Dark energy cosmology in $F(T)$ gravity

PLB 725, 368 (2013) [arXiv:1304.6191 [gr-qc]]

Kobayashi-Maskawa Institute for the Origin of Particles and the Universe

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Contents

I. Introduction

Research achievements after arriving at KMI

Collaborations with students

(1) Dark energy models

- **Curvature perturbations in *k*-essence models**

[KB, Matsumoto and Nojiri, PRD 85, 084026 (2012)]

- **Generalization of Galileon models**

[Shirai, KB, Kumekawa, Matsumoto and Nojiri, PRD 86, 043006 (2012)]

- **Scalar field theories with domain wall solutions**

[Toyozato, KB and Nojiri, PRD 87, 063008 (2013)]

(2) Modified gravity theories

- **A dark energy model of the hybrid symmetron leading to the spontaneous symmetry breaking in the universe**

[KB, Gannouji, Kamijo, Nojiri and Sami, JCAP 1307, 017 (2013)]

- **Cosmology and stability in scalar-tensor bigravity**

[KB, Kokusho, Nojiri and Shirai, arXiv:1310.1460 [hep-th]]

*** Other topic: Generation of large-scale magnetic fields from inflation**

Motivation and Subject

- **To investigate theoretical features as well as cosmology of modified gravity theories.**

- **Extended teleparallel gravity ($F(T)$ gravity)**

$F(T)$: Arbitrary function of the torsion scalar T

- **To explore the 4-dim. effective $F(T)$ gravity originating from the 5-dim. Randall-Sundrum (RS) model.**

II. $F(T)$ gravity

Teleparallel gravity

- $g_{\mu\nu} = \eta_{AB} e^A_\mu e^B_\nu$

η_{AB} : Minkowski metric

$e_A(x^\mu)$: Orthonormal tetrad components

- **Torsion tensor**

$$T^\rho_{\mu\nu} \equiv \Gamma^{\rho(W)}_{\mu\nu} - \Gamma^{\rho(W)}_{\nu\mu} = e^\rho_A \left(\partial_\mu e^A_\nu - \partial_\nu e^A_\mu\right)$$

$\Gamma^{\rho(W)}_{\mu\nu} \equiv e^\rho_A \partial_\mu e^A_\nu$: Weitzenböck connection

* μ and ν are coordinate indices on the manifold and also run over 0, 1, 2, 3, and $e_A(x^\mu)$ forms the tangent vector of the manifold.

* An index A runs over 0, 1, 2, 3 for the tangent space at each point x^μ of the manifold.

Torsion scalar

$$T \equiv S_{\rho}{}^{\mu\nu} T^{\rho}{}_{\mu\nu}$$

$$S_{\rho}{}^{\mu\nu} \equiv \frac{1}{2}\left(K^{\mu\nu}{}_{\rho} + \delta^{\mu}_{\rho}\, T^{\alpha\nu}{}_{\alpha} - \delta^{\nu}_{\rho}\, T^{\alpha\mu}{}_{\alpha}\right)$$

$$K^{\mu\nu}{}_{\rho} \equiv -\frac{1}{2}\left(T^{\mu\nu}{}_{\rho} - T^{\nu\mu}{}_{\rho} - T_{\rho}{}^{\mu\nu}\right)$$

[Hehl, Von Der Heyde, Kerlick and Nester, Rev. Mod. Phys. <u>48</u>, 393 (1976)]

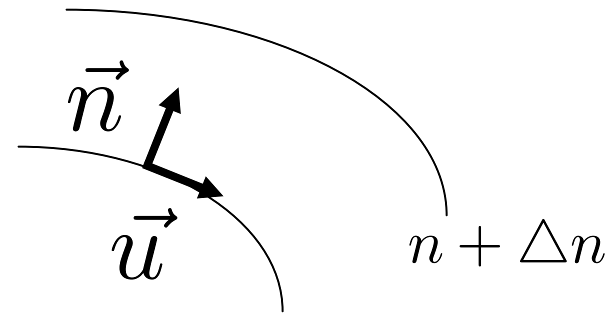
[Hayashi and Shirafuji, PRD <u>19</u>, 3529 (1979) [Addendum-ibid. D <u>24</u>, 3312 (1981)]]

Why teleparallel gravity?

- **General relativity**

  (with only curvature)

  Trajectories are determined by geodesics: $\vec{\nabla}_{\vec{u}}\vec{u} = 0$.



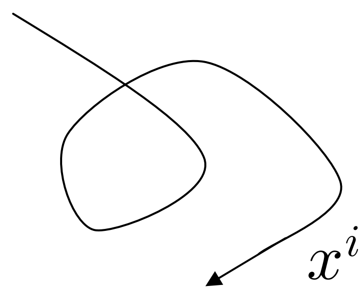
n : Selector parameter

From [Misner, Thorne and Wheeler, *Gravitation* (Friemann, New York, 1973)].

- **Teleparallel gravity**

  (with only torsion)

  Torsion acts as a force.



Coordinates (x^i) are twisted.

⇨ **Curvature and torsion represent the same gravitational field.**

[Aldrovandi and Pereira, *Teleparallel Gravity: An Introduction* (Springer, Dordrecht, 2012); http://www.ift.unesp.br/users/jpereira/tele.pdf]

Extended teleparallel gravity

Action

$$S = \int d^4x |e| \left(\underline{\frac{F(T)}{2\kappa^2}} + \mathcal{L}_{\mathrm{M}} \right)$$: ***F*(*T*) gravity**

Cf. $F(T) = T$: **Teleparallelism**

$$|e| = \det \left(e^A_\mu \right) = \sqrt{-g}$$

$\mathcal{L}_{\mathrm{M}}$: Matter Lagrangian

$$\kappa^2 \equiv 8\pi / {M_{\mathrm{Pl}}}^2$$

${T^{(\mathrm{M})}}_\rho{}^\nu$: Energy-momentum tensor of matter

M_{Pl} : Planck mass

Gravitational field equation

$$\frac{1}{e}\partial_\mu \left(e S_A{}^{\mu\nu}\right) F' - e_A^\lambda T^\rho{}_{\mu\lambda} S_\rho{}^{\nu\mu} F' + S_A{}^{\mu\nu} \partial_\mu \left(T\right) F'' + \frac{1}{4} e_A^\nu F = \frac{\kappa^2}{2} e_A^\rho T^{(\mathrm{M})}{}_\rho{}^\nu$$

∗ A prime denotes a derivative with respect to T .

[Bengochea and Ferraro, PRD <u>79</u>, 124019 (2009)]

- **Gravitational field equation in *F*(*T*) gravity is the 2nd order, while it is the 4th order in *F*(*R*) gravity.**

III. From the Randall-Sundrum (RS) model

The RS type-I and II models

- RS I model

A positive (Negative) tension brane exists at $y = 0$ $(y = \tilde{s})$.

y : 5th direction

$$ds^2 = \underline{e^{-2|y|/l}} g_{\mu\nu}(x) dx^\mu dx^\nu + dy^2, \qquad l = \sqrt{-6/\Lambda_5}$$

Warp factor $b(y)$

$\Lambda_5 (< 0)$: Negative cosmological constant in the bulk

$\Downarrow$ $\tilde{s} \to \infty$

- RS II model

There is a positive tension brane in the anti-de Sitter bulk space.

[Randall and Sundrum, PRL 83, 3370 (1999); 4690 (1999)]

Cf. [Garriga and Tanaka, PRL 84, 2778 (2000)]

Procedures in the RS II model

Pioneering work

[Shiromizu, Maeda and Sasaki, PRD 62, 024012 (2000)]

