LATE TIME COSMIC ACCELERATION: DARK ENERGY AND ALTERNATIVES

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Centre for theoretical physics (2006)

- CTP has a small library and comp center.
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Centre for theoretical physics , Jamia Millia Islamia

Established in 2006
Faculty: 6
Students: 11
A large number of visitors
Major theme of research: Dark energy and alternatives; neutrino physics.

COSMIC ACCELERATION: OVERVIEW

Dark Energy:

- Cosmological Constant: historical notes.
- Age crisis in the hot big bang model and its resolution.
- Fine tuning problem; Coincidence problem.
- Scalar fields: scaling solutions.
- Scaling solutions and transition to late time acceleration.
- Scalar field models: Trackers (freezing) and non-trackers (thawing).
- Can we detect the dark energy dynamics?
- Quintessential inflation: joining the two ends or reincarnation ?

Modified theories of gravity

- String inspired corrections to gravity: late time exit from scaling regime to dark energy universe.
- f (R) theories of gravity: Class of generic model.
- Galileon gravity : Relevance to late time acceleration.

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- M. Turner, D. Huterer, "**Cosmic Acceleration, Dark Energy and Fundamental Physics** "J.Phys.Soc.Jap.76:111015,2007.
- E. Copeland, **MS**, S. Tsujikawa, " **Dynamic of Dark Energy**", hep-th/0603057.

Homogeneous and isotropic universe

Hubble Law

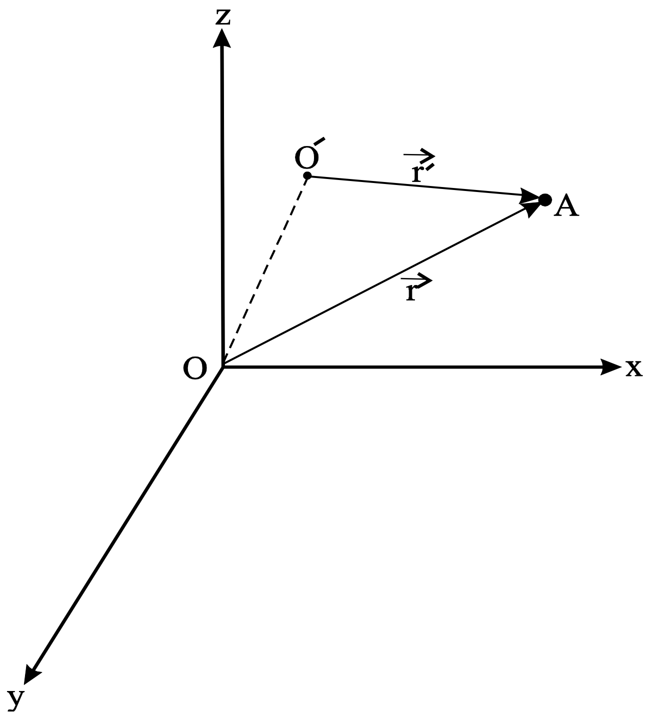
$$\mathbf{v}(\mathbf{r},t) = H(t)\mathbf{r}$$

$$\mathbf{r}'(t) = \mathbf{r}(\mathbf{t}) - \mathbf{r}_{o'}(t)$$

$$\mathbf{v}' = \mathbf{v} - \mathbf{v}_{o'} = H\mathbf{r}'$$

$$\mathbf{r}(t) = \mathbf{r}(t=0)e^{\int H(t)dt} \equiv a(t)\mathbf{x}$$

$$\frac{\dot{a}}{a} = H$$



NEWTONIAN COSMOLOGY

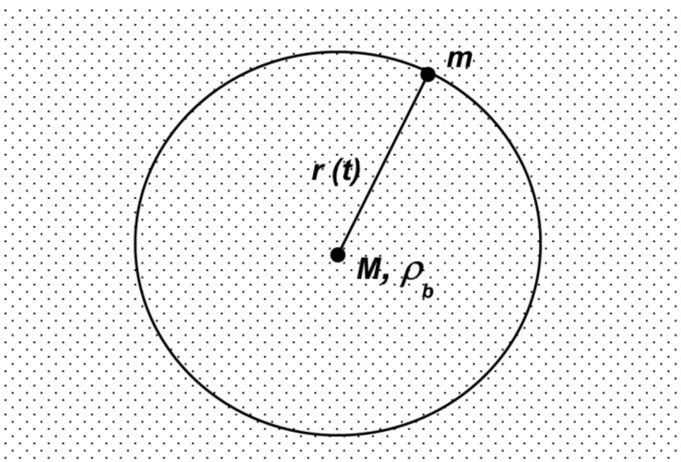
(1895- Neumann, 1896-Seeliger)

MS, arXiv: 0904.3445

$$\mathbf{F} = -\frac{4\pi G}{3}\rho_b(t)\mathbf{r}(t) \Rightarrow \quad \frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}\rho_b(t)$$

Co-moving system: $\mathbf{r}(t) \equiv a(t)\mathbf{x}$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}\rho_b(t) - \frac{K}{a^2}, \quad K \equiv a_0^2\left(\frac{8\pi G\rho_b^0}{3} - H_0^2\right)$$



$$\frac{\partial\rho_b(t)}{\partial t} + (\nabla.\rho_b\mathbf{v}) = 0$$

$$\frac{\partial\rho_b(t)}{\partial t} + 3H\rho_b = 0, \quad \rho_b(t) = \rho_b^{(0)}\left(\frac{a_0}{a}\right)^3$$

No static universe: $\dot{a} = \ddot{a} = 0$

NEWTONIAN COSMOLOGY

Cosmological constant a la Hook's law

$$\mathbf{F} = -\frac{4\pi G}{3}\rho_b \mathbf{r} + \frac{1}{3}\Lambda \mathbf{r}$$

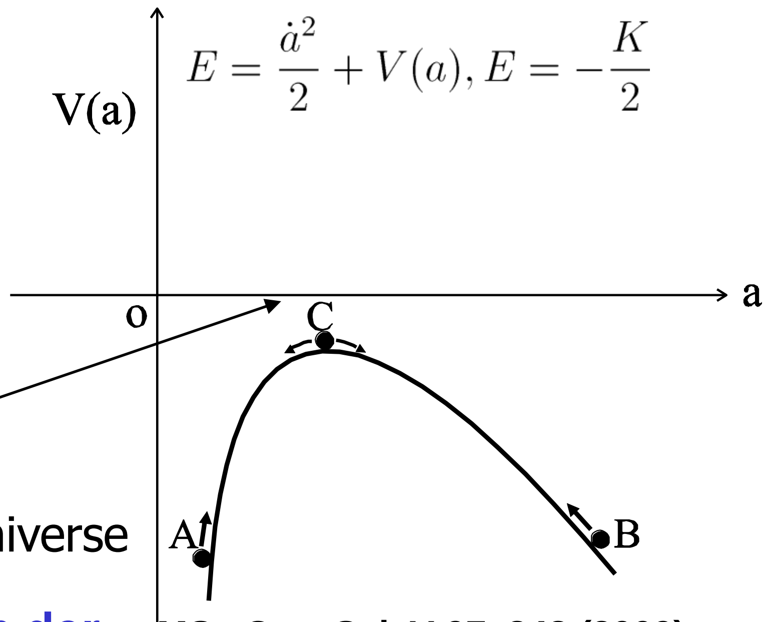
$$V(a) = -\left(\frac{4\pi G \rho_b a^2}{3} + \frac{\Lambda a^2}{6}\right)$$

$$\frac{1}{a}\frac{d^2 a}{dt^2} = -\frac{4\pi G}{3}\rho_b(t) + \frac{\Lambda}{3}$$

$$E = \frac{\dot{a}^2}{2} + V(a), E = -\frac{K}{2}$$

$$H^2 = \frac{8\pi G}{3}\rho_b(t) - \frac{K}{a^2} + \frac{\Lambda}{3}$$

(K>0) $\Lambda = \Lambda_c = 4\pi G \rho_b^{(0)}$ → Static universe



C. Neumann, Uber das Newtonische Prinzip der Fernwirkung, Leipzig, 1895.
H. Seeliger, Astron. Nachr. bf 137, 129(1896).

MS, Curr Sci, V 97, 848 (2009) [arXiv:0904.3445].

RALATIVISTIC COSMOLOGY

Einstein’s view on cosmological constant

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

$$\frac{1}{a}\frac{d^2 a}{dt^2} = -\frac{4\pi G}{3}\rho_b + \frac{\Lambda}{3}$$

$$H^2 = \frac{8\pi G}{3}\rho_b(t) - \frac{K}{a^2} + \frac{\Lambda}{3}$$

After the first paper of Friedmann 1922, Einstein published a brief note claiming an error in Friedmann's work; when it was pointed out to him that it was his error, Einstein published a retraction of his comment, with a sentence that luckily was deleted before publication: ``Friedmann's paper while mathematically correct is of no physical significance''. Einstein wrote to Weyl in 1923 : ``If there is no quasi-static world, then away with the cosmological term''.

