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Global Structure of Conformal Theories in the SU(3) Gauge Theory

Y. Iwasaki
U.Tsukuba and KEK

In Collaboration with
K.-I. Ishikawa(U.Hiroshima)
Yu Nakayama(CalTech)
T. Yoshie(U. Tsukuba)

Based on

Y. Iwasaki, to be published in Proceedings of SCGT 12
held in Nagoya on December 4 -7,2012:arXiv:1212.4343.

K.-I. Ishikawa, Y. Iwasaki, Yu Nakayama and T. Yoshie,
Phys. Rev. D 87 071503 (2013); arXiv:1301.4785.

K.-I. Ishikawa, Y. Iwasaki, Yu Nakayama and T. Yoshie,
arXiv:1304.4345.

K.-I. Ishikawa, Y. Iwasaki, Yu Nakayama and T. Yoshie,
arXiv:1310.5049

Y. Iwasaki, Contribution to the 31st International Symposium on Lattice Field Theory -
LATTICE 2013, July 29 - August 3, 2013, Mainz, Germany, arXiv:1311.2966

Objectives:

Clarify the global structure of Conformal theories in the SU(3) gauge theory, and thereby reveal the characteristics of each conformal theory.

SU(3) with Nf flavors in fundamental representation

Then confront the nature

Background:

Phenomenology:

nearly conformal theories are attractive candidates for BSM

Theoretical interest

Plan of Talk

(1) For N_f in the conformal window

$$N_f^c \leq N_f \leq 16$$

Propose a new concept “Conformal theories with an IR cutoff”

“conformal region” $m_H \leq c\,\Lambda_{IR}$

The vacuum is a vacuum with non-trivial Z(3) structure

Propagators behave as $G(t) = c\,\exp\left(-m_H\,t\right)/t^{\alpha}$

(2) For small Nf ($2 \leq N_f \leq 6$) at high temperature T/Tc > 1

“Conformal theories with an IR cutoff”

(3) Correspondence between the above two

Nf=7 <--> Nf=2;T/Tc ~ 2 <--> meson unparticle

STAGE and TOOLS

- Lattice gauge theory defined by two parameters:

$$\beta = 6/{g_0}^2 \qquad K = 1/2(m_0 a + 4)$$

- on a lattice with lattice size: Nx=Ny=Nz=N; Nt=r N

- Action: one-plaqutte gauge action + Wilson fermion action

- a --> 0, N --> infinity with keeping L= N a constant

- meson propagator $$G_H(t) = \sum_x \langle \bar{\psi}\gamma_H \psi(x,t) \bar{\psi}\gamma_H \psi(0) \rangle \ .$$

Conformal Window

$N_f \leq 16$ asymptotically free

$(g_0 = 0, m_o = 0) =$ UVFP

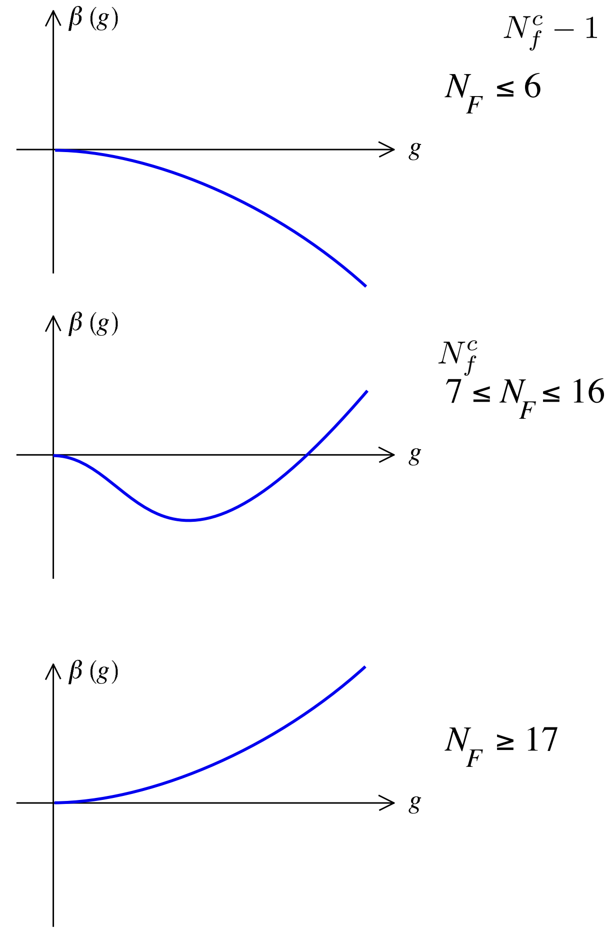
$N_f^c \leq N_f \leq 16$ conformal window

$(g_0 = g^*, m_o = 0) =$ IRFP

Banks and Zaks

N_f^c ?? controversial

Our conjecture $N_f^c = 7$



When $N_f^c \leq N_f \leq 16$

Propagators of mesons

When $g_0 = g^*$

$$G(t) = c\,\frac{1}{t^\alpha} \qquad \alpha = 3 - 2\gamma^*$$

scale invariant

When $0 \leq g_o < g^*$

$$G(t) = c\,\frac{1}{t^{\alpha(t)}}$$

$$\alpha(t) = 3 \qquad t \ll \Lambda_{CFT}$$

$$\alpha(t) = 3 - 2\gamma^* \qquad t \gg \Lambda_{CFT}$$

Two issues

scale invariance => conformal invariance ?

When an IR cutoff exists, what happens ?

scale invariance => conformal invariance ?

No general proof in 4d

J. Polchinski 1988
Yu Nakayama arXiv:1302.0884

Trace anomaly : massless quark at temperature T

$$\langle T^{\mu}_{\ \mu}\rangle|_T = \beta(g^{-2}(\mu))\langle \mathrm{Tr}(F_{\mu\nu}(\mu))^2\rangle|_T$$

$\beta(g^{-2}(\mu))$ beta function at T=0

Appendix B in arXiv:1310.5049
S. Adler arXiv:0405040(hep-th)

At T=0 the vanishing of the beta function means the energy momentum trace vanishes
Therefore conformal invariance holds

At finite T the vanishing of the beta function at T does not imply the energy momentum trace vanishes

When an IR cutoff exists, what happpnes ?

Note that all numerical simulations are with an IR cutoff

Even when $g_0 = g^*$

Meson propagator $G(t)$ is not scale invariant for massless quark

$$G(t) = c\,\frac{\exp\left(-m_H\, t\right)}{t^{\alpha}} \qquad\qquad m_H \leq c\,\Lambda_{IR}$$

When the quark is heavy

$$G(t) = c\,\exp\left(-m\, t\right)$$

Where is the boundary between the two decay forms ?

Propose a new concept

"Conformal theories with an IR cutoff"

Theories :

the beta function of the coupling constant possesses an IRFP

AND

there exists an IR cutoff

In the "Conformal theories with an IR cutoff"

There exists the "conformal region" for $m_H \leq c\,\Lambda_{IR}$

The vacuum is a vacuum with non-trivial Z(3) structure

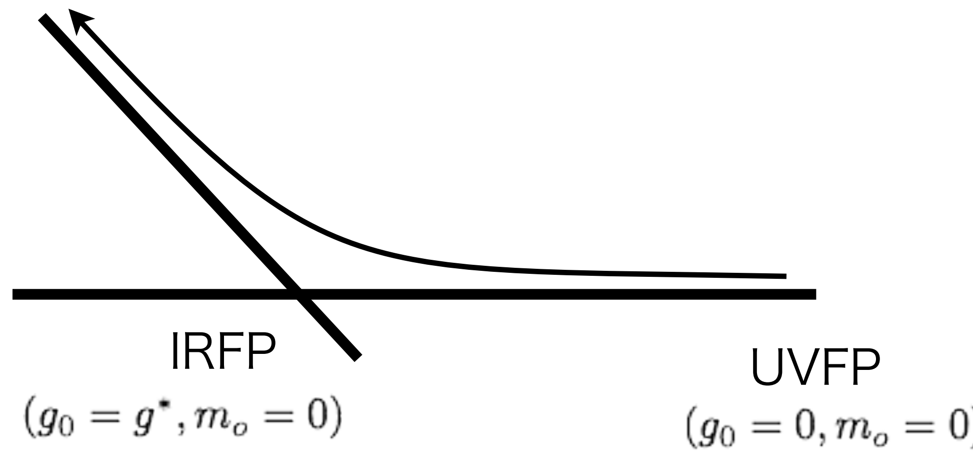
Meson propagators decay with a Yukawa-type decay form

$$G(t) = c\,\frac{\exp\left(-m_H\, t\right)}{t^{\alpha}}$$

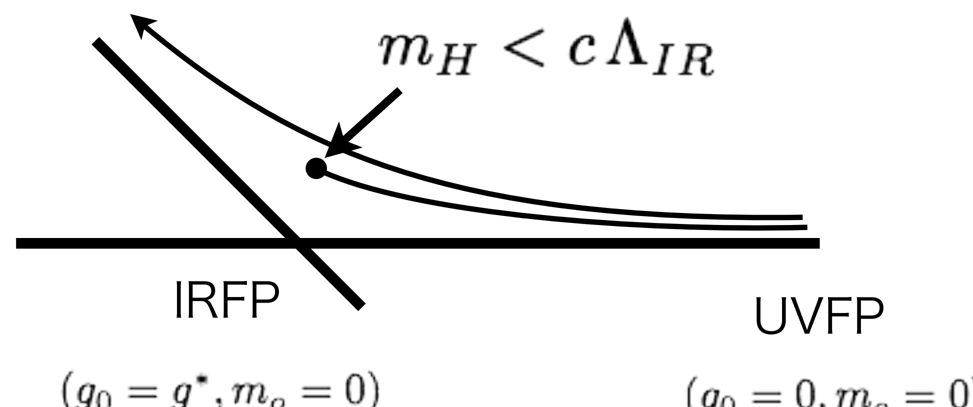
Transition at the boundary of the conformal region is first order

