The infrared regime of SU(2) with one adjoint Dirac Fermion

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Abstract

The $\mathrm{SU}(2)$ gauge theory with one Dirac flavour in the adjoint representation is investigated on the lattice for a value of the bare coupling in the region connected to the continuum limit. Results for the gluonic and mesonic spectrum, string tension from Wilson and Polyakov loops, and the anomalous dimension of the fermionic condensate from the Dirac mode number are presented. From these, we see evidence that the theory does not show conventional confining behaviour, instead seeming to lie within or very near the onset of the conformal window. The anomalous dimension of the fermionic condensate is found to lie in the range $0.9 \lesssim \gamma_* \lesssim 0.95$. Topological observables of the theory, and of the related 2-flavour theory, are also discussed both in terms of their phenomenology and of their use as a lattice diagnostic.

Outline

Introduction

- Motivation
- Dirac $\to$ Majorana decomposition
- Lattice formulation
- Quantum numbers
- Lattice topology

Results

- Phase diagram
- Spectrum
- Mass anomalous dimension
- Topological observables [arXiv:1209.5579]

What and why?

$\mathrm{SU}(2) + 1$ adjoint Dirac flavour

- $\equiv \mathrm{SU}(2) + 2$ adjoint Majorana flavours

Why?

- SUSY?
- Conformal window
- Technicolor?

What do we know?

- Conformal window:
  - $\mathrm{SU}(2) + 2$ flavours is conformal (e.g. arXiv:1104.4301 etc.)
  - $\mathrm{SU}(2) + 1$ flavour predicted to be confining (e.g. Bringoltz & Sharpe)
- Technicolor
  - $\mathrm{SU}(2) + 1$ flavour χSB $\mathrm{SU}(2) \to \mathrm{SO}(2) \Rightarrow 2$ Goldstones
    - Insufficient for EWSB
    - Not a walking technicolor candidate

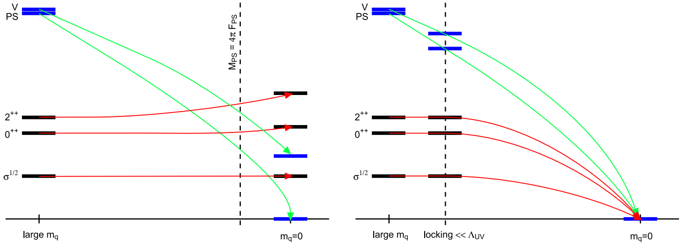
What don't we know?

A lot! Useful things we are able to calculate:

- First-principles determination of whether $\mathrm{SU}(2) + 1$ flavour is conformal or confining
- Spectroscopy
- Mass anomalous dimension

What do we expect?

- Confining: $m_{\mathsf{PS}} \to 0$, $m_{\mathsf{V}} \not\to 0$ as $m_{\mathsf{PCAC}} \to 0$.
- Conformal: Locking at scale m_{lock}:
  - $m_{\mathsf{PCAC}} < m_{\mathsf{lock}} \Rightarrow m_{\mathsf{state}} \sim m^{1/(1+\gamma_*)} \to 0$.
  - Ratios of spectral quantities in this regime *constant*.
  - Very different from QCD



(Figure: Agostino Patella, from arXiv:0911.0020)

  - Dynamical quenching in semiclassical dynamics—fermions decouple from e.g. topology

Chiral symmetry breaking?

- 1 flavour: no obvious chiral symmetry
- Real representation: real and imaginary parts factorise
- Reexpress action in terms of real and imaginary parts of field (c.f. e.g. Montvay, hep-lat/9510042)

$$\psi = \psi_{\mathsf{M}+} + i\psi_{\mathsf{M}-}$$

$$\begin{aligned} \psi_{\mathsf{M}+} &= \tfrac{1}{2}(\psi + C\overline{\psi}^{\mathrm{T}}) & \qquad \psi_{\mathsf{M}-} &= \tfrac{1}{2i}(\psi - C\overline{\psi}^{\mathrm{T}}) \\ \Rightarrow \overline{\psi}_{\mathsf{M}+} &= \tfrac{1}{2}(\overline{\psi} + \psi^{\mathrm{T}}C) & \qquad \overline{\psi}_{\mathsf{M}-} &= \tfrac{1}{2i}(\psi^{\mathrm{T}}C - \overline{\psi}) \end{aligned}$$

- Majorana constraint $\psi_{\mathsf{M}\pm\mathsf{C}} \equiv C\overline{\psi}^{\mathrm{T}}_{\mathsf{M}\pm} = \psi_{\mathsf{M}\pm}$ satisfied.

Action

- Mass term:
  - $\overline{\psi}_{\mathsf{M}\pm}\psi_{\mathsf{M}\pm} = \frac{1}{4}\left[2\overline{\psi}\psi \pm \psi^{\mathrm{T}}C\psi \pm \overline{\psi}C\overline{\psi}^{\mathrm{T}}\right]$
  - $\Rightarrow \overline{\psi}\psi = \overline{\psi}_{\mathsf{M}+}\psi_{\mathsf{M}+} + \overline{\psi}_{\mathsf{M}-}\psi_{\mathsf{M}-}$
- Kinetic term:
  - $\overline{\psi}_{\mathsf{M}\pm}\partial\!\!\!/\psi_{\mathsf{M}\pm} = \frac{1}{4}\left[\overline{\psi}\partial\!\!\!/\psi \pm \psi^{\mathrm{T}}C\partial\!\!\!/\psi \pm \overline{\psi}\partial\!\!\!/C\overline{\psi}^{\mathrm{T}} + \psi^{\mathrm{T}}C\partial\!\!\!/C\overline{\psi}^{\mathrm{T}}\right]$
  - $\psi^{\mathrm{T}}C\partial\!\!\!/C\overline{\psi}^{\mathrm{T}} = \overline{\psi}\partial\!\!\!/\psi$
  - $\overline{\psi}\partial\!\!\!/\psi = \overline{\psi}_{\mathsf{M}+}\partial\!\!\!/\psi_{\mathsf{M}+} + \overline{\psi}_{\mathsf{M}-}\partial\!\!\!/\psi_{\mathsf{M}-}$

$$\begin{aligned}\Rightarrow S_{1\ \mathsf{Dirac}} &= S_{2\ \mathsf{Majorana}} \\ &= \overline{\psi}_{\mathsf{M}+}\partial\!\!\!/\psi_{\mathsf{M}+} + \overline{\psi}_{\mathsf{M}-}\partial\!\!\!/\psi_{\mathsf{M}-} + m(\overline{\psi}_{\mathsf{M}+}\psi_{\mathsf{M}+} + \overline{\psi}_{\mathsf{M}-}\psi_{\mathsf{M}-})\end{aligned}$$

and the action needs no modification.

- $\mathrm{SU}(2)$ global chiral symmetry
  - Breaks to $\mathrm{SO}(2) \equiv \mathrm{U}(1)$
  - $\mathrm{U}(1)$ in Weyl basis $\leftrightarrow$ baryon number B in Dirac basis

Lattice formulation

- Lattice action: $S = S_\mathsf{g} + S_\mathsf{f}$
- Wilson gauge action: $\beta \sum_p (1 - \mathrm{Retr} U(p))$
- Wilson (Dirac) fermion action: $S_\mathsf{f}^{\mathsf{Dirac}} = \sum_{x,y} \overline{\psi}(x) D(x,y) \psi(y)$
  - Massive Dirac operator:

$$\delta_{x,y} - \frac{\kappa}{2} \left[(1-\gamma_\mu) U_\mu(x) \delta_{y,x+\mu} + (1+\gamma_\mu) U_\mu^\dagger(x-\mu) \delta_{y,x-\mu} \right]$$

- For observables, calculate correlation functions

$$\langle X \rangle = \frac{\int D\overline{\psi} D\psi dU X e^{-S}}{\int D\overline{\psi} D\psi dU e^{-S}}$$

- $X = O^\dagger(\mathbf{x}, t) O(\mathbf{0}, 0)$, operator O encodes quantum numbers
- $\lim_{t\to\infty} \langle X \rangle \sim e^{-mt}$