RALATIVISTIC COSMOLOGY

Dark Energy

$$H^2 = \frac{8\pi G}{3}\rho_b(t) - \frac{K}{a^2}$$

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}\left(\rho_b + 3P_b\right) \Rightarrow \ddot{a} > 0 \rightarrow w \equiv \frac{P_b}{\rho_b} < -\frac{1}{3}$$

$$\dot{\rho}_b + 3H\left(\rho_b + P_b\right) = 0$$

$$\rho_\Lambda = -P_\Lambda = \frac{\Lambda}{8\pi G}$$

AGE CRISIS IN HOT BIG BANG

Matter dominated universe:

$$\rho_b(t) = \rho_b^{(0)} \left(\frac{t_0}{t}\right)^2$$

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3} \longrightarrow H(t) = \frac{2}{3}\frac{1}{t} \Rightarrow t_0 = \frac{2}{3}\frac{1}{H_0}$$

$$H_0^{-1} = 9.8h^{-1}Gyr, \quad 0.64 \lesssim h \lesssim 0.8 \rightarrow t_0 = (8-10)Gyr$$

AGE CRISIS IN HOT BIG BANG

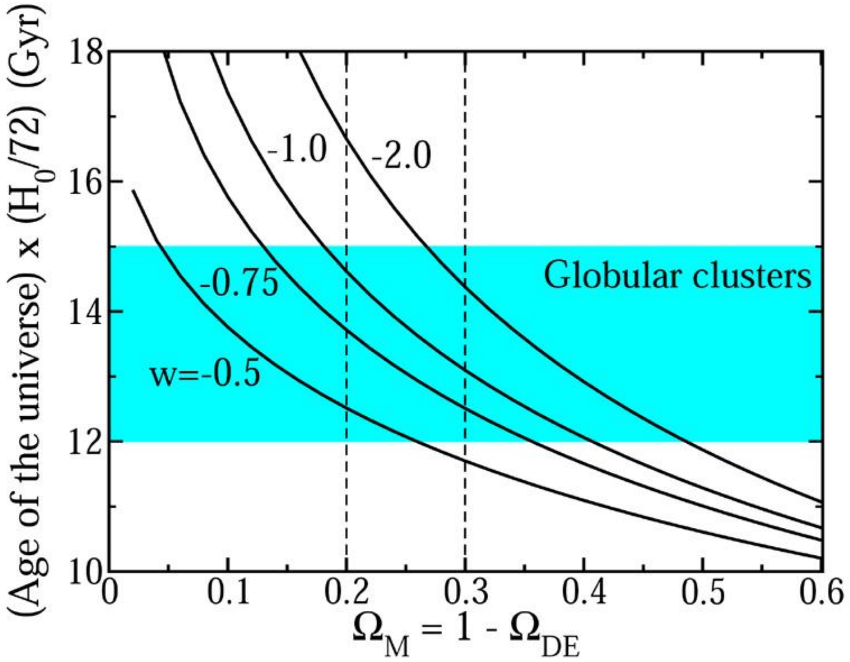
$$v = Hr(t)$$

Let us ignore gravity $v = const$

$$\frac{1}{t_0} = H_0$$

AGE CRISIS AND ITS RESOLUTION WITH Λ

Λ -dominated universe



OBSERVATIONS IN BRIEF

$$\Omega_{tot}(t) - 1 = \frac{K}{(aH)^2}, \ \Omega_b(t) = \frac{\rho_b(t)}{\rho_c(t)}$$

$$\rho_c(t) = 3H^2(t)/8\pi G$$

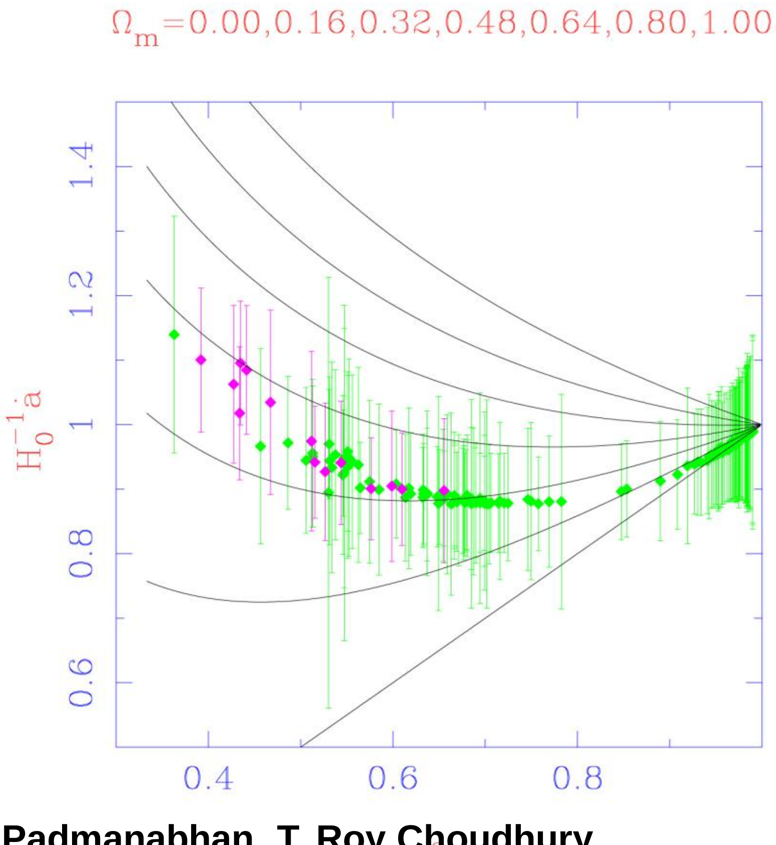
- **Universal is nearly critical -- CMB**

$$\Omega_{tot} \simeq 1$$

- **Dark matter contribution is nearly 30% ---LSS**

- **The missing component (about 70%) is Dark Energy—Supernovae observations**

$$\Omega_{tot} = \Omega_b + \Omega_{DE} \simeq 1$$



T. Padmanabhan, T. Roy Choudhury, Mon.Not.Roy.Astron.Soc.344:823-834,2003

COSMO-ILLOGICAL CONSTANT

THEORETICAL ISSUES

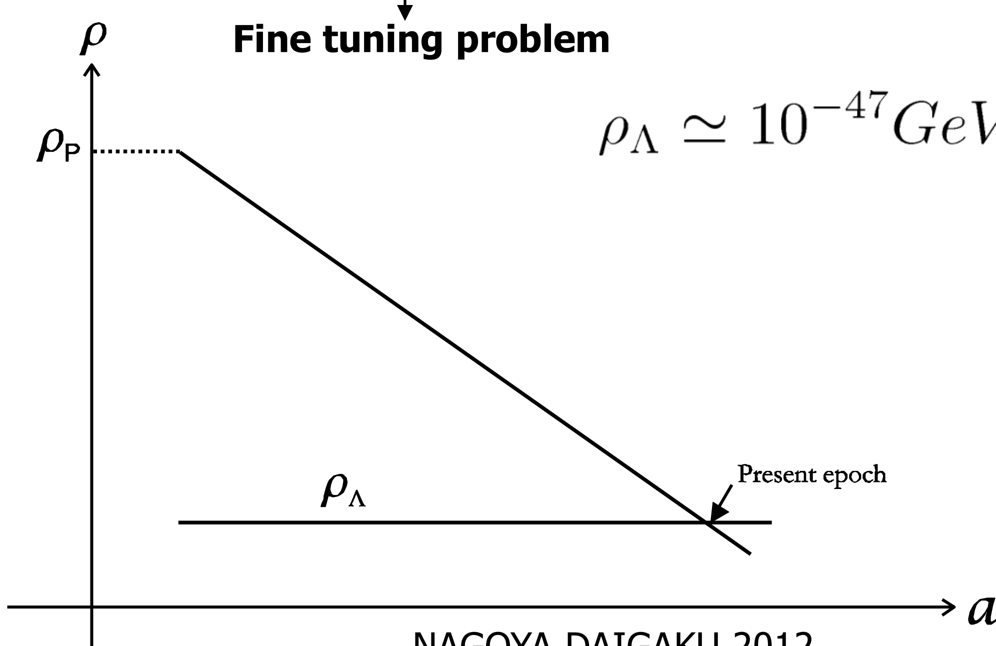
- **Cosmological constant as measure of vacuum energy.**

$$< T_{\mu\nu} >_0 = \Lambda g_{\mu\nu}, \ \ (8\pi G = 1)$$

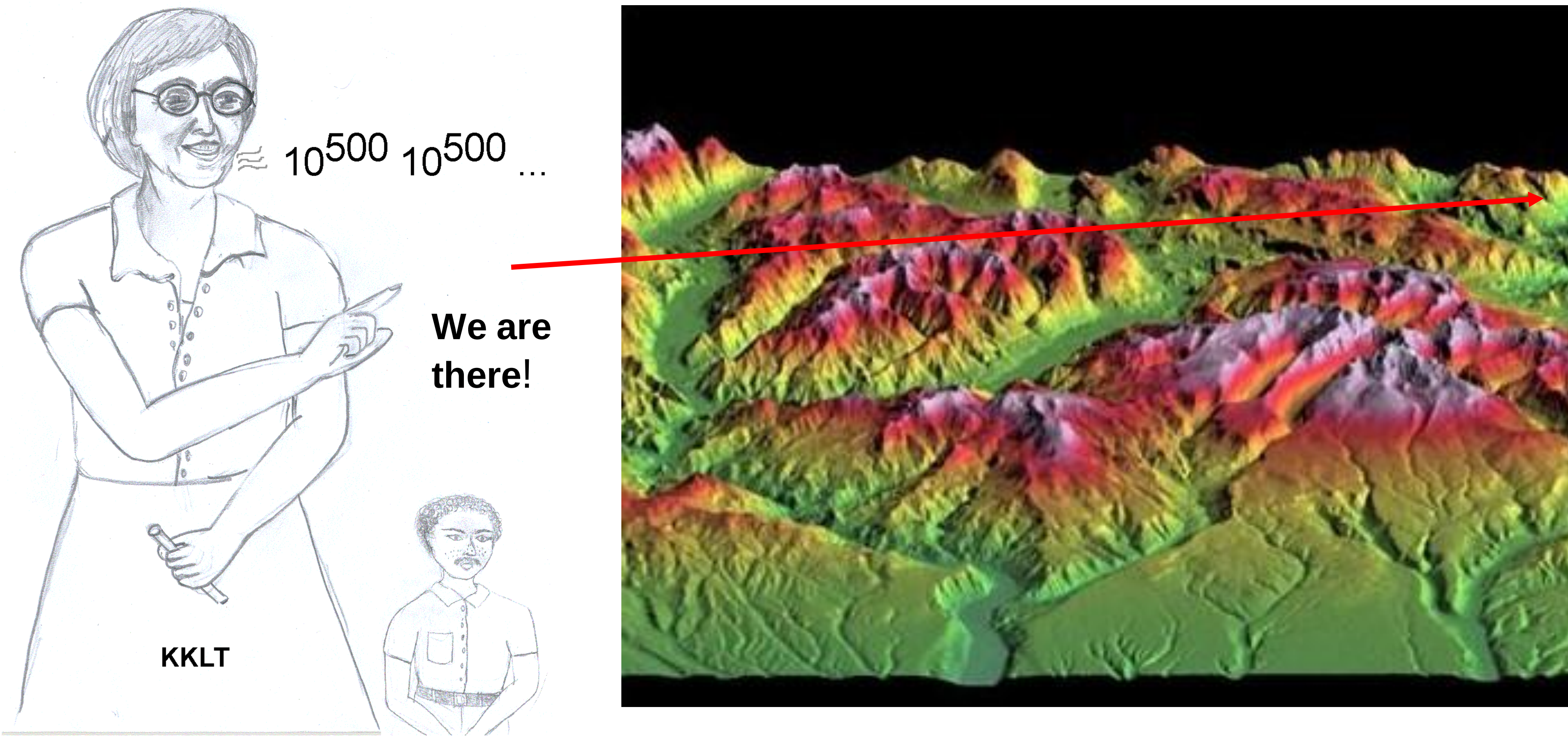
- **Cosmological constant from string theory--**10^{500} **! Vacua.**
- **Cosmological constant as a free parameter of GR.**

