Non-perturbative quantum field theory on curved manifolds

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KMI
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Outline

- BSM Motivation
- Our first attempt at lattice radial quantization (BFN)
  R.C. Brower, G.T. Fleming and H. Neuberger, Phys. Lett. B 721 (2013) 299
- What worked and what didn't
- Finite Element Method and ϕ^4 theory
- Future extensions to gauge theories with fermions.

Higgs Boson and Compositeness

- The discovery of the Higgs boson at 126 GeV and no other new physics is the leading theoretical puzzle of our time.
- Composite theories can naturally solve the hierarchy problem as asymptotic freedom makes IR physics insensitive to UV physics.
- The real challenge of composite theories is whether we know how to construct UV-complete versions of them.
- Even if we don’t know how to construct the full theory, do we have adequate low energy effective theories?

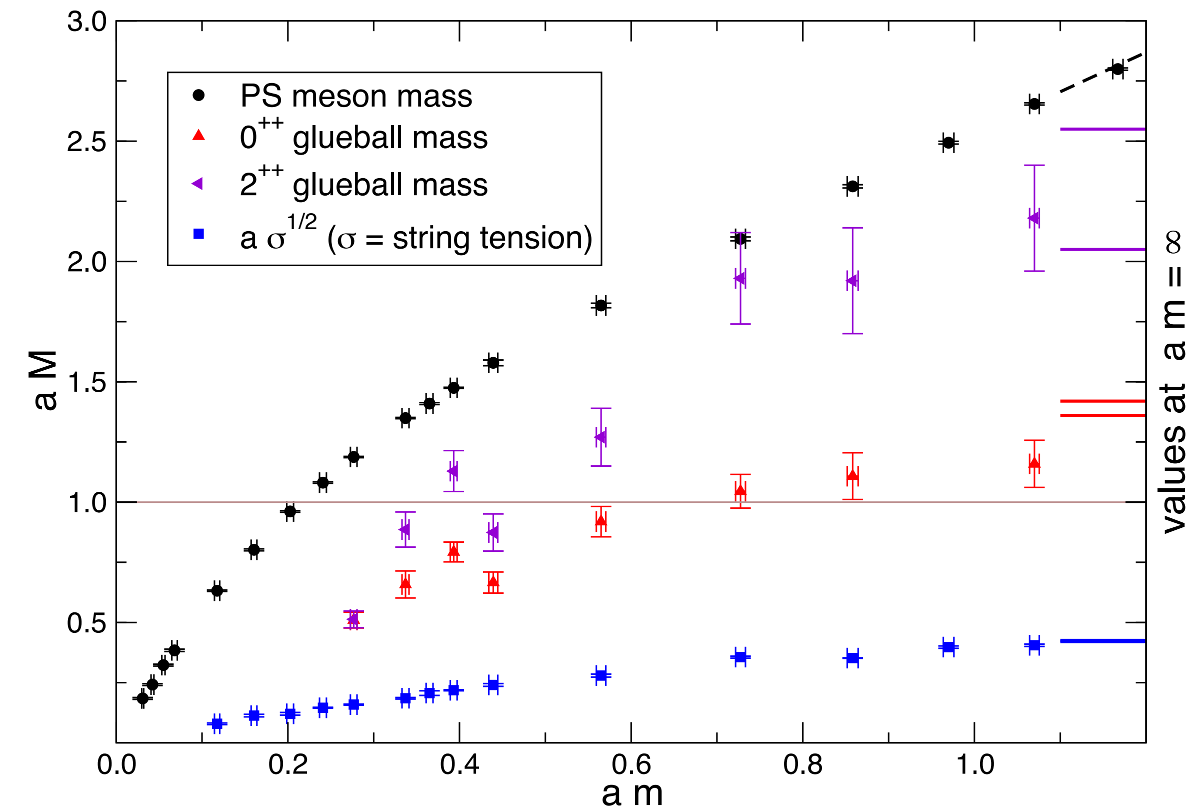
Scalar Sector of QCD

- QCD has five (or six) light isoscalar scalars below charm threshold: $f_0(500)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ [and maybe $f_0(1790)$].
- Only two of these states can be predominantly $(\bar{q}q)$, others might be $(\bar{q}\bar{q})(qq)$ or $(\bar{q}q)(\bar{q}q)$ or even glue balls or pseudo-dilatons. Which one is the σ?
- Example: A recent model [D. Rischke et al., PRD 87 104011 (2013)] has all these effective dof's yet can reproduce $f_0(500)$.
- Lattice QCD might help soon but is still beyond the state of the art.
- If we can't understand the mechanics of the scalar spectrum in QCD even knowing the answer a priori, what's the chance in a composite BSM theory where we've seen at most four energy levels (three NGBs and one Higgs) in the spectrum?

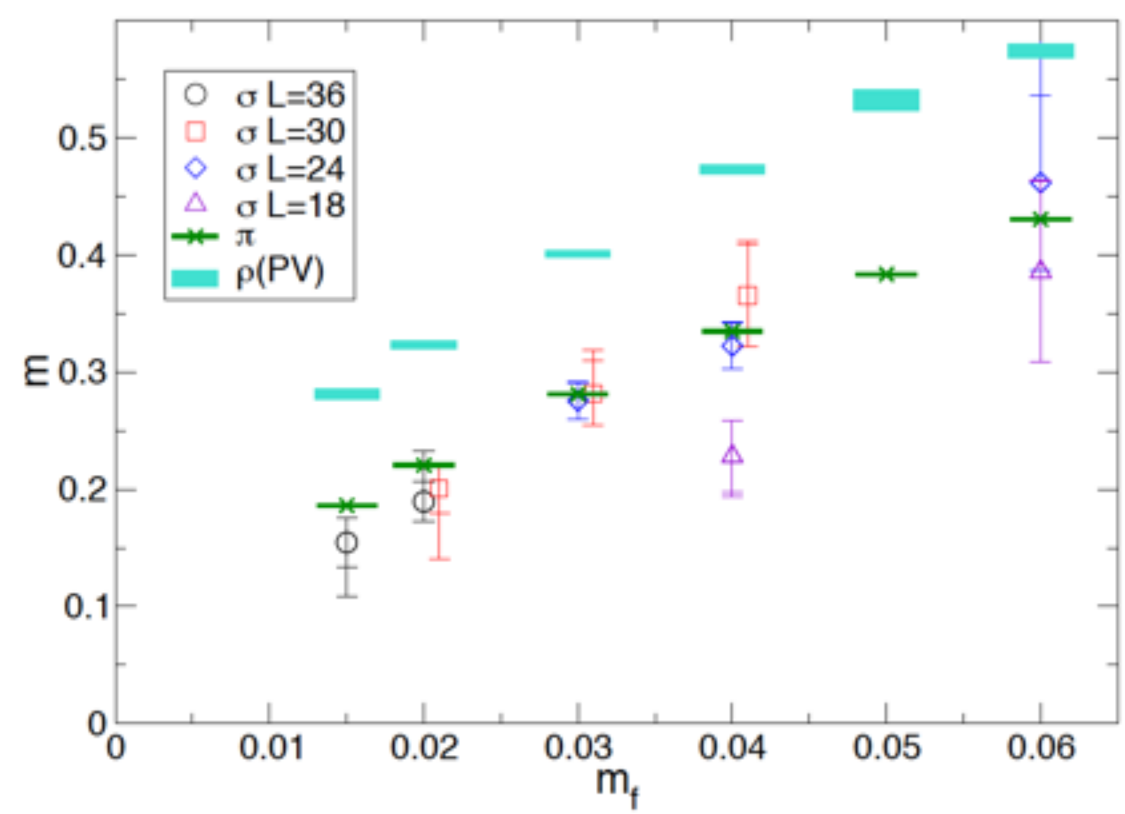
New Evidence for Pseudo-Dilatons

- There have been indications that isoscalar mesons could be lighter than pions in mass-deformed IRFP theories.
- SU(2) N_f=2 adj: Del Debbio et al. 2010
- SU(3) N_f=12: LatKMI Yamawaki et al. 2012
- The big surprise is that this seems to persist to SU(3) N_f=8: a slowly-running (walking?) confining gauge theory.
- SU(3) N_f=2 sym seems to also have a light scalar: LatHC Kuti et al. 2013.
- It's starting to look like slowly running theories have light composite scalars, completely different from QCD.

Phys. Rev. D 82, 014510 (2010)



LatKMI, arXiv:1403.5000

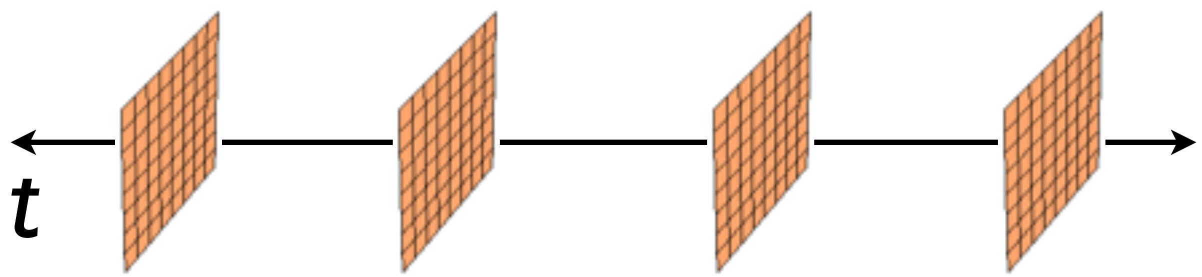


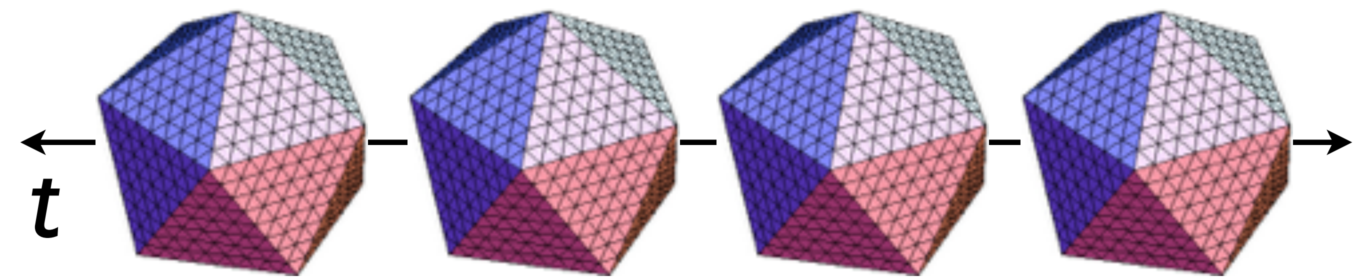
Strongly-coupled CFTs

- In general, a strongly-coupled CFT is a critical point in a strongly-coupled region of a quantum field theory.
- In BSM physics, near conformal theories might be important: e.g. Higgs as a pseudo-dilaton.
- Lattice discretized on R^d has linearly spaced elements but power-law correlations are relevant on exponentially separated scales.
- How can you compute a correlation across many orders of magnitude when the number of lattice sites in any dimension is restricted $O(100)$?
- Hopefully: Lattice Radial Quantization.

Spatial *vs.* Radial Quantization

Correlations of States on slices t *vs.* t′





Non-Conformal

$C(t,t') \propto e^{-E(\vec{p})|t-t'|}$

$E(\vec{p})^2 = |\vec{p}|^2 + m^2$

Conformal

$C(t,t') \propto |t-t'|^{\nu}$

Complicated $\vec{p}$ dependence

Non-Conformal

$C(t,t') \propto e^{-\mathcal{E}(\ell)|t-t'|}$

Complicated ℓ dependence

Conformal

$C(t,t') \propto e^{-\nu_\ell|t-t'|}$

$\nu_\ell = \nu_0 + \ell$

$\boxed{2+10\ s^2 \text{ points per sphere}}$

Why Wilson-Fisher FP in D=2,3?

- In D=2, Onsager solution to Ising model plus many results for φ^4 theory.
- In D=3, no analytic solution but many very efficient numerical algorithms.
- New independent field configurations generated in nearly $O(L^D)$ time: Swendsen-Wang, Wolff, Brower-Tamayo.
- 2-pt functions $<\varphi(x)\varphi(y)>$ computed in $O[(L \log L)^D]$ due to Fast Convolution: Cooley-Tukey (1965); Stockham, Helms, Sande (1966); Driscoll and Healy (1994).
- Combining clustering with Fast Convolution greatly reduces statistical noise: C. Ruge, F. Wagner (1993).
- Learn how to work with scalars, then add fermions and gauge fields.

