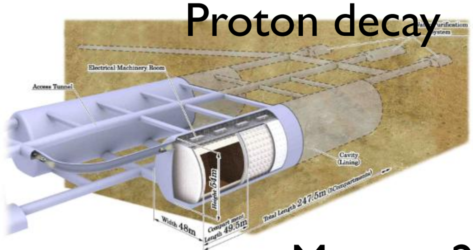
Role of lattice QCD in intensity frontier physics

Eigo Shintani (RIKEN-BNL) for RBC/UKQCD collaboration

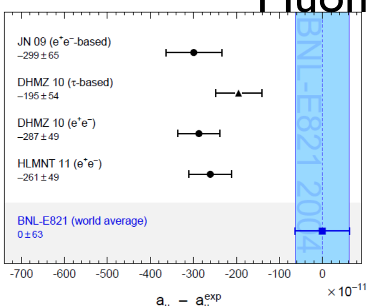
Frontier of particle physics

http://www.fnal.gov/pub/science/frontiers/

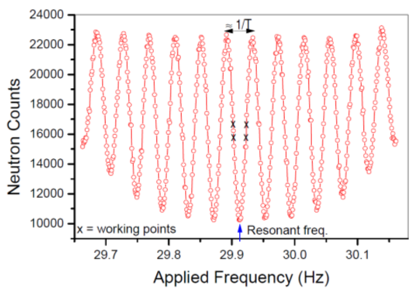
Proton decay

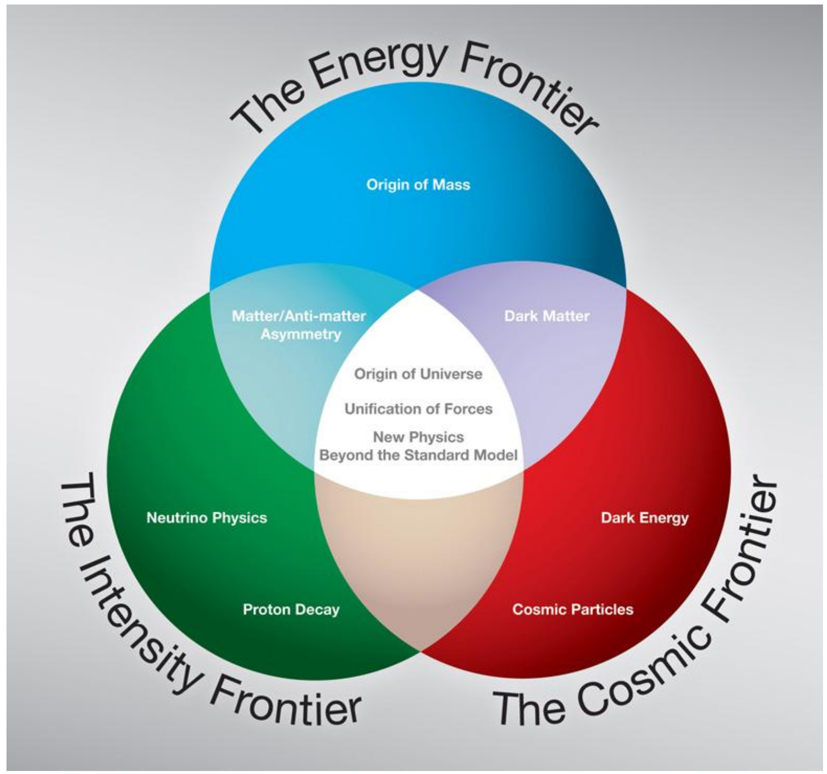


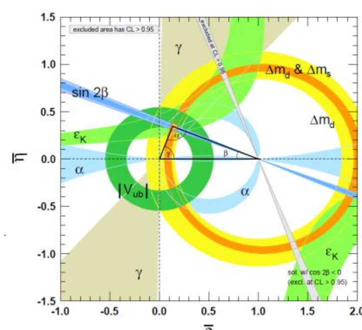
Muon g-2

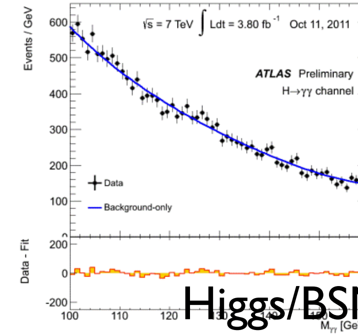


EDM

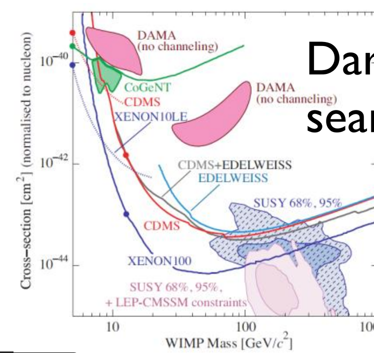




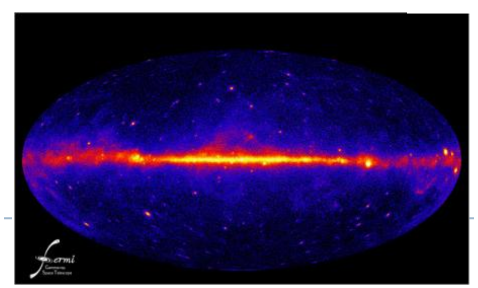




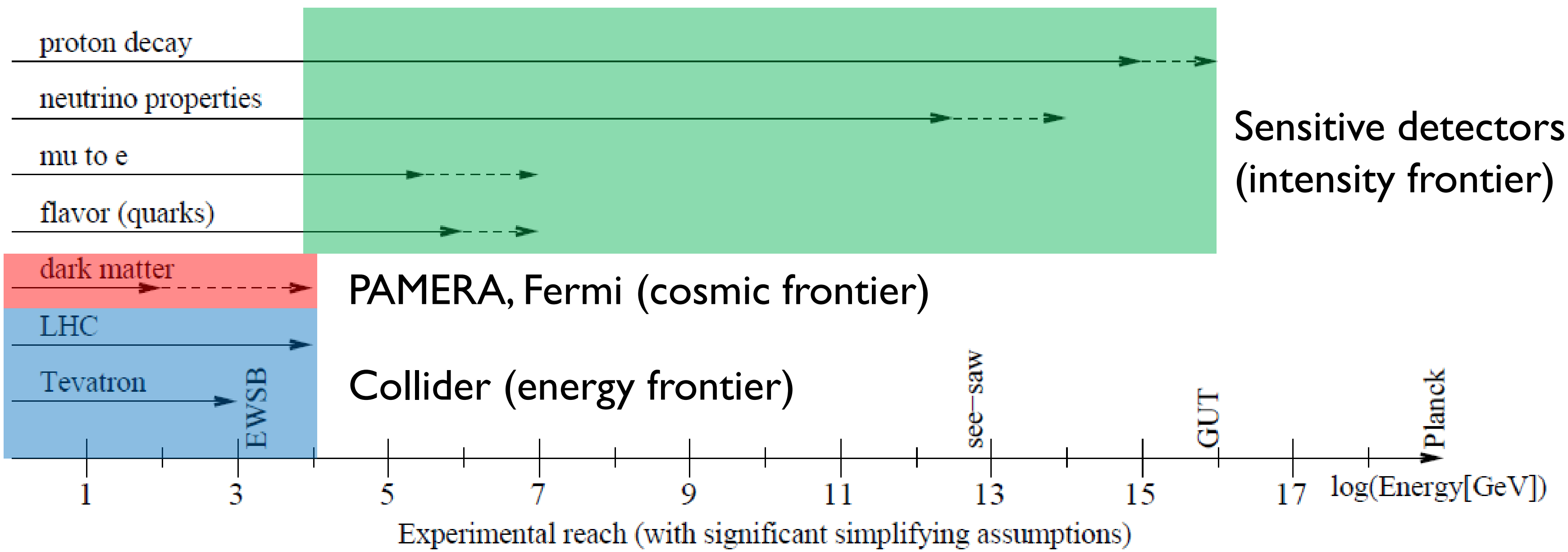
Higgs/BSM search



Dark matter search



Energy scale



Plotted by Zoltan Ligeti (LBL)

Grossman, ProjectX, 2012

Too rough picture or make sense ?

precision frontier ⇒ high energy scale (beyond SM)

Intensity frontier physics

- Exploration of fundamental physics using intense beam and massivelly sensitive detector
- Search the new physics from variation of the SM
  - Charged lepton

    Muon g-2/EDM @ BNL(E821) ⇒ FNAL, J-PARC

    Charged lepton flavor violating process of muon, tau @ BaBar, SuperKEKB
  - Nucleon, nuclei and atom EDM

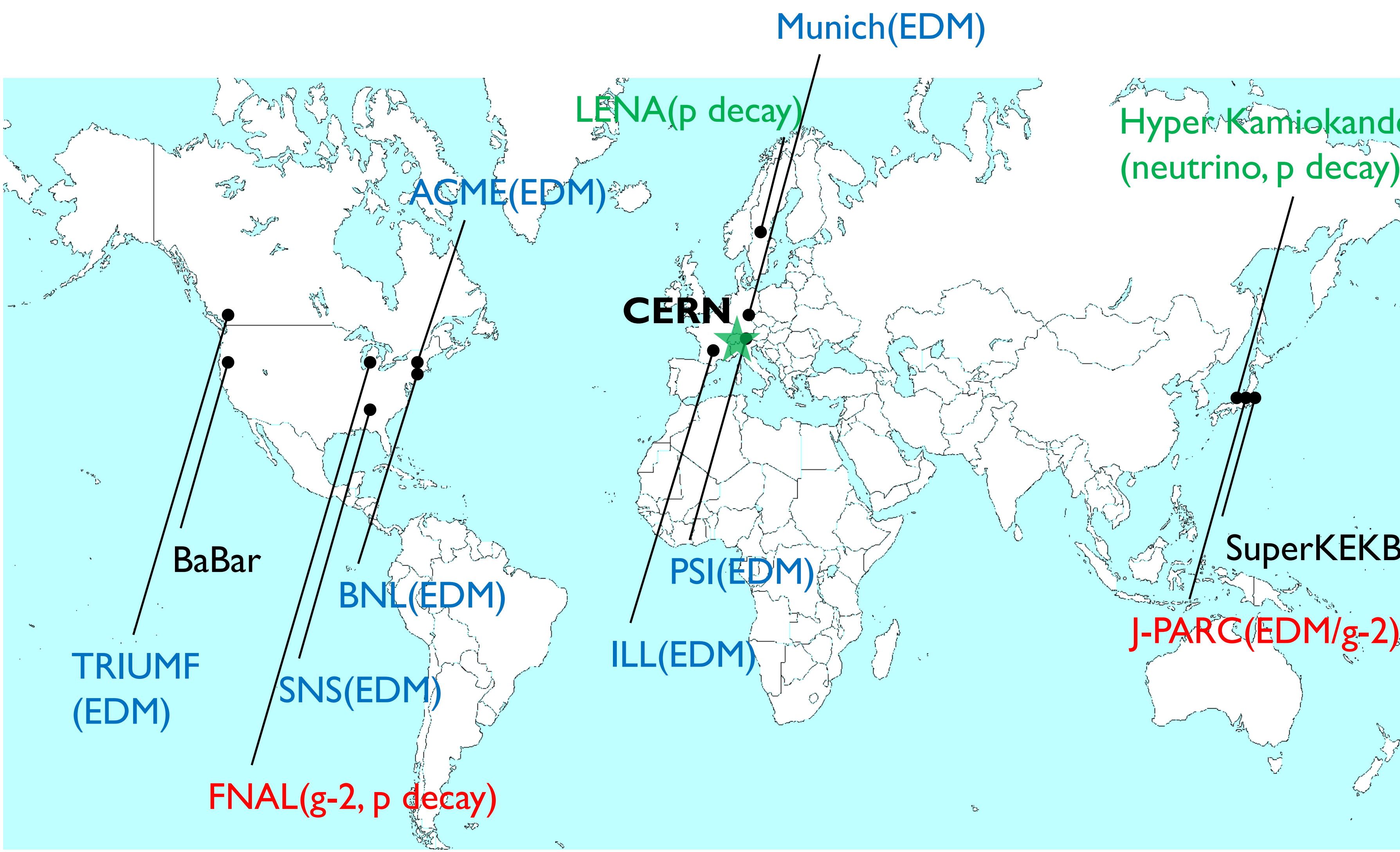
    Neutron/Proton EDM @ ILL, BNL, PSI, SNS, Munich, TRIUMF, ACME

    Mercury-199, Radon and Radium EDM
  - Proton decay

    SuperKamiokande, Hyper-Kamiokande, LBNE(FNAL), LENA

    N-Nbar oscillation @ FNAL
  - Heavy quark, neutrino oscillation, etc

(Future) experiments



Many other projects are planning under way

Search of NP from intensity frontier

- Variation from the SM predictions
  - $\Delta O_{SM} \sim 10^{-10} = O_{NP} / M_{NP}$?   Muon/electron g-2, Unitary triangle
  - Complementary signature of NP
  - Precisely theoretical value of the SM need to be known

    High-order perturbation, non-perturbative effect of QCD
- Bound of undetected observables
  - $O_{SM} < 10^{-15}$ which is direct constraint on NP

    EDM (nucleon (quark), electron, …), Proton(neutron) decay, NNbar oscillation, LFV, dark matter search, …
  - Whose signals are the signature of NP
  - Hadronic correction should be relevant for NP constraint

Lattice QCD plays a key role !