Fine tuning problem

$$\rho_\Lambda \simeq 10^{-47} GeV^4$$

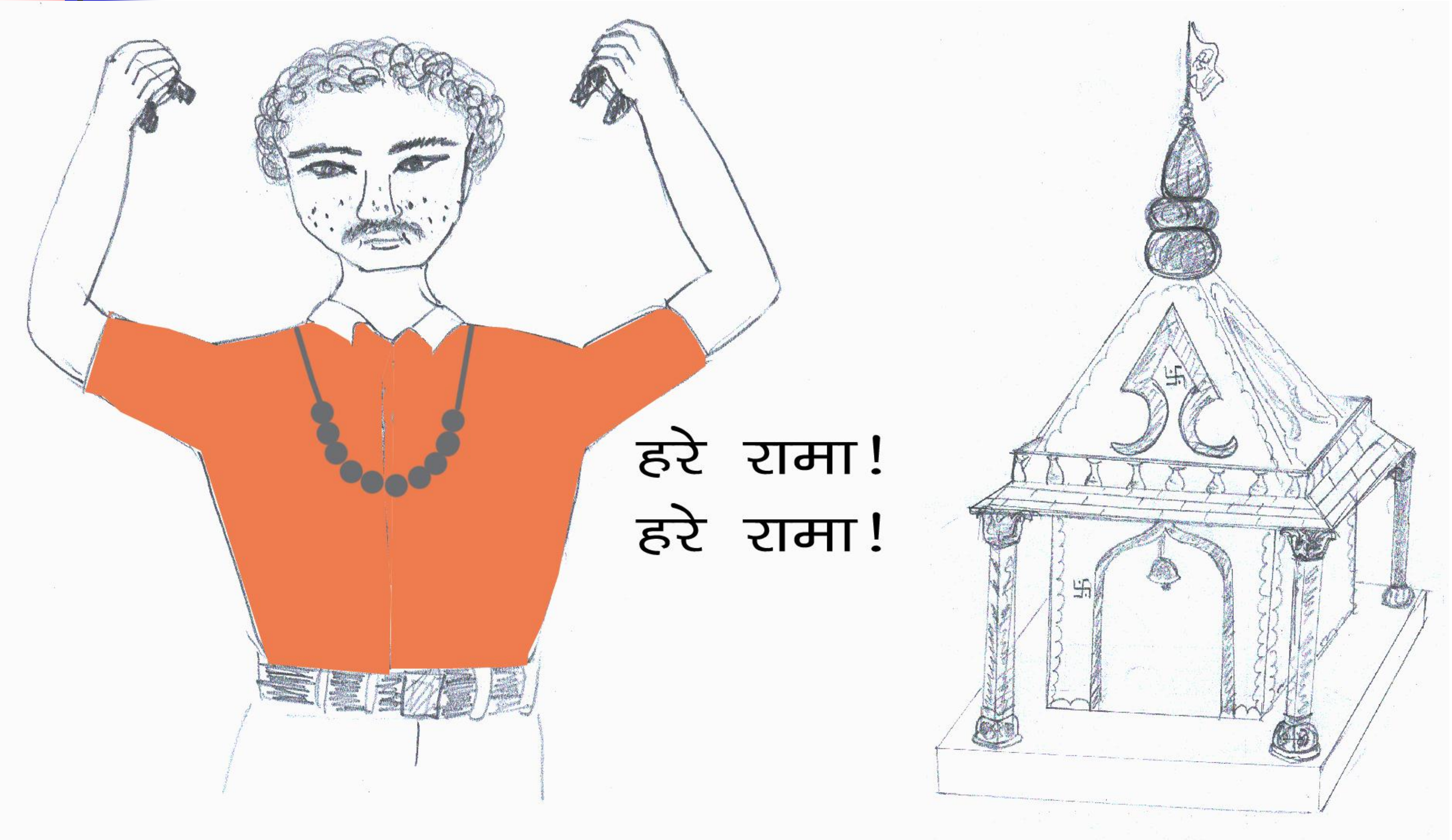


String Landscape



$10^{500} vacua!$

It is easier to believe in God!



FINE TUNING PROBLEM : TWO DISTANT SCALES TOGETHER

THE BIG AND THE SMALL
PUTTING THEM TOGETHER

SCALAR FIELD AS DARK ENERGY Quintessence

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \,, \qquad \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \;\; P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$H^2 = \frac{8\pi G}{3}\rho_\phi$$

$$\rho_\phi = \rho_\phi^0 \exp\left(-\int 3(1+w(\phi))\frac{da}{a}\right) \,, \qquad w(\phi) = P_\phi/\rho_\phi$$

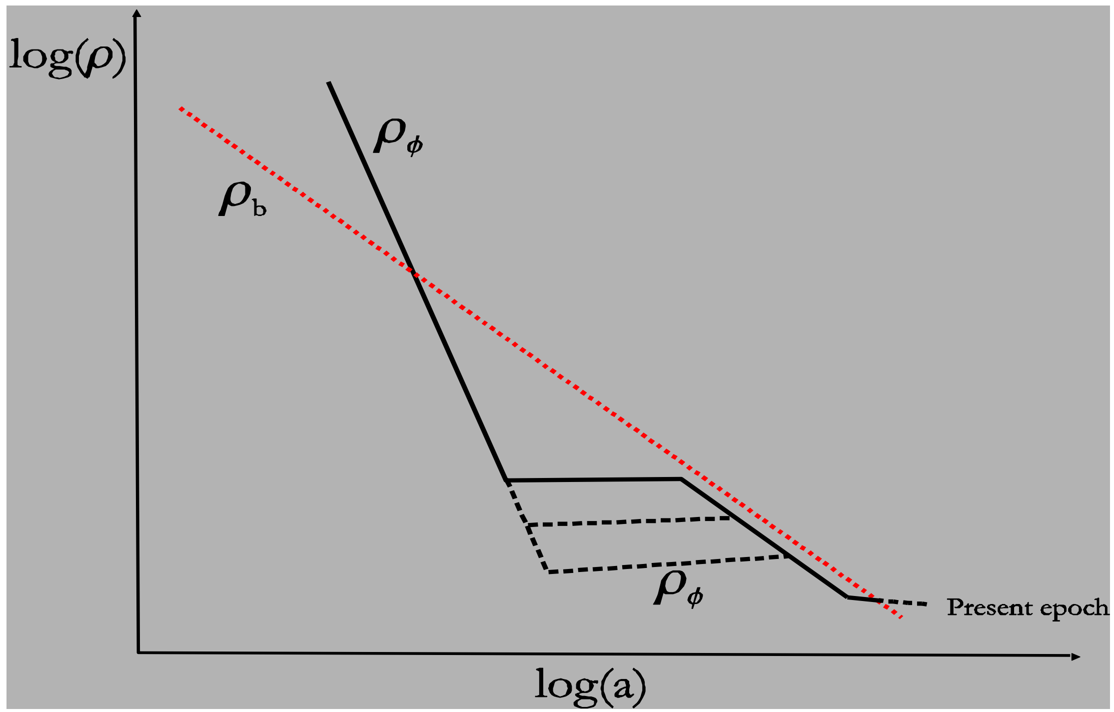
$$\rho_\phi \sim a^{-n}, \;\; 0 \leq n \leq 6 \,, \qquad \rho_\phi \sim 1/a^6 \longrightarrow$$ **for steep pot.**

Predictive power of scalar fields

For a priori given cosmic history, it is always possible to construct a field potential such that it gives rise to the desired result. Thus the scalar field models should be judged by their generic features.

Scalar Field Dynamics in presence of background matter : Tracker or Freezing Models

$$H^2 = \frac{8\pi G}{3}(\rho_\phi + \rho_b)$$



Scaling Solution:

$$\frac{\rho_\phi}{\rho_b} = Const$$

$$V = V_0 e^\alpha \phi / M_p (steep - \alpha^2 > 3(1 + w_b))$$

$$\Omega_\phi = \frac{3(1 + w_b)}{\alpha^2} \lesssim 0.13 \rightarrow \alpha \gtrsim 5$$

MS, Curr Sci, V 97, 848 (2009) [arXiv:0904.3445]

