Strongly coupled gauge theories: In and out of the conformal window

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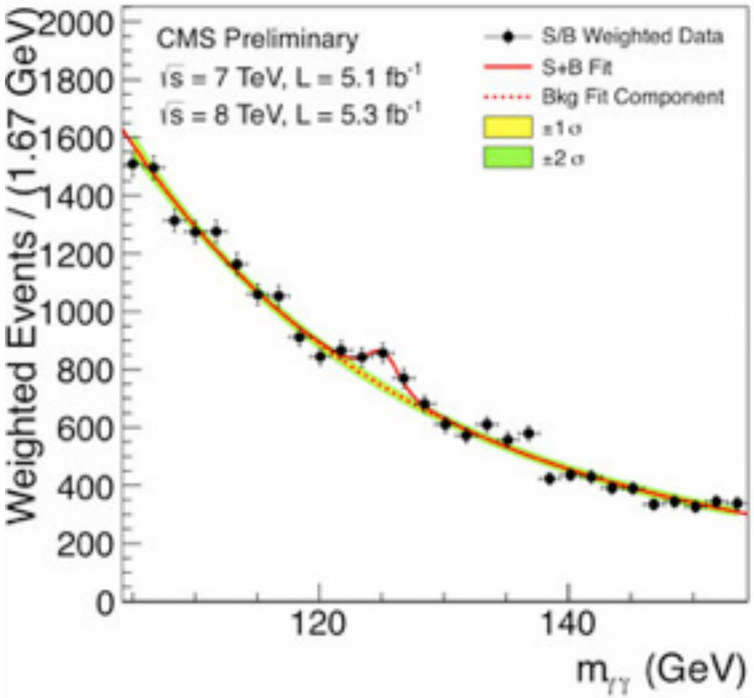
KMI
Feb 3, 2014

In collaboration with A. Cheng, Y. Liu, G. Petropoulos and D. Schaich

July 4th

July 4th 2012: Higgs boson “discovered”



0++ scalar at 126 GeV :

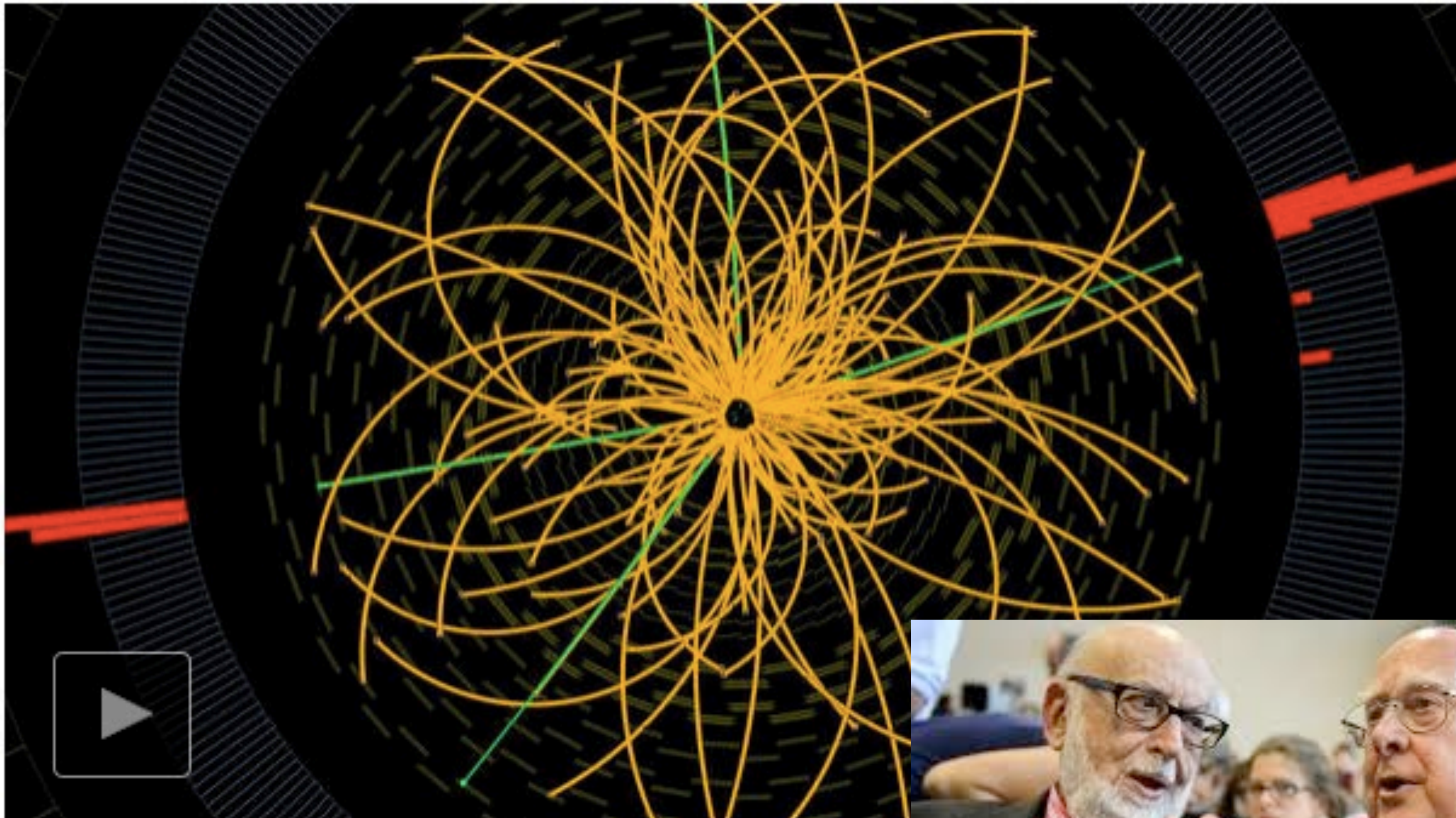
Standard Model like

- no sign of new TeV-scale physics!

Oct 8 2013

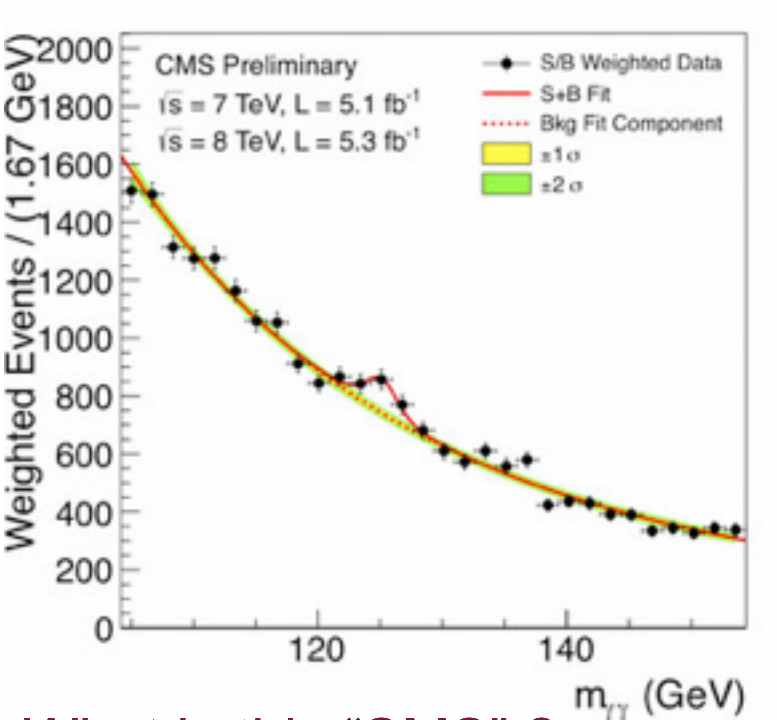
For Nobel, They Can Thank the 'God Particle'

Higgs and Englert Are Awarded Nobel Prize in Physics



Higgs Boson Particle Theory Wins Nobel: Dennis Overbye, a Times s
particle's significance — and insignificance — in understanding our universe.

July 4th 2012: Higgs boson "discovered"



0++ scalar at 126 GeV :

Standard Model like

- no sign of new TeV-scale physics!

What is this "SMS" ?

- Elementary scalar? and no new physics :
- SUSY? SMS is uncomfortably heavy
- Composite? SMS is uncomfortably light

  find strongly interacting model with light scalar

What's wrong with the SM Higgs?

.... nothing really

The Higgs sector

- Requires enormous fine tuning of the parameters (naturalness)
- Trivial: mathematically inconsistent: $\lambda(\mu) \rightarrow 0$ as $\Lambda \rightarrow \infty$
- Vacuum is metastable due to heavy top quark
- Provides no dynamical explanation for electroweak symmetry breaking or flavor physics

SUSY could solve/explain all this but

- no SUSY particles have been detected
- Higgs is uncomfortably heavy for most SUSY models

Composite Higgs:

Assume a new gauge-fermion system at high energies (techni-)

- If it is chirally broken the techni-pions are the Goldstone bosons of electroweak symmetry breaking, the 0^{++} meson is the Higgs

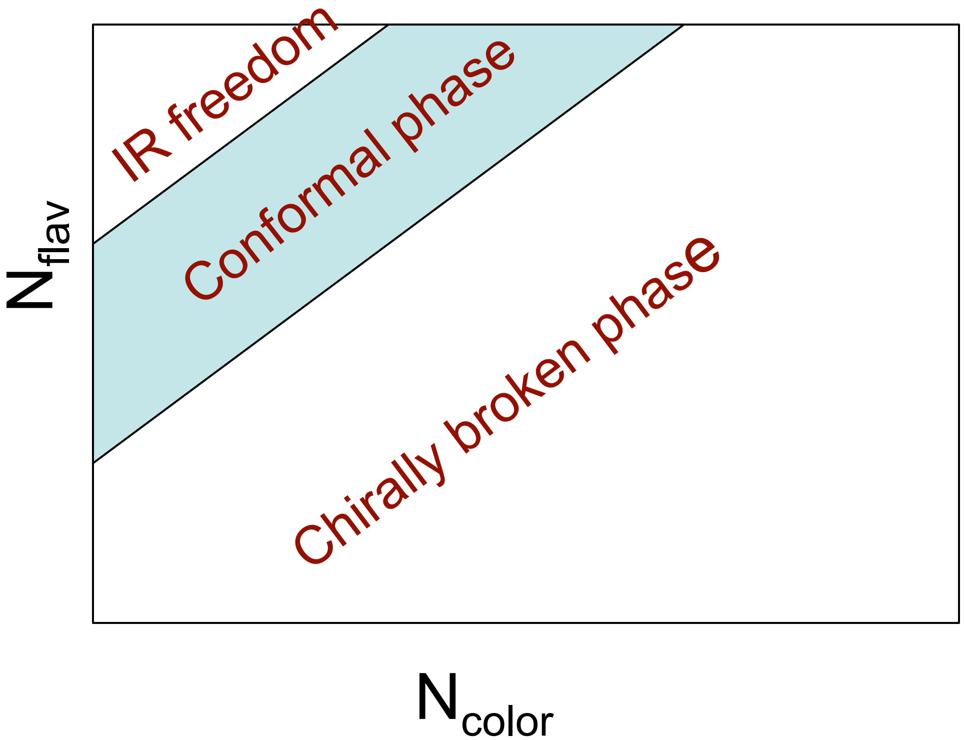
  Does it agree with experimental data?

- Scaled-up QCD models are out (were ruled out decades ago)!
  - EW measurements are violated (g^2 runs too fast)
- Walking TC models: gauge coupling evolves slowly over many magnitudes of energy scale with a large anomalous dimension could solve most these problems;

  Do they have a light Standard Model like scalar?
  - dilaton of spontaneously broken conformal symmetry
  - pseudo-Goldstone of expanded flavor symmetry

Composite Higgs in strongly coupled systems:

SU(N_{color} ≥2) gauge fields + N_{flavor} fermions in some representation



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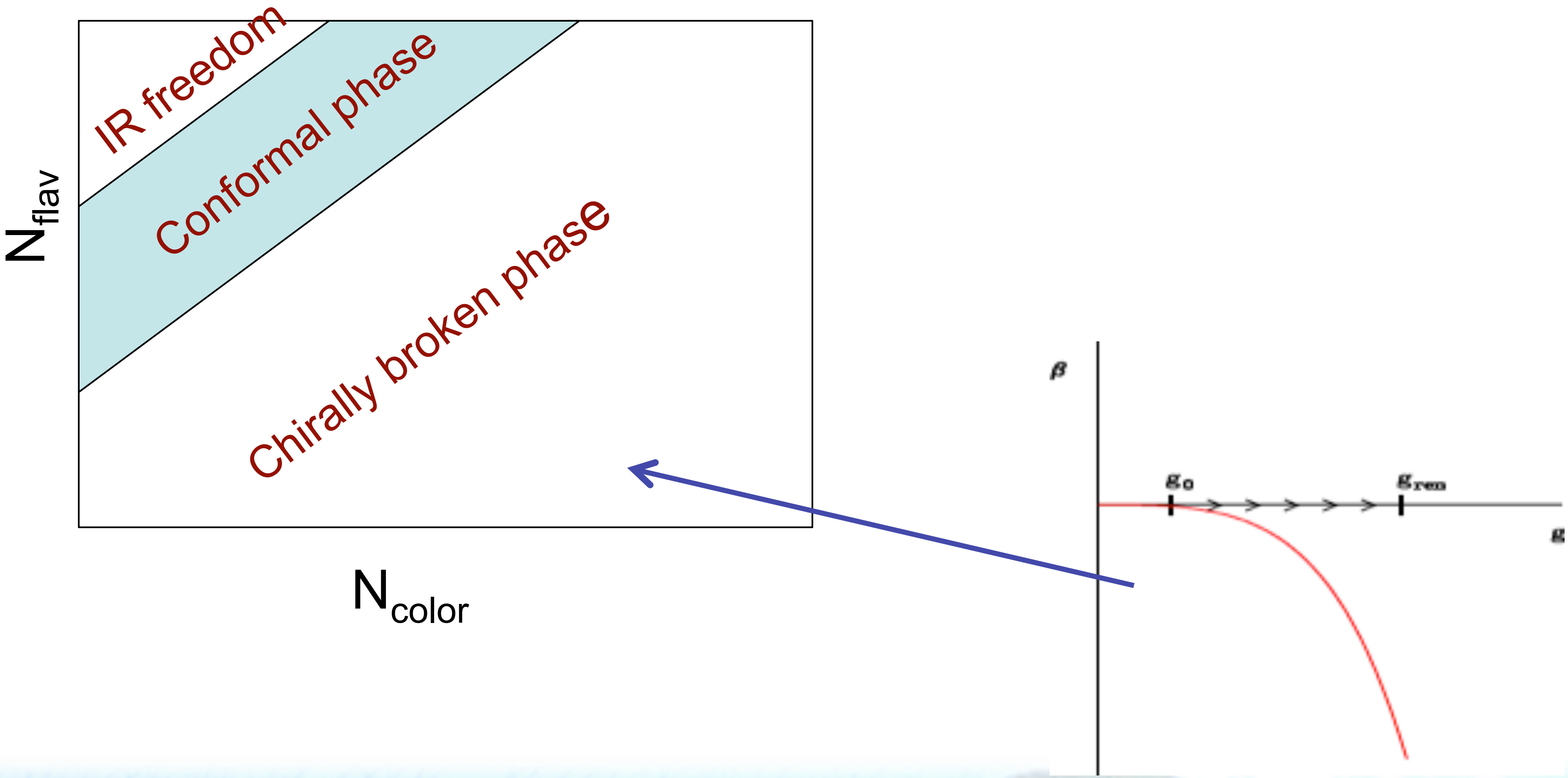

