Strongly coupled plasma - hydrodynamics, thermalization and nonequilibrium behavior

Romuald A. Janik

Jagiellonian University Kraków

work with M. Heller, P. Witaszczyk see RJ 1311.3966 [hep-ph] for a recent review

Outline

Why use AdS/CFT?

 $\mathcal{N}=4$ plasma versus QCD plasma

The AdS/CFT description of a plasma system

Boost-invariant flow

The transition to hydrodynamics and its characteristics

Nonequillibrium degrees of freedom

Conclusions

 $\mathcal{N} = 4$ Super Yang-Mills theory

strong coupling nonperturbative physics very difficult weak coupling 'easy' \equiv Superstrings on $AdS_5 \times S^5$

(semi-)classical strings or supergravity 'easy' highly quantum regime very difficult

- New ways of looking at nonperturbative gauge theory physics...
- Intricate links with General Relativity...
- Has been extended to many other cases

 $\mathcal{N} = 4$ Super Yang-Mills theory

strong coupling nonperturbative physics very difficult weak coupling 'easy' \equiv Superstrings on $AdS_5 \times S^5$

(semi-)classical strings or supergravity 'easy' highly quantum regime very difficult

New ways of looking at nonperturbative gauge theory physics...

Intricate links with General Relativity...

Has been extended to many other cases

 $\mathcal{N} = 4$ Super Yang-Mills theory

strong coupling nonperturbative physics very difficult weak coupling 'easy' \equiv Superstrings on $AdS_5 \times S^5$

(semi-)classical strings or supergravity 'easy' highly quantum regime very difficult

- New ways of looking at nonperturbative gauge theory physics...
- Intricate links with General Relativity...
- Has been extended to many other cases

 $\mathcal{N} = 4$ Super Yang-Mills theory

strong coupling nonperturbative physics very difficult weak coupling 'easy' \equiv Superstrings on $AdS_5 \times S^5$

(semi-)classical strings or supergravity 'easy' highly quantum regime very difficult

- New ways of looking at nonperturbative gauge theory physics...
- Intricate links with General Relativity...
- Has been extended to many other cases

Problem:

- QCD plasma produced at RHIC/LHC is most probably a strongly coupled system
- Nonperturbative methods applicable to real time dynamics are very scarce
- Conventional lattice QCD is inherently Euclidean

Problem:

- QCD plasma produced at RHIC/LHC is most probably a strongly coupled system
- Nonperturbative methods applicable to real time dynamics are very scarce
- Conventional lattice QCD is inherently Euclidean

Problem:

- QCD plasma produced at RHIC/LHC is most probably a strongly coupled system
- Nonperturbative methods applicable to real time dynamics are very scarce
- Conventional lattice QCD is inherently Euclidean

Problem:

- QCD plasma produced at RHIC/LHC is most probably a strongly coupled system
- Nonperturbative methods applicable to real time dynamics are very scarce
- Conventional lattice QCD is inherently Euclidean

Problem:

- QCD plasma produced at RHIC/LHC is most probably a strongly coupled system
- Nonperturbative methods applicable to real time dynamics are very scarce
- Conventional lattice QCD is inherently Euclidean

Problem:

- QCD plasma produced at RHIC/LHC is most probably a strongly coupled system
- Nonperturbative methods applicable to real time dynamics are very scarce
- Conventional lattice QCD is inherently Euclidean

Problem:

- QCD plasma produced at RHIC/LHC is most probably a strongly coupled system
- Nonperturbative methods applicable to real time dynamics are very scarce
- Conventional lattice QCD is inherently Euclidean

Problem:

- QCD plasma produced at RHIC/LHC is most probably a strongly coupled system
- Nonperturbative methods applicable to real time dynamics are very scarce
- Conventional lattice QCD is inherently Euclidean

$$\rightarrow$$
 \leftarrow Collision





Collision

Fireball

isotropization thermalization



Collision

Fireball

isotropization thermalization

hydrodynamic expansion



Collision

Fireball

isotropization thermalization

hydrodynamic expansion

freezout hadronization



Similarities:

- Deconfined phase
- Strongly coupled
- ► No supersymmetry!

Differences:

- ► No running coupling → Even at very high energy densities the coupling remains strong
- (Exactly) conformal equation of state \longrightarrow Perhaps not so bad around $T \sim 1.5 2.5 T_c$
- ► No confinement/deconfinement phase transition → The N = 4 plasma expands and cools indefinitely

Similarities:

- Deconfined phase
- Strongly coupled
- ► No supersymmetry!

Differences:

- ► No running coupling → Even at very high energy densities the coupling remains strong
- (Exactly) conformal equation of state \longrightarrow Perhaps not so bad around $T \sim 1.5 2.5 T_c$
- \blacktriangleright No confinement/deconfinement phase transition $~\longrightarrow~$ The ${\cal N}=4$ plasma expands and cools indefinitely

Similarities:

- Deconfined phase
- Strongly coupled
- ► No supersymmetry!

Differences:

- ► No running coupling → Even at very high energy densities the coupling remains strong
- (Exactly) conformal equation of state \longrightarrow Perhaps not so bad around $T \sim 1.5 2.5 T_c$
- ► No confinement/deconfinement phase transition → The N = 4 plasma expands and cools indefinitely

Similarities:

- Deconfined phase
- Strongly coupled
- No supersymmetry!

Differences:

- ▶ No running coupling → Even at very high energy densities the coupling remains strong
- (Exactly) conformal equation of state \longrightarrow Perhaps not so bad around $T \sim 1.5 2.5 T_c$
- ► No confinement/deconfinement phase transition → The N = 4 plasma expands and cools indefinitely

Similarities:

- Deconfined phase
- Strongly coupled
- No supersymmetry!

Differences:

- ► No running coupling → Even at very high energy densities the coupling remains strong
- (Exactly) conformal equation of state \longrightarrow Perhaps not so bad around $T \sim 1.5 2.5 T_c$
- ► No confinement/deconfinement phase transition → The N = 4 plasma expands and cools indefinitely

Similarities:

- Deconfined phase
- Strongly coupled
- No supersymmetry!

Differences:

- ► No running coupling → Even at very high energy densities the coupling remains strong
- (Exactly) conformal equation of state \longrightarrow Perhaps not so bad around $T \sim 1.5 2.5 T_c$
- \blacktriangleright No confinement/deconfinement phase transition $~\longrightarrow~$ The ${\cal N}=4$ plasma expands and cools indefinitely

Similarities:

- Deconfined phase
- Strongly coupled
- No supersymmetry!