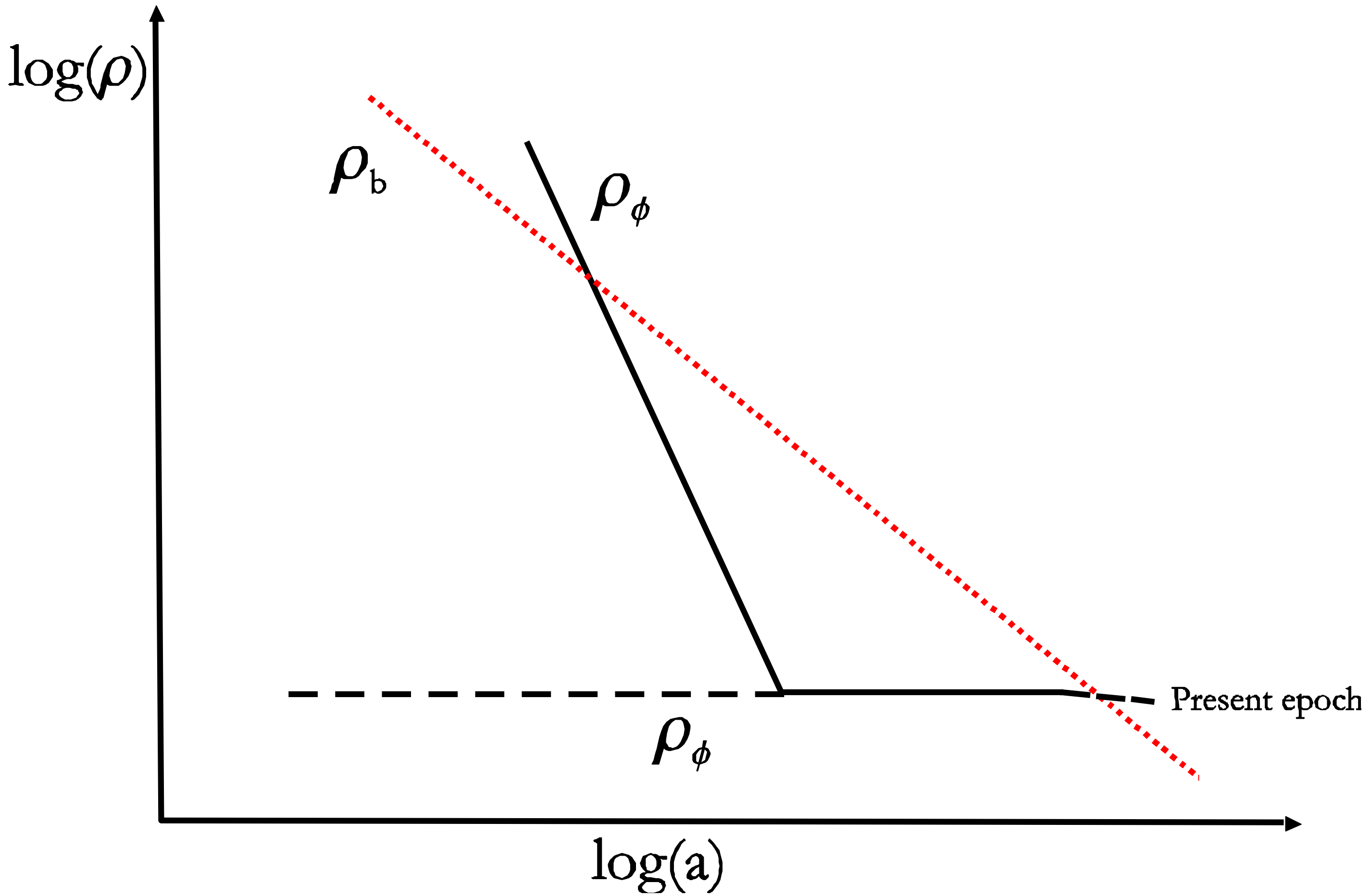
LATE TIME BEHAVIOR

Exit from scaling regime to late time acceleration.

- Potential gets shallow at late times: Inverse power law type of potentials.
- Potential acquires a particular power law type form

$$V(\phi) = V_0 \left(\frac{\phi}{M_p}\right)^{2p}, \ \langle w_\phi \rangle = \left\langle \frac{\frac{\dot{\phi}^2}{2} - V(\phi)}{\frac{\dot{\phi}^2}{2} + V(\phi)} \right\rangle = \frac{p-1}{p+1}$$

Absence of trackers: Thawing Models



Trackers and non-trackers

- **Trackers (Freezing models)**: **Potentials which mimic steep exponential at early epochs and become shallow at late times.**

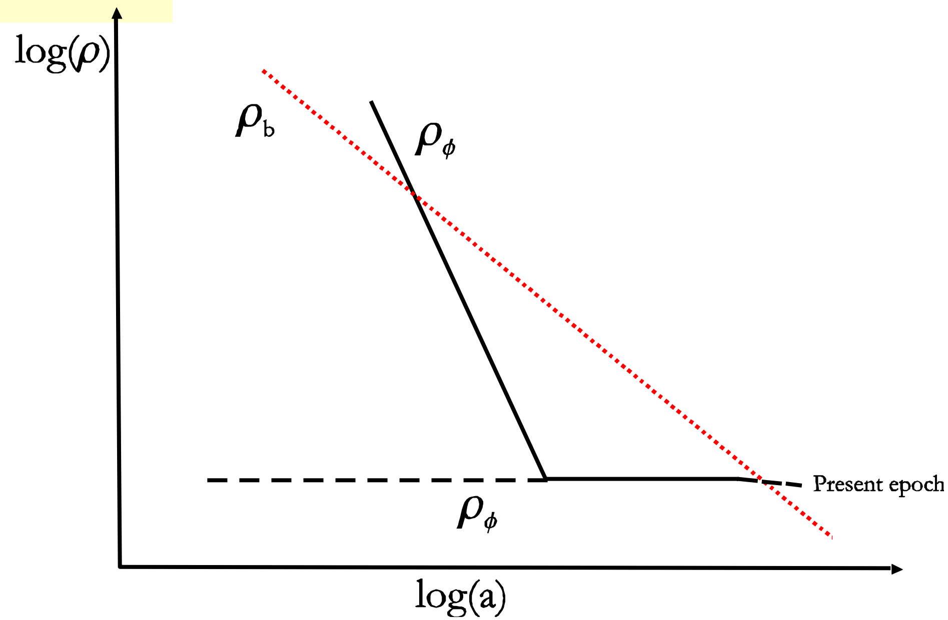
Thawing models: Most of the systems belong to this category

$$\mathcal{S} = \int -V(\phi)\sqrt{1-\partial^\mu\phi\partial_\mu\phi}\sqrt{-g}d^4x$$

$$w(\phi) = \dot{\phi}^2 - 1$$

$$V(\phi) \sim \frac{1}{\phi^2}$$, **Scaling Pot.**

Phantom field: $w(\phi) < -1$

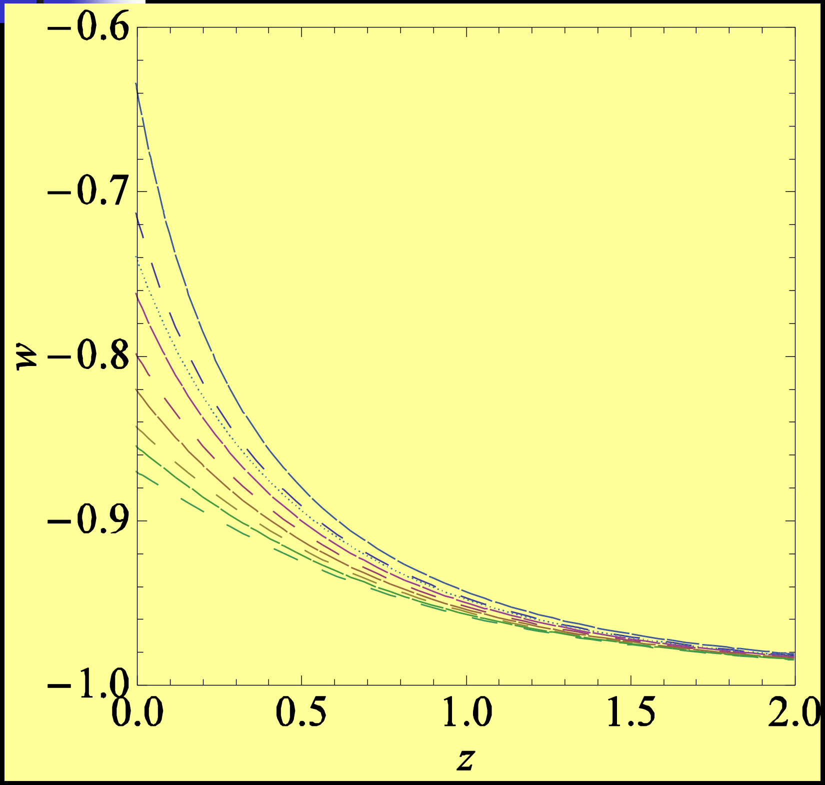


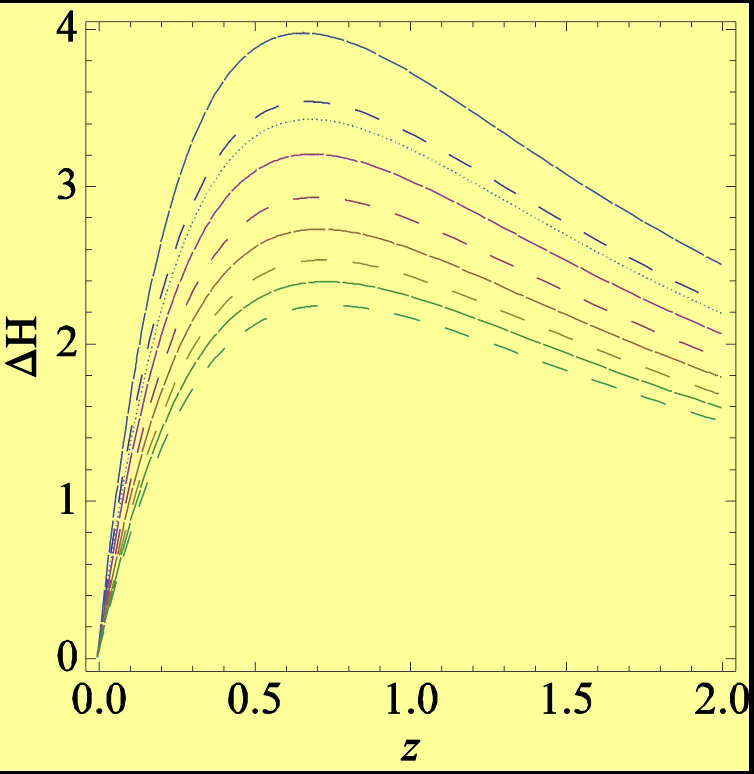
A. Sen, HEP0204:048,2002

N Mazumdar, S. Panda, A. Perez-Lorenzana, Phys.. B614,101(2001)

E.Copeland, M. Garousi, MS, S. Tsujikawa PRD71:043003,2005, S. Tsijikawa, MS, PRD71:043003,2005 .

