Energy momentum tensor on the lattice and the gradient flow

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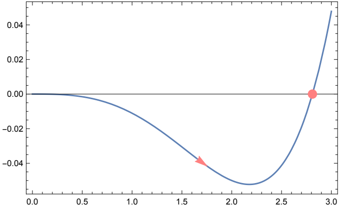
Motivations

Study of IR fixed points in gauge theories [Banks & Zaks 81]

$$\beta(g) = \mu \frac{d}{d\mu} g(\mu) = -b_0 g^3 - b_1 g^5 + \dots$$



More generally:

$$S[\phi] = \int \mathrm{d}^D x \; \hat{g}_k \, \mu^{D-d_k} \, O_k(x)$$

$$\hat{\beta}_k(\hat{g}) = \mu \frac{d}{d\mu} \hat{g}_k(\mu)$$

$$\hat{\beta}_k(\hat{g}^*) = 0$$

IRFP $\Longrightarrow$ scale-invariance at large distances

IRFP at strong coupling

Possible existence of IRFP at strong coupling

- nonperturbative formulation: lattice field theory
- large anomalous dimensions: phenomenology of composite Higgs

Identification of fixed points in numerical simulations

- test of scaling relations [LDD & Zwicky, LDD, Lucini, Patella, Pica & Rago, LDD et al]
- numerical computation of beta functions [Bursa, LDD, Keegan, Pica & Pickup]
- systematic errors in lattice simulations: finite volume, finite mass, finite lattice spacing
- dilatation Ward identities: study of the EM tensor

Ward identities

$$\phi \mapsto \phi' = \phi + \epsilon\delta\phi$$

$$\begin{aligned}\langle\mathcal{P}\rangle &= \frac{1}{Z}\int \mathcal{D}\phi\, e^{-S[\phi]}\,\mathcal{P} = \frac{1}{Z}\int \mathcal{D}\phi'\, e^{-S[\phi']}\,\mathcal{P}' \\ &= \frac{1}{Z}\int \mathcal{D}\phi\, e^{-S[\phi]}\left[1-\delta S\right]\,\left[\mathcal{P}+\delta\mathcal{P}\right]\end{aligned}$$

$$\boxed{\langle -(\delta S)\,\mathcal{P}\rangle + \langle\delta\mathcal{P}\rangle = 0}$$

if $\epsilon = \mathrm{const}$ is a global symmetry

$$\begin{aligned}\delta S &= \int \mathrm{d}^D x\,\epsilon(x)\left[-\partial^\mu J_\mu(x) + \Delta(x)\right] \\ \delta\mathcal{P} &= \int \mathrm{d}^D x\,\epsilon(x)\delta_x\mathcal{P}\end{aligned}$$

local Ward identities

local WI:

$$\langle \partial^\mu J_\mu(x)\, \mathcal{P} \rangle = \langle \Delta(x) \mathcal{P} \rangle - \langle \delta_x \mathcal{P} \rangle$$

usually:

$$\begin{aligned} \mathcal{P} &= \phi(x_1) \dots \phi(x_n) \\ \delta_x \mathcal{P} &= \sum_i \delta(x - x_i)\, \phi(x_1) \dots \delta\phi(x_i) \dots \phi(x_n) \end{aligned}$$

in the absence of explicit breaking:

$$\langle \int \mathrm{d}^D x\, \partial^\mu J_\mu(x)\, \mathcal{P} \rangle = 0 = -\langle \int \mathrm{d}^D x\, \delta_x \mathcal{P} \rangle$$

Translations

Finite transformations:

$$x \mapsto x' = x + a_\rho$$
$$\phi(x) \mapsto \phi'(x') = \phi(x)$$

Infinitesimal transformations:

$$\begin{aligned} \delta_\rho x^\mu &= \delta_\rho^\mu \\ \delta_\rho \phi(x) &= \frac{\phi'(x) - \phi(x)}{\epsilon} = -\frac{\partial}{\partial x_\rho}\phi(x) \end{aligned}$$