Differences:

- \blacktriangleright No running coupling \longrightarrow Even at very high energy densities the coupling remains strong
- (Exactly) conformal equation of state \longrightarrow Perhaps not so bad around $T \sim 1.5 2.5 T_c$
- ► No confinement/deconfinement phase transition → The N = 4 plasma expands and cools indefinitely

Similarities:

- Deconfined phase
- Strongly coupled
- No supersymmetry!

Differences:

- \blacktriangleright No running coupling \longrightarrow Even at very high energy densities the coupling remains strong
- (Exactly) conformal equation of state \rightarrow Perhaps not so bad around $T \sim 1.5 2.5T_c$
- ▶ No confinement/deconfinement phase transition → The N = 4 plasma expands and cools indefinitely

Similarities:

- Deconfined phase
- Strongly coupled
- No supersymmetry!

Differences:

- ► No running coupling → Even at very high energy densities the coupling remains strong
- ▶ (Exactly) conformal equation of state \longrightarrow Perhaps not so bad around $T \sim 1.5 2.5 T_c$
- \blacktriangleright No confinement/deconfinement phase transition $~\longrightarrow~$ The ${\cal N}=4$ plasma expands and cools indefinitely

Similarities:

- Deconfined phase
- Strongly coupled
- No supersymmetry!

Differences:

- ► No running coupling → Even at very high energy densities the coupling remains strong
- (Exactly) conformal equation of state \longrightarrow Perhaps not so bad around $T \sim 1.5 2.5 T_c$
- ► No confinement/deconfinement phase transition → The N = 4 plasma expands and cools indefinitely

Similarities:

- Deconfined phase
- Strongly coupled
- No supersymmetry!

Differences:

- ► No running coupling → Even at very high energy densities the coupling remains strong
- ▶ (Exactly) conformal equation of state \rightarrow Perhaps not so bad around $T \sim 1.5 2.5 T_c$
- \blacktriangleright No confinement/deconfinement phase transition $~\longrightarrow~$ The ${\cal N}=4$ plasma expands and cools indefinitely

One can pass to more complicated AdS/CFT setups and lift the above differences

Similarities:

- Deconfined phase
- Strongly coupled
- No supersymmetry!

Differences:

- ► No running coupling → Even at very high energy densities the coupling remains strong
- ▶ (Exactly) conformal equation of state \rightarrow Perhaps not so bad around $T \sim 1.5 2.5 T_c$
- \blacktriangleright No confinement/deconfinement phase transition $~\longrightarrow~$ The ${\cal N}=4$ plasma expands and cools indefinitely

Why study $\mathcal{N} = 4$ plasma?

- The applicability of using N = 4 plasma to model real world phenomenae dependes on the questions asked..
- Use it as a theoretical laboratory where we may compute from 'first principles' nonequilibrium nonperturbative dynamics
- Gain qualitative insight into the physics which is very difficult to access using other methods
- Discover some universal properties? (like η/s)
- For N = 4 plasma the AdS/CFT correspondence is technically simplest
- ► Use the results on strong coupling properties of N = 4 plasma as a point of reference for analyzing/describing QCD plasma
- Eventually one may consider more realistic theories with AdS/CFT duals...

Why study $\mathcal{N} = 4$ plasma?

- The applicability of using N = 4 plasma to model real world phenomenae dependes on the questions asked..
- Use it as a theoretical laboratory where we may compute from 'first principles' nonequilibrium nonperturbative dynamics
- Gain qualitative insight into the physics which is very difficult to access using other methods
- Discover some universal properties? (like η/s)
- For N = 4 plasma the AdS/CFT correspondence is technically simplest
- ► Use the results on strong coupling properties of N = 4 plasma as a point of reference for analyzing/describing QCD plasma
- Eventually one may consider more realistic theories with AdS/CFT duals...

Why study $\mathcal{N} = 4$ plasma?

- The applicability of using N = 4 plasma to model real world phenomenae dependes on the questions asked..
- Use it as a theoretical laboratory where we may compute from 'first principles' nonequilibrium nonperturbative dynamics
- Gain qualitative insight into the physics which is very difficult to access using other methods
- Discover some universal properties? (like η/s)
- For N = 4 plasma the AdS/CFT correspondence is technically simplest
- ► Use the results on strong coupling properties of N = 4 plasma as a point of reference for analyzing/describing QCD plasma
- Eventually one may consider more realistic theories with AdS/CFT duals...
- ► The applicability of using N = 4 plasma to model real world phenomenae dependes on the questions asked..
- Use it as a theoretical laboratory where we may compute from 'first principles' nonequilibrium nonperturbative dynamics
- Gain qualitative insight into the physics which is very difficult to access using other methods
- Discover some universal properties? (like η/s)
- For N = 4 plasma the AdS/CFT correspondence is technically simplest
- ► Use the results on strong coupling properties of N = 4 plasma as a point of reference for analyzing/describing QCD plasma
- Eventually one may consider more realistic theories with AdS/CFT duals...

- ► The applicability of using N = 4 plasma to model real world phenomenae dependes on the questions asked..
- Use it as a theoretical laboratory where we may compute from 'first principles' nonequilibrium nonperturbative dynamics
- Gain qualitative insight into the physics which is very difficult to access using other methods
- Discover some universal properties? (like η/s)

For N = 4 plasma the AdS/CFT correspondence is technically simplest

- ► Use the results on strong coupling properties of N = 4 plasma as a point of reference for analyzing/describing QCD plasma
- Eventually one may consider more realistic theories with AdS/CFT duals...

- ► The applicability of using N = 4 plasma to model real world phenomenae dependes on the questions asked..
- Use it as a theoretical laboratory where we may compute from 'first principles' nonequilibrium nonperturbative dynamics
- Gain qualitative insight into the physics which is very difficult to access using other methods
- Discover some universal properties? (like η/s)
- For N = 4 plasma the AdS/CFT correspondence is technically simplest
- Use the results on strong coupling properties of N = 4 plasma as a point of reference for analyzing/describing QCD plasma
- Eventually one may consider more realistic theories with AdS/CFT duals...

- ► The applicability of using N = 4 plasma to model real world phenomenae dependes on the questions asked..
- Use it as a theoretical laboratory where we may compute from 'first principles' nonequilibrium nonperturbative dynamics
- Gain qualitative insight into the physics which is very difficult to access using other methods
- Discover some universal properties? (like η/s)
- For N = 4 plasma the AdS/CFT correspondence is technically simplest
- Use the results on strong coupling properties of N = 4 plasma as a point of reference for analyzing/describing QCD plasma
- Eventually one may consider more realistic theories with AdS/CFT duals...