BFN Lessons Learned #1

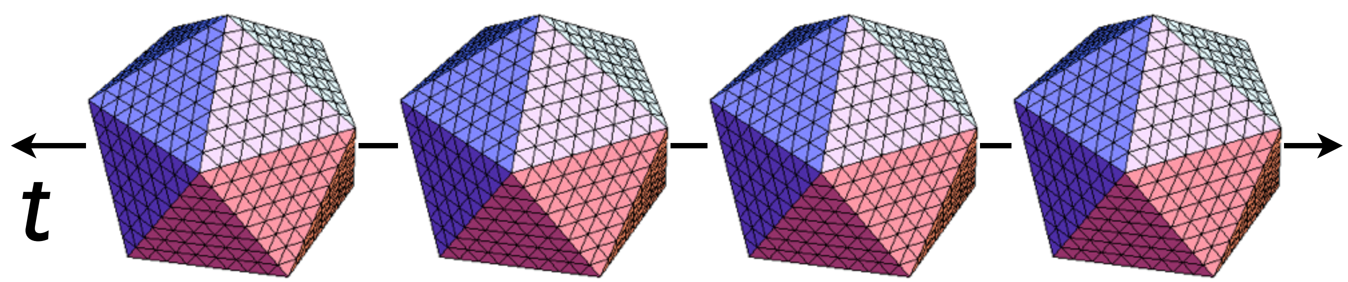
- Consider the D=2 O(N) sigma model at large N in radial quantization on R×S and compare with quantization on R^2.
- Theory is classically conformal broken by a quantum anomaly. How does the conformal breaking become manifest in each case?
- On R^2, one sees Lorentz invariance but not dilatation invariance.
- On R×S, on sees dilatation invariance but not Lorentz invariance

$$\langle \Phi^{*i}_{l'}(\tau)\Phi^j_l(\tau')\rangle = \delta^{ij}\delta_{ll'}\frac{e^{-\sqrt{l^2+\mu^2}|\tau-\tau'|}}{2\sqrt{l^2+\mu^2}} \ .$$

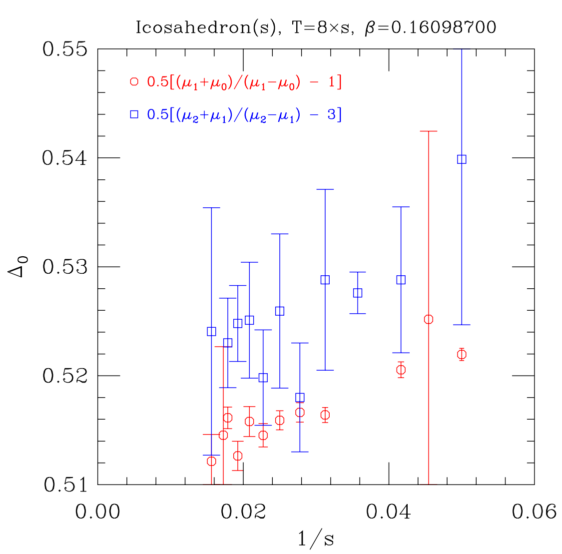
LESSON: Descendants don't have integer-spaced descendants. Consequently, we cannot construct translation generators satisfying the correct commutation relations with dilatations in the sector generated by the action of $\vec{\Phi}$ on the vacuum. The deviation of the dilatation spectrum from equal spacing is small if $l \gg \mu$. Because inversion has also been preserved in the quantization, if translations could be realized, special conformal transformations would come in automatically and the full conformal group would be realized. Because of rotation invariance, only one linear combination of translations needs to be considered in detail.

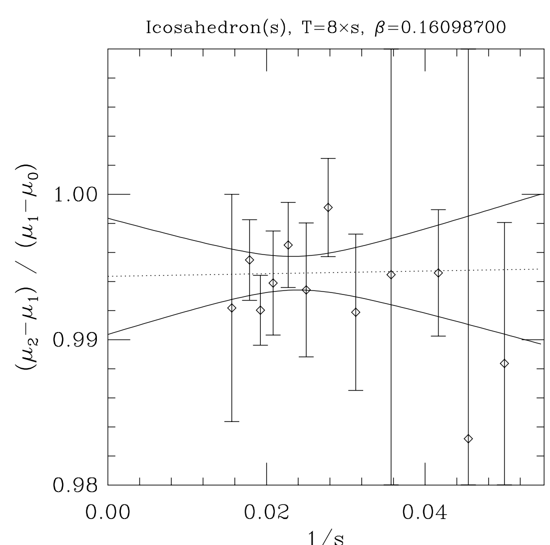
BFN Lessons Learned #2

$$Z_{\text{Ising}} = \sum_{\sigma(x,t)=\pm 1} \exp\left[\beta \sum_{t,\langle x,y\rangle} \sigma(t,x)\sigma(t,y) + \beta \sum_{t,x} \sigma(t,x)\sigma(t+1,x)\right]$$



Good:

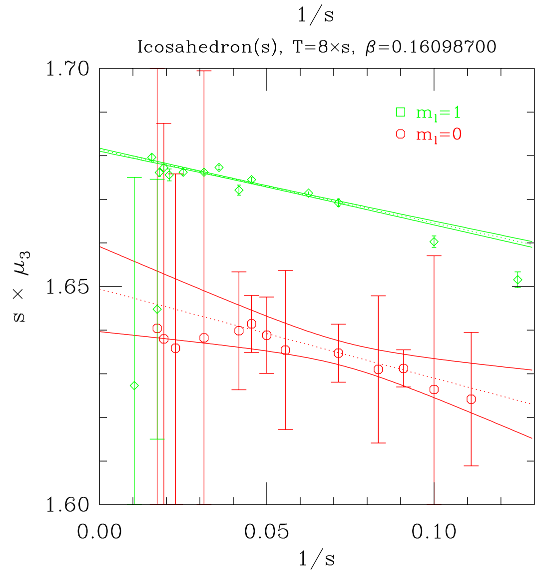




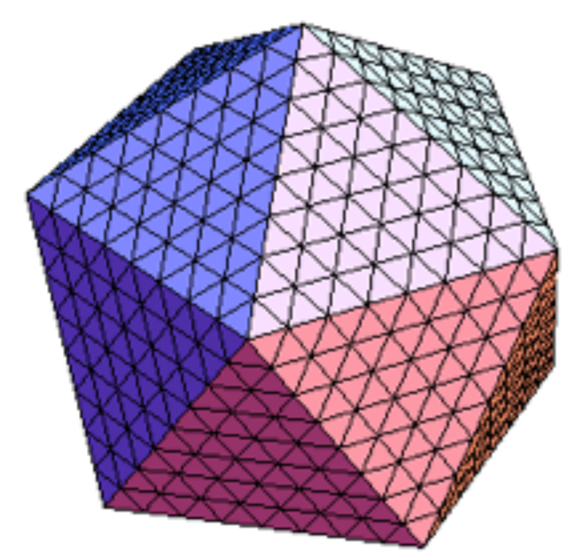
Operator	Spin l	$\mathbb{Z}$	Δ	Exponent
s	0	$-$	0.5182(3)	$\Delta = 1/2 + \eta/2$
s'	0	$-$	$\gtrsim 4.5$	$\Delta = 3 + \omega_A$
ε	0	$+$	1.413(1)	$\Delta = 3 - 1/\nu$
ε'	0	$+$	3.84(4)	$\Delta = 3 + \omega$
ε''	0	$+$	4.67(11)	$\Delta = 3 + \omega_2$
$T_{\mu\nu}$	2	$+$	3	$\Delta = 3$
$C_{\mu\nu\kappa\lambda}$	4	$+$	5.0208(12)	$\Delta = 3 + \omega_{\text{NR}}$

Low-lying primary operators of the 3D Ising model at criticality.

Bad:

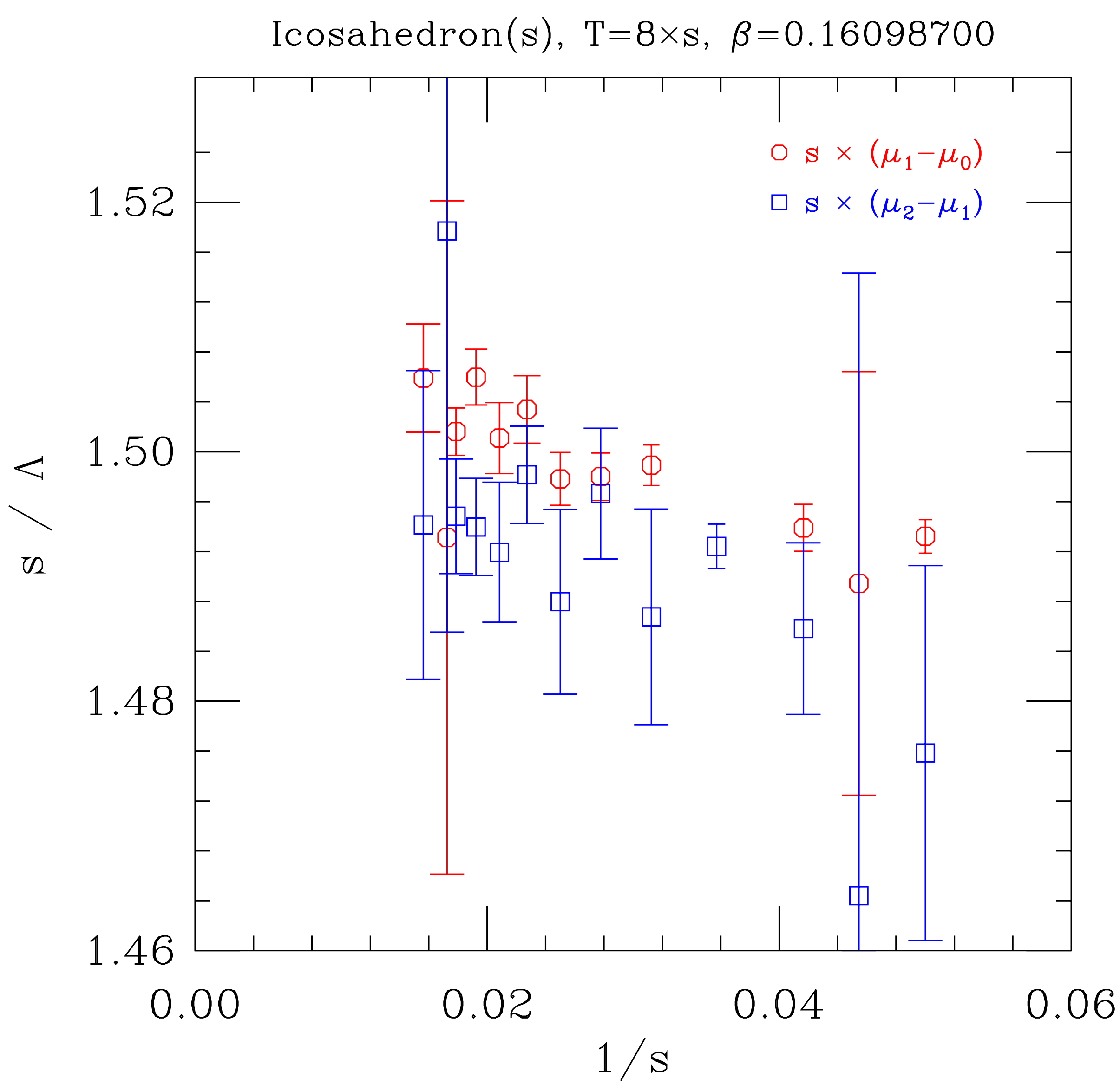


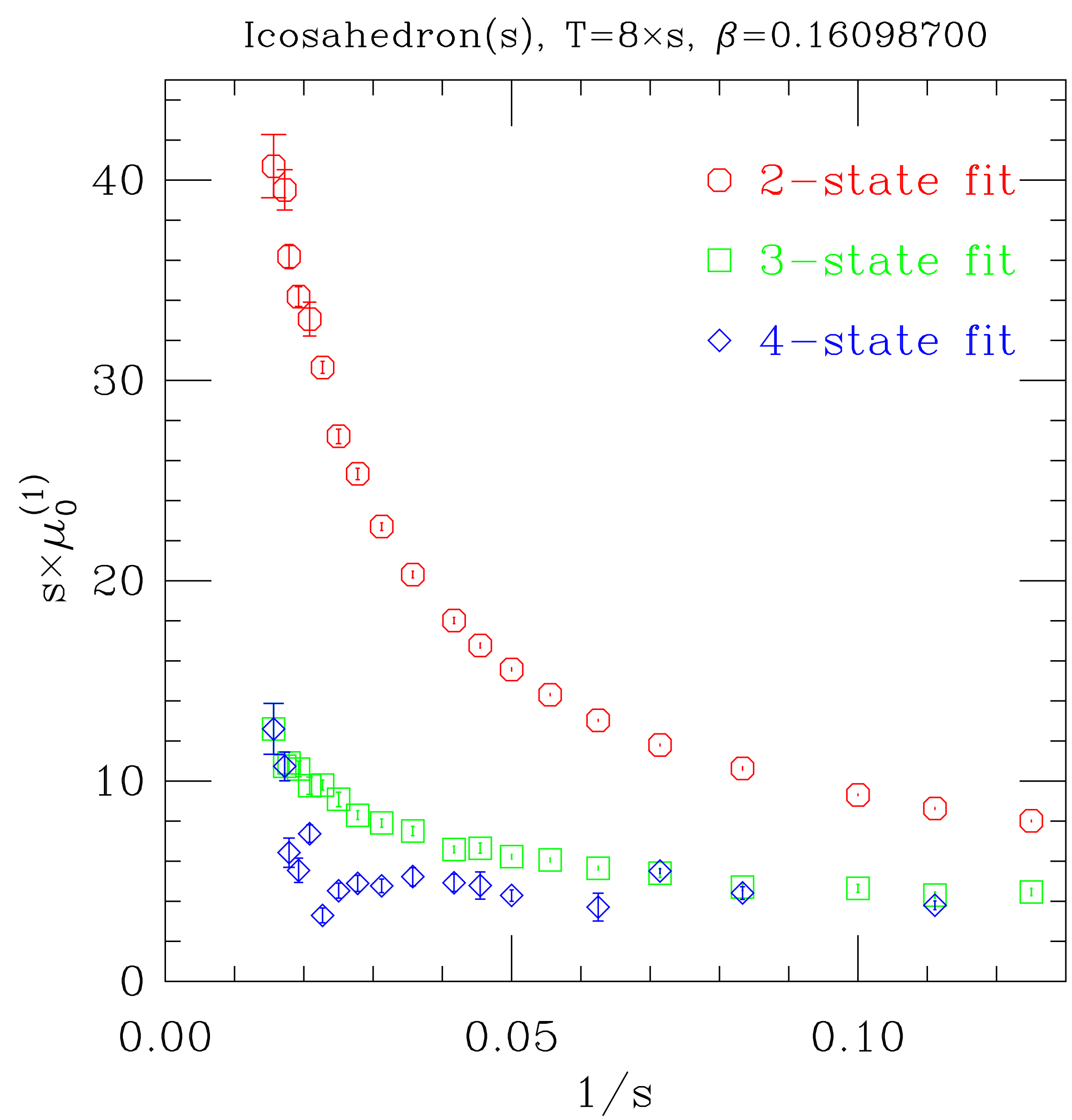
$$\nu_\ell = \nu_0 + \ell$$



BFN Bonus Plot: Higher primaries

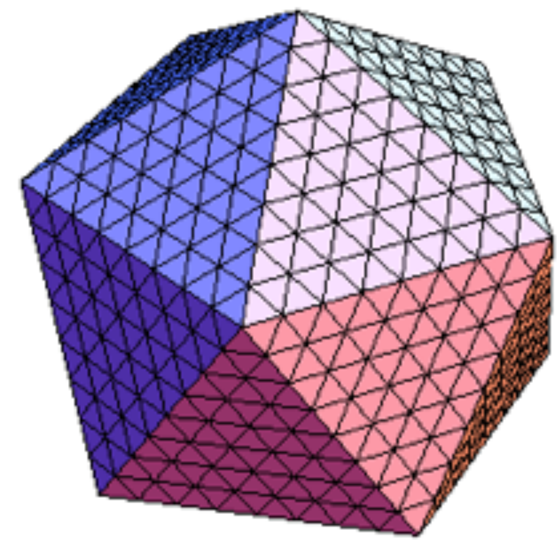
- We know our Ising calculation has some systematic error so we didn't bother to report results for $\Delta_{\sigma'}$

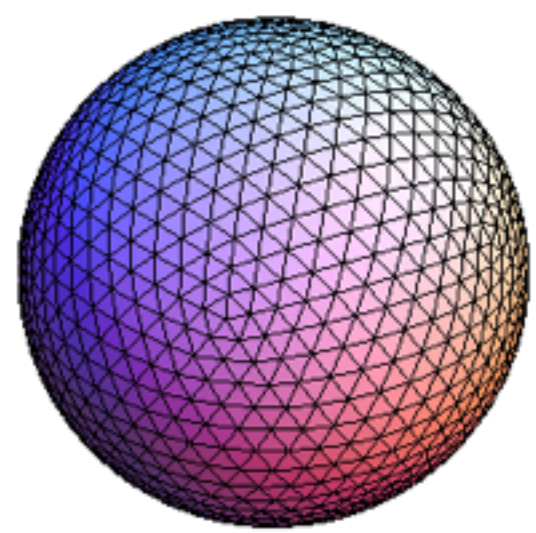


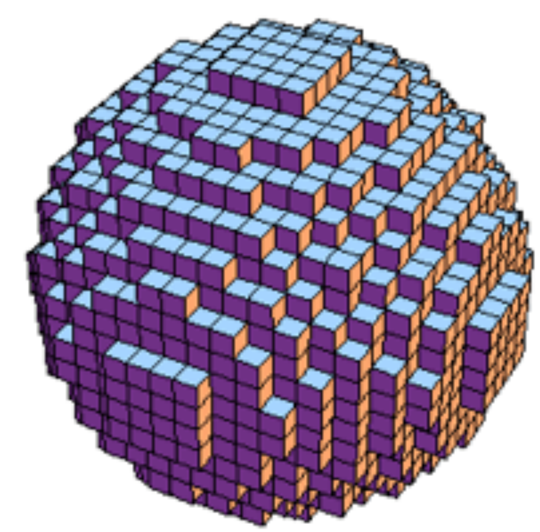


Different Discretization of Spheres

BFN:





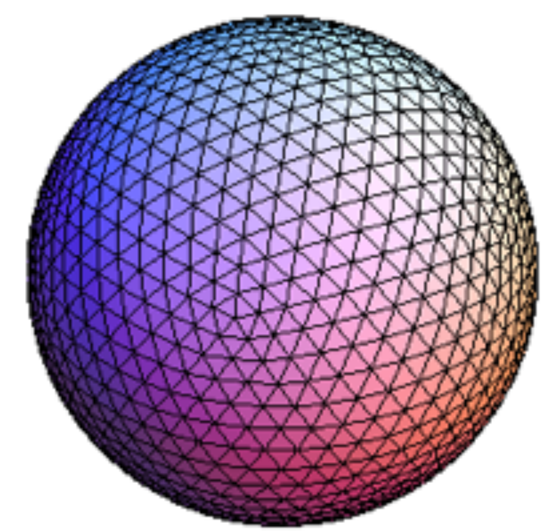


Good:	Very smooth.	Equal-sized elements. Simple action.
Challenge:	Unequal-sized elements. Very complicated action.	Very rough on small scales. May not recover continuum theory.

Scalar Field Theory with Unequal Elements

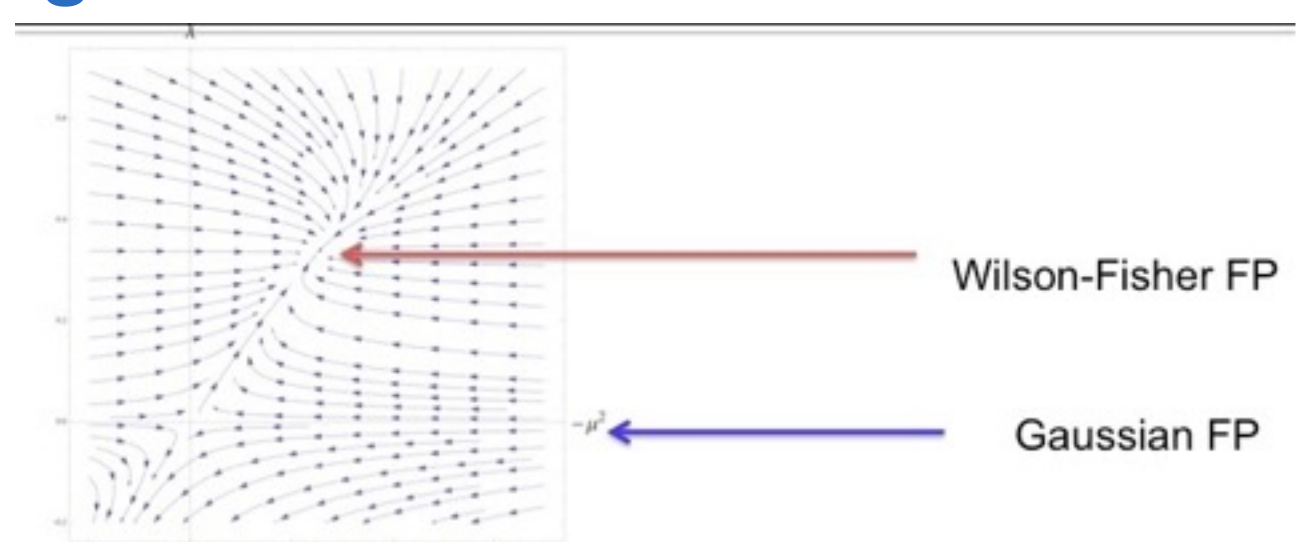
$$Z = \int [D\phi] \exp\left[-K_{ij}(\phi_i - \phi_j)^2 - \omega_i \lambda (\phi_i^2 - \mu^2/2\lambda)^2\right]$$

- The K_{ij} and ω_i are not couplings but they carry information about the geometry of the manifold: the curvature tensor Γ and the measure $g^{1/2}$.



- The couplings are μ^2 and λ and there is a critical point $(\mu^2{}_*, \lambda_*)$ that is the same Wilson-Fisher fixed point as the Ising model.

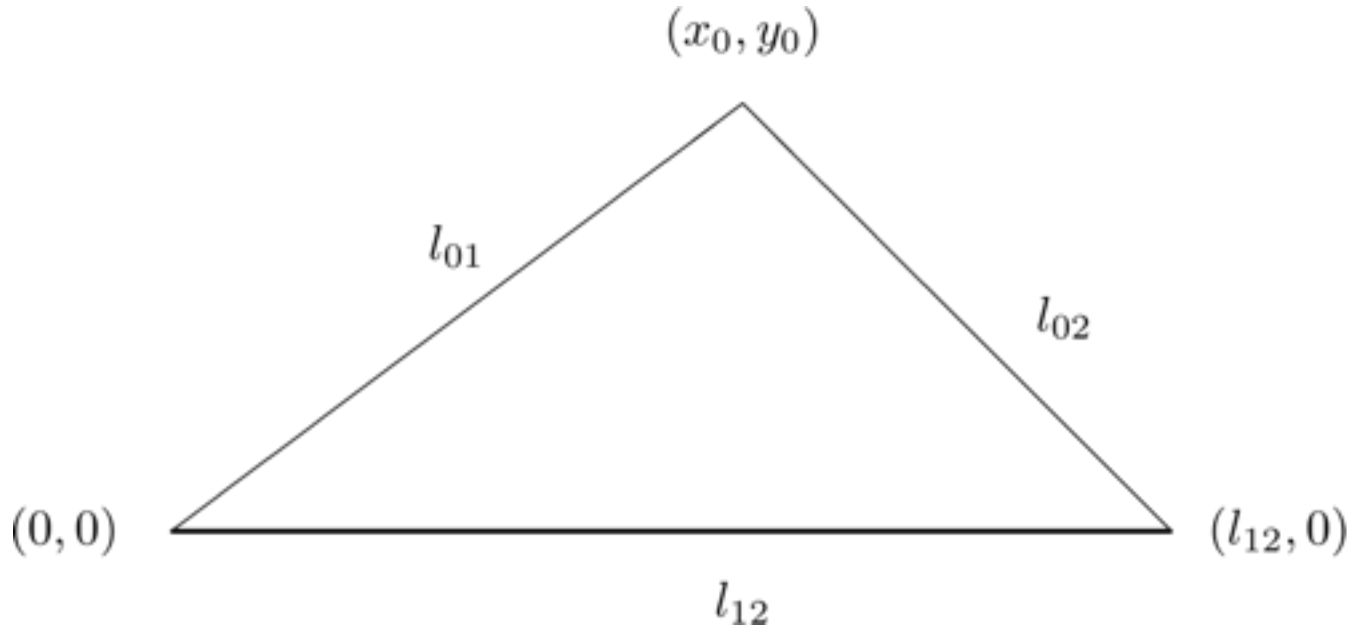
- Problem: find appropriate definition for K_{ij} and ω_i to study the Wilson-Fisher fixed point in the continuum limit, given a grid with varying edge lengths.

