

Modified theories of gravity and mechanisms of mass screening

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@ Nagoya Daigaku

Dark Energy in a nutshell

Chameleon

Symmetron

Vainshtein

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Dark Energy in a nutshell

- Friedmann equation

$$3H^2 = \rho$$

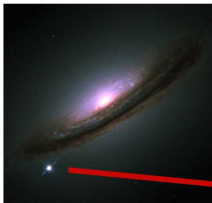
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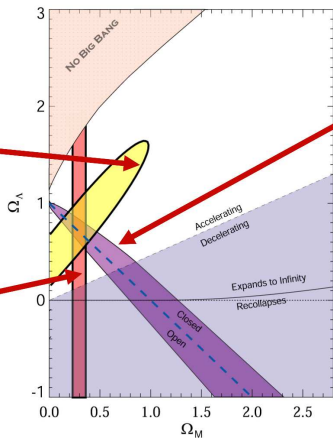
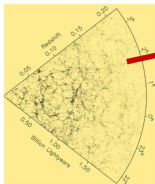
$$3H^2 = \rho + \Lambda$$

$$\Omega_{DE} = 0.72 \pm 0.015, \quad -1.11 < w_{DE} = P/\rho < -0.86$$

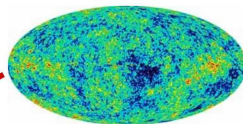
SNe Ia



LSS

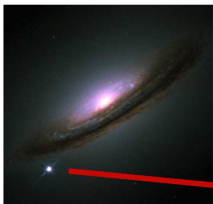


CMB

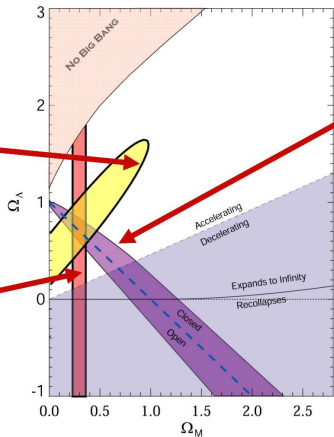
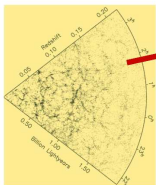


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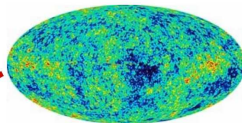
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⇒ **The Universe is accelerating**

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$$\rho_{vac}/\rho_{\Lambda} \approx 10^{55}$$

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- Add exotic matter (Dark Energy)

$$3H^2 = \rho + \rho_{vac}$$

- Modify the gravitational constant (Chameleon, Vainshtein)

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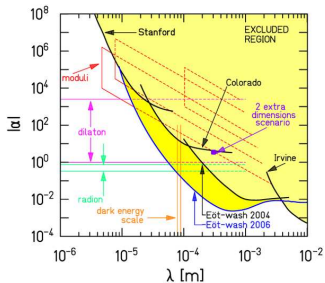
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Local bounds

• Laboratory Bounds

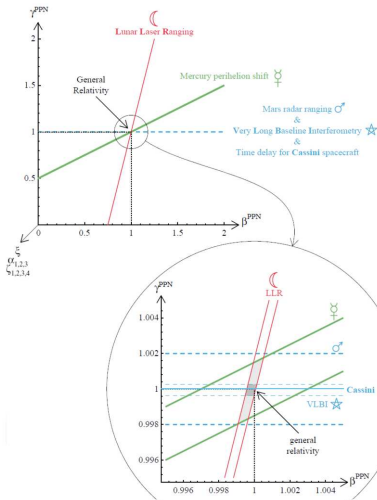
$$V(r) = -G \frac{m_1 m_2}{r} \left[1 + \alpha \exp^{-r/\lambda} \right]$$



$$g_{00} = -1 + 2 \frac{GM}{rc^2} - 2\beta^{\text{PPN}} \left(\frac{GM}{rc^2} \right)^2 + \dots$$

$$g_{ij} = \delta_{ij} \left[1 + 2\gamma^{\text{PPN}} \frac{GM}{rc^2} + \dots \right]$$

• Solar System Bounds



- **Problem?**

Solar System and lab experiments rule out any long range gravitationally coupled fifth force.

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- **How to modify the theory of gravity?**

Weinberg's theorem: A Lorentz-invariant theory of a massless spin-two field must be equivalent to GR in the low-energy limit.

Low-energy modifications of gravity therefore necessarily involve introducing new degrees of freedom, such as a scalar.

Giving the graviton a small mass width turns out to introduce a scalar.

Theories that invoke a Lorentz violating massive graviton also involve a scalar.