Aim: Describe the time dependent evolving strongly coupled plasma system

Describe it in terms of lightest degrees of freedom on the AdS side which are relevant at strong coupling

$$ds^{2} = \frac{g_{\mu\nu}(x^{\rho}, z)dx^{\mu}dx^{\nu} + dz^{2}}{z^{2}} \equiv g_{\alpha\beta}^{5D}dx^{\alpha}dx^{\beta}$$

$$\downarrow$$

Compute the time-evolution by solving (numerically) 5D Einstein's equations

$$R_{lphaeta} - rac{1}{2}g^{5D}_{lphaeta}R - 6\,g^{5D}_{lphaeta} = 0$$

Aim: Describe the time dependent evolving strongly coupled plasma system

Describe it in terms of lightest degrees of freedom on the AdS side which are relevant at strong coupling

 \downarrow

$$ds^{2} = \frac{g_{\mu\nu}(x^{\rho}, z)dx^{\mu}dx^{\nu} + dz^{2}}{z^{2}} \equiv g_{\alpha\beta}^{5D}dx^{\alpha}dx^{\beta}$$

Compute the time-evolution by solving (numerically) 5D Einstein's equations

$$R_{lphaeta} - rac{1}{2}g^{5D}_{lphaeta}R - 6\,g^{5D}_{lphaeta} = 0$$

Aim: Describe the time dependent evolving strongly coupled plasma system

Describe it in terms of lightest degrees of freedom on the AdS side which are relevant at strong coupling

 \downarrow

$$ds^{2} = \frac{g_{\mu\nu}(x^{\rho}, z)dx^{\mu}dx^{\nu} + dz^{2}}{z^{2}} \equiv g_{\alpha\beta}^{5D}dx^{\alpha}dx^{\beta}$$

Compute the time-evolution by solving (numerically) 5D Einstein's equations

$$R_{lphaeta} - rac{1}{2}g^{5D}_{lphaeta}R - 6\,g^{5D}_{lphaeta} = 0$$

Aim: Describe the time dependent evolving strongly coupled plasma system

Describe it in terms of lightest degrees of freedom on the AdS side which are relevant at strong coupling

 \downarrow

$$ds^{2} = \frac{g_{\mu\nu}(x^{\rho}, z)dx^{\mu}dx^{\nu} + dz^{2}}{z^{2}} \equiv g_{\alpha\beta}^{5D}dx^{\alpha}dx^{\beta}$$

$$\downarrow$$

Compute the time-evolution by solving (numerically) 5D Einstein's equations

$$R_{lphaeta} - rac{1}{2}g^{5D}_{lphaeta}R - 6\,g^{5D}_{lphaeta} = 0$$

Aim: Describe the time dependent evolving strongly coupled plasma system

\downarrow

Describe it in terms of lightest degrees of freedom on the AdS side which are relevant at strong coupling

$$ds^{2} = \frac{g_{\mu\nu}(x^{\rho}, z)dx^{\mu}dx^{\nu} + dz^{2}}{z^{2}} \equiv g_{\alpha\beta}^{5D}dx^{\alpha}dx^{\beta}$$

$$\downarrow$$

Compute the time-evolution by solving (numerically) 5D Einstein's equations

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}^{5D}R - 6\,g_{\alpha\beta}^{5D} = 0$$

\downarrow

- Asymptotics of g_{µν}(x^ρ, z) at z ~ 0 gives the energy-momentum tensor T_{µν}(x^ρ) of the plasma system
- We can test whether T_{µν}(x^ρ) is of a hydrodynamic form...
- We can check for local thermal equilibrium

- The area of the apparent horizon defines for us the entropy density
- ▶ We observe some *initial entropy*



- Asymptotics of g_{µν}(x^ρ, z) at z ~ 0 gives the energy-momentum tensor T_{µν}(x^ρ) of the plasma system
- We can test whether T_{µν}(x^ρ) is of a hydrodynamic form...
- We can check for local thermal equilibrium

- The area of the apparent horizon defines for us the entropy density
- We observe some initial entropy



- Asymptotics of g_{µν}(x^ρ, z) at z ~ 0 gives the energy-momentum tensor T_{µν}(x^ρ) of the plasma system
- We can test whether T_{µν}(x^ρ) is of a hydrodynamic form...
- We can check for local thermal equilibrium

- The area of the apparent horizon defines for us the entropy density
- We observe some initial entropy



- Asymptotics of g_{µν}(x^ρ, z) at z ~ 0 gives the energy-momentum tensor T_{µν}(x^ρ) of the plasma system
- We can test whether T_{µν}(x^ρ) is of a hydrodynamic form...
- We can check for local thermal equilibrium

- The area of the apparent horizon defines for us the entropy density
- We observe some initial entropy



- Asymptotics of g_{µν}(x^ρ, z) at z ~ 0 gives the energy-momentum tensor T_{µν}(x^ρ) of the plasma system
- We can test whether T_{µν}(x^ρ) is of a hydrodynamic form...
- We can check for local thermal equilibrium

- The area of the apparent horizon defines for us the entropy density
- We observe some initial entropy



- Asymptotics of g_{µν}(x^ρ, z) at z ~ 0 gives the energy-momentum tensor T_{µν}(x^ρ) of the plasma system
- We can test whether T_{µν}(x^ρ) is of a hydrodynamic form...
- We can check for local thermal equilibrium

- The area of the apparent horizon defines for us the entropy density
- We observe some initial entropy



- Asymptotics of g_{μν}(x^ρ, z) at z ~ 0 gives the energy-momentum tensor T_{μν}(x^ρ) of the plasma system
- We can test whether T_{μν}(x^ρ) is of a hydrodynamic form...
- We can check for local thermal equilibrium

- The area of the apparent horizon defines for us the entropy density
- We observe some *initial entropy*



- Asymptotics of g_{µν}(x^ρ, z) at z ~ 0 gives the energy-momentum tensor T_{µν}(x^ρ) of the plasma system
- We can test whether T_{µν}(x^ρ) is of a hydrodynamic form...
- We can check for local thermal equilibrium

- The area of the apparent horizon defines for us the entropy density
- We observe some initial entropy



- Asymptotics of g_{µν}(x^ρ, z) at z ∼ 0 gives the energy-momentum tensor T_{µν}(x^ρ) of the plasma system
- We can test whether T_{µν}(x^ρ) is of a hydrodynamic form...
- We can check for local thermal equilibrium

- The area of the apparent horizon defines for us the entropy density
- We observe some initial entropy

Key question:

Understand the features of the farfrom equilibrium stage of the dynamics of the strongly coupled plasma system Key question:

Understand the features of the farfrom equilibrium stage of the dynamics of the strongly coupled plasma system

Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



- ▶ In a conformal theory, $T^{\mu}_{\mu} = 0$ and $\partial_{\mu}T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- ▶ The longitudinal and transverse pressures are then given by

$$p_L = -\varepsilon - \tau \frac{d}{d\tau} \varepsilon$$
 and $p_T = \varepsilon + \frac{1}{2} \tau \frac{d}{d\tau} \varepsilon$.