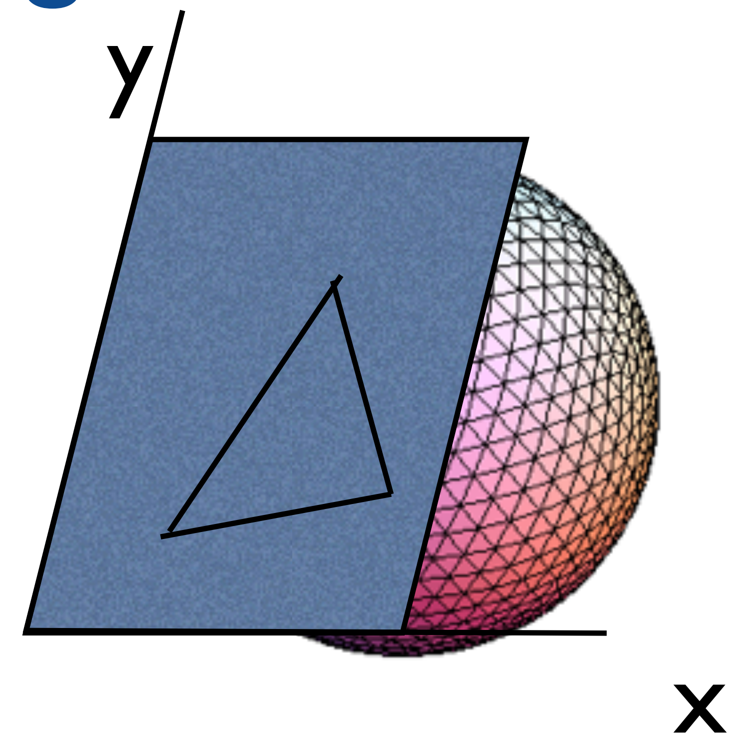


$$\beta_g = \epsilon g - \frac{3}{16\pi^2} g^2 + O(g^3, \epsilon g^2, \mu^4, \mu^2 g;)$$

$$\beta_{\mu^2} = 2\mu^2 + ag + \frac{9}{16\pi^2} g\mu^2 + O(\mu^4) \qquad \lambda = 4g/4!$$

Linear Finite Elements for Triangulated Manifolds





project spherical
triangle onto
local tangent plane

$$
\begin{aligned}
K_1(x,y) &= [l_{12} - x - \frac{(l_{12} - x_0)y}{y_0}]/l_{12} \\
K_2(x,y) &= [x - \frac{x_0 y}{y_0}]/l_{12} \\
K_0(x,y) &= \frac{y}{y_0}
\end{aligned}
$$

On each triangle expand: $\phi(x,y) = \sum_i K_i(x,y)\phi_i$ an integrate

$$
\int_{A_{012}} dxdy\partial_\mu\phi(x,y)\partial_\mu\phi(x,y) = \frac{1}{2A_{012}}[(l_{01}^2 + l_{20}^2 - l_{12}^2)(\phi_1 - \phi_2)^2 + \text{cyclic}]
$$

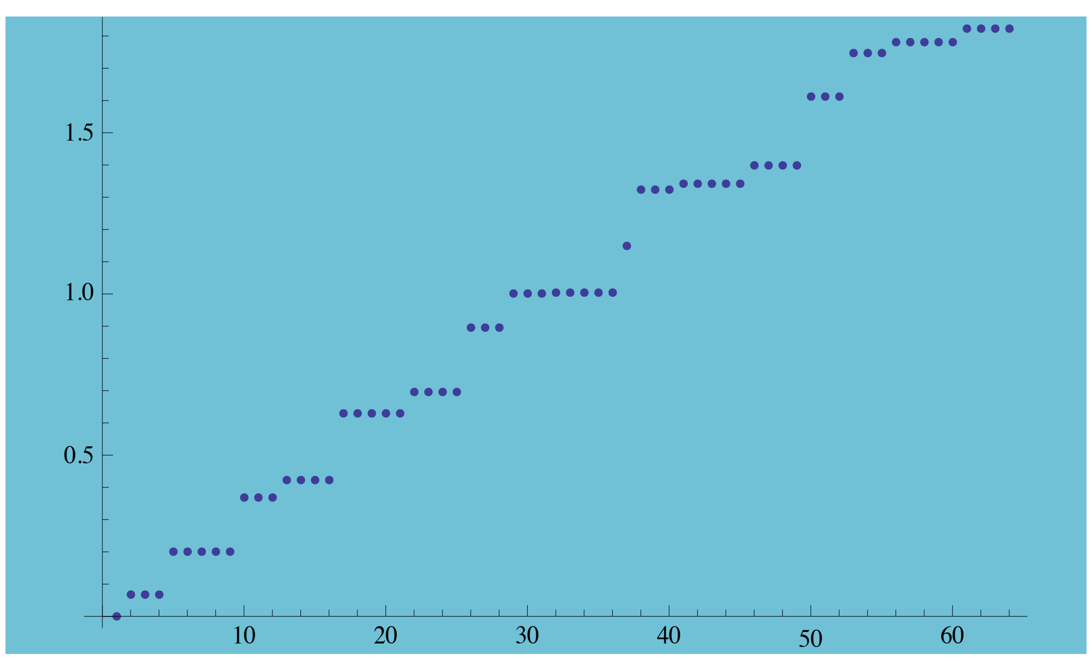
Same Laplacian from Regge calculus or random lattices (FCL)

Spectrum of FEM Laplacian

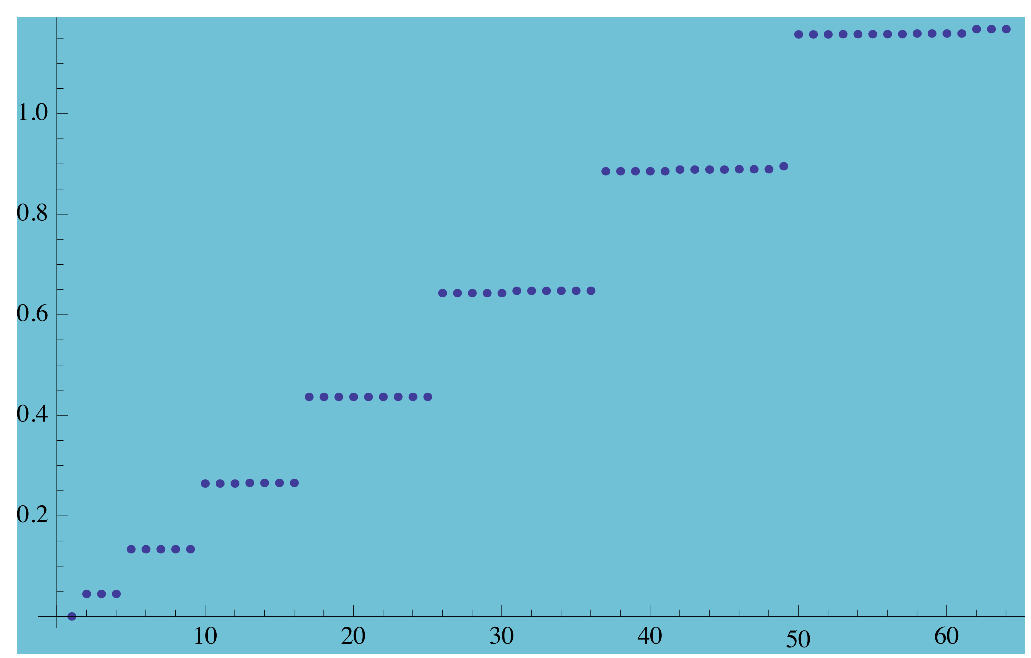
- Numerous proofs on spectral bounds in FEM literature.
- FEM works by guaranteeing rate of convergence to the continuum spectrum for *"shape regular"* triangles.
- But, no analog of OPE:

$$a^2 \sum_y K_{xy}(\phi_x - \phi_y)^2 \simeq c_{\mu\nu}\partial_\mu\phi\partial_\nu\phi + O(a^2), \;\; c_{\mu\nu} \not\propto \delta_{\mu\nu}$$