Topics

- Introduction
- Lattice QCD works
  - Muon g-2
  - Nucleon EDM
  - (Proton decay)
- Summary and prospects

Lattice QCD

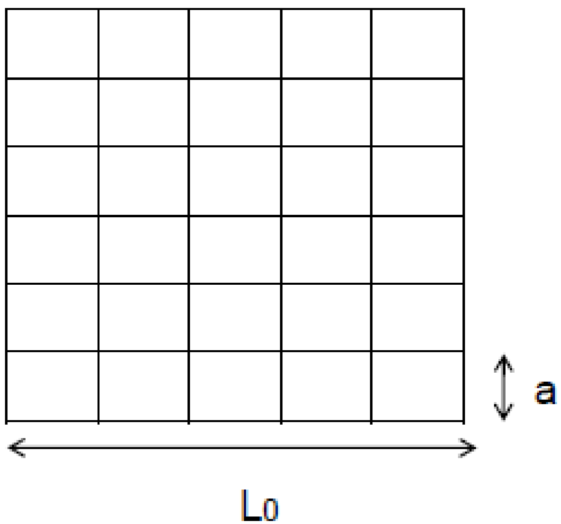
In lattice regularization, the path integral of $\langle O \rangle$ is computed by <u>Monte-Carlo integral</u>:

$$\langle O \rangle = Z^{-1} \int D\Psi\, O(\Psi) e^{-S(\Psi)} \simeq \frac{1}{N} \sum_i O(\Psi_i)$$

- Exact QCD calculation (enough large number of sampling N)
- Gauge invariant
- Translational invariant
- Ultraviolet cut-off *a* (lattice spacing)

  Infrared cut-off $V = L_0^D$ (lattice volume)
- Continuum limit, and infinite volume are important.
- The development of machine (BG, GPGPU, …)

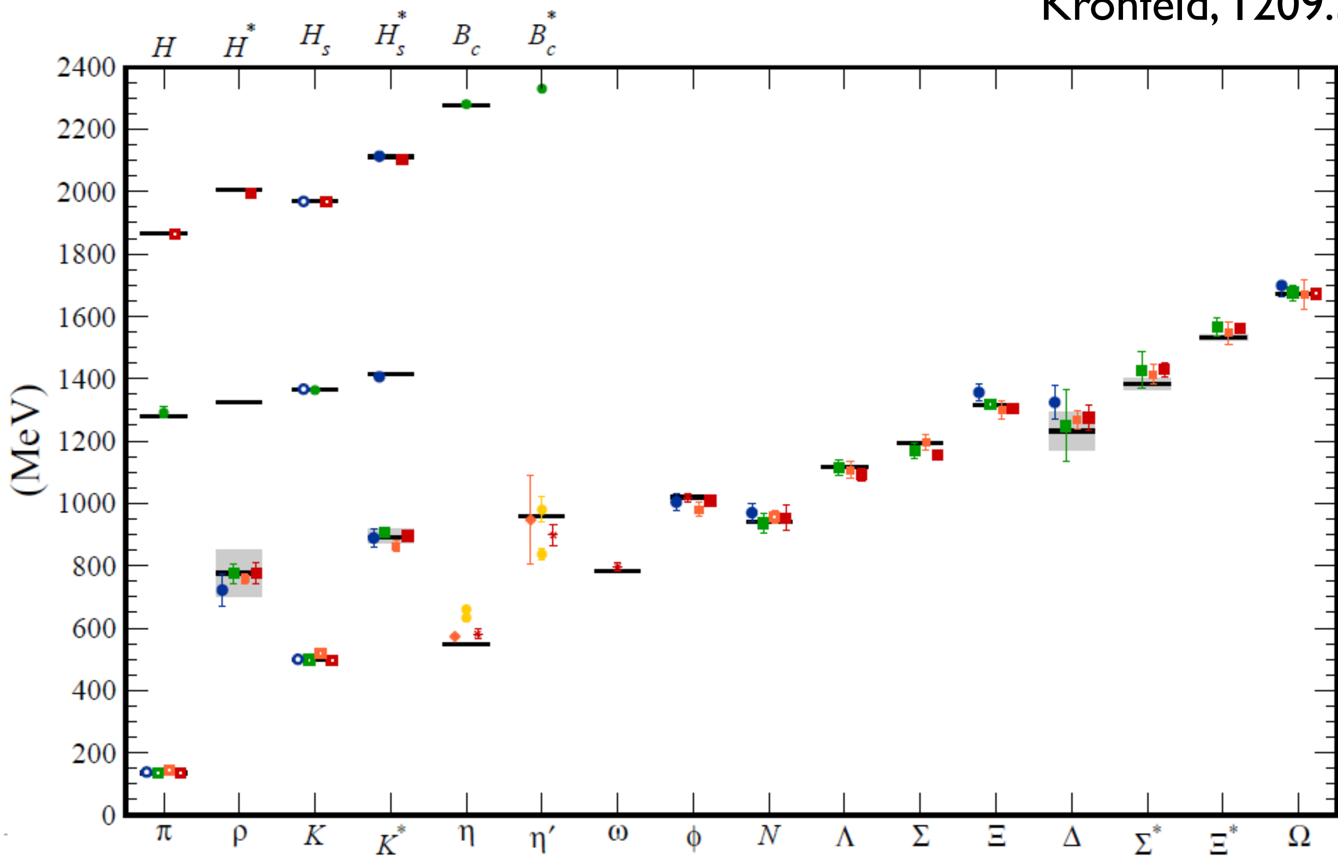
  and algorithm, which make much progress.



Lattice QCD

- Hadron spectrum in Nf=2+1 QCD
  - Good agreement with various lattice action and fermion with experimental results !

Kronfeld, 1209.3468



Choice of lattice fermion

- There are several kinds of fermion definition on the lattice

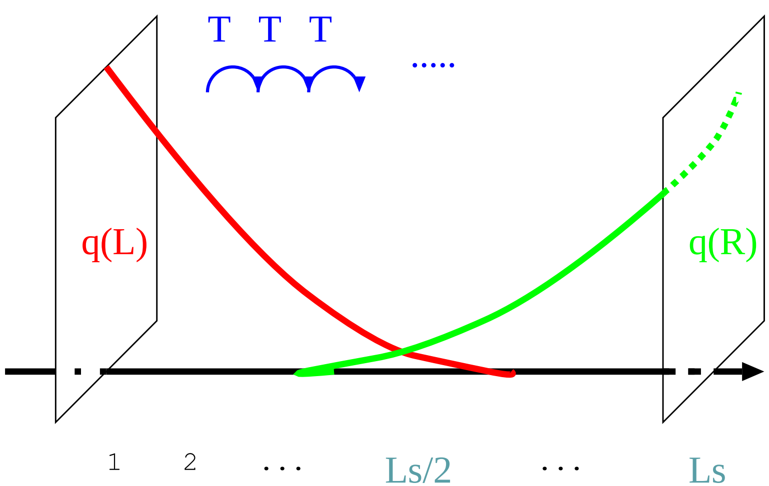
  Due to Nielsen-Ninomiya no-go theorem

- Require "realistic" fermion for the precise calculation
  - Wilson-clover and staggered fermions are not appropriate.
  - Domain-wall (and also overlap fermion) is even better.
- Domain-Wall fermion (DWF) [Blum Soni, (97), CP-PACS(99), RBC(00), RBC/UKQCD. (05 --)]
  - L, R fermion are localized on boundaries

    $\Rightarrow$ Chiral symmetry (if $L_s \to \infty$).
  - Good chiral symmetry

    Chiral symmetry breaking is suppressed as $am_{res} \sim \exp(-L_s)$.
  - Reasonable computational cost

    $10 \times$ [Wilson], but $1/10 \times$ [overlap]
  - Thanks to development of algorithm, it is possible to perform with recent machine.



DWF era

- Keon physics

"Lattice determination of the K→(ππ)I=2 Decay Amplitude A2", RBC/UKQCD, PRD86 (2012) 074513.
"The K→(ππ)I=2 Decay Amplitude from Lattice QCD", RBC/UKQCD, PRL108 (2012) 141601.
"K to ππ Decay amplitudes from Lattice QCD ", PRD84 (2011) 114503.

- B physics

"Nonperturbative tuning of an improved relativistic heavy-quark action with application to bottom spectroscopy", RBC/UKQCD, PRD86 (2012) 116003.
"Neutral B-meson mixing from unquenched lattice QCD with domain-wall light quarks and static b-quarks", PRD82 (2010) 014505.

- Nucleon physics

"Nucleon structure from 2+1-flavor dynamical DWF lattice QCD at nearly physical pion mass", RBC/UKQCD, Prog.Part.Nucl.Phys. 67 (2012) 218.

- QED+QCD

"Full QED+QCD low-energy constants through reweighting ", Ishikawa et al.PRL109 (2012) 072002.
"Electromagnetic mass splittings of the low lying hadrons and quark masses from 2+1 flavor lattice QCD+QED",Blum et al., Phys.Rev. D82 (2010) 094508.

On-going project with DWF

- ## Muon g-2

JSPS meeting, ES, 3/26/2013; Bolye et al., UKQCD, Phys.Rev. D85,074504(2012),
"Hadronic corrections to the muon anomalous magnetic moment from lattice QCD", Blum et al., PoS LATTICE2012 (2012) 022

- ## Neutron/proton EDM

"Lattice caclulation of neutron and proton EDM in full QCD", ES, PoS(Confinement X)330
"Electric Dipole Moment of the Neutron", ES, PoS(Confinement X)348

- ## Proton decay

"Proton decay matrix elements in 2+1 domain-wall fermion", ES et al., PoS(Lattice 2011)329
"Proton decay matrix elements from lattice QCD", Aoki and ES, International Workshop on Grand Unified Theories (GUT2012), AIP Conf. Proc. 1467, pp. 116-121, Mar 2012.

Preserving the chiral symmetry for DWF is important property to take extrapolation toward physical point, and avoid the systematic error due to lattice artifacts.

Error reduction techniques

Blum, Izubichi, ES, 1208.4349

- Covariant approximation averaging (CAA)
  - For original observables O, (unbiased) improved estimator

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g}, \quad \mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

which satisfies $\langle O\rangle = \langle O^{\mathrm{imp}}\rangle$ if approximation is covariant under lattice symmetry g, and error becomes $\mathrm{err}^{\mathrm{imp}} \simeq \mathrm{err}/\sqrt{N_G}$

  - All-mode averaging (AMA)

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

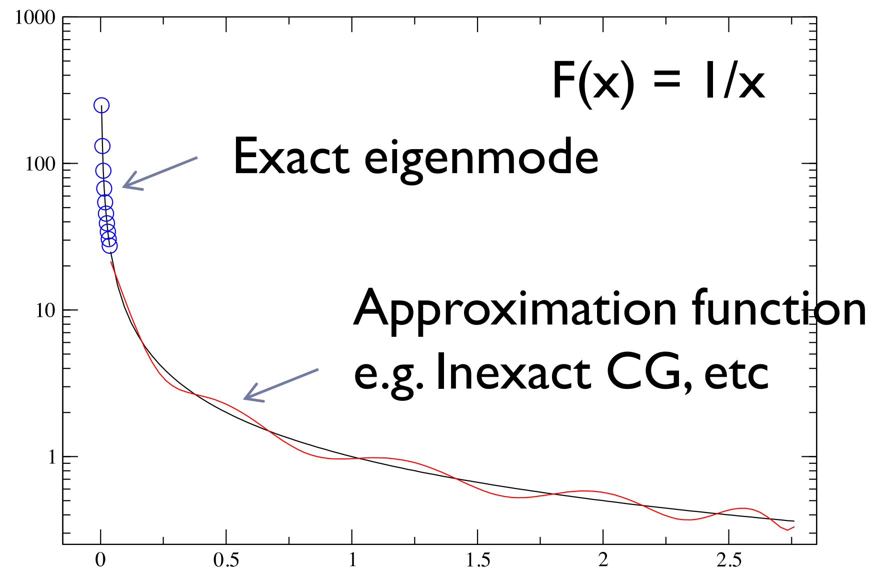
$$S_l = \sum_\lambda v_\lambda F(\lambda) v_\lambda^\dagger,$$

$$F(\lambda) = \begin{cases} \lambda^{-1}, & |\lambda| < \lambda_{\mathrm{cut}} \\ P_n(\lambda) \simeq \lambda^{-1} & |\lambda| > \lambda_{\mathrm{cut}} \end{cases}$$

accuracy control :

- low mode part : # of eig-mode
- mid-high mode : degree of poly.