Application to teleparallel gravity

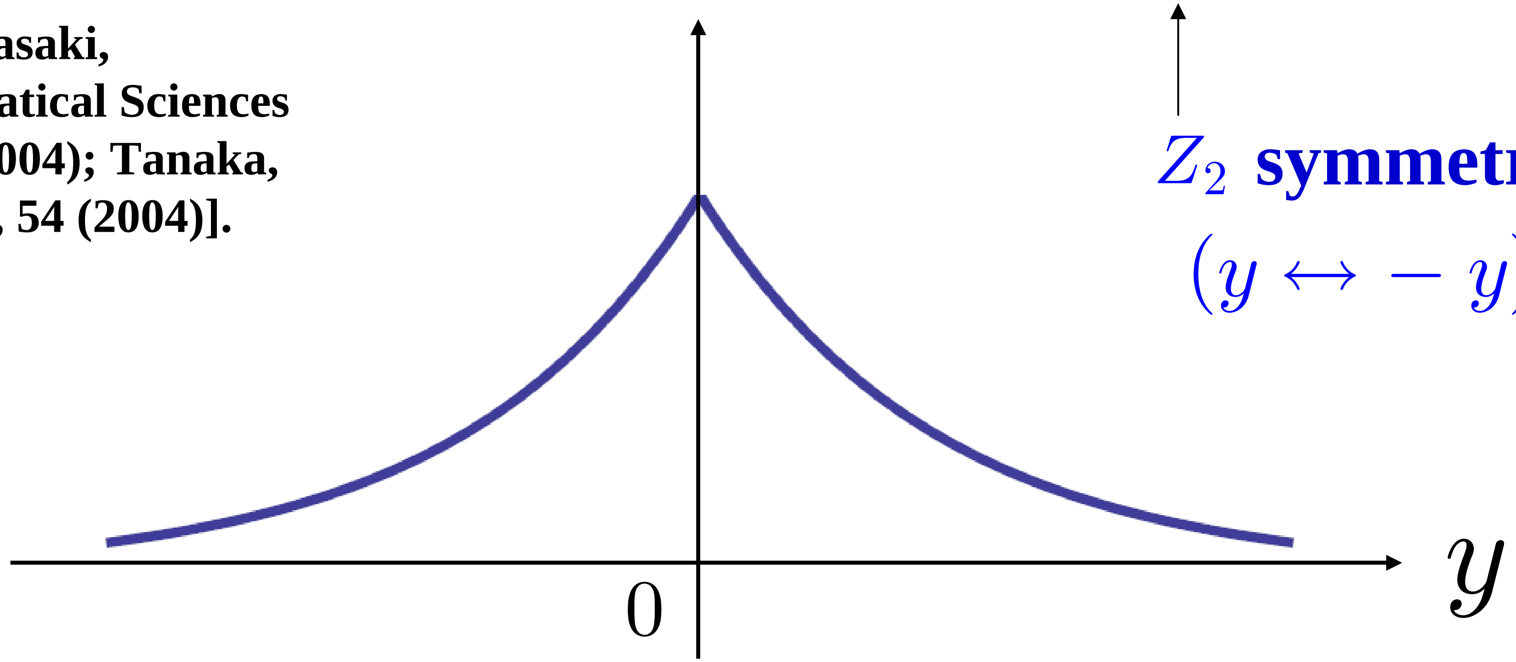
[Nozari, Behboodi and Akhshabi, PLB 723, 201 (2013)]

Warp factor $b(y) \equiv \exp(-2\,\underline{|y|}\,/l)$

From [Sasaki, Mathematical Sciences <u>487</u>, 5 (2004); Tanaka, *ibid.* <u>487</u>, 54 (2004)].

Z_2 **symmetry**

$(y \leftrightarrow -y)$



Left-side bulk **Brane at** $y = 0$ **Right-side bulk**

Induced (Gauss-Codazzi) equations on the brane

Israel's junction conditions

- **For the flat FLRW space-time with the metric:**

$$ds^2 = dt^2 - a^2(t) \sum_{i=1,2,3} \left(dx^i\right)^2$$

$$\Rightarrow \quad \underline{T = -6H^2}$$

$H \equiv \dfrac{\dot{a}}{a}$: Hubble parameter

* The dot denotes the time derivative of $\partial/\partial t$.

Cosmology in the flat FLRW space-time

Friedmann equation on the brane

$$H^2 \frac{dF(T)}{dT} = -\frac{1}{12}\left[F(T) \underline{- 4\Lambda} - 2\kappa^2 \rho_{\mathrm{M}} \underline{- \left(\frac{\kappa_5^2}{2}\right)^2 \mathcal{Q}\rho_{\mathrm{M}}^2}\right]$$

$$\mathcal{Q} \equiv \left(11 - 60w_{\mathrm{M}} + 93w_{\mathrm{M}}^2\right)/4 \longleftarrow$$

includes contributions from teleparallelism.

$$w_{\mathrm{M}} \equiv P_{\mathrm{M}}/\rho_{\mathrm{M}}$$

$$\Lambda \equiv \Lambda_5 + \left(\kappa_5^2/2\right)^2 \lambda^2$$: Eeffective cosmological constant on the brane

$$\lambda(> 0)$$: Tension of the brane

$$G = \left[1/\left(3\pi\right)\right]\left(\kappa_5^2/2\right)^2 \lambda$$

Example

$$F(T) = T - 2\Lambda_5 \qquad \text{(with } w_{\mathrm{M}} = 0 \text{)}$$

$$\longrightarrow H = H_{\mathrm{DE}} = \sqrt{\Lambda_5 + \kappa_5^4 \lambda^2/6} = \text{constant}$$

$$a(t) = a_{\mathrm{DE}} \exp\left(H_{\mathrm{DE}} t\right), \qquad a_{\mathrm{DE}} (> 0)$$

$\Rightarrow$ **A de Sitter solution on the brane can be realized.**

IV. Summary

- **4-dim. effective *F*(*T*) gravity coming from the 5-dim. RS space-time theories have been studied.**

- **For the RS II model, the contribution of *F*(*T*) gravity appears on the 4-dim. FLRW brane.**

- **The dark energy dominated stage can be realized in the RS II model.**

Further results

- **With the Kaluza-Klein (KK) reduction, the 4-dim. effective $F(T)$ gravity theory coupling to a scalar field has been built.**

  $\longrightarrow$ **Inflation can be realized in the KK theory.**

- **The dark energy dominated stage can be realized in the RS II model with $F(T)$ consisting of T^2 plus a cosmological constant.**

Backup Slides

Case (2)

$$F(T) = T^2/\bar{M}^2 + \alpha\Lambda_5$$

$\bar{M}$: Mass scale

α : Constant

$$\longrightarrow \quad H = H_{\mathrm{DE}} = \left[\left(\bar{M}^2/108\right)\mathcal{J}\right]^{1/4} = \mathrm{constant}$$

$$\mathcal{J} \equiv (\alpha - 4)\,\Lambda_5 - 4\left(\kappa_5^2/2\right)^2 \lambda^2$$

$$a(t) = a_{\mathrm{DE}} \exp\left(H_{\mathrm{DE}} t\right), \qquad a_{\mathrm{DE}} (> 0)$$

⇨ **A de Sitter solution on the brane can exist.**

General relativistic approach

(i) Cosmological constant

(ii) Scalar field :

- **x-matter, Quintessence** ← **Canonical field**

[Chiba, Sugiyama and Nakamura, Mon. Not. Roy. Astron. Soc. 289, L5 (1997)]
[Caldwell, Dave and Steinhardt, Phys. Rev. Lett. 80, 1582 (1998)]
Cf. Pioneering work: [Fujii, Phys. Rev. D 26, 2580 (1982)]

- **Phantom** ← **Wrong sign kinetic term**

[Caldwell, Phys. Lett. B 545, 23 (2002)]

