

Walking signals in $N_f=8$ QCD on the lattice



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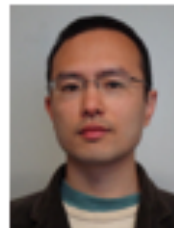
Nagoya University



KMI 2013, 11 December 2013 @ KMI, Nagoya

LatKMI collaboration

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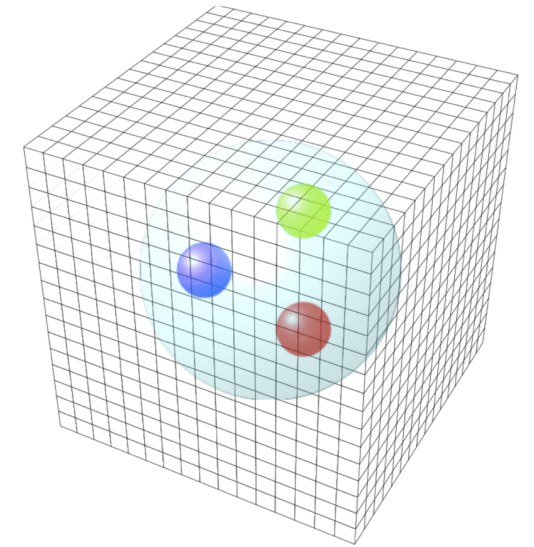


A. Shibata



Plan of the Talk:

1. Introduction
2. Lattice study of $N_f=8$ QCD
 - { Chiral Perturbation Theory (ChPT)
 - { Finite Size Hyperscaling (FSHS)
3. Summary



♠ $N_f=8$ is the candidate of the walking behavior.

Walking signals in $N_f=8$ QCD on the lattice

[Yasumichi Aoki](#), [Tatsumi Aoyama](#), [Masafumi Kurachi](#), [Toshihide Maskawa](#), [Kei-ichi Nagai](#), [Hiroshi Ohki](#), [Akihiro Shibata](#),
[Koichi Yamawaki](#), [Takeshi Yamazaki](#), Feb 27, 2013,

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1. Introduction

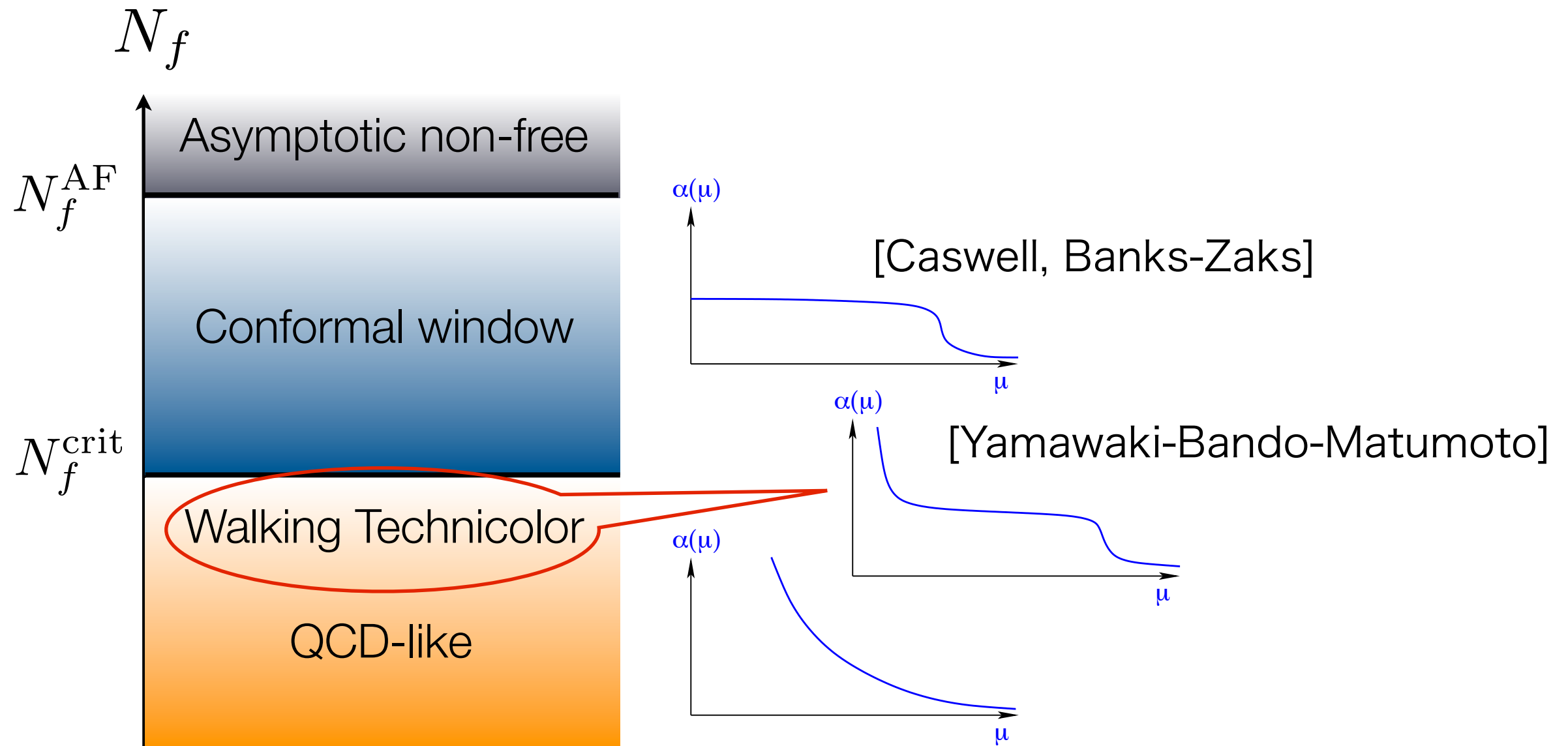
- LQCD with many fermions
- BSM (quark mass, FCNC, ..., model building)
- $\Rightarrow$ Candidate of the walking technicolor (WTC)

Requirements for the successful WTC theory

- spontaneous chiral symmetry breaking
 - running coupling “walks” = slowly changing with μ : $\rightarrow$ nearly conformal
 - large mass anomalous dimension: $\gamma_m \sim 1$
 - light scalar 0^{++} ($m_H = 126$ GeV @ LHC !)
 - with input $F_\pi = 246 / \sqrt{N}$ GeV (N: # weak doublet in techni-sector)
 - to reproduce $W^\pm$ mass
 - typical QCD like theory: $M_{\text{Had}} \gg F_\pi$ (ex.: QCD: $m_\rho/f_\pi \sim 8$)
 - Naive TC: $M_{\text{Had}} > 1,000$ GeV
 - 0^{++} is a special case: pseudo Nambu-Goldstone boson of scale inv.
- $\Rightarrow$ is it really so ?

conformal window and walking coupling

- non-Abelian gauge theory with N_f massless fermions -



- Walking Technicolor could be realized just below the conformal window
- crucial information: N_f^{crit} & mass anomalous dimension around N_f^{crit}

Many flavor QCD

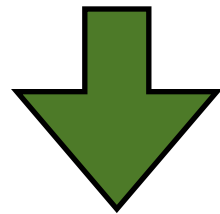
⇒ Candidate of walking/conformal

Our investigation in $N_f=12$ (Ohki's talk)

→ consistent with the conformal with $\gamma=0.4--0.5$.

not favor as WTC (model building)

Thus, we investigate $N_f=8$ QCD.
strong coupling dynamics and non-perturbative



Lattice simulation of $N_f=8$ QCD

Lattice studies of $N_f=8$:

Y. Iwasaki et al. ('92, '04) [pioneering work] $\rightarrow$ conformal

K. Ishikawa et al. ('12, '13) $\rightarrow$ conformal

(Yukawa-type correlator in $N_f=7$ and 16)

A. Cheng et al. ('13) $\rightarrow$ large γ_m over a wide range
of energy scales (slow-running?)

A. Deuzeman et al.('08), K. Miura et al.('12)

$\rightarrow S_\chi$ SB, but near the conformal edge
(thermodynamics)

Z. Fodor et al. ('09) $\rightarrow S_\chi$ SB

T. Appelquist et al. ('09) $\rightarrow$ no IRFP

LatKMI('13) $\rightarrow$ In this talk (walking?)

What is the signal of walking?

Scenario of Walking Dynamics ;

Case-1: probe $m_f \ll m_D \rightarrow S\chi SB$ -like

Case-2: probe $m_f \gg m_D \rightarrow$ conformal-like

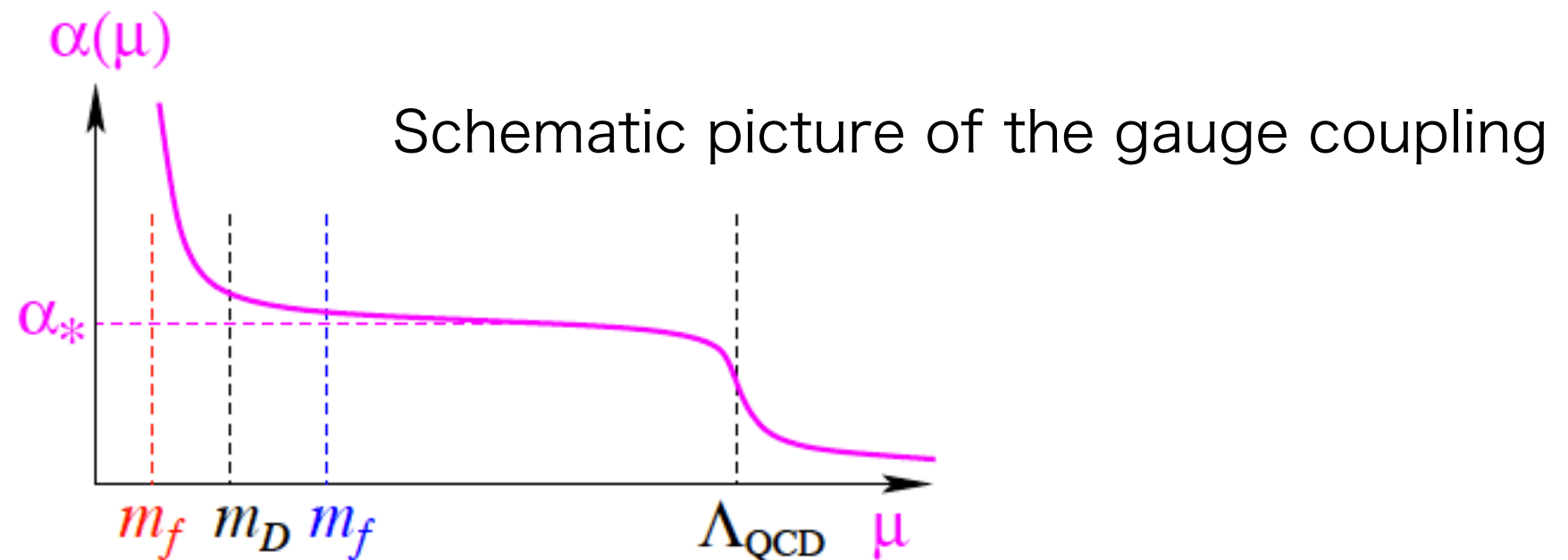


FIG. 1. Schematic two-loop/ladder picture of the gauge coupling of the massless large N_f QCD as a walking gauge theory in the $S\chi SB$ phase near the conformal window. m_D is the dynamical mass of the fermion generated by the $S\chi SB$. The effects of the bare mass of the fermion m_f would be qualitatively different depending on the cases: Case 1: $m_f \ll m_D$ (red dotted line) well described by ChPT, and Case 2: $m_f \gg m_D$ (blue dotted line) well described by the hyper scaling.

$\Rightarrow$ **Spectrum?**

$S\chi SB$ and/or conformal in some m_f region ?

2. Lattice Study of $N_f=8$ case

Walking signals in $N_f=8$ QCD on the lattice

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e-Print: [arXiv:1302.6859](#) [hep-lat].