$\Rightarrow$ speed-up more than 5



Muon g-2 from lattice QCD

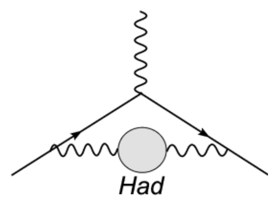
Muon g-2

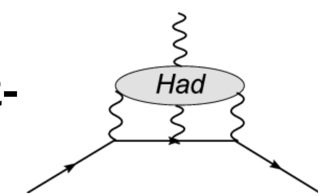
- Discrepancy from the SM

$$a_\mu^{Exp} - a_\mu^{SM} = +287(63)_{Exp}\,(49)_{SM} \times 10^{-11} \sim 3.6\ \sigma \text{ discrepancy !}$$

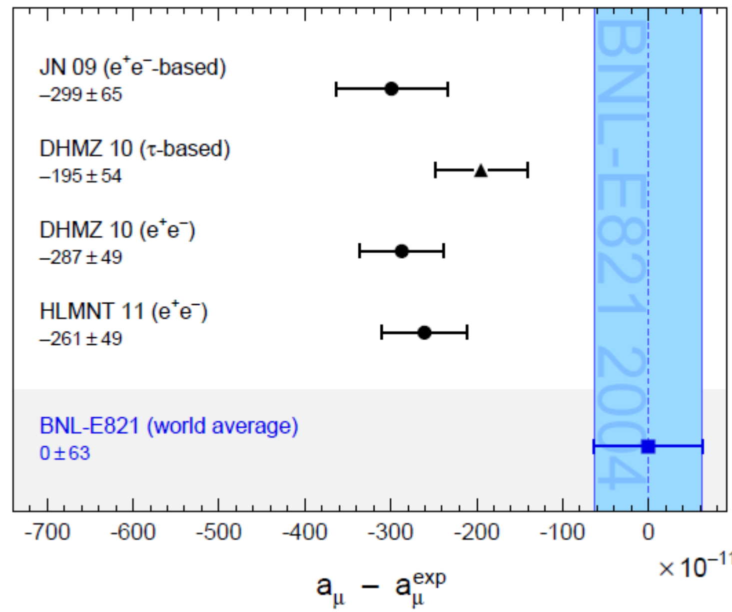
New physics model may explain what is a source of this discrepancy;
SUSY particle, dark photon, … ?

- Main uncertainties in the SM
  - Leading order of hadronic contribution (HVP);
    ~90% of error
  - Next-to-leading order of hadronic contribution;(light-by-light) ~ *unknown*, may be large uncertainty





PDG 2012



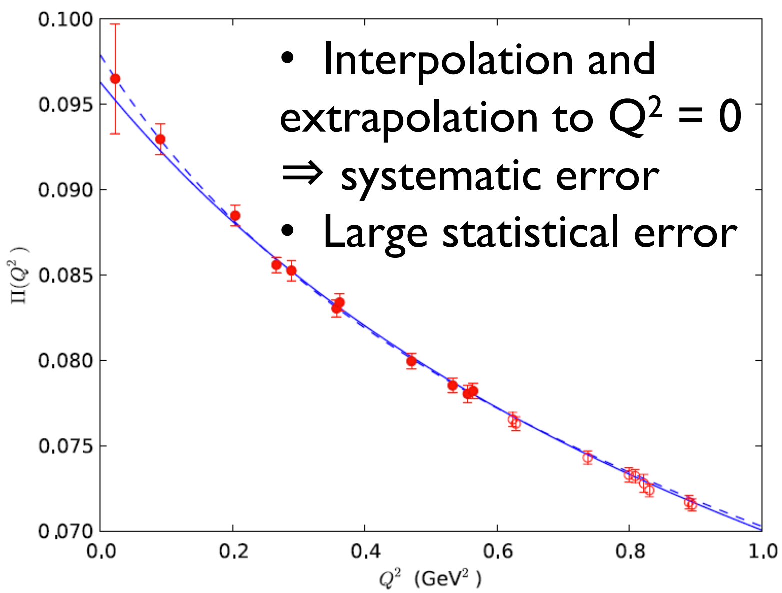
Lattice QCD is able to precisely calculate HVP and LbyL, being independent from data set and models.

a_μ(HVP)

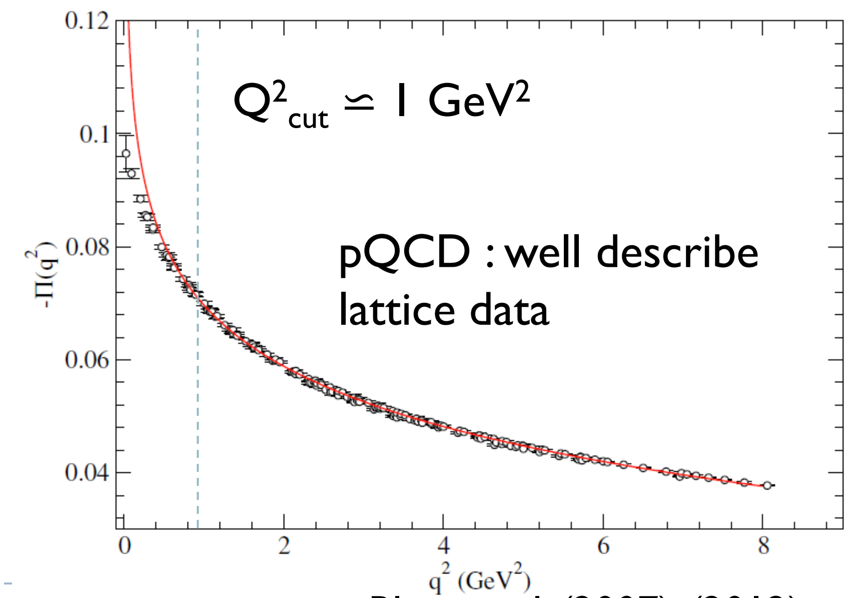
$$\int d^4x \langle T\{V_\mu^{\rm em}(x)V_\nu^{\rm em}(0)\}\rangle e^{iQx} = (Q^2\delta_{\mu\nu} - Q_\mu Q_\nu)\Pi_V(Q^2)$$

$$a_\mu^{\rm had} = \frac{\alpha}{\pi^2}\int_{m_\pi^2}^{\infty}\frac{ds}{s}{\rm Im}\Pi(s)K(s) = \left(\frac{\alpha}{\pi}\right)^2\int_0^\infty dQ^2 f(Q^2)4\pi^2\left[\Pi_V(0) - \Pi_V(Q^2)\right]$$

Aubin, Blum, Phys. Rev.D75,114502 (2007), Feng, et al., Phys.Rev.Lett. 107, 081802 (2011), Bolye et al., Phys.Rev. D85,074504(2012), Della Morte, et al., JHEP 1203,055(2012),Aubin et al.,Phys.Rev.D86, 054509(2012)



- Interpolation and extrapolation to $Q^2 = 0$ $\Rightarrow$ systematic error
- Large statistical error



$Q^2_{\rm cut} \backsimeq 1$ GeV^2

pQCD : well describe lattice data

Blum et al. (2007), (2012)

a_μ(HVP)

- Comparison in physical point

Errors are still large ! (far from precision of phenomenological one)

Statistical error

• Using AMA algorithm error is reduced to factor 1/4 -- 1/5 !

Blum, Izubichi, ES, 1208.4349

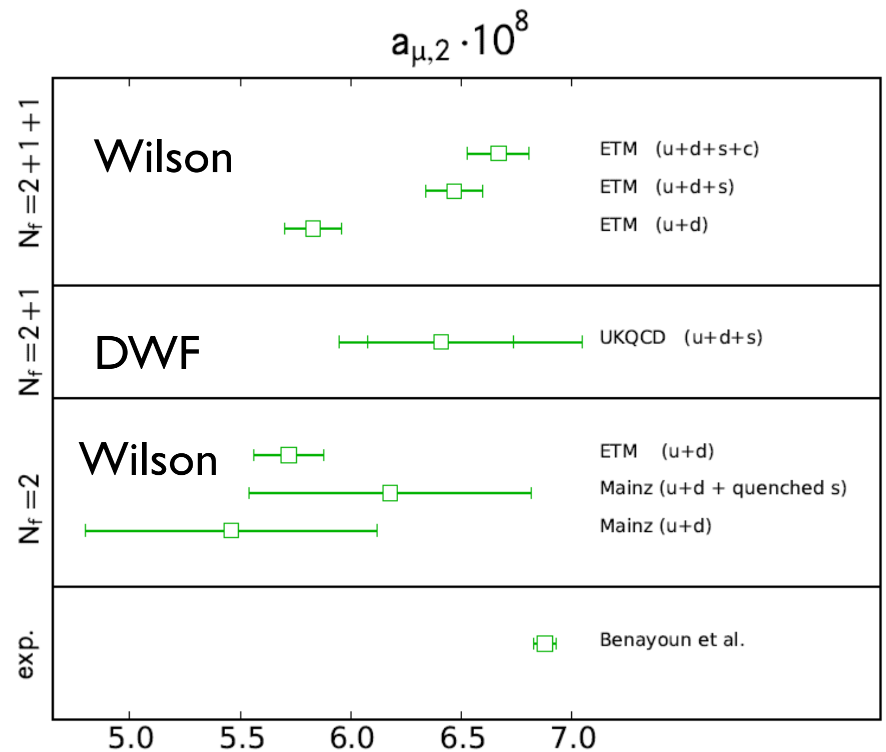
Q^2 dependence

• Direct measurement at $Q^2 = 0$

de Divitiis, et al.arXiv:1208.5914

• Time-like momentum trick

ES, Blum, Kim, Izubuchi, under way



Andreas Juttner (2012)

Chiral extrapolation

• Direct measurement in physical point

in progress of DWF $48^3 \times 96$ lattice $\Rightarrow$ going to a few % uncertainty

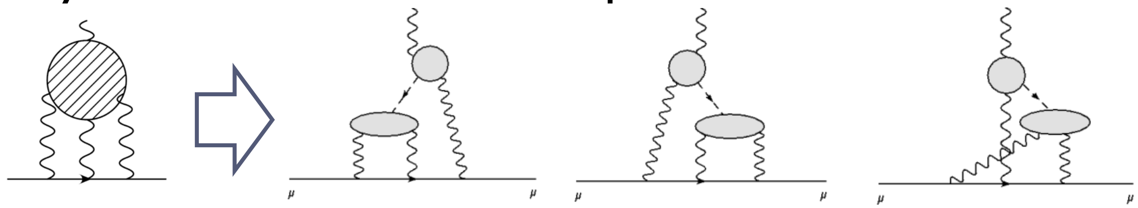
a_μ(LbyL)

- Not-yet established in lattice QCD
- Possible two ways
  - Indirect measurement

ES et al., PoS LATTICE2010 (2010) 159,
Feng et al.(JLQCD),PRL109, 182001 (2012).

Separate two $\pi^0 \rightarrow \gamma\gamma$ decay diagrams, and connect with pion, eta props.

Easy calculation, but assume pion-dominance-model.



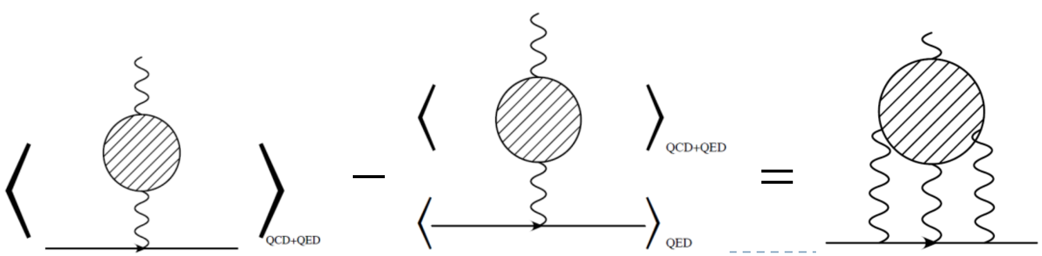
Knecht, Nyffeler, Phys.Rev. D65 (2002) 073034

  - Direct measurement

Basically there requires four-point function, which is hard to compute.