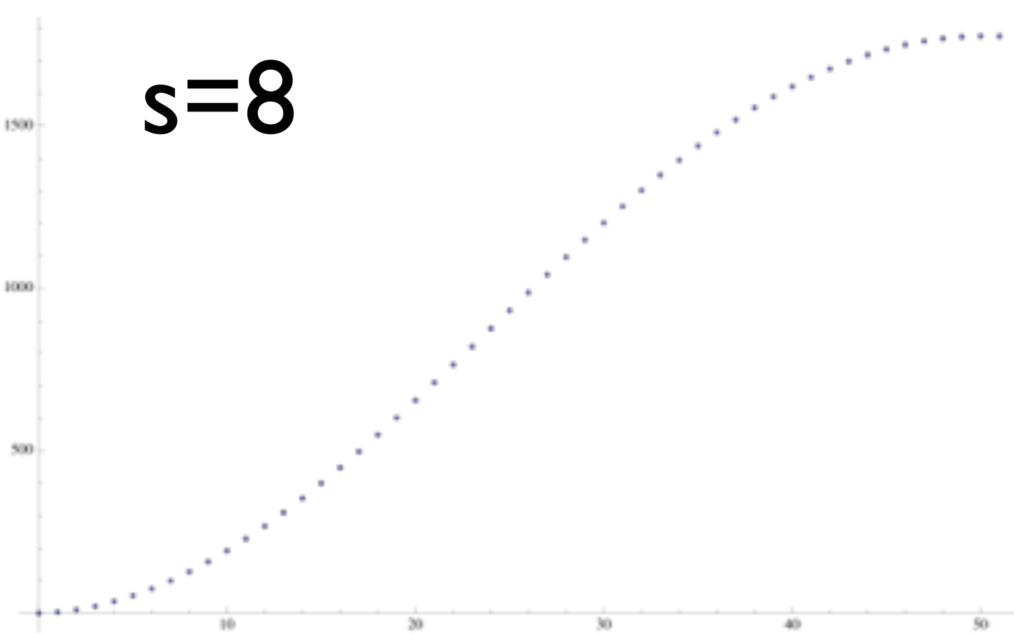
Ising-like Laplacian

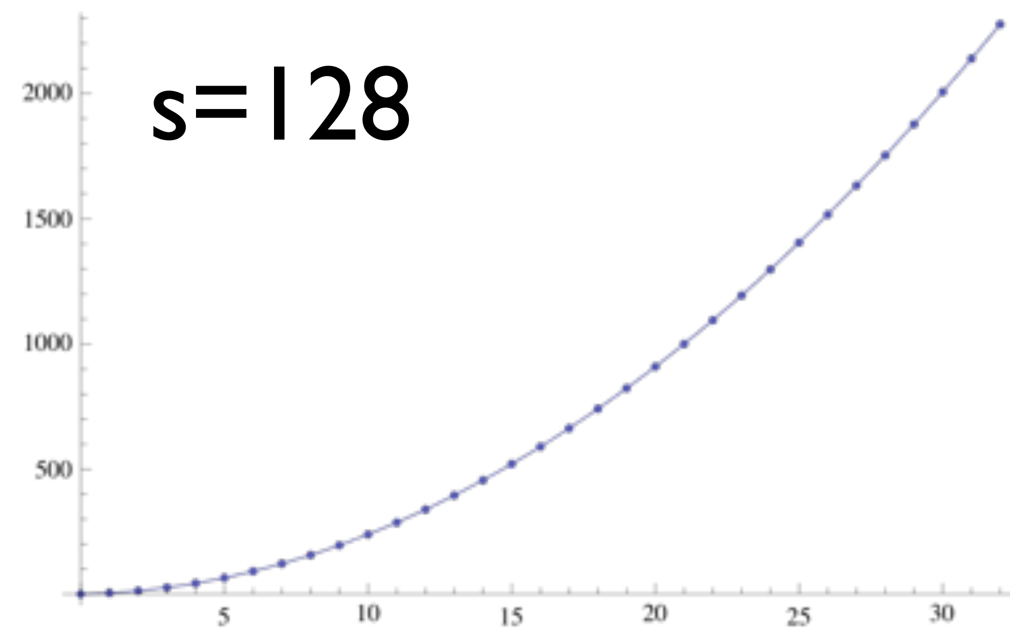


FEM Lapacian



Spectrum of FEM Laplacian



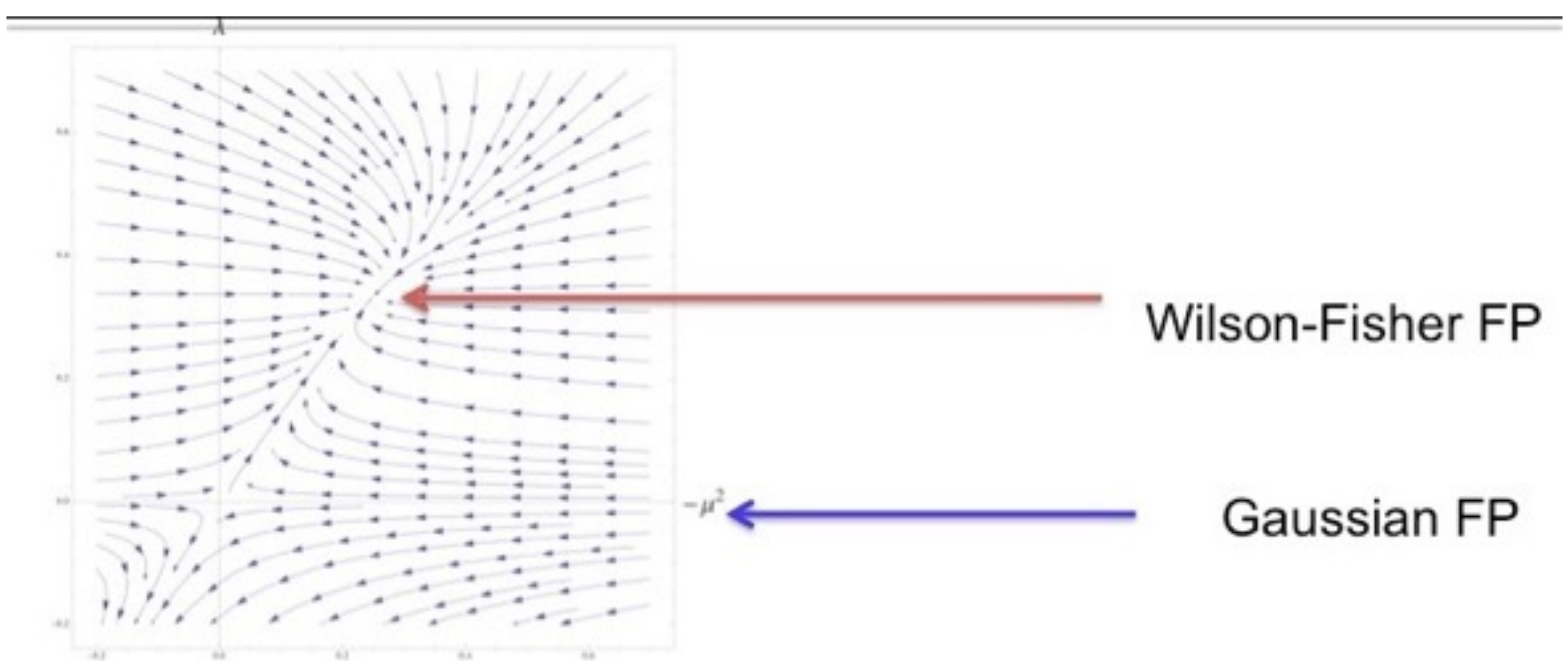


fit: $\ell + 1.00012\,\ell^2 - 1.3\times10^{-5}\,\ell^3 - 5\times10^{-5}\,\ell^4$

continuum: $\ell + \ell^2$

Finding Wilson-Fisher Fixed Point

- Unlike Ising model, there are now two couplings μ^2 and λ. The critical surface is some 1-d curve $f(\mu^2, \lambda)$ in the 2-d plane.
- The relevant direction controls flow tangent to the critical surface. The orthogonal direction is irrelevant and affects flow along the critical surface.



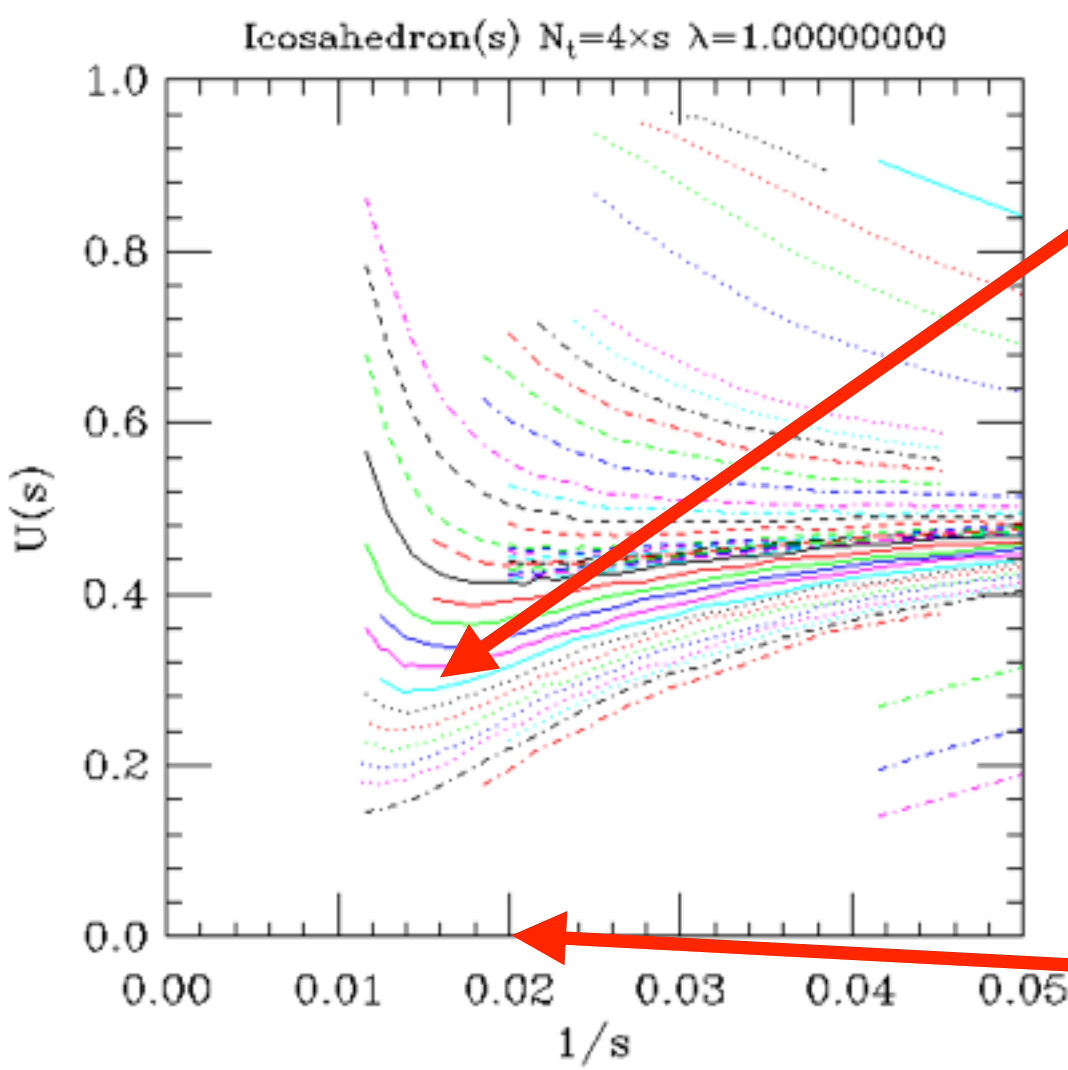
$$\begin{aligned} \beta_g &= \epsilon g - \frac{3}{16\pi^2} g^2 + O(g^3, \epsilon g^2, \mu^4, \mu^2 g;) \\ \beta_{\mu^2} &= 2\mu^2 + a g + \frac{9}{16\pi^2} g \mu^2 + O(\mu^4) \end{aligned} \qquad \lambda = 4g/4!$$

Using the Binder cumulant

$$U_4 = \frac{3}{2}\left(1 - \frac{\langle \phi^4 \rangle}{3\langle \phi^2 \rangle^2}\right)$$

Zero in disordered phase
One in ordered phase

$$U(s, \mu^2, \lambda) = U_c + a(\mu^2 - \mu_c^2)s^{1/\nu} + b(\lambda - \lambda_c)s^{-\omega}$$



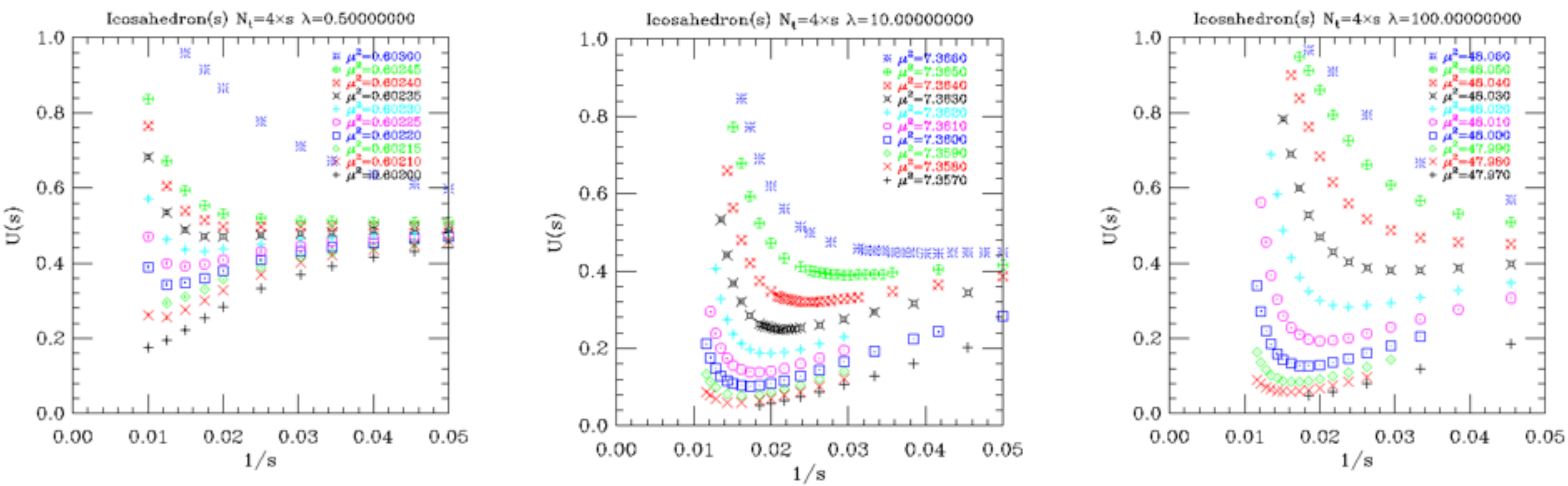
What's causing wiggle on $R \times S^2$?

Can we tune $\lambda \sim \lambda_c$?

Note problem starts about $s=50$.

Suppressing irrelevant terms?

$$U(s,\mu^2,\lambda) = U_c + a(\mu^2 - \mu_c^2)s^{1/\nu} + b(\lambda - \lambda_c)s^{-\omega}$$

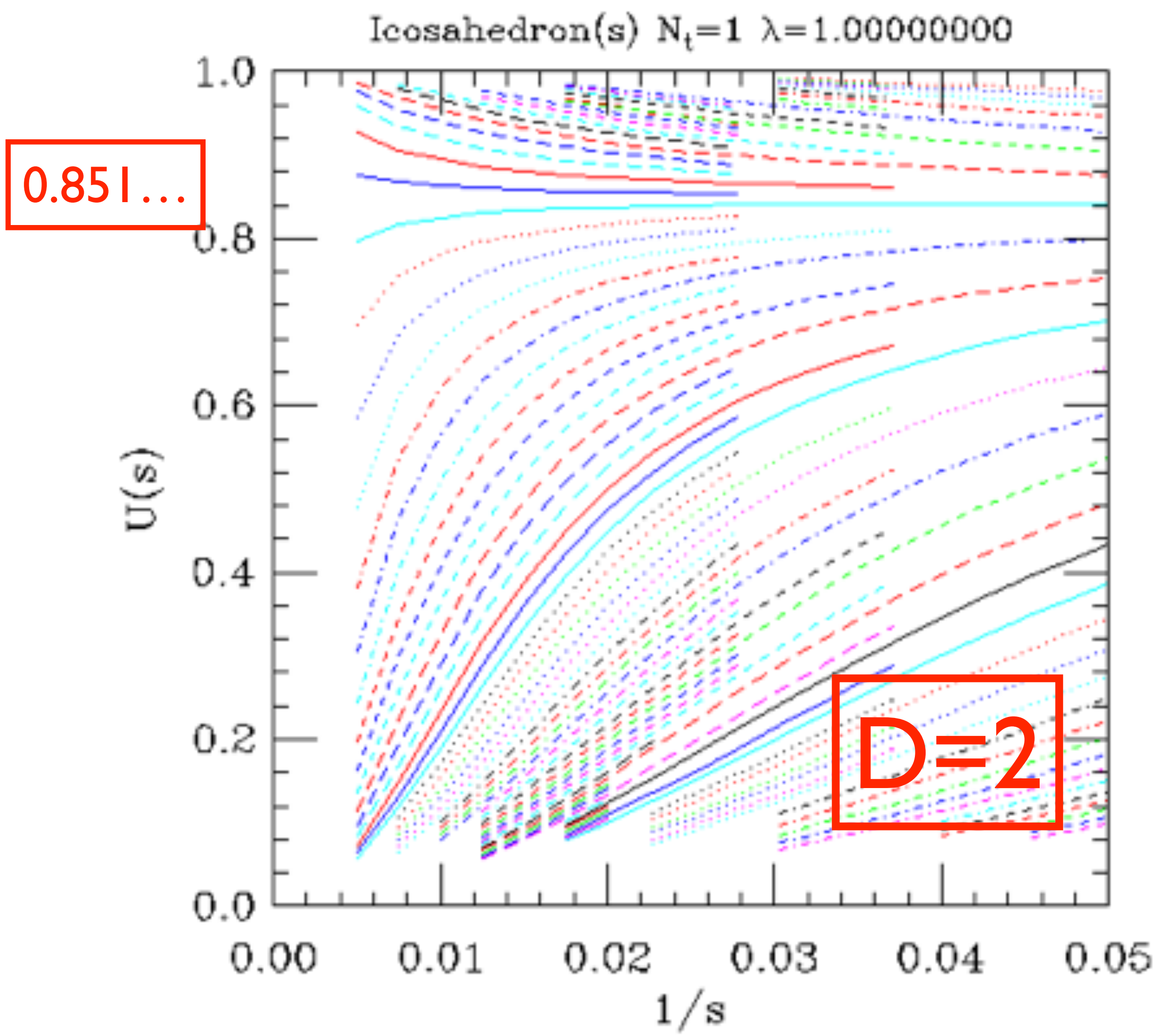


- There doesn't seem to be a λ that can speed the approach to the critical surface. What's going on?
- Is there a problem with our sphere? Let's try φ^4 on S^2.