Simulation for $N_f=8$ (same setup with $N_f=12$)

lattice action (Hybrid Monte-Carlo simulation)

- Tree-level Symanzik gauge action
- Highly Improved Staggered Quarks = **HISQ**
(without tadpole improvement and mass correction in Naik term)

★ parameter set

- $\beta (\equiv 6/g^2) = \mathbf{3.8}, \quad V=L^3 \times T, T/L=4/3 \text{ fixed.}$

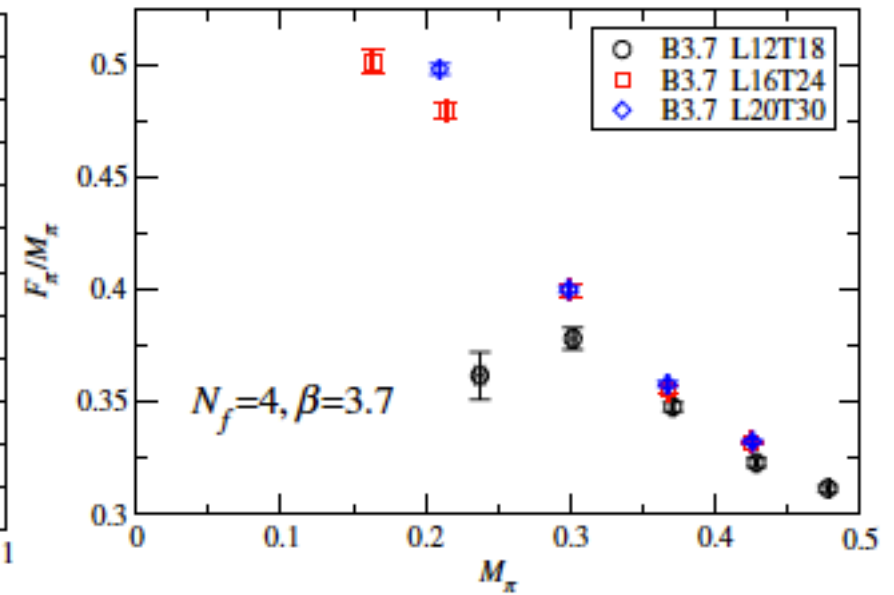
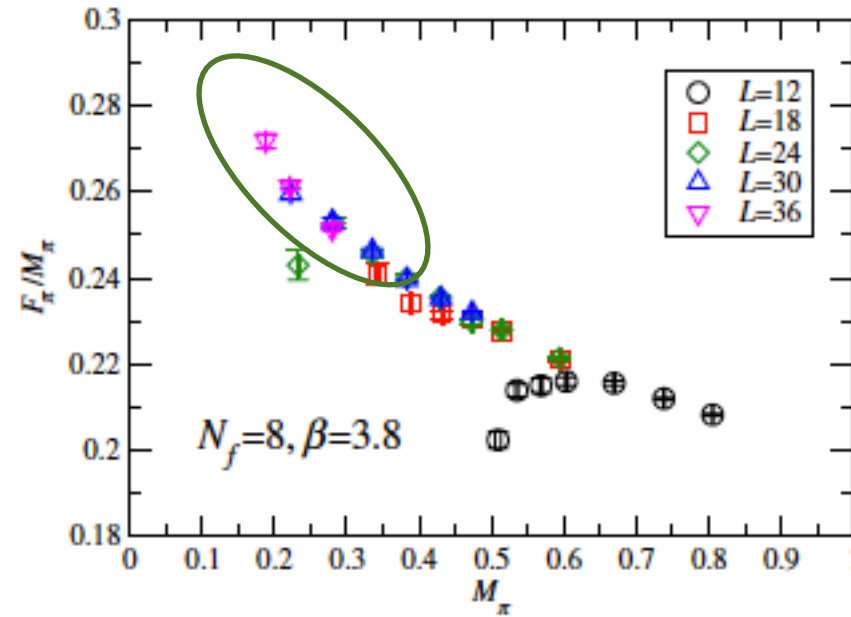
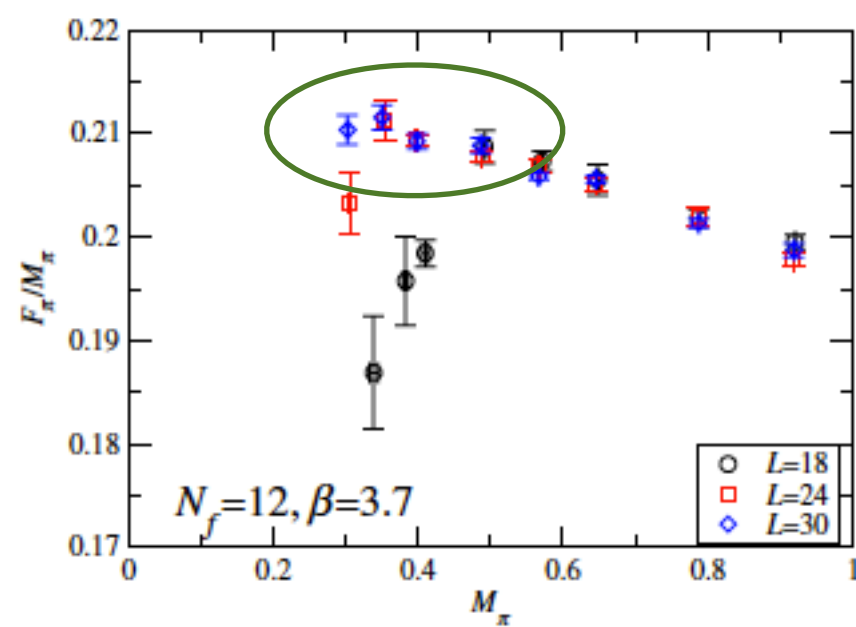
V	$12^3 \times 16$	$18^3 \times 24$	$24^3 \times 32$	$30^3 \times 40$	$36^3 \times 48$
mf	0.01~0.16	0.04~0.1	0.02~0.1	0.02~0.07	0.015~0.03

★ Measurements (P+AP method $\Rightarrow$ double size in T-dir.)

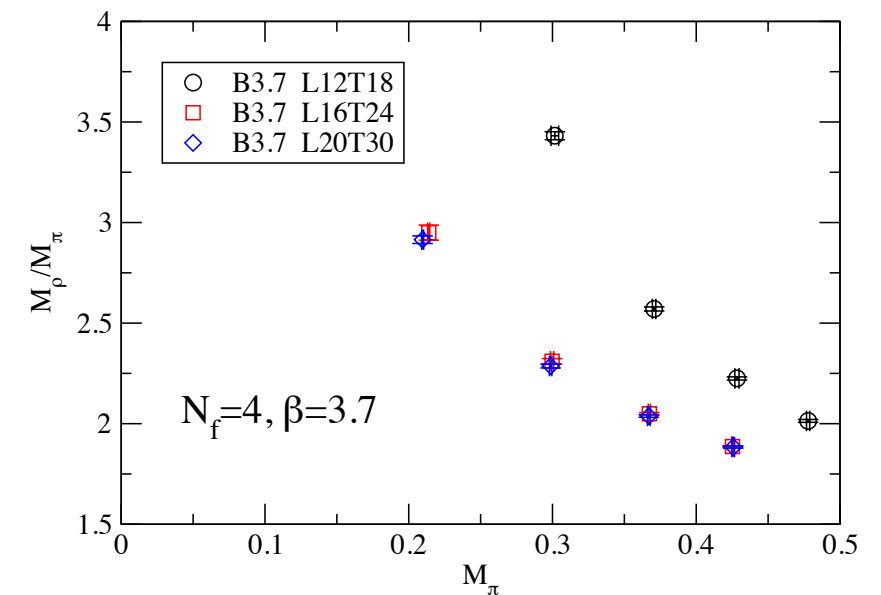
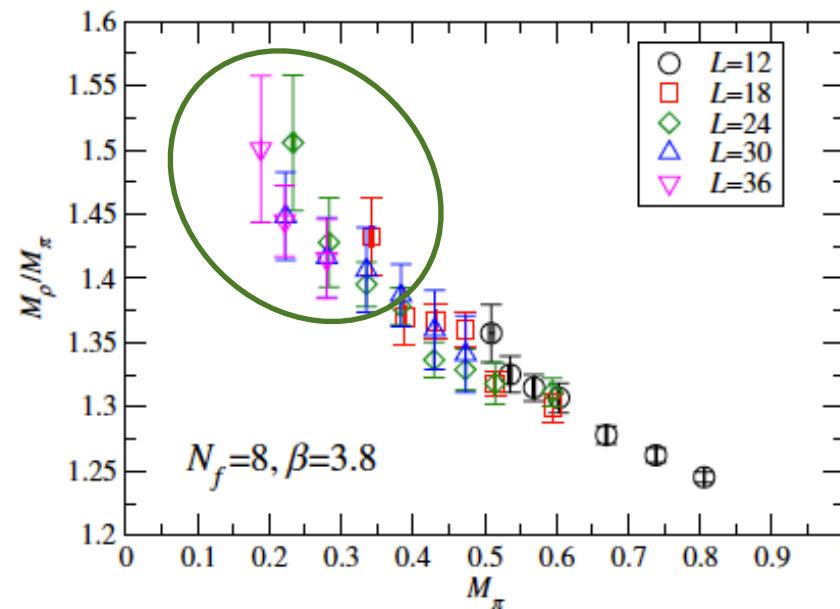
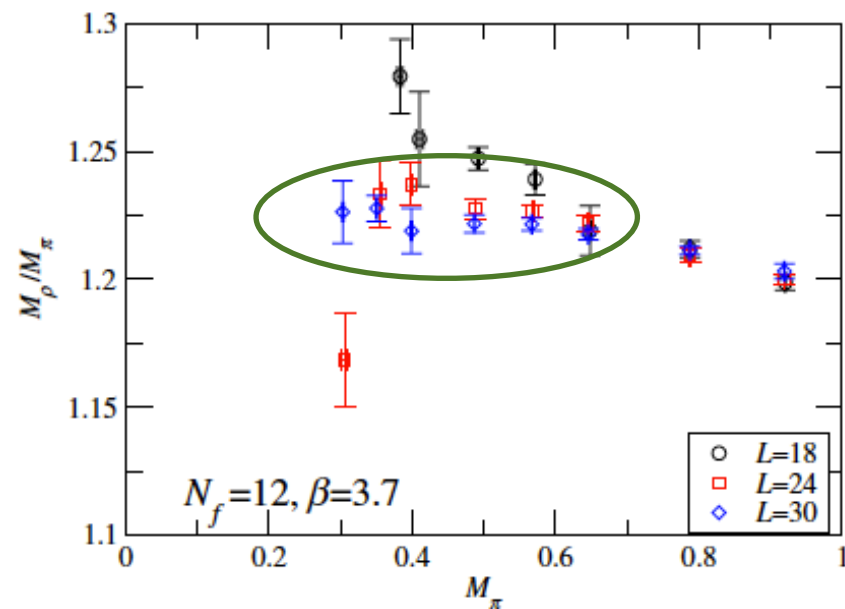
★ about 1000 trajectories after thermalization

- M_π, F_π, M_ρ , chiral condensate
- analysis for $M_\pi L > 6$ (to cut the finite size effect)

F_π/M_π for $N_f=12$, 8 and 4 (flat or divergent in χ -limit?)



M_ρ/M_π for $N_f=12$, 8 and 4 (flat or divergent in χ -limit?)