RG argument

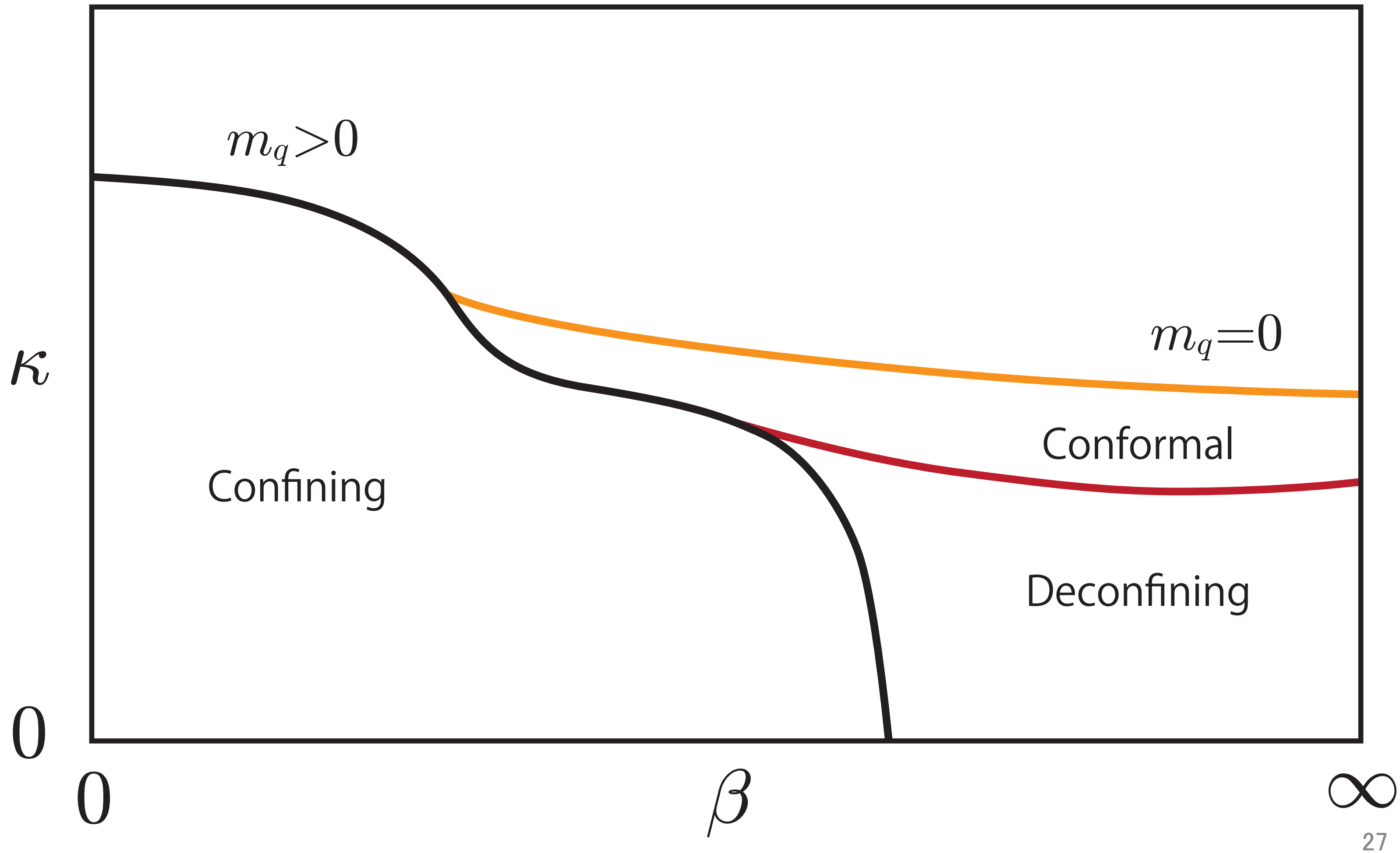
When $\Lambda_{IR} = 0$



When $\Lambda_{IR} = finite$



Phase structure on a finite lattice

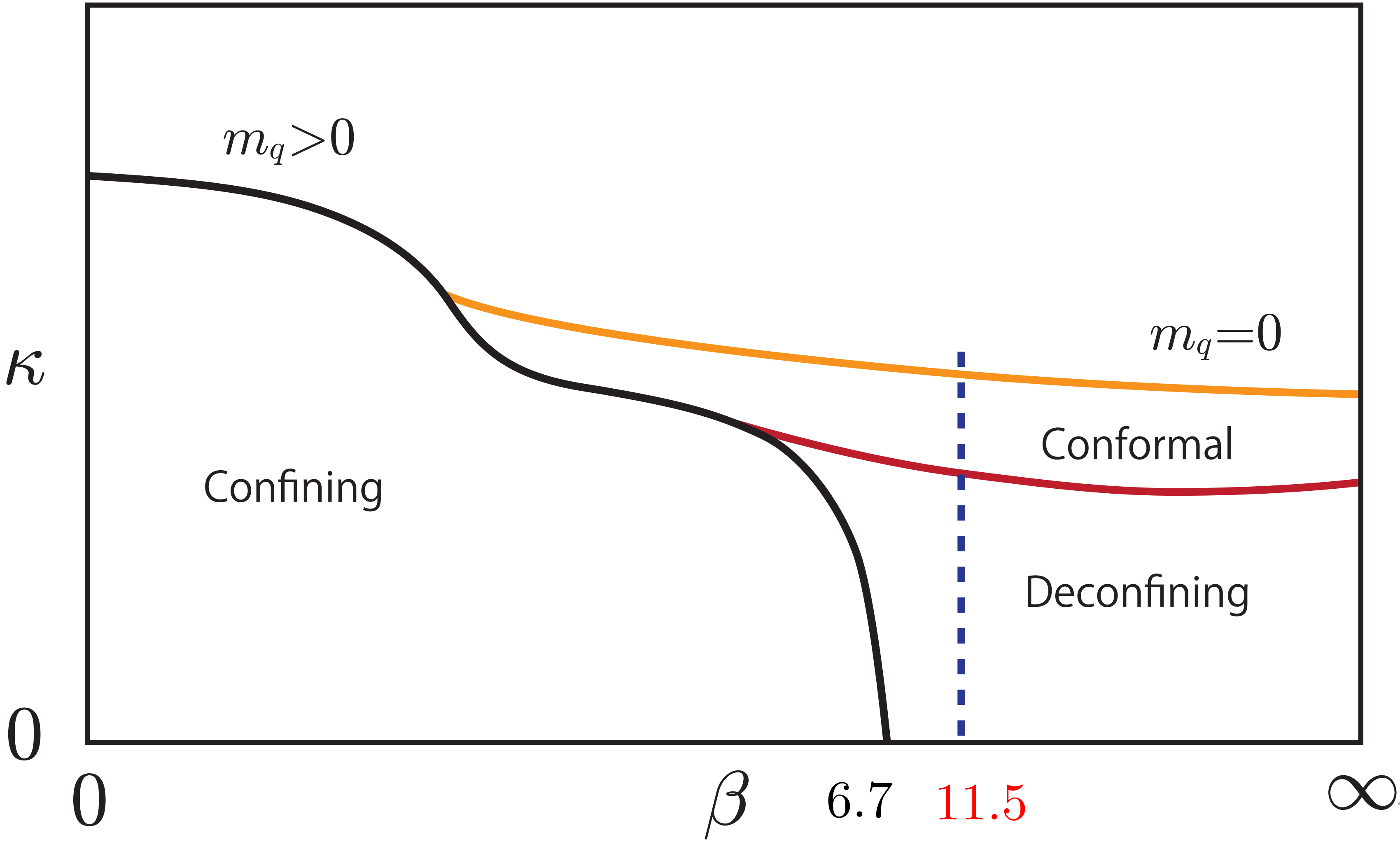


Verify the existence of the conformal region, and the vacuum structure and the Yukawa type decay form
For:

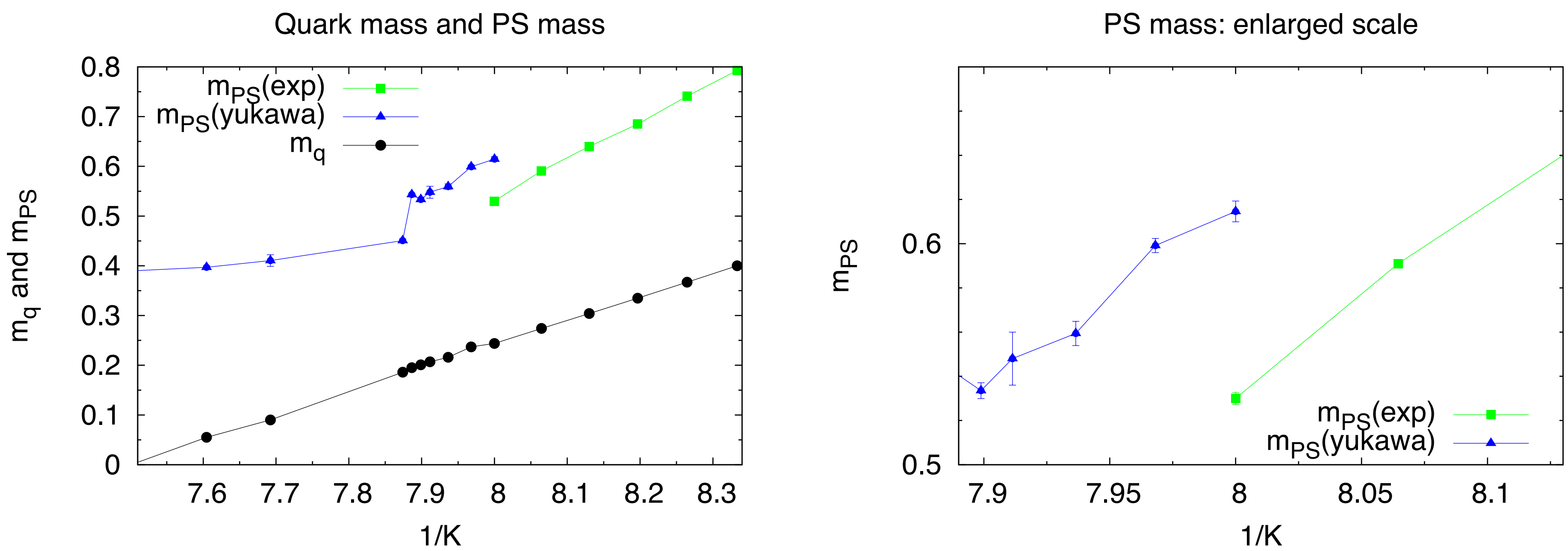
Nf=16
Nf=7
Nf=12

on a lattice16^3 x 64

Phase structure on a finite lattice: Nf=16



$N_f = 16;\ \beta = 11.5$

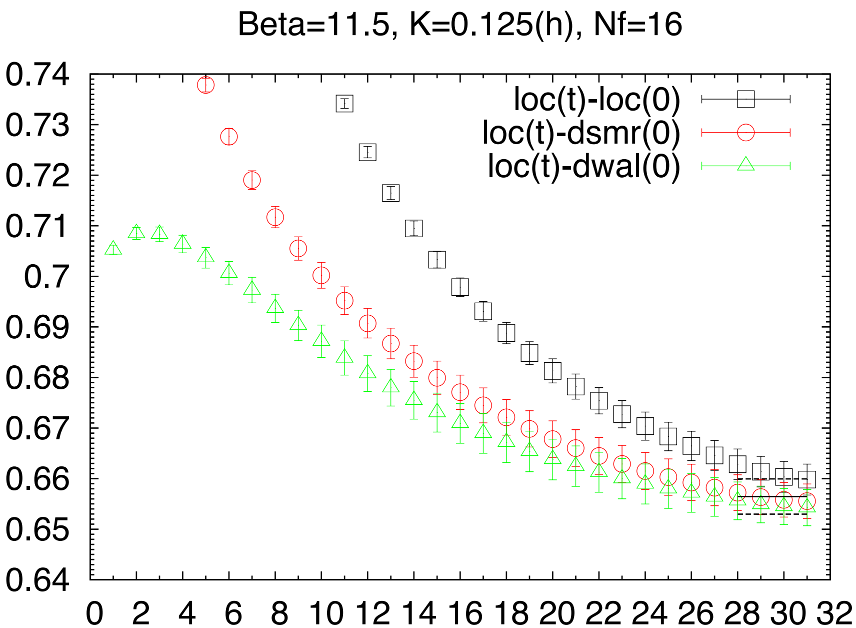


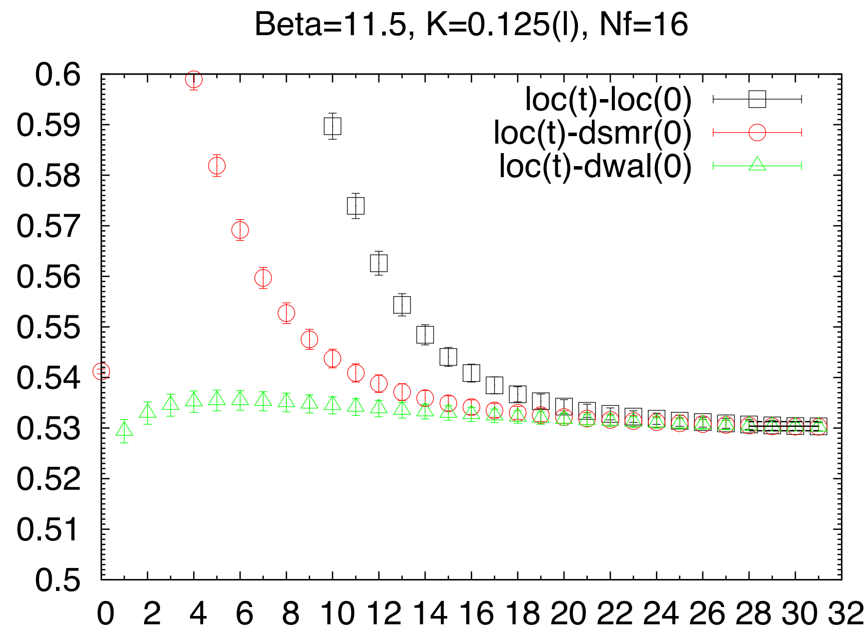
For K>0.125 yukawa-type decay:

For K<0.125 exponential-type decay

Two states at K=0.125; First order transition

Two states at K=0.125: effective mass





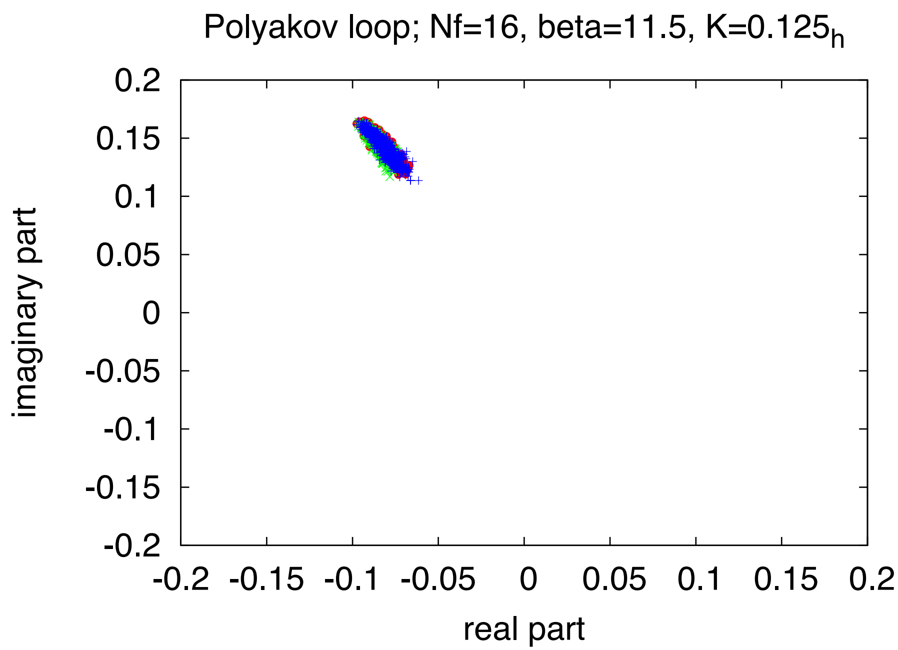
Yukawa-type decay

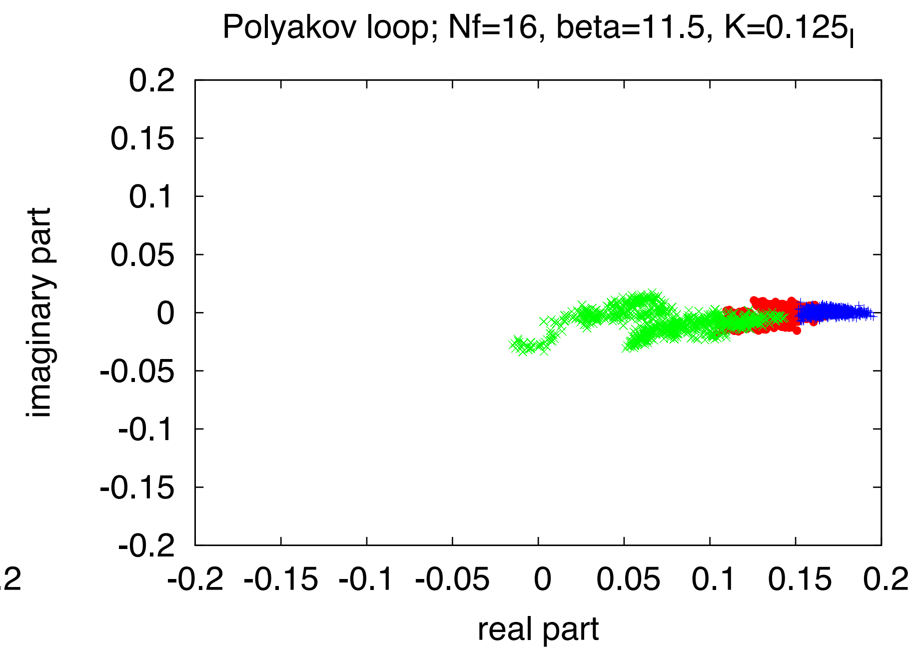
m→0.66

Exponential-type decay

m→ 0.53

Nf=16:Two states at K=0.125: Polyakov loops





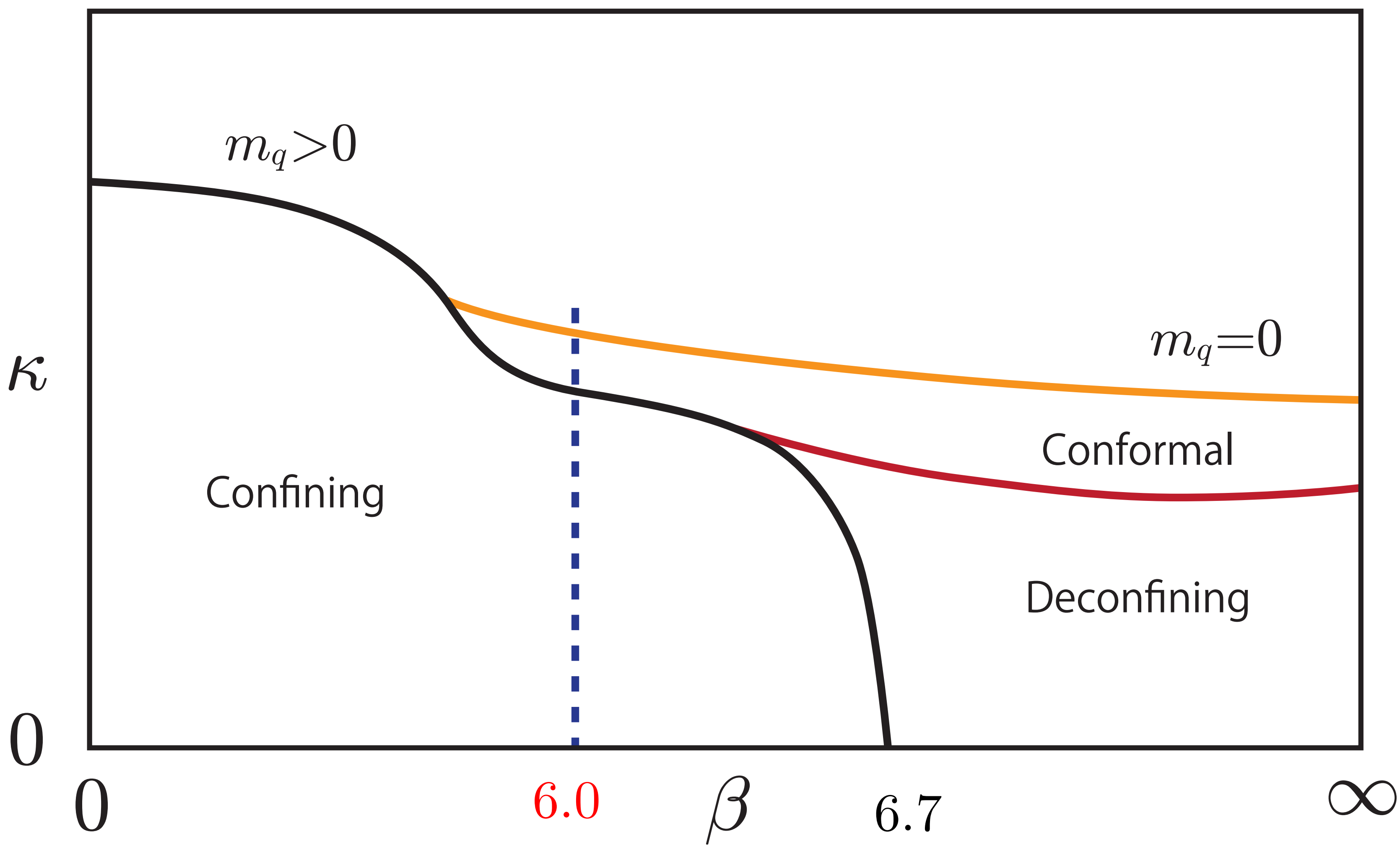
$$P_x, P_y, P_z \simeq 0.2 \exp\left(\pm i\, 2\pi/3\right)$$

$$P_x, P_y, P_z \simeq 0.05 \sim 0.2$$

Close to the twisted vacuum,
But not equal

characteristic in the deconfining
region

Phase structure on a finite lattice: Nf=7



For small Nf at high temperature T/Tc > 1

$$2 \le N_f \le 6$$

Define a running coupling constant $g(\mu; T)$ at temperature T

For example, a running coupling constant g(r; T) is defined in terms of the quark anti-quark free energy