Scalar Field Models: Observations

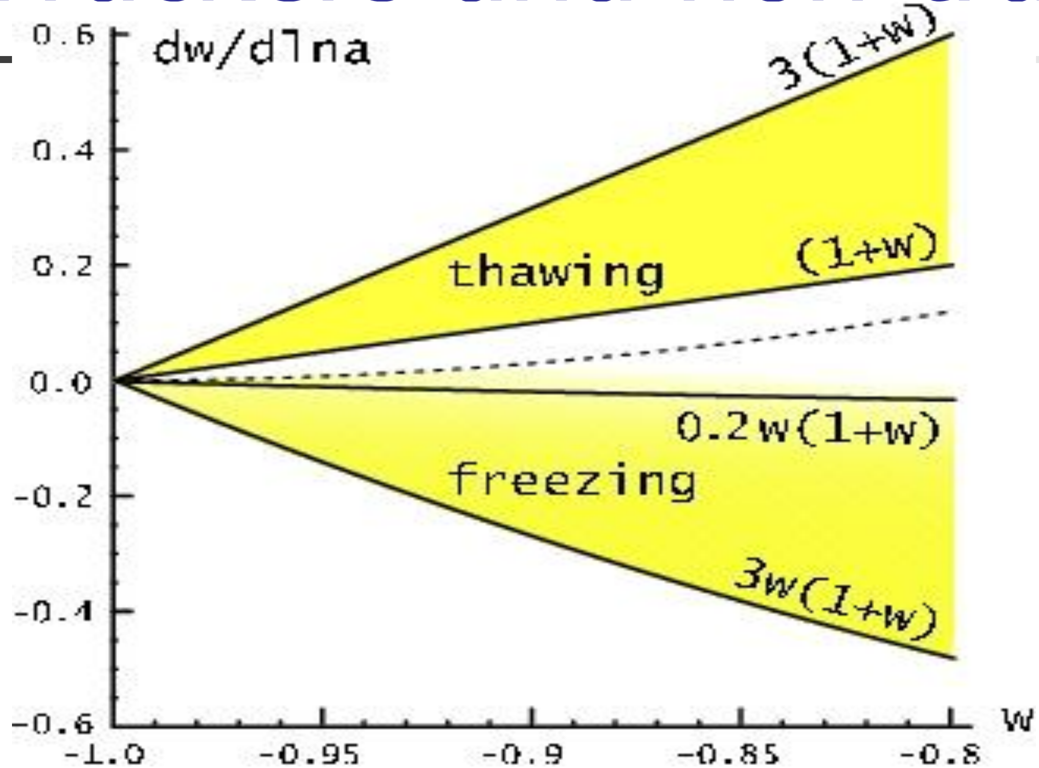




$$\Delta H = H_{\phi} - H_{\Lambda CDM}, \quad H_0 = 73 \pm 3 kms^{-1} MPc^{-1}$$

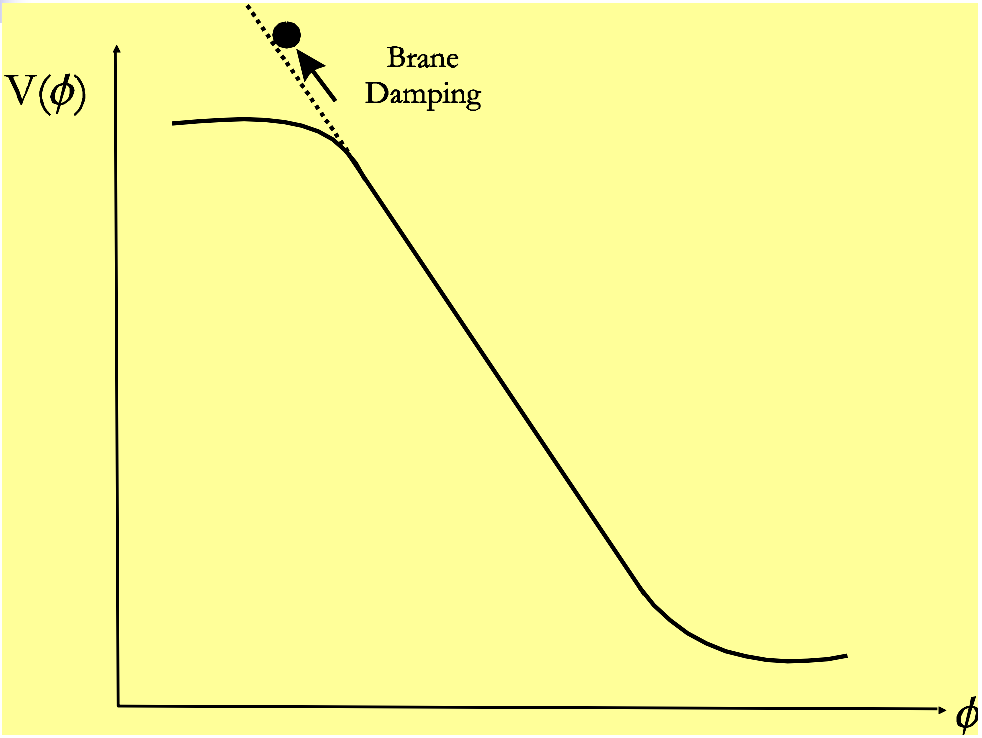
A.Ali, MS, A. A. Sen, Phys.Rev.D79:123501,2009
S.Sen, A. A. Sen, MS,arXiv: 0907.2814

Trackers and non-trackers



Caldwell, Eric V. R.R Linder, Phys.Rev.Lett.95:141301,2005

Quintessential inflation-Model Of reincarnation



$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda_B}\right)$$

$$\epsilon = \epsilon_{FRW}\frac{1 + V/\lambda_B}{(1 + V/2\lambda_B)^2}$$

$$\eta = \eta_{FRW}(1 + V/2\lambda_B)^{-1}$$

$$\epsilon \simeq 4\epsilon_{FRW}\,(V/\lambda_B)^{-1}$$

$$\eta \simeq 2\eta_{FRW}\,(V/\lambda_B)^{-1}$$

P.J.E. ***Peebles, A.Vilenkin, PRD59,06350(1999)***
MS, V. Sahni, Phys. Rev. D **70, 083513 (2004);**
Phys. Rev. D **65, 023518 (2002); MS, N. Dadhich, T.Shiromizu, PLB568, 118(2003).**

TRACKER POTENTIAL

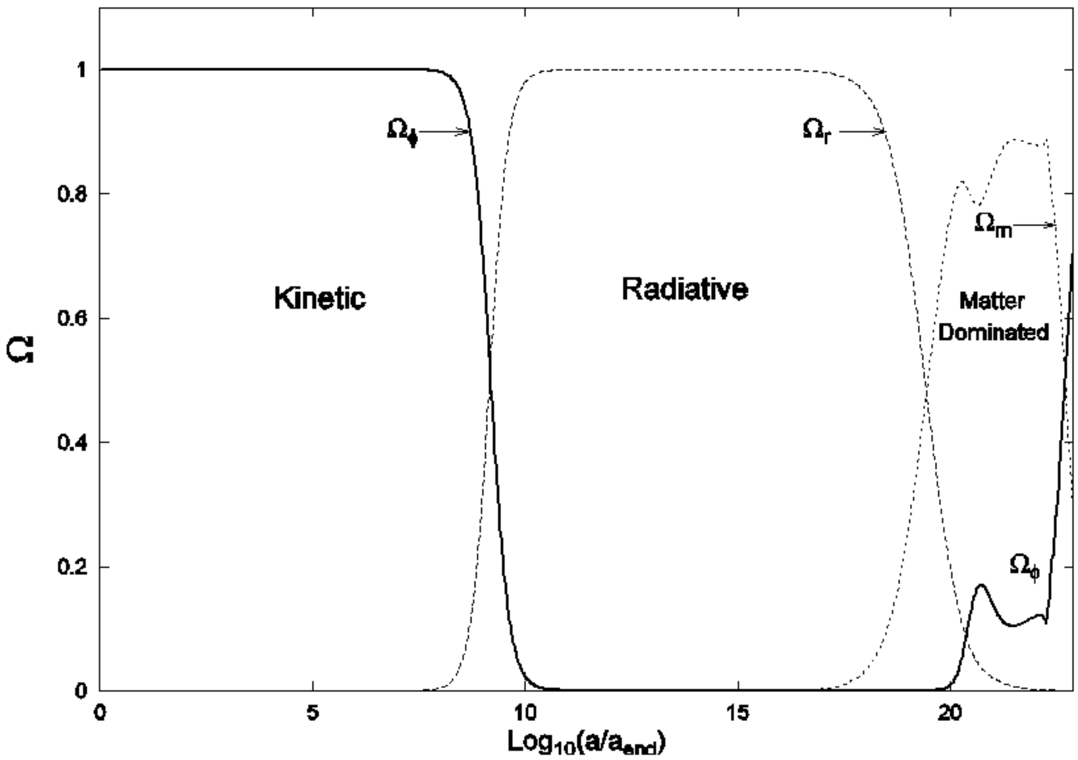
$$V(\phi) = V_0\Big[\cosh(\tilde{\alpha}\phi/M_P) - 1\Big]^p, \ 0 < p < 1/2$$

$$V(\phi) = \frac{V_0}{2^p} e^{\tilde{\alpha}p\phi/M_P}, \quad \tilde{\alpha}|\phi/M_P >> 1$$

$$V(\phi) = \frac{V_0}{2^p}\left[\frac{\tilde{\alpha}\phi}{M_P}\right]^{2p}, \quad \tilde{\alpha}|\phi/M_P << 1$$