Nf=8 $\Rightarrow$ spontaneous chiral symmetry breaking?
(S χ SB)

Chiral Perturbation Theory (ChPT)

In S χ SB; $M_\pi^2 = C_1^\pi m_f + C_2^\pi m_f^2 + \dots$, $F_\pi = F + C_1^F m_f + C_2^F m_f^2 + \dots$,

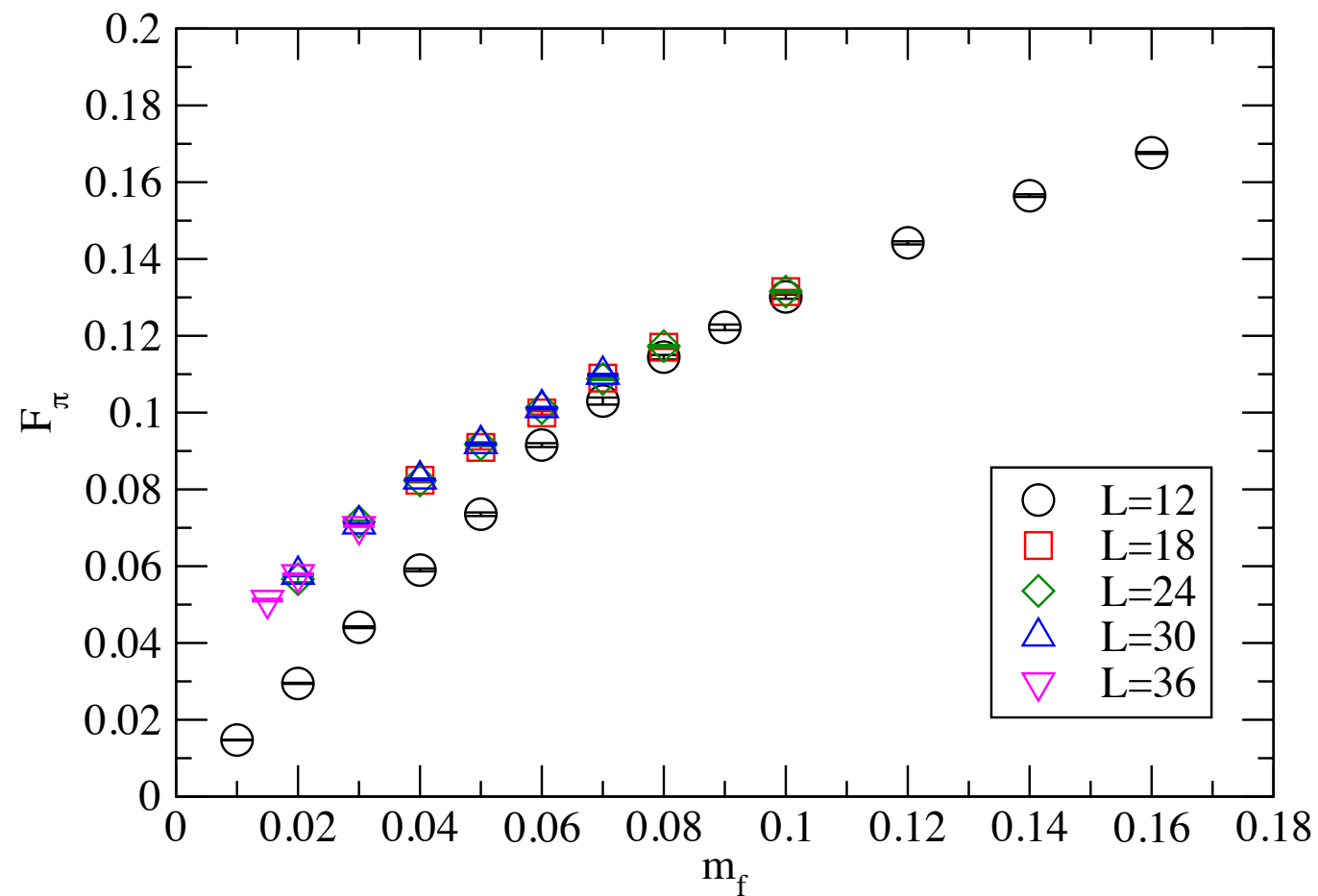
$\Rightarrow$ Polynomial fit in small mf region

We regard the data on the largest volume at each mf
as the ones on the infinite volume. (Backup figs.)

We don't discuss the chiral log behavior in this talk.

However, we discussed the chiral log in the published paper.

F_π vs m_f on various Lattice sizes



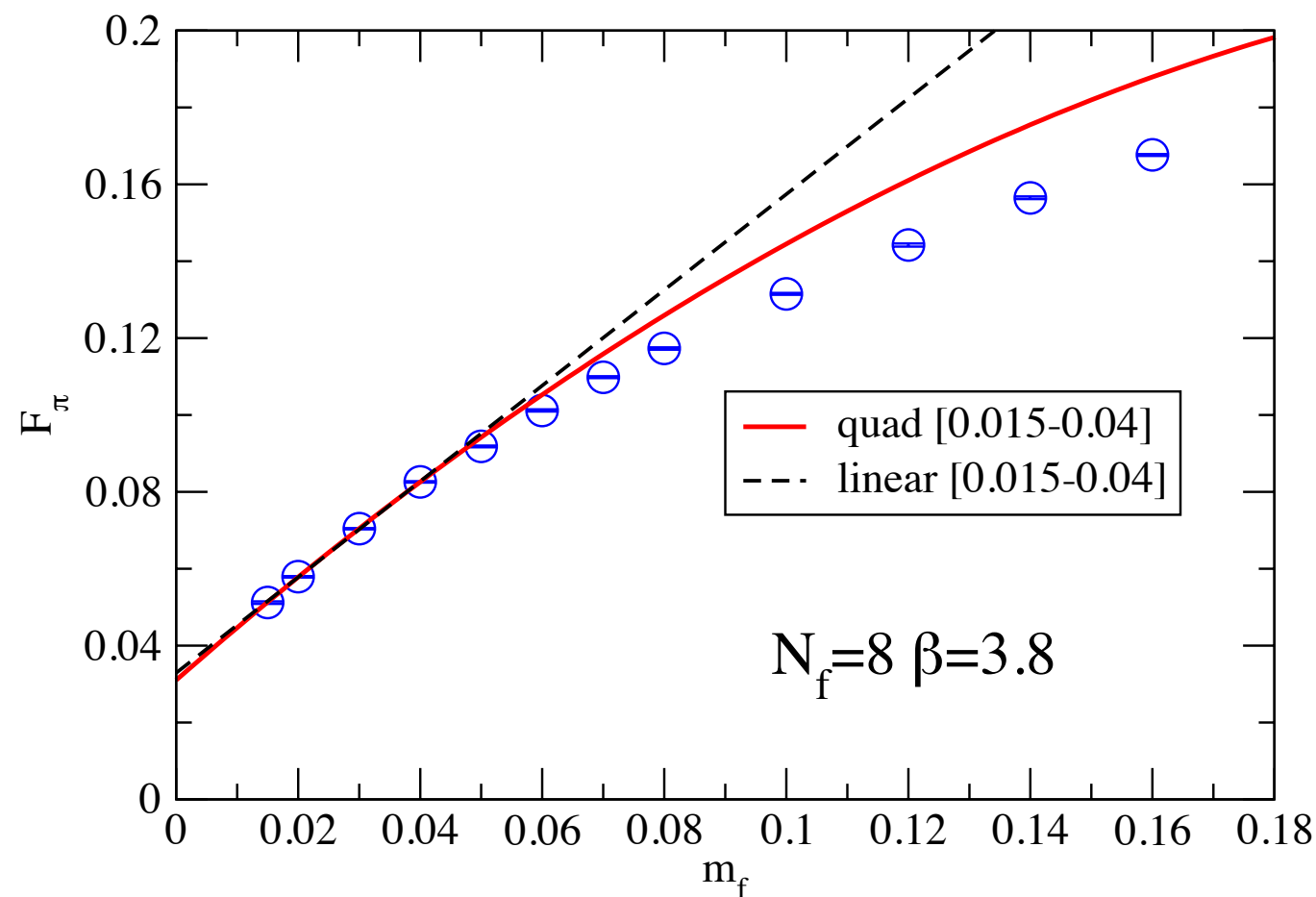
for the ChPT fit (to cut the size effect) ;
 F_π data on the largest volume at each m_f

$$F_\pi$$

Quadratic fit in $m_f=[0.015, 0.04]$

Linear fit in $m_f=[0.015, 0.04]$

In wider fit range, χ^2/dof becomes worse.



$$F_\pi = 0.0310(13) + 1.4(1) m_f - 2.6(1.7) m_f^2 \quad (\chi^2/\text{dof}=0.46)$$

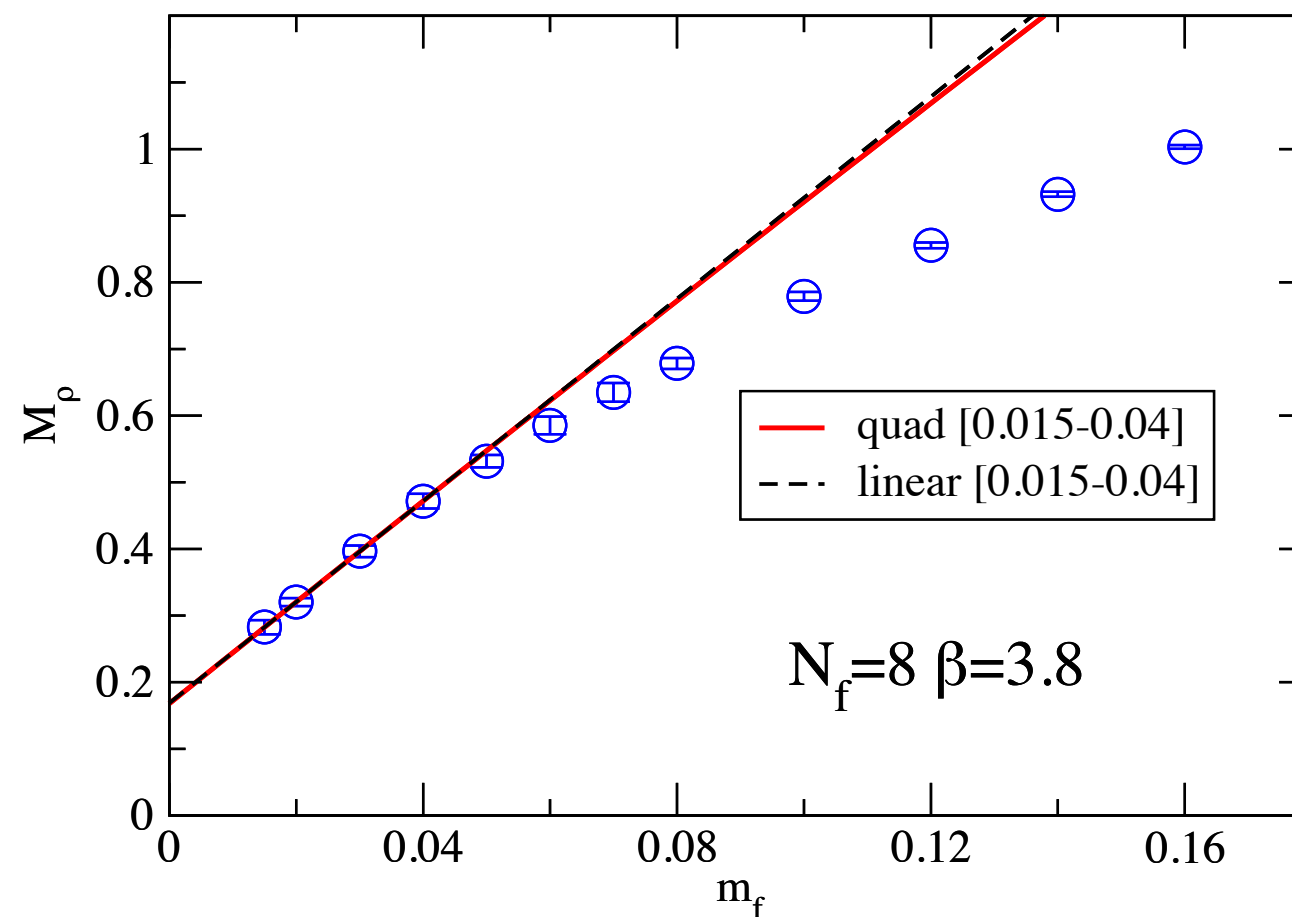
$$F_\pi = 0.0329(3) + 1.24(1) m_f \quad (\chi^2/\text{dof}=1.4)$$

$$M_\rho$$

Quadratic fit in $mf=[0.015, 0.04]$

Linear fit in $mf=[0.015, 0.04]$

$N_f=8$ (F_π and M_ρ) in small mf is consistent with ChPT.