► The assumptions of symmetry fix uniquely the flow velocity

Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



▶ In a conformal theory, $T^{\mu}_{\mu} = 0$ and $\partial_{\mu} T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.

▶ The longitudinal and transverse pressures are then given by

$$p_L = -\varepsilon - \tau \frac{d}{d\tau} \varepsilon$$
 and $p_T = \varepsilon + \frac{1}{2} \tau \frac{d}{d\tau} \varepsilon$.

▶ The assumptions of symmetry fix uniquely the flow velocity

Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



- ▶ In a conformal theory, $T^{\mu}_{\mu} = 0$ and $\partial_{\mu} T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- > The longitudinal and transverse pressures are then given by

$$p_L = -\varepsilon - \tau \frac{d}{d\tau} \varepsilon$$
 and $p_T = \varepsilon + \frac{1}{2} \tau \frac{d}{d\tau} \varepsilon$.

► The assumptions of symmetry fix uniquely the flow velocity

Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



- ▶ In a conformal theory, $T^{\mu}_{\mu} = 0$ and $\partial_{\mu} T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- > The longitudinal and transverse pressures are then given by

$$p_L = -\varepsilon - \tau \frac{d}{d\tau} \varepsilon$$
 and $p_T = \varepsilon + \frac{1}{2} \tau \frac{d}{d\tau} \varepsilon$.

> The assumptions of symmetry fix uniquely the flow velocity

• Structure of the analytical result for large τ :

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1+2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \dots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

Leading term — perfect fluid behaviour second term — 1st order viscous hydrodynamics third term — 2nd order viscous hydrodynamics fourth term — 3rd order viscous hydrodynamics..

► In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

Currently we know 240 terms in this expansion

Heller, RJ, Witaszczyk

The hydrodynamic series is only asymptotic and has zero radius of convergence...

• Structure of the analytical result for large τ :

 $\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1+2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \dots$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

Leading term — perfect fluid behaviour

second term — 1^{st} order viscous hydrodynamics third term — 2^{nd} order viscous hydrodynamics fourth term — 3^{rd} order viscous hydrodynamics..

► In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

Currently we know 240 terms in this expansion

Heller, RJ, Witaszczyk

The hydrodynamic series is only asymptotic and has zero radius of convergence...

• Structure of the analytical result for large τ :

 $\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1+2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \dots$ RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller • Leading term — perfect fluid behaviour second term — 1st order viscous hydrodynamics third term — 2nd order viscous hydrodynamics fourth term — 3rd order viscous hydrodynamics... • In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

Currently we know 240 terms in this expansion

Heller, RJ, Witaszczyk

The hydrodynamic series is only asymptotic and has zero radius of convergence...

• Structure of the analytical result for large τ :

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1+2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \dots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

 Leading term — perfect fluid behaviour second term — 1st order viscous hydrodynamics third term — 2nd order viscous hydrodynamics fourth term — 3rd order viscous hydrodynamics...

► In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

Currently we know 240 terms in this expansion

Heller, RJ, Witaszczyk

The hydrodynamic series is only asymptotic and has zero radius of convergence...

• Structure of the analytical result for large τ :

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1+2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \dots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

 Leading term — perfect fluid behaviour second term — 1st order viscous hydrodynamics third term — 2nd order viscous hydrodynamics fourth term — 3rd order viscous hydrodynamics...

► In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

Currently we know 240 terms in this expansion

Heller, RJ, Witaszczyk

The hydrodynamic series is only asymptotic and has zero radius of convergence...

• Structure of the analytical result for large τ :

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1+2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \dots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

- Leading term perfect fluid behaviour second term — 1st order viscous hydrodynamics third term — 2nd order viscous hydrodynamics fourth term — 3rd order viscous hydrodynamics...
- ► In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

Currently we know 240 terms in this expansion

Heller, RJ, Witaszczyk

The hydrodynamic series is only asymptotic and has zero radius of convergence...

• Structure of the analytical result for large τ :

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1+2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \dots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

- Leading term perfect fluid behaviour second term — 1st order viscous hydrodynamics third term — 2nd order viscous hydrodynamics fourth term — 3rd order viscous hydrodynamics...
- ► In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

Currently we know 240 terms in this expansion

Heller, RJ, Witaszczyk

The hydrodynamic series is only asymptotic and has zero radius of convergence...

• Structure of the analytical result for large τ :

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}} \frac{1}{\tau^2} + \frac{1+2\log 2}{12\sqrt{3}} \frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}} \frac{1}{\tau^{\frac{10}{3}}} + \dots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

- Leading term perfect fluid behaviour second term — 1st order viscous hydrodynamics third term — 2nd order viscous hydrodynamics fourth term — 3rd order viscous hydrodynamics...
- In general:

$$\varepsilon(\tau) = \sum_{n=2}^{\infty} \frac{\varepsilon_n}{\tau^{\frac{2n}{3}}}$$

Currently we know 240 terms in this expansion

Heller, RJ, Witaszczyk

The hydrodynamic series is only asymptotic and has zero radius of convergence...

- ▶ In order to study far-from equilibrium behaviour for small τ we have to use numerical relativity methods
- We get rid of the dependence on the number of degrees of freedom by parametrizing the energy density through an effective temperature given by

$$\varepsilon(\tau) = \frac{3}{8} N_c^2 \pi^2 T_{eff}^4(\tau)$$

Previously, we normalized our initial data by setting

 $T_{eff}(au=0)=1$

but this is generically unknown in realistic heavy-ion collisions...