- **K-essence** ← **Non canonical kinetic term**

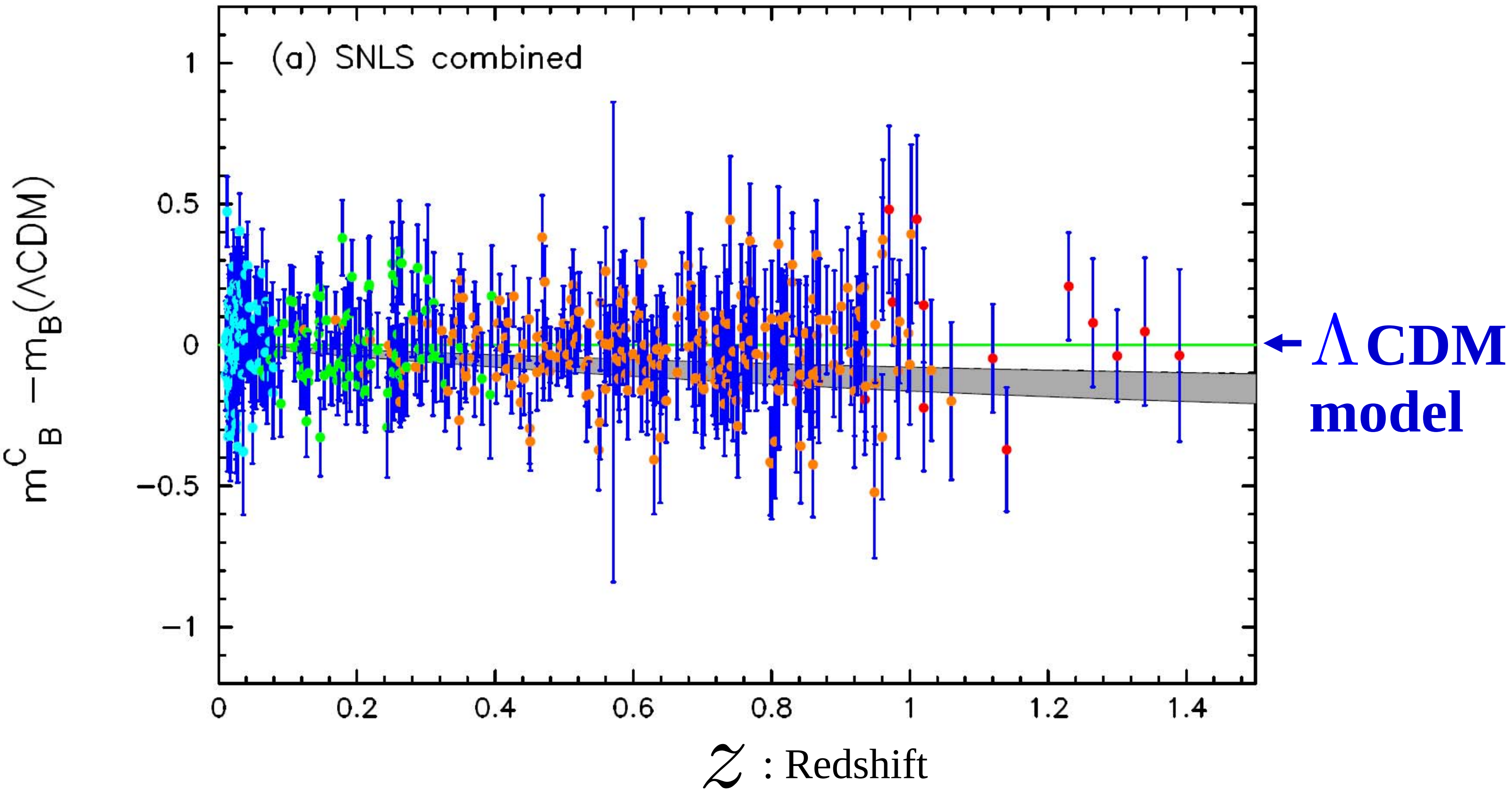
[Chiba, Okabe and Yamaguchi, Phys. Rev. D 62, 023511 (2000)]
[Armendariz-Picon, Mukhanov and Steinhardt, Phys. Rev. Lett. 85, 4438 (2000)]

- **Tachyon** ← **String theories** * **The mass squared is negative.**

[Padmanabhan, Phys. Rev. D 66, 021301 (2002)]

PLANCK 2013 results of SNLS

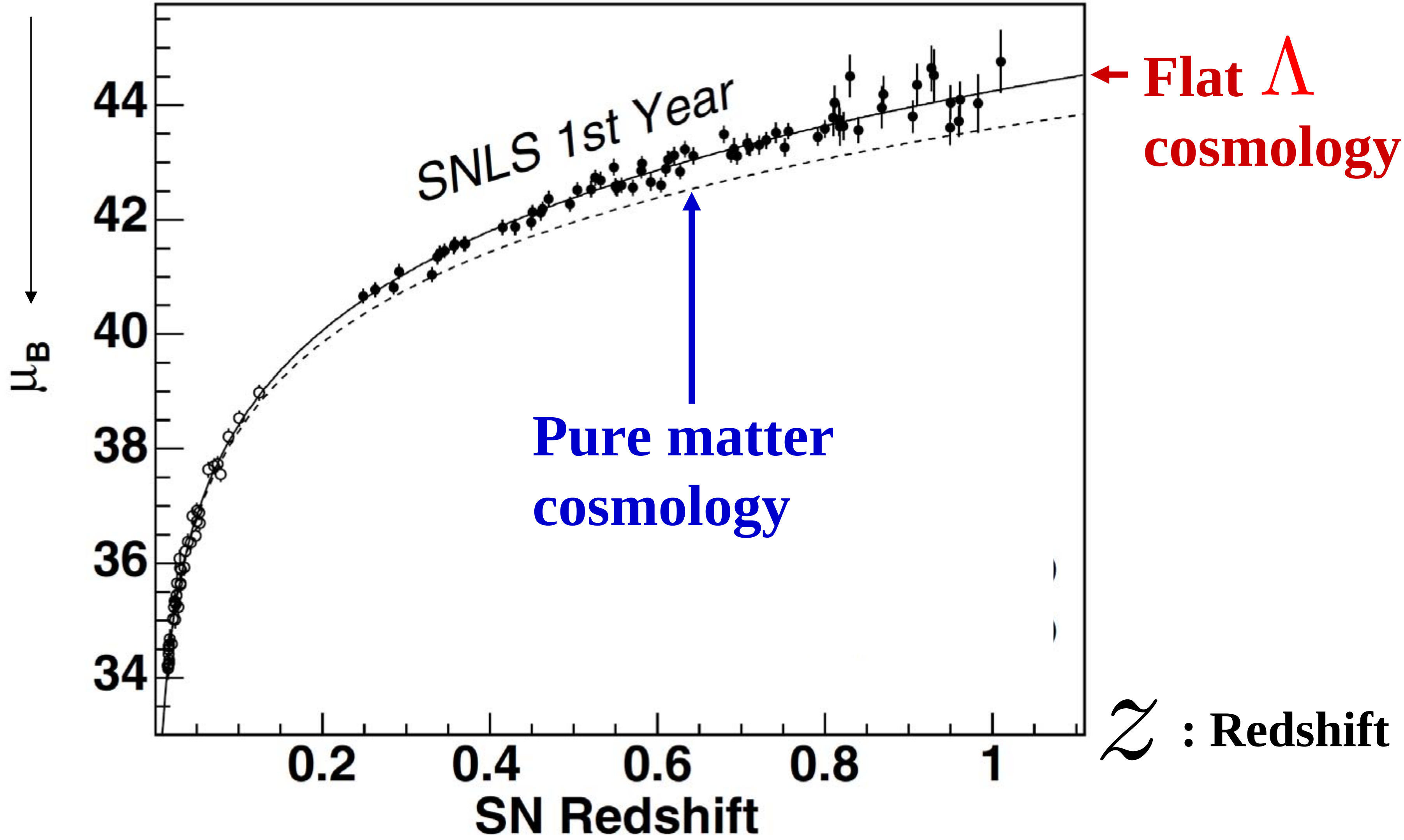
Magnitude residuals of the Λ CDM model that best fits the SNLS combined sample



From [Ade *et al*. [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO]].