$$M_\rho = 0.168(32) + 7.6(4.1) m_f - 1.2(73.4) m_f^2 \quad (\chi^2/\text{dof} = 0.0017)$$

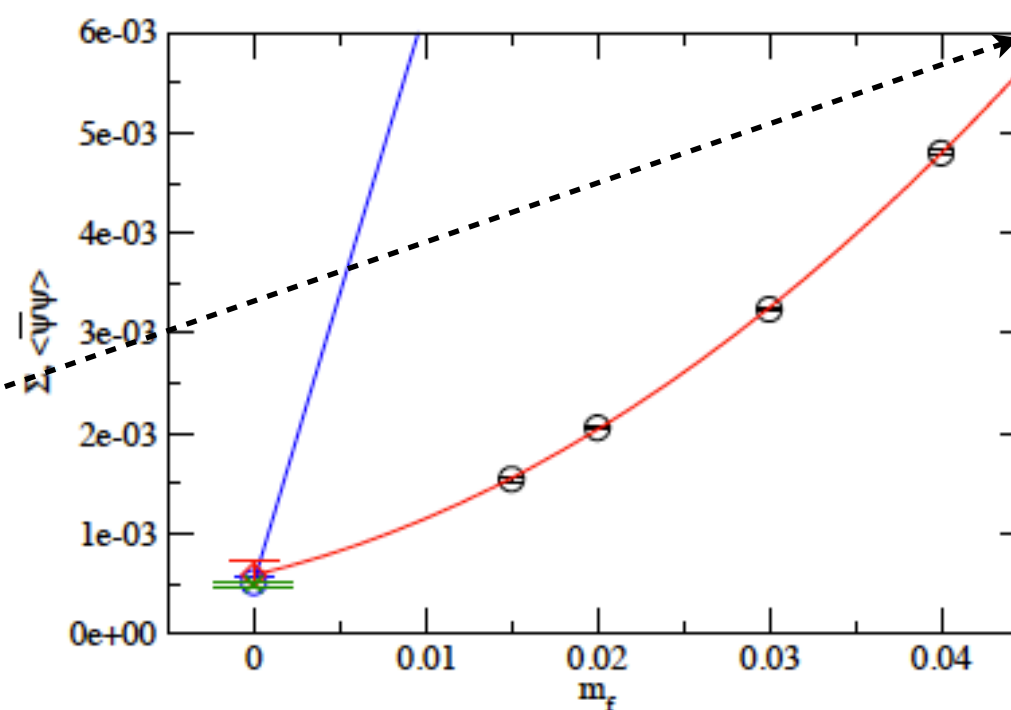
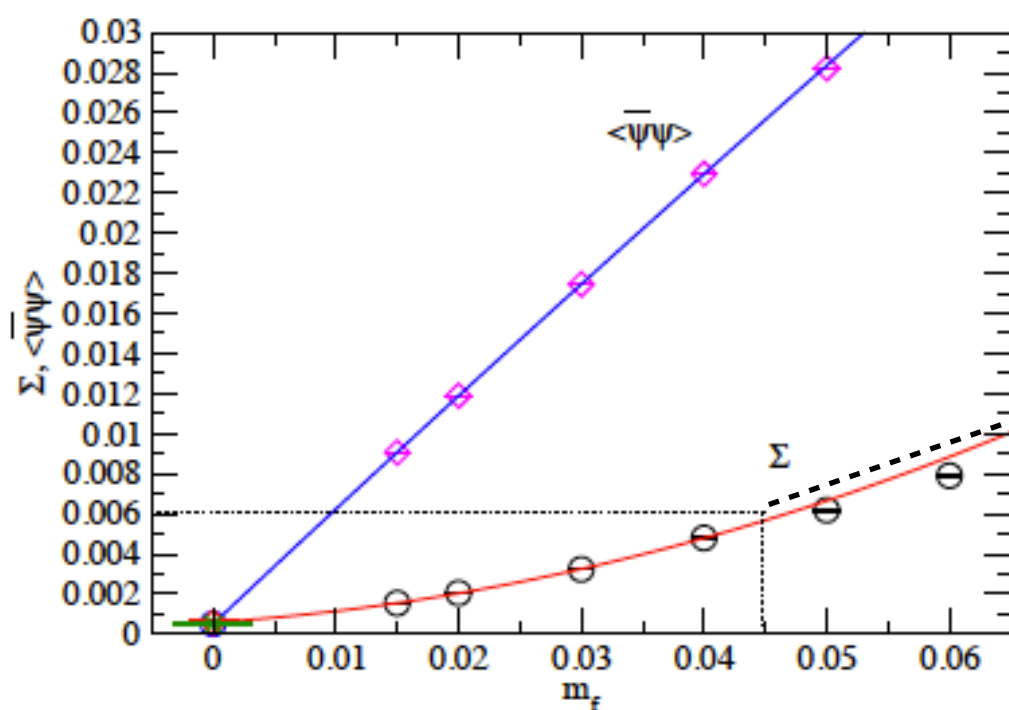
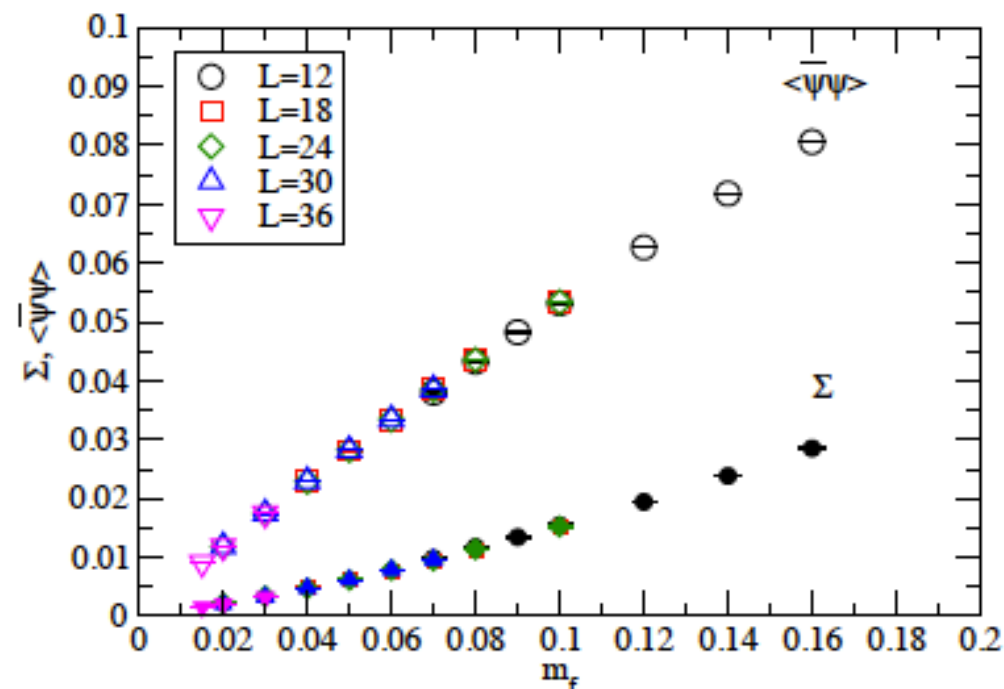
$$M_\rho = 0.169(13) + 7.6(5) m_f \quad (\chi^2/\text{dof} = 0.0010)$$

Chiral condensate (direct and indirect calc.)

direct: $= \text{Tr}[D_{\text{HISQ}}^{-1}(x,x)]/4$

indirect: $\Sigma = F_\pi^2 M_\pi^2 / (4m_f)$

based on the GMOR relation



In chiral limit (quadratic fit in $0.015 \leq m_f \leq 0.04$)

$$\langle \bar{\psi}\psi \rangle|_{m_f \rightarrow 0} = 0.00052(5), \quad \Sigma|_{m_f \rightarrow 0} = 0.00059(13). \quad F^2 \cdot \left(\frac{M_\pi^2}{4m_f} \right) \Big|_{m_f \rightarrow 0} = 0.00050(3)$$

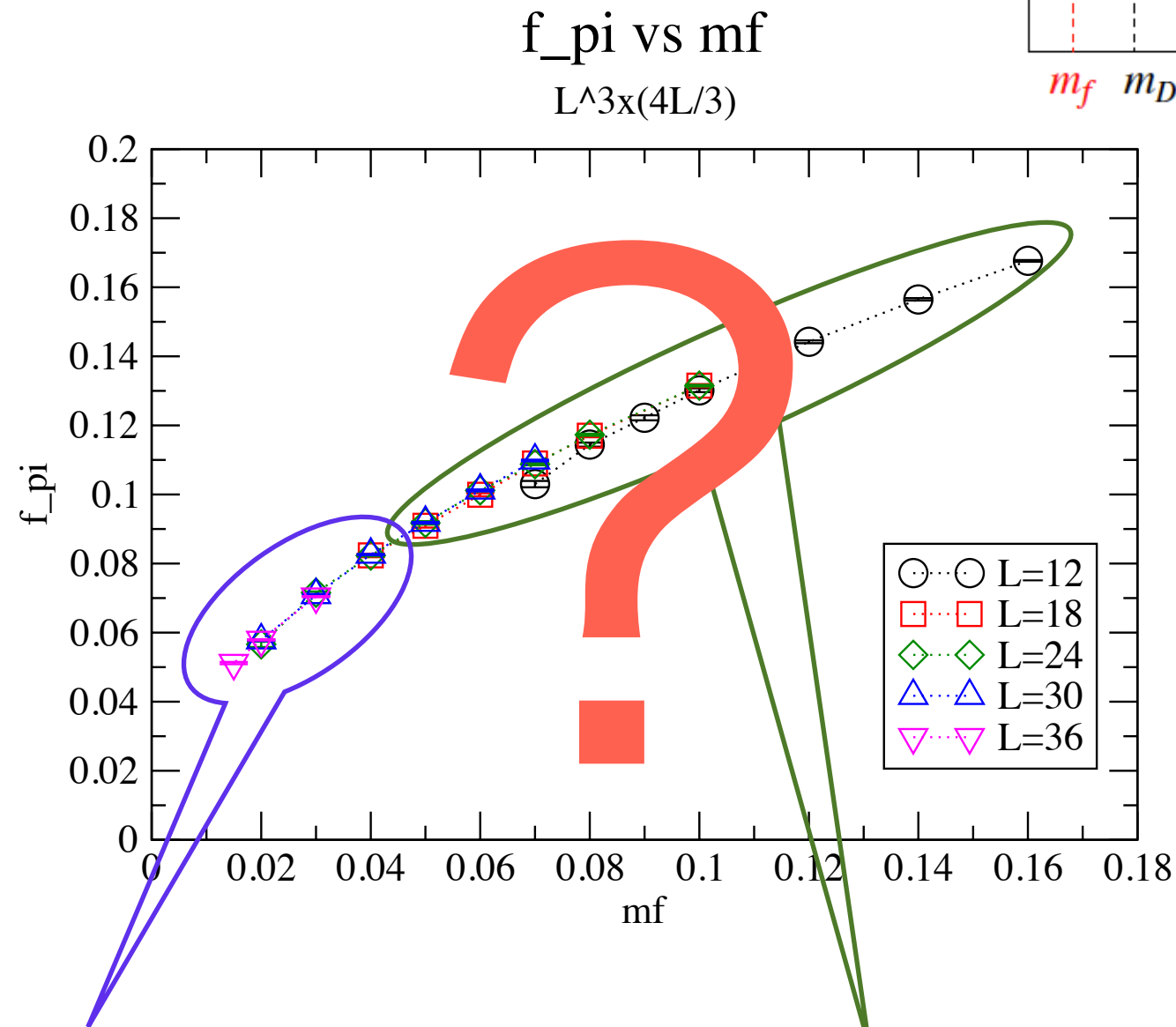
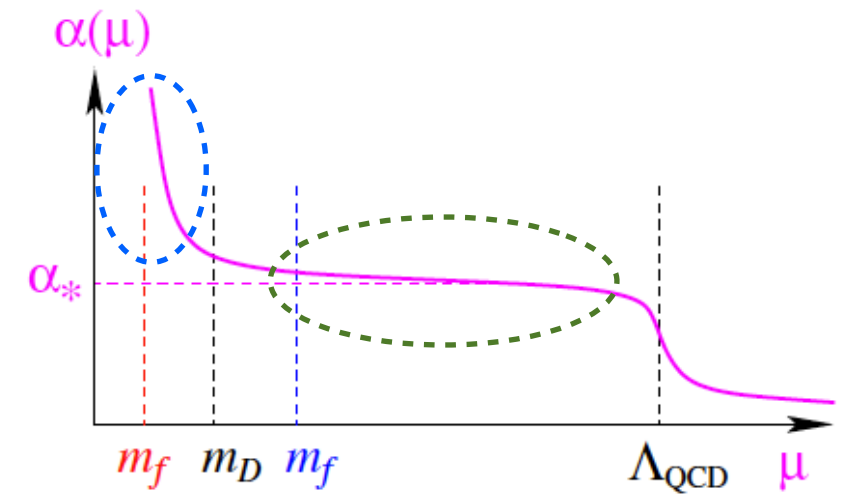
Summary-1, ChPT analysis