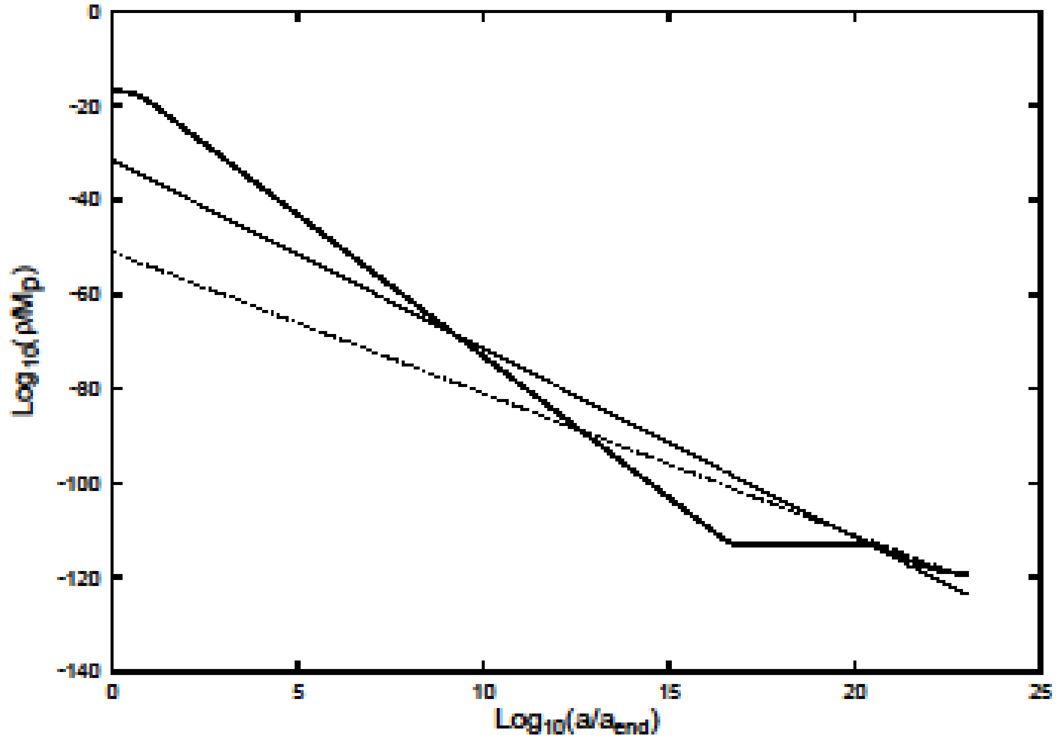
$$\alpha = \tilde{\alpha}p$$

Quintessential Inflation

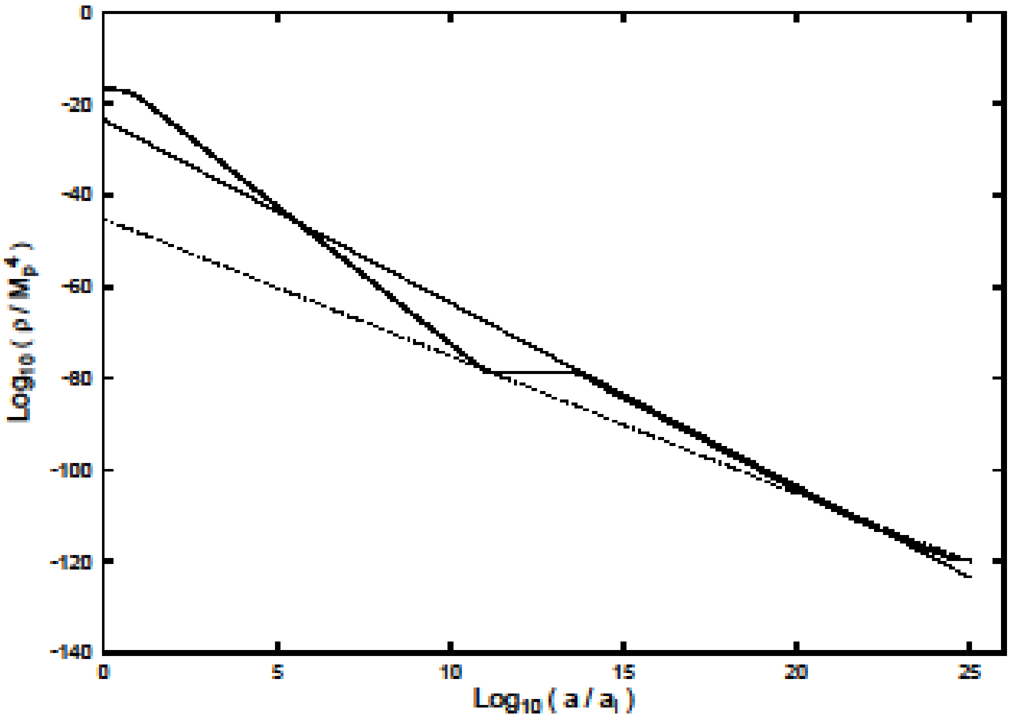


***V. Sahni, MS,T Souradeep, PRD* 65, 023518 (2002).**

Quintessential Inflation (**gravitational particle production**)



Quintessential Inflation (**instant preheating**)



OBSERVATIONAL CONSTRAINTS

$$n_S = 1 - \frac{4}{N}$$

$$r = \frac{24}{N}$$

$$N = 70, \quad n_S = 0.94, \quad r = 0.34$$

GAUSS BONNET IN THE BULK—efforts to rescue the model

$$\rho >> M_{GB}^4 \;\Rightarrow H^2 \simeq \rho^{2/3} \quad (GB)$$

$$M_{GB}^4 >> \rho >> \lambda \;\Rightarrow H^2 \simeq \rho^2 \quad (RS)$$

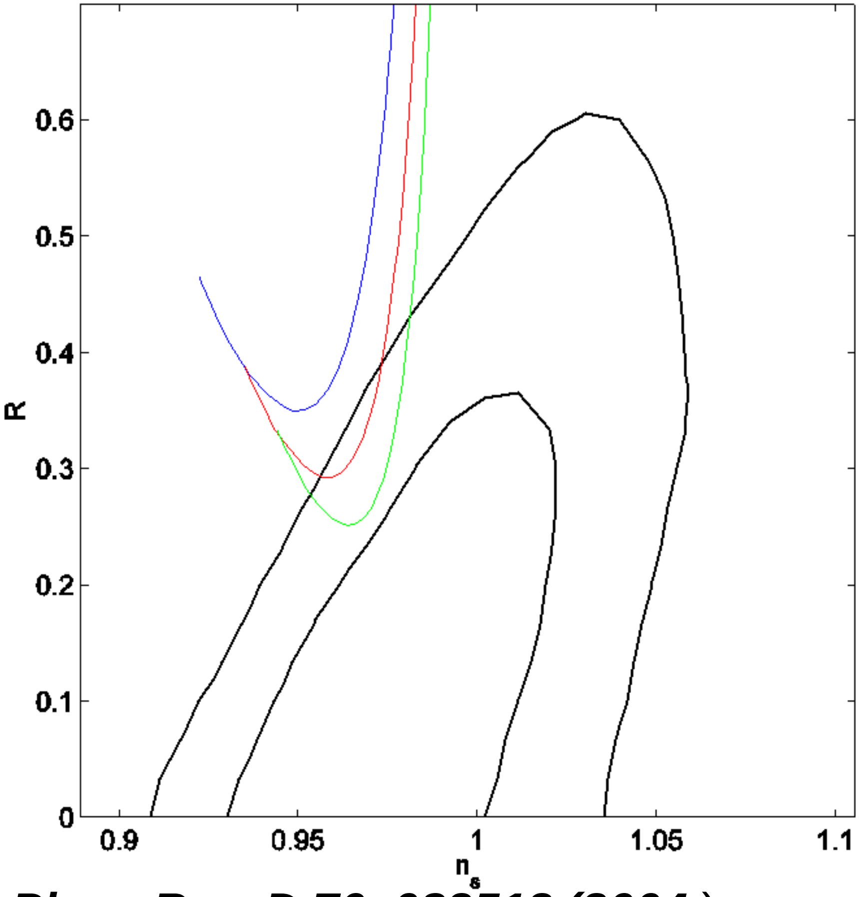
$$\rho << \lambda \;\Rightarrow H^2 \simeq \rho \qquad (GR)$$

J-F. Dufaux, J. Lidsey, Roy Maartens, MS, PR D70 (2004) 083525

Gauss-Bonnet term in the bulk

J-F. Dufaux, J. Lidsey, Roy Maartens, MS, PR D70 (2004) 083525

S. Tsujikawa, MS, Roy Maartens, PRD70 (2004) 063525



MS, V. Shani, Phys. Rev. D 70, 083513 (2004)

MODIFIED THEORIES OF GRAVITY
String curvature corrections to GR

- **Higher order curvature corrections to GR with fixed dilaton.**
- **S. Nojiri, S. Odintsov, M. Sasaki, hep-th/0504052.**
- **MS, A. Toporensky, Peter V. Tretjakov, Shinji Tsujikawa**
  Phys.Lett. B619:193,2005; G. Calcagni, Shinji Tsujikawa, MS, Class.Quant.Grav.22:3977-4006,2005 ; Sergei D. Odintsov, MS, Phys. Rev.D74:046004,2006

- **Curvature corrections to GR with dynamical dilaton.**
  E. Elizalde, S. Jhingan, S. Nojiri, S. D. Odintsov, MS, I. Thongkool,
  Eur. Phys.J.C53:447-457,2008
- **Gauss-Bonnet correction : Tracker solution.**
  S. Tsujikawa, MS, JCAP0701:006,2007

String inspired dark energy

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big[\frac{1}{2}R - (1/2)g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi) - f(\phi)R_{GB}^2\Big] + S_m$$