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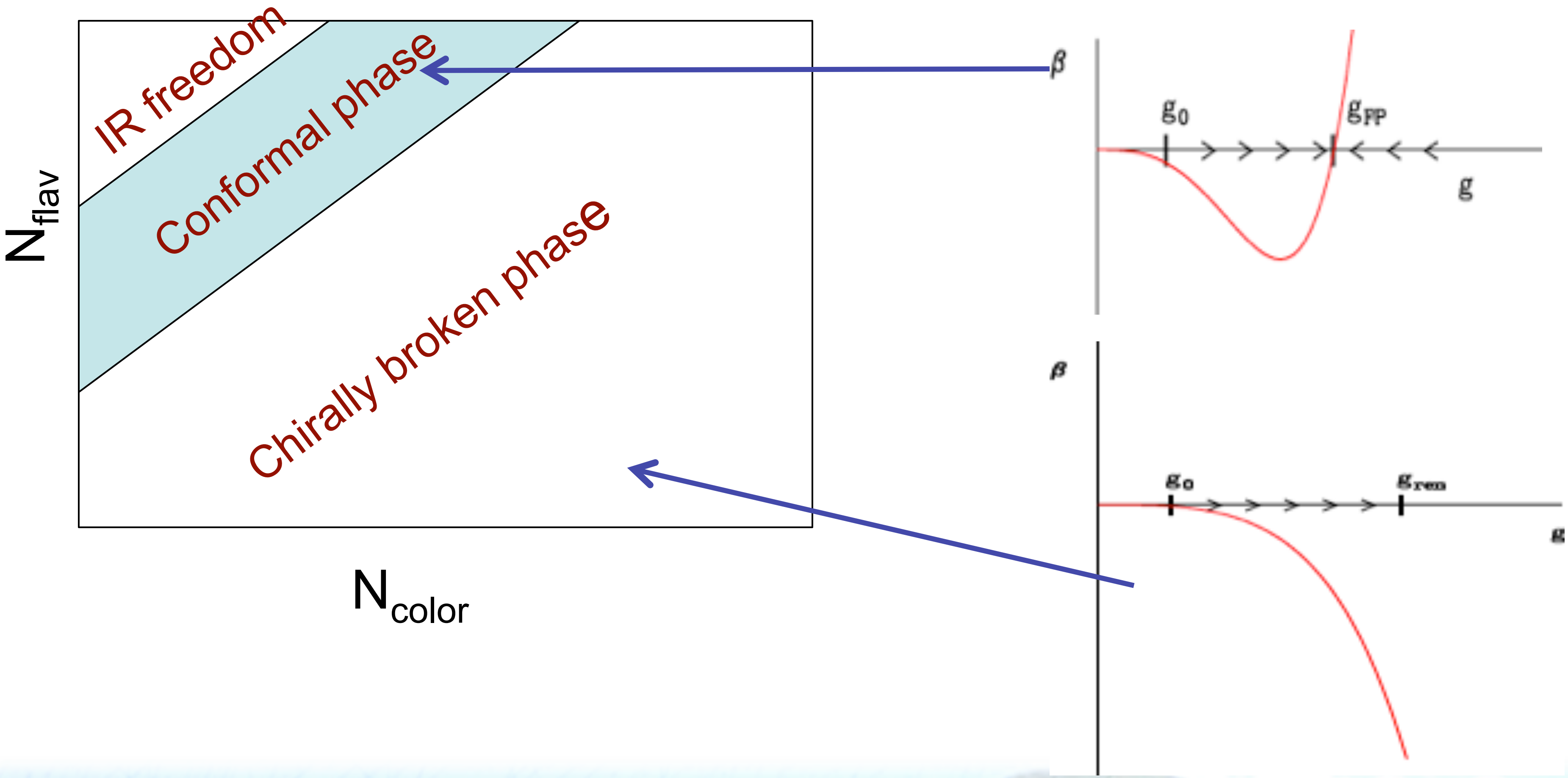

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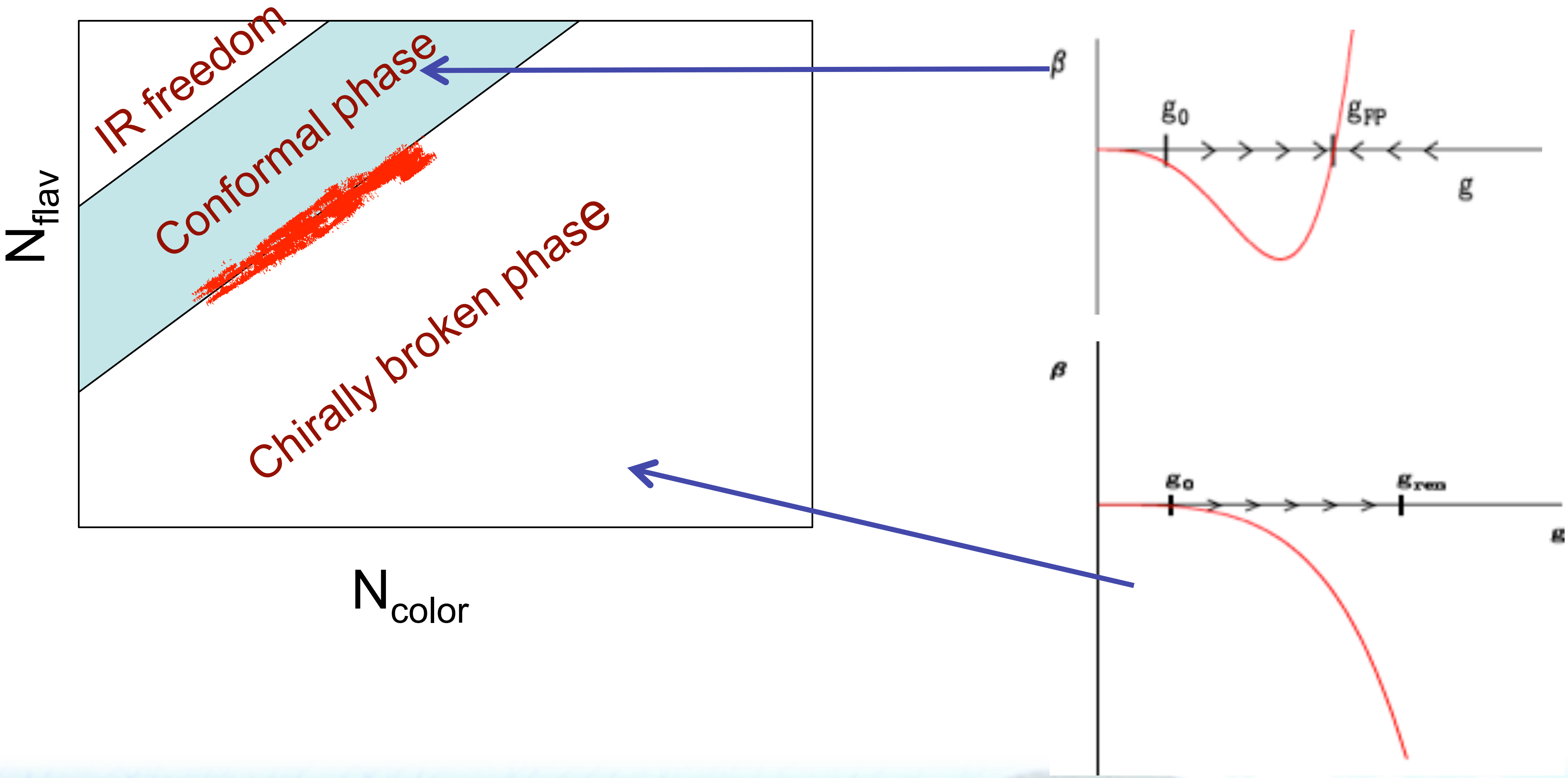

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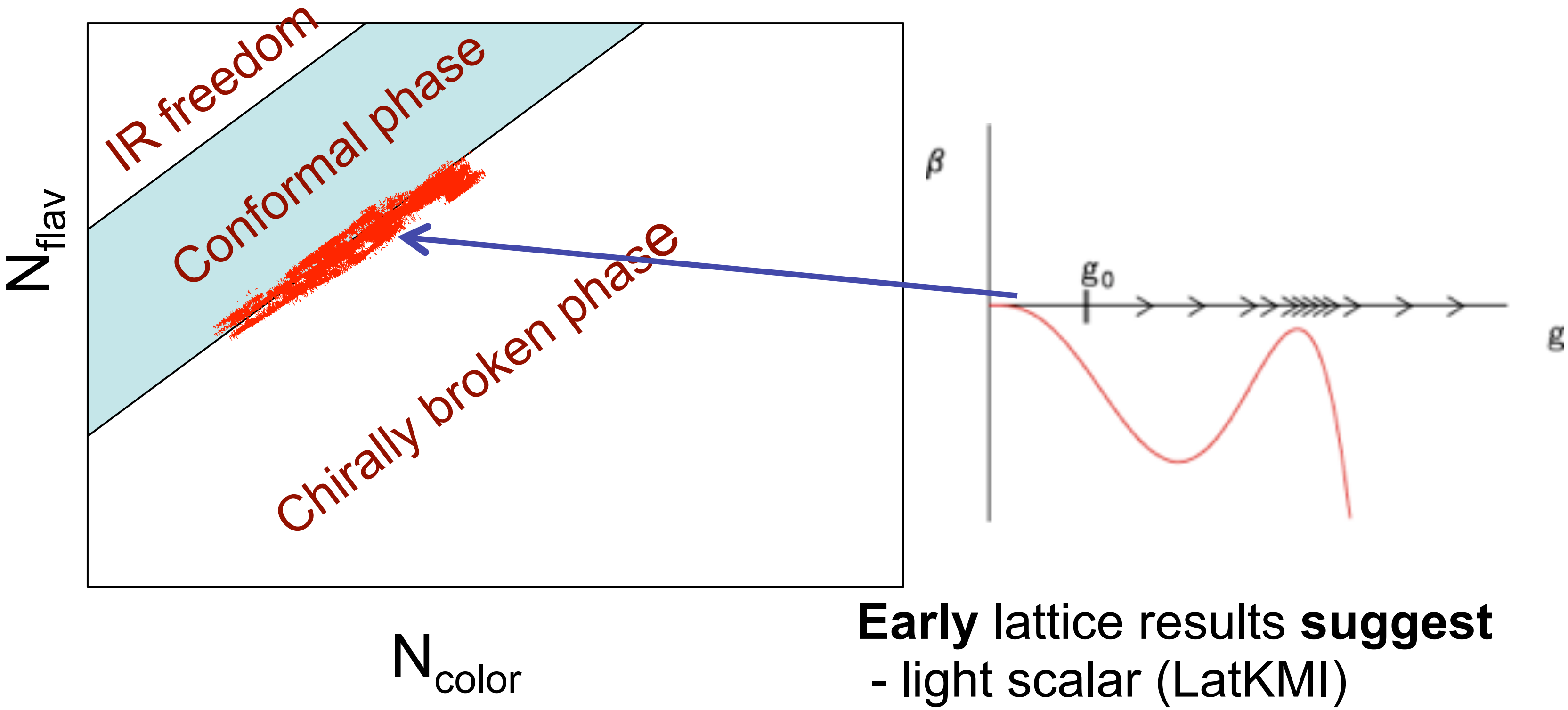
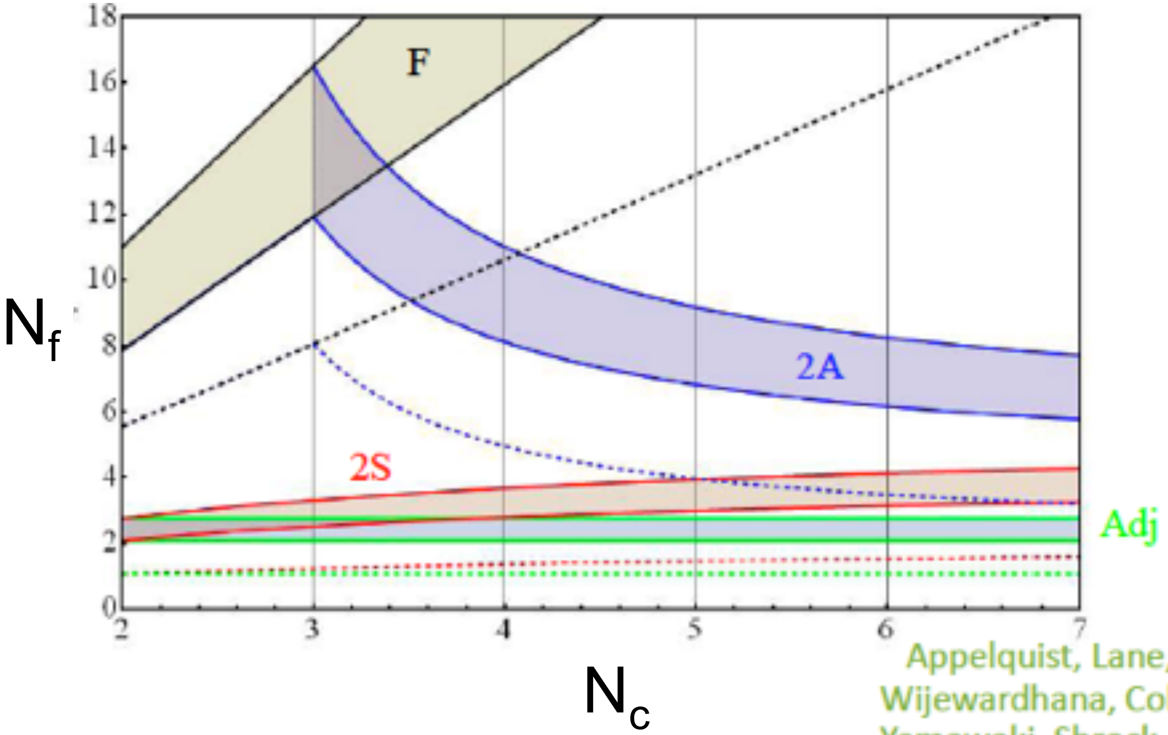


Early lattice results **suggest**
- light scalar (LatKMI)
- enhanced chiral condensate
- suppressed S parameter

Roadmap for the conformal window

S-D type calculations



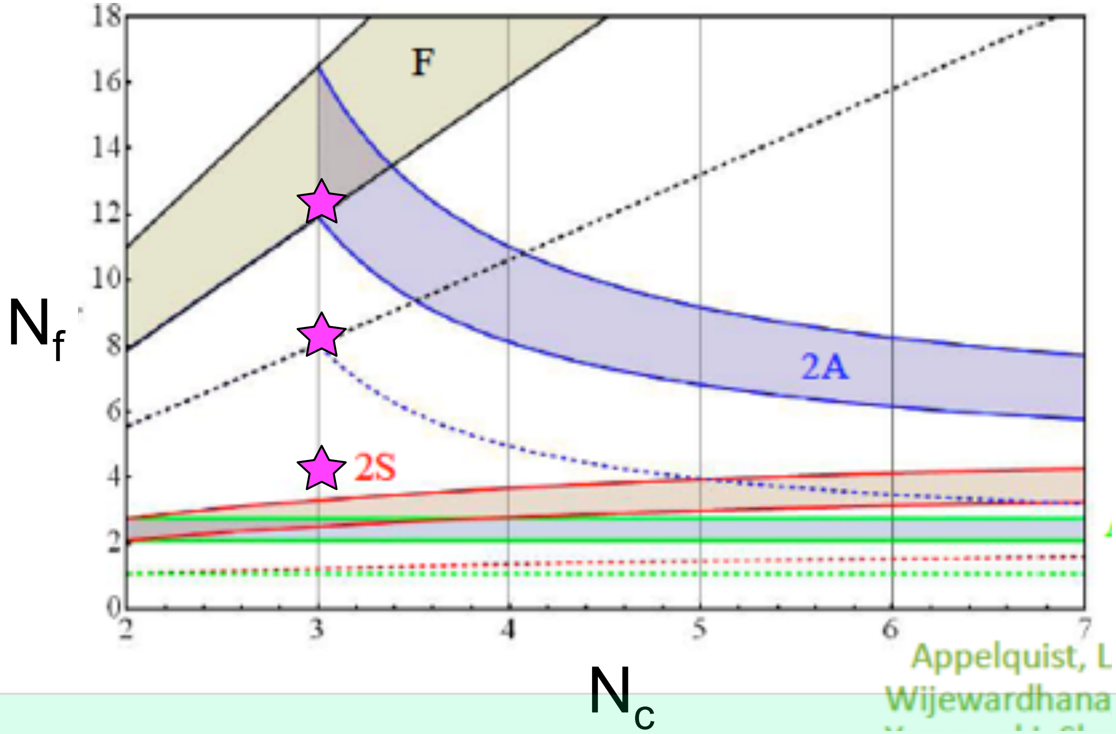
Shaded: conformal
Below : confining
Above: IR free
Dotted lines: 2-loop PT

fermion representation:
Fundamental
Adjoint
2Symmetric
2Antisymm

Appelquist, Lane, Mahanta, Wijewardhana, Cohen, Georgi, Yamawaki, Shrock, Dietrich, Sannino, Tuominen

Needs non-perturbative verification! → **LATTICE**

In this talk: N_f = 4, 8 and 12 fundamental fermions



Concentrate on

N_f=12:
- controversial system near the conformal boundary

N_f=8:
- most likely chirally broken but could be walking

Questions to answer:

- Is the system conformal or chirally broken (and walking)?
- Is there a light scalar?
- Is the S parameter small? What is the anomalous mass dim.?
- ....