O. Kaczmarek, F. Karsch, F. Zantow and P. Petreczky, Phys.Rev. 70, 074505 (2004)

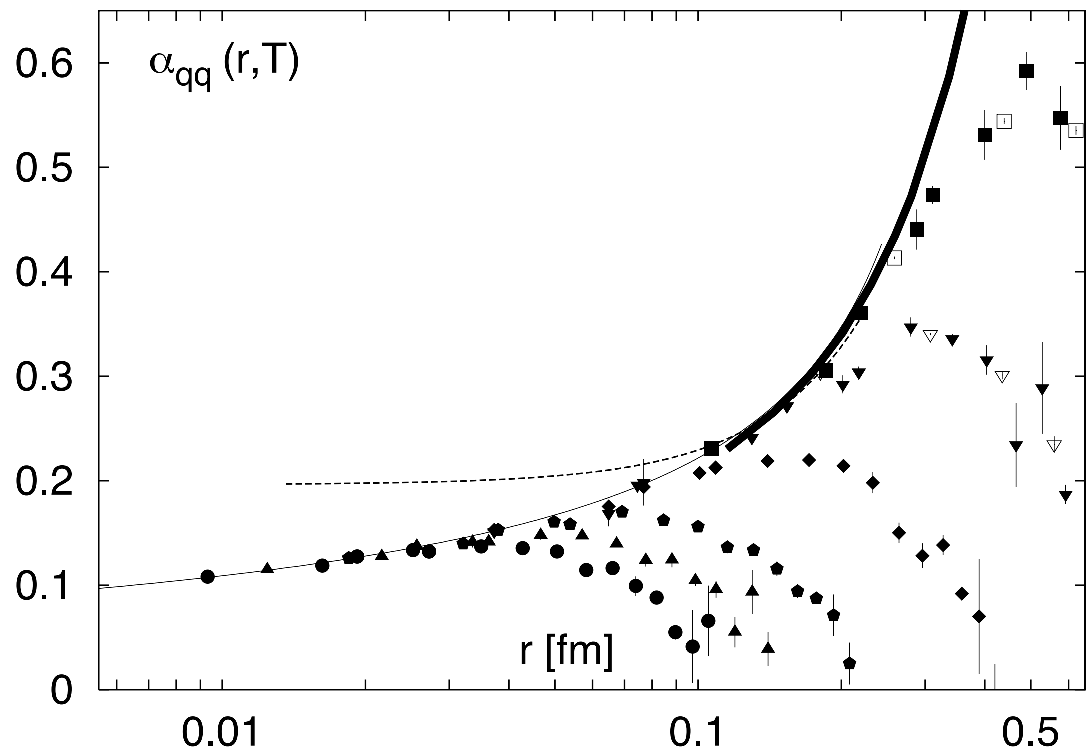
If there were no IRFP of the running coupling constant,
the quarks would be confined
therefore there should exist an IRFP for T/Tc > 1

temperature T provide an IR cutoff

Thus QCD at T/Tc >1 are “conformal theories with an IR cutoff”

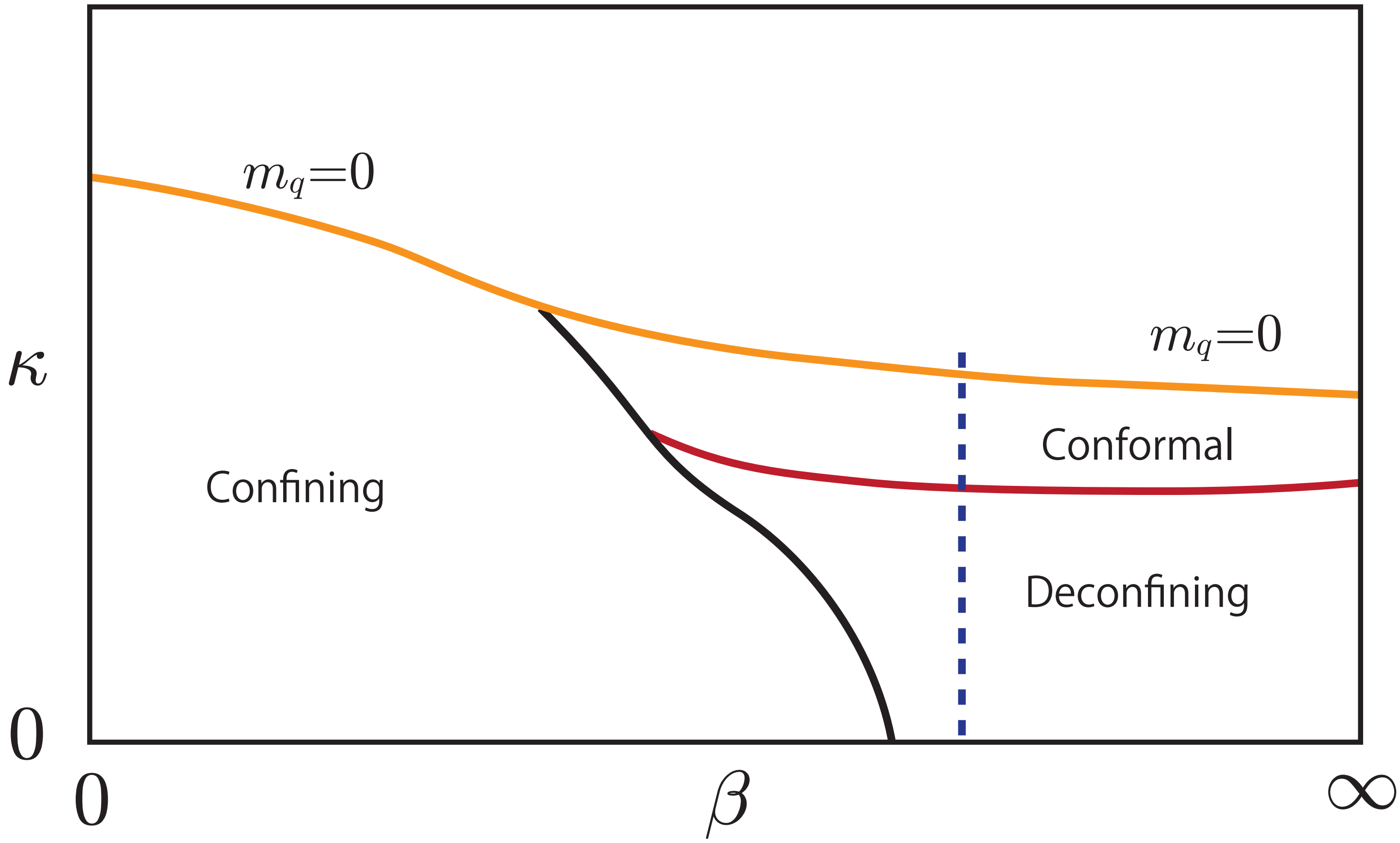
Numerical results for the running coupling constant

Fig.2 of O. Kaczmarek et al. T/Tc=1.05, 1.50, 3.00, 6.00, 9.00, 12.0



the running coupling constant g(r; T) increases as r increases up to some value and does not further increases more than that, and the maximum value decrease as T/T_c increases.

Phase structure on a finite lattice; Nf=2, beta=10.0

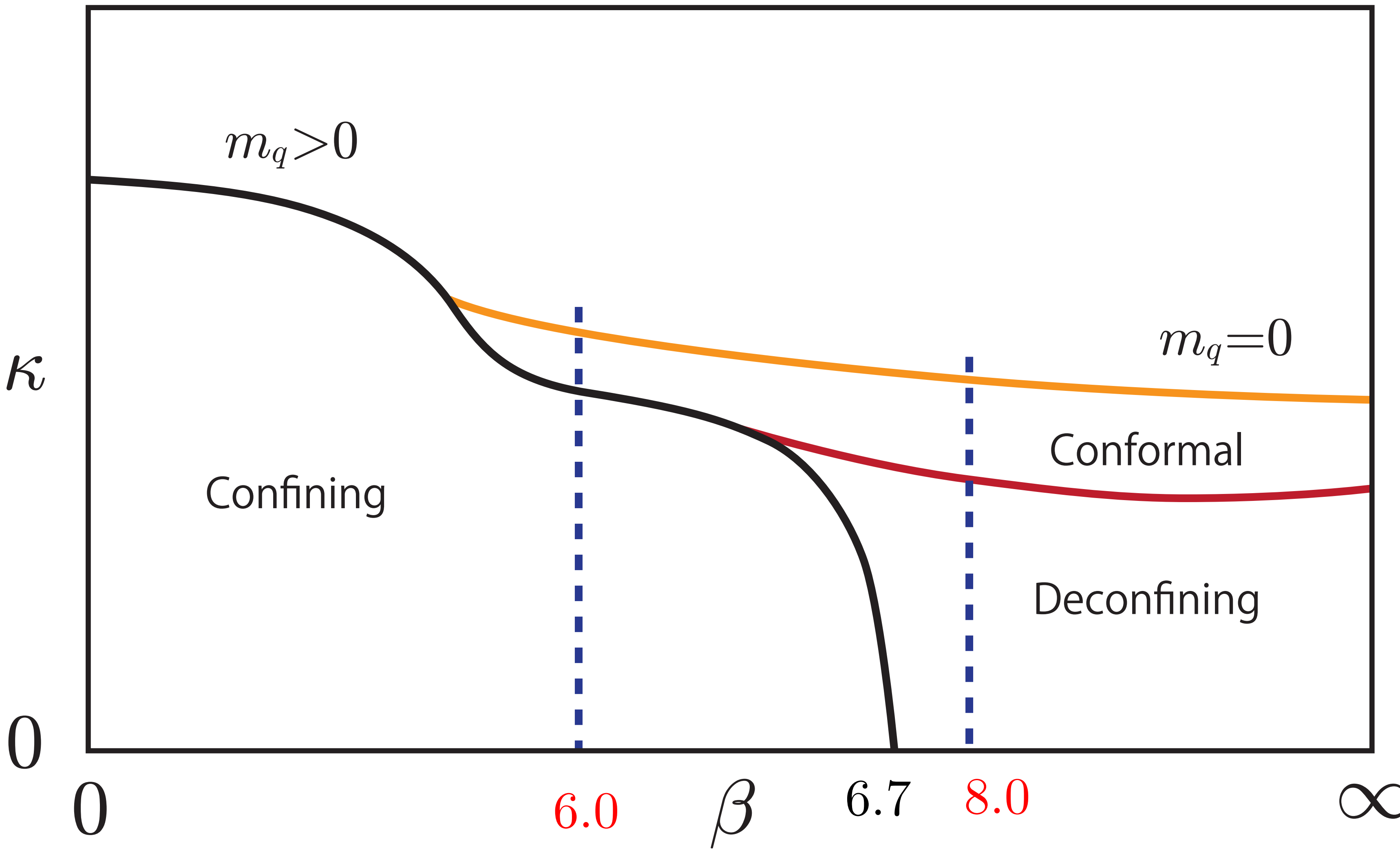


Cautious remark:

for the determination of conformality or non-conformality

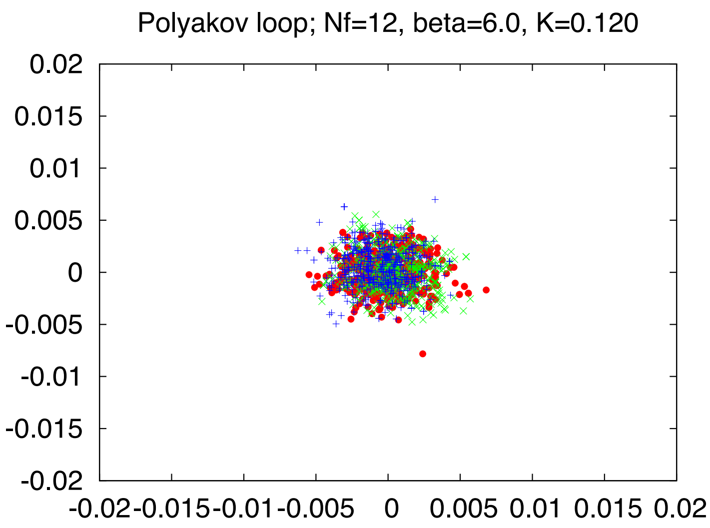
from mass spectroscopy

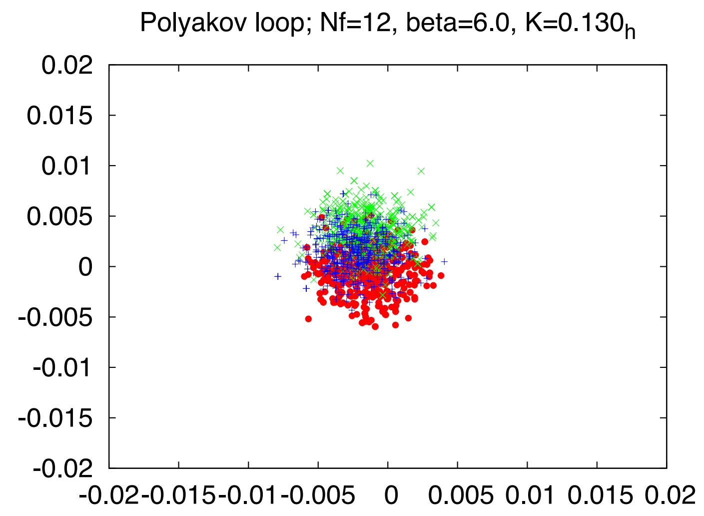
Phase structure on a finite lattice: Nf=12



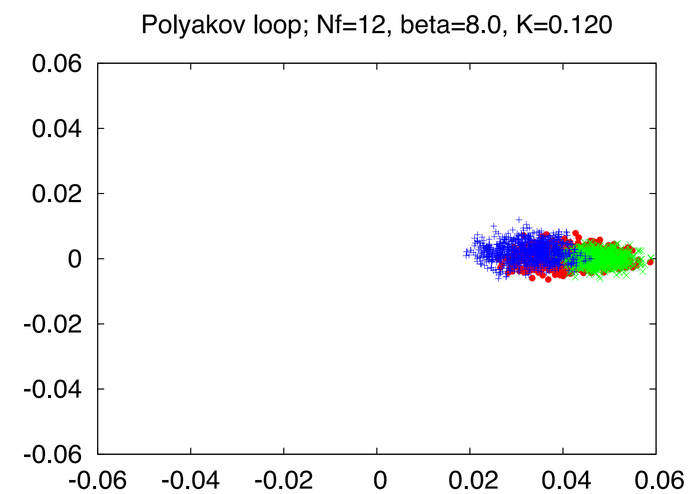
Polyakov loops outside the conformal region :Nf=12

