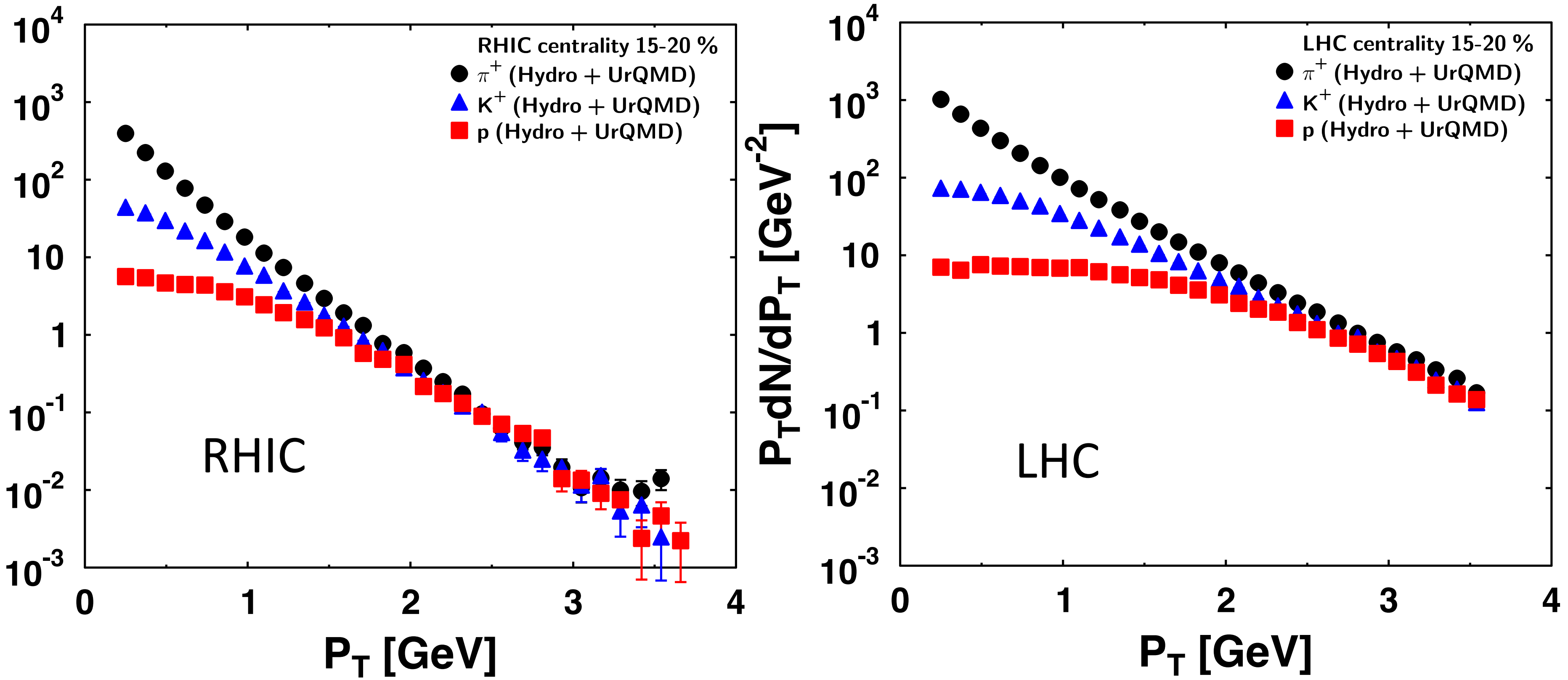
Hydro + UrQMD

- Transverse momentum spectrum

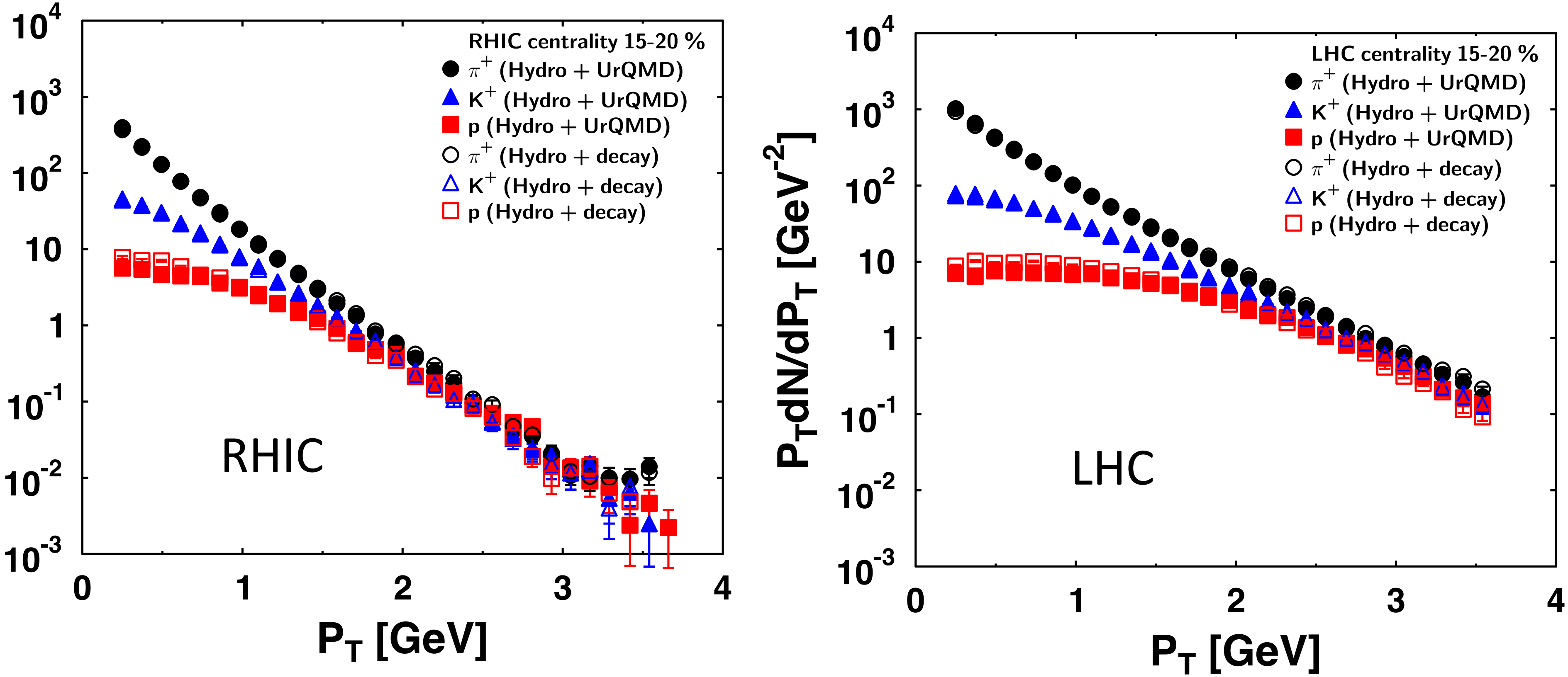


- Pt distribution at LHC has flatter slope

Larger radial flow at LHC

Effect of Hadronic Interaction

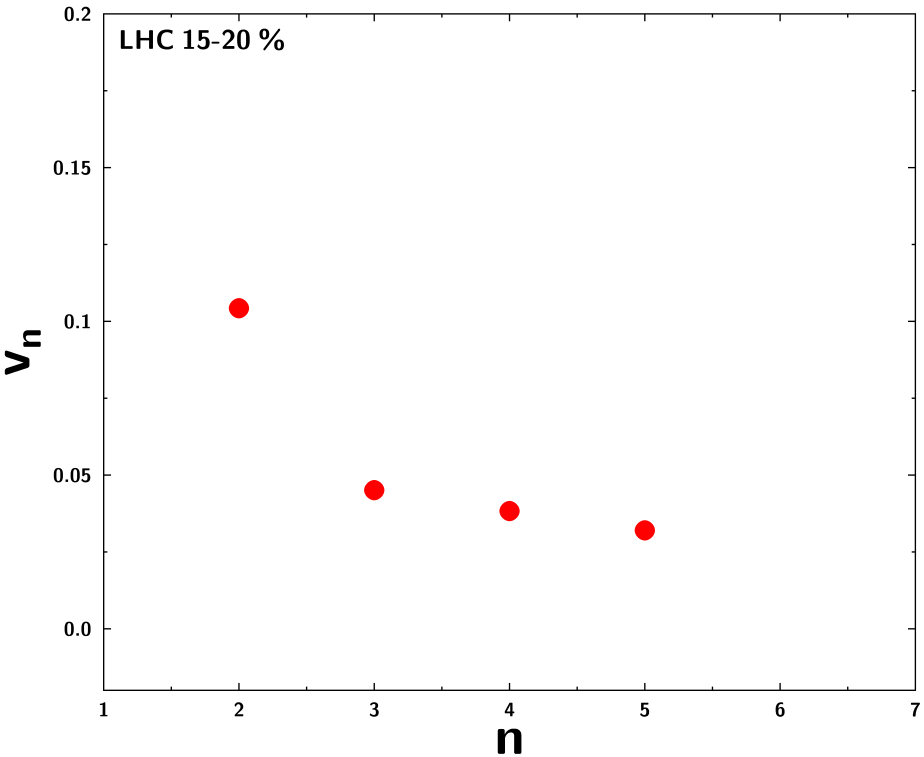
- Transverse momentum distribution