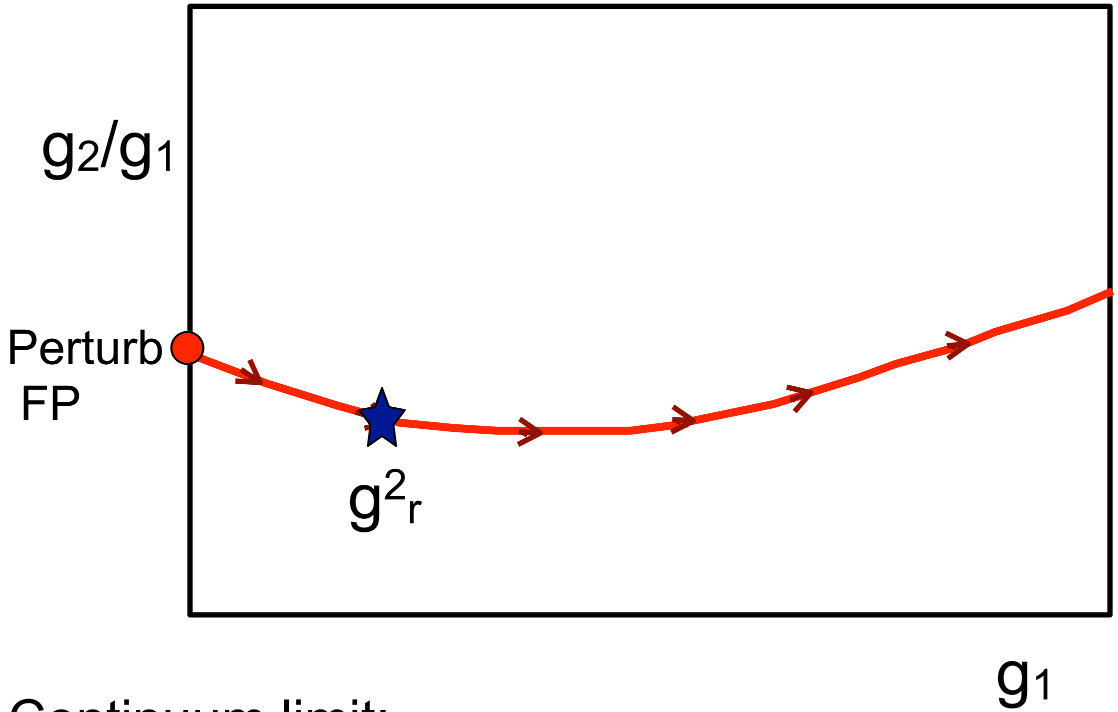
Simple enough .... cannot be much harder than QCD

It is surprisingly difficult to distinguish conformal, walking, and chirally broken systems on the lattice

Fixed point structure of a **chirally broken system**

Wilson RG

m=0 critical surface: **one fixed point**



g_1: gauge coupling
g_2,... : irrelevant couplings

Perturbative FP
g_1=0,m=0 : 2 relevant directions

Continuum limit:

Tune bare $g^2 \to 0$ and $m \to 0$: renormalized g^2 anywhere on renormalized trajectory

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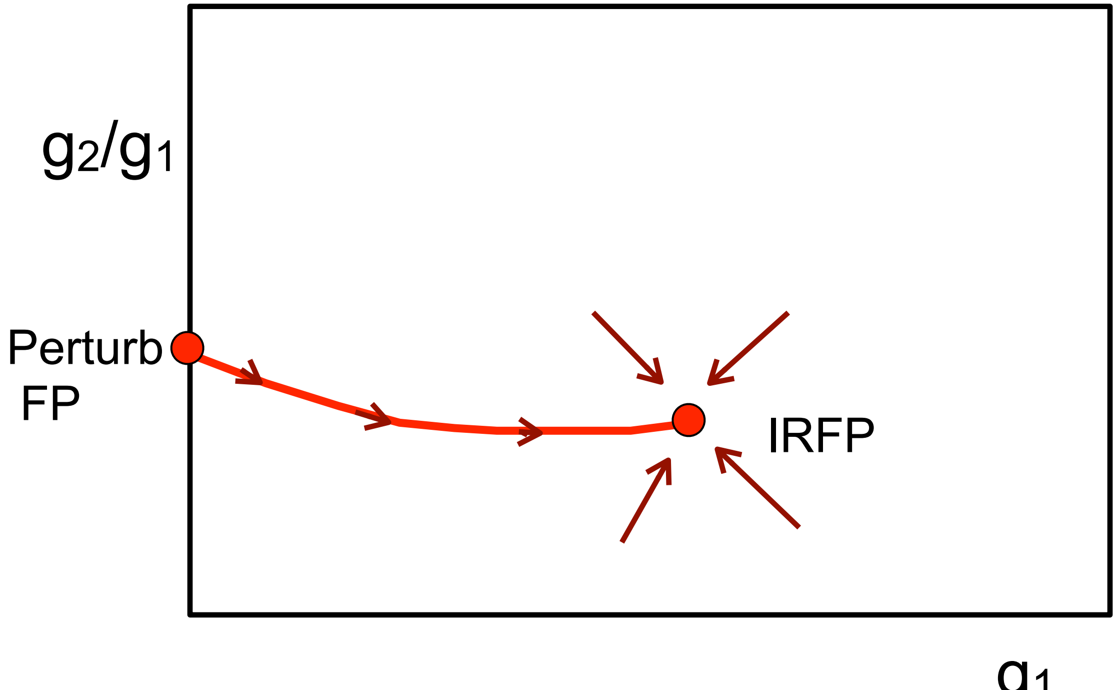

Perturbative FP

g_1=0,m=0 : 2 relevant directions

IRFP

$g_1=g_{IRFP}$,m=0 : 1 relevant direction

Two possible continuum limits:

1. Tune bare $g^2 \to 0$ and $m \to 0$: renormalized g^2 anywhere on renormalized trajectory
2. Tune only $m \to 0$: renormalized $g^2 = g^2_{IRFP}$

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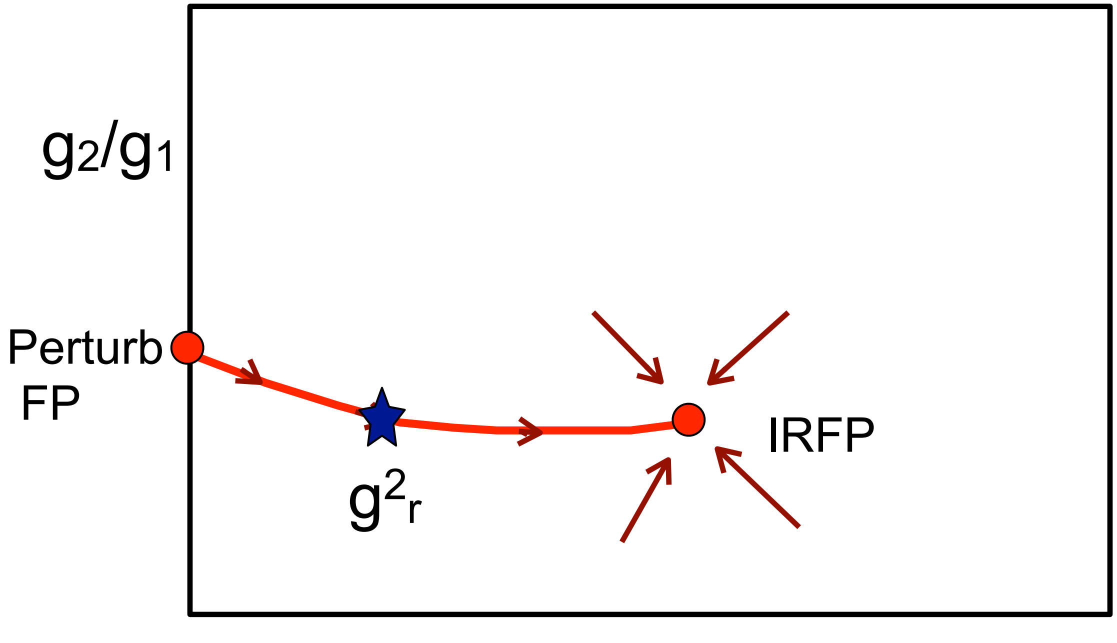

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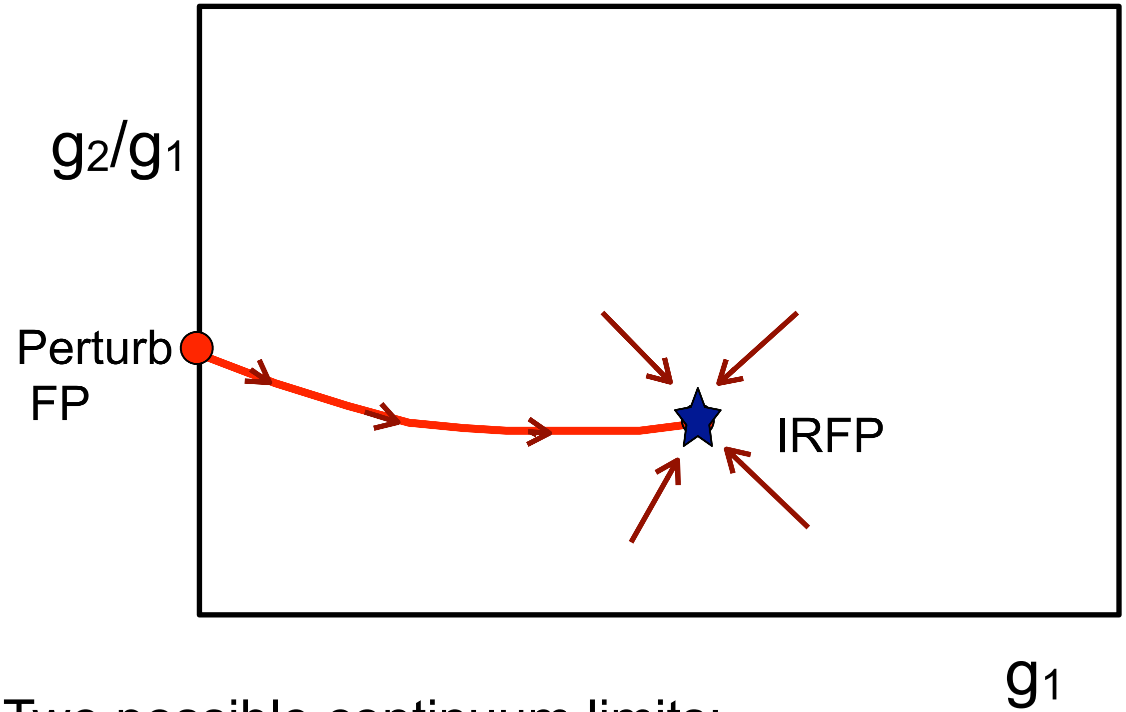

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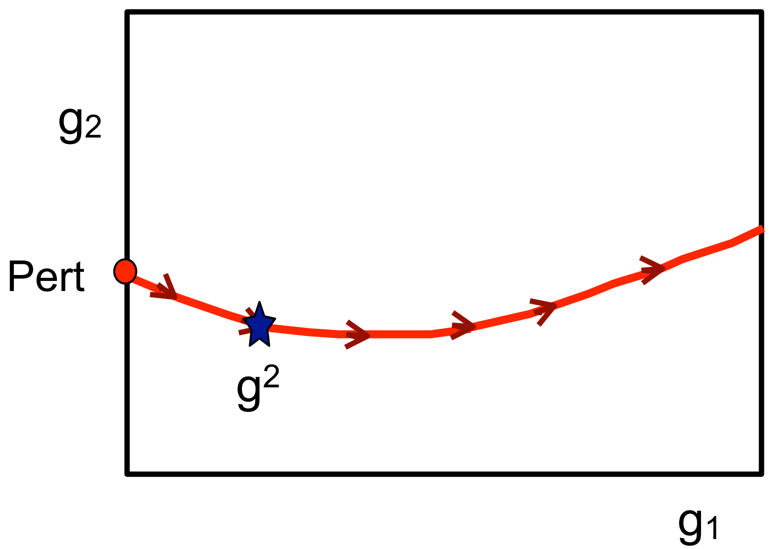

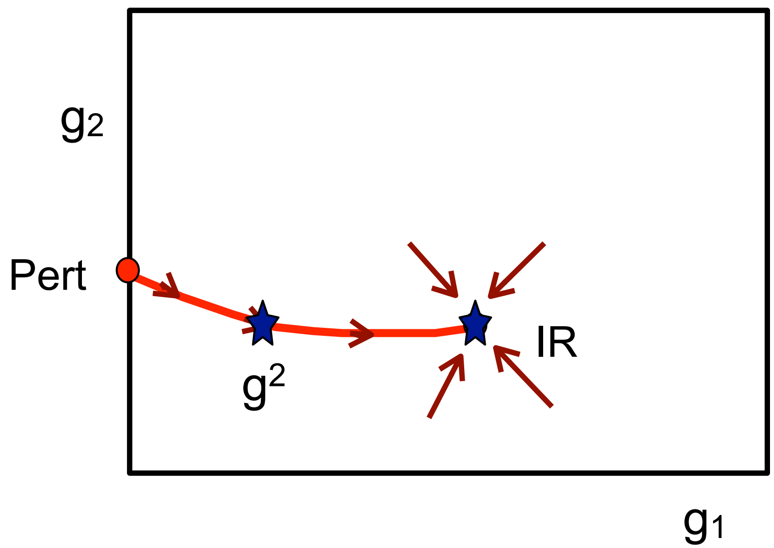

- they look very similar along the RT
- if the gauge coupling "walks" : g is nearly **marginal !** **(non-QCD like)**

SU(3) gauge with N_f= 4, 8 and 12 fundamental flavors

Discuss 2 methods:

1. Study of Dirac eigenmodes and spectral density $\rho(\lambda)$
   Distinguishes weak & strong coupling regions

2. Finite size scaling analysis
   Shows the effect of the near marginal gauge coupling

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   m,L
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   m,L
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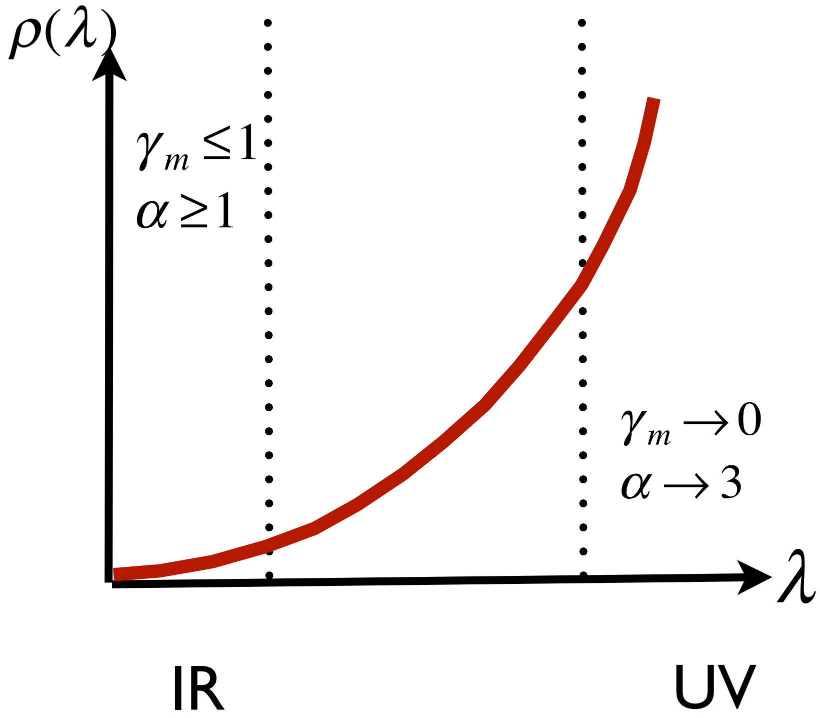
Mostly N_f=4 and 12 flavor to test the methods and understand/resolve existing controversies.
Some N_f=8 : preliminary but exciting!