There is idea of easier calculation with QED+QCD:

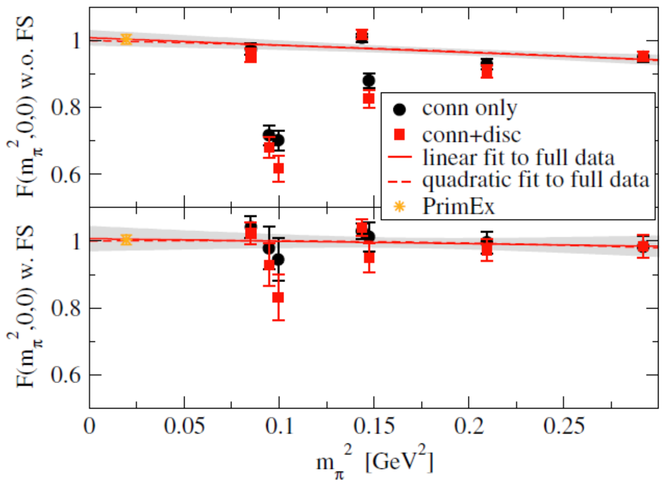
Blum et al., arXiv:1301.2607



$+ O(\alpha^4)$

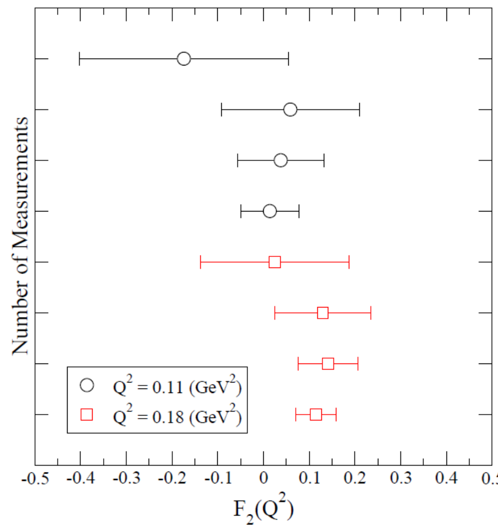
a_μ(LbyL)

Feng et al.(JLQCD),PRL109, 182001 (2012).



- Indirect measurement
  - Form factor of $\pi^0 \rightarrow \gamma\gamma$ is in agreement with PrimEx.
  - Next step is off-shell photon decay amplitude

- Direct measurement
  - There is some progress to reduce statistical error
  - Finite signal appears
  - AMA is helpful.



Blum et al., arXiv:1301.2607

Neutron/proton EDM from lattice QCD

Nucleon EDM in the SM and BSM

- Sensitive to P, CP violation
- Upper limit from experiment: $< 2.9 \times 10^{-26}$ e·cm
- Contribution from weak boson: CKM phase

  Very tiny, which is 3-loop : $d_N^{KM} \backsimeq 10^{-30}$ -- 10^{-32} e·cm

  Khriplovich and Zhitnitsky, PLB109, 490 (1982); Czarnecki, Krause, PRL78, 4339 (1997)

- Contribution from QCD: θ term

  Unnaturally small (strong CP problem): $\bar{\theta} < 10^{-9\pm1}$

  Crewther, et al. (1979), Ellis, Gaillard (1979)

- Contribution from BSM: dim-5,6 operator

$$\mathcal{O}_{\rm qEDM} = d_q \bar{q}(\sigma\cdot F)\gamma_5 q,\ \mathcal{O}_{\rm cEDM} = d_q^c \bar{q}(\sigma\cdot G)\gamma_5 q,\ \mathcal{O}_{\rm Weinberg} = d^G GG\widetilde{G}$$

$$\begin{aligned} d_N &= d_N^{\rm QCD}\bar{\theta} + d_N(d_q, d_q^c) + d_N(d^G) \\ &\sim 10^{-17}[\mathrm{e}\cdot\mathrm{cm}]\bar{\theta} + (1.4\text{--}0.47)d_d - (0.12\text{--}0.35)d_u + O(10^{-2})d_q^c \\ &\sim O(10^{-25}\text{--}10^{-27})\,\mathrm{e}\cdot\mathrm{cm} \end{aligned}$$

Hisano, Shimizu (04), Ellis, Lee, Pilaftsis (08), Hisano, Lee, Nagata, Shimizu (12)

Nucleon EDM from lattice QCD

- Non-perturbative determination of QCD effect
  - L_θ
    Lattice QCD provides d_N in $d_N^{\rm QCD} = \bar{\theta} d_N$
    θ parameter can be estimated by $d_N^{exp}/d_N = \theta$ (if there is no BSM)
  - $L_{\rm qEDM}$, $L_{\rm cEDM}$
    Lattice QCD provides $C_{\rm qEDM}$, $C_{\rm cEDM}$

$$d_N^{\rm BSM} = \sum_q \left[d_q^{\rm qEDM} C_{\rm qEDM}^q + d_q^{\rm cEDM} C_{\rm cEDM}^q \right]$$

$d^{\rm qEDM}$, $d^{\rm cEDM}$ depend on model parameters.

Result of lattice QCD is an important input value of strong interaction contribution inside nucleon.

Methods

Aoki and Gocksch, PRL63, 1125 (1989); ES, et al., (CP-PACS) PRD75, 034507 (2007); ES et al.,PRD78, 014503 (2008)

- Spectrum

$m_{\uparrow \text{spin}} - m_{\downarrow \text{spin}} = 2 d_N \theta E$

$$R_3 = \frac{\langle N(t)\bar{N}(0)\rangle^{\text{up}}_{\theta,E}}{\langle N(t)\bar{N}(0)\rangle^{\text{down}}_{\theta,E}} \simeq 1 + d_N E\theta t$$

Direct measurement of EDM from 2-pt function with external E field, which is defined as $U_t \rightarrow U_t e^{qEt}$, $U_t^\dagger \rightarrow U_t^\dagger e^{-qEt}$ (although boundary effect is significant)

- Form factor

ES, et al., (CP-PACS), PRD72, 014504 (2005); Berruto, et al. (RBC) PRD73, 05409 (2006).

$$\langle n(P_1)|J^{\text{EM}}_\mu|n(P_2)\rangle_\theta = \bar{u}^\theta_N \Big[\underbrace{\frac{F_3^\theta(Q^2)}{2m_N}\gamma_5\sigma_{\mu\nu}Q_\nu}_{\text{P,T-odd}} + \underbrace{F_1\gamma_\mu + \frac{F_2}{2m_N}\sigma_{\mu\nu}Q_\nu}_{\text{P,T-even}} + \cdots \Big] u^\theta_N$$

$$d_N = \lim_{Q^2\to 0} F_3(Q^2)/2m_N$$

Extraction of CP-odd form factor from 3-pt function, and take into $Q^2 \rightarrow 0$

- Imaginary θ

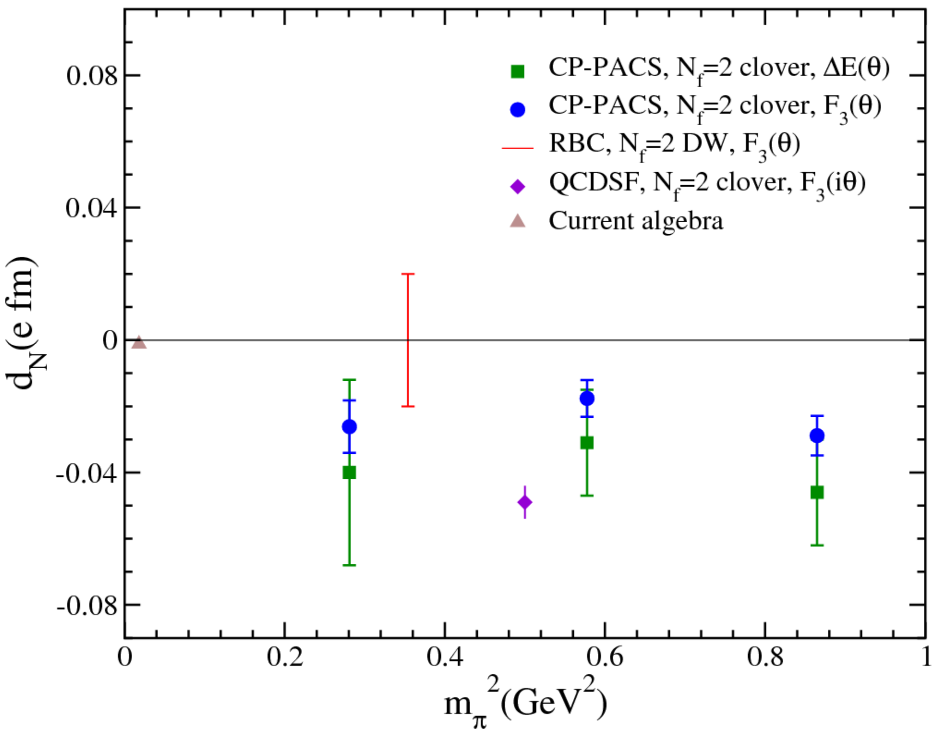
T. Izubuchi, Lattice 2007

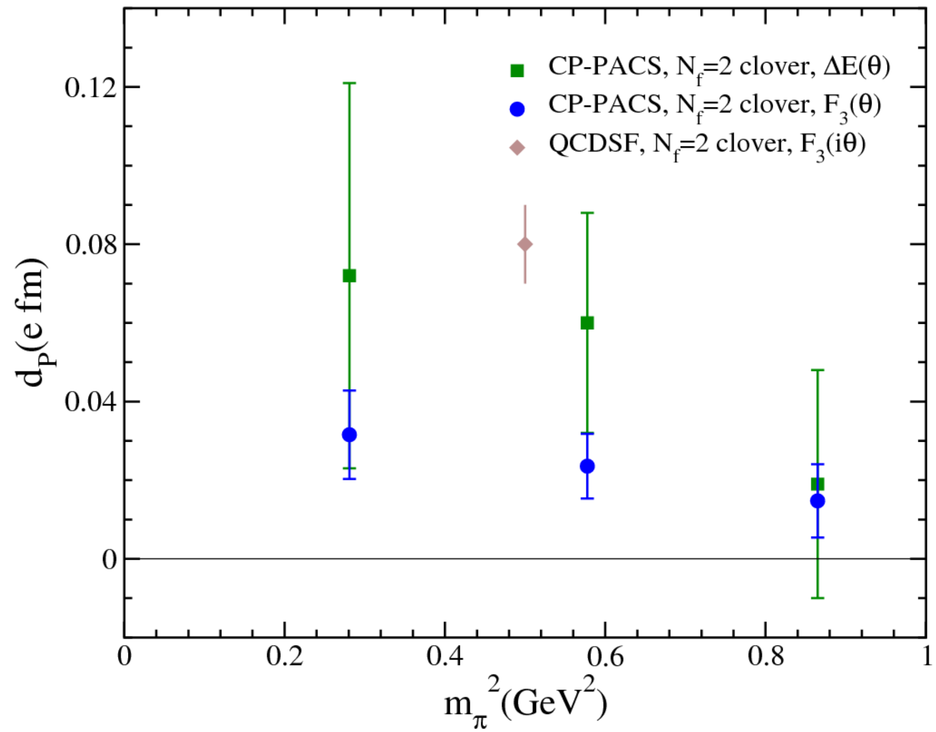
New generation of imaginary θ action: $\langle O e^{i\theta Q}\rangle \rightarrow \langle O e^{-\theta^I Q}\rangle$

Clear signal is expected, but huge computational cost is needed.

Numerical results in Nf=2

- Wilson-clover fermion [spectrum, form factor, imaginary θ]
- DWF [form factor]





- Statistical error only.

In result of QCDSF large systematic error due to lattice artifact will be hidden.