Fermionic bilinears

- Majorana mesons: $O(\mathbf{x},t) = \overline{\psi}_{\mathsf{M}i}\Gamma\psi_{\mathsf{M}j} = O_{ij}(\Gamma)$, $i,j \in \{+,-\}$
- Reexpress these in Dirac form:

$$O_{\pm\mp}(\Gamma) = \begin{cases} \frac{1}{4i}\left(\psi^{\mathrm{T}}C\Gamma\psi - \overline{\psi}\Gamma C\overline{\psi}^{\mathrm{T}}\right) & \Gamma = \mathbb{1}, \gamma_5\gamma_\mu, \gamma_5 \\ \pm\frac{1}{2i}\overline{\psi}\Gamma\psi & \Gamma = \gamma_\mu, \gamma_0\gamma_5\boldsymbol{\gamma} \end{cases}$$

$$O_{\pm\pm}(\Gamma) = \begin{cases} \frac{1}{4}\left(2\overline{\psi}\Gamma\psi \pm \psi^{\mathrm{T}}C\Gamma\psi \pm \overline{\psi}\Gamma C\overline{\psi}^{\mathrm{T}}\right) & \Gamma = \mathbb{1}, \gamma_5\gamma_\mu, \gamma_5 \\ 0 & \Gamma = \gamma_\mu, \gamma_0\boldsymbol{\gamma}, \gamma_0\gamma_5\boldsymbol{\gamma} \end{cases}$$

- Then take correlation functions; e.g. for $\Gamma \in \{\mathbb{1}, \gamma_5\gamma_\mu, \gamma_5\}$,

$$\begin{aligned} &\left\langle O^\dagger_{+-}(x)O_{+-}(0)\right\rangle \\ &= -\mathrm{tr}\overline{\Gamma}CD^{-1\mathrm{T}}(0;x)C\Gamma D^{-1}(0;x) + \mathrm{tr}(\overline{\Gamma}C)^{\mathrm{T}}D^{-1\mathrm{T}}(0;x)C\Gamma D^{-1}(0;x) \\ &\qquad - \mathrm{tr}C\overline{\Gamma}D^{-1}(x;0)\Gamma CD^{-1\mathrm{T}}(x;0) + \mathrm{tr}(C\overline{\Gamma})^{\mathrm{T}}D^{-1}(x;0)\Gamma CD^{-1\mathrm{T}}(x;0) \\ &= -\frac{1}{4}\mathrm{tr}\overline{\Gamma}D^{-1}(x;0)\Gamma D^{-1}(0;x) \end{aligned}$$

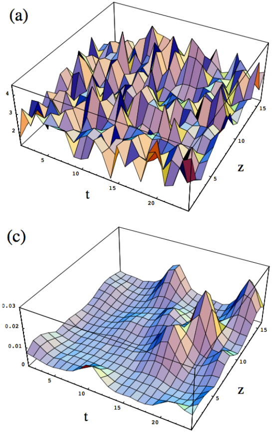
Quantum numbers

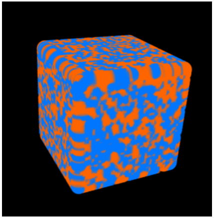
Dirac bilinears	Majorana bilinears	$U(1)^P$	correlators
$\bar\psi\gamma_0\gamma_5\psi$	$O_{++}(\gamma_0\gamma_5)+O_{--}(\gamma_0\gamma_5)$	0^-	singlet γ_5, $\gamma_0\gamma_5$
$\bar\psi\gamma_5\psi$	$O_{++}(\gamma_5)+O_{--}(\gamma_5)$		
$\psi^{\mathrm{T}}C\gamma_5\psi$	$-i(O_{++}(1)-O_{--}(1)+2iO_{+-}(1))$	2^-	triplet 1
$\psi^\dagger C\gamma_5\psi^*$	$-i(O_{++}(1)-O_{--}(1)-2iO_{+-}(1))$	-2^-	
$\bar\psi\psi$	$O_{++}(1)+O_{--}(1)$	0^+	singlet 1, γ_0
$\bar\psi\gamma_0\psi$	$O_{+-}(\gamma_0)$		
$\psi^{\mathrm{T}}C\psi$	$-i(O_{++}(\gamma_5)-O_{--}(\gamma_5)+2iO_{+-}(\gamma_5))$	2^+	triplet γ_5, $\gamma_0\gamma_5$
$\psi^{\mathrm{T}}C\gamma_0\psi$	$-i(O_{++}(\gamma_5\gamma_0)-O_{--}(\gamma_5\gamma_0)+2iO_{+-}(\gamma_5\gamma_0))$		
$\psi^\dagger C\psi^*$	$-i(O_{++}(\gamma_5)-O_{--}(\gamma_5)-2iO_{+-}(\gamma_5))$	-2^+	
$\psi^\dagger C\gamma_0\psi^*$	$-i(O_{++}(\gamma_5\gamma_0)-O_{--}(\gamma_5\gamma_0)-2iO_{+-}(\gamma_5\gamma_0))$		
$\bar\psi\gamma_5\boldsymbol{\gamma}\psi$	$O_{++}(\gamma_5\boldsymbol{\gamma})+O_{--}(\gamma_5\boldsymbol{\gamma})$	0^+	singlet $\gamma_5\boldsymbol{\gamma}$, $\gamma_0\gamma_5\boldsymbol{\gamma}$
$\bar\psi\gamma_0\gamma_5\boldsymbol{\gamma}\psi$	$O_{+-}(\gamma_0\gamma_5\boldsymbol{\gamma})$		
$\bar\psi\gamma_0\boldsymbol{\gamma}\psi$	$O_{+-}(\gamma_0\boldsymbol{\gamma})$	0^-	singlet $\boldsymbol{\gamma}$, $\gamma_0\boldsymbol{\gamma}$
$\bar\psi\boldsymbol{\gamma}\psi$	$O_{+-}(\boldsymbol{\gamma})$		
$\psi^{\mathrm{T}}C\boldsymbol{\gamma}\psi$	$-i(O_{++}(\gamma_5\boldsymbol{\gamma})-O_{--}(\gamma_5\boldsymbol{\gamma})+2iO_{+-}(\gamma_5\boldsymbol{\gamma}))$	2^-	triplet $\gamma_5\boldsymbol{\gamma}$
$\psi^\dagger C\boldsymbol{\gamma}\psi^*$	$-i(O_{++}(\gamma_5\boldsymbol{\gamma})-O_{--}(\gamma_5\boldsymbol{\gamma})-2iO_{+-}(\gamma_5\boldsymbol{\gamma}))$	-2^-	

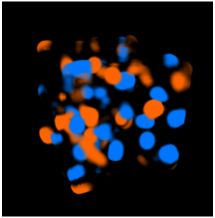
Lattice topology

- Relevant topological objects: instantons
- Continuum topological charge: $Q = \frac{1}{32\pi^2} \int d^4x F^a_{\mu\nu} \tilde{F}^a_{\mu\nu}$
- Define lattice topological charge density:
  $Q_{\mathsf{L}}(i) = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{tr}\left\{U_{\mu\nu}(i) U_{\rho\sigma}(i)\right\}$
- Then total topological charge: $Q_{\mathsf{T}} = \sum_i Q_{\mathsf{L}}(i)$
- Problem: realistic gauge fields are hot and noisy
- One solution: Cool gauge fields
  - Minimize local action for each site
  - Local fluctuations smoothed out
  - Excessive cooling risks shrinking instantons
- Observables:
  - Topological susceptibility $\chi_{\mathsf{T}} = \langle Q^2_{\mathsf{T}} \rangle / V \equiv (\langle Q^2_{\mathsf{T}} \rangle - \langle Q_{\mathsf{T}} \rangle^2)/V$
  - Instanton size: $Q_{\text{peak}} = 6/(\pi^2 \rho^4)$
    - Instanton size distribution
    - Average instanton size (correcting for cut-off)

Gauge noise







(left: from Schäfer & Shuryak arXiv:hep-ph/9610451 §III.B.2)