Noether current:

$$\boxed{J^\mu_{(\rho)}(x) = T^{\mu\rho}(x)}$$

TWI

$$\boxed{\langle \partial_\mu T^{\mu\rho}(x)\phi(x_1)\ldots\phi(x_n)\rangle = -\sum_j \delta(x-x_j)\langle\phi(x_1)\ldots\delta_\rho\phi(x)\ldots\phi(x_n)\rangle}$$

where the *canonical* EMT

$$T_c^{\mu\nu} = \Pi^\mu \partial^\nu \phi - g^{\mu\nu}\mathcal{L}$$
$$\Pi^\mu = \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi)}$$

alternatively one can use the *symmetric Belinfante* EMT

$$T_B^{\mu\nu} = T_c^{\mu\nu} + \partial_\alpha X^{\alpha\mu\nu}$$

integrated TWI

$$0 = \int \mathrm{d}^D x \, \langle \partial_\mu T^{\mu\nu}(x)\mathcal{P}\rangle = -\int \mathrm{d}^D x \, \langle \delta_x \mathcal{P}\rangle = \langle \delta\mathcal{P}\rangle$$

Dilatations

Finite transformations:

$$x \mapsto x' = \lambda x$$
$$\phi(x) \mapsto \phi'(x') = \lambda^{-d_\phi} \phi(x)$$

Infinitesimal transformations:

$$\delta x^\mu = x^\mu$$
$$\delta\phi(x) = \frac{\phi'(x) - \phi(x)}{\epsilon} = -\left[x_\mu \frac{\partial}{\partial x_\mu} + d_\phi \right] \phi(x)$$

Noether current:

$$\boxed{D^\mu(x) = x_\nu T_B^{\mu\nu}(x) + V^\mu(x) = x_\nu T^{\mu\nu}}$$

$$V^\mu = \Pi_\nu \left[g^{\mu\nu} d_\phi - \mathbf{\Sigma}^{\nu\mu} \right] \phi$$

DWI

$$\langle \partial_\mu D^\mu(x) \phi(x_1) \dots \phi(x_n) \rangle = \langle \Delta(x) \phi(x_1) \dots \phi(x_n) \rangle -$$
$$- \sum_j \delta(x - x_j) \langle \phi(x_1) \dots \delta\phi(x) \dots \phi(x_n) \rangle$$

yields

$$\int \mathrm{d}^D x \, \langle T^\mu_\mu(x) \phi(x_1) \dots \phi(x_n) \rangle = \int \mathrm{d}^D x \, \langle \Delta(x) \phi(x_1) \dots \phi(x_n) \rangle +$$
$$+ n d_\phi \langle \phi(x_1) \dots \phi(x_n) \rangle \, .$$

and

$$\int \mathrm{d}^D x \, \langle T^\mu_\mu(x) \phi(x_1) \dots \phi(x_n) \rangle = - \int \mathrm{d}^D x \, x_\nu \langle \partial_\mu T^{\mu\nu}(x) \phi(x_1) \dots \phi(x_n) \rangle$$
$$= \sum_{j=1}^n x_j \cdot \partial_j \langle \phi(x_1) \dots \phi(x_n) \rangle \, .$$

Trace anomaly

$$\boxed{\begin{aligned}&\langle \int \mathrm{d}^D x \; T^\mu_\mu(x)\, \phi(x_1) \ldots \phi(x_n) \rangle = \\ &\qquad\qquad = \left[- \sum_k \frac{\hat{\beta}_k(\hat{g})}{\hat{g}_k} \, \hat{g}_k \frac{\partial}{\partial \hat{g}_k} + n \Delta_\phi \right] \langle \phi(x_1) \ldots \phi(x_n) \rangle\end{aligned}}$$

where: $\Delta_\phi = d_\phi + \gamma_\phi$

$\boxed{T^\mu_\mu \text{ probes the beta functions of the theory}}$

[Adler, Collins & Duncan, 1977]