Three ways to hide

$$\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi)\partial_\mu\phi\partial_\nu\phi - V(\phi) + A(\phi)T$$

- The scalar field is weakly coupled to matter
 - Quintessence
- The scalar field is massive in vicinity of matter
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$$S = \int d^4x \sqrt{-g} \left(R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right) + S_m[\psi_m; A^2(\phi) g_{\mu\nu}]$$

- Equation of motion
- Dynamics governed by effective potential:

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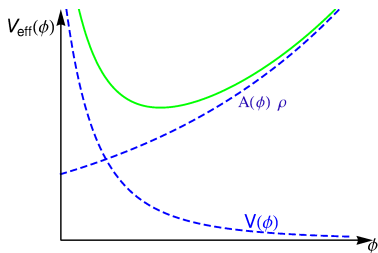
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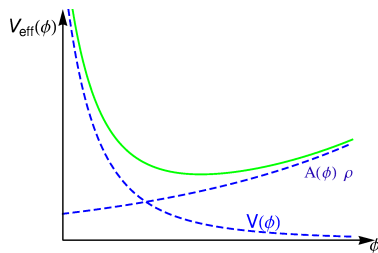
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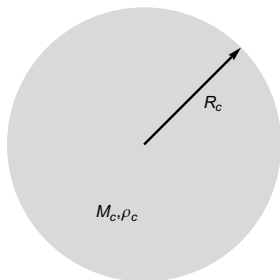


Large ρ



Small ρ

Solution for a compact object

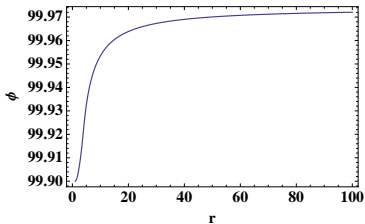
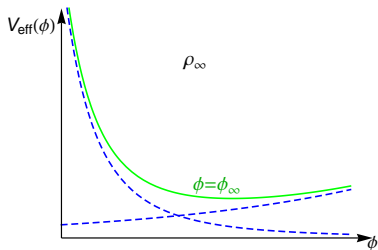
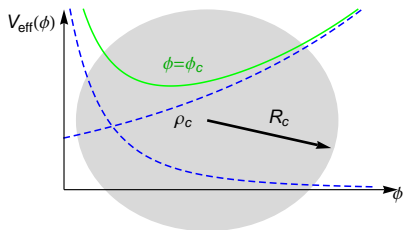

 ρ_∞

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = V_{,\phi} + A(\phi)\rho(r)$$

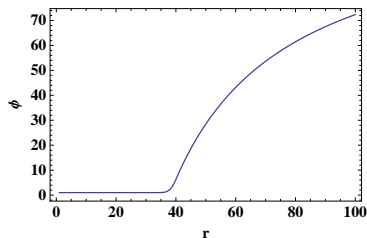
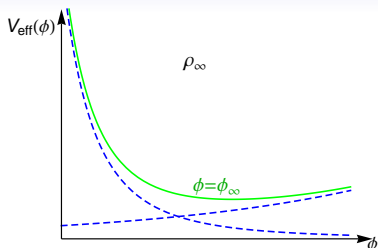
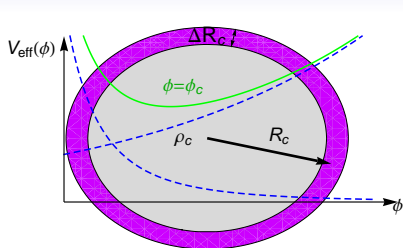
The boundary conditions specify that the solution be nonsingular at the

origin: $\frac{d\phi}{dr} = 0$ at $r = 0$

And that the force on a test particle vanishes at infinity: $\phi \rightarrow \phi_\infty$ as $r \rightarrow \infty$



$$\phi(r) \approx - \left(\frac{\beta}{4\pi M_{pl}} \right) \frac{M_c e^{-m_\infty(r-R_c)}}{r} + \phi_\infty$$



$$\phi(r) \approx - \left(\frac{\beta}{4\pi M_{pl}} \right) \left(3 \frac{\Delta R_c}{R_c} \right) \frac{M_c e^{-m_\infty (r-R_c)}}{r} + \phi_\infty$$

- Field reaches effective minimum almost everywhere in the body
- All field variations confined to small region near body surface
- This region is called a thin-shell $\frac{\Delta R_c}{R_c} \ll 1$

Fifth Force on Body with Thin-Shell

A test mass M_1 in a static chameleon field ϕ experiences a force \vec{F}_ϕ

$$\frac{\vec{F}_\phi}{M_1} = -\frac{\beta}{M_{pl}} \vec{\nabla} \phi$$

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$$\Rightarrow F_{12} = \frac{G_N m_1 m_2}{r^2} (1 + \alpha e^{-mr})$$

- deviations from Newton's law are given by $\alpha = 6\beta^2 \frac{\Delta R_c}{R_c}$

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with

$$\frac{\Delta R_c}{R_c} \approx \frac{1}{\text{Newton's potential}}$$

for large objects (sun, earth, moon) , Newton's potential is large and a thin shell is always present

⇒ Planetary motion unaffected by the chameleon field

- Example (Earth)

Modeled as a sphere of radius $R_c = 6000 \text{ Km}$ and density $\rho_c = 10 \text{ g/cm}^3$.