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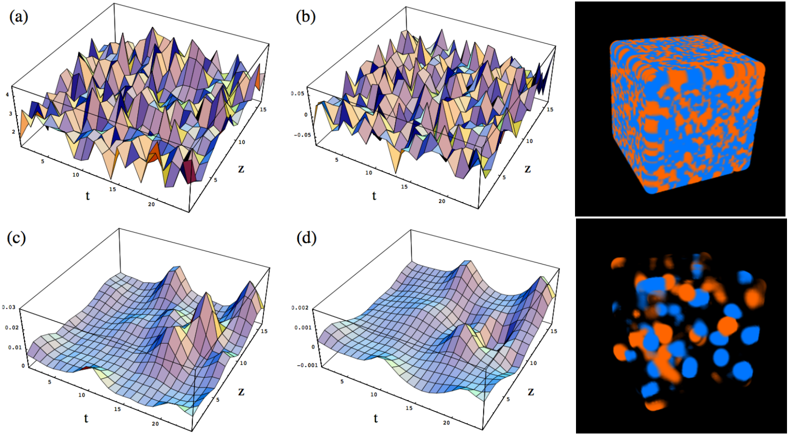

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- Phase diagram: plaquette on 4^4 lattice; $1.4 \leq \beta \leq 2.8$, $-1.7 \leq am \leq -0.1$
- Spectroscopy at 16×8^3, 24×12^3, 32×16^3, 48×24^3; $\beta = 2.05$, $-1.523 \leq am \leq -1.475$.
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  - RHMC: HiRep; observables: HiRep + Münster code
- 16×8^3 & lighter 32×16^3 data finite-volume afflicted; others OK.
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  - PCAC mass
  - Meson masses
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  - Spin-$\frac{1}{2}$ state ($\sim$gluion-glue)
  - Fundamental string tension (Polyakov loops)
- Spectral ratios roughly constant—consistent with conformality
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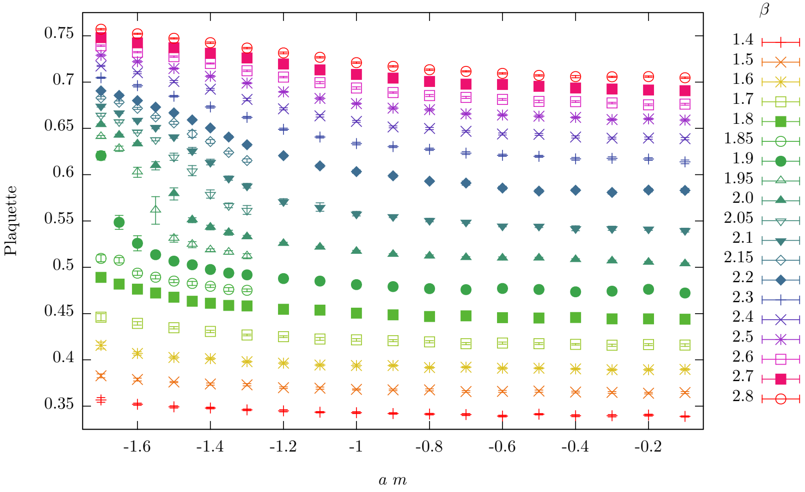
Phase diagram



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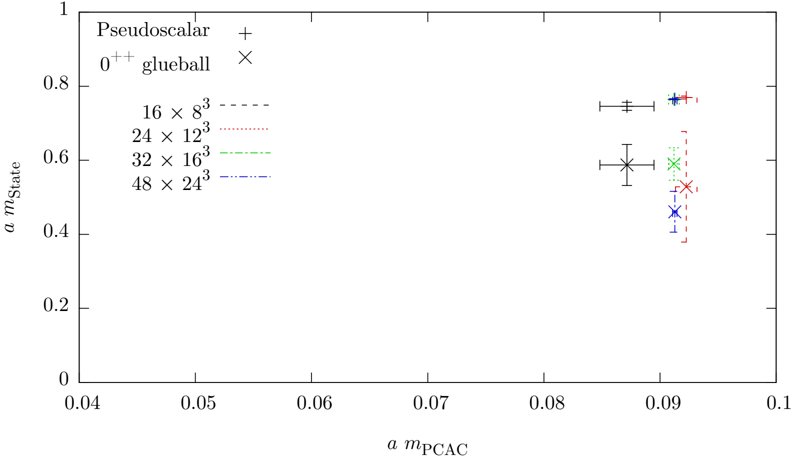
Lattice parameters

Lattice	V	$-am_0$	$N_{\rm conf}$	Acceptance	$N_{\rm pf}$	$t_{\rm len}$	$n_{\rm steps}$	Machine
A1	16×8^3	1.475	2400	91.4%	1	1.0	10	SC
A2	16×8^3	1.500	2200	90.9%	1	1.0	10	SC, UL
A3	16×8^3	1.510	2400	89.8%	1	1.0	10	SC, UL
A4	16×8^3	1.510	4000	92.4%	2	1.0	8	SC
B1	24×12^3	1.475	2400	79.9%	1	1.0	10	SC, UL
B2	24×12^3	1.500	2300	78.7%	1	1.0	10	SC, UL
B3	24×12^3	1.510	4000	88.5%	2	1.0	10	SC, UL
C1	32×16^3	1.475	2100	90.6%	1	1.0	20	SC
C2	32×16^3	1.490	2300	90.0%	1	1.0	20	SC, UL
C3	32×16^3	1.510	2200	89.4%	1	1.0	20	UL
C4	32×16^3	1.510	2300	83.2%	2	4.0	45	BGP
C5	32×16^3	1.514	2300	89.8%	1	1.0	20	UL, BGP
C6	32×16^3	1.519	2300	81.8%	1	1.0	20	UL, BGP
C7	32×16^3	1.523	2200	88.0%	1	1.0	20	SC
D1	48×24^3	1.510	1534	80.5%	2	4.0	65	BGP
D2	48×24^3	1.523	2168	91.4%	1	1.0	40	BGP

Finite-volume study



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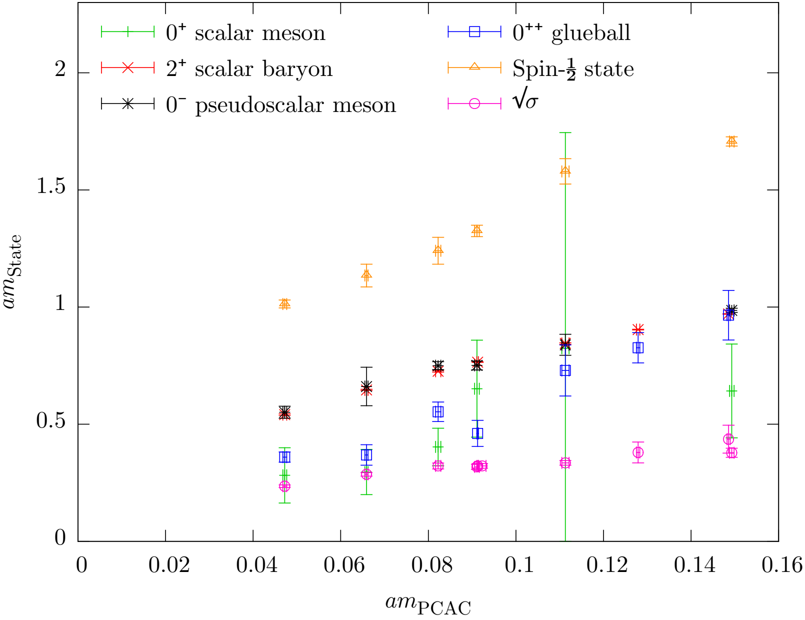
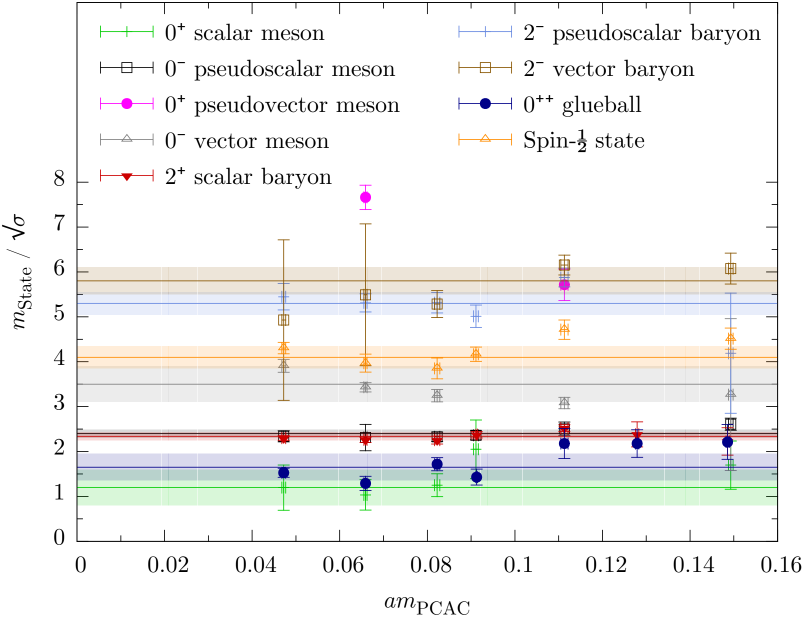
Spectrum