Renormalization of $T_{\mu\nu}$

- if the regulator preserves translation invariance $T_{\mu\nu}$ does not renormalize
- lattice regularization breaks translation invariance
- $T_{\mu\nu}$ needs to be renormalized
- renormalization by imposing the validity of Ward identities [Caracciolo et al 90]

- brief review of Ward identities and broken symmetries
- translation WI, finiteness of $T_{\mu\nu}$
- gradient flow and WI along the flow
- lattice space-time transformations
- renormalization conditions

Ward identities & renormalization

For a symmetry-preserving regulator:

$$\langle -(\delta S)\, \mathcal{P} \rangle + \langle \delta \mathcal{P} \rangle = 0$$

local WI:

$$\langle \partial_\mu J^\mu(x)\, \mathcal{P} \rangle = -\langle \delta_x \mathcal{P} \rangle$$

when the theory is renormalized, then $\delta_x \mathcal{P}$ is finite

$$\Longrightarrow \partial_\mu J^\mu \text{ is finite.}$$

Breaking by the regulator

Well-known examples in QFT: restored in the continuum limit
anomalies

Explicit breaking in the WI:

$$\langle \partial_\mu J^\mu(x)\, \mathcal{P} \rangle = \langle \Delta(x)\, \mathcal{P} \rangle + \langle X(x) \mathcal{P} \rangle - \langle \delta_x \mathcal{P} \rangle$$

breaking by irrelevant operators, e.g.:

$$X(x) = aO(x) = a \left[\frac{1}{Z_O} O_R(x) - \frac{1}{a} \bar{\Delta}(x) - \frac{1}{a} (Z_J - 1) \partial^\mu J_\mu \right]$$

hence:

$$\langle \partial^\mu \mathbf{J}_{\mathbf{R},\mu}\, \mathcal{P} \rangle = \langle \Delta_R(x) \mathcal{P} \rangle - \langle \delta_x \mathcal{P} \rangle + O(a)$$

symmetry is recovered/Noether current renormalizes [Bochicchio et al 84]

Translation Ward identities

Pure gauge theory, e.g. using dim reg:

$$S = -\frac{1}{2g_0^2} \int \mathrm{d}^D x \; \mathrm{Tr}\; F_{\sigma\tau} F_{\sigma\tau}$$

TWI:

$$\delta_\rho A_\mu(x) \stackrel{\mathrm{def}}{=} \epsilon_\rho(x) F_{\rho\mu}(x)\,, \quad \delta_\rho S = \int \mathrm{d}^D x \; \partial_\mu \epsilon_\rho(x)\, T_{\mu\rho}(x)$$

$$\boxed{T_{\mu\rho} = -\frac{2}{g_0^2} \mathrm{Tr} \left[F_{\sigma\mu} F_{\sigma\rho} - \frac{\delta_{\mu\rho}}{4} F_{\sigma\tau} F_{\sigma\tau} \right]}$$

$$\delta_{x,\rho} \mathcal{P} \stackrel{\mathrm{def}}{=} \frac{\delta \mathcal{P}}{\delta A_\mu^A(x)} F_{\rho\mu}^A(x)$$

finiteness of $T_{\mu\nu}$:

$$\boxed{\langle \partial_\mu T_{\mu\rho}(x)\, \mathcal{P} \rangle = -\langle \delta_{x,\rho} \mathcal{P} \rangle}$$

Gradient flow: the essential toolkit

Flow of fields $\bar{\varphi}_t(x),\ t \geq 0$:

$$\bar{\varphi}\big|_{t=0} = \phi(x)$$
$$\partial_t \bar{\varphi}_t(x) = -\left.\frac{\delta S}{\delta \phi(x)}\right|_{\phi(x)=\bar{\varphi}_t(x)}$$

[Luscher 10]

Recursive solution by expanding in powers of the fundamental field:

$$\bar{\varphi}_t(x) = \mathcal{F}_t[\phi(x)] = \int \mathrm{d}^D y\, K_t(x-y)\,\phi(y) + \text{non linear terms}$$
$$K_t(z) = \frac{1}{(4\pi t)^{D/2}} \exp\left[-\frac{z^2}{4t}\right]$$