- The quadratic fit was done in $0.015 \leq mf \leq 0.04$.
- $N_f=8$ is consistent with $S \chi$ SB in the small mf region.
- $F_\pi > 0$, $M_\rho > 0$, Condensate > 0 , $M_\pi = 0$ in the χ -limit.
- In the χ -limit. $F_\pi = 0.0310(13)$, $M_\rho / (F_\pi / \sqrt{2}) = 7.7(1.5)$.
- The expansion parameter $\chi = O(1)$ of ChPT in the smallest mf (self-consistent), in contrast to $N_f=12$.
- $\Rightarrow$ simple $S \chi$ SB phase?

$$\chi = N_f \left(\frac{M_\pi}{4\pi F} \right)^2$$

F_π vs mf :

How is in the region $mf \geq 0.05$?



Polynomial-like behavior
(ChPT-like)

Power-like behavior ?
(Remnant of conformality ?)

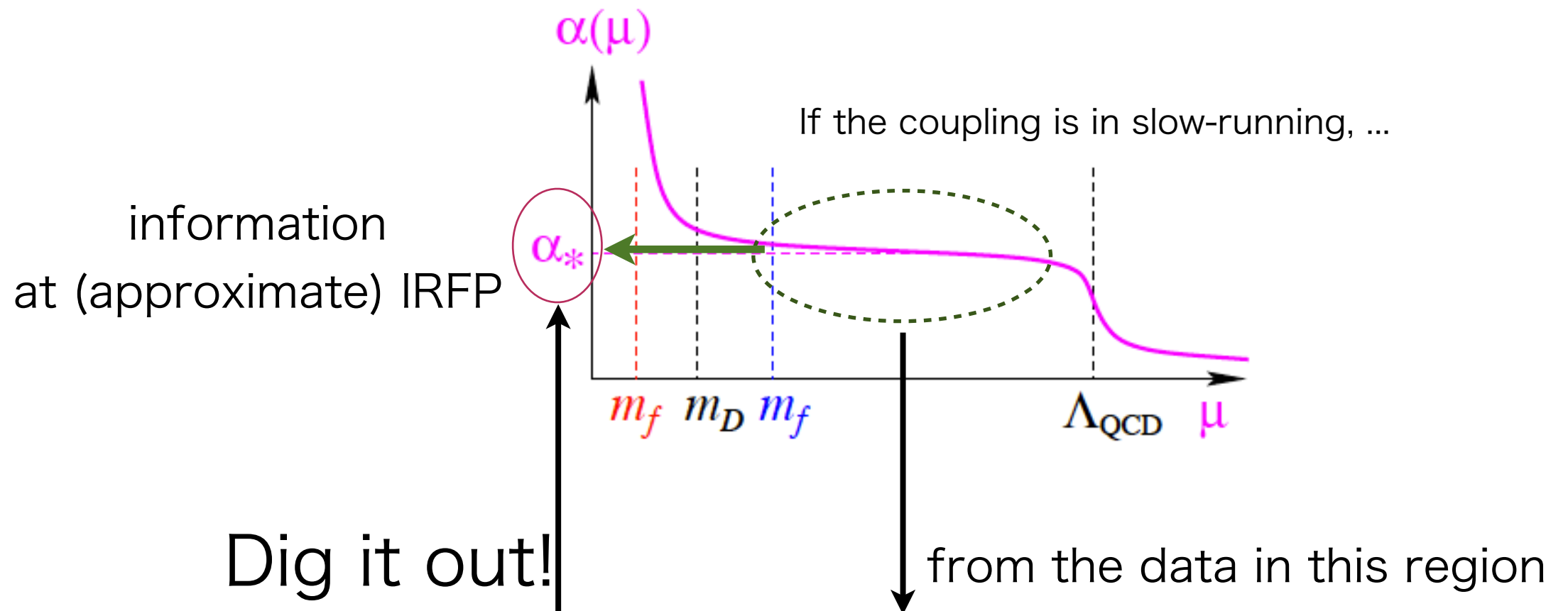
Finite size Hyperscaling analysis

(critical phenomena in conformal transition)

$$LM_H = \mathcal{F}_H(x), LF_H = \mathcal{G}_F(x)$$

(DeGrand, Del Debbio et al.)

$$x \equiv L m^{1/1+\gamma} \text{ (: universal } \gamma \text{)}$$



The hyperscaling with mass corrections is needed, from the lesson of Schwinger-Dyson analysis (our paper, '12).

Finite size Hyperscaling analysis

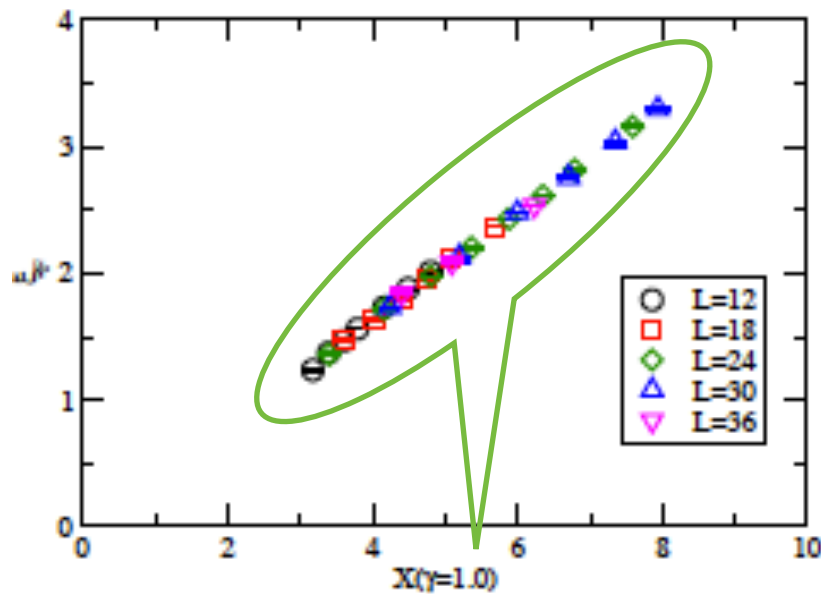
Comformal $\rightarrow$ Finite size Hyperscaling behavior with universal γ
 (Critical exponent obtained from the finite volume setup)

$$LM_H = \mathcal{F}_H(x), LF_H = \mathcal{G}_F(x)$$

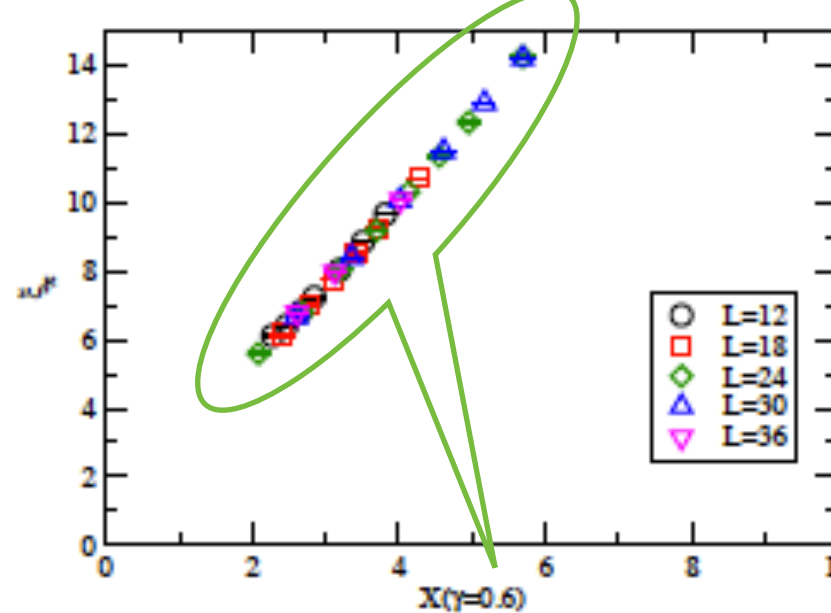
$$x \equiv L m^{1/1+\gamma}$$

data aligned $\rightarrow$ different from Nf=4

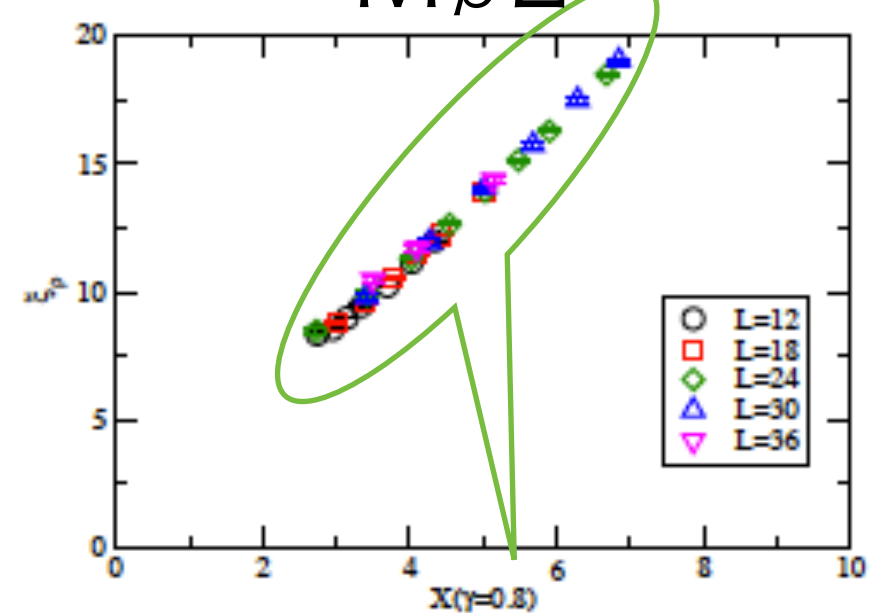
$F_\pi L$



$M_\pi L$



$M_\rho L$



$$\gamma(F_\pi) \sim 1.0, \gamma(M_\pi) \sim 0.6, \gamma(M_\rho) \sim 0.8$$

different from Nf=12

Simultaneous fit of Hyperscaling with mass corrections

$$\xi_H = C_0^H + C_1^H X + C_2^H L m_f^\alpha \quad \text{in the middle region of } m_f \geq 0.05 \text{ and } \xi_\pi (=M_\pi L) \geq 8$$

(Schwinger-Dyson eq. with large mass)

$\xi_H (=M_H L)$ vs m_f (not X): $\alpha=1$ fixed (example)

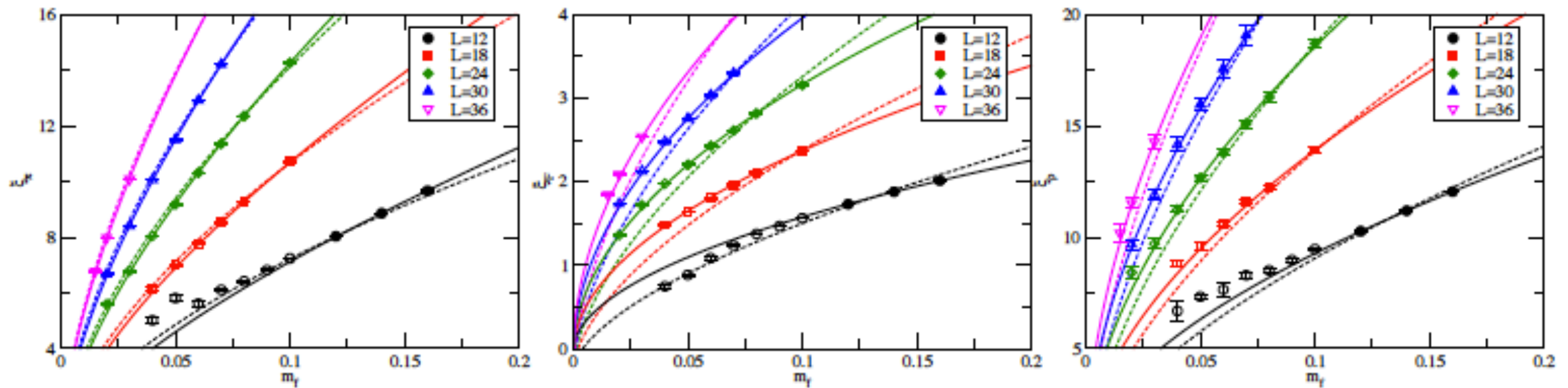


Fig. 5. Simultaneous FSHS fit in ξ_π (left), ξ_F (center) and ξ_ρ (right) with $\alpha = 1$. The filled symbols are included in the fit, but the open symbols are omitted. The fitted region is $m_f \geq 0.05$ and $\xi_\pi \geq 8$. The solid curve is the fit result. For a comparison, the simultaneous fit result without correction terms is also plotted by the dashed curve, whose $\chi^2/dof = 83$.