It is much better to fix the normalization through the hydrodynamic tail...

$$\pi T_{eff}(au) \sim rac{1}{ au^{rac{1}{3}}}$$
in the $au o \infty$ limit

- ► In order to study far-from equilibrium behaviour for small τ we have to use numerical relativity methods
- We get rid of the dependence on the number of degrees of freedom by parametrizing the energy density through an effective temperature given by

$$\varepsilon(\tau) = \frac{3}{8} N_c^2 \pi^2 T_{eff}^4(\tau)$$

Previously, we normalized our initial data by setting

 $T_{eff}(au=0)=1$

but this is generically unknown in realistic heavy-ion collisions...

It is much better to fix the normalization through the hydrodynamic tail...

$$\pi T_{eff}(au) \sim rac{1}{ au^{rac{1}{3}}}$$
in the $au o \infty$ limit

- ► In order to study far-from equilibrium behaviour for small τ we have to use numerical relativity methods
- We get rid of the dependence on the number of degrees of freedom by parametrizing the energy density through an effective temperature given by

$$arepsilon(au) = rac{3}{8} N_c^2 \pi^2 T_{eff}^4(au)$$

Previously, we normalized our initial data by setting

 $T_{eff}(au=0)=1$

but this is generically unknown in realistic heavy-ion collisions...

It is much better to fix the normalization through the hydrodynamic tail...

$$\pi T_{eff}(au) \sim rac{1}{ au^{rac{1}{3}}}$$
in the $au o \infty$ limit

- ► In order to study far-from equilibrium behaviour for small τ we have to use numerical relativity methods
- We get rid of the dependence on the number of degrees of freedom by parametrizing the energy density through an effective temperature given by

$$arepsilon(au) = rac{3}{8} N_c^2 \pi^2 T_{eff}^4(au)$$

Previously, we normalized our initial data by setting

 $T_{eff}(\tau=0)=1$

but this is generically unknown in realistic heavy-ion collisions...

It is much better to fix the normalization through the hydrodynamic tail...

$$\pi T_{eff}(au) \sim rac{1}{ au^{rac{1}{3}}}$$
in the $au o \infty$ limit
Remarks:

- ► In order to study far-from equilibrium behaviour for small τ we have to use numerical relativity methods
- We get rid of the dependence on the number of degrees of freedom by parametrizing the energy density through an effective temperature given by

$$arepsilon(au)=rac{3}{8}N_c^2\pi^2T_{eff}^4(au)$$

Previously, we normalized our initial data by setting

 $T_{eff}(\tau=0)=1$

but this is generically unknown in realistic heavy-ion collisions...

It is much better to fix the normalization through the hydrodynamic tail...

$$\pi T_{eff}(au) \sim rac{1}{ au^{rac{1}{3}}}$$
in the $au o \infty$ limit

The coefficient '1' fixes the units of τ .

Remarks:

- ► In order to study far-from equilibrium behaviour for small τ we have to use numerical relativity methods
- We get rid of the dependence on the number of degrees of freedom by parametrizing the energy density through an effective temperature given by

$$arepsilon(au) = rac{3}{8} N_c^2 \pi^2 T_{eff}^4(au)$$

Previously, we normalized our initial data by setting

 $T_{eff}(\tau=0)=1$

but this is generically unknown in realistic heavy-ion collisions...

It is much better to fix the normalization through the hydrodynamic tail...

$$\pi T_{eff}(au) \sim rac{1}{ au^{rac{1}{3}}}$$
in the $au o \infty$ limit

The coefficient '1' fixes the units of τ .

green line: 3rd order hydro
Very clear transition to a hydrodynamic behaviour
Very little information on the initial energy density at τ = 0 (unless we have some specific information on the initial state)







we have some specific information on the initial state)



we have some specific information on the initial state)



Very clear transition to a hydrodynamic behaviour

Very little information on the initial energy density at \(\tau = 0\) (unless we have some specific information on the initial state)

1. Form the dimensionless product $w \equiv T_{eff} \cdot \tau$

2. For all initial conditions considered, viscous hydrodynamics works very well for $w \equiv T_{eff} \cdot \tau > 0.7$

(natural values for RHIC: ($\tau_0 = 0.25 \text{ fm}$, $T_0 = 500 \text{ MeV}$) assumed in [Broniowski, Chojnacki, Florkowski, Kisiel] correspond to w = 0.63)

$$\Delta p_L \equiv 1 - \frac{p_L}{\varepsilon/3} \sim 0.7$$

1. Form the dimensionless product $w \equiv T_{eff} \cdot \tau$

2. For all initial conditions considered, viscous hydrodynamics works very well for $w \equiv T_{eff} \cdot \tau > 0.7$

(natural values for RHIC: ($\tau_0 = 0.25 \text{ fm}$, $T_0 = 500 \text{ MeV}$) assumed in [Broniowski, Chojnacki, Florkowski, Kisiel] correspond to w = 0.63)

$$\Delta p_L \equiv 1 - \frac{p_L}{\varepsilon/3} \sim 0.7$$

- 1. Form the dimensionless product $w \equiv T_{eff} \cdot \tau$
- 2. For all initial conditions considered, viscous hydrodynamics works very well for $w \equiv T_{eff} \cdot \tau > 0.7$

(natural values for RHIC: ($\tau_0 = 0.25 \text{ fm}$, $T_0 = 500 \text{ MeV}$) assumed in [Broniowski, Chojnacki, Florkowski, Kisiel] correspond to w = 0.63)

$$\Delta p_L \equiv 1 - \frac{p_L}{\varepsilon/3} \sim 0.7$$

- 1. Form the dimensionless product $w \equiv T_{eff} \cdot \tau$
- 2. For all initial conditions considered, viscous hydrodynamics works very well for $w \equiv T_{eff} \cdot \tau > 0.7$



(natural values for RHIC: ($\tau_0 = 0.25 \text{ fm}$, $T_0 = 500 \text{ MeV}$) assumed in [Broniowski, Chojnacki, Florkowski, Kisiel] correspond to w = 0.63)

$$\Delta p_L \equiv 1 - \frac{p_L}{\varepsilon/3} \sim 0.7$$

- 1. Form the dimensionless product $w \equiv T_{eff} \cdot \tau$
- 2. For all initial conditions considered, viscous hydrodynamics works very well for $w \equiv T_{eff} \cdot \tau > 0.7$

$$\begin{array}{c} \mathbf{w}^{(th)} \\ 0.75 \\ 0.60 \\ 0.45 \\ 0.30 \\ 0.15 \\ 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ s_{n-eq}^{(i)} \\ s$$