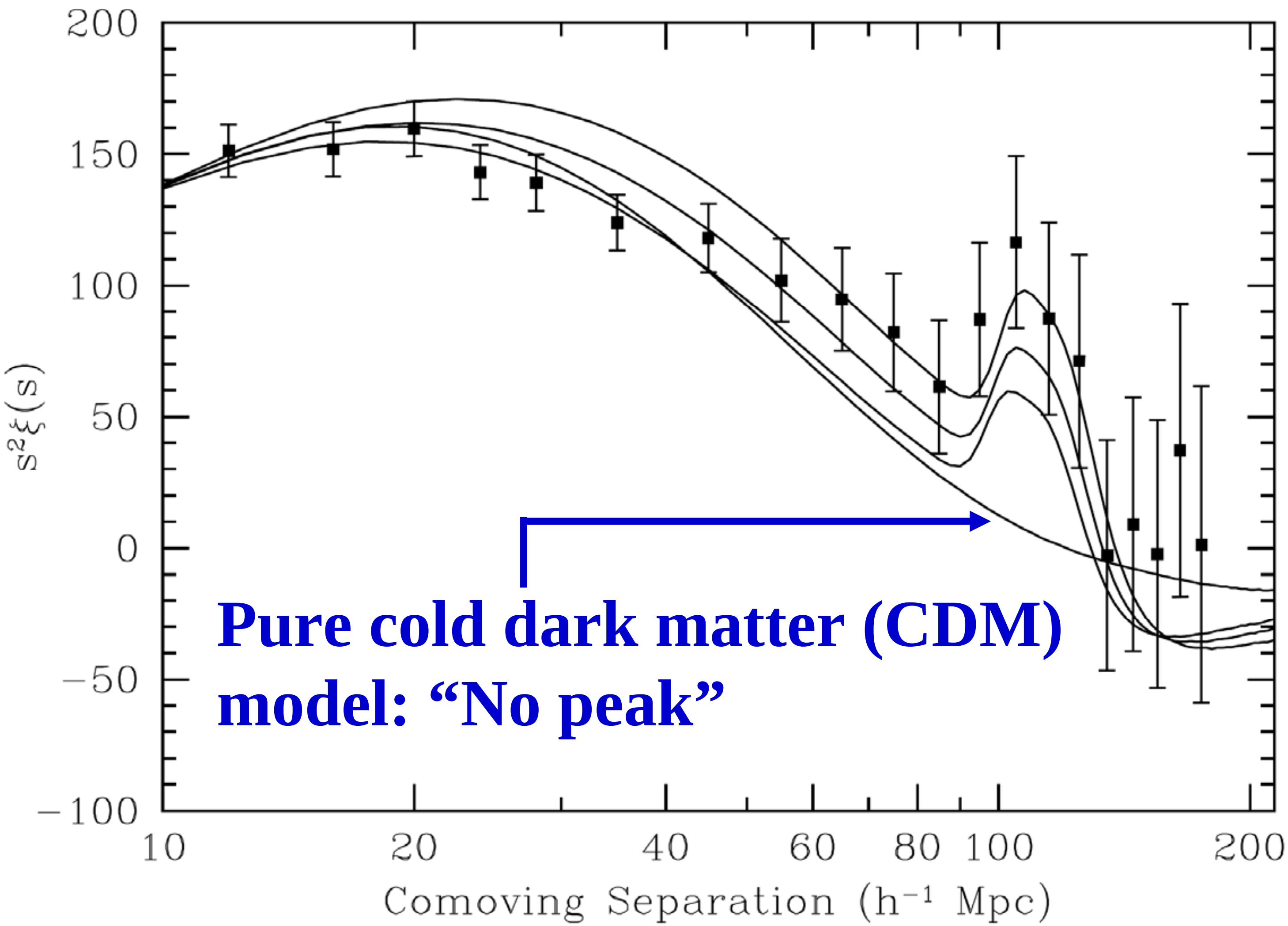
SNLS data

Distance estimator



From [Astier *et al*. [The SNLS Collaboration], Astron. Astrophys. 447, 31 (2006)].

Baryon acoustic oscillation (BAO)



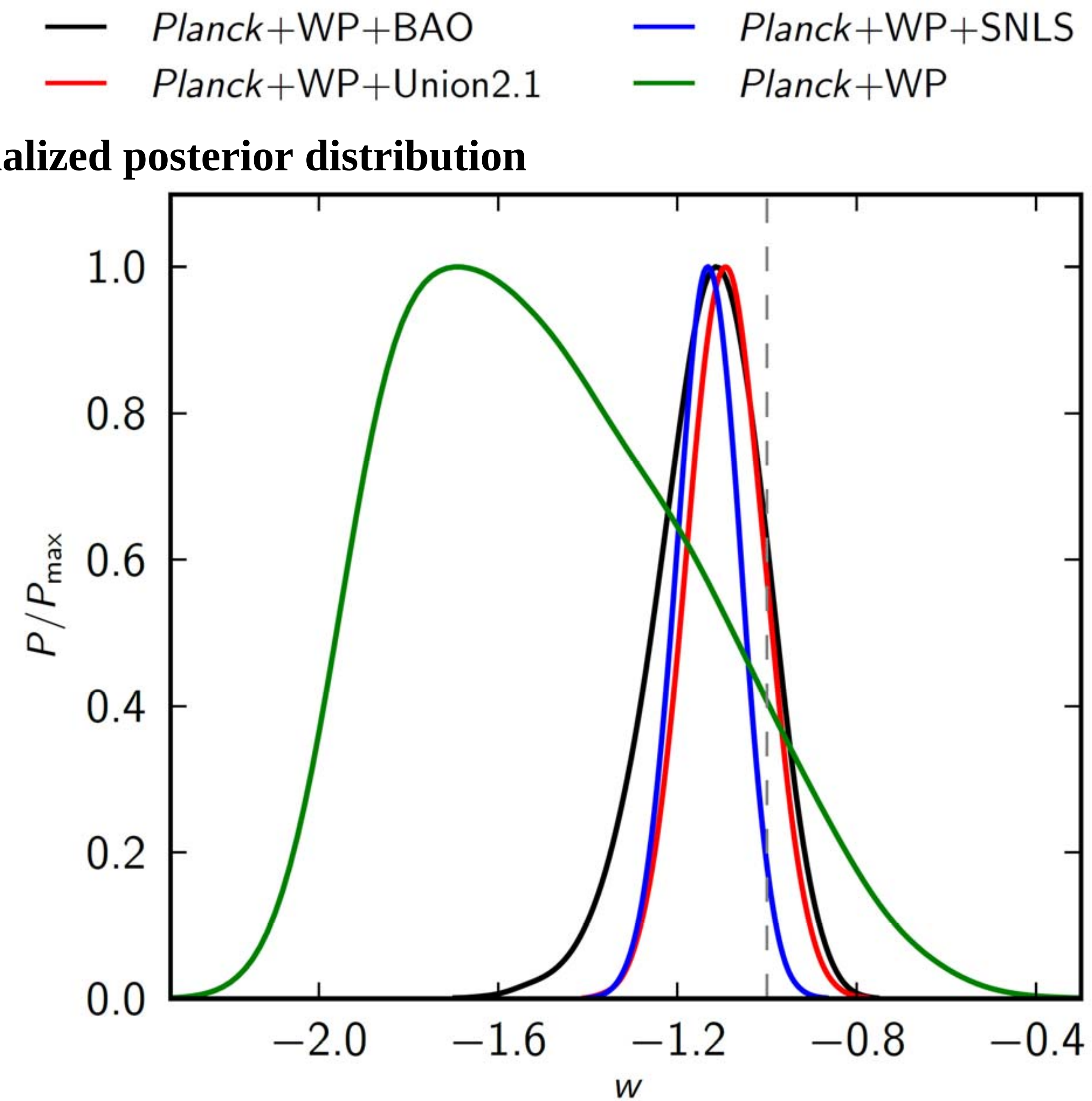
Special pattern in the large-scale correlation function of Sloan Digital Sky Survey (SDSS) luminous red galaxies

From [Eisenstein *et al.* [SDSS Collaboration], Astrophys. J. 633, 560 (2005)].

Cf. [Yamamoto, astro-ph/0110596; Astrophys. J. 595, 577 (2003)]
[Matsubara and Szalay, Phys. Rev. Lett. 90, 021302 (2003)]

PLANCK data for the current w

Marginalized posterior distribution



From [Ade *et al*. [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO]].