Scaling of the Dirac eigenvalue spectrum - **conformal system**

Eigenvalue density $\rho(0)=0$, scales as $\rho(\lambda) \propto \lambda^{\alpha(\lambda)}$

RG invariance implies $\frac{4}{1+\alpha} = y_m = 1+\gamma_m$

λ provides an energy scale



IR – small λ region:

$$\gamma_m(\lambda \to 0) = \gamma_m^*$$

predicts the universal anomalous dimension at the IRFP

UV – large λ =O(1) region:
if governed by the asymptotically free perturbative FP

$$\gamma_m(\lambda = \mathcal{O}(1)) = \gamma_0 g^2 + \ldots$$

In between:
scale dependent effective γ_m

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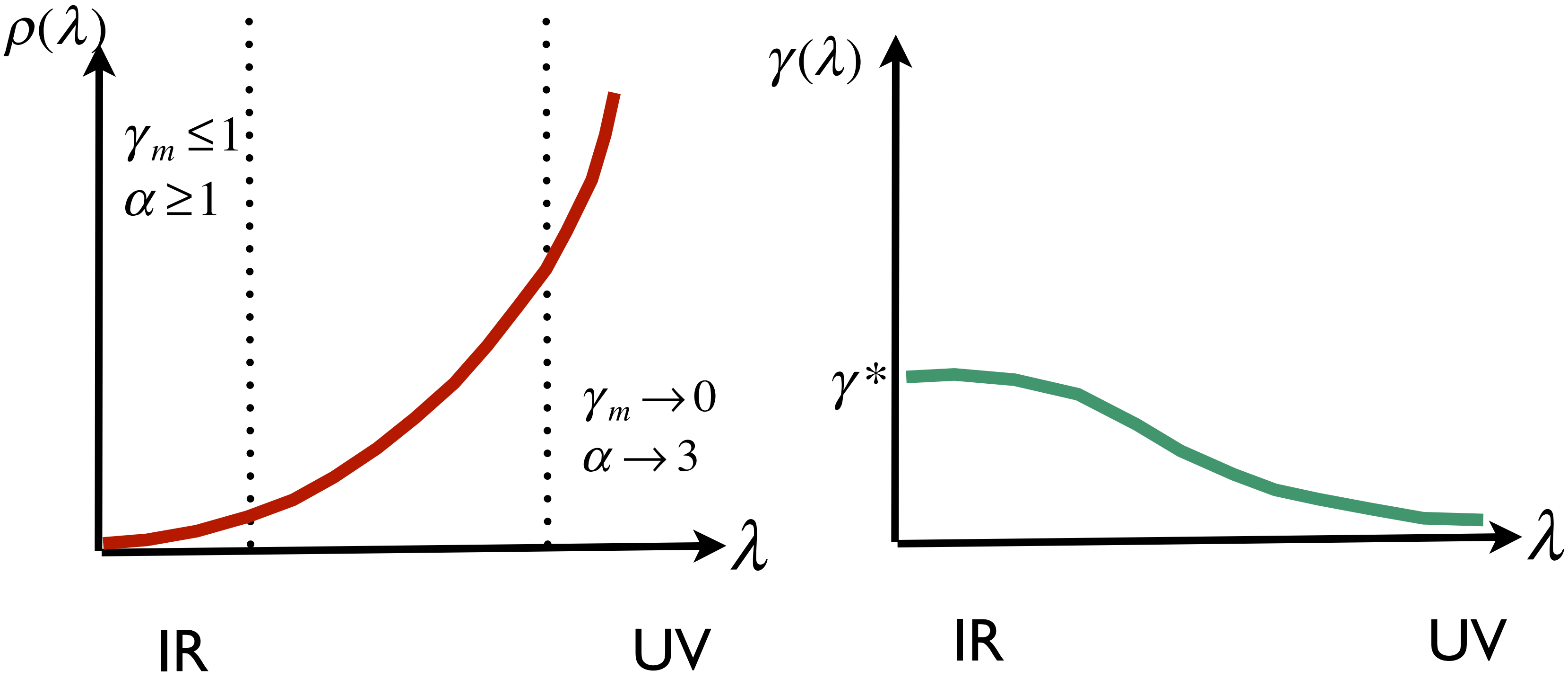

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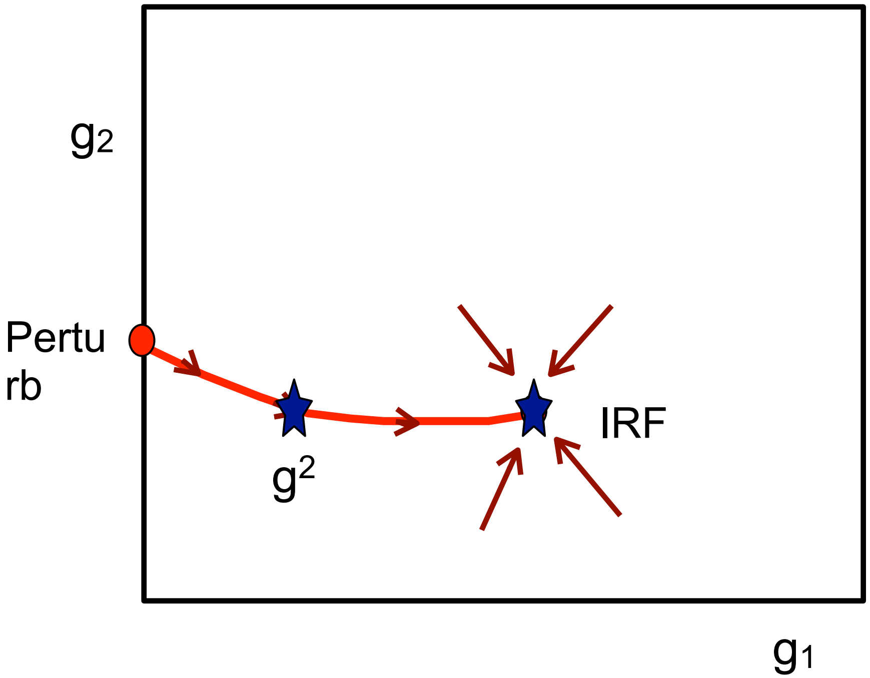

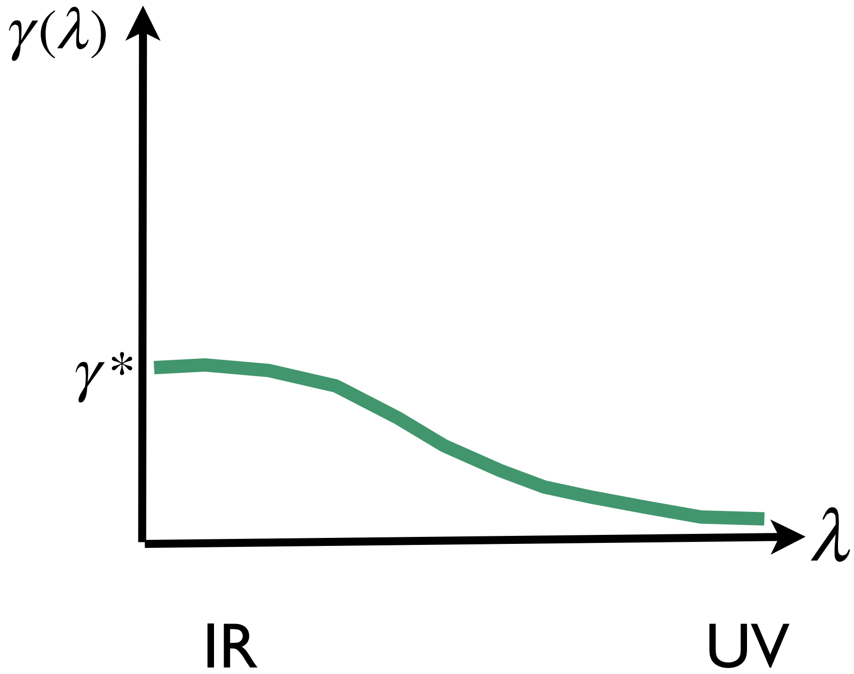

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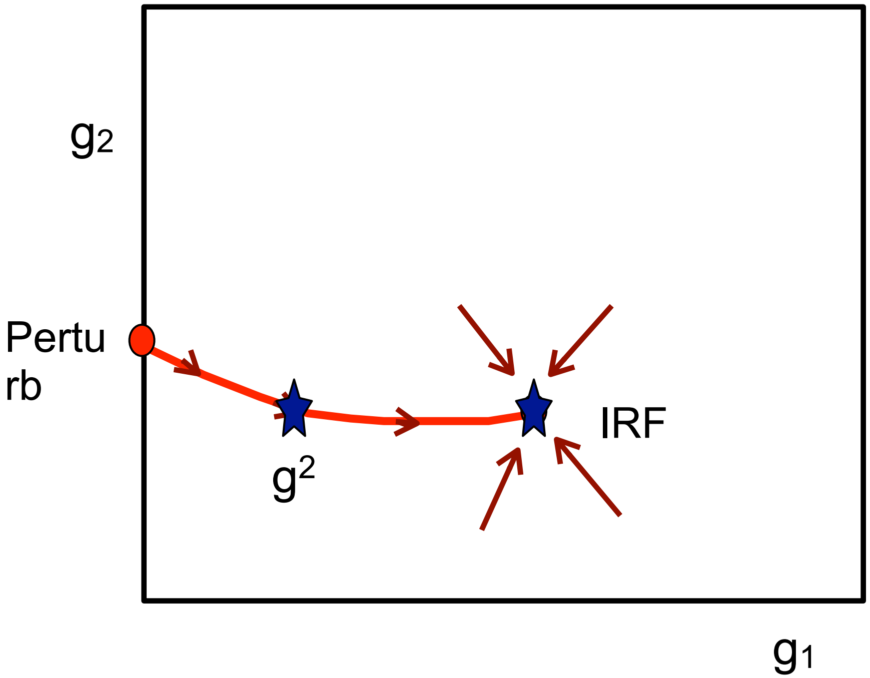
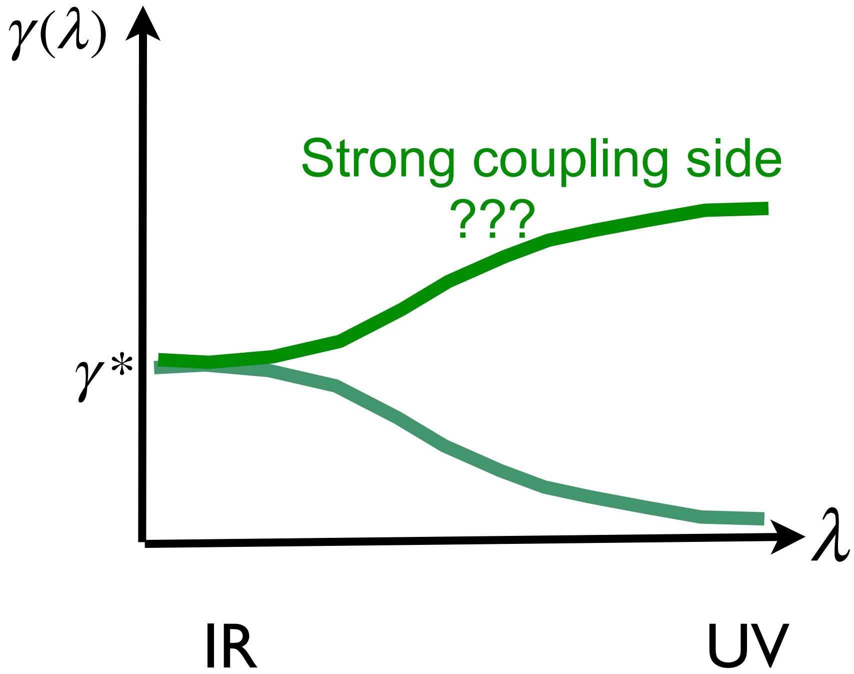
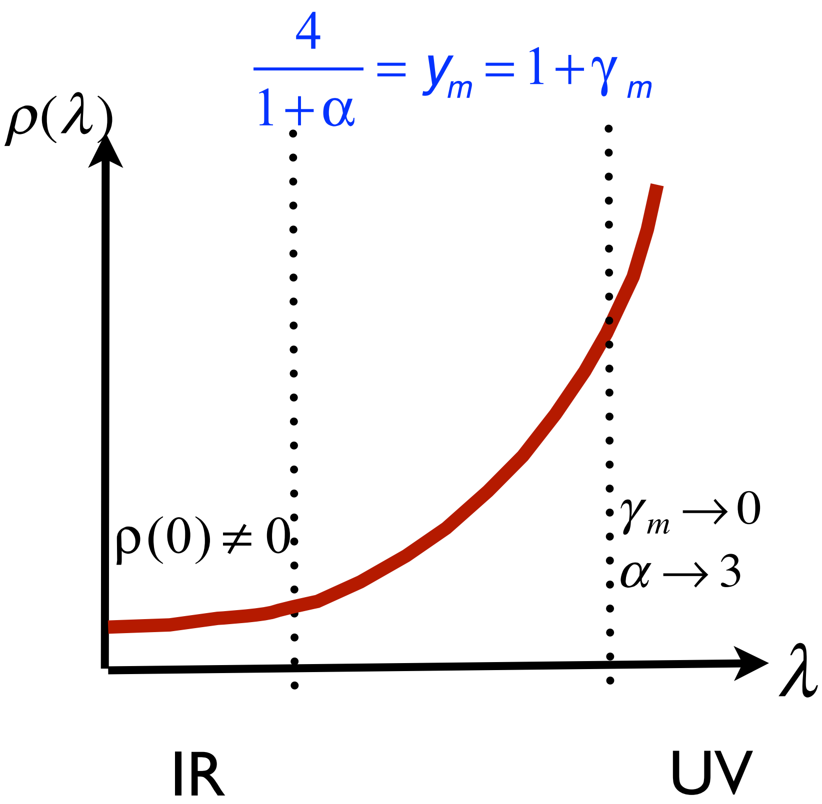
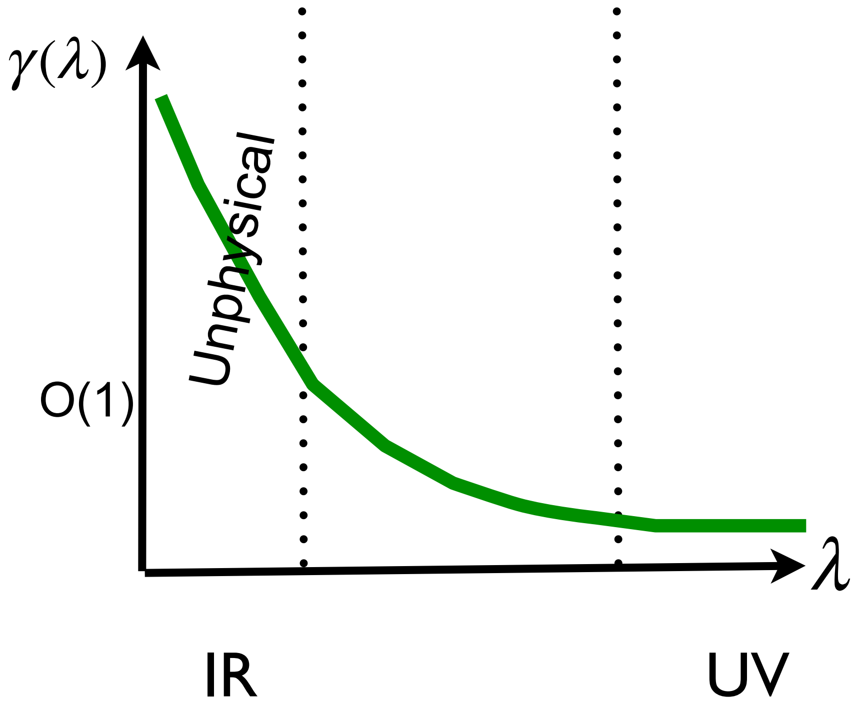