φ^4 on S^2: Dirty Laundry (I)

$$U(s,\mu^2,\lambda) = U_c + a(\mu^2 - \mu_c^2)s^{1/\nu} + b(\lambda - \lambda_c)s^{-\omega}$$

$$Z = \int [D\phi] \exp\left[-K_{ij}(\phi_i - \phi_j)^2 - \omega_i \lambda(\phi_i^2 - \mu^2/2\lambda)^2\right]$$



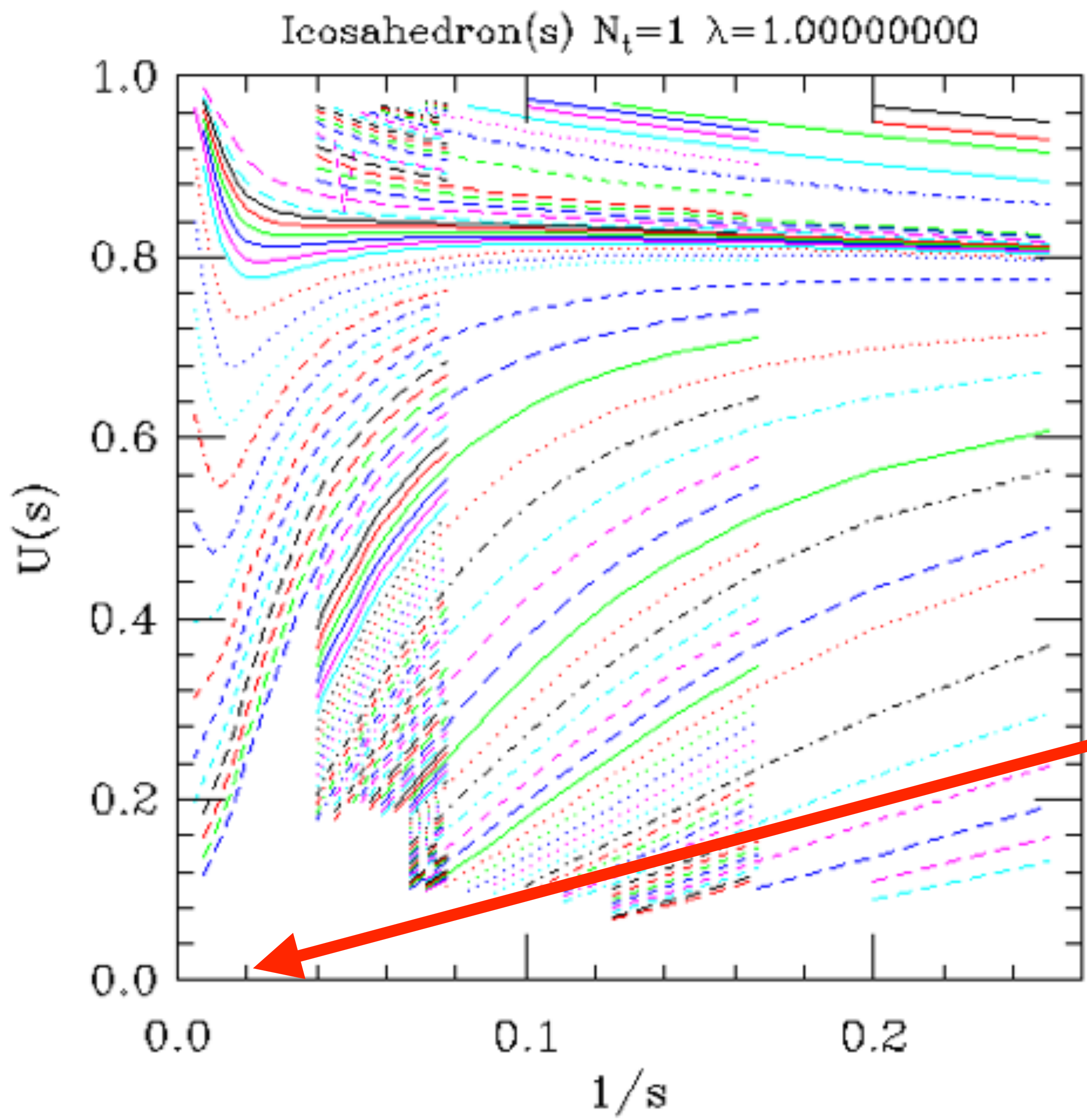
- Everything looks great! But…
- A bug in the code was giving wrong weights to the potential.
- The weights [$\omega_i \sim \sqrt{g(x_i)}$] were more uniform than they should have been [big hint!].
- When that bug was fixed…

U known exactly: Deng and Blöte (2003)

φ^4 on S^2: Dirty Laundry (II)

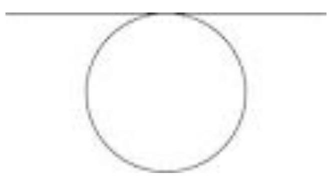
$$U(s,\mu^2,\lambda) = U_c + a(\mu^2 - \mu_c^2)s^{1/\nu} + b(\lambda - \lambda_c)s^{-\omega}$$

0.851…



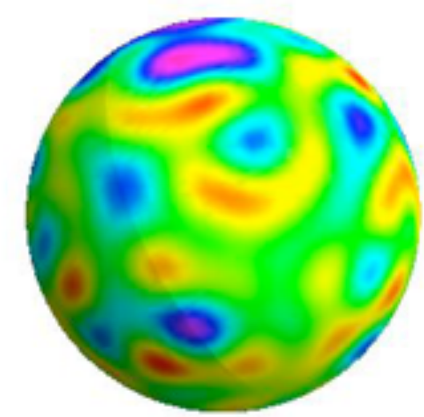
Note again problem starts about s=50.

UV terms affect rotational invariance

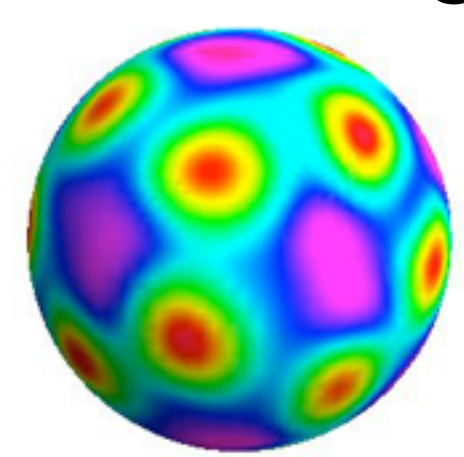


$$\delta m^2 = \lambda \langle \phi(x)\phi(x) \rangle \to \frac{1}{K_{xx}}$$

- Points do not occupy equal area elements on the sphere, so short difference contributions to the renormalized mass could be different at each point.
- Ignoring this effect leads to regions becoming critical at different couplings.
- For d=2-3, we can compute numerically in pert. thy.

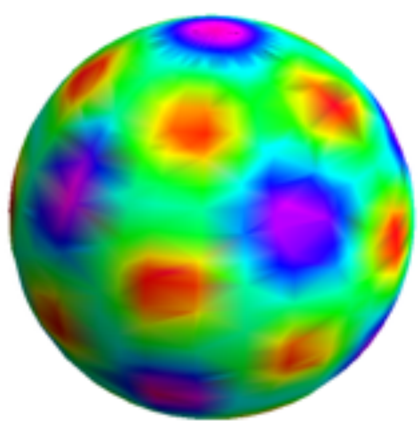


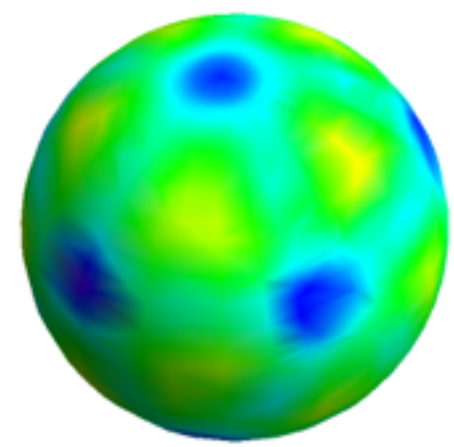
One Config

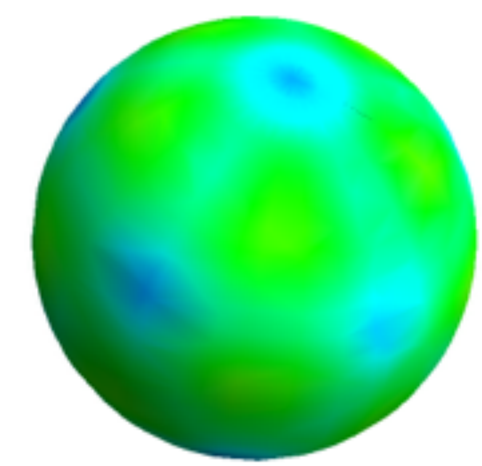


Ensemble average

Effect of CT on rotational symmetry


s=4


s=8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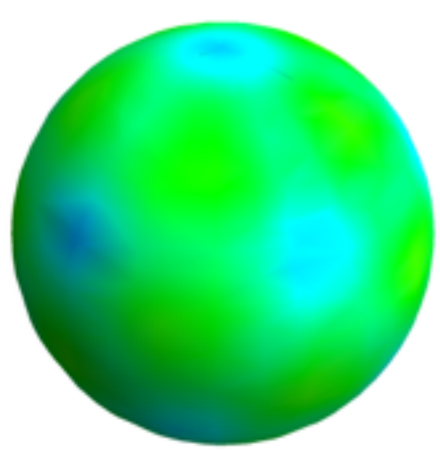
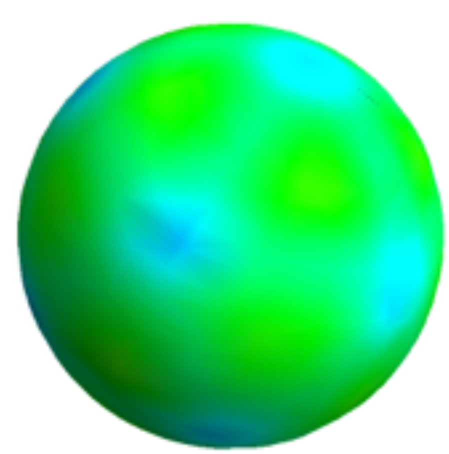
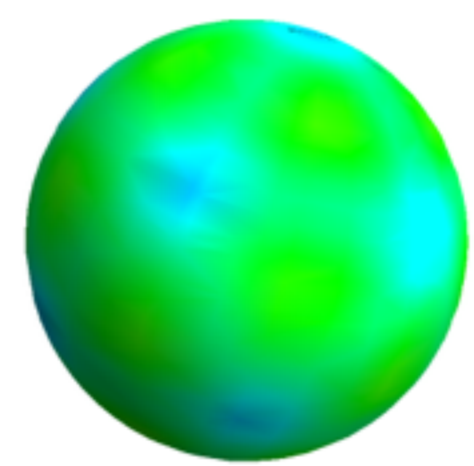
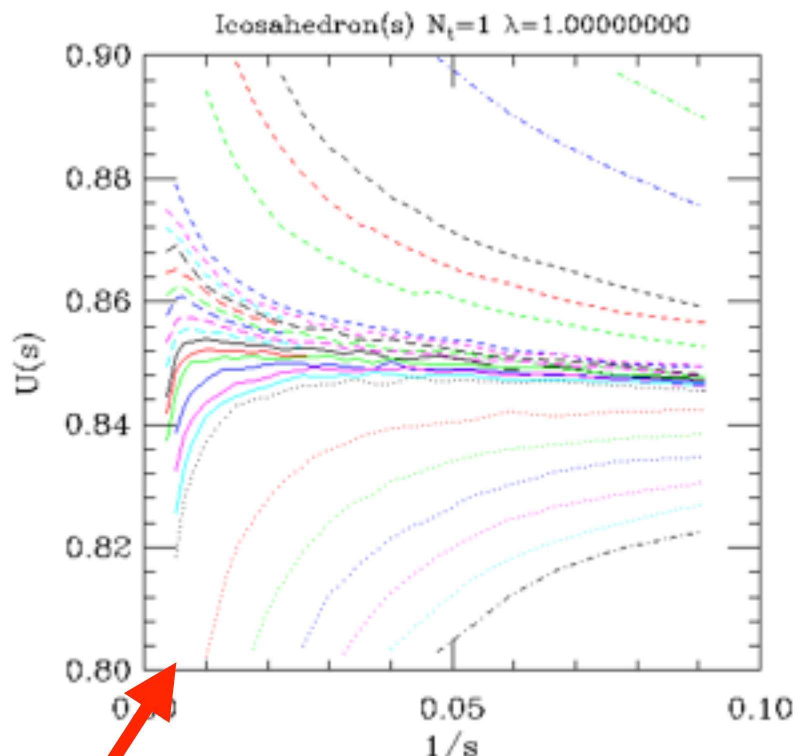