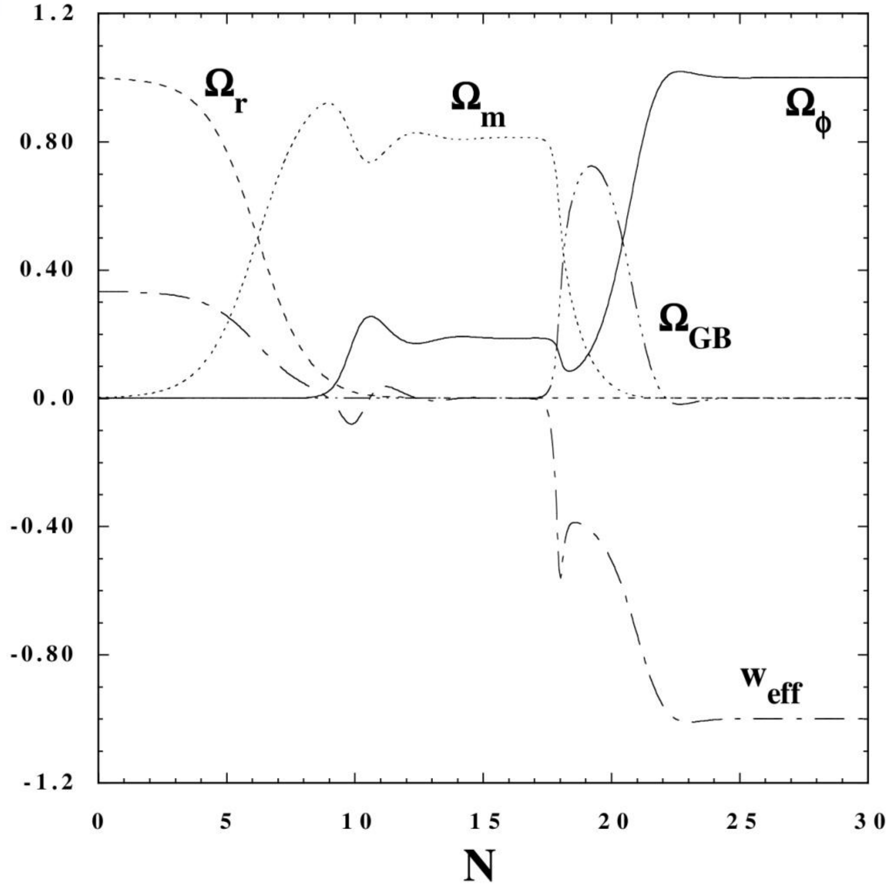
$$R_{GB}^2{}^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$$

$$\rho_\phi/\rho_B = C \rightarrow \quad scaling\ solution$$

$$V(\phi) \sim e^{(\alpha\phi)}, \quad f(\phi) \sim e^{-(\mu\phi)}, \ \alpha = \mu$$

$$\alpha \neq \mu \rightarrow \ de - Sitter\ solution$$

String Inspired dark energy



S. Tsujikawa, MS, JCAP0701:006,2007;
T.Koivisto, D. Mota, PLB644, 104(07)

f(R) GRAVITY- REFERENCES

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A.~D.~Dolgov and M.~Kawasaki, Phys.Lett.B {573}, 1 (2003) .

S. Nojiri and S. D. Odintsov, Phys. Rev. D 68, 123512 (2003) …………..

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f(R) theories of gravity

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{f(R)}{16\pi G} + \mathcal{L}_m \right]$$

$$f' R_{\mu\nu} - \nabla_\mu \nabla_\nu f' + \left(\Box f' - \frac{1}{2} f \right) g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$H^2 = \frac{8\pi G}{3f'} \rho_R \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{f'} (\rho_R + 3P_R)$$

$$\rho_R = \frac{Rf' - f}{2} - 3H\dot{R}f''; \quad P_R = 2H\dot{R}f'' + \ddot{R}f'' + \frac{1}{2}(f - f'R) + f'''\dot{R}^2$$

$$f(R) = R - \alpha_n / R^n \quad w_R = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -1 + \frac{2(n+2)}{3(2n+1)(n+1)}$$

f(R) theories

$$\Box f_{,R} = \frac{1}{3}(2f - f_{,R}R) + \frac{8\pi G}{3}T$$

$$\varphi = f_{,R}(R) \Rightarrow \ \Box\varphi = \frac{8\pi G}{3}T + \frac{dV}{d\varphi}$$

$$\frac{dV}{dR} = \frac{dV}{d\varphi}\frac{d\varphi}{dR} = \frac{1}{3}(2f - f_{,R}R)\, f''$$

de Sitter solution: R=Const

$$f_{,R}R - 2f(R) = 0$$

f(R) Gravity as scalar tensor theory

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[f(\chi) + f_{,\chi}\,(R-\chi)\Big] + \int d^4x\sqrt{-g}L_m(g_{\mu\nu})$$

$$f_{,\chi\chi}(\chi))\,(R-\chi) = 0 \qquad \Rightarrow f(R) \quad action$$

$$\varphi \equiv f_{,\chi}: \qquad S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[\varphi R - U(\varphi\Big] + \int d^4x\sqrt{-g}L_m(g_{\mu,\nu})$$

$$U = \chi(\varphi)\varphi - f(\chi(\varphi))$$

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[\varphi R - \frac{w_{BD}}{2\varphi}(\Delta\varphi)^2 - U(\varphi\Big] + \int d^4x\sqrt{-g}L_m(g_{\mu,\nu})$$

Local gravity constraints: Screening Mechanism

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[\varphi R-U(\varphi)\Big]+\int d^4x\sqrt{-g}L_m(g_{\mu,\nu})$$

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\Big[\varphi R-\frac{w_{BD}}{2\varphi}(\Delta\varphi)^2-U(\varphi)\Big]+\int d^4x\sqrt{-g}L_m(g_{\mu,\nu})$$

$$f(R) \quad \Rightarrow \quad w_{BD}=0$$

$$\varphi \ - light \ \ (m_\varphi << m_{AU} > \quad m_\varphi r_{AU} << 1) \Rightarrow \quad \gamma=\frac{1+w_{BD}}{2+w_{BD}}$$

$$Observation: \quad |\gamma-1|\simeq 10^{-5}$$

$$r_{AU}\simeq 10^8 km, \quad M_{AU}\simeq 10^{-27}GeV$$

$$\gamma=\frac{1-e^{-mr}/(2w_{BD}+3)}{1+e^{-mr}/(2w_{BD}+3)}; \qquad m_\varphi >> m_{AU}, \quad \Rightarrow \gamma\to 1$$

$\varphi \ - \ Quintessence \Rightarrow m_\varphi \sim H_0 \ \Rightarrow$ **Chameleon Field**

Chameleon Screening

Conformal Transformation: $\tilde{g}_{\mu\nu} = \varphi g_{\mu\nu}$ $\phi = \sqrt{3/2}\ln\varphi$

$$S_E = \int d^4x\sqrt{-g}\Big[\frac{\tilde{R}}{2} - \frac{1}{2}(\tilde{\nabla}\phi)^2 - V(\phi)\Big] + \int d^4x\sqrt{-g}L_m(e^{2\beta\phi}\tilde{g}_{\mu\nu})$$

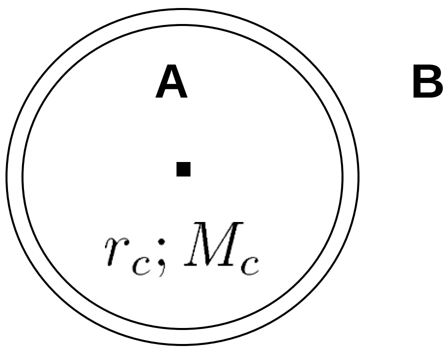
$$V(\phi) = \frac{f_{,R}R - f}{2f_{,R}^2}; \qquad \beta = -\frac{1}{6}$$

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - \frac{dV_{eff}}{d\phi} = 0; \qquad V_{eff} = V(\phi) + e^{\beta\phi}\rho$$

Chameleon Screening

Inside body: $\phi_{min} = \phi_A; \quad m_A^2 = V_{eff,\phi\phi}(\phi_A)$

Out side: $\phi_{min} = \phi_B; \quad m_B^2 = V_{eff,\phi\phi}(\phi_B)$



$$\phi(r) \simeq \phi_B - \frac{6\beta\epsilon_{th}}{\kappa}\frac{GM_c}{r}e^{-m_B(r-r_c)}, \quad m_A r >> 1$$

$$\epsilon_{th} = 3\frac{\Delta r}{r_c} = \frac{\kappa(\phi_B - \phi_A)}{6\beta\Phi_c}; \gamma = \frac{1+6\beta^2\epsilon_{th}}{1-6\beta^2\epsilon_{th}\left(1-r/r_c\right)} \quad \Rightarrow \quad \epsilon_{th} \lesssim 10^{-5}$$

$$\frac{\Delta r}{r_c} \equiv \frac{r_c - r}{r_c}$$

VIABLE f(R) MODELS

Stability conditions: $$G_{eff} = \frac{G}{f'} \ , \qquad G'_{eff} = -\frac{G f''}{f'^2} \ ,$$

$$R^{(0)}, \rho^{(0} : \quad M^2 \simeq f_{,R}/(3f_{,RR})|_{R=R^{(0)}}$$

Stable de Sitter: $$Rf_{,R} = 2f \quad : \ 0 < Rf_{,RR}/f_{,R} < 1$$

Local gravity constraints: $$f(R) \to R - 2\Lambda \quad for \quad R >> R_c$$

$$\Delta = \alpha R_c \left(\left[1 + \frac{R^2}{R_c^2} \right]^{-n} - 1 \right); \quad \alpha, R_c, n > 0$$

Starobinsky Model

$$f(R) = R + \Delta(R)$$

$$\varphi = \frac{\partial f}{\partial R} = 1 + \Delta_{,R}.$$

$$\Delta = \alpha R_c \left(\left[1 + \frac{R^2}{R_c^2} \right]^{-n} - 1 \right),$$ n>0

$$\varphi(R) = 1 - \frac{2n\alpha R}{R_0 (1 + \frac{R^2}{R_0^2})^{n+1}}, \qquad \varphi \to 1, \;\; R \to \infty$$

A.Starobinsky, JETP Lett.86,157(2007); W. Hu, I. Sawicki, PRD76,64004(07); S.A.Appleby, R.A., Battye, PLB654,7(2007);
A. Dev, D. Jain, S. Jhingan, S. Nojiri, M.S, I. Thongkool, Phys.Rev.D78:083515,2008

LCAL GRAVITY CONSTRAINTS

$$\varphi = 1 + \Delta(R)$$

$$\Delta_{,R} \simeq \alpha n \left(\frac{R_c}{R}\right)^{2n+1} \lesssim 10^{-15}$$

$$n > 1 \qquad \left(\rho_c \simeq 10^{-29} g/cm^3, \quad \rho_g \simeq 10^{-24} g/cm^3\right)$$