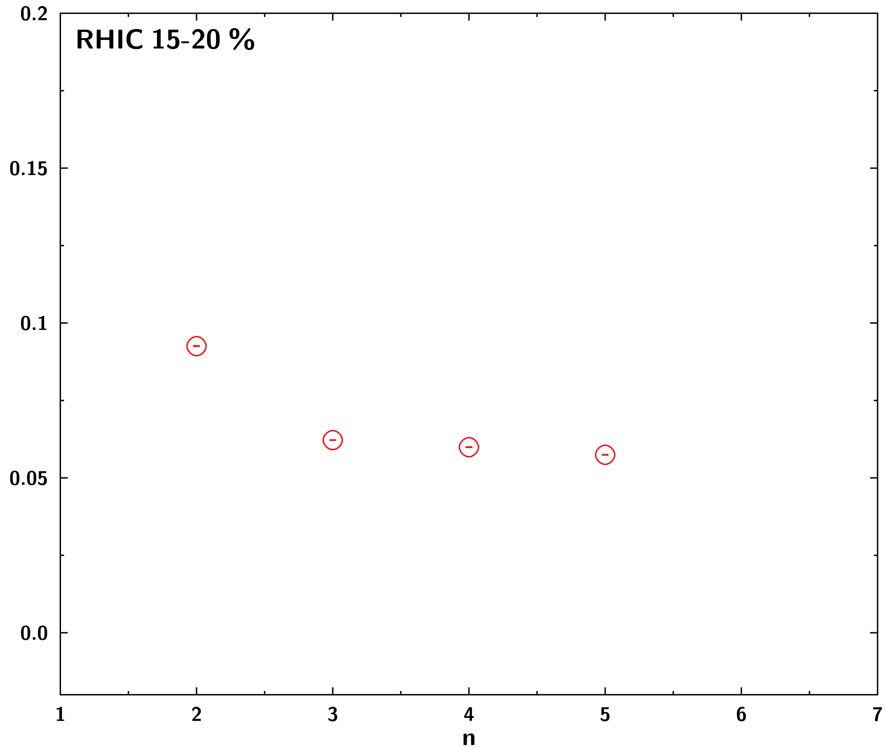


- Effect of final state interactions is small
- Slope of proton Pt spectra become flatter

Higher harmonics from Hydro + UrQMD

- Effect of hadronic interaction





Summary

Importance of numerical scheme in Hydrodynamic Models

- We develop a state-of-the-art numerical scheme
  - Shock wave capturing scheme: Godunov method

Our algorithm

- Less artificial diffusion: crucial for viscosity analyses
- Stable for strong shock wave

- Construction of a hybrid model
  - Fluctuating initial conditions + Hydrodynamic evolution + UrQMD
- Higher Harmonics
  - Time evolution, hadron interaction