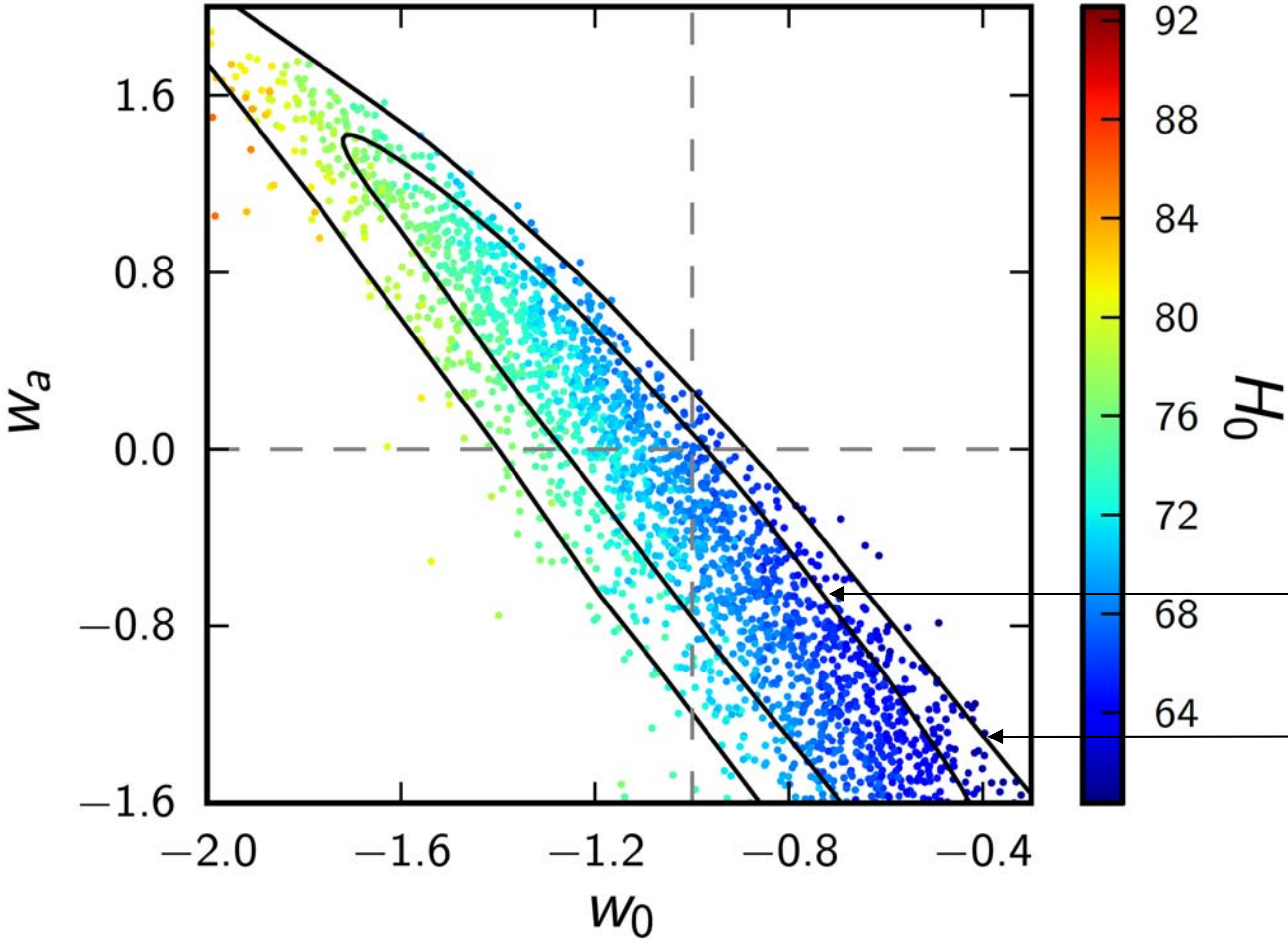
$$w = \text{constant}$$

WP: WMAP

BAO: Baryon Acoustic Oscillation

PLANCK data for the time-dependent w

2D Marginalized posterior distribution



From [Ade *et al.* [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO]].

$$w(a) = w_0 + w_a(1 - a)$$

(68% CL)

(95% CL)

$$w_0 = -1.04^{+0.72}_{-0.69} \quad (95\%; \textit{Planck}+\text{WP}+\text{BAO})$$

$$w_a < 1.32 \quad (95\%; \textit{Planck}+\text{WP}+\text{BAO})$$

9-year WMAP data of current w

[Hinshaw *et al.*, arXiv:1212.5226 [astro-ph.CO]]

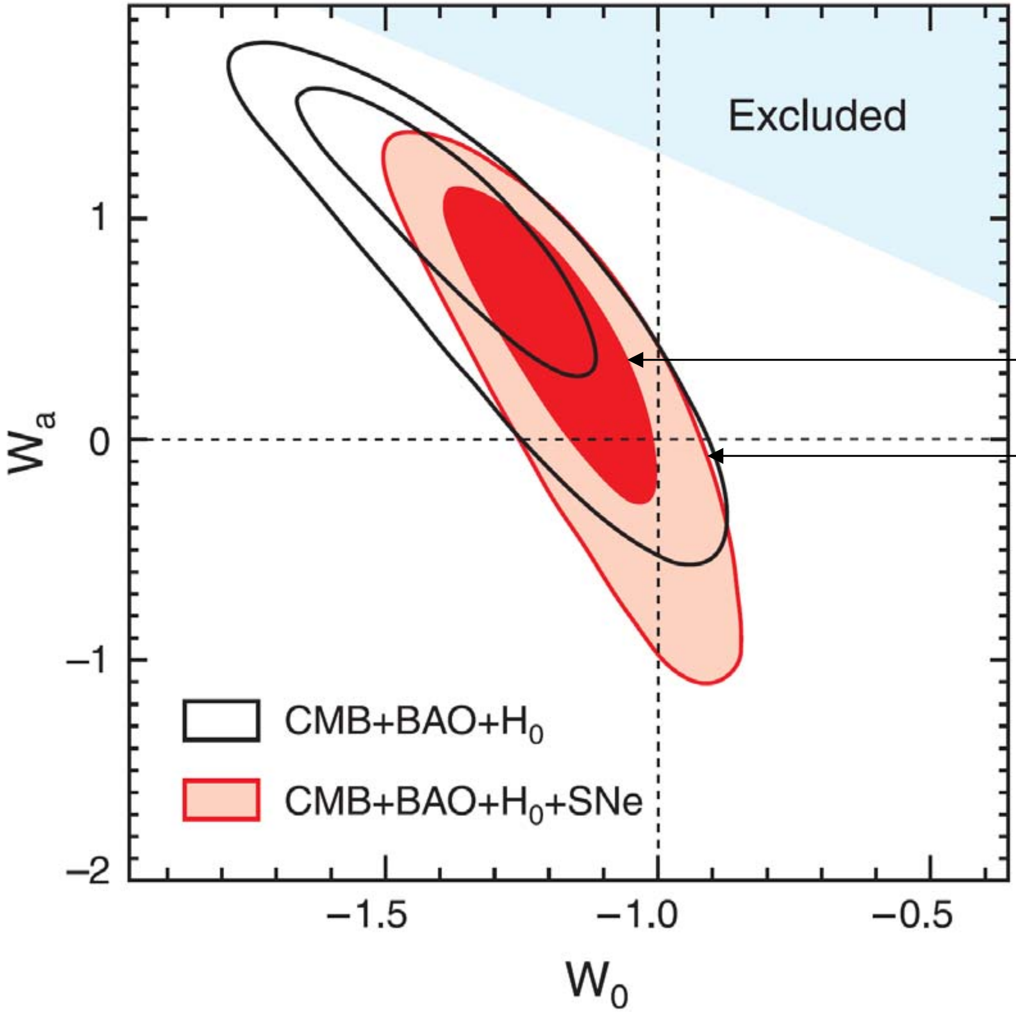
For constant w :

$$w = \begin{cases} -1.084 \pm 0.063 & \text{(flat)} \\ -1.122^{+0.068}_{-0.067} & \text{(non-flat)} \end{cases} \quad \text{(68\% CL)}$$

(From $WMAP$+eCMB+BAO+H_0+SNe.)

* Hubble constant (H_0) measurement

Time-dependent w



$$w(a) = w_0 + w_a(1-a)$$

(68% CL)

(95% CL)

From [Hinshaw *et al.*, arXiv:1212.5226 [astro-ph.CO]].

w_0: Current value of w

(From *WMAP*+eCMB +BAO+H_0+SNe.)

For the flat universe:

$$w_0 = -1.17^{+0.13}_{-0.12}, \quad w_a = 0.35^{+0.50}_{-0.49} \quad (68\% \text{ CL})$$

(iii) Fluid :

- **(Generalized) Chaplygin gas**

Equation of state (EoS): $P = -A/\rho^u$

$A > 0,\ u$: Constants

ρ : Energy density

P : Pressure

[Kamenshchik, Moschella and Pasquier, Phys. Lett. B 511, 265 (2001)] $\longleftarrow$ $(u = 1)$

[Bento, Bertolami and Sen, Phys. Rev. D 66, 043507 (2002)]

Extension of gravitational theory

- ***F*(*R*) gravity** $\longleftarrow$ $F(R)$: Arbitrary function of the Ricci scalar R

Cf. Application to inflation: **[Starobinsky, Phys. Lett. B 91, 99 (1980)]**

[Capozziello, Cardone, Carloni and Troisi, Int. J. Mod. Phys. D 12, 1969 (2003)]

[Carroll, Duvvuri, Trodden and Turner, Phys. Rev. D 70, 043528 (2004)]

[Nojiri and Odintsov, Phys. Rev. D 68, 123512 (2003)]

- **Scalar-tensor theories** $\longleftarrow$ $f_1(\phi)R$

$f_i(\phi)$ $(i = 1, 2)$: Arbitrary function of a scalar field ϕ

[Boisseau, Esposito-Farese, Polarski and Starobinsky, Phys. Rev. Lett. 85, 2236 (2000)]

[Gannouji, Polarski, Ranquet and Starobinsky, JCAP 0609, 016 (2006)]

- **Ghost condensates scenario**

[Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405, 074 (2004)]

- **Higher-order curvature term**

Gauss-Bonnet invariant with a coupling to a scalar field: $f_2(\phi)\mathcal{G}$

$$\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

: Gauss-Bonnet invariant

$R_{\mu\nu}$: Ricci curvature tensor

$R_{\mu\nu\rho\sigma}$: Riemann tensor

[Nojiri, Odintsov and Sasaki, Phys. Rev. D 71, 123509 (2005)]

- $f(\mathcal{G})$ **gravity** $\longleftarrow$ $\dfrac{R}{2\kappa^2} + f(\mathcal{G})$

$$\kappa^2 \equiv 8\pi G$$

G : Gravitational constant

[Nojiri and Odintsov, Phys. Lett. B 631, 1 (2005)]

· **DGP braneworld scenario**

[Dvali, Gabadadze and Porrati, Phys. Lett B 485, 208 (2000)]

[Deffayet, Dvali and Gabadadze, Phys. Rev. D 65, 044023 (2002)]

· **Non-local gravity** $\longleftarrow$ $\frac{1}{2\kappa^2} f\left(\Box^{-1} R\right)$ **: Quantum effects**

[Deser and Woodard, Phys. Rev. Lett. 99, 111301 (2007)]

$\Box$: Covariant d'Alembertian

[Nojiri and Odintsov, Phys. Lett. B 659, 821 (2008)]

· ***F*(*T*) gravity** $\longleftarrow$ **Extended teleparallel Lagrangian described by the torsion scalar T.**

[Bengochea and Ferraro, Phys. Rev. D 79, 124019 (2009)]

[Linder, Phys. Rev. D 81, 127301 (2010) [Erratum-ibid. D 82, 109902 (2010)]]

* "Teleparallelism" : One could use the Weitzenböck connection, which has no curvature but torsion, rather than the curvature defined by the Levi-Civita connection.