0^+ scalar meson
2^+ scalar baryon
0^- pseudoscalar meson
0^{++} glueball
Spin-$\frac{1}{2}$ state
$\sqrt{\sigma}$
am_{State}
am_{PCAC}
0
0.5
1
1.5
2
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16

Spectral ratios



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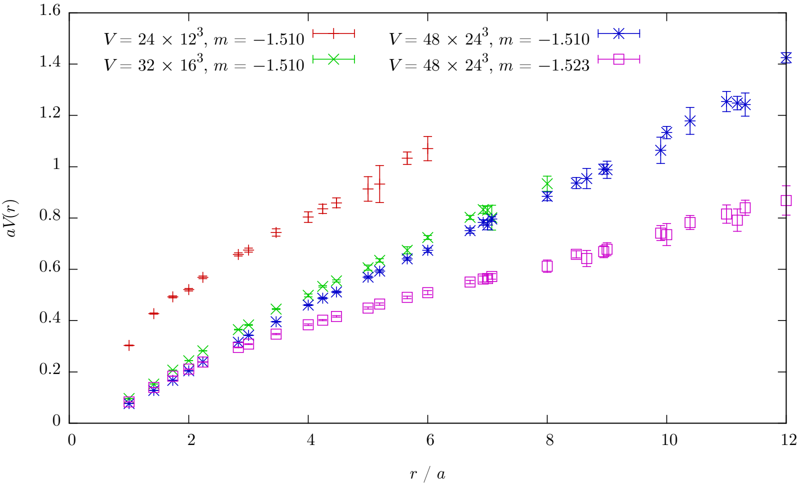
Wilson loops



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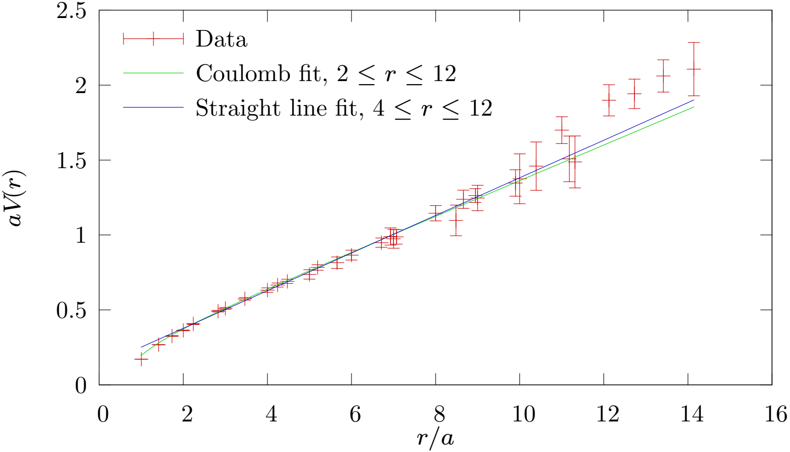

$$\Rightarrow \sqrt{\sigma} = 0.354(5)$$

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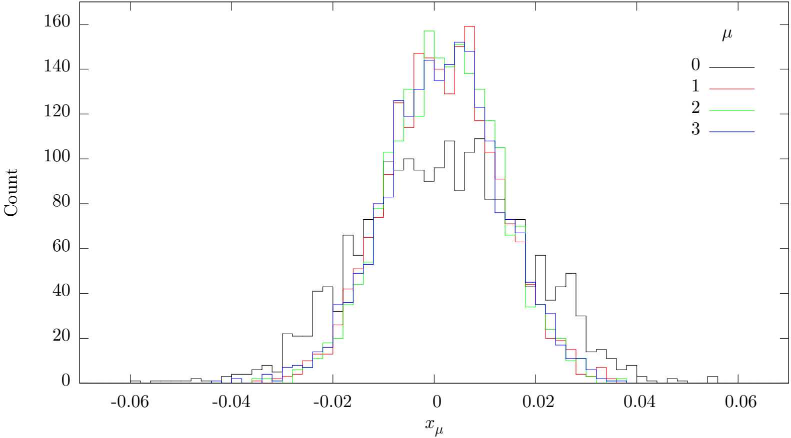
Center symmetry



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Mass anomalous dimension

- Mass anomalous dimension $\gamma_* \sim 1$ important for WTC
- Observing large γ_* here indicates viability for other WTC candidates
- By inspection, fitting $Lam_{\gamma_5} \sim L(am_{\mathsf{PCAC}})^{\frac{1}{1+\gamma_*}}$
  - $\Rightarrow 0.9 \lesssim \gamma_* \lesssim 1.1$
- Fitting the Dirac mode number per unit volume $\overline{\nu}(\Omega)$

$$a^{-4}\overline{\nu}(\Omega) \approx a^{-4}\nu_0(m) + A\left[(a\Omega)^2 - (am)^2\right]^{\frac{2}{1+\gamma_*}}$$

from Patella [arxiv:1204.4432]
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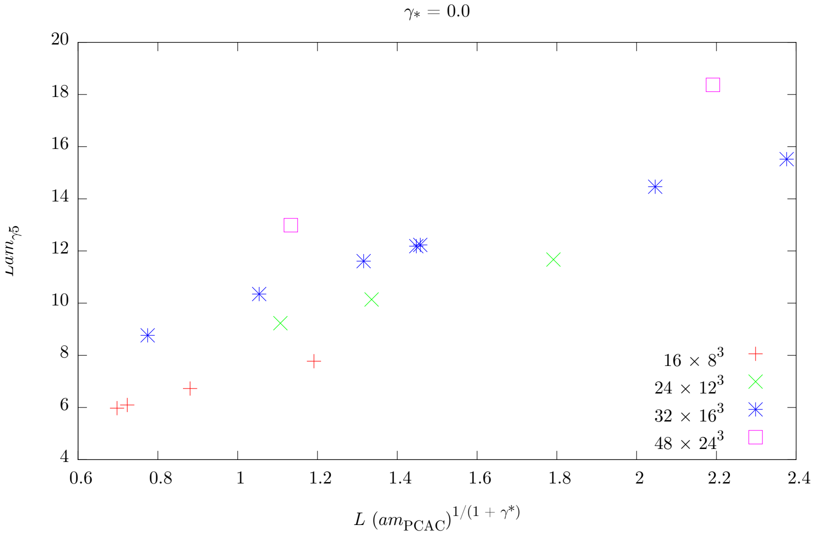
γ_* inspection fit



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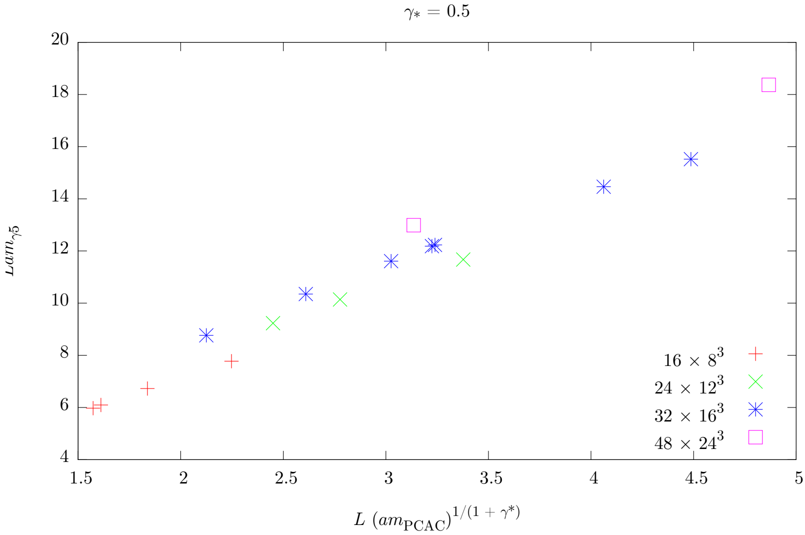