Field correlators of $\bar{\varphi}$ can be computed e.g. in perturbation theory

Gradient flow in D+1 dimensions

$D+1$ dimensional theory with independent fields $\varphi(t,x)$ and $L(t,x)$

$$S_{D+1}[\varphi, L] = S[\varphi(0)] + S_{\rm flow}[\varphi, L]$$
$$S_{\rm flow} = \int_0^\infty \mathrm{d}t \int \mathrm{d}^D x \ \left[L(t,x) \left(\partial_t \varphi(t,x) + \frac{\delta S}{\delta \varphi(t,x)} \right) \right]$$

Integrating out the Lagrange multiplier L:

$$\boxed{\left.\langle \mathcal{O}(\varphi) \rangle\right|_{\varphi, L} = \left.\langle \mathcal{O}(\bar{\varphi}) \rangle\right|_{\phi}}$$

Local theory in $D+1$ dimensions

All divergencies are renormalized by renormalizing the boundary theory

[Luscher & Weisz 11, Zinn-Justin & Zwanziger 86]

Ward identities along the flow [LDD, A Patella, A Rago 13]

Consider a probe made of fields at $T > 0$:

$$\delta_{x,\rho}\mathcal{P}_T = \int d^D y \, \frac{\delta \mathcal{P}_T}{\delta \bar{B}^B_{T,\nu}(y)} \boxed{\frac{\delta \bar{B}^B_{T,\nu}(y)}{\delta \bar{B}^A_{0,\mu}(x)}} F^A_{\rho\mu}(x)$$

Divergences in $\delta_{x,\rho}\mathcal{P}_T$ are difficult to study when working with A_μ fields

BUT

$$\langle \delta_{x,\rho}\mathcal{P}_T \rangle = \langle \tilde{T}_{0\rho}(0,x)\mathcal{P}_T \rangle$$
$$\tilde{T}_{0\rho}(t,x) = -2\,\mathrm{Tr}\; L_\mu(t,x) G_{\rho\mu}(t,x)$$

$\tilde{T}_{0\rho}(t,x)$ is a local composite operator in the $D+1$ dim theory

$$\boxed{\tilde{T}_{0\rho,R} = Z_\delta \tilde{T}_{0\rho}}$$

Ward identities along the flow

TWI:

$$\boxed{\langle \partial_\mu T_{\mu\rho}(x)\,\mathcal{P}_T\rangle = -\langle \delta_{x,\rho}\mathcal{P}_T\rangle = -\langle \tilde{T}_{0\rho}(0,x)\mathcal{P}_T\rangle}$$

in a regularization that does not break translation symmetry, e.g. dim reg:

$$\boxed{\tilde{T}_{0\rho}(0,x) \text{ is finite, } Z_\delta = 1}$$

Dilatations along the flow

Local dilatations are generated by:

$$\epsilon_\rho(x) = x_\rho \lambda(x)$$

i.e. they are a special case of the translations discussed above

DWI

$$\langle \partial^\mu D_\mu(x) \mathcal{P}_T \rangle - \langle T^\mu_\mu(x) \mathcal{P}_T \rangle = -\langle x^\rho \delta_{x,\rho} \mathcal{P}_T \rangle$$

$$\int \mathrm{d}^D y \, y^\rho \delta_{y,\rho} \phi_T(x) = \left[2T \frac{d}{dT} + x^\rho \partial_\rho + d_\phi \right] \phi_T(x)$$

$$\langle \int \mathrm{d}^D y \, T^\mu_\mu(y) \phi_T(x) \rangle = \left[2T \frac{d}{dT} + d_\phi \right] \langle \phi_T(x) \rangle$$

T^μ_μ can be probed just by looking at the T dependence.