Far away the matter density is approximately that of baryonic gas and dark matter: $\rho_\infty = 10^{-24} \text{ g/cm}^3$

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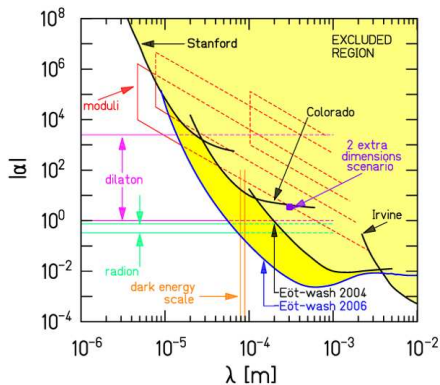
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$$\alpha = 10^{-8}$$

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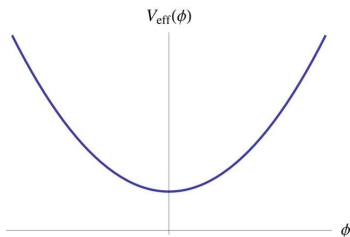
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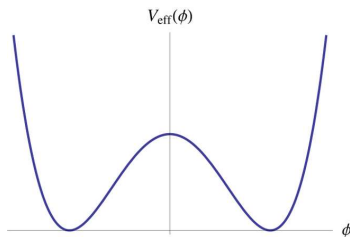
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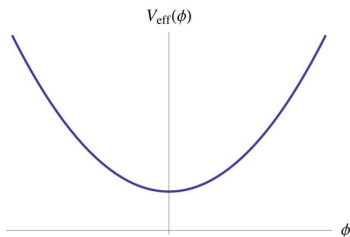


Low density

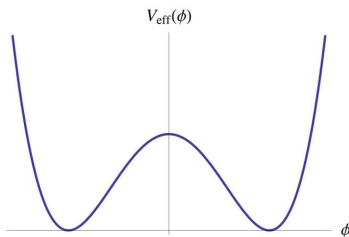
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Large density



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$$\text{Coupling to matter} \simeq \frac{\phi^2}{M^2} \rho$$

$$\text{Fluctuations } (\delta\phi) \text{ around the local background } (\phi_0) \simeq \frac{\phi_0}{M^2} \delta\phi \rho$$

Conclusion

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 - Useful for many models: $f(R, G)$
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- Strengthens:
 - Useful for many models: $f(R, G)$
 - Fifth force is suppressed locally
 - Interesting cosmological consequences
- Weaknesses
 - Singularity problem ?
 - We do not solve the cosmological constant problem:

$$m_\phi < 10^{-33} \text{eV}$$

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$$L = -\frac{1}{2}Z^{\mu\nu}(\phi)\partial_\mu\phi\partial_\nu\phi - V(\phi) + A(\phi)T$$

- At low energy $Z_{\mu\nu} \approx g_{\mu\nu}$
- At high energy $Z_{\mu\nu} \gg 1$
- The field is strongly coupled to itself and becomes weakly coupled to external sources

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Vainshtein

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DGP

- Dvali, Gabadadze, Porrati (2000).
- The 3-brane is embedded in a Minkowski bulk spacetime with infinitely large 5th extra dimensions:
- Boundary effective action (Luty,Porrati,Rattazzi: 2003)

$\int_{\Sigma} d^4x \sqrt{-g} [-\frac{1}{2} \text{tr} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)]$

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• ADM decomposition ($g_{\mu\nu}, N, N_\mu$)

• Gauss constraint

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Decoupling limit

$$L_\phi = -3(\partial\phi)^2 - \frac{1}{\Lambda^3}\square\phi(\partial\phi)^2 + 2\frac{\phi}{M_{pl}}T$$

- The effective action is local
- The equation of motion remain second order in derivative
- The equations of motion are invariant under shift and Galilean global transformations $\phi \rightarrow \phi + c + v_\mu x^\mu$

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\Rightarrow Galileon models

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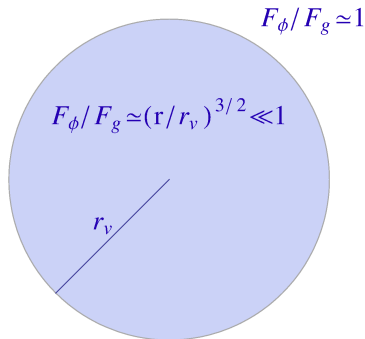
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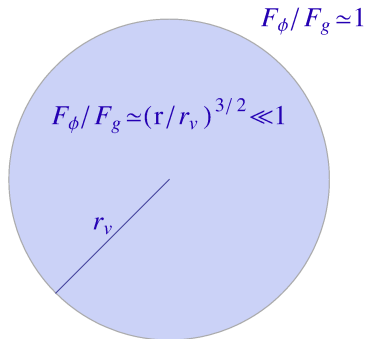


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\Rightarrow The fifth force is suppressed locally

Conclusion

- Three ways to hide the additional degrees of freedom
 - Chameleon
 - Symmetron
 - Vainshtein
- Nontrivial interactions between the scalar field and matter allowed
- Candidate for dark energy
- Bounds from laboratories, solar system and astrophysical experiments are exponentially relaxed
- The best model is the cosmological constant

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