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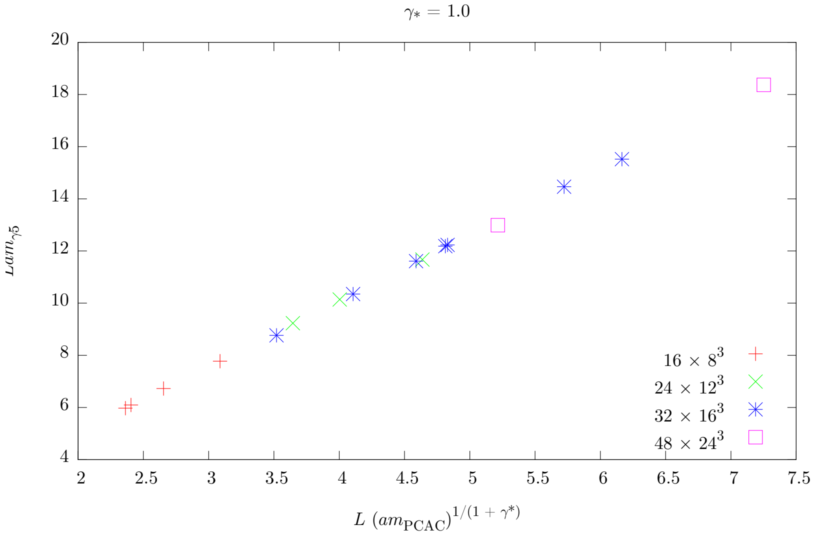

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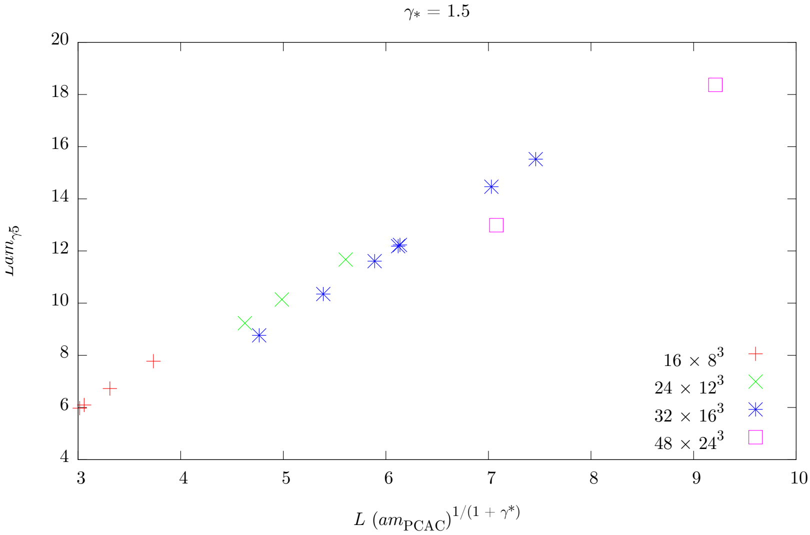

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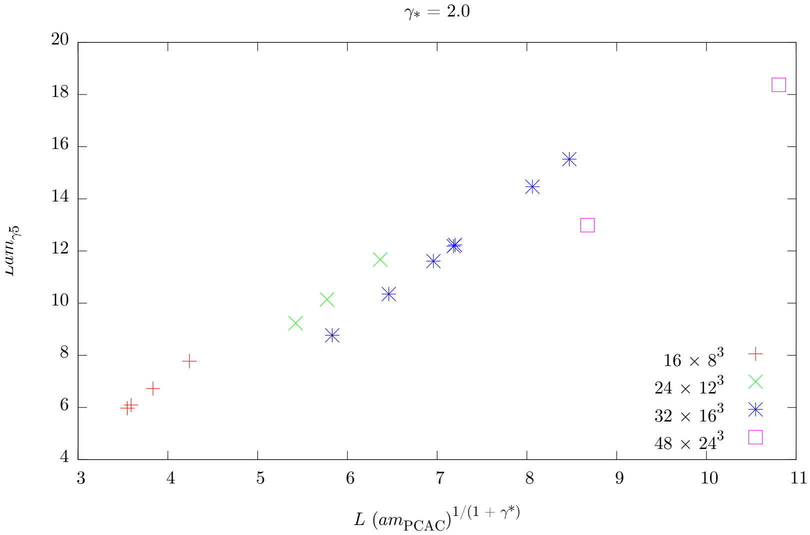

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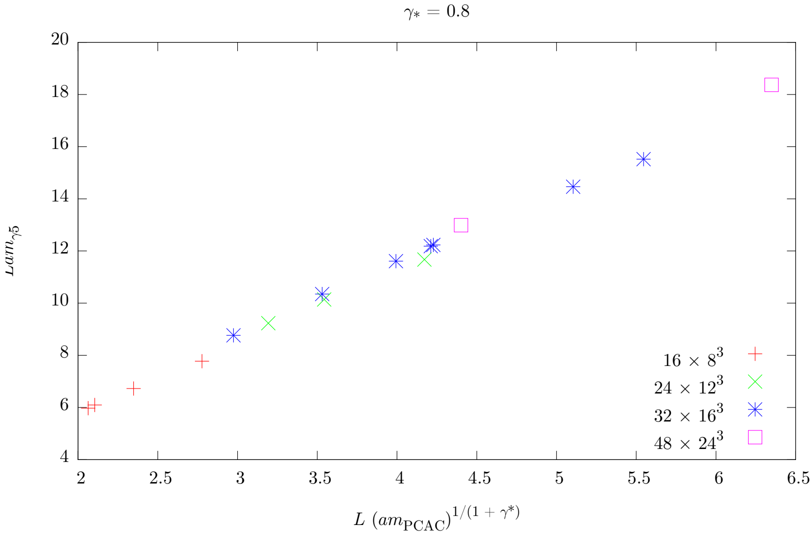

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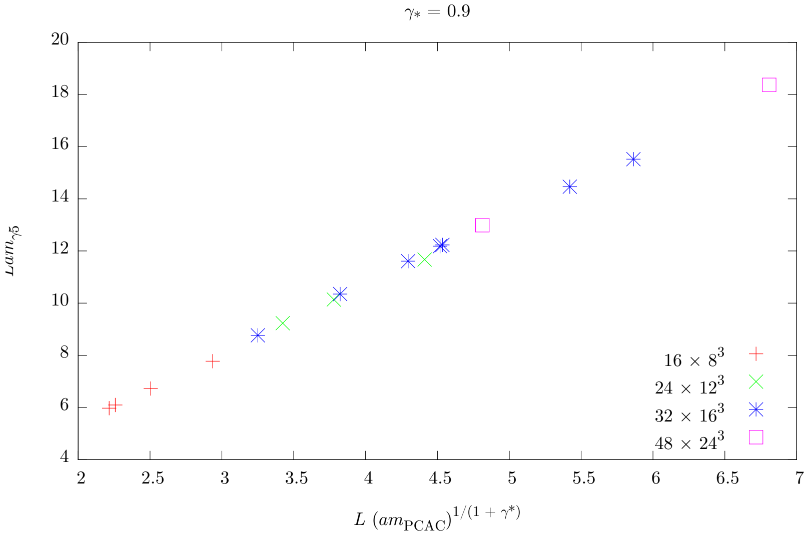