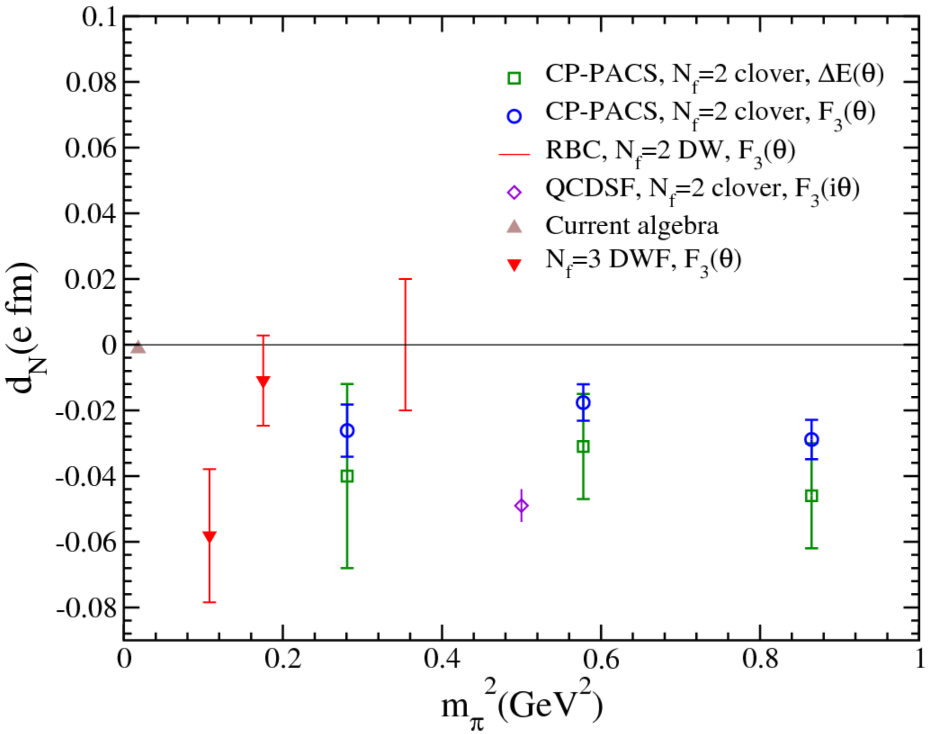
Recent results

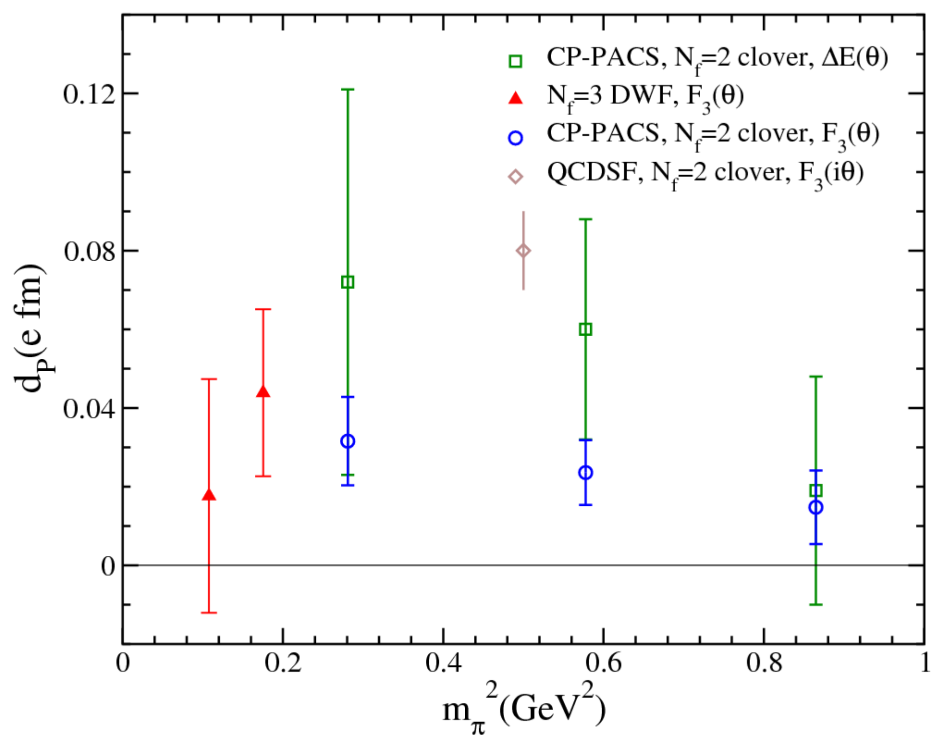
- DWF in Nf=2+1 (RBC/UKQCD)

  $24^3 \times 64$ (2.5 fm^3) at a^{-1} = 1.73 GeV

  using m=0.005 ($m_\pi = 0.3$ GeV), m=0.01 ($m_\pi = 0.4$ GeV)

  AMA is very helpful, cost is reduced to 1/5 or less.





Proton decay from lattice QCD

Smoking gun

- Baryon number is accidental symmetry in the SM ?

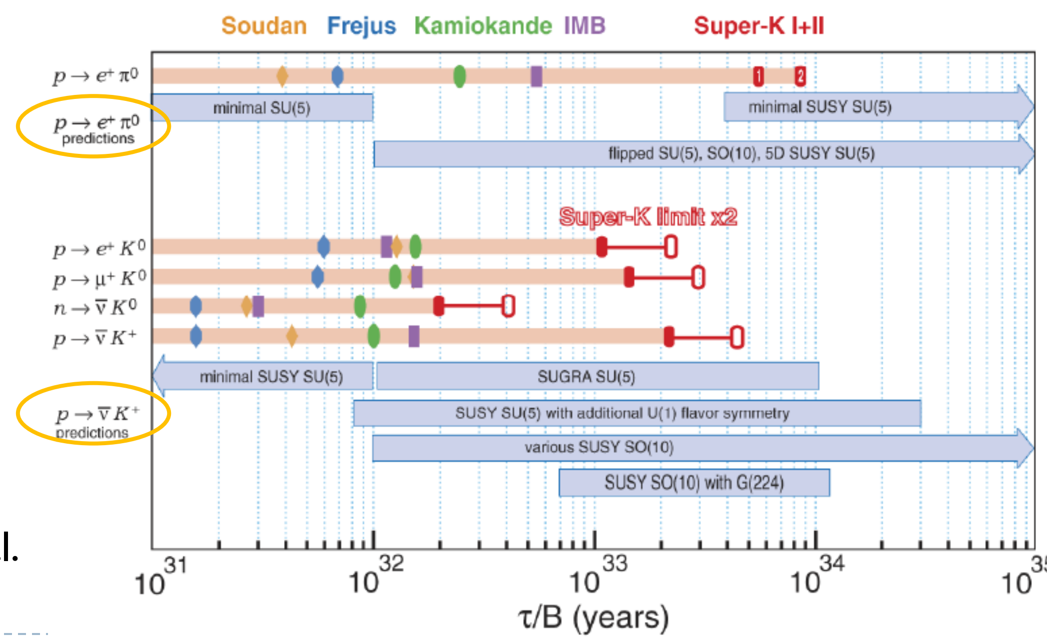
  via anomaly, it is very rare event (‘tHooft 1976):

  $\Delta B = \Delta L = 2$: $\tau(d \rightarrow e^+ \nu_\mu) \sim 10^{120}$ years,

  $\Delta B = \Delta L = 3$: $\tau(^3He \rightarrow e^+ \nu_\tau \nu_\mu) \sim 10^{150}$ years
- Universe looks like made of only baryons
- (SUSY-) GUTs
  - Coupling unification
  - Proton decay
- Experiments
  - $\tau(pe^+\pi^0) > 8.2 \times 10^{33}$ years
  - $\tau(p\nu K^+) > 2.3 \times 10^{33}$ years

  Nishino et al. (Super-Kamiokande), PRD85, 112001(2012), Kobayashi et al. (Super-Kamiokande), PRD72, 052007 (2005)

arXiv:1205.2671v1



Effective operator

- Dimension-6 operator

$$\mathcal{L}_{\mathrm{GUT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i C_i(\mu) O_i(\mu)/\Lambda^2_{\mathrm{GUT}} + \mathcal{O}((O(\mu)/\Lambda^2_{GUT})^2)$$

$O_i(\mu) = (qq)_\Gamma (ql)_{\Gamma'}$ "i" labels chirality (Γ) and flavor (q,l)

C_i depends on type of GUTs model

- Decay rate

$$\Gamma_{p\to\pi^0 e^+} = \frac{m_p}{32\pi^2}\left[1-\left(\frac{m_e}{m_p}\right)^2\right]^2 \left|\sum_i C_i W_0^i(p\to\pi^0)\right|^2$$

W_0^i : determine from QCD matrix element (model independent)

Precision of W_0 is significant, since the decay rate is affected by twice of that.

- Matrix element

Lattice QCD provides each decay channels of W_0 from <u>matrix element</u>;

$$\langle \pi^0 | (ud)_\Gamma u_{\Gamma'} | p \rangle = P_{\Gamma'}\left[W_0^\Gamma - \frac{i\not{q}}{m_p} W_q^\Gamma\right] u_p$$

Aoki et al. (JLQCD), PRD62, 014506 (2000); Aoki et al.(RBC), PRD75, 014507 (2007)

which is extracted from 3-pt function.

Numerical results

- Works on RBC collaboration with DWFs
  - Quenched QCD (direct/indirect)

    Y. Aoki, C. Dawson, J. Noaki, and A. Soni, Phys. Rev. D75, 014507 (2007)
  - Nf=2+1 (indirect)

    Y. Aoki et al. (RBC-UKQCD), Phys. Rev. D78, 054505 (2008)

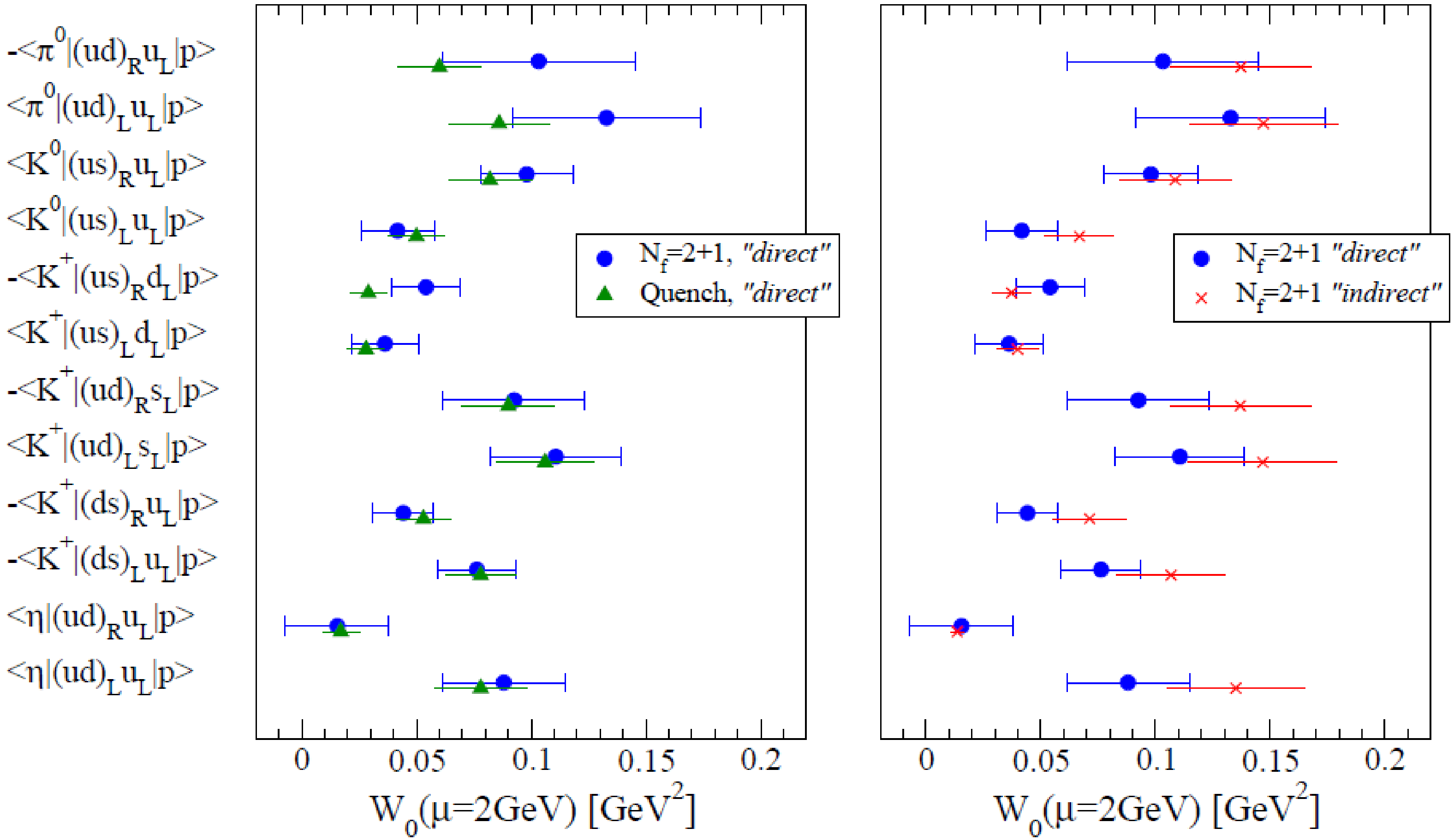
  - Direct : measurement of matrix element.
  - Indirect: compute low-energy constant in W_0 , possibly including model dependence.
- DWFs in Nf=2+1 (direct)

  $24^3 \times 64$ lattice in RBC/UKQCD collaboration

  m=0.005, 0.01, 0.02, 0.03 (m_π = 0.3 -- 0.8 GeV)

  Determination of W_0 at each channels

W_0



- Estimate all systematic errors

Summary and prospects

- There are many proposals of experiment for Intensity Frontier Physics.
- Theoretical uncertainties of muon g-2, EDM and proton decay may be critical issue for precision test and search of NP.
- Lattice QCD makes it possible

  DWFs is even better to pursue high precision of these observables
- RBC/UKQCD plans the big projects:
  - $48^3 \times 96$ lattice (5 fm^3) in physical points
  - No need chiral extrapolation, and almost ignore lattice artifacts
  - Using above configs (and also AMA) we will reach

    muon g-2 (HVP) ~ 1% error, n and p EDM ~ 10% stat error, sys study

    proton decay ~ 5% error

$\Rightarrow$ essential input values for NP search from intensity physics.