[Hayashi and Shirafuji, Phys. Rev. D 19, 3524 (1979) [Addendum-ibid. D 24, 3312 (1982)]]

- **Galileon gravity** ← $\Box\phi\left(\partial^\mu\phi\partial_\mu\phi\right)$

Longitudinal graviton (a branebending mode ϕ)

[Nicolis, Rattazzi and Trincherini, Phys. Rev. D 79, 064036 (2009)]

- **Massive gravity** ← **Graviton with a non-zero mass**

[de Rham and Gabadadze, Phys. Rev. D 82, 044020 (2010)]

[de Rham and Gabadadze and Tolley, Phys. Rev. Lett. 106, 231101 (2011)]

Review: [Hinterbichler, Rev. Mod. Phys. 84, 671 (2012)]

Example of *F*(*T*) gravity model

- **The model contains only one parameter u if one has the value of $\Omega_{\mathrm{m}}^{(0)}$.**

[KB, Geng, Lee and Luo, JCAP 1101, 021 (2011)]

Gravitational field equations

$$H^2 = \frac{\kappa^2}{3}\left(\rho_{\mathrm{M}} + \rho_{\mathrm{DE}}\right)$$

$$\dot{H} = -\frac{\kappa^2}{2}\left(\rho_{\mathrm{M}} + P_{\mathrm{M}} + \rho_{\mathrm{DE}} + P_{\mathrm{DE}}\right)$$

$$\rho_{\mathrm{DE}} = \frac{1}{2\kappa^2}\left(- T - F + 2TF'\right)$$

$$P_{\mathrm{DE}} = -\frac{1}{2\kappa^2}\left[4\left(1 - F' - 2TF''\right)\dot{H} - T - F + 2TF'\right]$$

ρ_{DE} : Dark energy density

P_{DE} : Pressure of dark energy

$\rho_{\mathrm{M}}, \; P_{\mathrm{M}}$

: Energy density and pressure of dark energy

Continuity equation: $\dot{\rho}_{\mathrm{DE}} + 3H\left(\rho_{\mathrm{DE}} + P_{\mathrm{DE}}\right) = 0$

Example of *F*(*T*) gravity model

$$F(T) = T + \gamma \left[T_0 \left(\frac{uT_0}{T} \right)^{-1/2} \ln \left(\frac{uT_0}{T} \right) - T \left(1 - \mathrm{e}^{uT_0/T} \right) \right]$$

[KB, Geng, Lee and Luo, JCAP <u>1101</u>, 021 (2011)]

$$\gamma \equiv \frac{1 - \Omega_{\mathrm{m}}^{(0)}}{2u^{-1/2} + \left[1 - (1 - 2u)\, \mathrm{e}^{u} \right]}$$

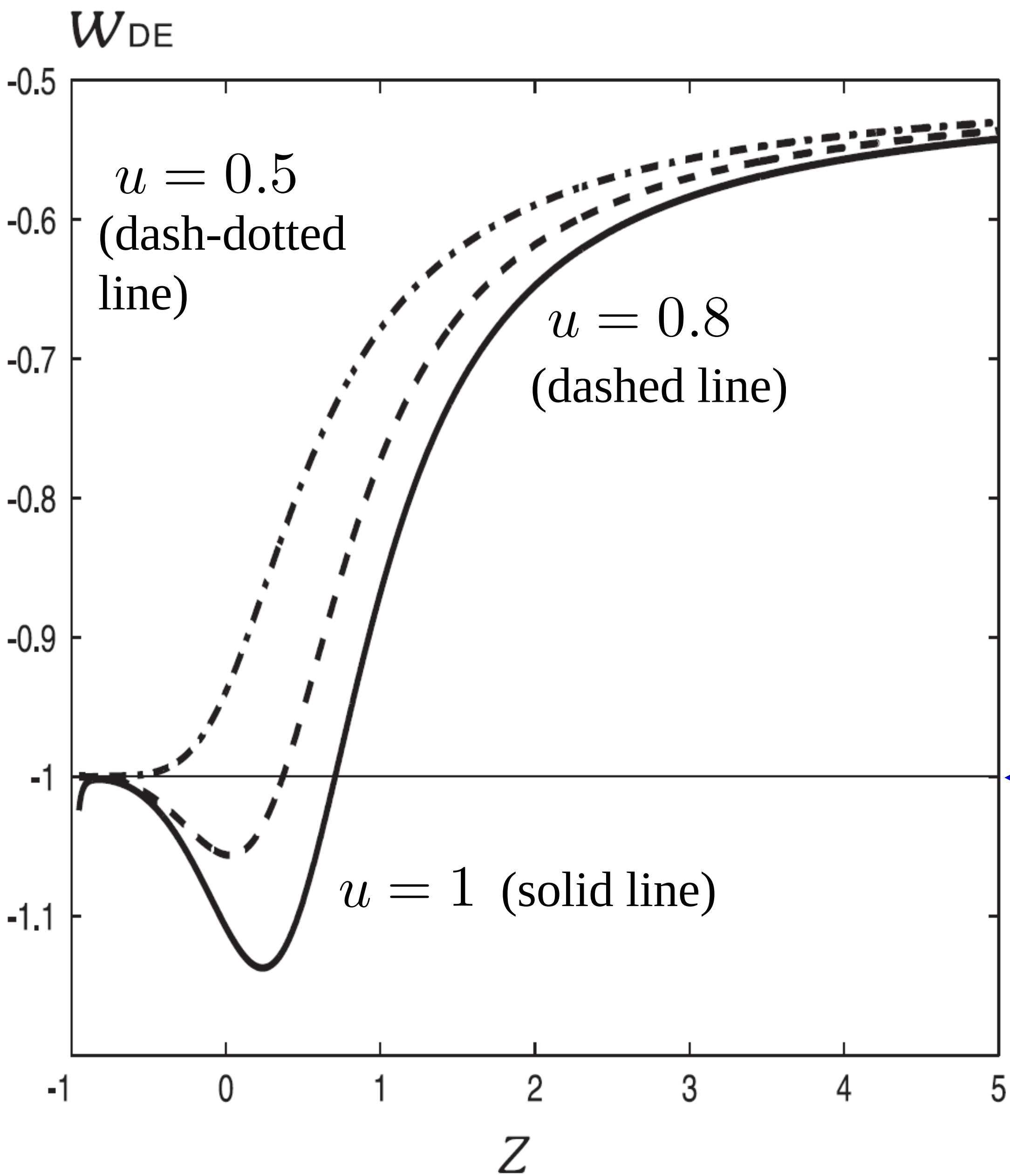
$u (> 0)$: Positive constant

$$\Omega_{\mathrm{m}}^{(0)} \equiv \rho_{\mathrm{m}}^{(0)} / \rho_{\mathrm{crit}}^{(0)} \,, \qquad T_0 = T(z = 0)$$

$$\rho_{\mathrm{crit}}^{(0)} = 3H_0^2 / \kappa^2 \,, \qquad z = \frac{1}{a} - 1$$