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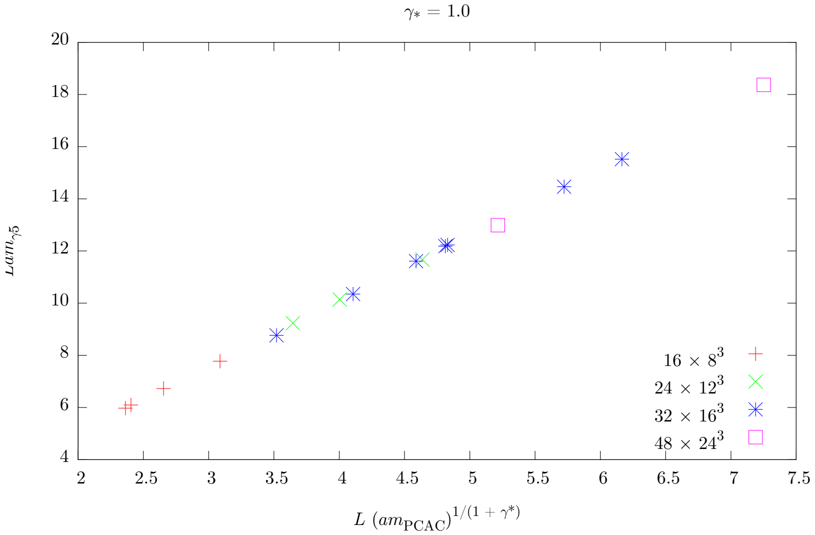

Mass anomalous dimension

- Mass anomalous dimension $\gamma_* \sim 1$ important for WTC
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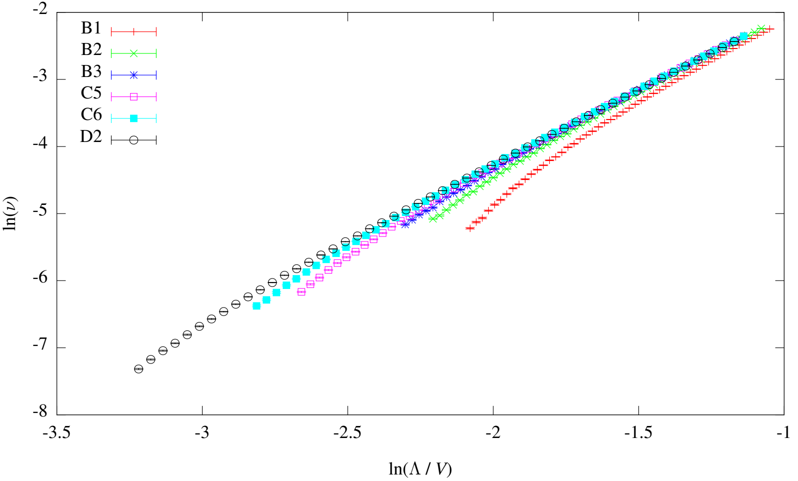

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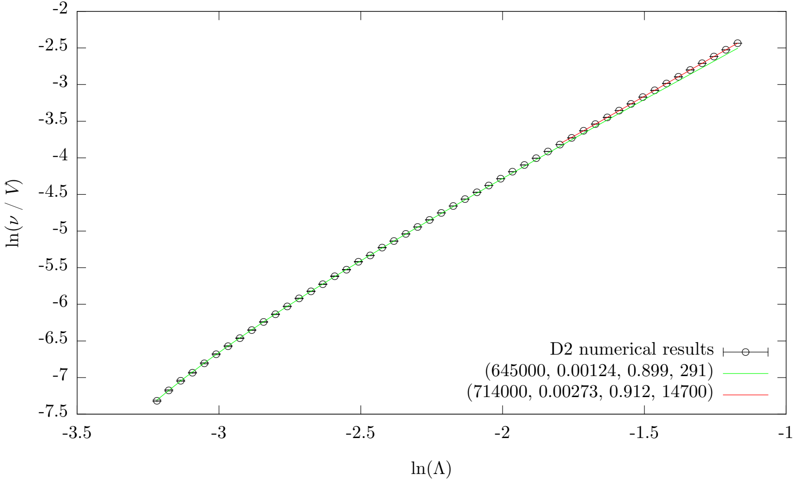

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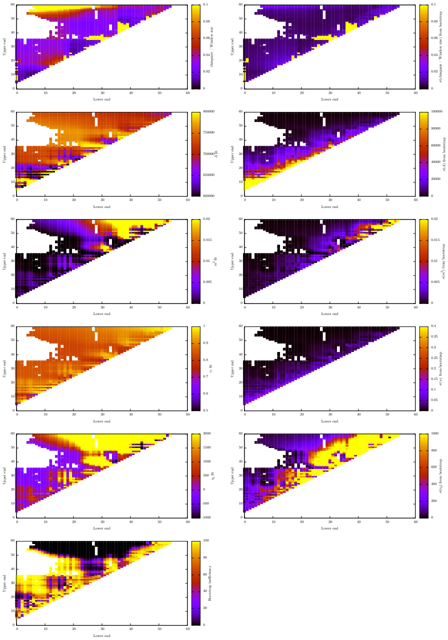

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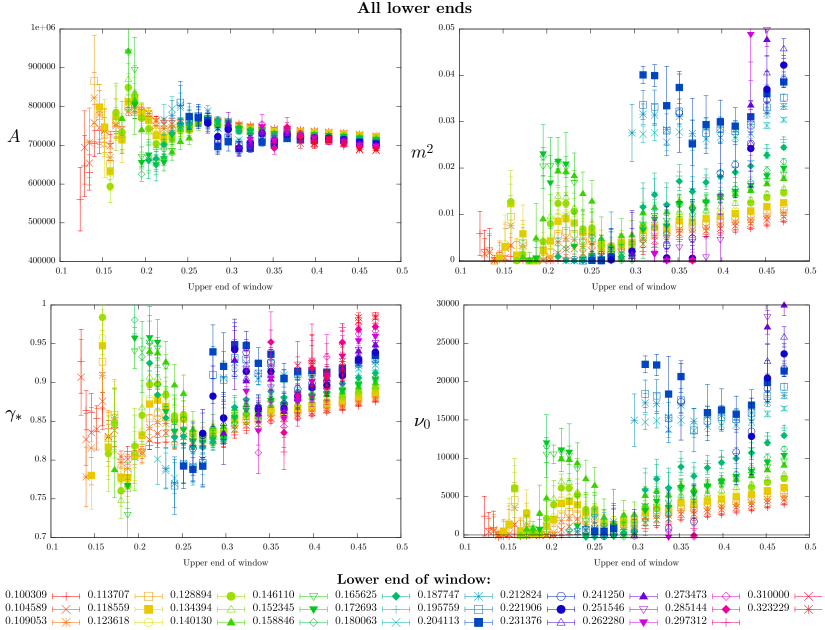

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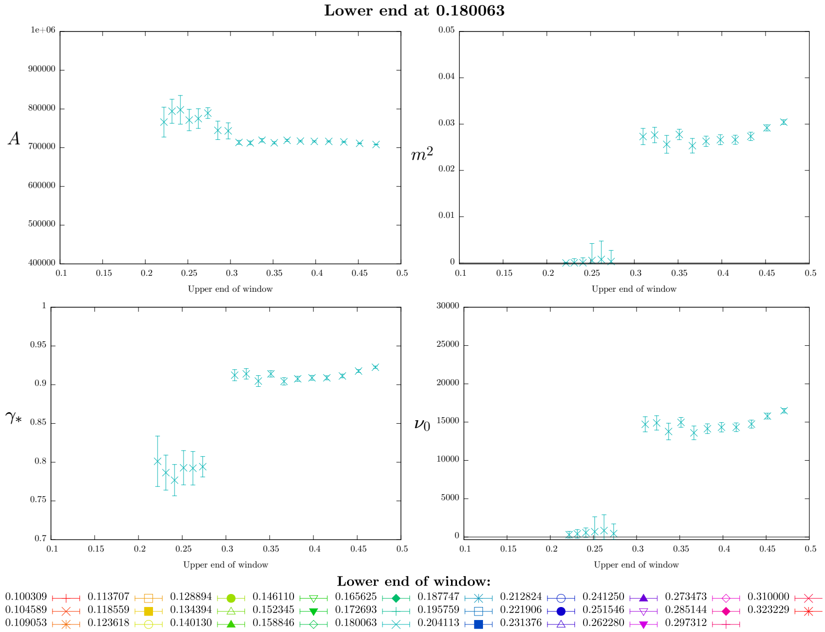