s=32

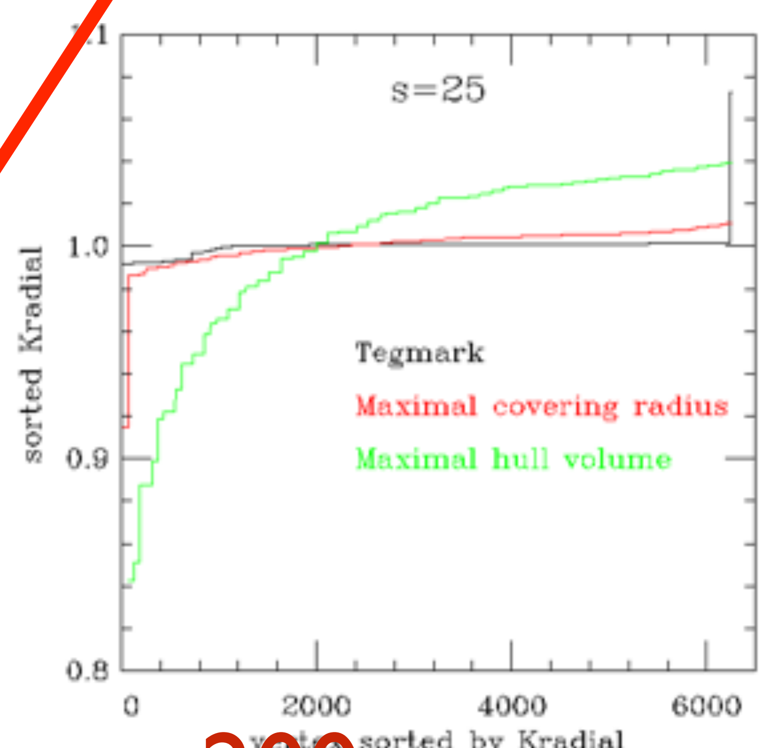

s=64


s=128

φ^4 on S^2: Area Equalization

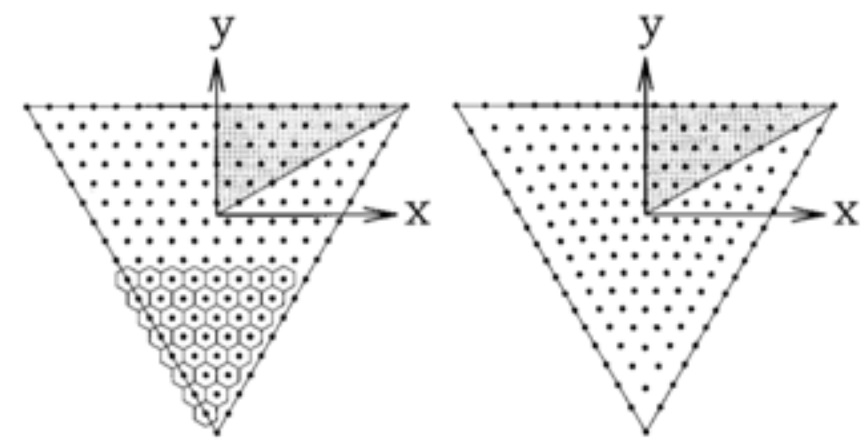
0.851…





Problem at s=200.

Max Tegmark (1996)



- Tegmark moves the points around on the sphere to make equal-area pixels (not unique).
- Our bug suggested this was a good idea.
- Tegmark's lattices improve the problem, but his method has defects at boundaries.

Summary

- Lack of discrete rotational invariance that grows with number of points on sphere requires a position-dependent renormalization of the mass term to restore spherical symmetry.
- Three ways forward:
  - Compute the counter term directly (only needs to be done once at each s).
  - Find an exact equal area partition of the sphere that respects icosahedral symmetry for each s (challenging numerical problem).
  - Use a Pauli-Villars regulator to suppress UV contributions. Works fine but no cluster algorithm. Useful for d>3?
- All methods seem to work which indicates we have identified the cause of rotational symmetry breaking.

Backup Slides

Composite Higgs Scenarios

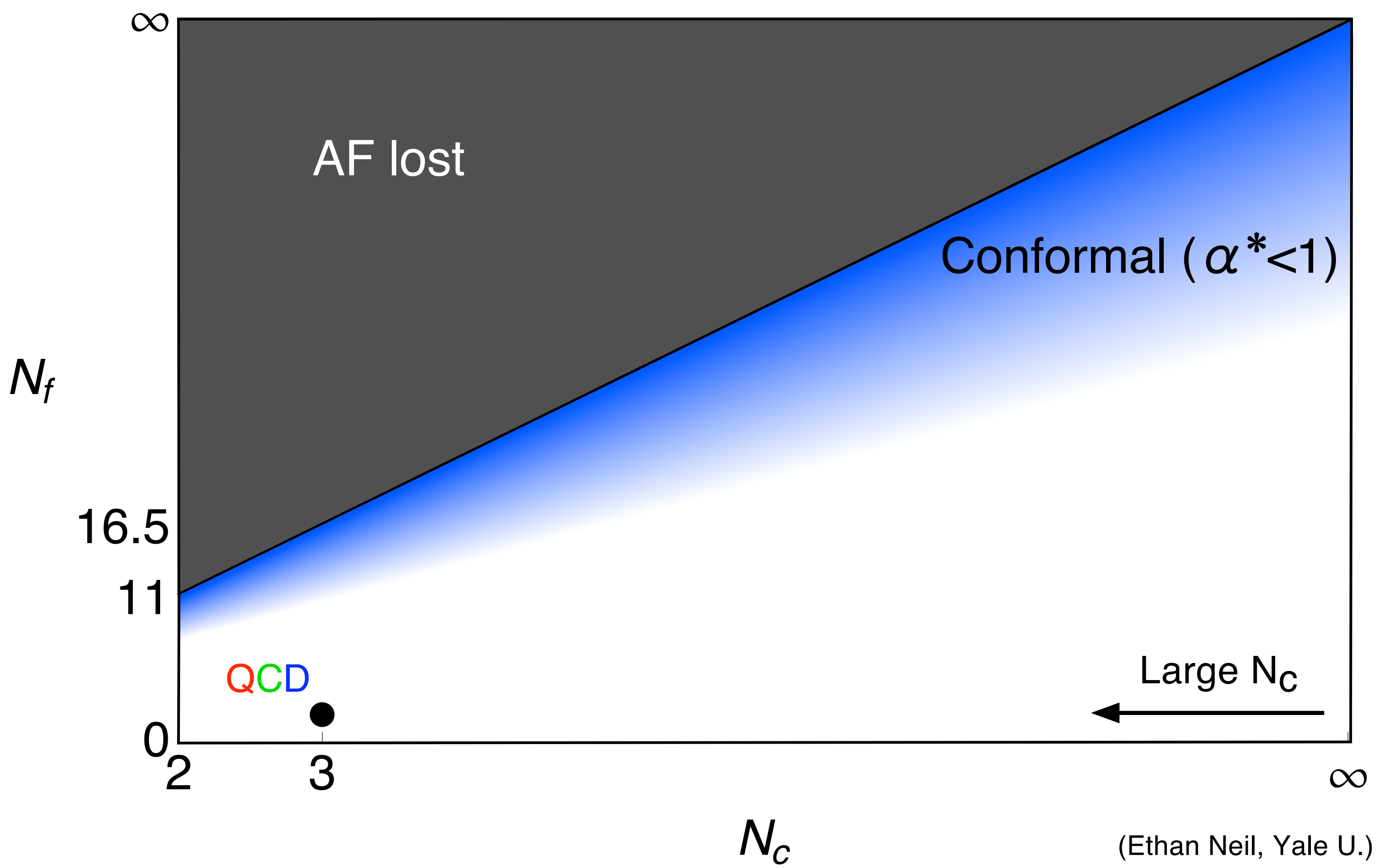
- For strong interactions, 126 GeV is really light. Typically, F ~ 246 GeV and typical hadron masses are at 4πF ~ 3 TeV.
- It is very unlikely the LHC can discover weakly produced resonances with 3 TeV masses even at 14 TeV and 3000 fb^{-1}.
- A "Higgs impostor" could be a pseudo-dilaton arising from spontaneous breaking of scale invariance.
- Something this light could be a pseudo-NGB (pNGB), which generally go by the name "composite Higgs".
- A light scalar glueball could arise from a confinement scale significantly below the chiSB scale. Could that also be a "Higgs impostor"?

Looking for Pseudo-Dilatons

- For $N_f = 0–1$, confinement but no NG bosons.

- For $N_c = 2$, enhanced chiral symmetry $SU(2N_f)/Sp(2N_f)$. pNGB Higgs?

- Pert. theory indicates IRFP for $N_f \lesssim 5.5 \cdot N_c$.

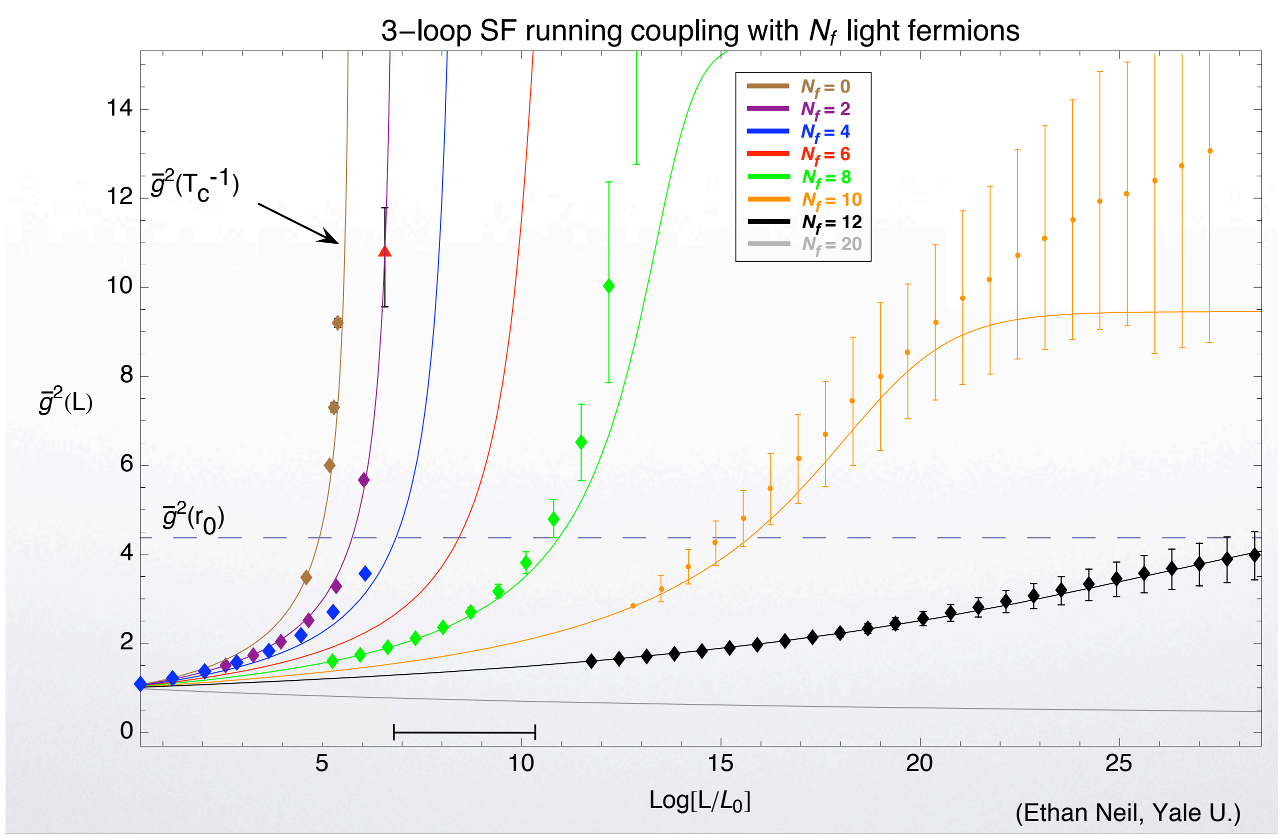


- What is the nature of the quantum phase transition at the bottom of the conformal window? Are pseudo-dilatons an order parameter?