Backup

New EDM experiment proposal @ BNL

Storage Ring EDM Collaboration

Y. K. SEMERTZIDIS, ProjectX, 2012

>20 Institutions
>80 Collaborators

- Aristotle University of Thessaloniki, Thessaloniki/Greece
- Research Inst. for Nuclear Problems, Be
- Brookhaven National Laboratory, Upton
- Budker Institute for Nuclear Physics, No
- Royal Holloway, University of London,
- Cornell University, Ithaca, NY/USA
- Institut für Kernphysik and Jülich Centre Jülich, Jülich/Germany
- Institute of Nuclear Physics Demokritos,
- University and INFN Ferrara, Ferrara/Ita
- Laboratori Nazionali di Frascati dell'INF
- Joint Institute for Nuclear Research, Dub
- Indiana University, Indiana/USA
- Istanbul Technical University, Istanbul/T
- University of Massachusetts, Amherst, M
- Michigan State University, East Lansing
- Dipartimento do Fisica, Universita' "To
- University of Patras, Patras/Greece
- CEA, Saclay, Paris/France
- KEK, High Energy Accel. Res. Organiz
- University of Virginia, Virginia/USA

http://www.bnl.

Summary

- ✓ Proton EDM physics is a must do, > order of magnitude improvement over the neutron EDM
- ✓ E-field issues well understood
- ✓ Working EDM lattice with long SCT and large enough acceptance (1.3×10^{-29} e•cm/year)
- ✓ Polarimeter work
- Planning BPM-prototype demonstration including tests at RHIC
- Old accumulator ring could house the proton EDM ring at Fermi; significant cost savings. Upgrade possibilities...

$$d_N^{\theta} < 10^{-29}\,\mathrm{e}\cdot\mathrm{cm} \Rightarrow \bar{\theta} < 10^{-13}?$$

Time-like momentum

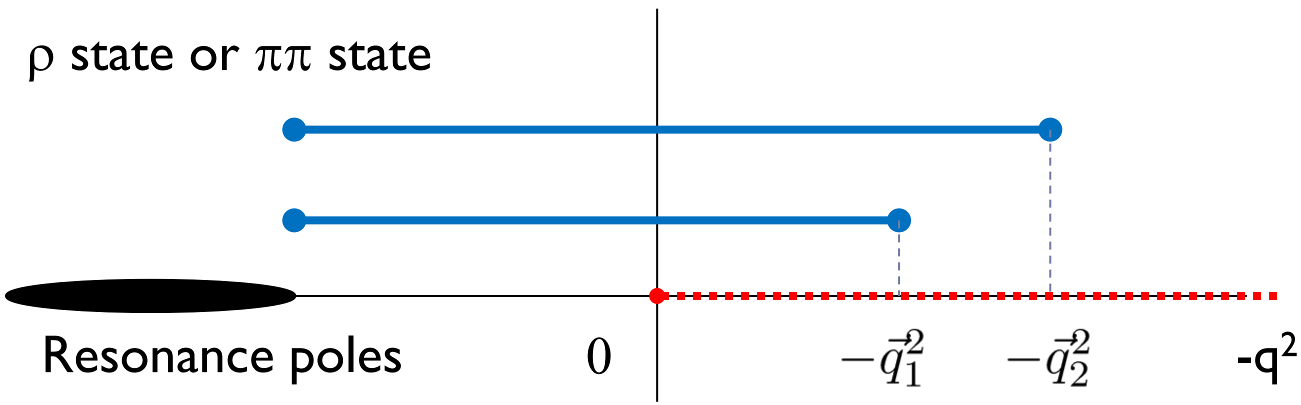
- $Q_4 = i\omega$

$$\int d^4x \langle T\{V_\mu^{\rm em}(x)V_\nu^{\rm em}(0)\}\rangle e^{iqx} = \Pi_{\mu\nu}(\vec{q},\omega) = (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi_V(q^2)$$

$$q = (\omega, \vec{q}), \quad g_{\mu\nu} = {\rm diag}(1,-1,-1,-1), \quad q^2 = \omega^2 - \vec{q}^2 = -Q^2$$

  - ω is "photon energy" which can be controlled by hand.
  - Temporal integral from $-\infty < t < \infty$:

$$\Pi_{\mu\nu}(\vec{q},\omega) = \int_0^\infty dt \sum_{\vec{x}} e^{-\omega t - i\vec{x}\vec{q}} \langle V_\mu(\vec{x},t)V_\nu(0)\rangle_c + \int_{-\infty}^0 dt \sum_{\vec{x}} e^{-\omega t - i\vec{x}\vec{q}} \langle V_\nu(0)V_\mu(\vec{x},t)\rangle_c$$



Time-like momentum

- Modeling

To perform the temporal integral, we use a modeling procedure

$$\sum_{\vec{x}} e^{i\vec{q}\vec{x}} \langle V_\mu(x) V_\nu(0) \rangle \simeq g_v e^{-E_V t}$$

(asymptotic state dominance at t $\geqq$ t_{cut})

$$\int_0^{t_{\rm cut}} dt e^{-\omega t} \sum_{\vec{x}} e^{i\vec{q}\vec{x}} \langle V_\mu(x) V_\nu(0) \rangle \simeq \sum_{t=0}^{t_{\rm cut}} C_{VV}(\vec{q},\omega;t)$$

(numerical integral with lattice data from 0 $\leqq$ t $\leqq$ t_{cut})

Longitudinal part will be

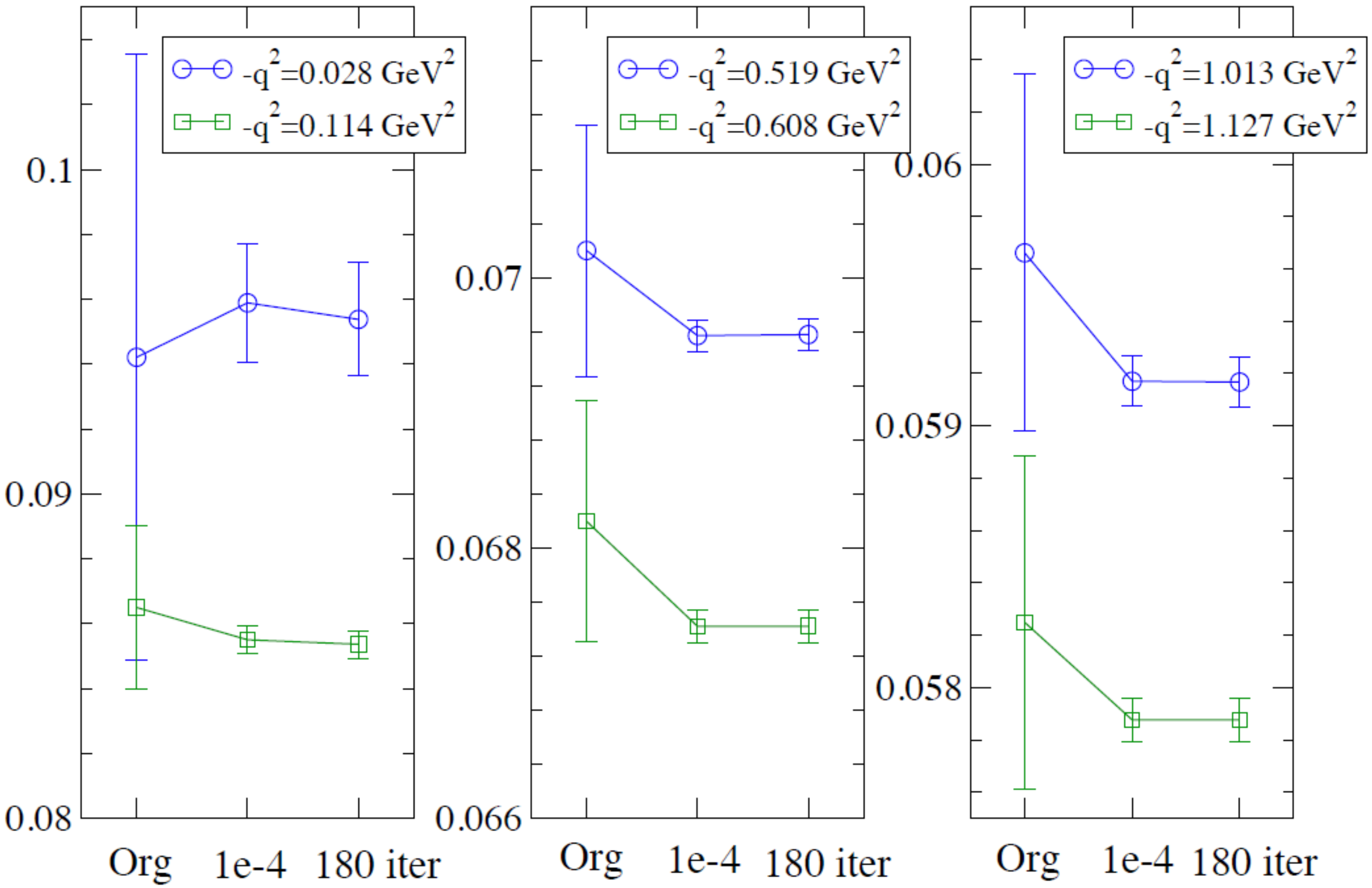
$$\Pi_{\rm long}(\vec{q},\omega) = \frac{g_V}{E_V+\omega} e^{-(E_V+\omega)t_{\rm cut}} + \frac{g_V}{E_V-\omega} e^{-(E_V-\omega)t_{\rm cut}} + \sum_{t=0}^{t_{\rm cut}} 2F(t)\cosh\omega t$$

Finally we consider the particular momentum $q_\mu \neq 0,\ q_{j\neq\mu} = 0$

$$\Pi_{\rm long}(\vec{q},\omega) = -\omega^2 \Pi_V(q^2), \quad q^2 = \omega^2 - q_\mu^2$$

Application of AMA

In $24^3 \times 64$, 300 MeV pion, Nf=2+1 DWF (37 configs)



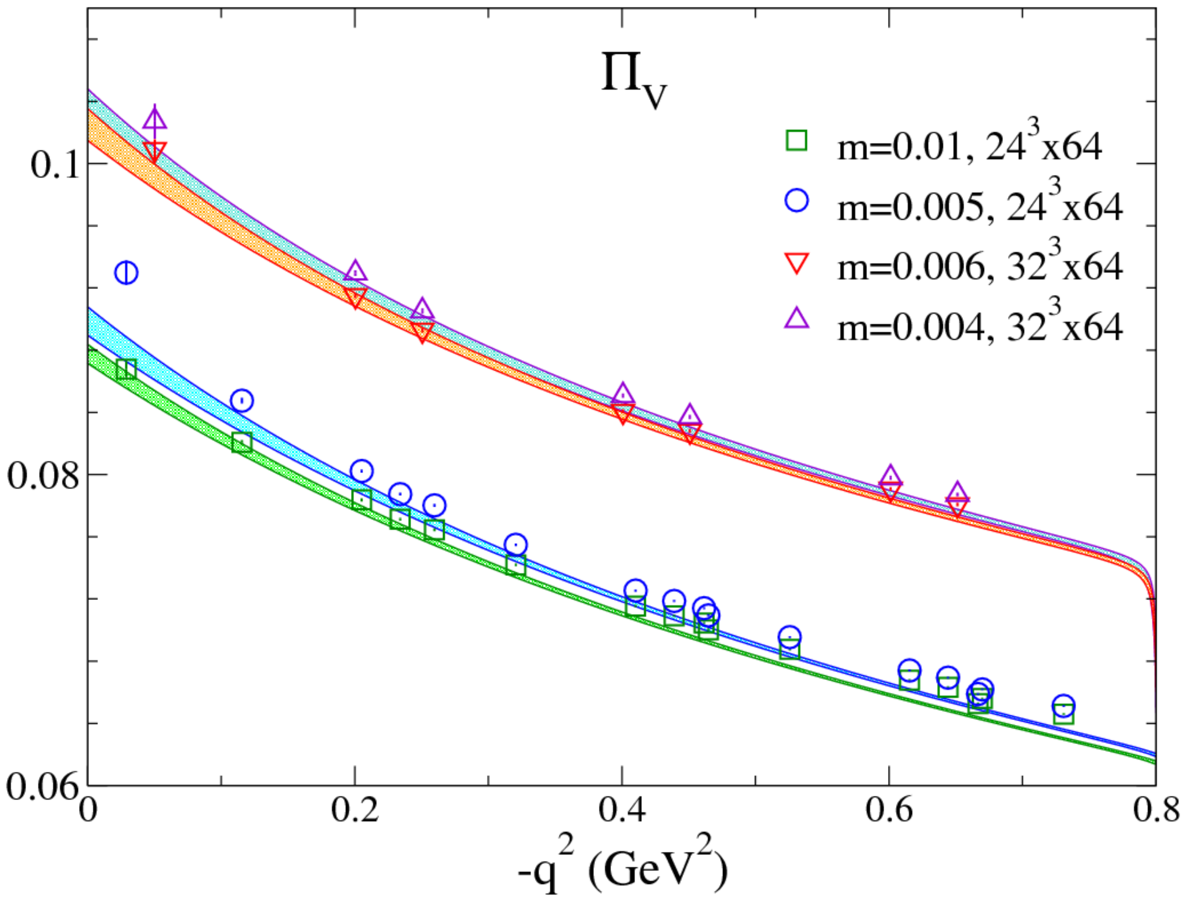
180 lowmode + inexact CG w/ deflation (N_G = 32)

Error reduction ~ 1/5 -- 1/6
Cost reduction ~ 1/5

For light quark, error reduction is even better.