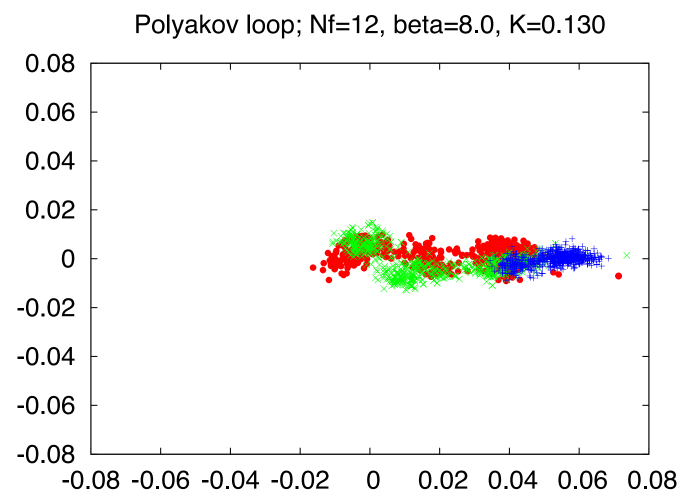
Beta=6.0





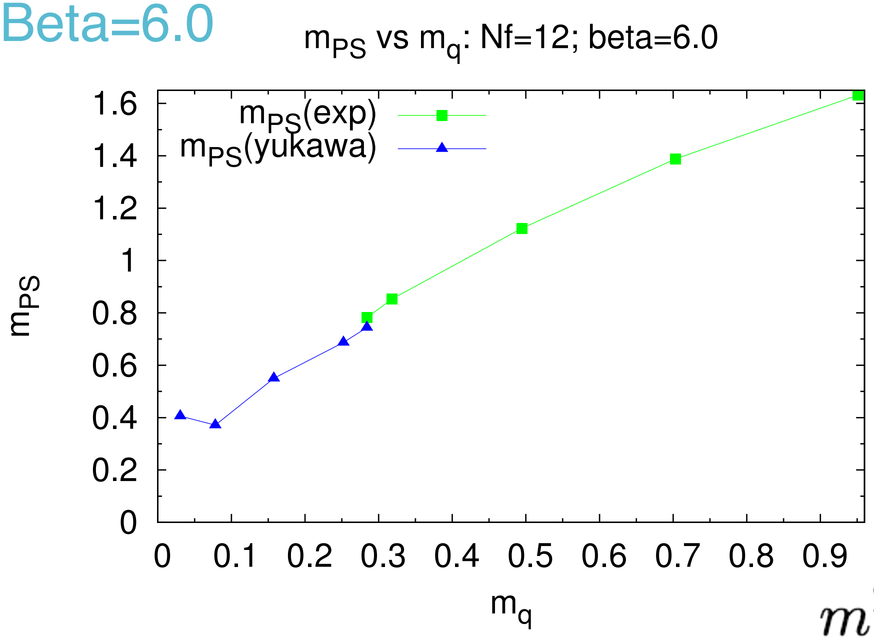
Beta=8.0

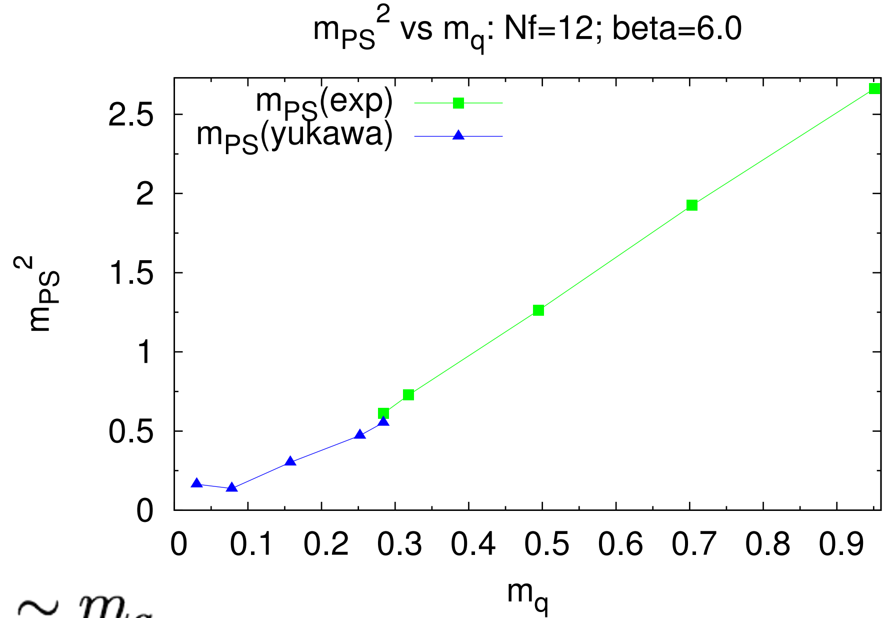




m_{PS} vs. m_q

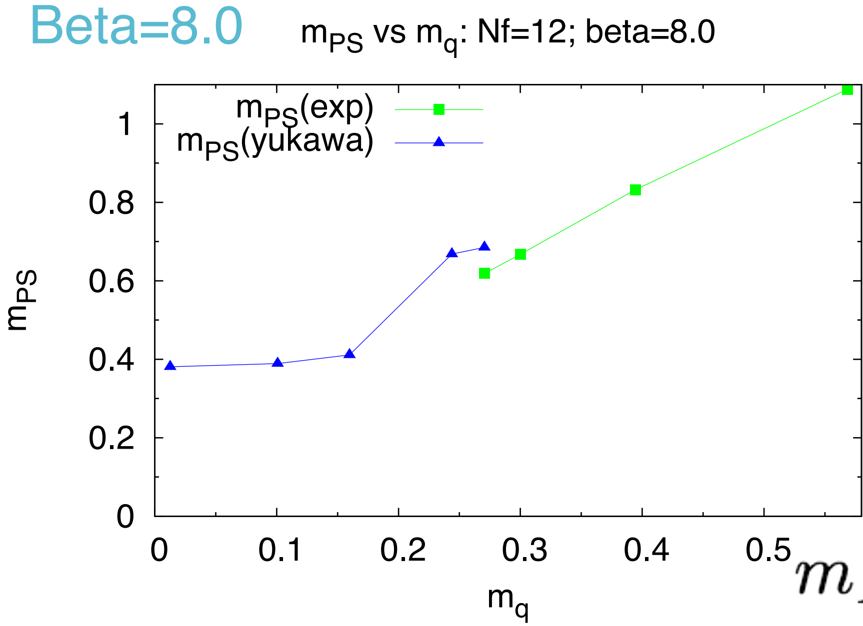
Beta=6.0

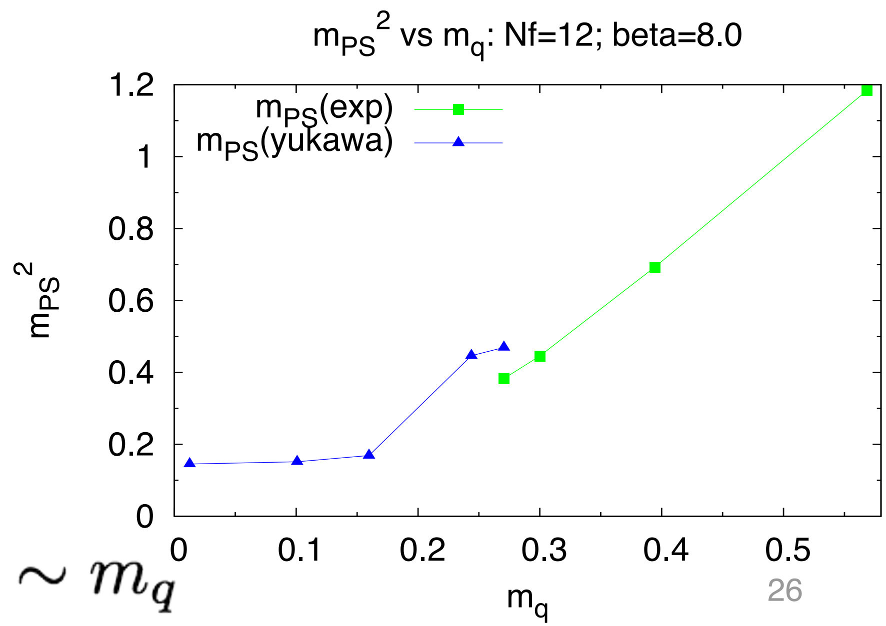




$$m_{PS}^2 \sim m_q$$

Beta=8.0





$$m_{PS} \sim m_q$$

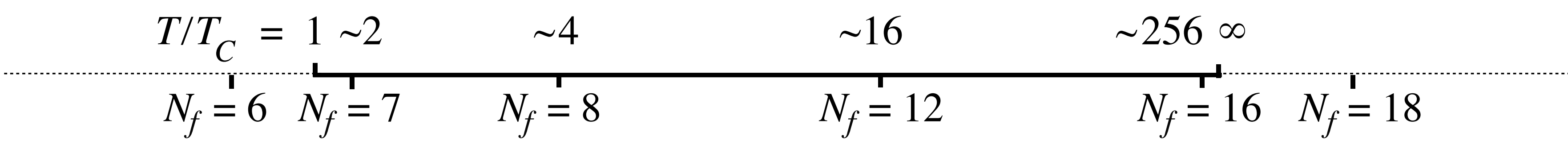
Cautious remark

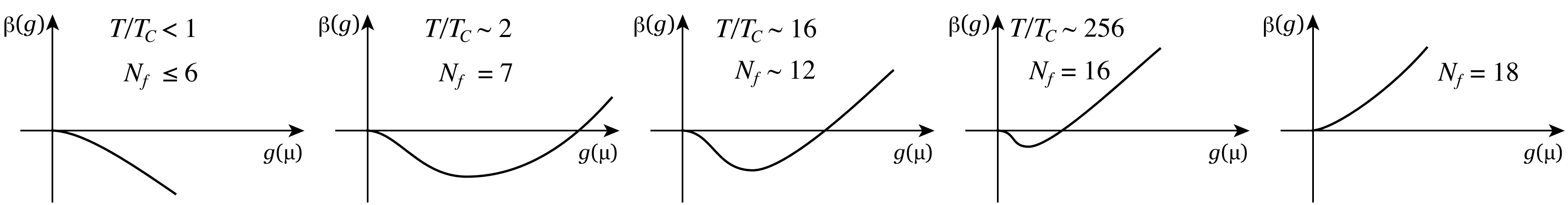
The mq dependence of m_{PS} outside the conformal region is determined by the lattice size and the beta

It is irrelevant to the conformal behavior

In order to obtain conformal properties, one should be inside the conformal region

The correspondence between Conformal QCD and high temperature QCD

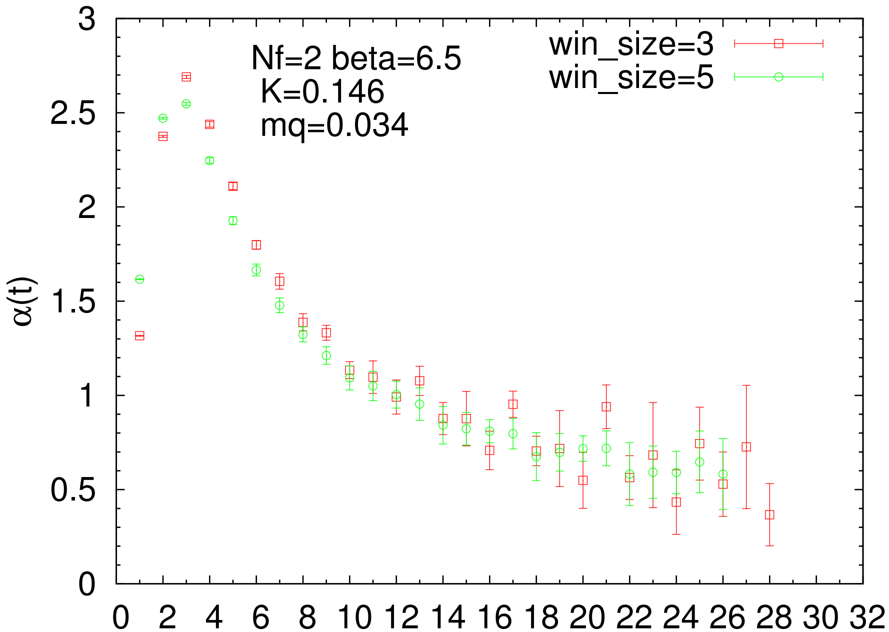


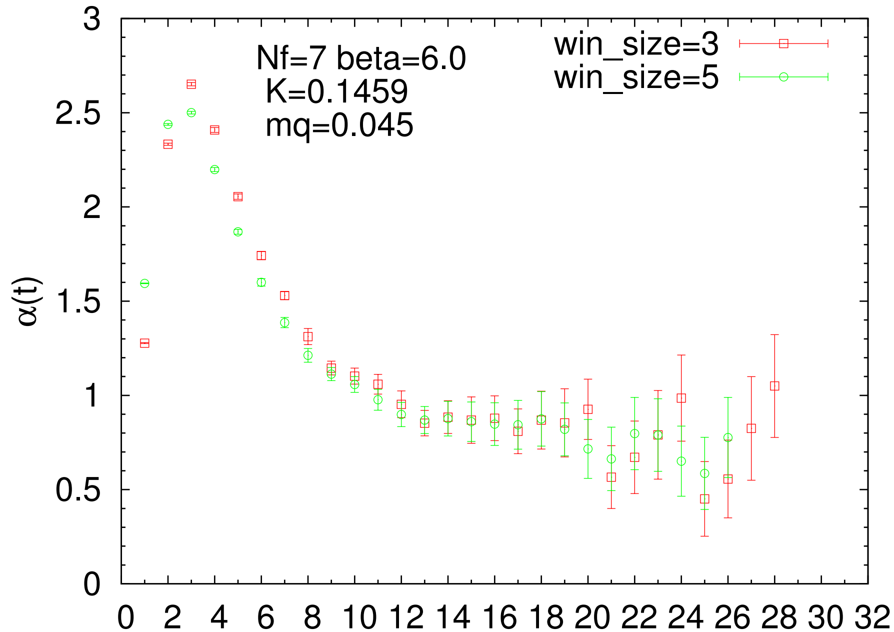


Local analysis of propagators

Parametrize

$$G(t) = c\,\frac{\exp(-m(t)\,t)}{t^{\,\alpha(t)}}$$





Nf=2;T/Tc ~ 2 <--> Nf=7

Anomalous mass dimension for Nf=7

plateau in $\alpha(t)$ at t=16 - 31

$$\alpha(t) \simeq 0.8(1)$$

$$\gamma^* = 2 - \alpha$$

$$\gamma^* \simeq 1.2(1)$$

derivation:

$$\langle O(p)O(-p)\rangle = \frac{1}{(p^2+m^2)^{2-\Delta}} \qquad \Delta = 3 - \gamma_m$$

fourier transform

$$\frac{e^{-mt}}{t^{\Delta-1}} \qquad \Delta - 1 = 2 - \gamma_m$$

Implications for physics

hint for models for BSM

$\gamma^* \simeq 1.2(1)$ for Nf=7

possible solutions for long standing issues in high temperature QCD

the order of the chiral transition for Nf=2

the slow approach of the free energy to the
Stefan-Boltzmann ideal gas limit.

“meson state” above T>Tc

Summary

We have verified

- The existence of the conformal region in Conformal QCD (Nf=7, 12, 16) and High temperature QCD (Nf=2) on a 16^3 x 64 lattice
- The vacuum in the conformal region is a modified Z(3) twisted vacuum
- The boundary between the conformal region and the confining (deconfining) region is first order transition
- The mq dependence of m_{PS} outside the conformal region is irrelevant to the conformal behavior

Summary (Cont.)

- The correspondence between Conformal QCD and High temperature QCD (Nf=2) on a 16^3 x 64 lattice
- Implications for physics

Thank you very much

BACKUP

Structure of the vacuum

Effective potential in the one-loop approximation

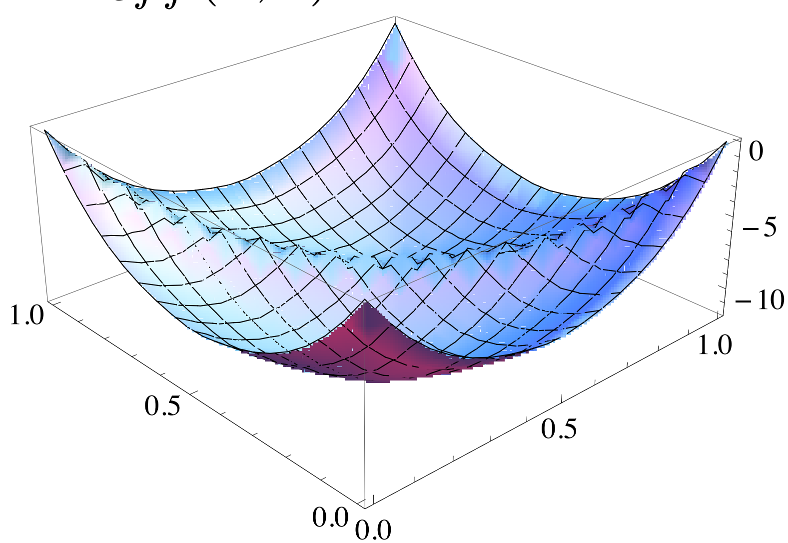
Parametrize the loop of link variables in spatial directions