- One simple search strategy: start from QCD and increase N_f.

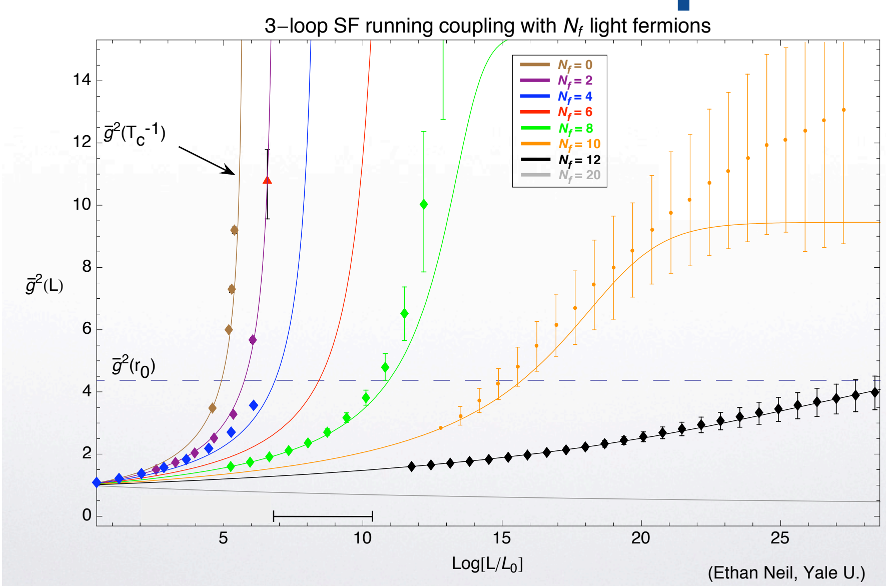
What happens to QCD with increasing N_f?

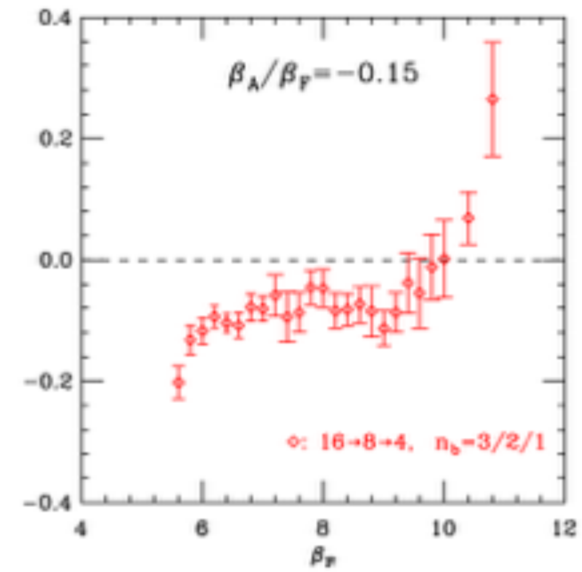
- In QCD, $\overline{g}(L)$ is asympotically free and runs rapidly until SSB and confinement: $\overline{g}(L_c)=\overline{g}_c$.
- As N_f increases, the running slows down.
- For large N_f, $\overline{g}(L)$ flows to $\overline{g}^*$ at IR fixed point(IRFP). No EWSB, no 126 GeV boson.



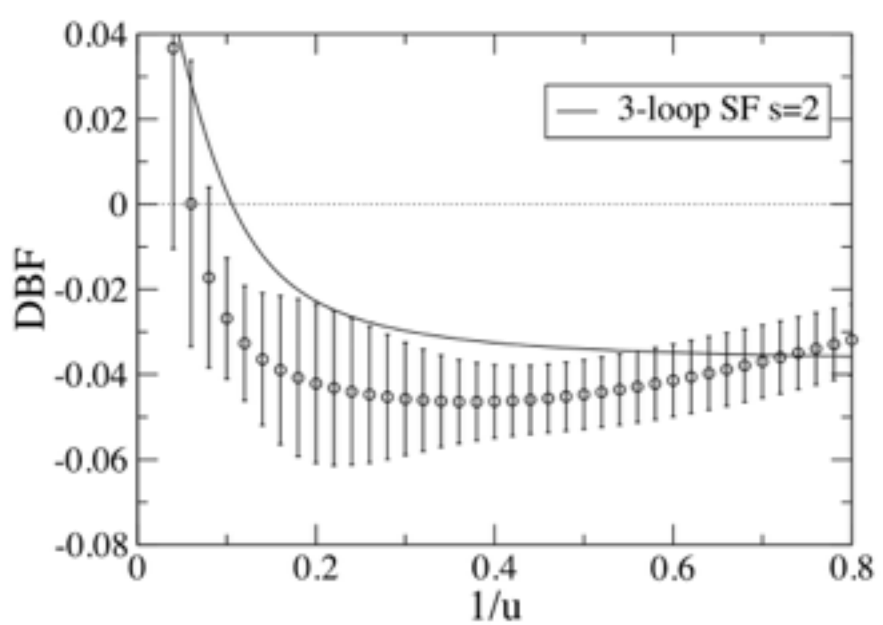
- Walking theories may exist nearby theories with strongly-coupled IRFP: $\overline{g}^* \lesssim \overline{g}_c$.
- Unlike QCD, walking theories have two dynamically generated scales: $\Lambda_{IR} << \Lambda_{UV}$ and the theory is nearly conformal between IR and UV scales.
- In composite Higgs models, usually Λ_{UV} > 1000 TeV is preferred and Λ_{IR} is related to scale of EWSB.

Non-perturbative running coupling

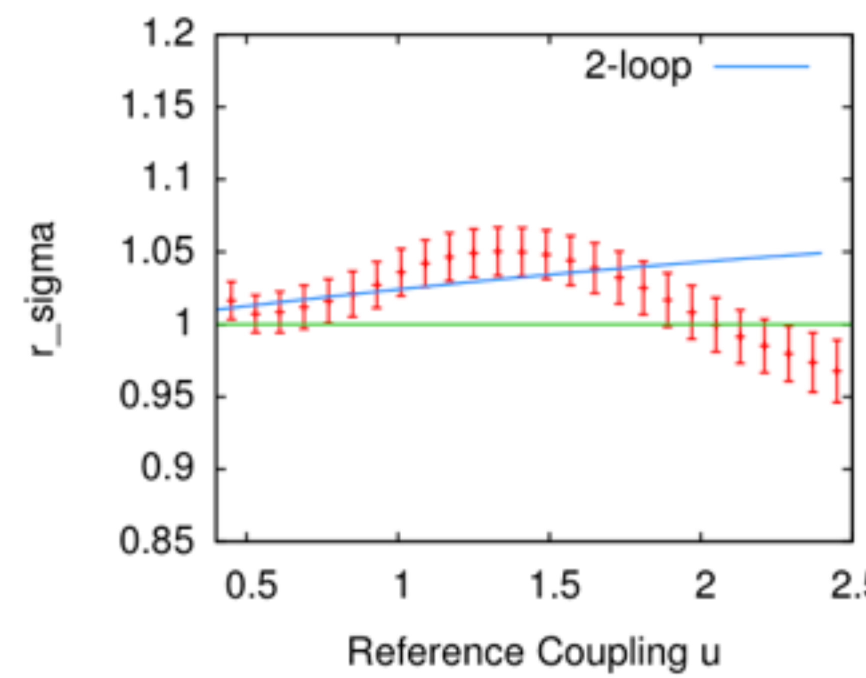




A. Hasenfratz SU(3) N_f=12



N. Yamada *et al.* SU(3) N_f=10



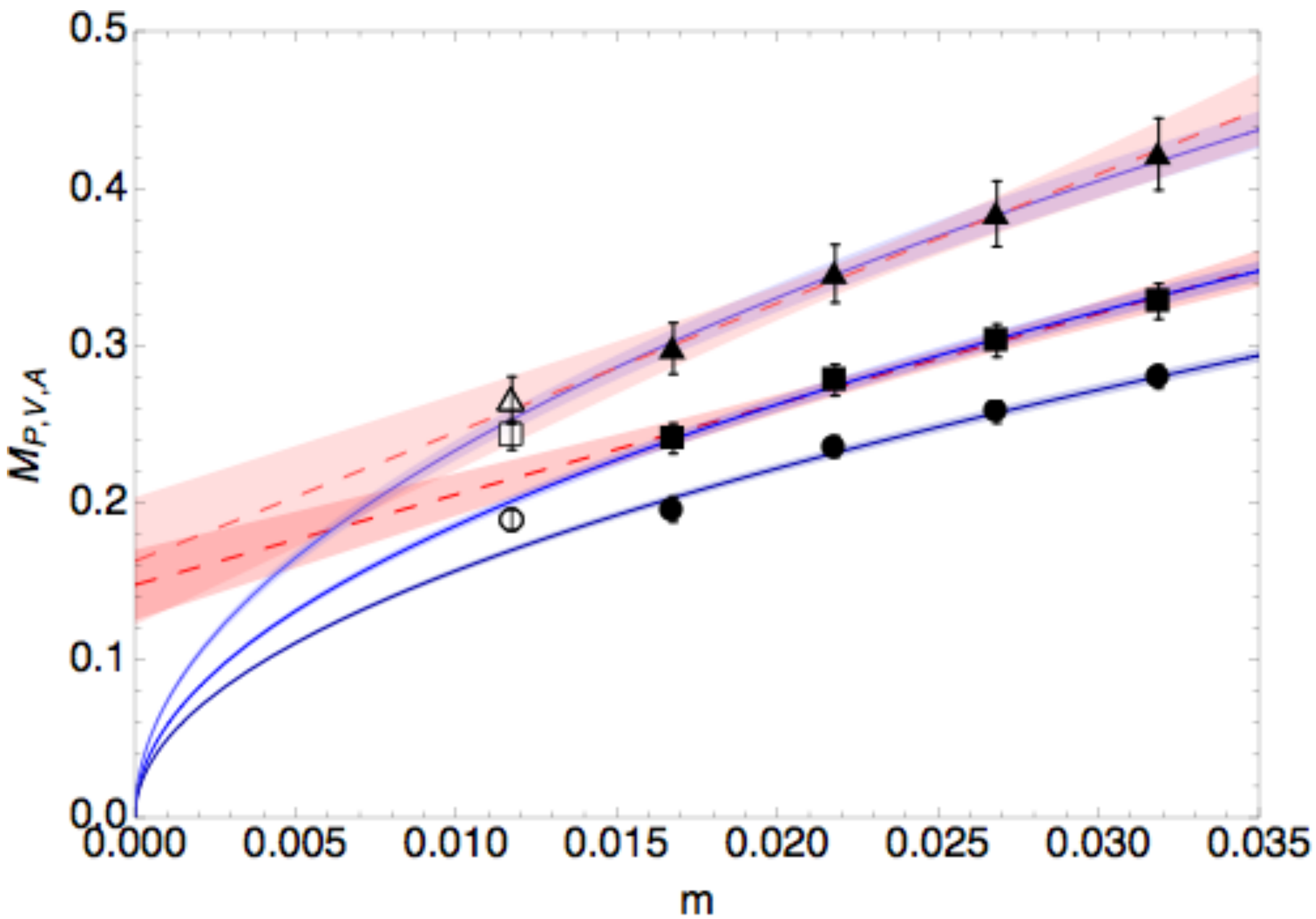
D. Lin *et al.* SU(3) N_f=12

Mass-deformed IRFP

- Adding a small mass to an IRFP Yang-Mills theory will produce a confining spectrum with a particular scaling behavior: $M_X = C_X\, m^{1/(1+\gamma)}$, $F_X = C_{FX}\, m^{1/(1+\gamma)}$, ...
- The effective γ extracted from fitting can be meaningfully compared to values from fits for other N_f.
- In particular, all lattice results are consistent with γ increasing as N_f decreases from $5.5 \cdot N_c$.
- At some point, $\gamma \sim O(1)$ will occur and it is long term goal to test the conjecture that these are walking theories.

Mass-deformed IRFP

SU(3), N_f=10, γ~1



SU(3), N_f=12, γ~0.3-0.5