Lattice Ward identities

$$\hat{\delta}_\rho U_\mu(x) = \epsilon_\rho(x)\hat{F}_{\rho\mu}(x)U_\mu(x)$$
$$\hat{\delta}_{x,\rho}\mathcal{P} = \frac{1}{a^3}\hat{F}^A_{\rho\mu}(x)\partial^A_{U_\mu(x)}\hat{\mathcal{P}}$$

Variation of the action:

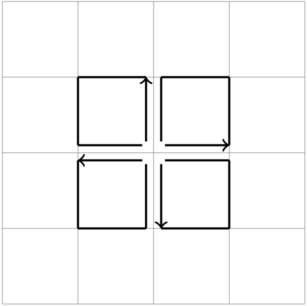
$$\hat{\delta}_\rho \hat{S} = -a^4 \sum_x \epsilon_\rho(x)\left\{\nabla_\mu \hat{T}_{\mu\rho}(x) + X_\rho(x)\right\}$$
$$\hat{T}_{\mu\rho}(x) = \hat{T}^{[1]}_{\mu\rho}(x) + \hat{T}^{[3]}_{\mu\rho}(x) + \hat{T}^{[6]}_{\mu\rho}(x)$$

$X_\rho(x) = a\mathcal{O}_\rho(x)$ irrelevant operators

$$X_\rho = a\left[\frac{1}{Z_\delta}O_{R,\rho} + \frac{1}{a}\left(\frac{c_1}{Z_\delta} - 1\right)\nabla_\mu \hat{T}^{[1]}_{\mu\rho} + \frac{1}{a}\left(\frac{c_3}{Z_\delta} - 1\right)\nabla_\mu \hat{T}^{[3]}_{\mu\rho} + \right.$$
$$\left. + \frac{1}{a}\left(\frac{c_6}{Z_\delta} - 1\right)\nabla_\mu \hat{T}^{[6]}_{\mu\rho}\right]$$

[Caracciolo et al 90]

Lattice Ward identities



$$\hat{F}^A_{\mu\nu}(x) = F^A_{\mu\nu}(x) + O(a^2)$$

$$\hat{T}^{[1]}_{\mu\rho} = -\frac{2}{g_0^2}\delta_{\mu\rho}\sum_{\sigma\tau}\mathrm{Tr}\ \hat{F}_{\sigma\tau}\hat{F}_{\sigma\tau}$$

$$\hat{T}^{[3]}_{\mu\rho} = -\frac{2}{g_0^2}\delta_{\mu\rho}\sum_{\sigma}\mathrm{Tr}\left[\hat{F}_{\sigma\mu}\hat{F}_{\sigma\mu} - \frac{1}{4}\sum_{\tau}\hat{F}_{\sigma\tau}\hat{F}_{\sigma\tau}\right]$$

$$\hat{T}^{[6]}_{\mu\rho} = -\frac{2}{g_0^2}(1-\delta_{\mu\rho})\sum_{\sigma}\mathrm{Tr}\ \hat{F}_{\mu\sigma}\hat{F}_{\rho\sigma}$$

Lattice Ward identities

$$X_\rho = a\left[\frac{1}{Z_\delta} O_{R,\rho} + \frac{1}{a}\left(\frac{c_1}{Z_\delta} - 1\right)\nabla_\mu \hat{T}^{[1]}_{\mu\rho} + \frac{1}{a}\left(\frac{c_3}{Z_\delta} - 1\right)\nabla_\mu \hat{T}^{[3]}_{\mu\rho} + \right.$$
$$\left. + \frac{1}{a}\left(\frac{c_6}{Z_\delta} - 1\right)\nabla_\mu \hat{T}^{[6]}_{\mu\rho}\right]$$

Terms appearing in the renormalization of X_ρ renormalize $T_{\mu\rho}$:

$$(\hat{T}_{\mu\rho})_R = \sum_{i=1,3,6} c_i \hat{T}^{[i]}_{\mu\rho}$$

$$\boxed{\langle \nabla_\mu \hat{\mathbf{T}}_{\mathbf{R},\mu\rho}(x)\hat{\mathcal{P}}\rangle = -\langle Z_\delta \hat{\delta}_{x,\rho}\hat{\mathcal{P}} + O_{R,\rho}(x)\hat{\mathcal{P}}\rangle}$$