HVP with time-like momentum

- Very preliminary



t_{cut} = 9 (24^3), 10 (32^3)
Fitting range at large t
[8,13] (24^3), [10,15] (32^3)

- Similar behavior with results obtained in Euclid momentum

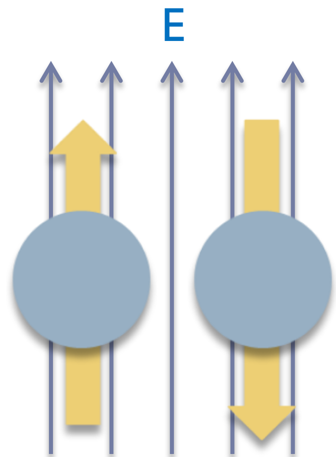
- Slight discrepancy from HVP in space-like momentum, especially for light mass.

More carefully systematic study is necessary !

Spectrum method

- Given by 2-pt function: $m_{\uparrow \text{spin}} - m_{\downarrow \text{spin}} = 2d_N\theta E$
- Direct measurement of EDM.

  It is simple extraction method from 2-pt function
- Ratio of spin up and down



$$R_3 = \frac{\langle N(t)\bar{N}(0)\rangle^{\text{up}}_{\theta,E}}{\langle N(t)\bar{N}(0)\rangle^{\text{down}}_{\theta,E}} \simeq 1 + d_N E\theta t$$

$\rightarrow$ Linear response, its slope is a signal of EDM.

- Reweighting with small θ: $\langle O\rangle_\theta = \langle O e^{i\theta Q}\rangle$

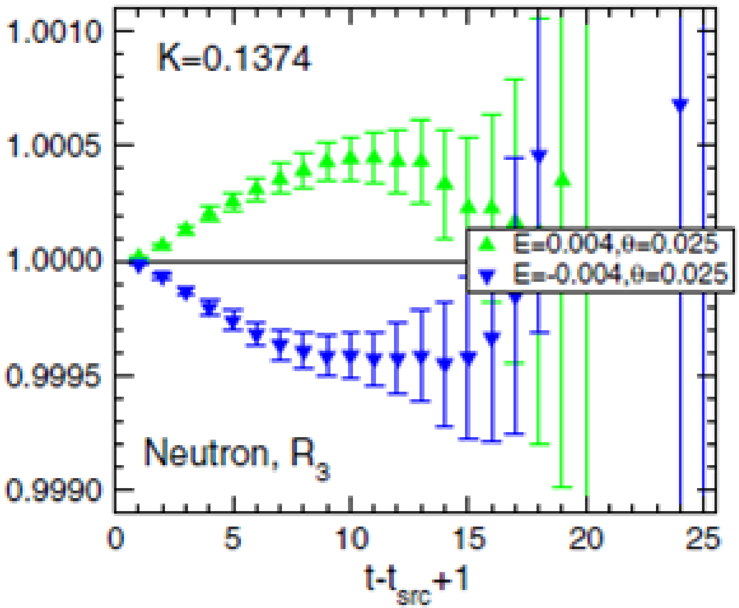
  and introduce external Minkowski E field: $U_t \rightarrow U_t e^{qEt}$, $U_t^\dagger \rightarrow U_t^\dagger e^{-qEt}$
- Temporal periodicity is broken by Minkowski electric field.

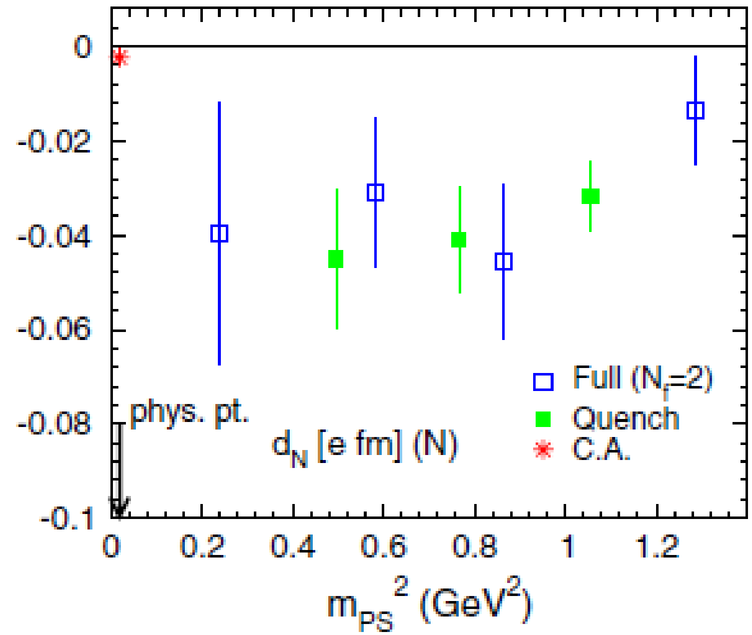
  $\Rightarrow$ additional systematic error

  In imaginary θ method we can avoid this issue.

Spectrum method

ES et al. (06, 07)





Nf=2 Clover (Wilson-type) fermion:

- $24^3 \times 48$ lattice (~2 fm^3), pion mass ~ 500 MeV
- Signal of EDM in full QCD ensembles, O(1000) statistics
- Central value is larger than other phenomenological model.
- Statistical noise (and boundary effect) is still large contribution.

Form factor

- Matrix element

$$\langle n(P_1)|J_\mu^{\rm EM}|n(P_2)\rangle_\theta = \bar{u}_N^\theta \Big[\underbrace{\frac{F_3^\theta(Q^2)}{2m_N}\gamma_5\sigma_{\mu\nu}Q_\nu}_{\text{P,T-odd}} + \underbrace{F_1\gamma_\mu + \frac{F_2}{2m_N}\sigma_{\mu\nu}Q_\nu}_{\text{P,T-even}} + \cdots \Big] u_N^\theta$$

$$\sum_s u_N^\theta(s)\bar{u}_N^\theta(s) = \frac{ip\cdot\gamma + m_N e^{i\alpha_N^\theta\gamma_5}}{2E_N}$$

$$\langle\theta|\eta_N J_\mu^{\rm EM}\bar{\eta}_N|\theta\rangle \quad = \quad \langle 0|\eta_N J_\mu^{\rm EM}\bar{\eta}_N|0\rangle + i\theta\langle 0|\eta_N J_\mu^{\rm EM} Q\bar{\eta}_N|0\rangle$$

$$\begin{aligned}
&\langle 0|\eta_N(t_1)J_\mu^{\rm EM}(t)Q\bar{\eta}_N(t_0)|0\rangle && \big\}\ \text{Computation} \\
&= \frac{\alpha_N}{2}\gamma_5\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{ip\cdot\gamma+m_N}{2E_N} + \frac{1+\gamma_4}{2}\left[F_1\gamma_\mu + F_2\frac{q_\nu\sigma_{\mu\nu}}{2m_N}\right]\frac{\alpha_N}{2}\gamma_5 && \Big\}\ \text{Subtraction} \\
&+\frac{1+\gamma_4}{2}\left[F_3\frac{q_\nu\gamma_5\sigma_{\mu\nu}}{2m_N} + F_A(iq^2\gamma_\mu\gamma_5 - 2m_N q_\mu\gamma_5)\right]\frac{ip\cdot\gamma+m_N}{2E_N} && \Big\}\ \text{Extraction}
\end{aligned}$$

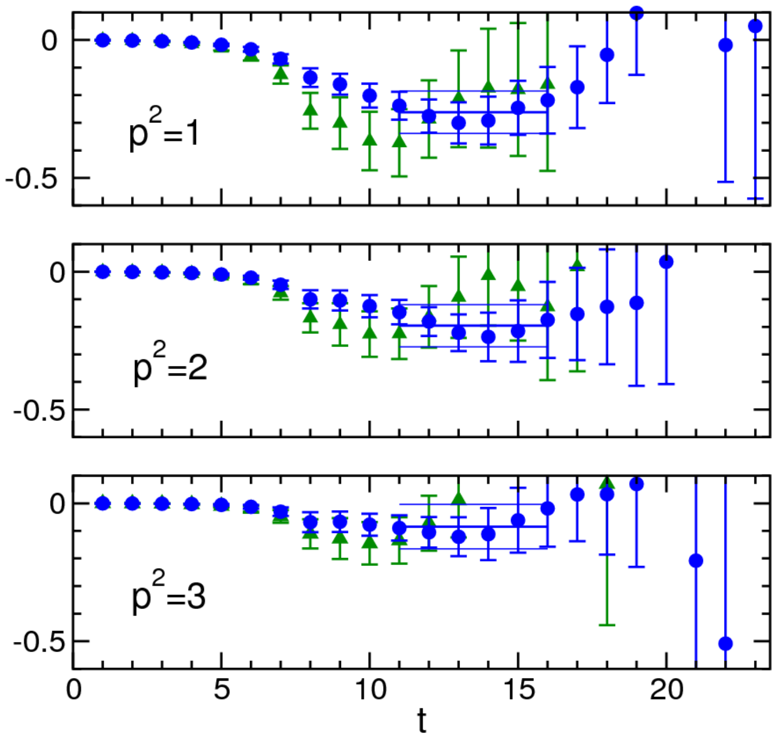
- Subtraction of CP-odd phase, α_N, in n propagator and CP-even part $F_{1,2}$

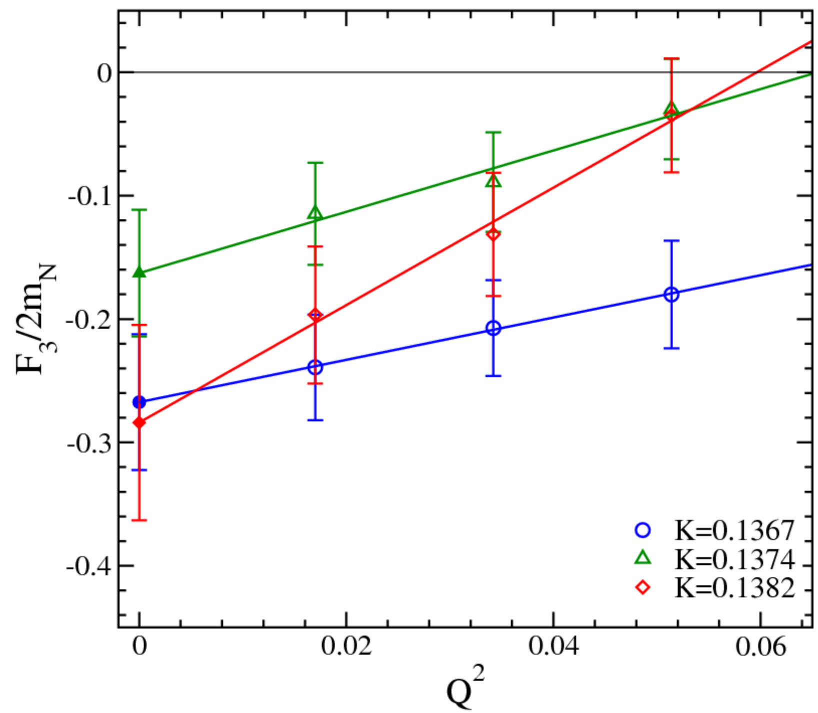
$$d_N = \lim_{Q^2\to 0} F_3(Q^2)/2m_N$$

Form factor

ES et al.(05, 08)

- Result of Nf=2 clover fermion
  - Size is $24^3 \times 48$ lattice (~2 fm^3), pion mass is around 500 MeV
  - Ignoring disconnected diagram in 3-pt function
  - momentum transfer $Q^2 \to 0$ limit is with linear func.