In this case, the mass correction works well
with $r=0.874(25)$, $\chi^2/dof=0.75$, $dof=32$.

- From various trials of this analysis: $r=0.78-0.93 \sim 1$

Summary-2, Finite-size Hyperscaling analysis

- In the region of $mf \geq 0.05$, hyperscaling is seen. (different from $N_f=4$)
- non-universal γ for each observable in the finite-size hyperscaling (different from $N_f=12$)
- Lesson from the Schwinger-Dyson analysis with the (large) mass.
- Simultaneous fit of hyperscaling with mass correction gives the universal $\gamma = 0.78 - 0.93 \sim 1$. [requirement for the successful walking technicolor.]
- $N_f=8$ has the “remnant” of the conformality in the middle range of mf .

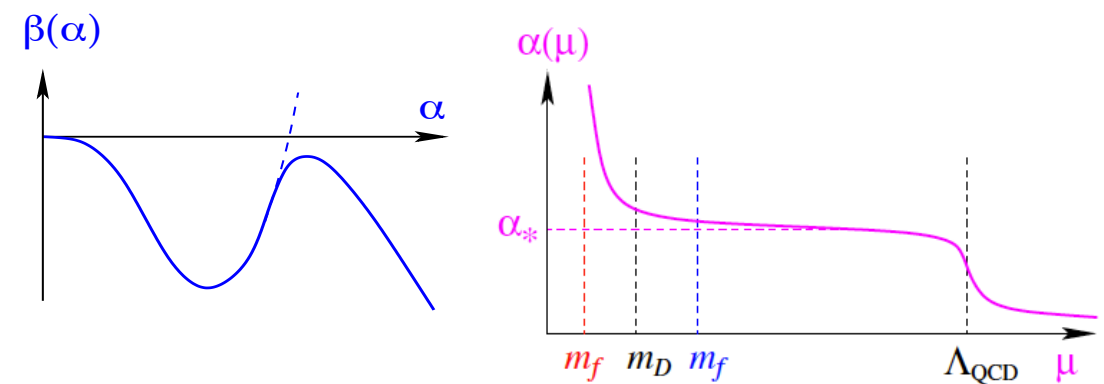
Summary

- ◆ SU(3) gauge theories with 4, 12 and **8 HISQ quarks**.
- ◆ $N_f=8$; consistent with S χ SB in the small mass region of our simulation and the remnant of the conformality in the middle region of m_f with $\gamma \sim 1$. (In contrast to $N_f=4$ and 12 cases.)

$N_f=8 \rightarrow$ Candidate of Walking dynamics

In Progress:

- ◆ Simulation on larger volumes at lighter masses
- ◆ Finite Size Effect (due to the difficulty to take $V=\infty$)
- ◆ Lattice spacing dependence (Enhancement) $\leftarrow$ many β
- ◆ Spectroscopy (M_{glueball} , $M_{\text{"scalar"}}$, M_{baryon} , M_{meson} , $F_{\rho/\sigma}$, S-param. etc.)
- ◆ String tension
- ◆ $M_{\text{"flavor-singlet light scalar"}} \Rightarrow 125\text{GeV?}$ ($\rightarrow$ **Yamazaki's talk**; next)

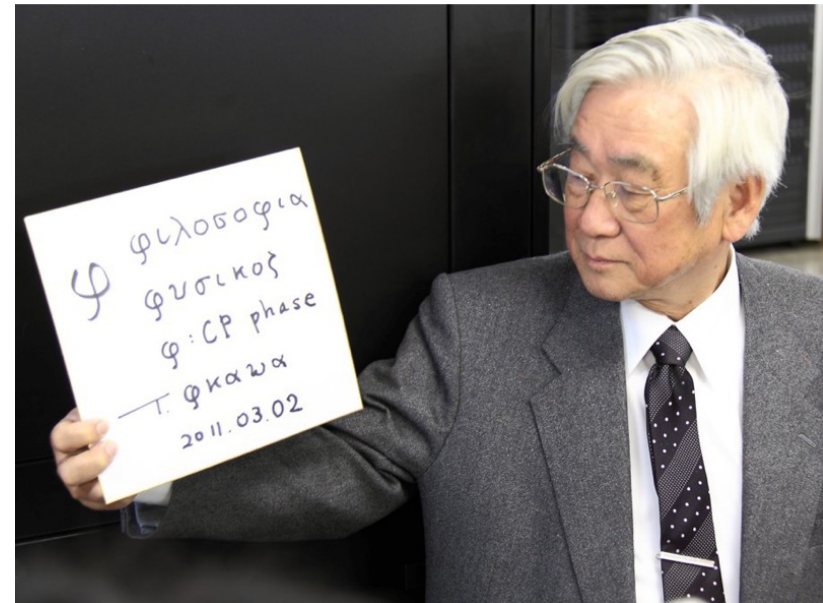


Thank you

Backup

KMI computer, φ

- non GPU nodes
 - 148 nodes
 - 2x Xenon 3.3 GHz
 - 24 TFlops (peak)
- GPU nodes
 - 23 nodes
 - 3x Tesla M2050
 - 39 TFlops (peak)



Size dependence of M_π and F_π at $\beta=3.8$

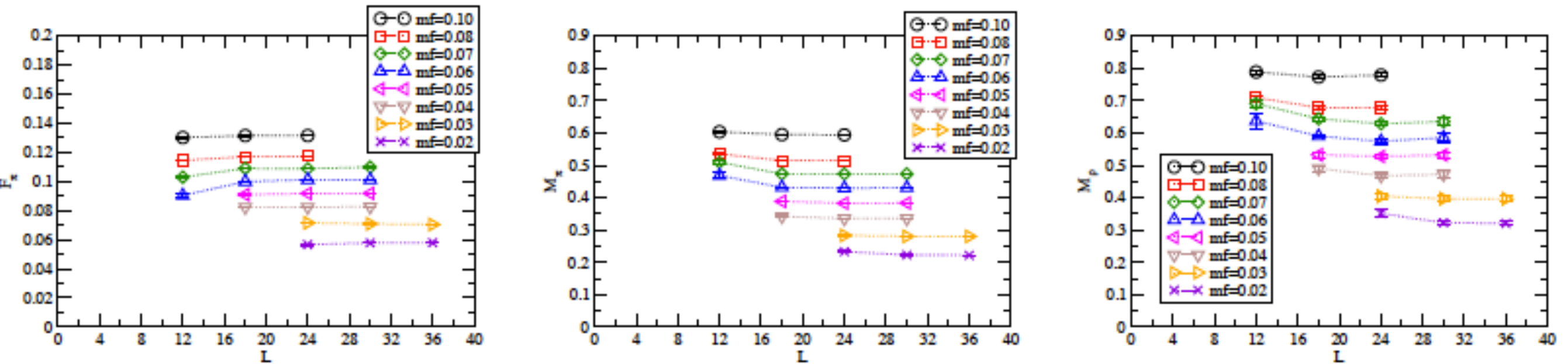


FIG. 7. F_π (left), M_π (center) and M_ρ (right) as functions of L .

On the larger volume, there is not (or very tiny) size dependence.

We use the data on the largest volume at each mf .

HISQ with Nf=8:

effective mass for the lowest $m_f(=0.015)$
on the largest size($L=36$)

$$M_{\pi}^{\text{PS}} = M_{\pi}^{\text{SC}}, M_{\rho}^{\text{PV}} = M_{\rho}^{\text{VT}} \\ \rightarrow \text{good flavor symmetry}$$

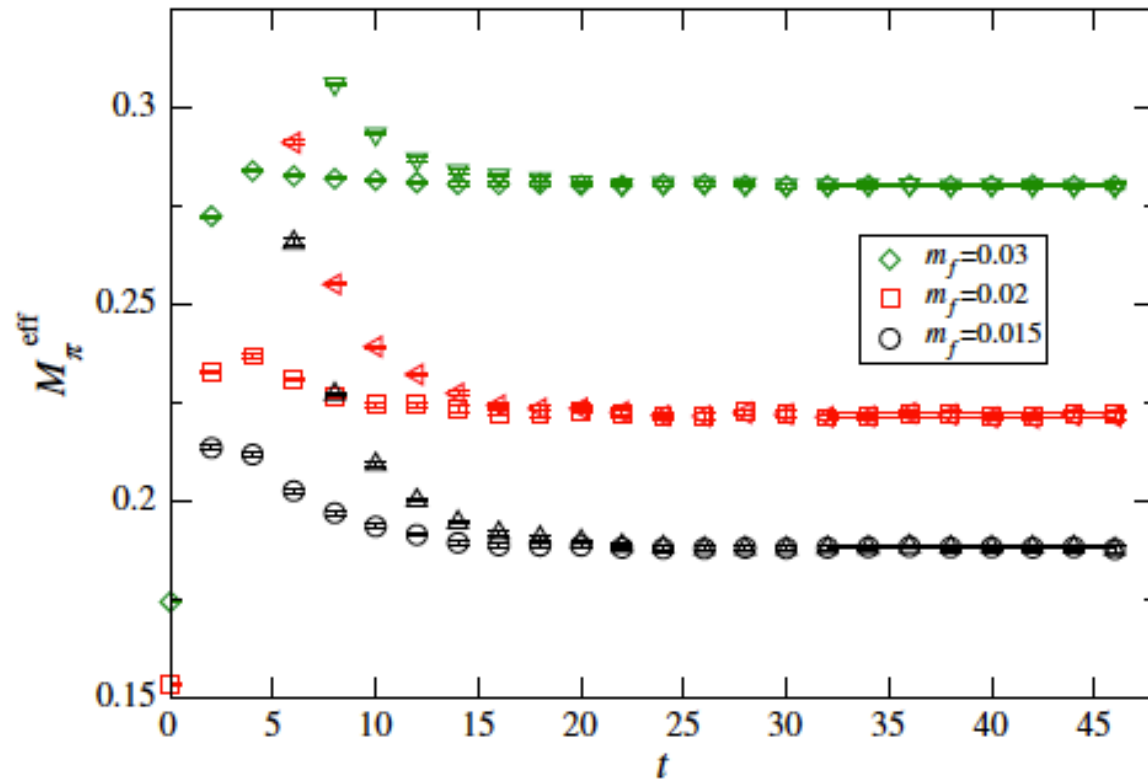


FIG. 2 (color online). Effective masses of PS meson, M_{π}^{eff} , at $L = 36$. Triangles and other symbols denote results from point sink correlators with random wall source and corner wall source, respectively. Fit results with error band obtained from random wall source correlator are also plotted by solid lines.