Probe at the boundary

For a probe defined at the boundary

$$\hat{\mathcal{P}} = \hat{\phi}(x_1)_R \ldots \hat{\phi}(x_n)_R$$

$$\lim_{a\to 0} \langle \hat{T}_{\mu\rho}(x)_R \hat{\phi}_1(x_1)_R \cdots \hat{\phi}_k(x_k)_R \rangle = \langle T_{\mu\rho}(x) \phi_1(x_1)_R \cdots \phi_k(x_k)_R \rangle$$

$$\lim_{a\to 0} \langle \left\{ Z_\delta \hat{\delta}_{x,\rho} + a O_{R,\rho}(x) \right\} \hat{\phi}_1(x_1)_R \cdots \hat{\phi}_k(x_k)_R \rangle =$$
$$= \sum_j \delta(x - x_j) \frac{\partial}{\partial x_j} \langle \phi_1(x_1)_R \cdots \phi_k(x_k)_R \rangle$$

Probe at the boundary

Fixing the renormalization coefficients:

$$\langle \nabla_\mu \hat{T}_{\mu\rho}(0)_R \hat{\phi}(x_1)_R \dots \hat{\phi}(x_n)_R \rangle = 0\,, \quad \text{for } x_i \neq 0$$

Set $c_1 = 1$, choose several probes $\mathcal{P}^j$:

$$\begin{aligned} &M^{ij} = \langle \nabla_\mu \hat{T}^{(i)}_{\mu\rho}(0)_R\, \mathcal{P}^j \rangle \\ &c_i\, M^{ij} = 0 \end{aligned}$$

Overall normalization:

$$\langle H | \int \mathrm{d}^{D-1}x\, T_{00}(0,x) | H \rangle = M_H$$

[Caracciolo et al 90]

Probe in the bulk

$$(\hat{T}_{\mu\rho})_R = \sum_i c_i \hat{T}^{[i]}_{\mu\rho}$$

$$\boxed{\langle \nabla_\mu \hat{T}_{\mu\rho}(x)_R \hat{\mathcal{P}}_T \rangle = -\langle Z_\delta \hat{\delta}_{x,\rho} \hat{\mathcal{P}}_T + a O_{R,\rho}(x) \hat{\mathcal{P}}_T \rangle}$$

For a probe defined in the bulk:

$$\langle \delta_{x,\rho} \mathcal{P}_T \rangle = Z_\delta \langle \hat{\delta}_{x,\rho} \hat{\mathcal{P}}_T \rangle = Z_\delta \left(-2 \langle \mathrm{Tr}\ \hat{L}_\mu(0,x) \hat{F}_{\rho\mu}(x) \hat{\mathcal{P}}_T \rangle \right)$$

$$\lim_{a \to 0} a \langle O_{R,\rho}(x) \hat{\mathcal{P}}_T \rangle = 0$$

NP renormalization

Determination of the ratios c_i/Z_δ similar to Caracciolo et al:

$$\boxed{\frac{c_i}{Z_\delta}\langle \nabla_\mu \hat{T}^{(i)}_{\mu\rho}(x)\phi^{[j]}_{T,\rho}(0)\rangle = -\langle \hat{\delta}_{x,\rho}\phi^{[j]}_{T,\rho}(0)\rangle}$$

with

$$\phi^{[j]}_{T,\rho}(x) = \nabla_\mu \, \hat{T}^{[j]}(x)\Big|_{U_\mu = V_{t\mu}}$$