Dirac eigenvalue spectrum - **chirally broken system**

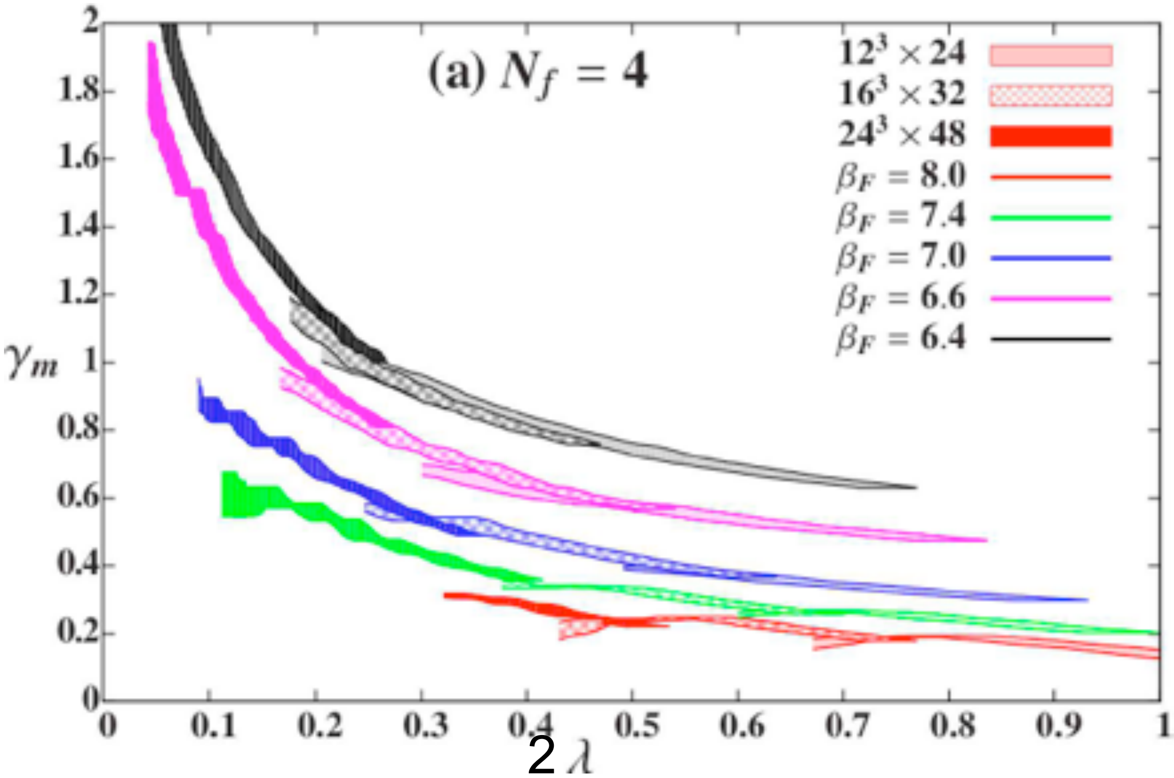
Chirally broken systems show only the asymptotically free region





Results: N_f =4

Broken chiral symmetry in IR, asymptotic freedom in UV



Lattice spacing from Wilson flow:

$$a_{6.4} \;/\; a_{7.4} = 2.84(3)$$
$$a_{6.6} \;/\; a_{7.4} = 2.20(5)$$
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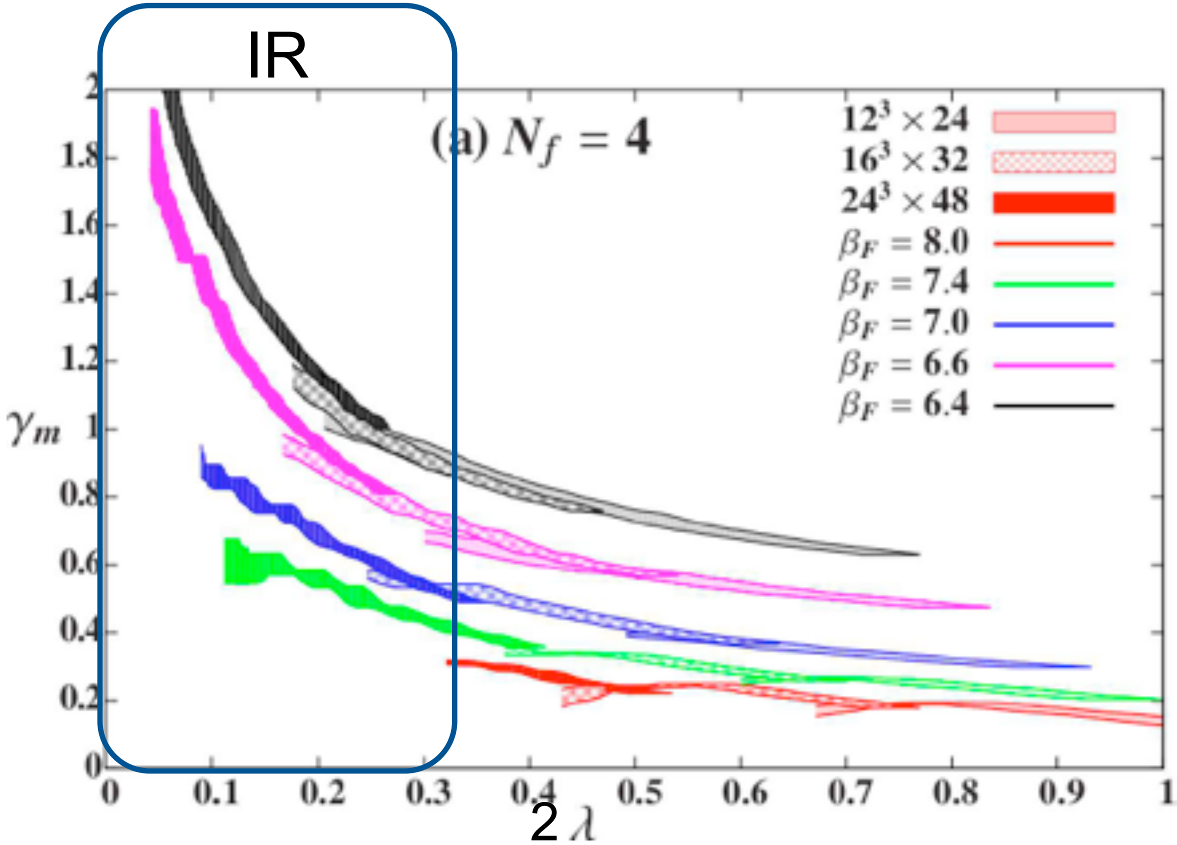

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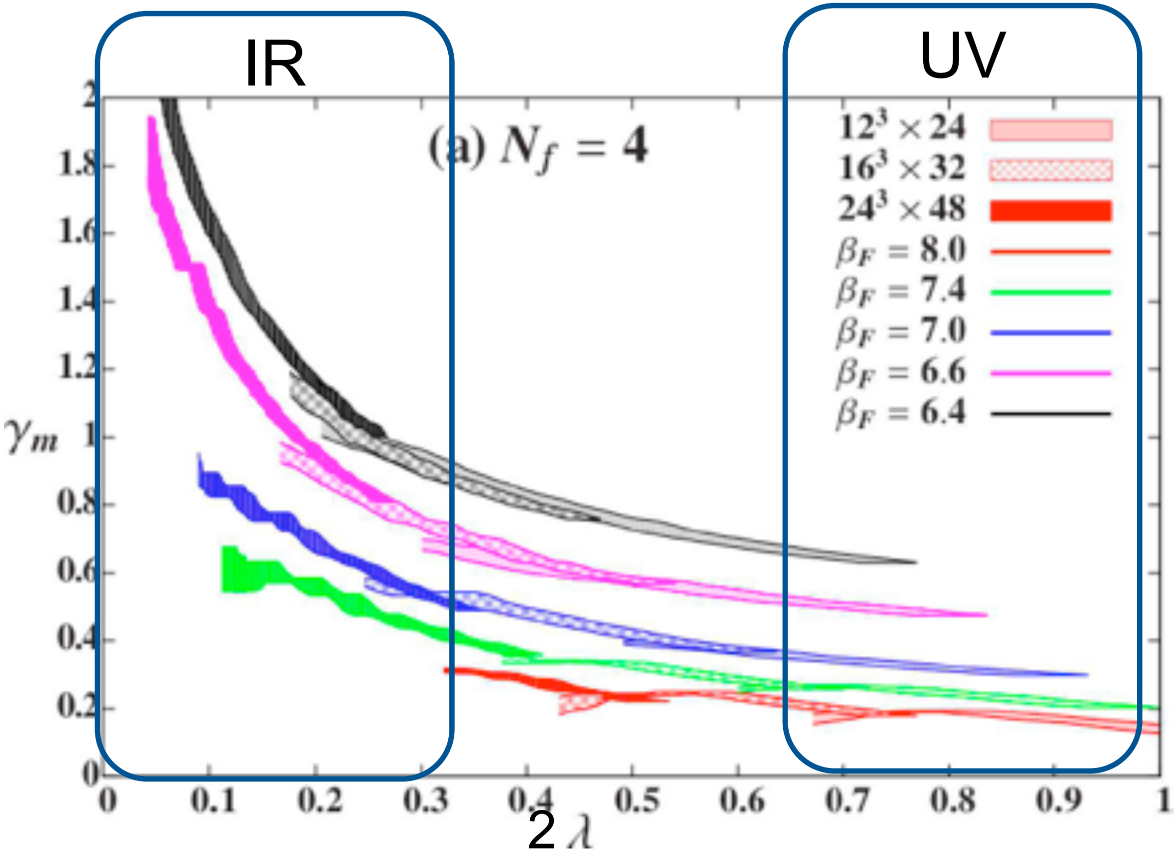

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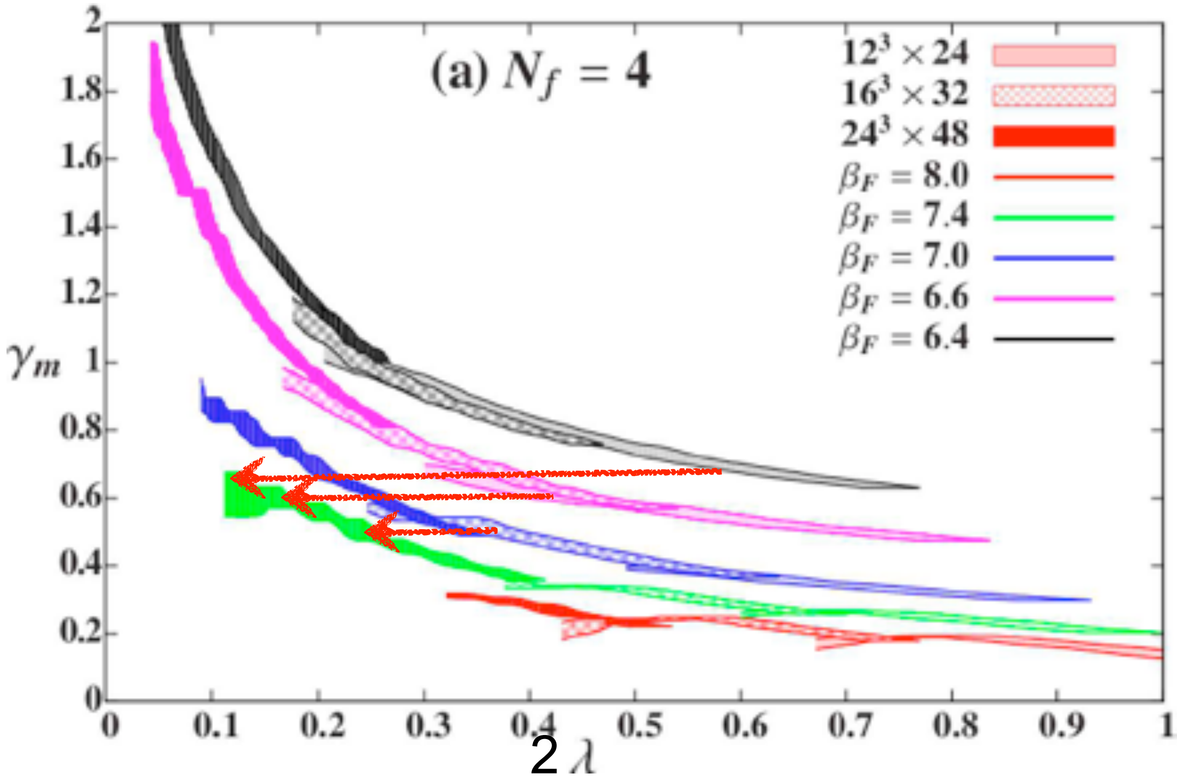
Rescaling: N_f =4

The dimension of λ is carried by the lattice spacing: $\lambda_{lat} = \lambda_p a$
Rescale to a common physical scale:

$$\lambda_\beta \to \lambda_\beta \left(\frac{a_{7.4}}{a_\beta} \right)^{1+\gamma_m(\lambda_\beta)}$$



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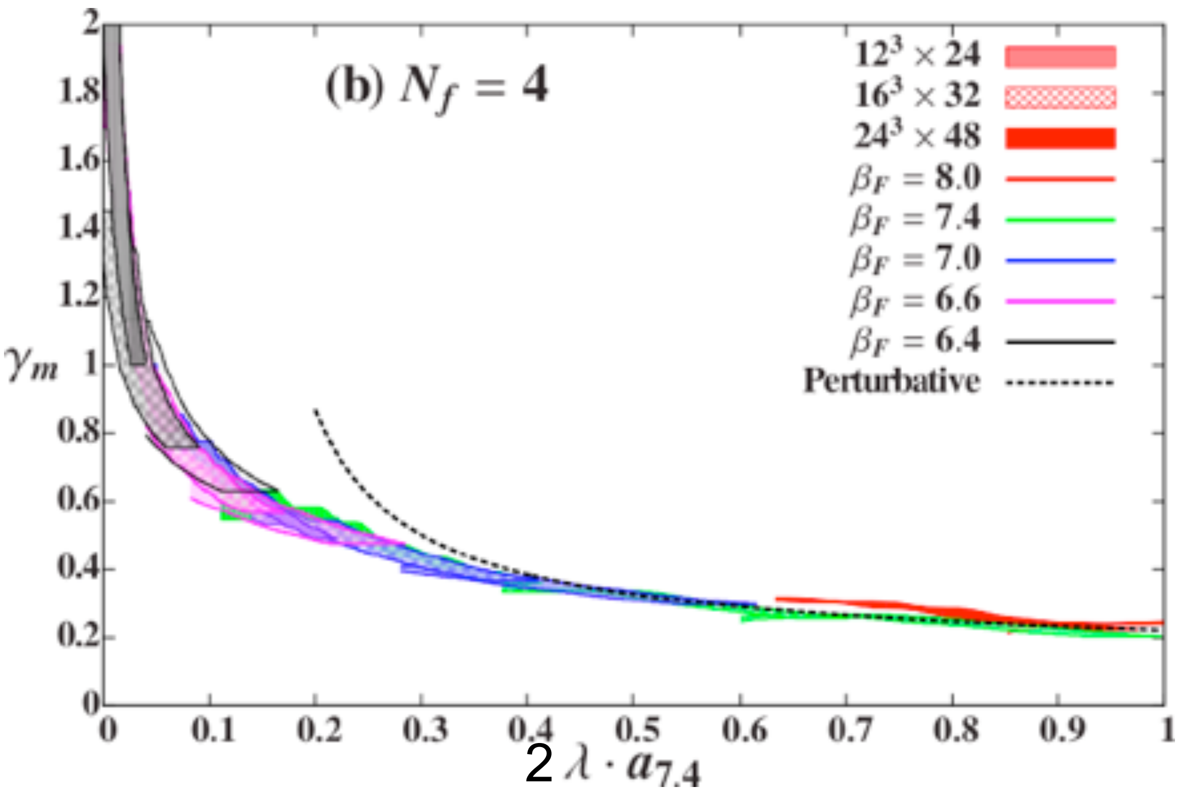
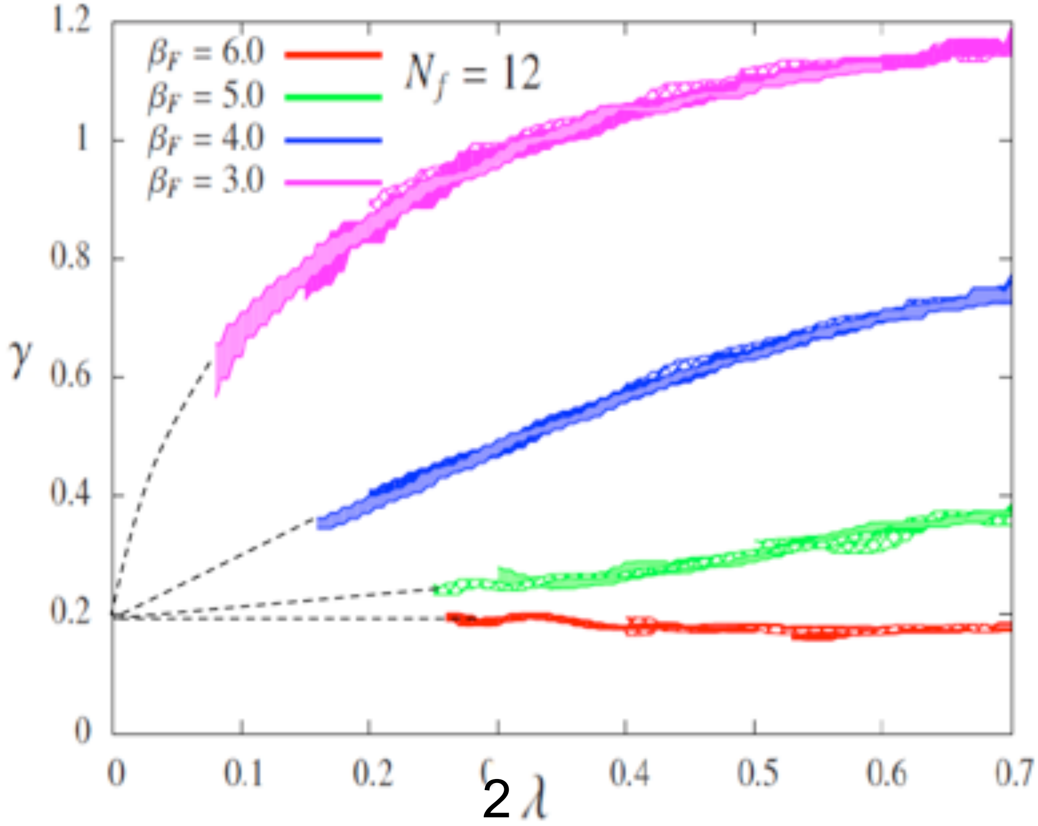


Universal curve covering almost 2 orders of magnitude in energy!