$$U = \mathrm{diag}(e^{i2\pi a}, e^{i2\pi b}, e^{-i2\pi(a+b)})$$

x, y, z directions

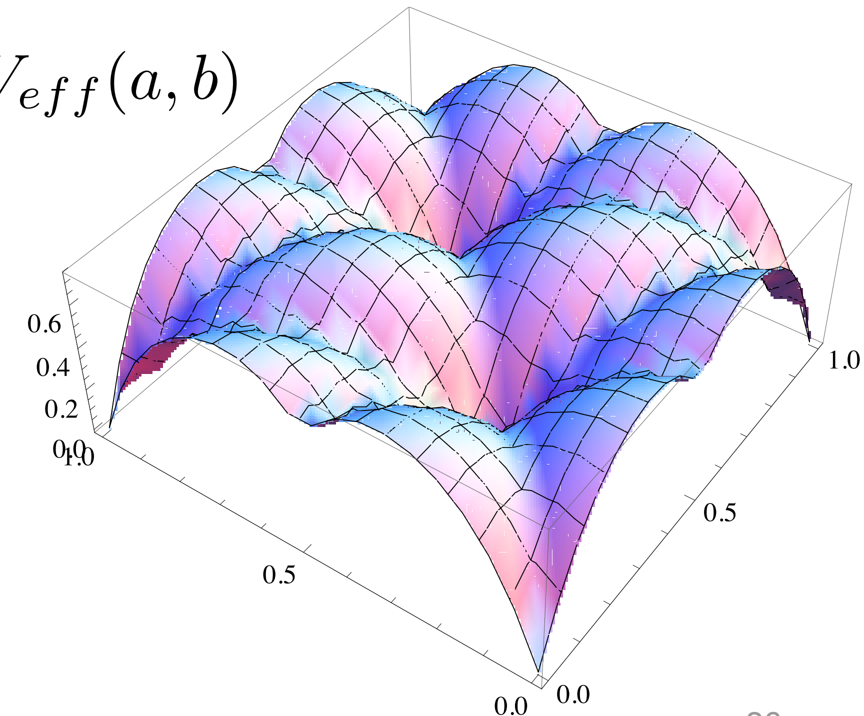
Nf=16, mq=0.000

$V_{eff}(a,b)$



Nf=16, mq=1.0

$V_{eff}(a,b)$



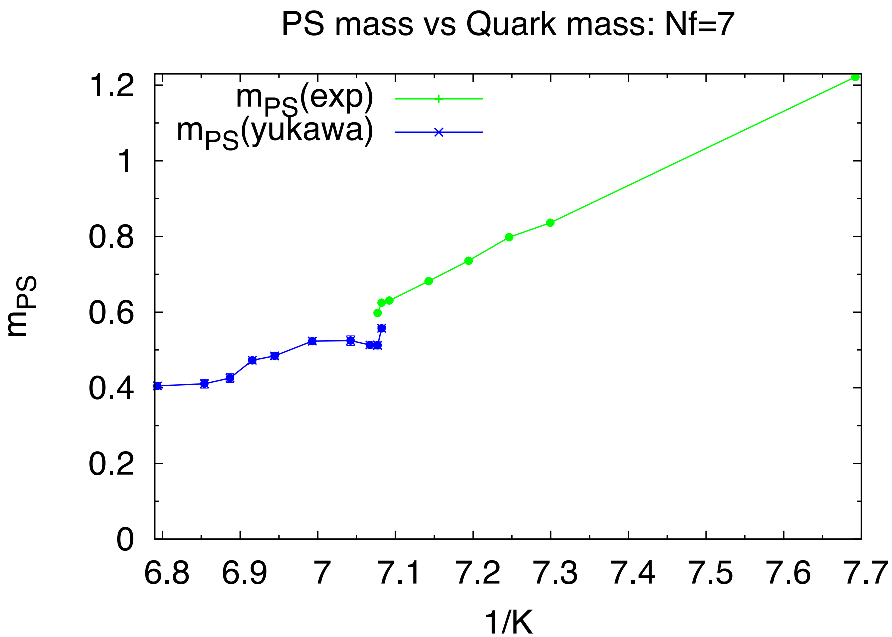
Z(3) twisted vacuum

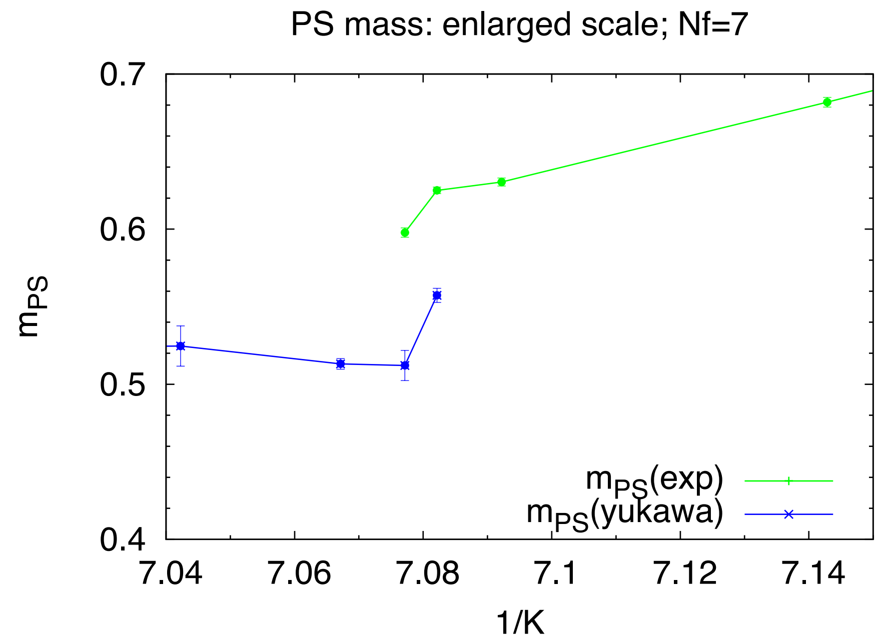
Polyakov loops in spatial directions: Px, Py, Pz

Lowest energy state $a = b = 1/3$ and $= 2/3 (= -1/3)$

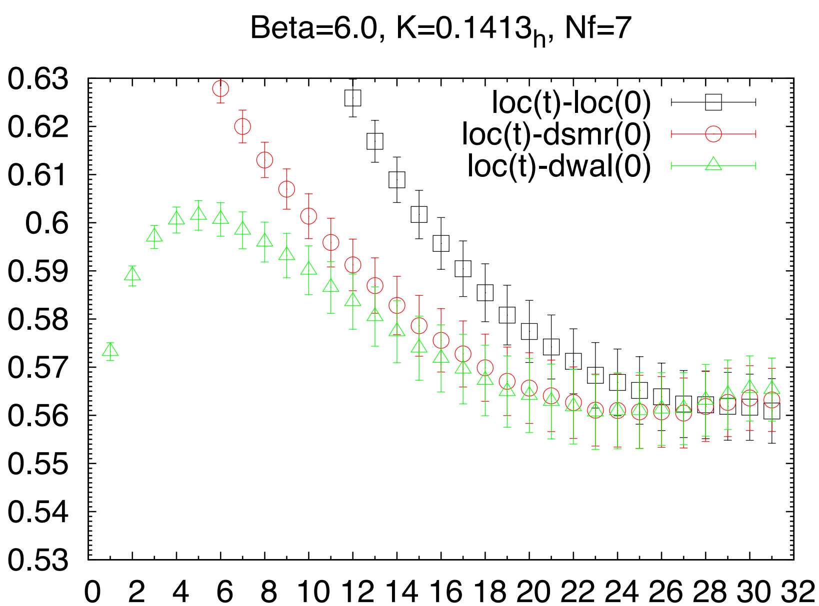
$$P_x, P_y, P_z = \exp\left(\pm i\, 2\pi/3\right)$$

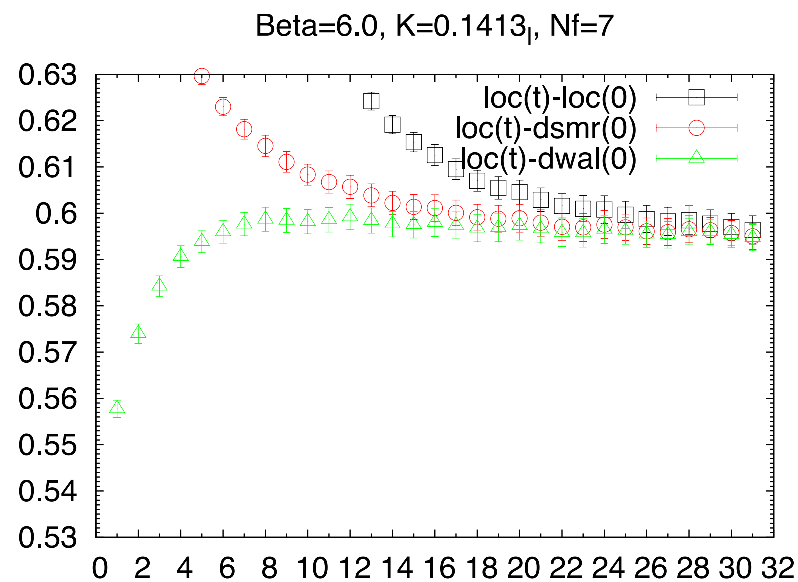
N_f=7: beta = 6.0: Two states at K=0.1413