(natural values for RHIC: ($\tau_0 = 0.25 \text{ fm}$, $T_0 = 500 \text{ MeV}$) assumed in [Broniowski, Chojnacki, Florkowski, Kisiel] correspond to w = 0.63)

$$\Delta p_L \equiv 1 - \frac{p_L}{\varepsilon/3} \sim 0.7$$

- 1. Form the dimensionless product $w \equiv T_{eff} \cdot \tau$
- 2. For all initial conditions considered, viscous hydrodynamics works very well for $w \equiv T_{eff} \cdot \tau > 0.7$

$$\begin{array}{c} w^{(\text{ff})} \\ 0.75 \\ 0.60 \\ 0.45 \\ 0.30 \\ 0.15 \\ 0 \\ 0.1 \\ 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ s^{(i)}_{n-eq} \\ s^{(i)}_$$

(natural values for RHIC: ($\tau_0 = 0.25 \text{ fm}$, $T_0 = 500 \text{ MeV}$) assumed in [Broniowski, Chojnacki, Florkowski, Kisiel] correspond to w = 0.63)

$$\Delta p_L \equiv 1 - \frac{p_L}{\varepsilon/3} \sim 0.7$$

Key observation:

Hydrodynamization \neq **Thermalization**

Key observation:

Hydrodynamization \neq Thermalization

Initial entropy turns out to be a key characterization of the initial state

There is a clear correlation of produced entropy with the initial entropy...

Similar conclusion holds for e.g. (effective) thermalization time (understood here as the transition to a viscous hydrodynamic description)

Initial entropy turns out to be a key characterization of the initial state

There is a clear correlation of produced entropy with the initial entropy...

Similar conclusion holds for e.g. (effective) thermalization time (understood here as the transition to a viscous hydrodynamic description)



Initial entropy turns out to be a key characterization of the initial state

There is a clear correlation of produced entropy with the initial entropy...

Similar conclusion holds for e.g. (effective) thermalization time (understood here as the transition to a viscous hydrodynamic description)



Initial entropy turns out to be a key characterization of the initial state

There is a clear correlation of produced entropy with the initial entropy...

Similar conclusion holds for e.g. (effective) thermalization time (understood here as the transition to a viscous hydrodynamic description)



Recall $T_{eff}(\tau)$

How to model deviations from (all-order) hydrodynamics?

Recall $T_{eff}(\tau)$



How to model deviations from (all-order) hydrodynamics?

Recall $T_{eff}(\tau)$



How to model deviations from (all-order) hydrodynamics?



Is there a simple phenomenological model simpler than 5D Einstein's equations??

 $egin{array}{lll} T_{\mu
u}(au,u^
ho) &\sim & ext{hydrodynamics} \ T_{\mu
u}(au,u^
ho,???) &\sim & ext{hydrodynamics} + ext{additional DOF} \end{array}$

c.f. anisotropic hydrodynamics of Florkowski, Strickland and collaborators



Is there a simple phenomenological model simpler than 5D Einstein's equations??

 $T_{\mu
u}(T, u^{
ho}) \sim ext{hydrodynamics}$ $T_{\mu
u}(T, u^{
ho}, ???) \sim ext{hydrodynamics} + ext{additional DOF}$

c.f. anisotropic hydrodynamics of Florkowski, Strickland and collaborators



Is there a simple phenomenological model simpler than 5D Einstein's equations??

 $T_{\mu
u}(T, u^{
ho}) \sim ext{hydrodynamics}$ $T_{\mu
u}(T, u^{
ho}, ???) \sim ext{hydrodynamics} + ext{additional DOF}$

c.f. anisotropic hydrodynamics of Florkowski, Strickland and collaborators



Is there a simple phenomenological model simpler than 5D Einstein's equations??

 $T_{\mu
u}(T, u^{
ho}) \sim ext{hydrodynamics}$ $T_{\mu
u}(T, u^{
ho}, ???) \sim ext{hydrodynamics} + ext{additional DOF}$

c.f. anisotropic hydrodynamics of $\mathsf{Florkowski},\,\mathsf{Strickland}$ and collaborators



Is there a simple phenomenological model simpler than 5D Einstein's equations??

 $T_{\mu
u}(T, u^{
ho}) \sim ext{hydrodynamics}$ $T_{\mu
u}(T, u^{
ho}, ???) \sim ext{hydrodynamics} + ext{additional DOF}$

c.f. anisotropic hydrodynamics of Florkowski, Strickland and collaborators

- On the gravity side, deviations from hydrodynamics may be represented by metric perturbations (quasinormal modes)
- Each quasinormal mode represents an independent degree of freedom from the 4D perspective...
- The generic structure of QNM modes for a boost-invariant flow (including first viscous corrections)

 $\deltaarepsilon(au)\sim au^{-2}e^{-i\omega_{ extsf{QNM}}\int\pi\,T(au)d au}$

- One can estimate that $s = \int_0^{\tau} \pi T(\tau) d\tau$ at the transition to hydrodynamics would set the scale for how many QNM would be relevant there...
- ▶ It turns out that s = 1.6...3 and $Im \omega_{QNM} = 2.75, 4.76, 6.77, ...$
- A few additional DOF might suffice?

- On the gravity side, deviations from hydrodynamics may be represented by metric perturbations (quasinormal modes)
- Each quasinormal mode represents an independent degree of freedom from the 4D perspective...
- The generic structure of QNM modes for a boost-invariant flow (including first viscous corrections)

 $\deltaarepsilon(au)\sim au^{-2}e^{-i\omega_{ extsf{QNM}}\int\pi\,T(au)d au}$

- One can estimate that $s = \int_0^\tau \pi T(\tau) d\tau$ at the transition to hydrodynamics would set the scale for how many QNM would be relevant there...
- ▶ It turns out that s = 1.6...3 and $Im \omega_{QNM} = 2.75, 4.76, 6.77, ...$
- A few additional DOF might suffice?

- On the gravity side, deviations from hydrodynamics may be represented by metric perturbations (quasinormal modes)
- Each quasinormal mode represents an independent degree of freedom from the 4D perspective...
- The generic structure of QNM modes for a boost-invariant flow (including first viscous corrections)

 $\delta arepsilon(au) \sim au^{-2} e^{-i\omega_{ extsf{QNM}}\int \pi T(au)d au}$

- One can estimate that $s = \int_0^\tau \pi T(\tau) d\tau$ at the transition to hydrodynamics would set the scale for how many QNM would be relevant there...
- ▶ It turns out that s = 1.6...3 and $Im \omega_{QNM} = 2.75, 4.76, 6.77, ...$
- A few additional DOF might suffice?