Perturbative: functional form from 1-loop PT, relative scale is fitted

Most of these data were obtained on deconfined (small) volumes at m=0!

Spectral density results: N_f =12



β=3.0, 4.0, 5.0, 6.0

- There is no sign of asymptotic freedom behavior for β<6.0,
  γ_m grows towards UV

- Not possible to rescale different β's to a single universal curve

Looks as if there was an IRFP between β=5.0 -6.0

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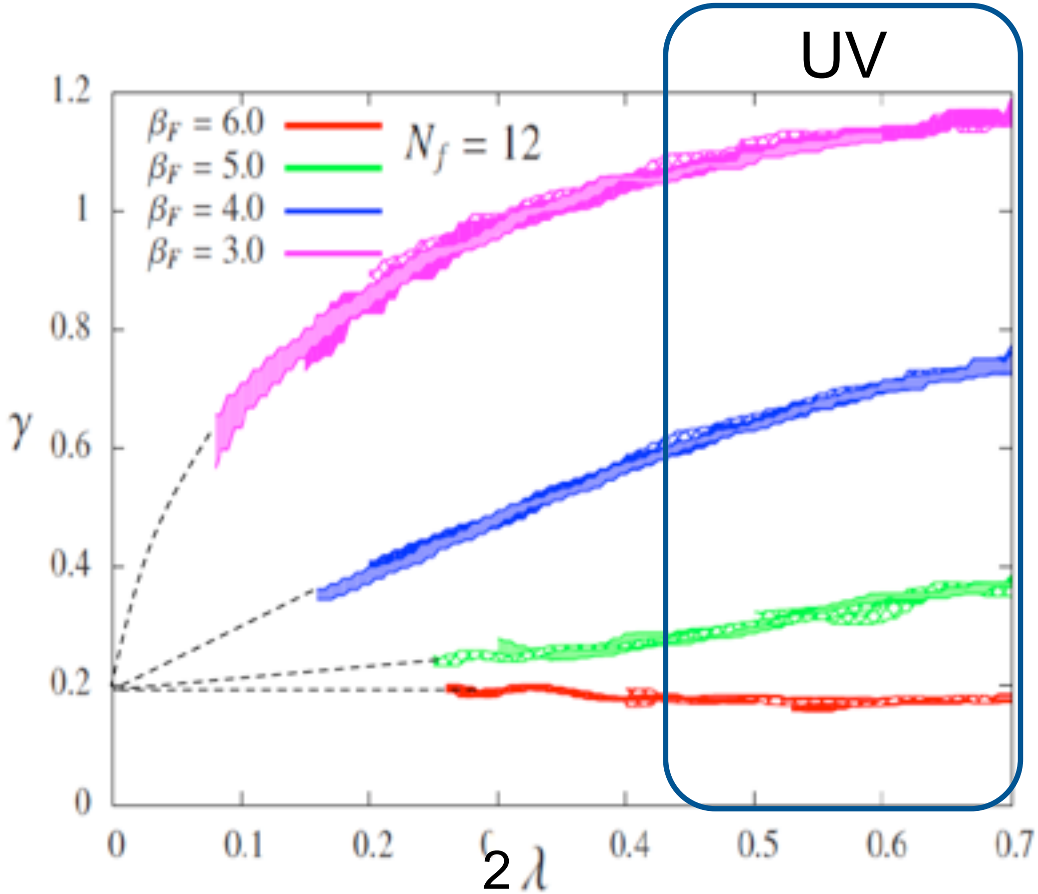

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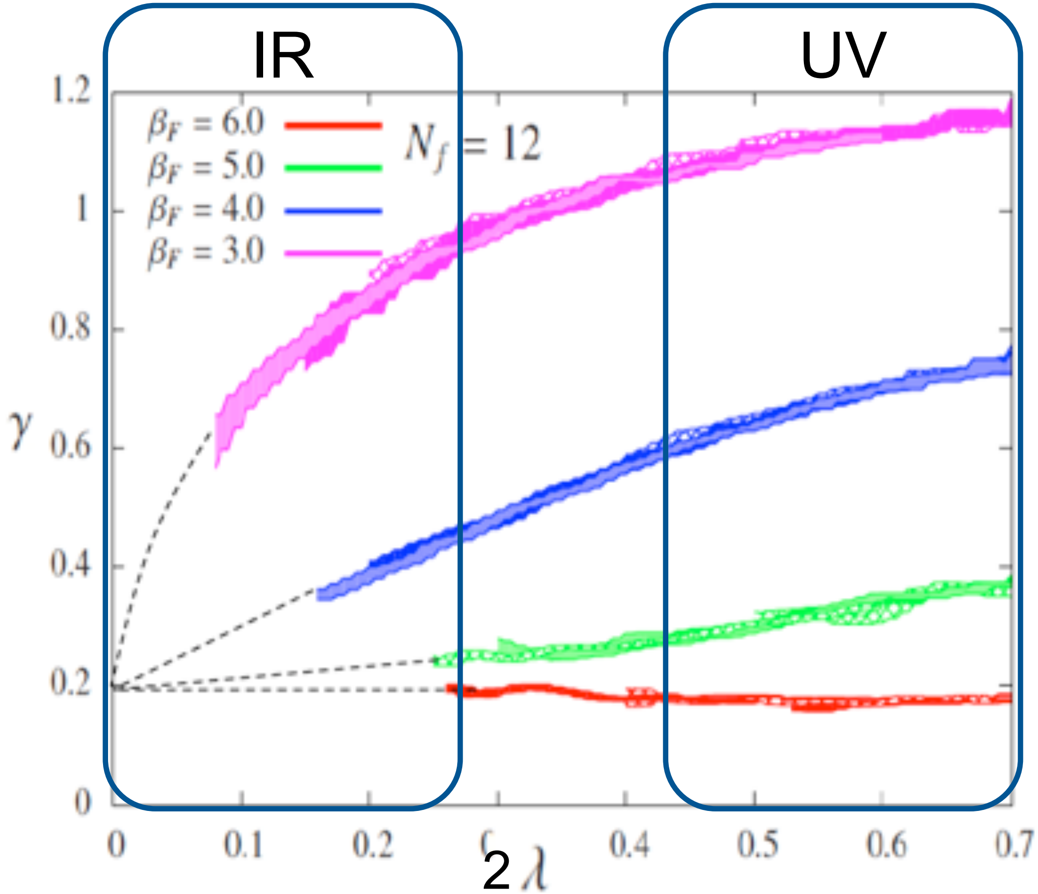

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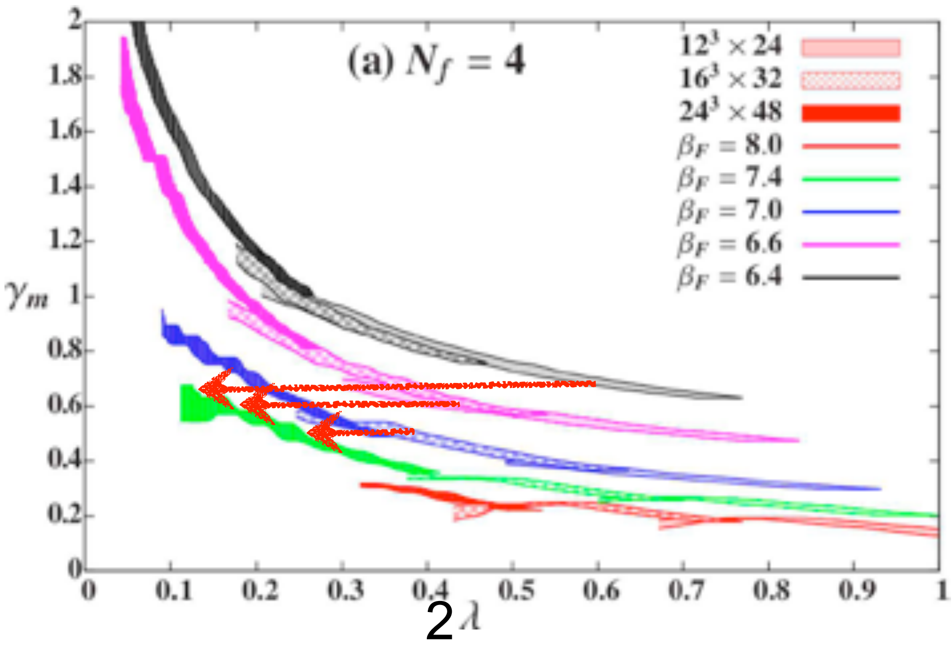
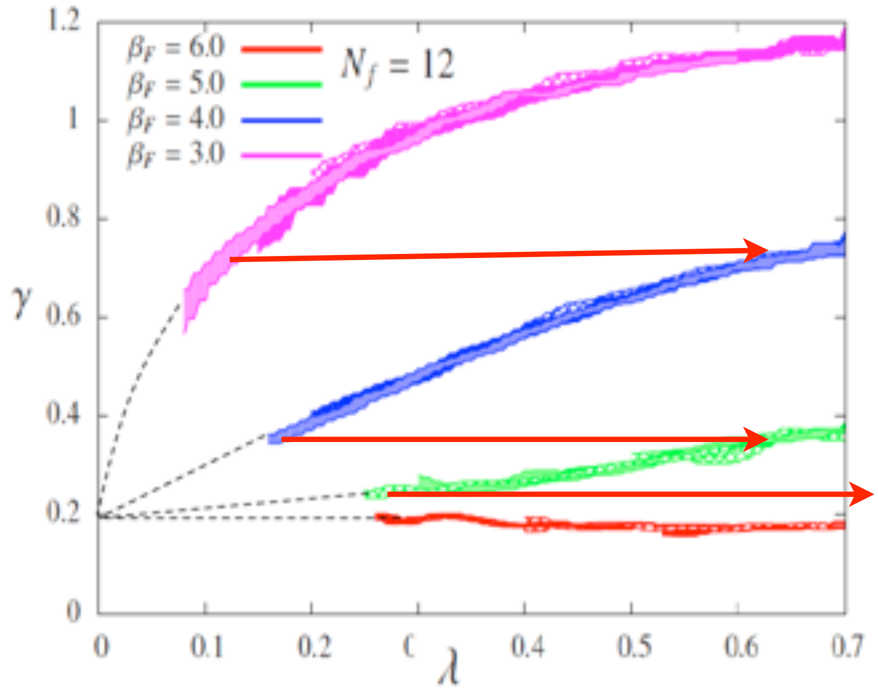
Rescaling N_f=4 vs N_f=12



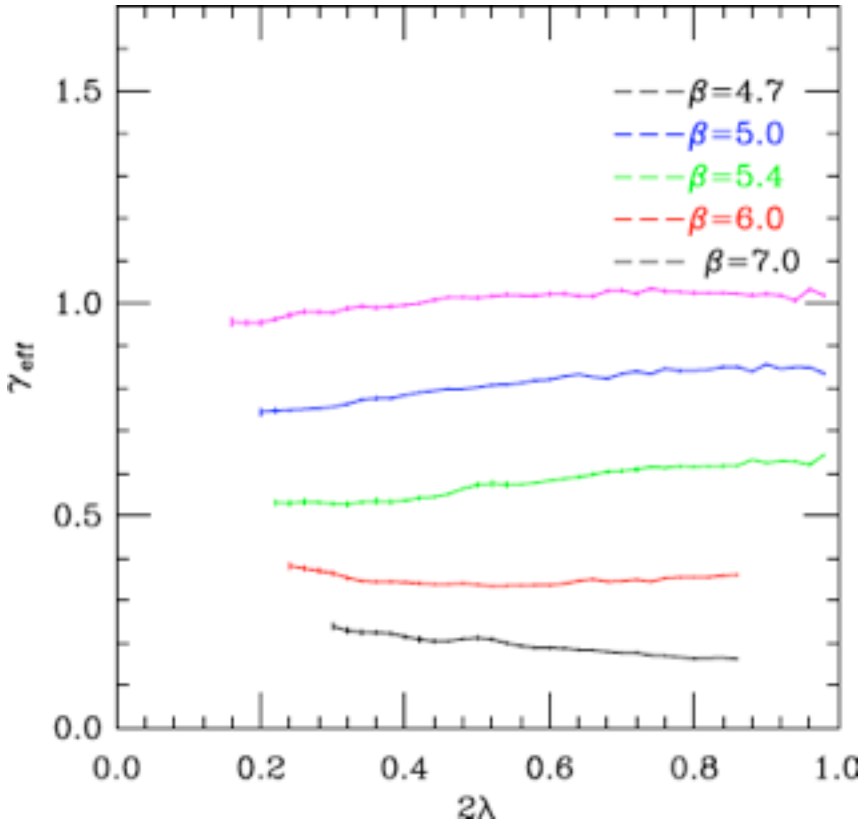


N_f=4 : smaller β matches to the left (forward flow)

N_f=12 : no consistent rescaling but even an approximate one matches to the right of β<6.0

Anomalous dimension, N_f =8

Expected to be chirally broken - looks like walking!