N_f=7: beta = 6.0: effective masses at K=0.1413

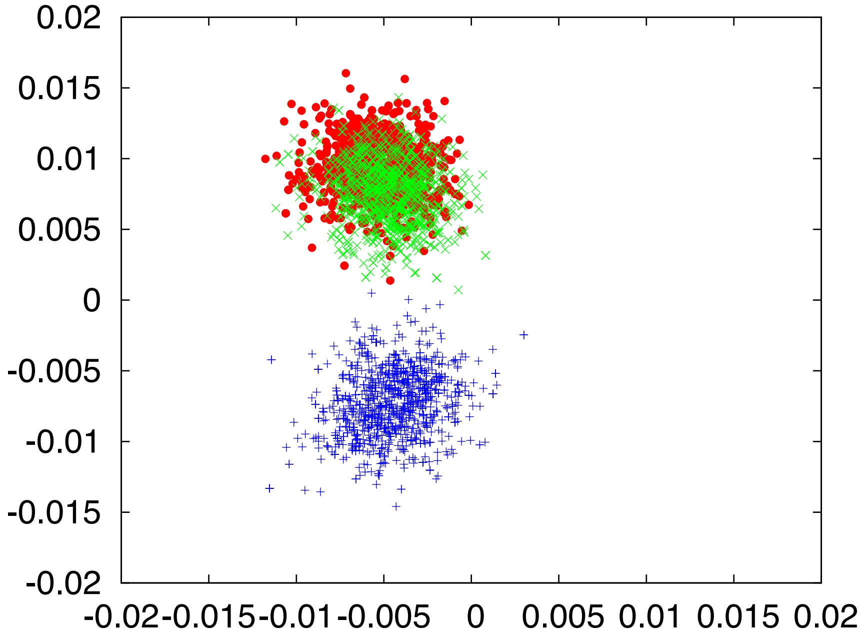


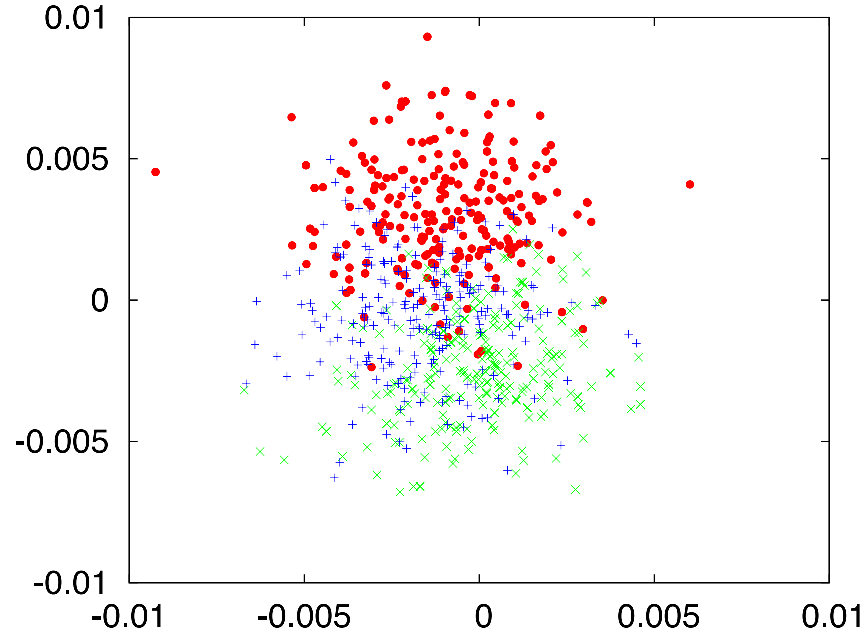


Yukawa-type decay
m→0.56

Exponential-type decay
m→0.59

Polyakov loops : Nf=7

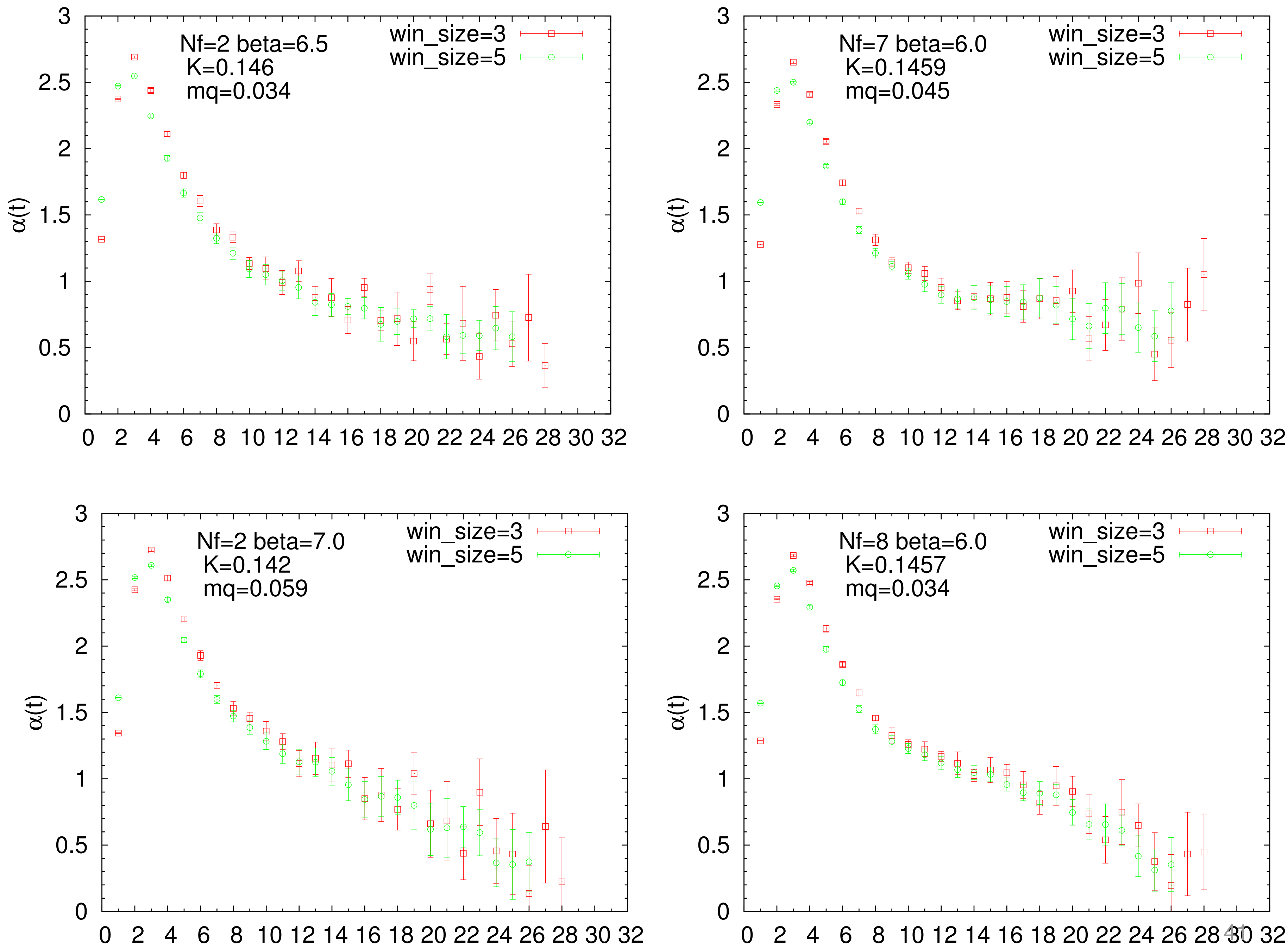




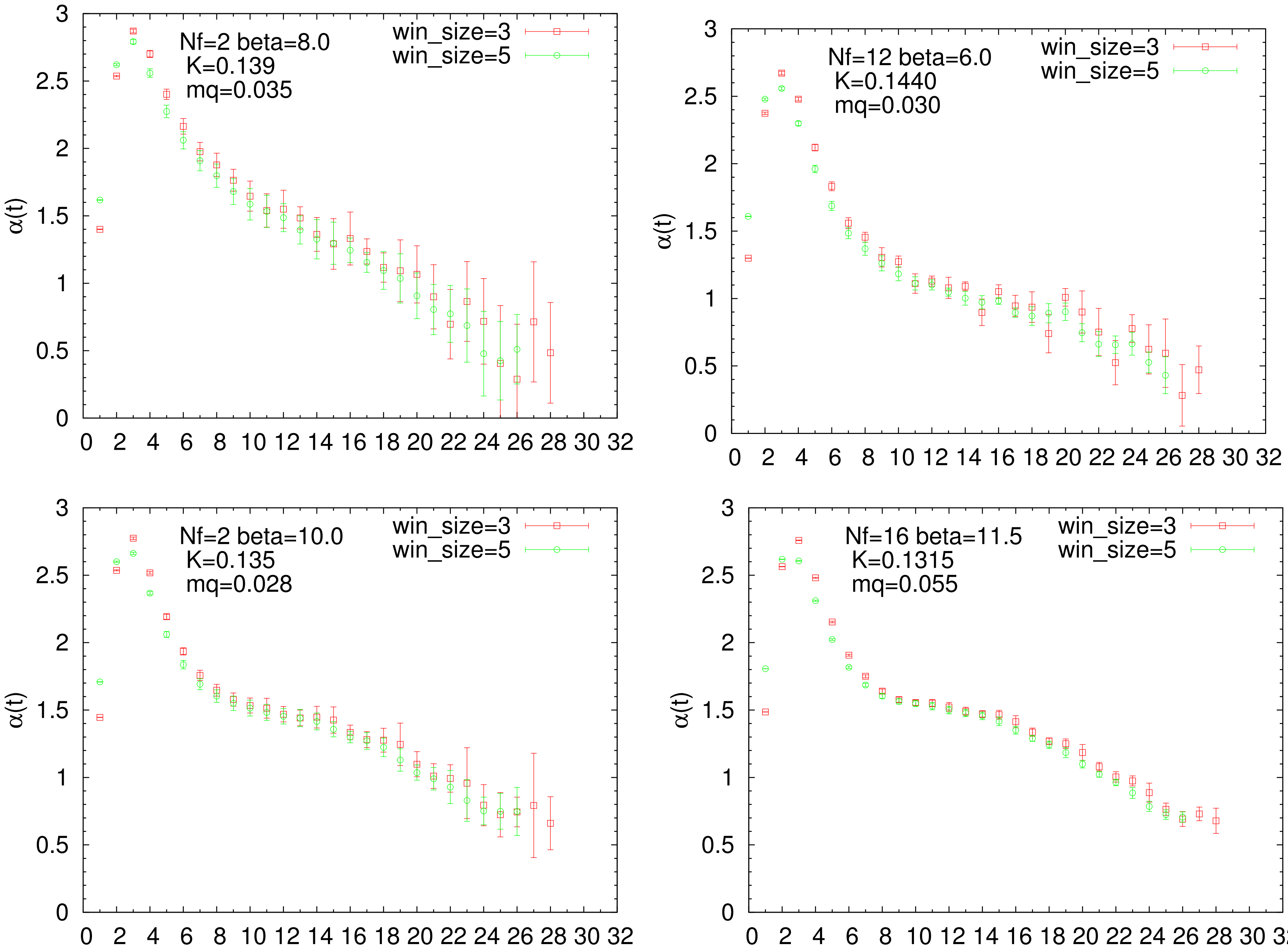
Inside the conformal region

$$P_x, P_y, P_z \simeq 0.01 \exp\left(\pm i\, 2\pi/3\right)$$

Outside the conformal region
Typical in the confining region



Nf=2 beta=6.5
K=0.146
mq=0.034
win_size=3
win_size=5
α(t)
Nf=7 beta=6.0
K=0.1459
mq=0.045
win_size=3
win_size=5
α(t)
Nf=2 beta=7.0
K=0.142
mq=0.059
win_size=3
win_size=5
α(t)
Nf=8 beta=6.0
K=0.1457
mq=0.034
win_size=3
win_size=5
α(t)



Nf=2 beta=8.0
K=0.139
mq=0.035
win_size=3
win_size=5
α(t)
Nf=12 beta=6.0
K=0.1440
mq=0.030
win_size=3
win_size=5
α(t)
Nf=2 beta=10.0
K=0.135
mq=0.028
win_size=3
win_size=5
α(t)
Nf=16 beta=11.5
K=0.1315
mq=0.055
win_size=3
win_size=5
α(t)