- On the gravity side, deviations from hydrodynamics may be represented by metric perturbations (quasinormal modes)
- Each quasinormal mode represents an independent degree of freedom from the 4D perspective...
- The generic structure of QNM modes for a boost-invariant flow (including first viscous corrections)

 $\delta \varepsilon(\tau) \sim \tau^{-2} e^{-i\omega_{\text{QNM}}\int \pi T(\tau)d\tau}$

- One can estimate that $s = \int_0^\tau \pi T(\tau) d\tau$ at the transition to hydrodynamics would set the scale for how many QNM would be relevant there...
- ▶ It turns out that s = 1.6...3 and $Im \omega_{QNM} = 2.75, 4.76, 6.77, ...$
- A few additional DOF might suffice?

- On the gravity side, deviations from hydrodynamics may be represented by metric perturbations (quasinormal modes)
- Each quasinormal mode represents an independent degree of freedom from the 4D perspective...
- The generic structure of QNM modes for a boost-invariant flow (including first viscous corrections)

$$\delta arepsilon(au) \sim au^{-2} e^{-i\omega_{ extsf{QNM}}\int \pi T(au)d au}$$

- One can estimate that $s = \int_0^\tau \pi T(\tau) d\tau$ at the transition to hydrodynamics would set the scale for how many QNM would be relevant there...
- ▶ It turns out that s = 1.6...3 and $Im \omega_{QNM} = 2.75, 4.76, 6.77, ...$
- A few additional DOF might suffice?

- On the gravity side, deviations from hydrodynamics may be represented by metric perturbations (quasinormal modes)
- Each quasinormal mode represents an independent degree of freedom from the 4D perspective...
- The generic structure of QNM modes for a boost-invariant flow (including first viscous corrections)

$$\delta \varepsilon(au) \sim au^{-2} e^{-i\omega_{\text{QNM}}\int \pi T(au)d au}$$

where $\omega_{QNM} = \omega_R - i\omega_I$

- One can estimate that $s = \int_0^\tau \pi T(\tau) d\tau$ at the transition to hydrodynamics would set the scale for how many QNM would be relevant there...
- It turns out that s = 1.6...3 and $Im \omega_{QNM} = 2.75, 4.76, 6.77, ...$

A few additional DOF might suffice?

- On the gravity side, deviations from hydrodynamics may be represented by metric perturbations (quasinormal modes)
- Each quasinormal mode represents an independent degree of freedom from the 4D perspective...
- The generic structure of QNM modes for a boost-invariant flow (including first viscous corrections)

$$\delta \varepsilon(au) \sim au^{-2} e^{-i\omega_{\text{QNM}}\int \pi T(au)d au}$$

- One can estimate that $s = \int_0^\tau \pi T(\tau) d\tau$ at the transition to hydrodynamics would set the scale for how many QNM would be relevant there...
- It turns out that s = 1.6...3 and $Im \omega_{QNM} = 2.75, 4.76, 6.77, ...$
- A few additional DOF might suffice?

How to write an equation of motion for a scalar nonhydrodynamic degree of freedom (e.g. $tr F^2$) in a generic hydrodynamic background?

- ► The scalar dof corresponds to a scalar QNM with frequencies $\omega_{QNM} = \omega_R i\omega_I$
- ► Key feature: the frequencies have only very mild dependence on the spatial momentum the dynamics seems 'ultralocal'
- ▶ We may write the equation of motion in a generic hydrodynamic flow

$$\mathcal{D}^{2}\phi + 2\omega_{I}\mathcal{D}\phi + \left(\omega_{I}^{2} + \omega_{R}^{2}\right)\phi = 0$$

where

$$\mathcal{D}\equiv\frac{1}{T}u^{\mu}\partial_{\mu}$$

Extension to nonhydrodynamic modes of $T_{\mu\nu}$ in progress...
- ► The scalar dof corresponds to a scalar QNM with frequencies $\omega_{QNM} = \omega_R i\omega_I$
- ► Key feature: the frequencies have only very mild dependence on the spatial momentum the dynamics seems 'ultralocal'
- ▶ We may write the equation of motion in a generic hydrodynamic flow

$$\mathcal{D}^{2}\phi + 2\omega_{I}\mathcal{D}\phi + \left(\omega_{I}^{2} + \omega_{R}^{2}\right)\phi = 0$$

where

$$\mathcal{D} \equiv \frac{1}{T} u^{\mu} \partial_{\mu}$$

- ► The scalar dof corresponds to a scalar QNM with frequencies $\omega_{QNM} = \omega_R i\omega_I$
- ► Key feature: the frequencies have only very mild dependence on the spatial momentum the dynamics seems 'ultralocal'
- ▶ We may write the equation of motion in a generic hydrodynamic flow

$$\mathcal{D}^{2}\phi + 2\omega_{I}\mathcal{D}\phi + \left(\omega_{I}^{2} + \omega_{R}^{2}\right)\phi = 0$$

where

$$\mathcal{D} \equiv \frac{1}{T} u^{\mu} \partial_{\mu}$$

- ► The scalar dof corresponds to a scalar QNM with frequencies $\omega_{QNM} = \omega_R i\omega_I$
- ► Key feature: the frequencies have only very mild dependence on the spatial momentum the dynamics seems 'ultralocal'
- ▶ We may write the equation of motion in a generic hydrodynamic flow

$$\mathcal{D}^{2}\phi + 2\omega_{I}\mathcal{D}\phi + \left(\omega_{I}^{2} + \omega_{R}^{2}\right)\phi = 0$$

where

$$\mathcal{D}\equiv\frac{1}{T}u^{\mu}\partial_{\mu}$$

- ► The scalar dof corresponds to a scalar QNM with frequencies $\omega_{QNM} = \omega_R i\omega_I$
- ► Key feature: the frequencies have only very mild dependence on the spatial momentum the dynamics seems 'ultralocal'
- ▶ We may write the equation of motion in a generic hydrodynamic flow

$$\mathcal{D}^{2}\phi + 2\omega_{I}\mathcal{D}\phi + \left(\omega_{I}^{2} + \omega_{R}^{2}\right)\phi = 0$$

where

$$\mathcal{D}\equiv\frac{1}{T}u^{\mu}\partial_{\mu}$$

- ► The scalar dof corresponds to a scalar QNM with frequencies $\omega_{QNM} = \omega_R i\omega_I$
- Key feature: the frequencies have only very mild dependence on the spatial momentum — the dynamics seems 'ultralocal'
- We may write the equation of motion in a generic hydrodynamic flow