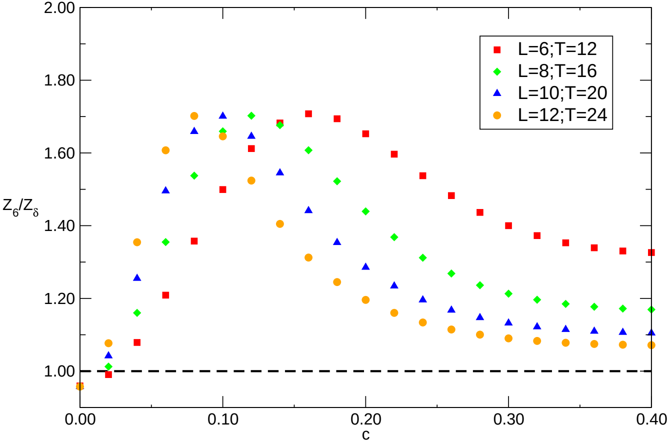
We need to solve a linear system

$$\boxed{M_{ij}\frac{c_i}{Z_\delta} = b_i}$$

$$M_{ij} = \langle \nabla_\mu \hat{T}^{(i)}_{\mu\rho}(x)\phi^{[j]}_{T,\rho}(0)\rangle$$
$$b_i = -\langle \hat{\delta}_{x,\rho}\phi^{[j]}_{T,\rho}(0)\rangle$$

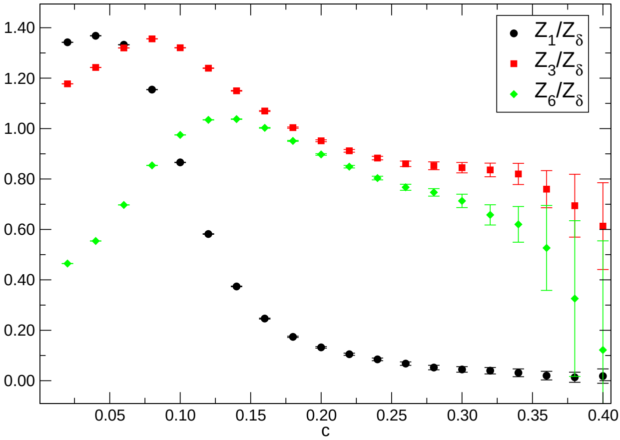
Numerical results

open bc, free theory calculation, $c = \sqrt{8t}/L$



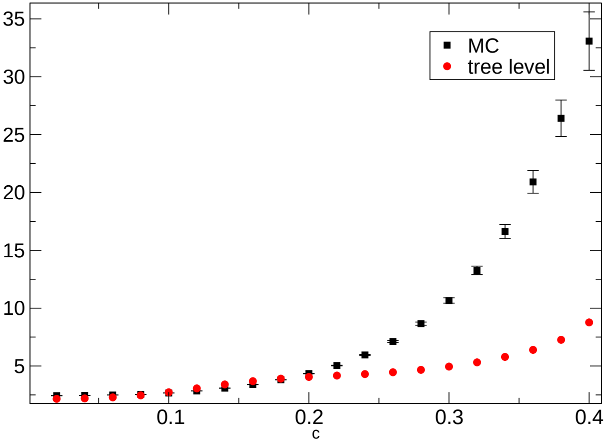
Numerical results

open bc, $\beta = 6.0056$, $16^3 \times 8$ lattice, $c = \sqrt{8t}/L$



Numerical results

open bc, condition number



NP renormalization

Determination of Z_δ using two-point functions:

$$\Phi_T(x_4) = \frac{a^3}{L^3} \sum_{\mathbf{x}} \phi_T(\mathbf{x}, x_4)$$

$$f_\Phi(d, t, z_4) = \langle \Phi_T(z_4)\, a^4 \sum_{y_4=-d}^{+d} \sum_{\mathbf{y}} \hat{\delta}_{y,4} \Phi_T(0) \rangle$$

$$\boxed{Z_\delta = \frac{\langle \Phi_T(z_4) \nabla_4 \Phi_T(0) \rangle}{f_\Phi(d, t, z_4)} + O(e^{-\bar{r}^2/16t})\,,} \quad \bar{r} = \min(d, |z_4 - d|)$$