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  - Pure gauge $\mathrm{SU}(2)$
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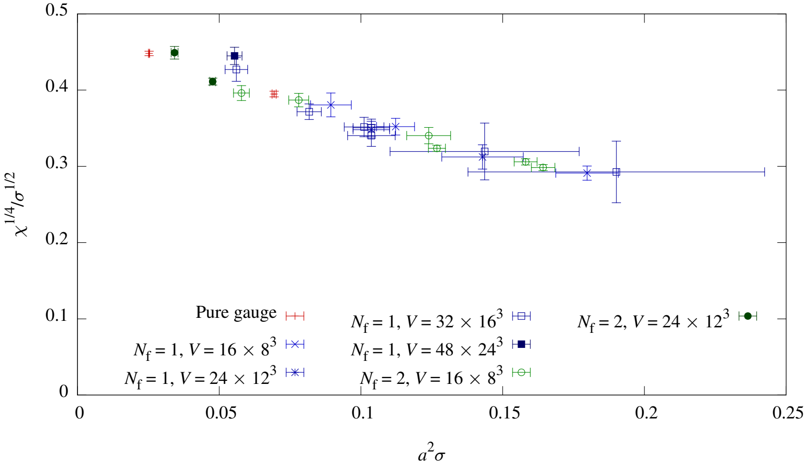
Topological susceptibility



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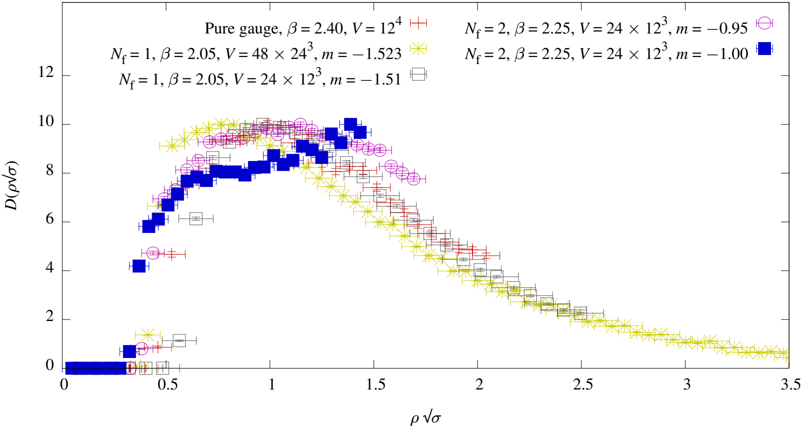
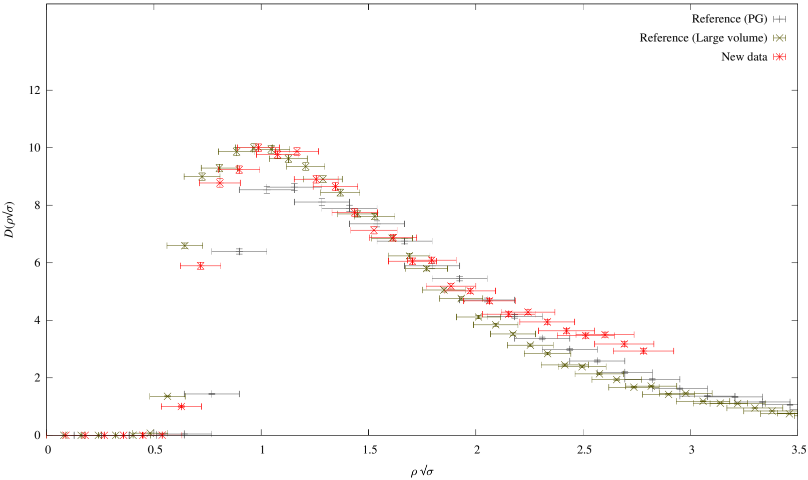
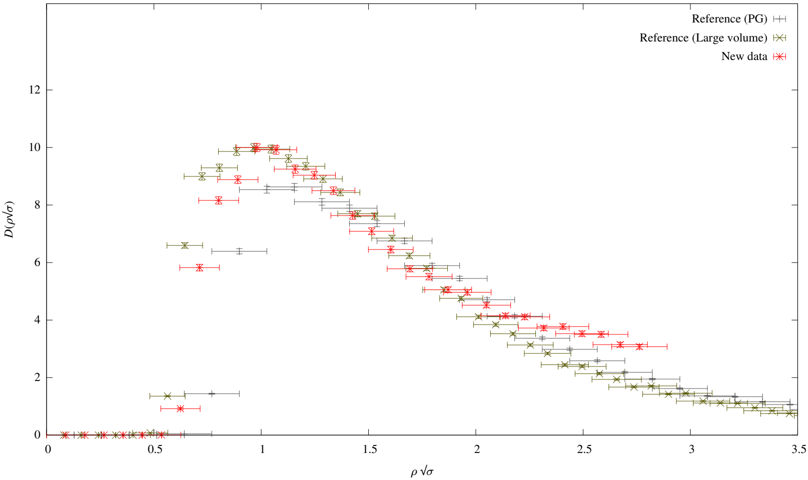


Finite-volume effects from instanton size distribution



Finite volume effects under control

Finite-volume effects from instanton size distribution



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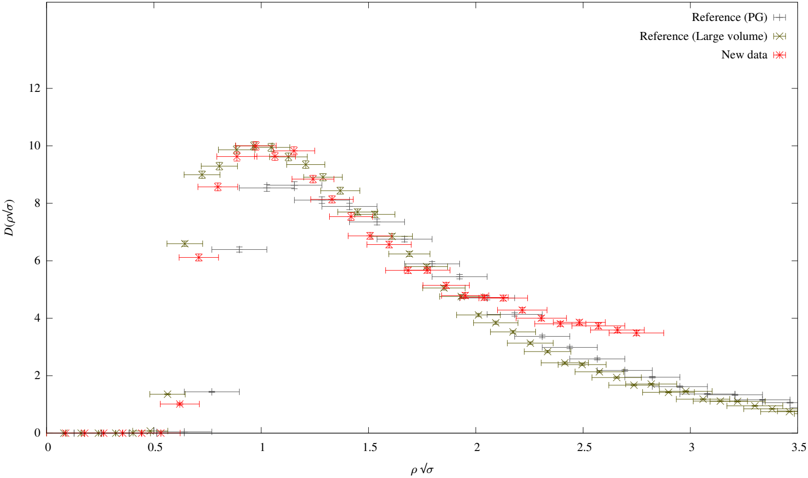

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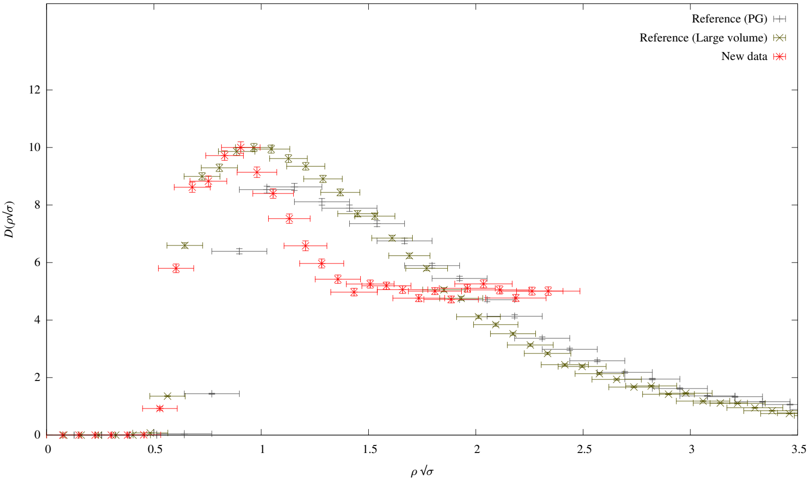

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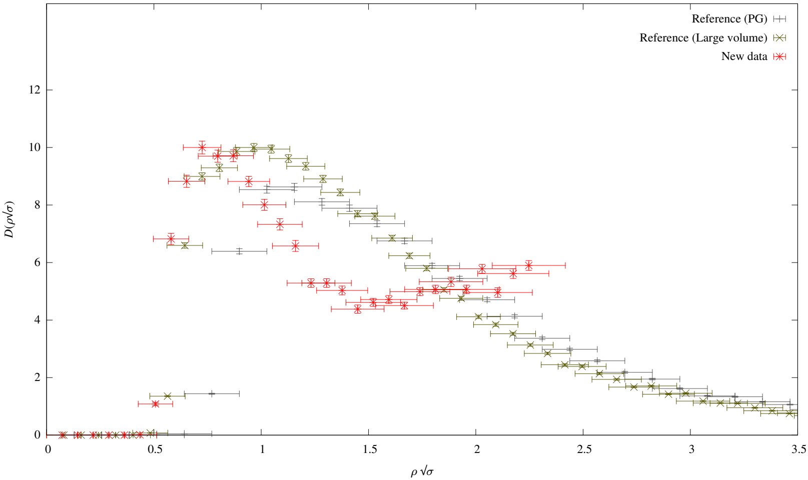