Imaginary θ

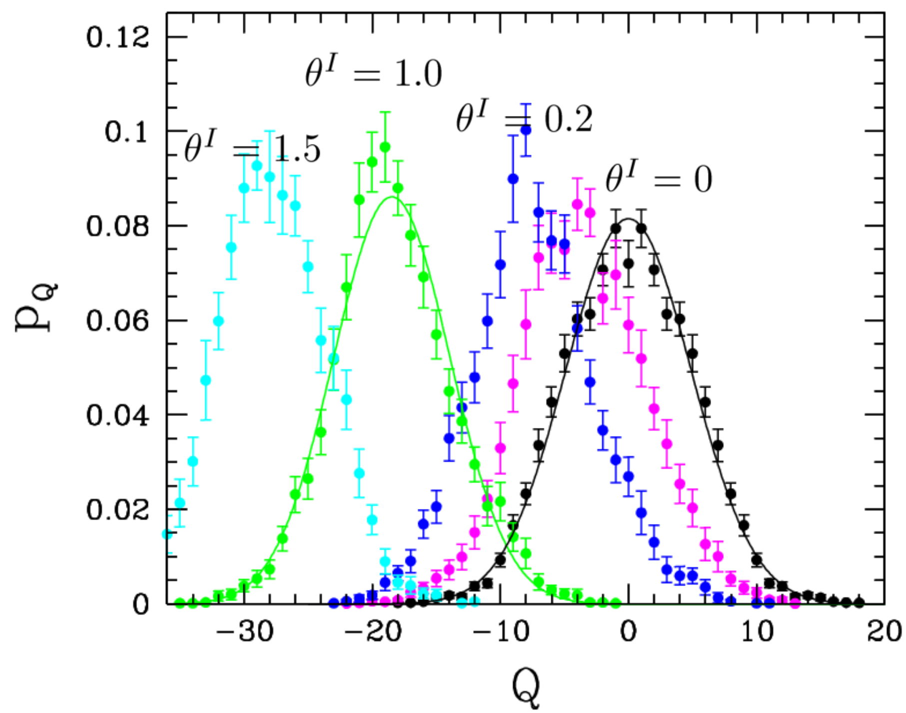
Izubuchi(07), Horsley et al. (08)

- Analytical continuation to pure imaginary

$$\langle O e^{i\theta Q} \rangle \to \langle O e^{-\theta^I Q} \rangle$$

  - There is <u>no sign problem</u>, then expect better signal.
  - Distribution of Q is shifted by θ^I
  - EDM is regarded as the slope of θ
  - <u>Need to generate the new QCD ensemble</u> for each θ^I

  ⇒ it will be challenging work when going to realistic lattice (larger lattice and physical quark mass)



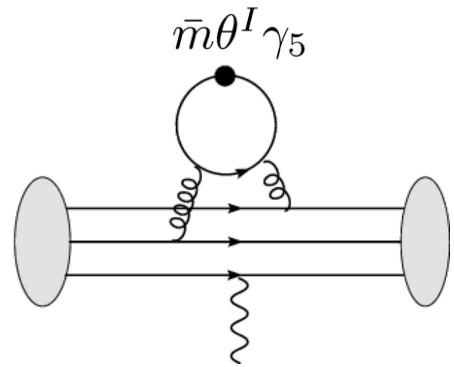
Imaginary θ

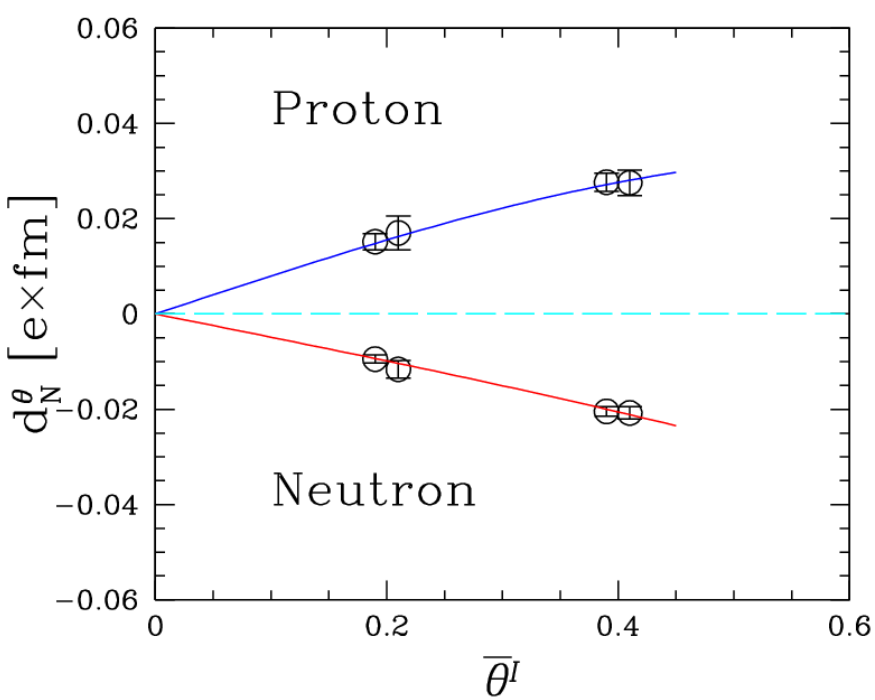
Izubuchi(07), Horsley et al. (08)

- Results with Nf=2 Wilson fermion

$16^3 \times 32$ lattice, m_π = 700 MeV (heavey)
Fermionic insertion of imaginary theta:

$$\mathcal{L}_\theta = \bar{m}\theta^I \bar{q}\gamma_5 q/2$$





- Measuring EDM form factor
- generate ensemble with 4 different θ^I
- Clear signal, but systematic error (lattice artifacts) due to chiral symmetry breaking of clover fermion has not been taken into account.

⇒ need careful check with chiral fermion (DWF etc)

Imaginary θ

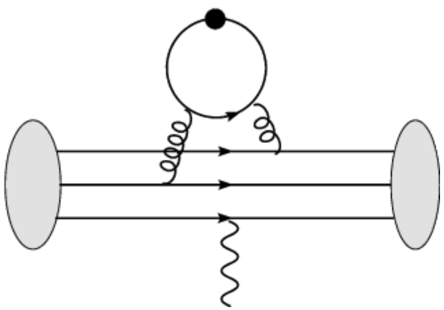
Izubuchi(07), Horsley et al. (08)

- Problem with Wilson fermion

Fermionic insertion of imaginary theta should be changed by Wilson term:

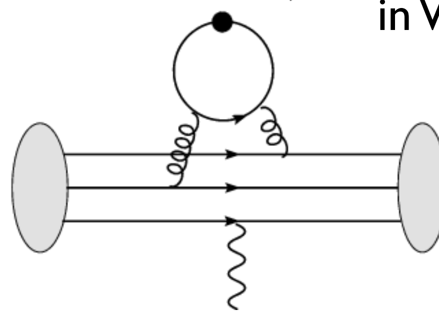
$$\mathcal{L}_\theta = \bar{m}\theta^I \bar{q}\gamma_5 q/2 \to \mathcal{L}_\theta^W = \bar{m}(1+\kappa_P)\theta^I \bar{q}\gamma_5 q,\ \kappa_P \sim \mathcal{O}(a) : \text{renom. const.}$$

$\bar{m}\theta^I\gamma_5$ Continuum



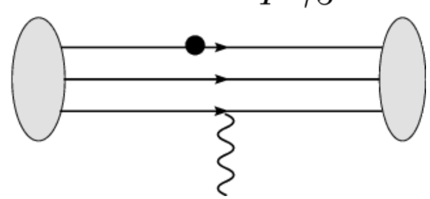
$\bar{m}\theta^I\kappa_P\gamma_5$

Lattice artifact (systematic error) in Wilson-type fermion



$\sim \mathcal{O}(am_{\text{sea}}\alpha^2\theta^I)$

$\bar{m}\theta^I\kappa_P\gamma_5$



$\sim \mathcal{O}(am_{\text{val}}\theta^I)$

Need subtraction !

Cf. discussion in Aoki, Gocksh, Manohar,Sharpe (1990)

Recent results (preliminary)

- Nf=2+1 DWF configurations

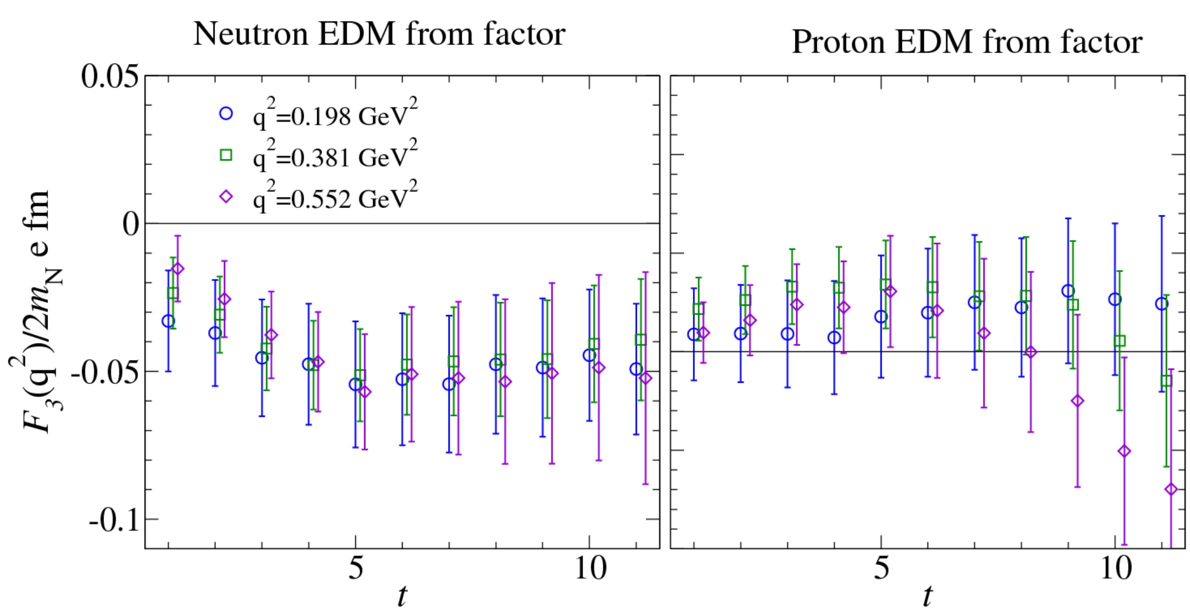
Blum, Izubuchi, ES (2012)

All-mode-averaging (AMA) which is a new error reduction techniques

⇒ reduction of computational cost is more than 15 times

(in bigger lattice AMA can do more large error reduction)

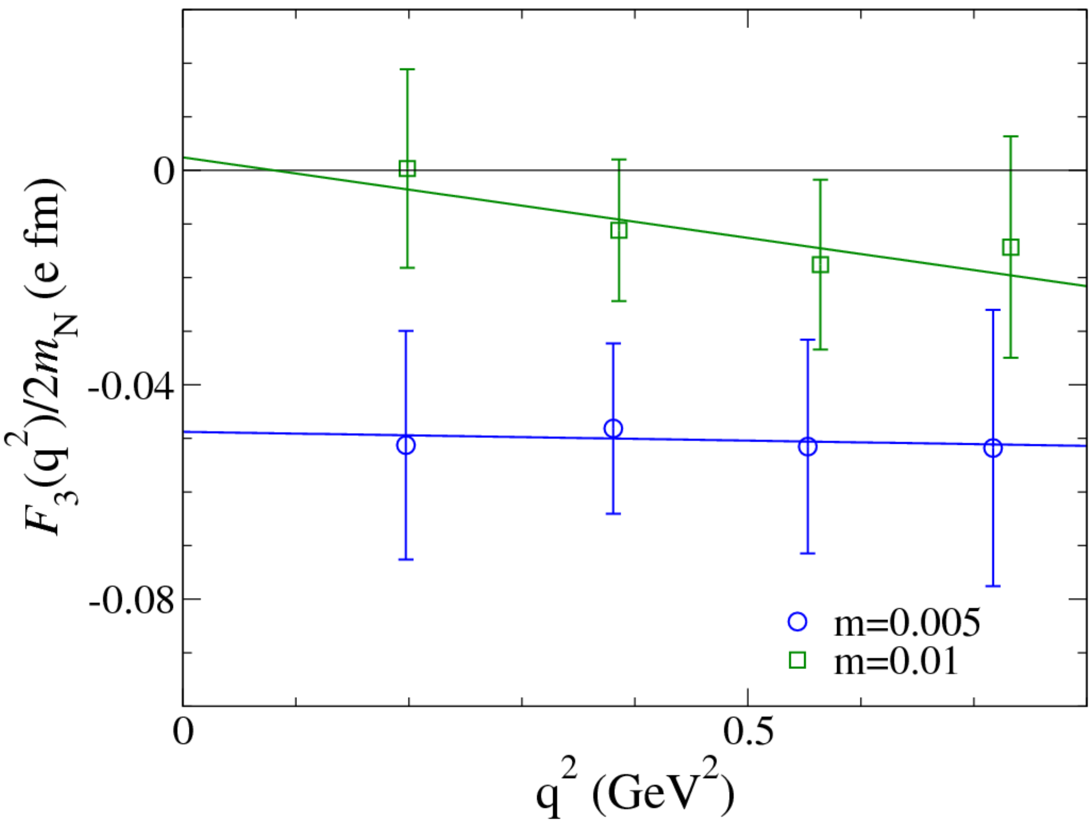
  - $24^3 \times 64$ lattice (3 fm^3), $m_\pi = 0.3$ GeV, 384 configs with AMA



Using AMA, signal of neutron (and proton) EDM (plateau region) can be observed.

Recent results (preliminary)

- Nf=2+1 DWF configurations
  - Linear extrapolation to zero transfer momentum



Small slope of q^2 dependence (one of the input parameter of effective model. Vries, Timmermans, Mereghetti and Kolck 1006.2304)