$$\mathcal{D}^{2}\phi + 2\omega_{I}\mathcal{D}\phi + \left(\omega_{I}^{2} + \omega_{R}^{2}\right)\phi = 0$$

where

$$\mathcal{D}\equiv\frac{1}{T}u^{\mu}\partial_{\mu}$$

- ► The scalar dof corresponds to a scalar QNM with frequencies $\omega_{QNM} = \omega_R i\omega_I$
- Key feature: the frequencies have only very mild dependence on the spatial momentum — the dynamics seems 'ultralocal'
- We may write the equation of motion in a generic hydrodynamic flow

$$\mathcal{D}^{2}\phi + 2\omega_{I}\mathcal{D}\phi + \left(\omega_{I}^{2} + \omega_{R}^{2}\right)\phi = 0$$

where

$$\mathcal{D} \equiv \frac{1}{T} u^{\mu} \partial_{\mu}$$

- AdS/CFT provides a very general framework for studying time-dependent dynamical processes
- The AdS/CFT methods do not presuppose hydrodynamics so are applicable even to very out-of-equilibrium configurations
- AdS/CFT may fill in gaps in our knowledge of the early nonequilibrium stage of plasma evolution
- Thermalization \neq hydrodynamization
- Simple dimensionless criterion for applicability of hydrodynamics
- Important role of 'initial entropy' as a characterization of the initial state
- One can perhaps understand better the dynamics of lowest nonhydrodynamic degrees of freedom from the 4D perspective
- ► Key role of quasinormal frequencies...

- AdS/CFT provides a very general framework for studying time-dependent dynamical processes
- The AdS/CFT methods do not presuppose hydrodynamics so are applicable even to very out-of-equilibrium configurations
- AdS/CFT may fill in gaps in our knowledge of the early nonequilibrium stage of plasma evolution
- Thermalization \neq hydrodynamization
- Simple dimensionless criterion for applicability of hydrodynamics
- Important role of 'initial entropy' as a characterization of the initial state
- One can perhaps understand better the dynamics of lowest nonhydrodynamic degrees of freedom from the 4D perspective
- ► Key role of quasinormal frequencies...

- AdS/CFT provides a very general framework for studying time-dependent dynamical processes
- The AdS/CFT methods do not presuppose hydrodynamics so are applicable even to very out-of-equilibrium configurations
- AdS/CFT may fill in gaps in our knowledge of the early nonequilibrium stage of plasma evolution
- Thermalization \neq hydrodynamization
- Simple dimensionless criterion for applicability of hydrodynamics
- Important role of 'initial entropy' as a characterization of the initial state
- One can perhaps understand better the dynamics of lowest nonhydrodynamic degrees of freedom from the 4D perspective
- ► Key role of quasinormal frequencies...

- AdS/CFT provides a very general framework for studying time-dependent dynamical processes
- The AdS/CFT methods do not presuppose hydrodynamics so are applicable even to very out-of-equilibrium configurations
- AdS/CFT may fill in gaps in our knowledge of the early nonequilibrium stage of plasma evolution
- Thermalization \neq hydrodynamization
- Simple dimensionless criterion for applicability of hydrodynamics
- Important role of 'initial entropy' as a characterization of the initial state
- One can perhaps understand better the dynamics of lowest nonhydrodynamic degrees of freedom from the 4D perspective
- ► Key role of quasinormal frequencies...

- AdS/CFT provides a very general framework for studying time-dependent dynamical processes
- The AdS/CFT methods do not presuppose hydrodynamics so are applicable even to very out-of-equilibrium configurations
- AdS/CFT may fill in gaps in our knowledge of the early nonequilibrium stage of plasma evolution
- Thermalization \neq hydrodynamization
- Simple dimensionless criterion for applicability of hydrodynamics
- Important role of 'initial entropy' as a characterization of the initial state
- One can perhaps understand better the dynamics of lowest nonhydrodynamic degrees of freedom from the 4D perspective
- ► Key role of quasinormal frequencies...

- AdS/CFT provides a very general framework for studying time-dependent dynamical processes
- The AdS/CFT methods do not presuppose hydrodynamics so are applicable even to very out-of-equilibrium configurations
- AdS/CFT may fill in gaps in our knowledge of the early nonequilibrium stage of plasma evolution
- Thermalization \neq hydrodynamization
- Simple dimensionless criterion for applicability of hydrodynamics
- Important role of 'initial entropy' as a characterization of the initial state
- One can perhaps understand better the dynamics of lowest nonhydrodynamic degrees of freedom from the 4D perspective
- ► Key role of quasinormal frequencies...

- AdS/CFT provides a very general framework for studying time-dependent dynamical processes
- The AdS/CFT methods do not presuppose hydrodynamics so are applicable even to very out-of-equilibrium configurations
- AdS/CFT may fill in gaps in our knowledge of the early nonequilibrium stage of plasma evolution
- Thermalization \neq hydrodynamization
- Simple dimensionless criterion for applicability of hydrodynamics
- Important role of 'initial entropy' as a characterization of the initial state
- One can perhaps understand better the dynamics of lowest nonhydrodynamic degrees of freedom from the 4D perspective
- ► Key role of quasinormal frequencies...

- AdS/CFT provides a very general framework for studying time-dependent dynamical processes
- The AdS/CFT methods do not presuppose hydrodynamics so are applicable even to very out-of-equilibrium configurations
- AdS/CFT may fill in gaps in our knowledge of the early nonequilibrium stage of plasma evolution
- Thermalization \neq hydrodynamization
- Simple dimensionless criterion for applicability of hydrodynamics
- Important role of 'initial entropy' as a characterization of the initial state
- One can perhaps understand better the dynamics of lowest nonhydrodynamic degrees of freedom from the 4D perspective
- Key role of quasinormal frequencies...

- AdS/CFT provides a very general framework for studying time-dependent dynamical processes
- The AdS/CFT methods do not presuppose hydrodynamics so are applicable even to very out-of-equilibrium configurations
- AdS/CFT may fill in gaps in our knowledge of the early nonequilibrium stage of plasma evolution
- Thermalization \neq hydrodynamization
- Simple dimensionless criterion for applicability of hydrodynamics
- Important role of 'initial entropy' as a characterization of the initial state
- One can perhaps understand better the dynamics of lowest nonhydrodynamic degrees of freedom from the 4D perspective
- Key role of quasinormal frequencies...