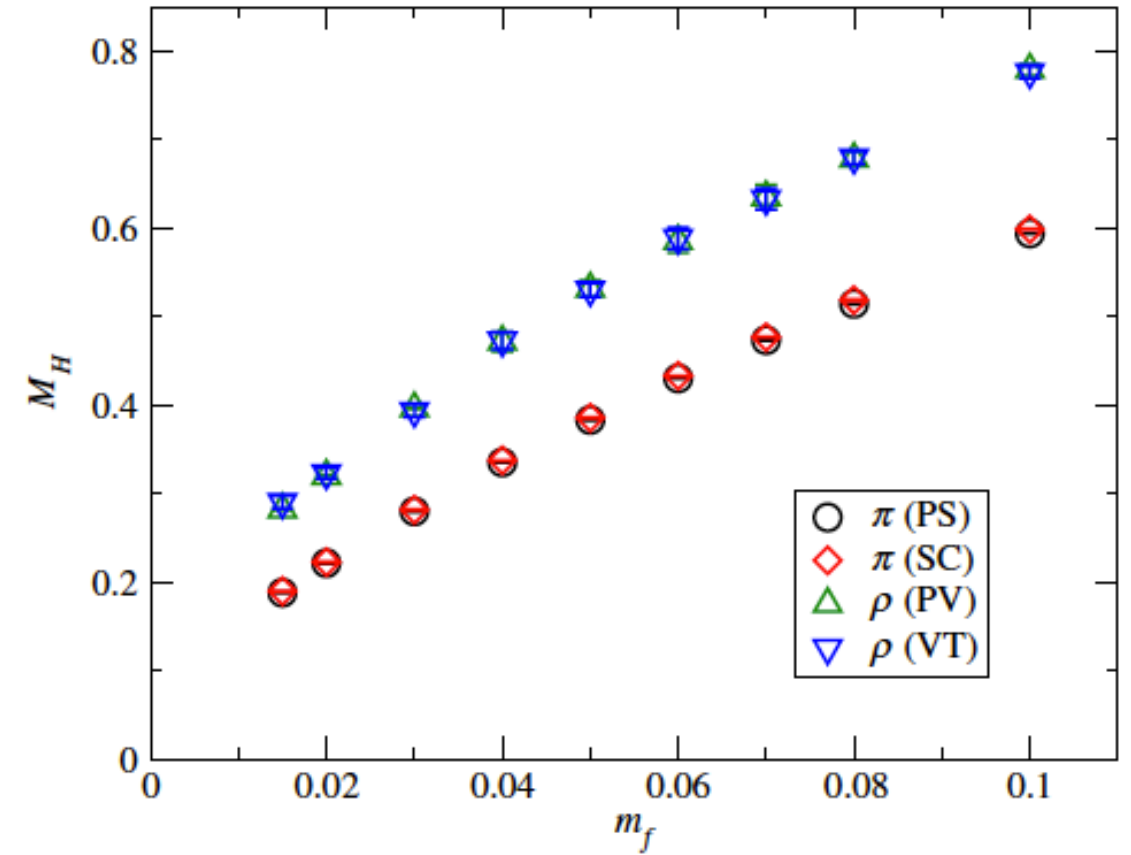
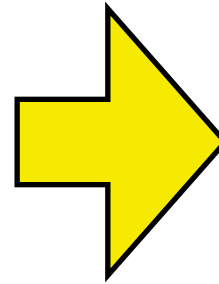
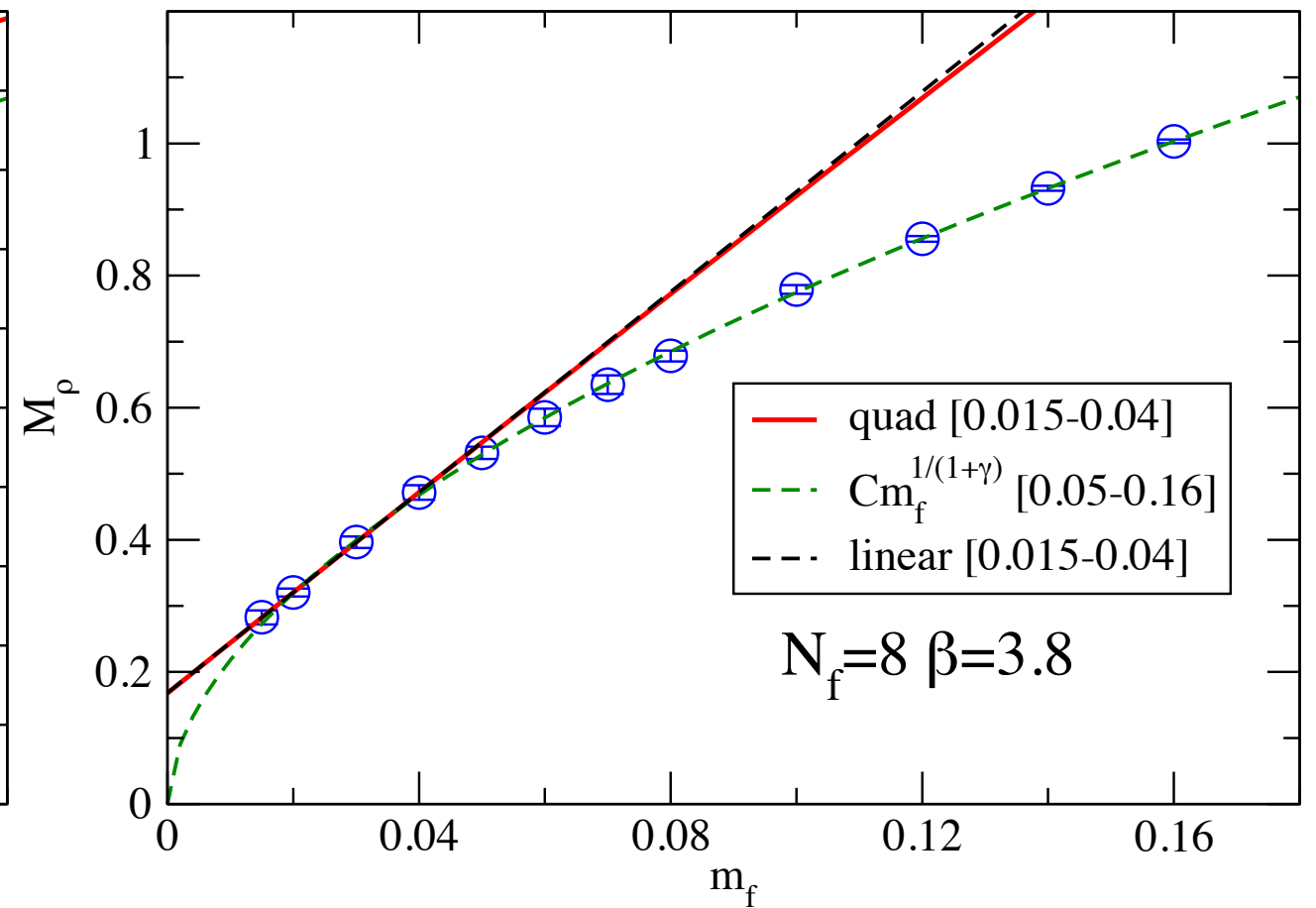
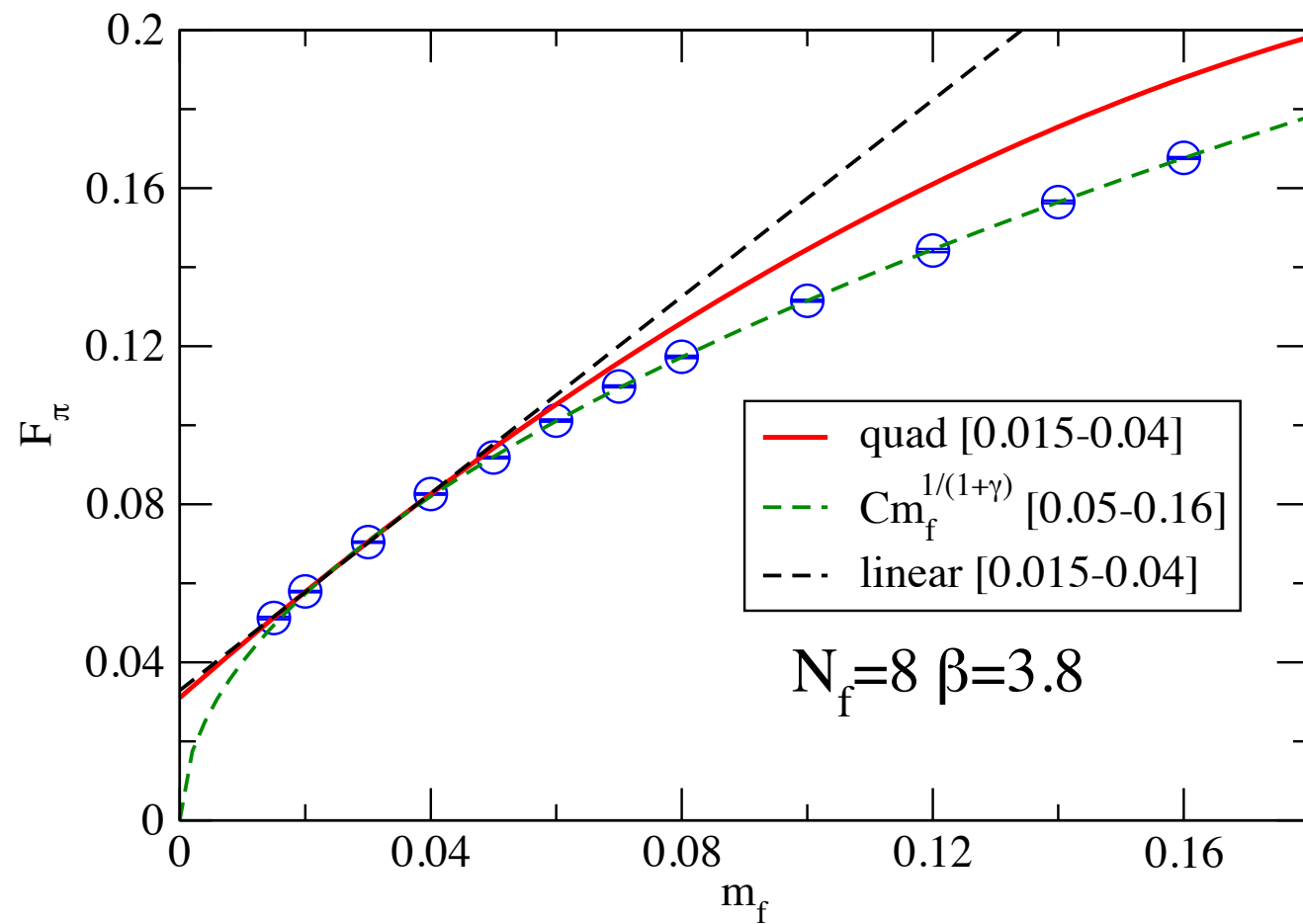


FIG. 4 (color online). Comparisons of M_{π} and M_{SC} , and of $M_{\rho(\text{PV})}$ and $M_{\rho(\text{VT})}$ as a function of m_f with largest volume data at each m_f .

F_π (left panel) and M_ρ (right panel):

quadratic and linear fit in $0.015 \leq m_f \leq 0.04$

power function fit (critical phenomena) in $0.05 \leq m_f$.



non-universal:

$$\gamma(F_\pi) \sim 1.0, \quad \gamma(M_\rho) \sim 0.8$$

different from $N_f=4$ and 12

Simultaneous fit of hyperscaling with mass corrections

$$\xi_H = C_0^H + C_1^H X + C_2^H L m_f^\alpha. \quad (\text{same method with } N_f=12)$$

Hyperscaling? in the middle region of m_f ($m_f \geq 0.05$ and $\xi_\pi (=M\pi L) \geq 8$)

The mass corrections might be needed, as done in $N_f=12$,
from the lesson in SD analysis.