: Redshift

Cosmological evolutions of w_{DE}



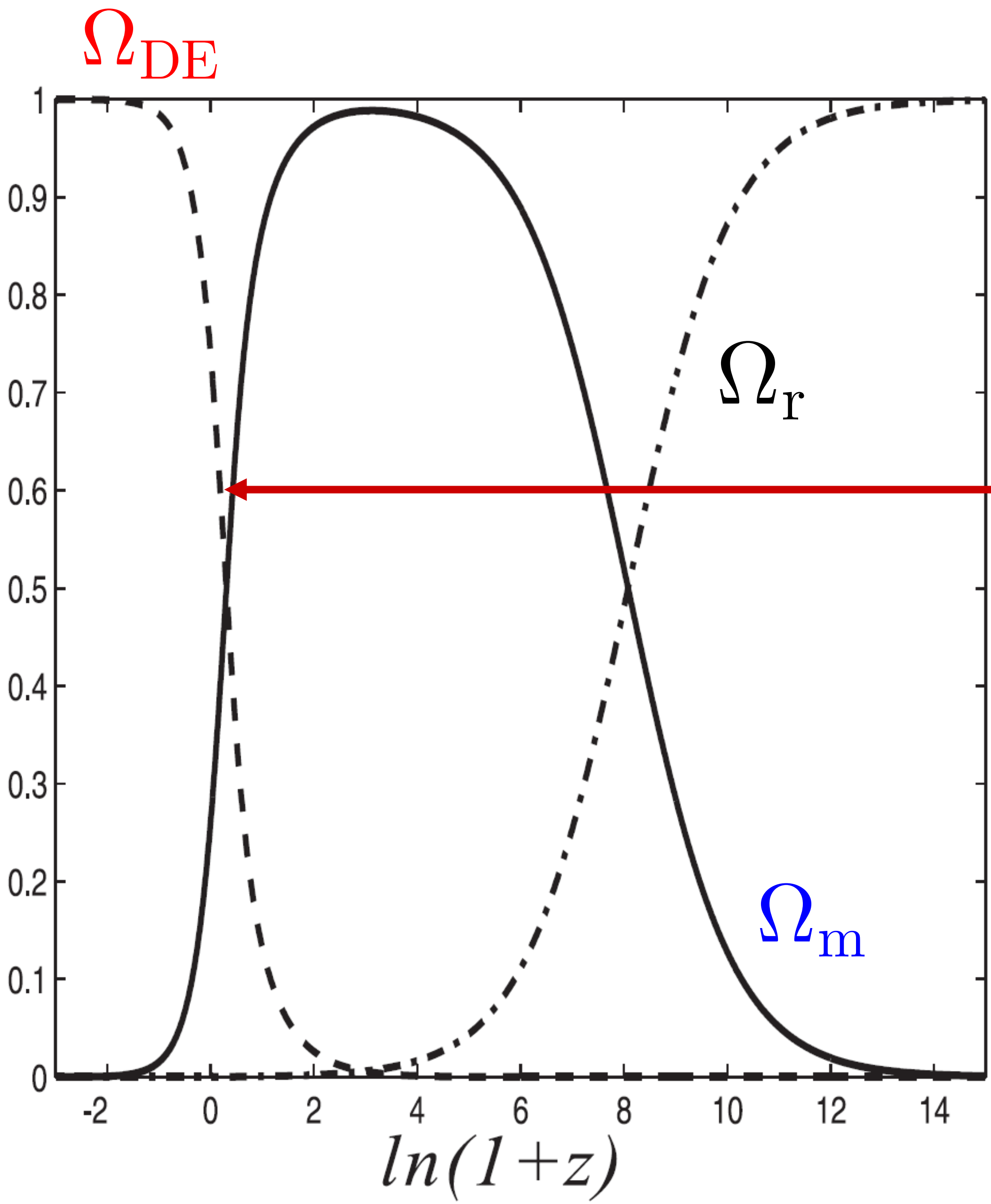
From [KB, Geng, Lee and Luo, JCAP 1101, 021 (2011)].

$w_{\mathrm{DE}} = -1$

Crossing of the phantom divide

z : Redshift

Cosmological evolutions of $\Omega_{\rm DE}$, $\Omega_{\rm m}$ and $\Omega_{\rm r}$



$u = 1$

Dark energy dominated stage

From [KB, Geng, Lee and Luo, JCAP 1101, 021 (2011)].

Metric in five-dimensional space-time

$$ {}^{(5)}g_{ab} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & -\phi^2 \end{pmatrix}, $$

$\phi \equiv \varphi/\varphi_*$: Dimensionless homogeneous scalar field

φ_* : Fiducial value of φ

$$ \phi^2 = \mathcal{R}^2\theta^2 $$

$\mathcal{R}$: Rradius of the compactified space

θ : Dimensionless coordinates such as an angle

$$ \sqrt{{}^{(5)}g} = \sqrt{-g}\mathcal{R}\sqrt{\hat{g}} $$

$\hat{g}$: Determinant of the metric corresponding to the pure geometrical part represented by θ

$V_{\rm com} = \int \hat{g} d\theta$: Compactified space volume

Settings in the RS type-II model

- We start with the equation in the five-dimensional space-time with the brane whose tension is a positive constant.

- We consider that the vacuum solution in the five-dimensional space-time is the AdS one, and that the brane configuration is consistent with the equation in the five-dimensional space-time.

⇨ This implies that the brane configuration with a positive constant tension connecting two vacuum solutions in the five-dimensional space-time, namely, the condition of the configuration is nothing but the equation for the brane.

(ii) The Israel's junction conditions to connect the left-side and right-side bulk spaces sandwiching the brane are investigated.

- The first junction condition is that the vierbeins induced on the brane from the left side and right side of the brane should be the same with each other.

- Moreover, the second junction condition is that the difference of the tensor $S_\rho{}^{\mu\nu}$ between the left side and right side of the brane comes from the energy-momentum tensor of matter, which is confined in the brane.

(iii) Provided that there exists Z_2 symmetry, i.e., $y \leftrightarrow -y$, in the five-dimensional space-time, the quantities on the left and right sides of the brane are explored.

- The difference between the scalar curvature and the torsion scalar is a total derivative of the torsion tensor.

  $\longrightarrow$ This may affect the boundary.

- It has been shown that in comparison with the gravitational field equations in general relativity, the induced gravitational field equations on the brane have new terms, which comes from the additional degrees of freedom in teleparallelism.

- These extra terms correspond to the projection on the brane of the vector portion of the torsion tensor in the bulk.

Cf. Other solution

For $F(T)=T$, $\Lambda=0$, $\mathcal{Q}=8/3$, and $w_{\mathrm{M}}=-5.5\times 10^{-3}$,

$$H^2 = \left(\kappa^2/3\right)\rho_{\mathrm{M}}\left[1+\rho_{\mathrm{M}}/\left(2\lambda\right)\right]$$

[Astashenok, Elizalde, de Haro, Odintsov and Yurov, Astrophys. Space Sci. 347, 1 (2013)]

Case (2)

$\bar{M}$: Mass scale

α : Constant

$$F(T)=T^2/\bar{M}^2+\alpha\Lambda_5$$

$$\longrightarrow \quad H = H_{\mathrm{DE}} = \left[\left(\bar{M}^2/108\right)\mathcal{J}\right]^{1/4} = \text{constant}$$

$$\mathcal{J} \equiv \left(\alpha-4\right)\Lambda_5 - 4\left(\kappa_5^2/2\right)^2\lambda^2$$

$$a(t)=a_{\mathrm{DE}}\exp\left(H_{\mathrm{DE}}t\right), \qquad a_{\mathrm{DE}}(>0)$$

$$\mathcal{J}(\geq 0) \Longrightarrow \alpha \geq 4+\left(\kappa_5^2\lambda^2\right)/\Lambda_5$$

III. From Kaluza-Klein (KK) theory

Action in the 5-dim. space-time

[Capozziello, Gonzalez, Saridakis and Vasquez, JHEP 1302, 039 (2013)]

$$ {}^{(5)}S = \int d^5x \left| {}^{(5)}e \right| \frac{F({}^{(5)}T)}{2\kappa_5^2} $$

$$ {}^{(5)}T \equiv \frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{cba} - T_{ab}{}^{a} T^{cb}{}_{c} $$

$$ {}^{(5)}e = \sqrt{{}^{(5)}g} $$

$$ \kappa_5^2 \equiv 8\pi G_5 = \left({}^{(5)}M_{\mathrm{Pl}} \right)^{-3} $$

* $a, b, \ldots$ run over $0, 1, 2, 3, 5$.

* “5” denotes the component of the fifth coordinate.

KK compactification scenario

- One of the dimensions of space is compactified to a small circle and the 4-dim. space-time is extended infinitely.
- The radius of the 5-dim. is taken to be of order of the Planck length in order for the KK effects not to be seen.