-No asymptotic free scaling
-No rescale of different couplings

-When $\gamma_m \sim 1$ in the UV, the S^4b phase develops

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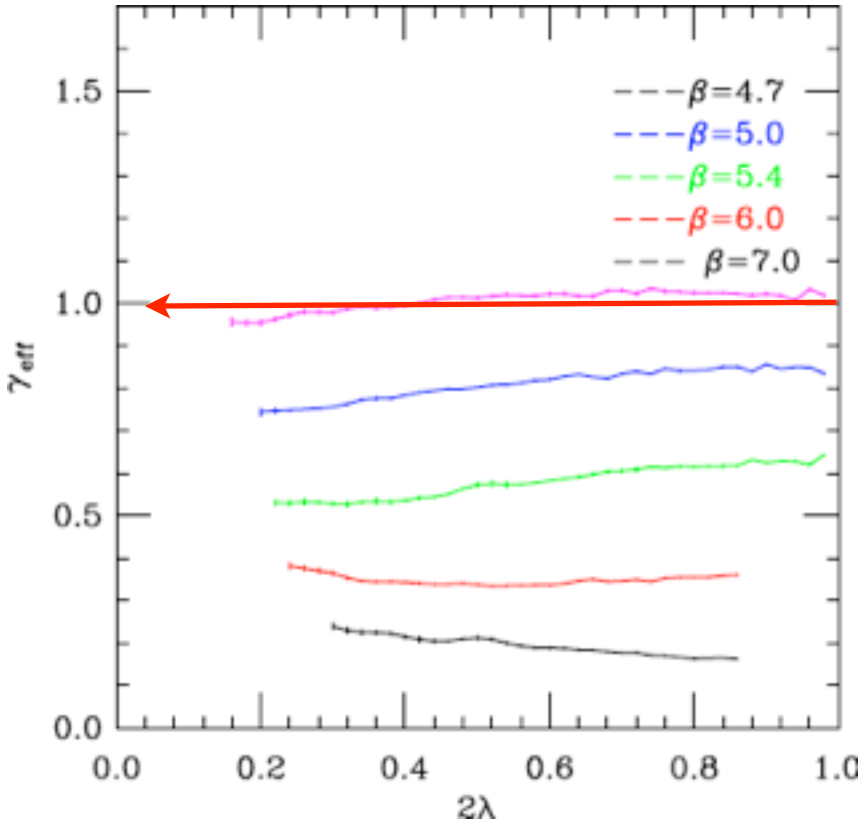

-No asymptotic free scaling
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-When γ_m ~ 1 in the UV, the S^4b phase develops

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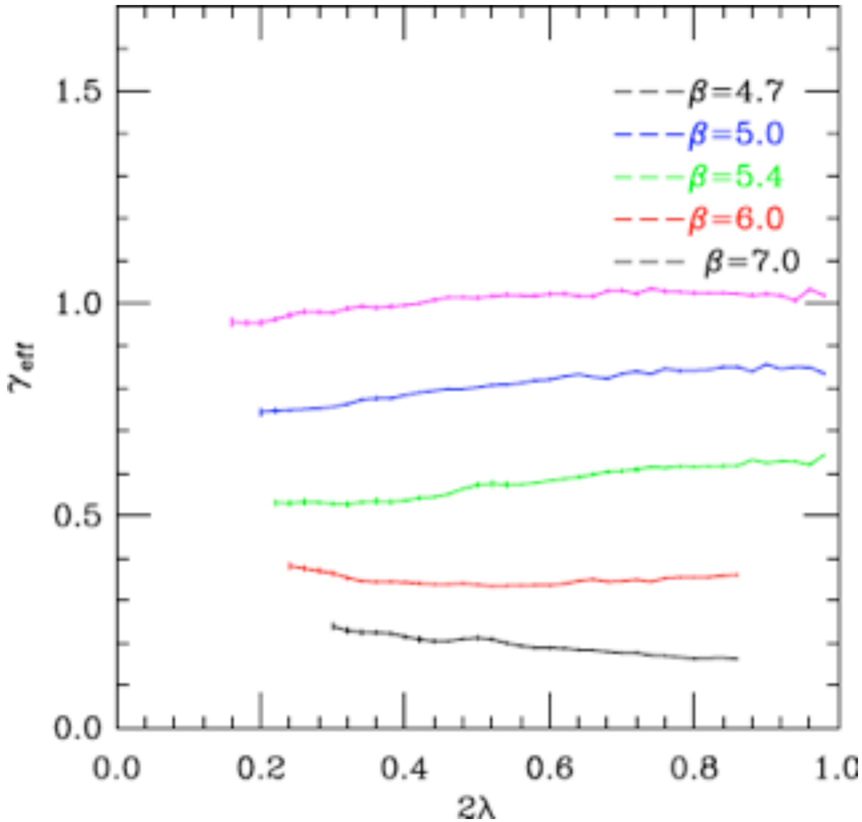

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- No rescale of different couplings

- When γ_m ~ 1 in the UV, the S^4b phase develops

Dirac operator eigenvalue spectrum and spectral density

Unique & promising method !

- Can distinguish strong and weak coupling region of conformal /chirally broken systems

Predictions:

N_f=4 : scaling & anomalous dimension

N_f=12 : looks conformal

N_f=8 : could be walking with large anomalous dimension!

II : Finite size scaling

Well understood method in systems governed by one relevant operator

→ in conformal systems it could predict the mass anomalous dimension

Is this prediction internally consistent? Is it consistent with results of spectral density?

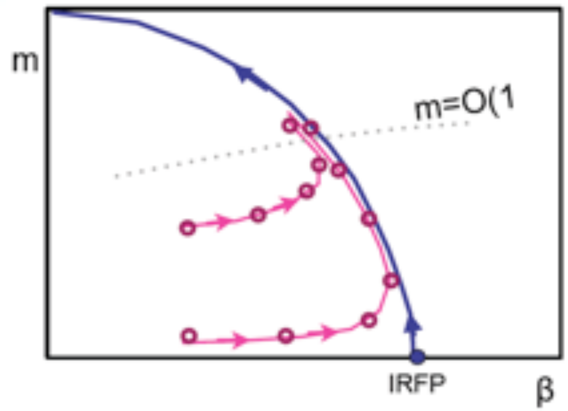
Finite size scaling - textbook case

Consider a FP with one relevant operator

$m \approx 0$ with scaling dimension $y_m > 0$

and irrelevant operators

g_i with scaling dimensions $y_i < 0$.



Renormalization group arguments in volume L^3 predict scaling of physical masses as

$$M_H L = f(L m^{1/y_m}, g_i m^{-y_i/y_m}) \quad \text{as} \quad m \approx 0$$

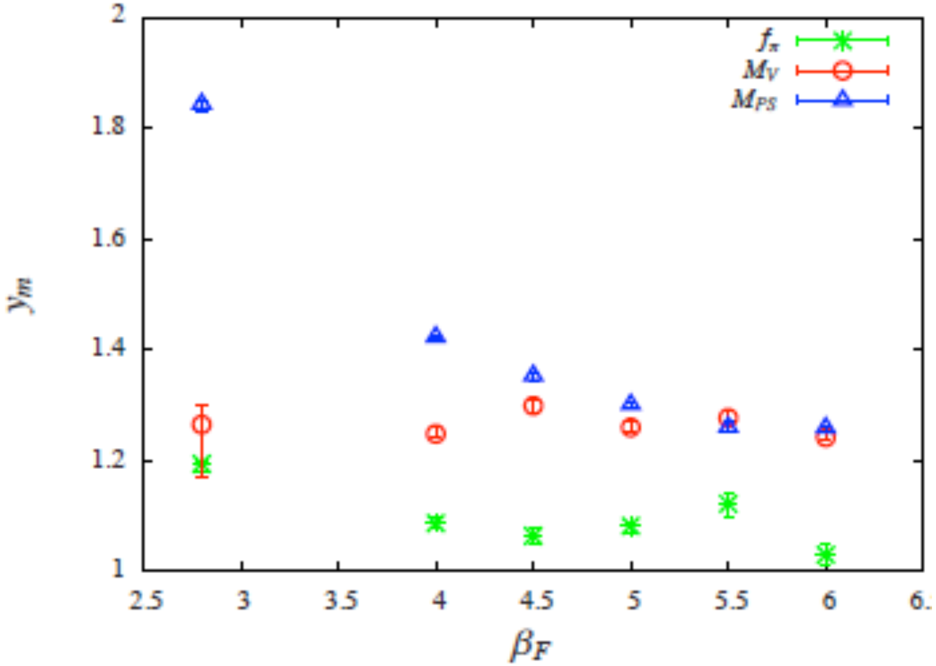
as $m \to 0, \quad L \to \infty: \quad g_i m^{-y_i/y_0} \to 0$

$$M_H L = f(x), \quad x = L m^{1/y_m}$$

–tune y_m until different volumes “collapse”

Scaling exponents

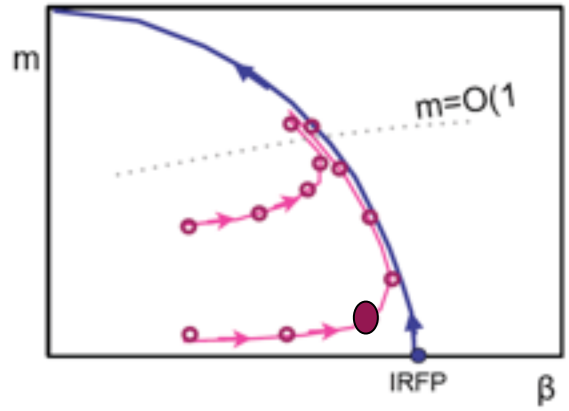
Result of “curve collapse” for pseudo-scalar, vector and f_{π}:



y_m depends strongly on β and the operator considered

→ Internally inconsistent !!!

Finite size scaling with a **near-marginal operator**



Consider a FP with one relevant operator

$m \approx 0$ with scaling dimension $y_m > 0$

and irrelevant operators

g_i with scaling dimensions $y_i < 0$

g_0 (near) marginal, $y_0 \lesssim 0$

Renormalization group arguments in volume L^3 predict

$$M_H L = f(Lm^{1/y_m}, g_i m^{-y_i/y_m}) \quad \text{as} \quad m \approx 0$$

as $m \to 0, \quad L \to \infty: \quad g_i m^{-y_i/y_0} \to 0$

$$g_0 \to g_0 m^{\omega}, \quad \omega = -y_0 / y_m \gtrsim 0$$

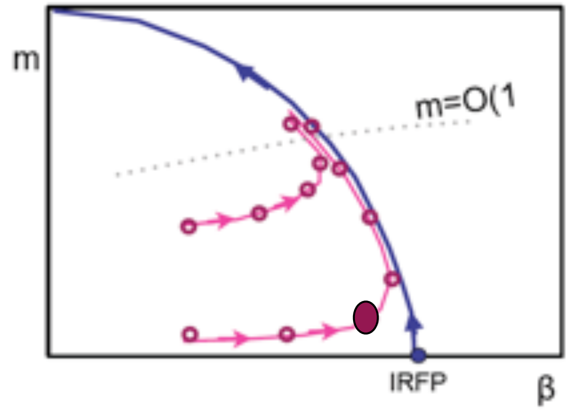
$$M_H L = f(x, g_0 m^{\omega}), \quad x = Lm^{1/y_m}$$

The scaling function depends on two variables now!

Corrections to finite size scaling

Physical masses scale as

$$M_H = L^{-1} f(x, g_0 m^{\omega}), \quad \omega = -y_0 / y_m$$



If the $g_0 m^{\omega}$ corrections are small, expand

$$LM_H = F(x)\left(1 + g_0 m^{\omega} G(x)\right)$$

Approximate $G(x) = c$ (should be checked) →

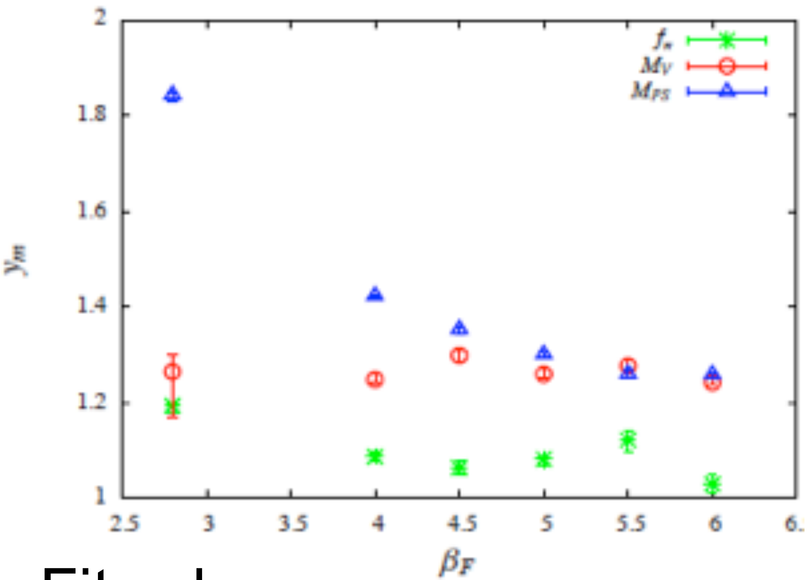
$$\frac{LM_H}{1 + c\, g_0 m^{\omega}} = F(x)$$

Fit needs minimization in y_m, ω, and $c_0 = c g_0$

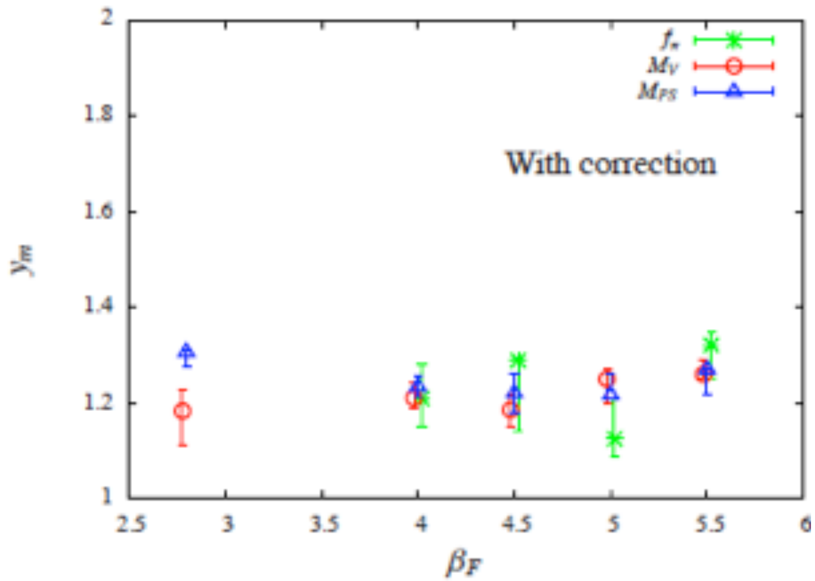
Scaling exponent **with** corrections

Include all data $M_\pi L$, $M_V L$, $f_\pi L$ points

Leading operator only



With correction



Fits show

- good curve collapse
- consistent scaling exponent y_m=1.22(2)
- can we constrain the fit parameters better?

Scaling exponent **with** corrections

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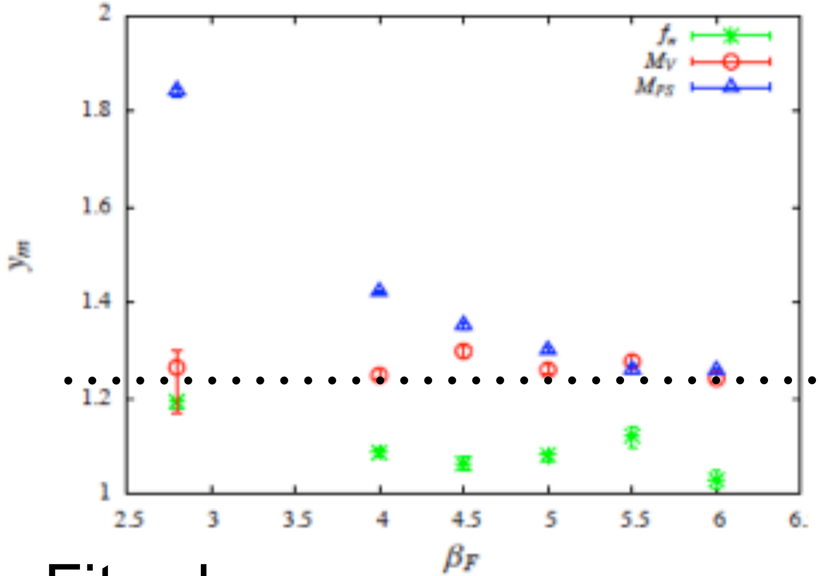
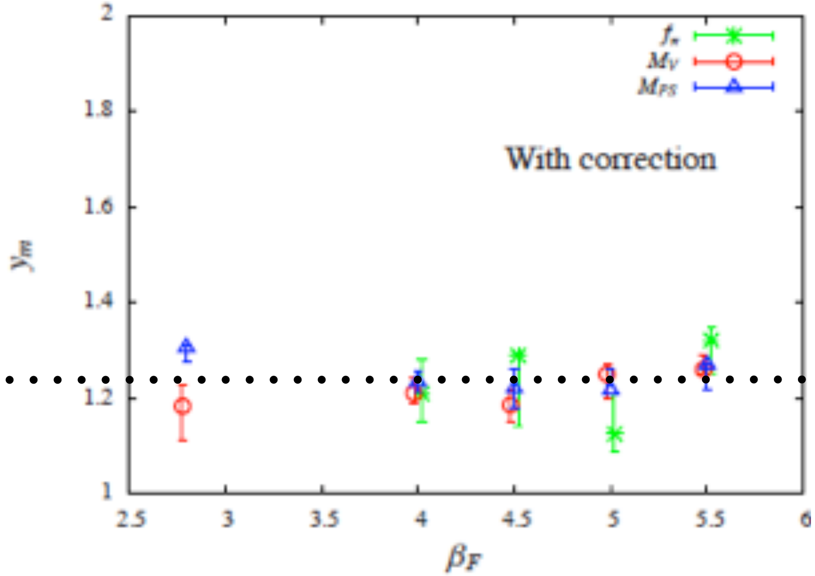


With correction



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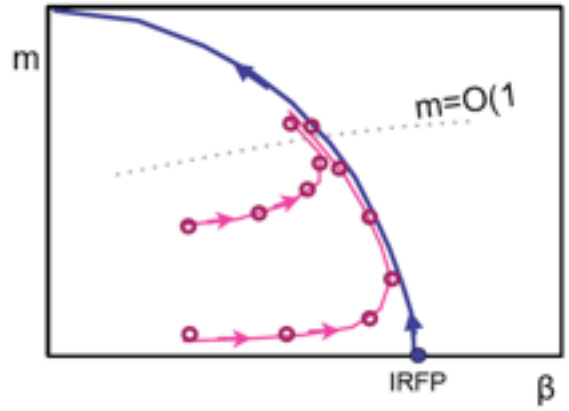
Combining data sets:



If the gauge coupling is irrelevant,
the scaling function F(x)

$$\frac{LM_H}{1+c\,g_0 m^{\omega}} = F(x)$$

is unique, independent of

- gauge coupling β
- lattice action (nHYP or stout or HISQ)

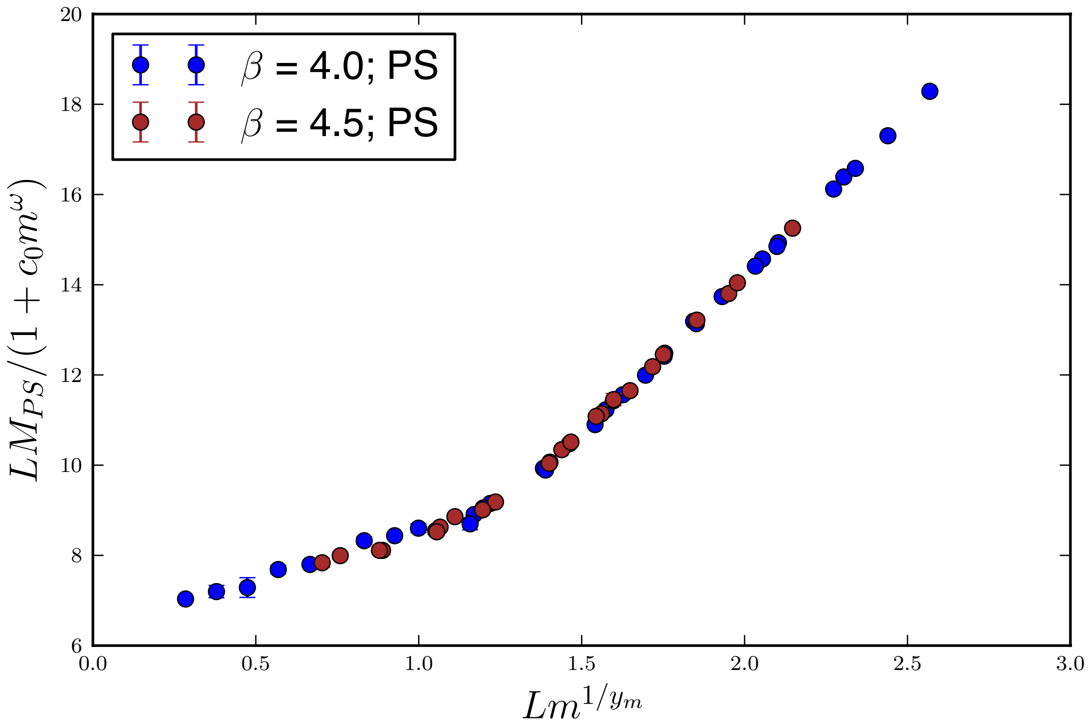
Combine different data sets

- we need to rescale the bare fermion mass m(β)→ s m(β)
- remnant scaling violations could be different for different sets
  → most noticeable at small x (or L)

Combining gauge couplings:

pion at β=4.0,4.5 (all available volumes):

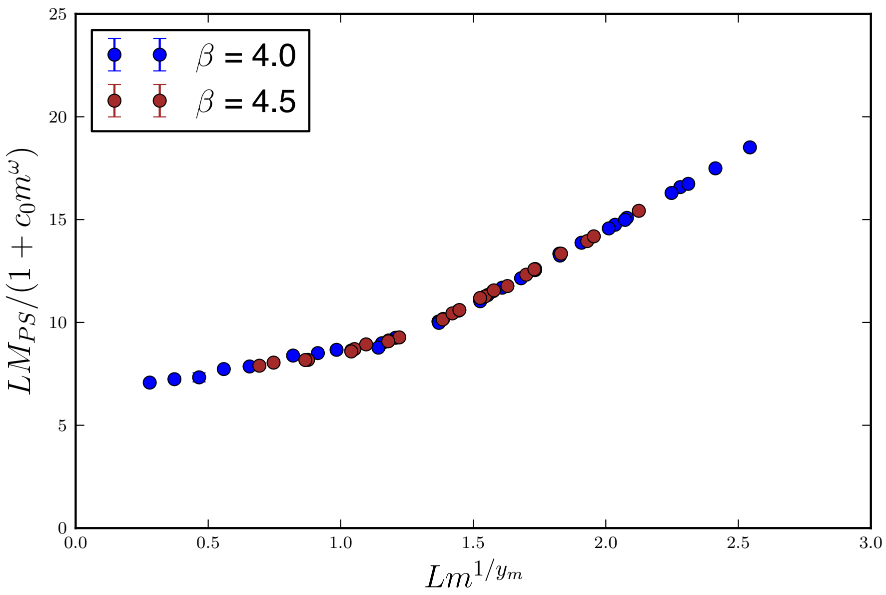
y_m=1.23[2], y_0=-0.47[6] ; χ^2/dof =1.2 [60]

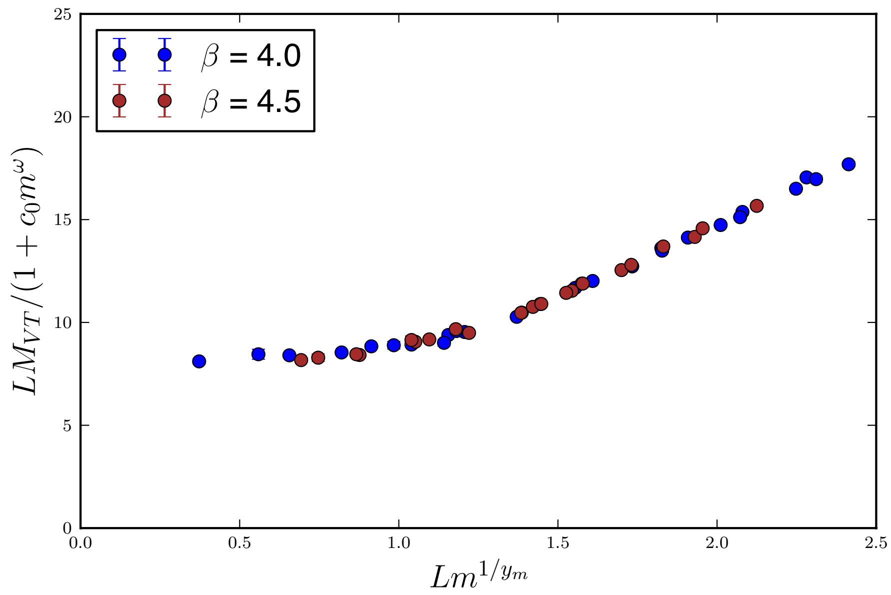


Combining gauge couplings AND operators

pion and vector at β=4.0,4.5 (new fit!)

y_m=1.22[2], y_0=-0.50[5] ; χ^2 /dof =1.4 [108]

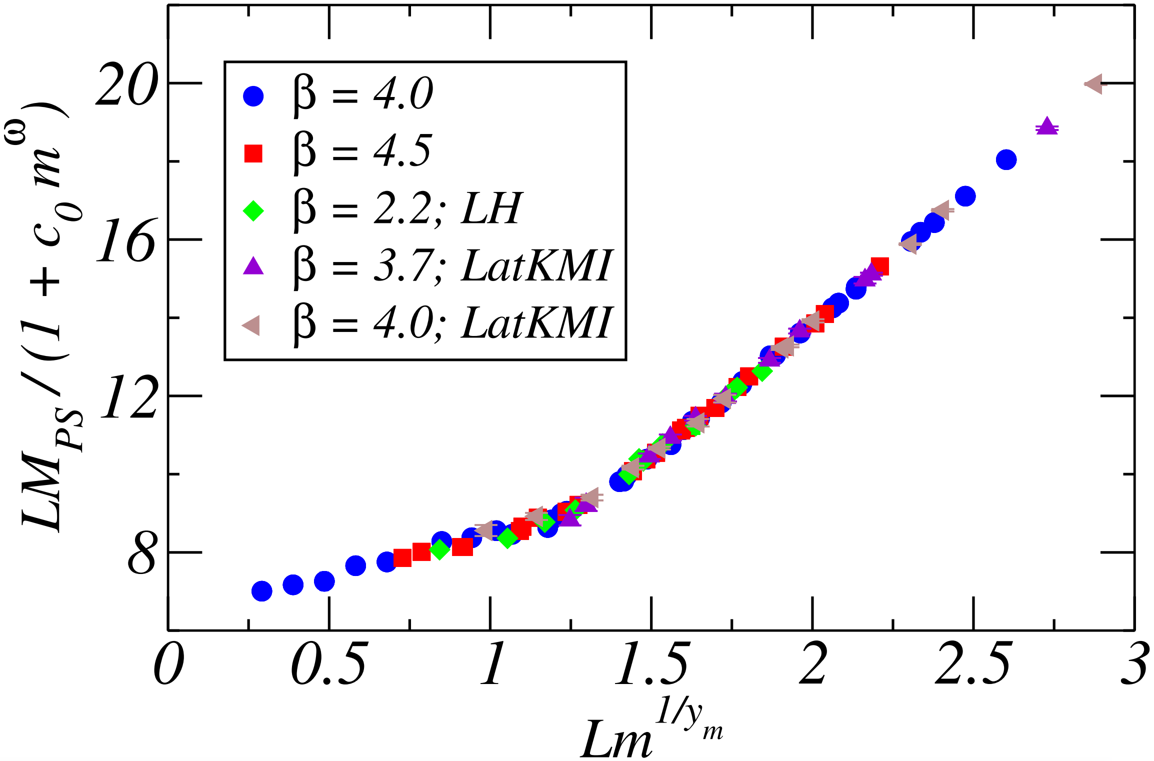




Combining gauge couplings AND actions

pion at β=4.0,4.5, LH, KMI :

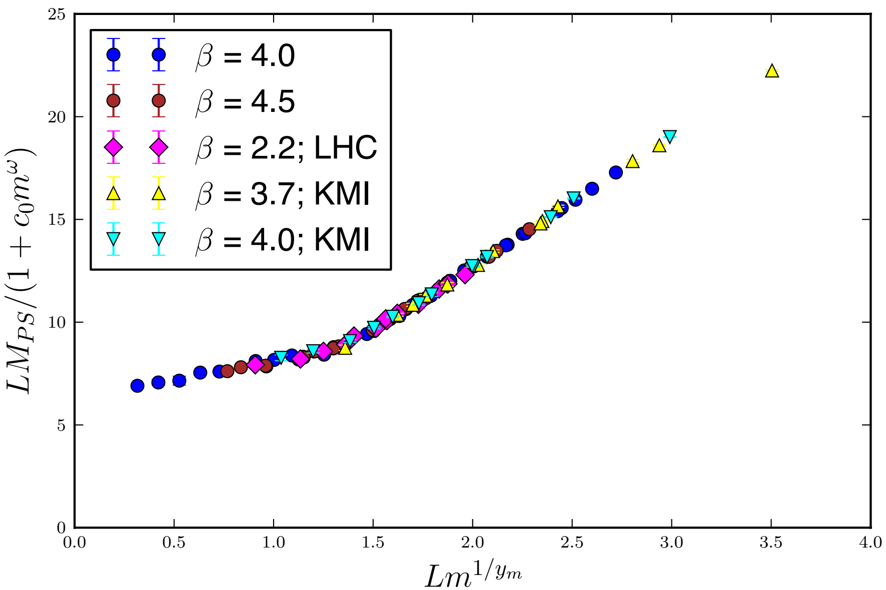
y_m=1.24[1], y_0=-0.51[5] ; χ^2 /dof =1.4 [95]

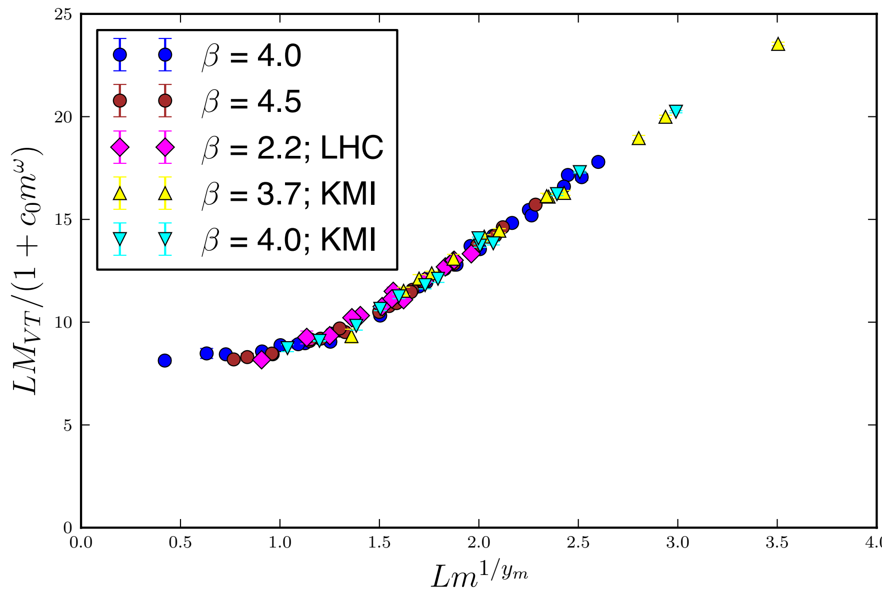


Combining gauge couplings AND actions AND operators

pion and vector at β=4.0,4.5, LHC, KMI :

y_m=1.27[1], y_0=-0.51[5] ; χ^2/dof =2.7 [188]





Consistency:

Fit 30-300 points with 10 - 20 parameters ...

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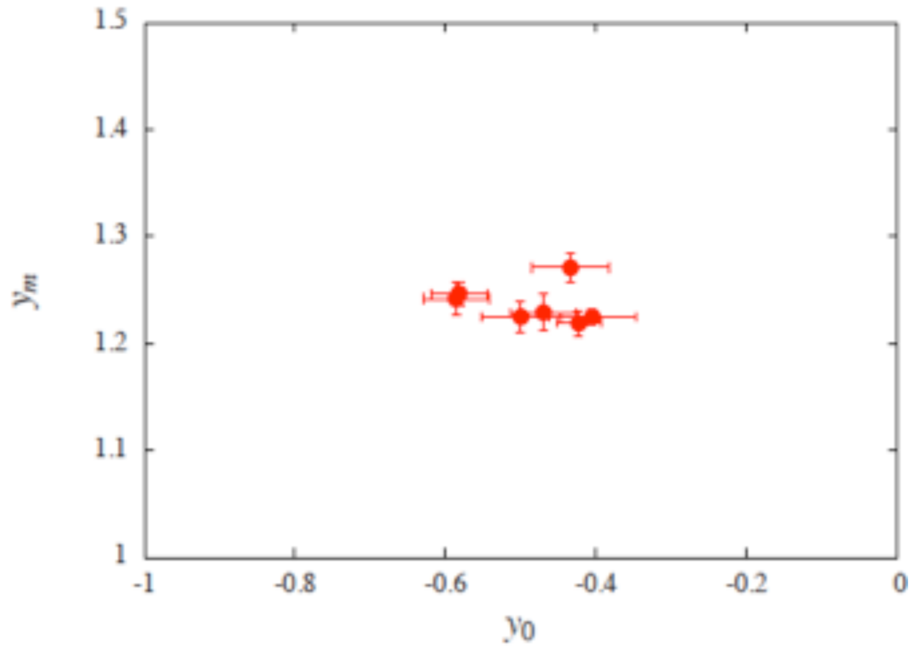
“With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.”

John von Neumann

Consistency:

Fit 30-300 points with 10 - 20 parameters ...

yet y_m, y_0, are consistent



"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk."

John von Neumann

Fits combining different data sets, operators, predict

$y_m = 1 + \gamma_m = 1.235[15]$ with $\chi^2/\mathrm{dof} \approx 1 - 3$

Message from FSS

The gauge coupling of strongly coupled conformal systems are expected to run slowly ("walking")

→ scaling is strongly influenced by this near-marginal coupling

This is universal in every walking system!

- In finite size scaling analysis the marginal coupling can be accounted for
- Its effect should be considered in every other approach

Summary

Strongly coupled gauge-fermion systems are exciting

— show non-perturbative dynamics with unusual properties

— can offer BSM description with composite Higgs

Near the conformal window they (could)

— walk : slowly changing gauge coupling

— large anomalous dimension

— dilaton: light scalar ?

Lattice studies are only starting to understand these systems