TABLE XI. Simultaneous FSHS fit with a correction term, $\xi = C_0^H + C_1^H X + C_2^H L m_f^\alpha$ using several choices of α . The fitted region is $m_f \geq 0.05$ and $\xi_\pi \geq 8$.

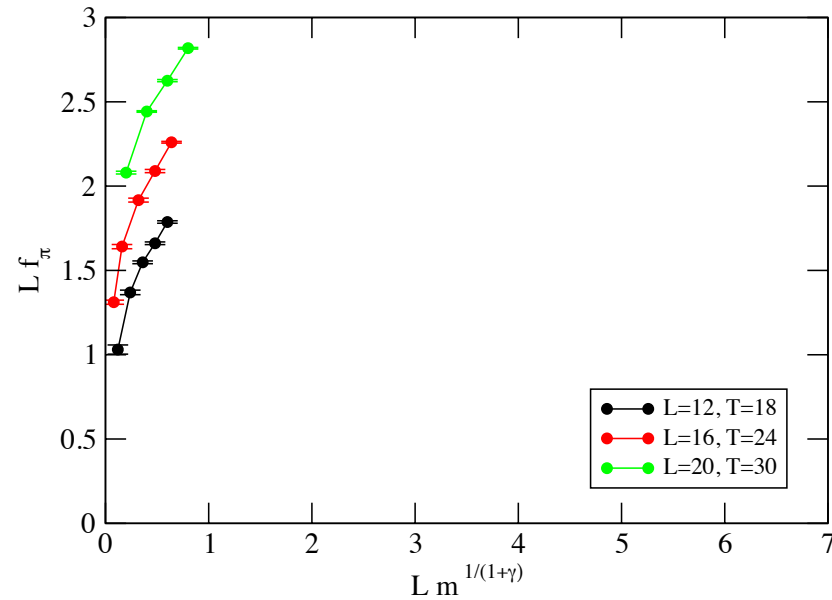
$\alpha = 0.889(55)$	C_0^H	C_1^H	C_2^H
ξ_π	-0.005(25)	1.338(96)	1.494(37)
ξ_F	-0.0275(98)	0.4435(36)	—
ξ_ρ	0.53(16)	2.476(39)	—
$\gamma = 0.9130(76), \chi^2/\text{dof} = 1.73, \text{dof} = 33$			
$\alpha = 1 \text{ fixed}$	C_0^H	C_1^H	C_2^H
ξ_π	-0.014(24)	1.61(10)	1.31(15)
ξ_F	-0.012(10)	0.484(30)	-0.068(44)
ξ_ρ	0.01(19)	2.60(17)	0.25(24)
$\gamma = 0.874(25), \chi^2/\text{dof} = 0.75, \text{dof} = 32$			
$\alpha = \frac{3-2\gamma}{1+\gamma} \text{ fixed}$	C_0^H	C_1^H	C_2^H
ξ_π	0.020(24)	1.52(39)	1.17(35)
ξ_F	-0.011(10)	0.572(34)	-0.158(52)
ξ_ρ	0.03(19)	2.91(30)	-0.15(36)
$\gamma = 0.775(56), \chi^2/\text{dof} = 0.93, \text{dof} = 32$			

$\Rightarrow$ good χ^2/dof , but unclear which α is better.

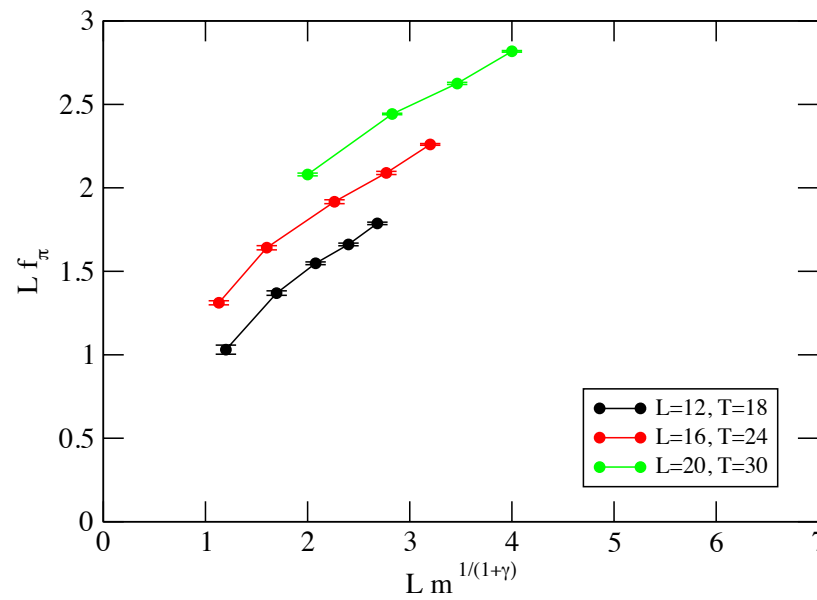
Comparison with $N_f = 4$

: trial of hyperscaling in $F_\pi L$ and $M_\rho L$ (in $S\chi SB$)

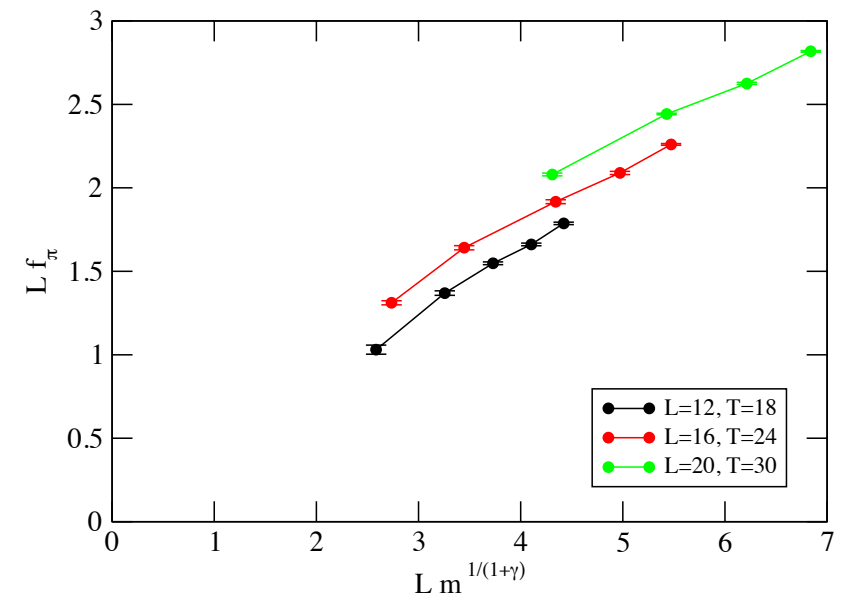
$\beta=3.7, \gamma = 0.0$



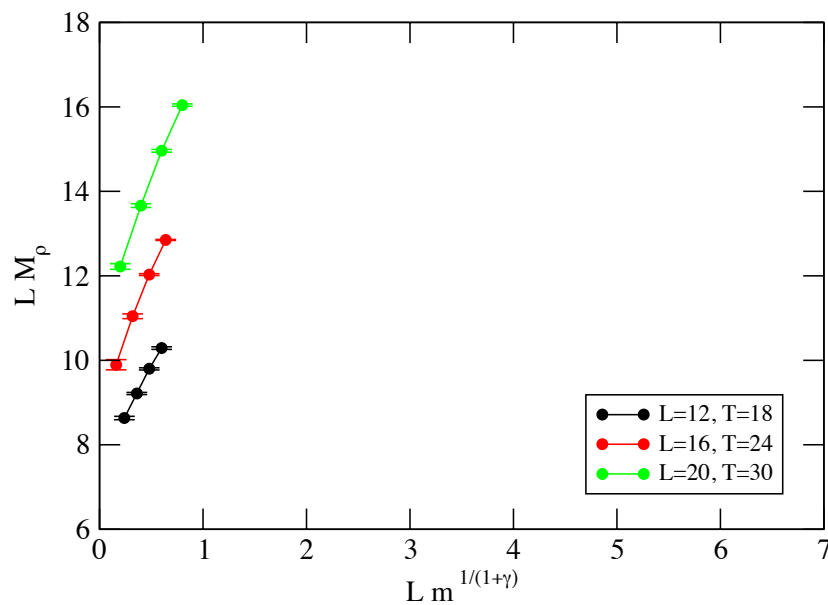
$\beta=3.7, \gamma = 1.0$



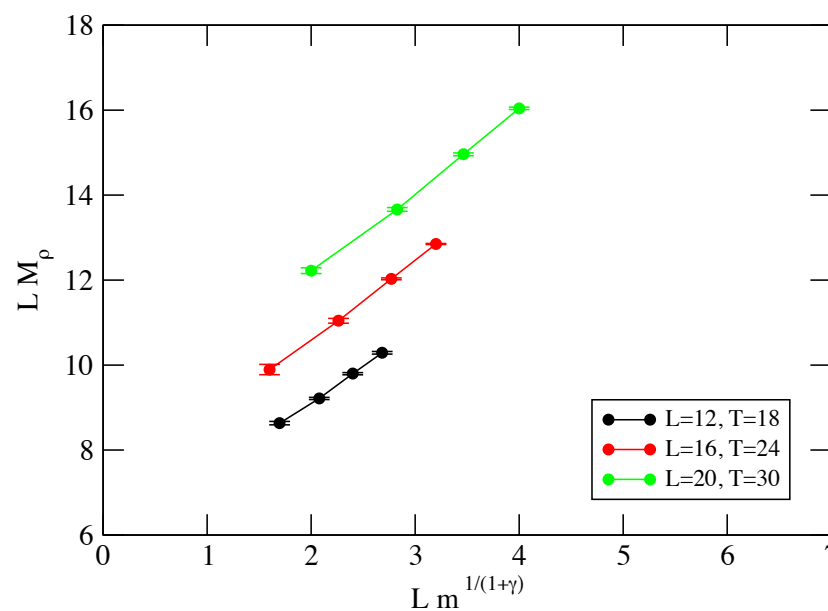
$\beta=3.7, \gamma = 2.0$



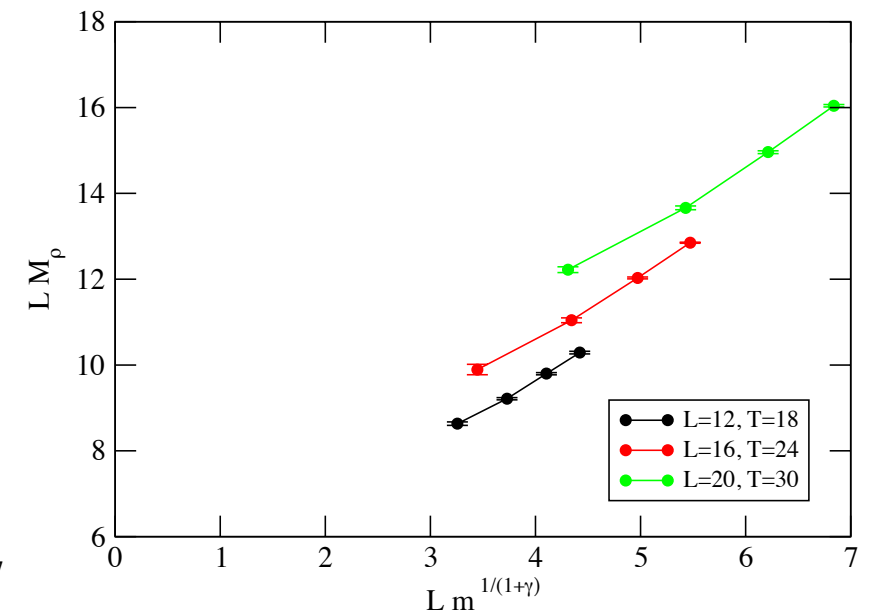
$\beta=3.7, \gamma = 0.0$



$\beta=3.7, \gamma = 1.0$



$\beta=3.7, \gamma = 2.0$



no scaling in $0 < \gamma < 2$