[Appelquist, Chodos and Freund, *Modern Kaluza-Klein Theories* (1987)]

[Fujii and Maeda, *The Scalar-Tensor Theory of Gravitation* (2003)]

Effective action in the 4-dim. space-time

- **Metric in five-dimensional space-time**

$$ {}^{(5)}g_{ab} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & -\phi^2 \end{pmatrix} $$

ϕ : Dimensionless homogeneous scalar field

$$ \Longrightarrow \quad S_{\mathrm{KK}}^{\mathrm{eff}} = \int d^4x |e| \frac{1}{2\kappa^2} \phi F(T + \phi^{-2} \partial_\mu \phi \partial^\mu \phi) $$

Cf. [Fiorini, Gonzalez and Vasquez, arXiv:1304.1912 [gr-qc]]

Cosmology in the flat FLRW space-time

- $F(T) = T - 2\Lambda_4$, $\Lambda_4(> \ 0)$: Cosmological constant

- We define σ as $\phi \equiv \xi\sigma^2$, $\xi = 1/4$

$$\Rightarrow \ S_{\mathrm{KK}}^{\mathrm{eff}}|_{F(T)=T-2\Lambda_4} =$$

$$\int d^4x|e| \left(1/\kappa^2\right) \left[(1/8)\, \sigma^2 T + \underline{(1/2)\, \partial_\mu \sigma \partial^\mu \sigma} - \Lambda_4\right]$$

Canonical kinetic term

From gravitational field equations, we have

$$(3/2)\, H^2\sigma^2 - 2\Lambda_4 + H\sigma\dot{\sigma} + (1/2)\, \dot{H}\sigma^2 = 0$$

Equation of motion of σ

$$\ddot{\sigma} + 3H\dot{\sigma} + (3/2)\, H^2\sigma = 0$$

Cf. [Geng, Lee, Saridakis and Wu, PLB 704, 384 (2011)]

Solution

$$H = H_{\rm inf} = {\rm constant}(> 0)$$

$$\sigma = b_1 \left(t/t_1 \right) + b_2$$

$b_1, \quad b_2(> 0), \quad t_1$: Constants

- **In the limit $t \rightarrow 0$:**

$$H_{\rm inf} \approx (2/b_2) \sqrt{\Lambda_4/3}$$

$$\sigma \approx b_2$$

$$b_1 \approx -(1/2)\, b_2 H_{\rm inf} t_1 \approx -\sqrt{\Lambda_4/3} t_1$$

$$a \approx \bar{a} \exp \left(H_{\rm inf} t \right), \qquad \bar{a}(> 0)$$

⇨ **Exponential inflation can be realized.**

Case (2)

$$F(T) = T^2/\bar{M}^2 + \alpha\Lambda_5$$

$\bar{M}$: Mass scale

α : Constant

$$\longrightarrow \quad H = H_{\mathrm{DE}} = \left[\left(\bar{M}^2/108\right)\mathcal{J}\right]^{1/4} = \mathrm{constant}$$

$$\mathcal{J} \equiv (\alpha - 4)\,\Lambda_5 - 4\left(\kappa_5^2/2\right)^2 \lambda^2$$

$$a(t) = a_{\mathrm{DE}} \exp\left(H_{\mathrm{DE}} t\right), \qquad a_{\mathrm{DE}} (> 0)$$

⇨ **A de Sitter solution on the brane can exist.**

PLANCK data for the current *w* (=constant)

[Ade *et al*. [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO]]

- $w \equiv \dfrac{P}{\rho}$ **: Equation of state (EoS) parameter**

ρ : Energy density, P : Pressure

$\Longrightarrow$ $\underline{w = -1.13^{+0.24}_{-0.25}}$ (95%; *Planck*+WP+BAO)

WP: WMAP

BAO: Baryon Acoustic Oscillation

($w = -1$ **: Cosmological constant)**

(i) **The induced equations (Gauss-Codazzi equations) on the brane are examined by using the projection vierbein of the five-dimensional space-time quantities into the four-dimensional space-time brane.**

(ii) **The Israel's junction conditions to connect the left-side and right-side bulk spaces sandwiching the brane are investigated.**

(iii) **Provided that there exists Z_2 symmetry, i.e., $y \leftrightarrow -y$, in the five-dimensional space-time, the quantities on the left and right sides of the brane are explored.**

- **Recent observations of Type Ia Supernova (SNe Ia) has supported that the current expansion of the universe is accelerating.**

[Perlmutter *et al*. [Supernova Cosmology Project Collaboration], ApJ <u>517</u>, 565 (1999)]

[Riess *et al*. [Supernova Search Team Collaboration], Astron. J. <u>116</u>, 1009 (1998)]

2011 Nobel Prize in Physics

- **Suppose that the universe is strictly homogeneous and isotropic.**

There are two approaches to explain the current accelerated expansion of the universe.

Reviews, e.g.,

[Nojiri and Odintsov, Phys. Rept. 505, 59 (2011); Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007)]

[KB, Capozziello, Nojiri and Odintsov, Astrophys. Space Sci. 342, 155 (2012)]

Gravitational field equation

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

Gravity | **Matter**

$\Uparrow$ | $\Uparrow$

(2) Modified gravity | **(1) Dark energy**

$G_{\mu\nu}$: Einstein tensor, $T_{\mu\nu}$: Energy-momentum tensor

$\kappa^2 \equiv 8\pi / {M_{\rm Pl}}^2$ $M_{\rm Pl}$: Planck mass

(1) Dark energy

Cosmological constant, Scalar field, Fluid

(2) Modified gravity

- ***F*(*R*) gravity** $F(R)$: Arbitrary function of the Ricci scalar R

- **DGP braneworld scenario**

- **Massive/Bimetric gravity**

- **Extended teleparallel gravity (*F*(*T*) gravity)**

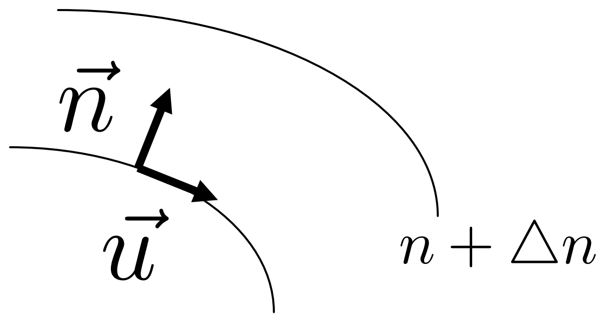
$F(T)$: Arbitrary function of the torsion scalar T

Why teleparallel gravity?

- **General relativity**

  (with only curvature)

  Trajectories are determined by geodesics: $\vec{\nabla}_{\vec{u}}\vec{u} = 0$



n : Selector parameter

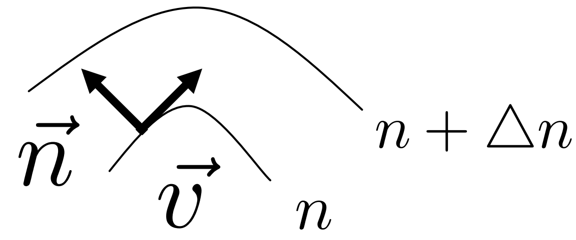
- **Teleparallel gravity**

  (with only torsion)

  Torsion acts as a force:

$$\left(\frac{\partial^2 x^j}{\partial t^2}\right)_n + \frac{\partial \Phi}{\partial x^j} = 0$$

Φ : Newton potential



From [Misner, Thorne and Wheeler, *Gravitation* (Friemann, New York, 1973)].

⇨ **Curvature and torsion represent the same gravitational field.**

[Aldrovandi and Pereira, *Teleparallel Gravity: An Introduction* (Springer, Dordrecht, 2012); http://www.ift.unesp.br/users/jpereira/tele.pdf]

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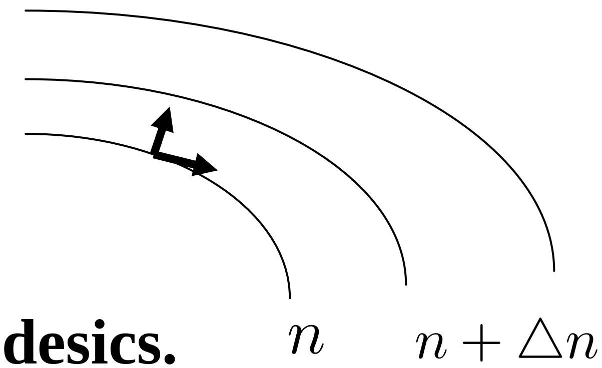

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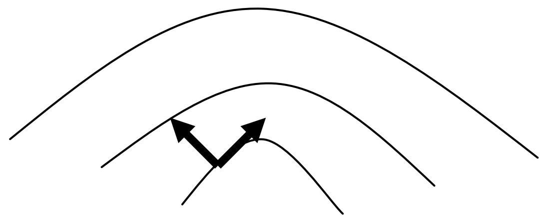

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Why teleparallel gravity?

- **General relativity**

  Only curvature described by the Levi-Civita connection exists and there is no torsion.

- **Teleparallel gravity**

  Only torsion written with the Weitzenböck connection exists and the curvature vanishes.

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The projection of the 5-dim. space-time quantities into the 4-dim. space-time brane.

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To connect the left-side and right-side bulk spaces sandwiching the brane.

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