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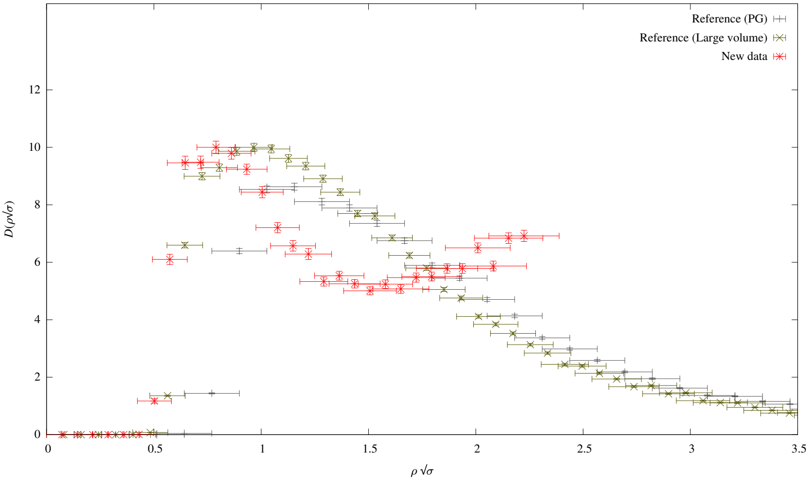

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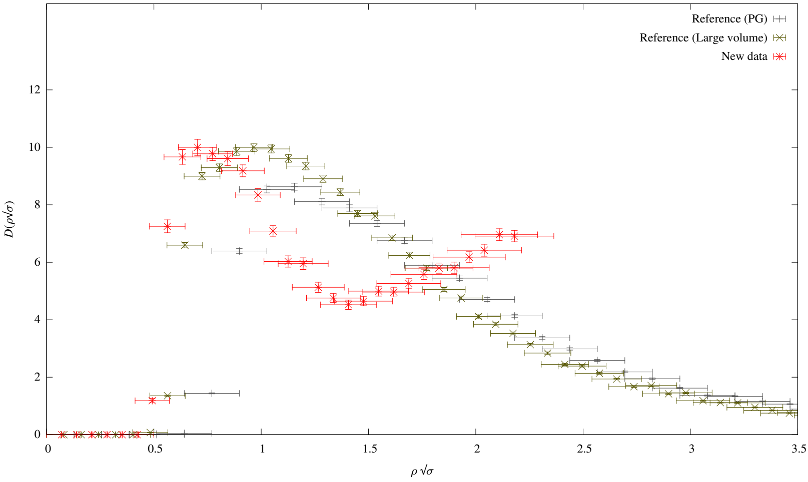

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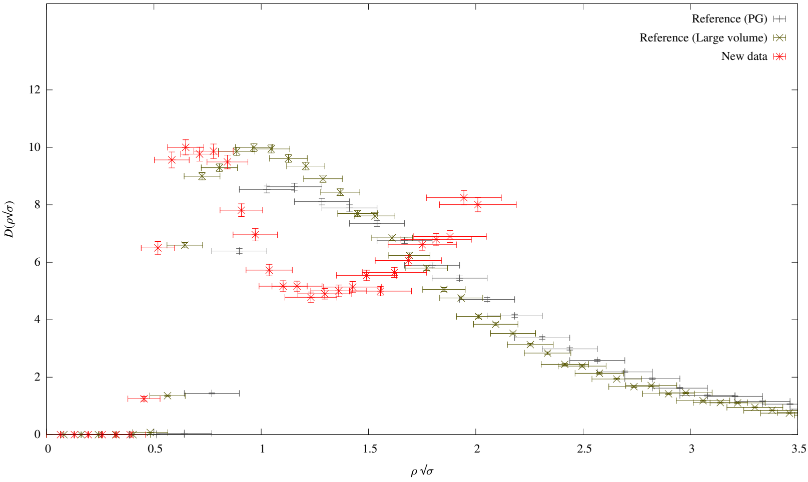

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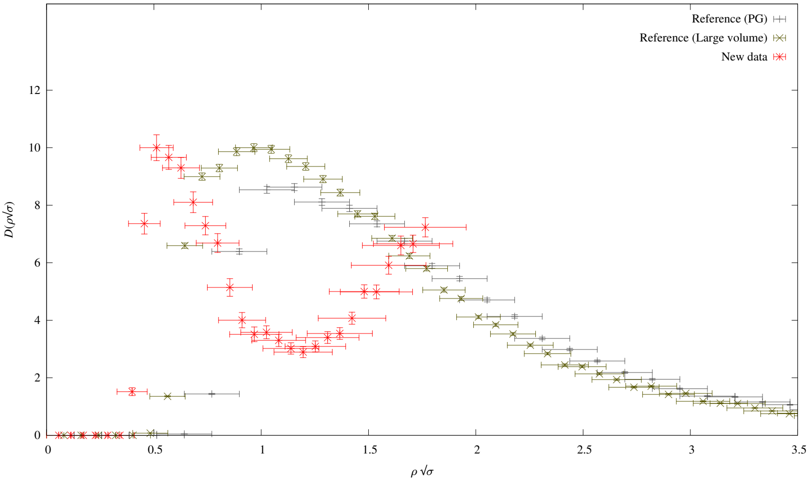
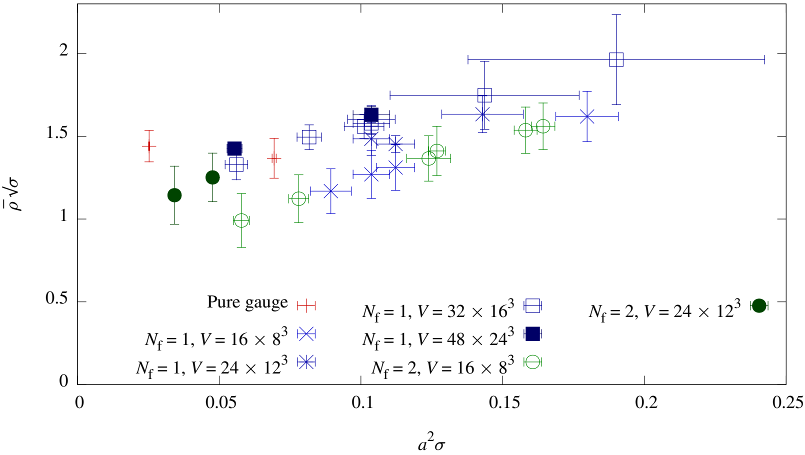


Finite volume effects severely uncontrolled

Average instanton size



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Conclusions

- First lattice study of $\mathrm{SU}(2) + 1$ adjoint Dirac flavour
- Constant mass ratios, topology consistent with (near-)conformality
- Light scalar present in spectrum
- Mass anomalous dimension is large, ~ 1
- Topology of MWT confirms existing findings of conformality
- Instanton size distribution gives clear indication of finite-volume artefacts

Current work

This project

- $\beta = 2.05$ work to be published "soon"
- Higher values of β to confirm continuum limit (currently $\beta = 2.2$)
- Larger volumes ($V = 64 \times 32^3, 96 \times 48^3$
- Lower m (towards chiral limit, look for signs of χSB)
- Look to running of coupling via Wilson flow

With LatKMI

- $N_\mathsf{f} = 4$ QCD is chorally broken, $N_\mathsf{f} = 8$ walking, $N_\mathsf{f} = 12$ conformal
- Compare topological observables to $\mathrm{SU}(3)$ YM using gradient flow smoothing
- Expect $N_\mathsf{f} = 12 \equiv$ YM, $N_\mathsf{f} = 4 \neq$ YM, $N_\mathsf{f} = 12 \approx$ YM
- Use mode number to verify existing calculations of γ_*


ご清聴ありがとうございました！
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