NP renormalization

Determination of Z_δ using one-point functions:

$$\Phi_T(x_4) = \frac{a^3}{L^3} \sum_{\mathbf{x}} \phi_T(\mathbf{x}, x_4)$$

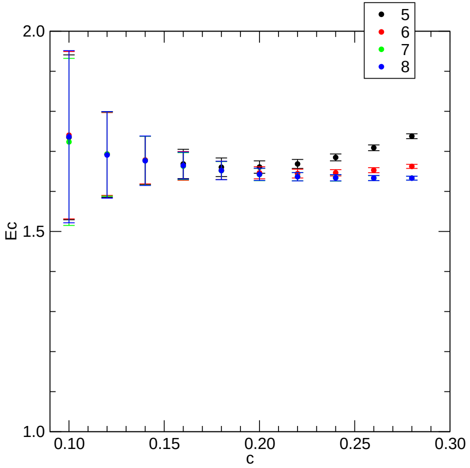
$$h_\Phi(d,t) = \langle a^4 \sum_{y_4=-d}^{+d} \sum_{\mathbf{y}} \hat{\delta}_{y,4} \Phi_T(0) \rangle_{\mathrm{BC}}$$

$$\boxed{Z_\delta = \frac{\langle \nabla_4 \Phi_T(0) \rangle_{\mathrm{BC}}}{h_\Phi(d,t)} + O(e^{-\bar{r}^2/16t})\,,} \quad \bar{r} = \min(d, |z_4 - d|)$$

Several possible choices for the operator ϕ.

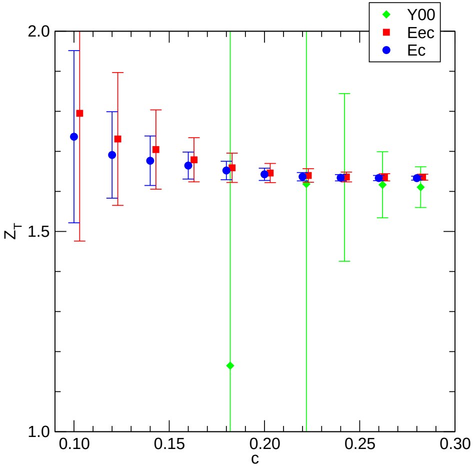
Example: determination of Z_δ

$$\phi = E\,, \quad \sqrt{8t} = cL$$



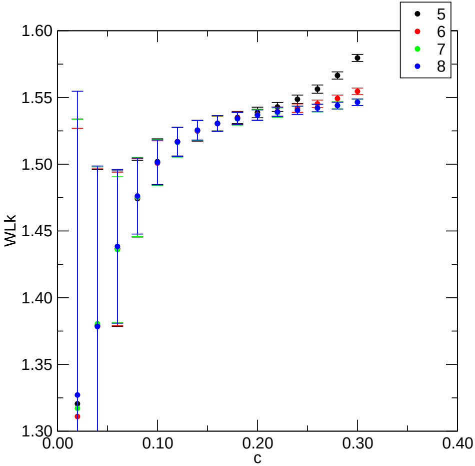
Example: determination of Z_δ

$$\phi = Y_{00}\,, E\,, \quad \sqrt{8t} = cL$$



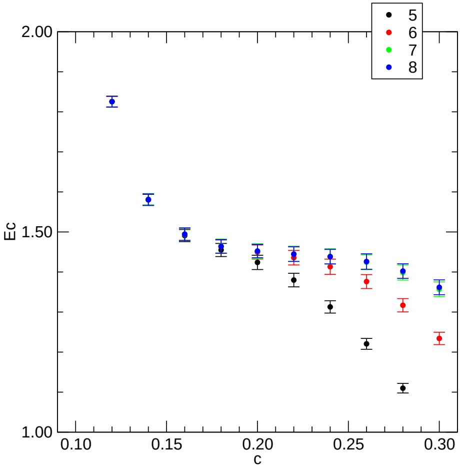
Example: determination of Z_δ

$$\phi = L_k\,, \quad \sqrt{8t} = cL$$



Example: determination of Z_δ

$\phi = E\,, \quad \sqrt{8t} = cL$ using DWI



TODO

- Wilson flow provides a new way to implement Ward identities
- WI along the flow involve finite correlators, no contact terms
- translation Ward identities allow to compute the renormalized EM tensor (more efficiently?)
- a lot of ideas... too many? work in progress...
- use EM tensor to study IRFP, compute anomalous dimensions