Local gravity constraints are violated

Model is not distinguished from cosmological constant

Capozziello,S.Tsujikawa, PRD 77, (2008) 107501;
I. Thongkool, MS.
R. Gannouji, S. Jhingan PR.D80:043523,2009

Observational signatures

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{eff}\rho_m\delta_m = 0 \qquad k/(aH) >> 1$$

$$G_{eff} = \frac{G}{f_R}\frac{1+4mk^2/(a^2R)}{1+4mk^2/(a^2R)}$$

During matter era:

$$z < z_k, \quad G_{eff} \simeq G, \quad \delta_m \sim t^{2/3}$$

$$z > z_k, \quad G_{eff} \simeq 4G/3, \quad \delta_m \sim t^{(\sqrt{33}-1)/6}$$

$$\delta n_s = \frac{\sqrt{33}-5}{2(3n+2)} \lesssim 0.05 \quad \to \quad n > 2$$

Growth index:

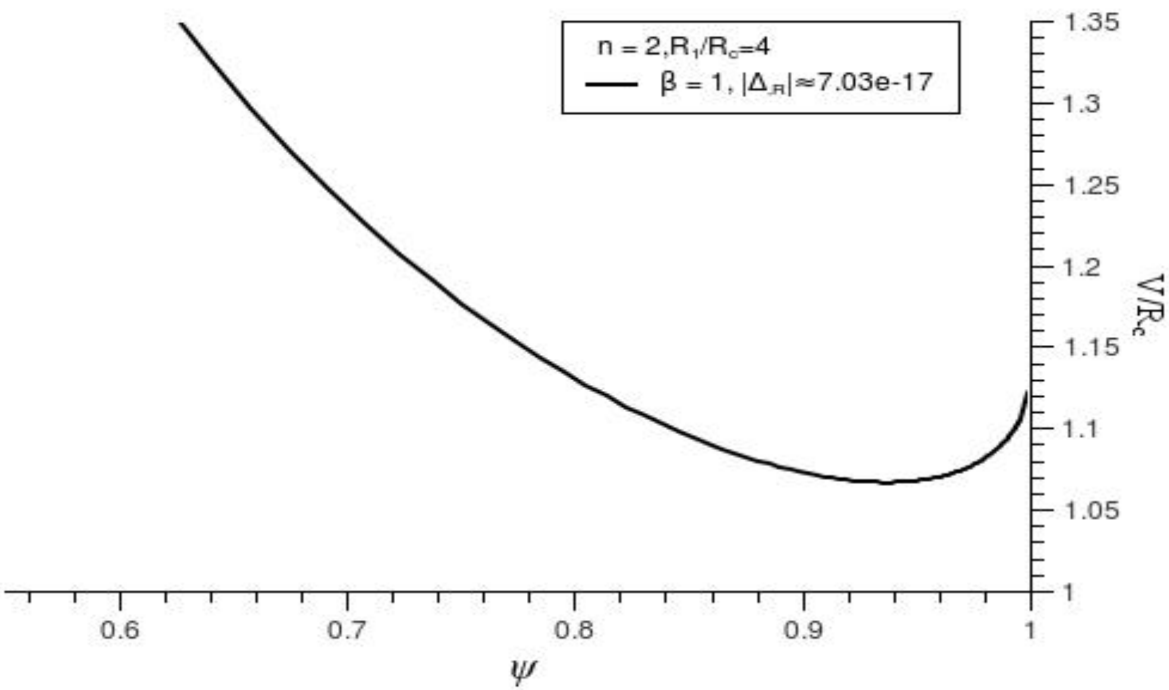
$$\frac{\dot{\delta}_m}{H\delta_m} = (f_R\Omega_m)^\gamma \qquad 0.4 \lesssim \gamma \lesssim 0.55$$

Curvature Singularity

$$V = \frac{R\Delta_{,R} - \Delta}{2(1+\Delta_{,R})^2}$$

A.V. Frolov, Phys.Rev.Lett.101:061103,2008

$$\varphi(R) = 1 - \frac{2n\lambda R}{R_0(1+\frac{R^2}{R_0^2})^{n+1}}, \quad \varphi \to 1, \; R \to \infty$$



Extended Starobinsky Model

$$\Delta = \alpha\beta R_c\left(\left[1+\left(\frac{R}{R_c}\right)^n\right]^{-1/\beta}-1\right)$$

$V_{min}/R_c \simeq \mathcal{O}(1)$ at $R \simeq R_1$

$H_{pb} \simeq \frac{\alpha\beta}{2}$

$2/\beta \gtrsim 1$

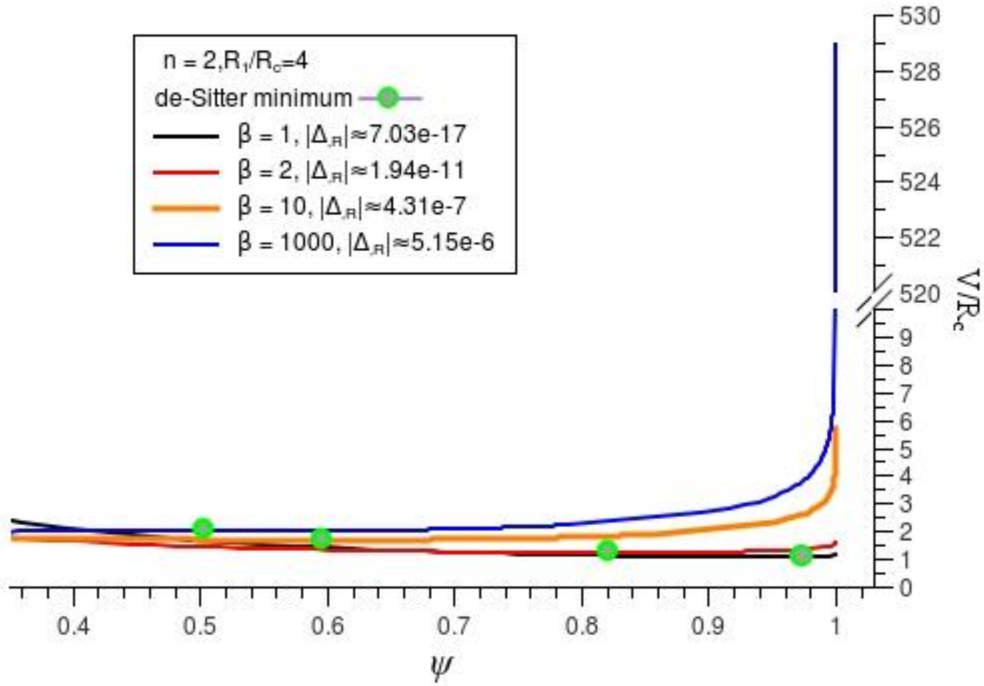
V.Miranda et al

PRL.102:221101,2009

I. Thongkool, MS.

R. Gannouji, S. Jhingan

PR.D80:043523,2009



Problems of f(R)

Scalaron Mass:

$$\mathrm{m}_\phi \sim \sqrt{R_c}\left(\frac{R}{R_c}\right)^{n+1} \quad (0)$$

$$\frac{\sqrt{R_c}}{M_p} \sim \frac{H_0}{M_p} = \frac{2.13 \times 0.74 \times 10^{-42} GeV}{1.22 \times 10^{19} GeV} = 1.292 \times 10^{-61},$$

$$\rho/\rho_c \simeq 10^{43} \qquad \phi \simeq 1 - \left(\frac{\rho}{\rho_c}\right)^{2n+1}$$

$$\phi \approx \begin{cases} 1 - \mathcal{O}\left(10^{-129}\right) & n = 1, \\ 1 - \mathcal{O}\left(10^{-217}\right) & n = 2. \end{cases}$$

Fine tuning problem

$$\delta\phi(r_*) = \delta\phi_{(0)} \frac{\sinh(m_\phi r_*)}{m_\phi r_*}$$

$$m_\phi r_* = r_*/\lambda_c \sim \left(\frac{\rho}{\rho_\Lambda}\right)^{n+1/2} \sim 10^{43(n+1/2)}$$

$$\delta\phi(r_*) \approx \delta\phi(0) \frac{\exp[10^{63}]}{10^{63}}, \; m_\phi r_* \simeq 10^{63}, \quad n = 1$$

Thongkool, MS, S. Rai Choudhury,
arXiv:0908.1663

CURING THE SINGULARITY ?

Countering Term: $\frac{\mu}{R_c} R^2$

$$m_\phi^2 \simeq \frac{1}{3} (\Delta_{,RR}) = \frac{R_c}{6\mu}$$

$$\delta\phi(r_*) = \delta\phi_{(0)} \frac{\sinh(m_\phi r_*)}{m_\phi r_*}$$

$$m_\phi r_* \sim 1 \Rightarrow \mu \simeq 10^{-43}$$

Inflation: $m_\phi = 10^{-6} M_P$

Thongkool, MS, S. Rai Choudhury, arXiv:0908.1663

Thongkool, MS, R. Gannouji, S. Jhingan, Phys.Rev.D80:043523,2009 ;
S A. Appleby, R. A. Battye, A. Starobinsky, arXiv:0909.1737

Classicality of scalarons

$$\Delta_{1-loop}(\phi) = \frac{m_\phi^4(\phi)}{64\pi^2} \ln\left(\frac{m_\phi^2(\phi)}{\mu_0^2}\right)$$

$$m_\phi(\rho) \leq 0.0073 \left(\frac{\beta\rho}{\rho_{\rm lab}}\right)^{1/3} = 0.0542 \left(\frac{\rho}{\rho_{\rm lab}}\right)^{1/3} \quad \text{eV},$$

$$m_\phi(\rho) \simeq \sqrt{R_c} \left(\frac{\rho}{R_c}\right)^{(n+1)}$$

MODIFIED GRAVITY– a la Galileon

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{R}{2} + c_i\ L^{(i)}\right) + \mathcal{S}_m[\psi_m, e^{2\beta\pi}g_{\mu\nu}]$$

$$L^{(1)} = \pi$$

$$L^{(2)} = -\frac{1}{2}(\nabla\pi)^2 \equiv -\frac{1}{2}\pi_{;\mu}\pi^{;\mu}$$

$$L^{(3)} = -\frac{1}{2}(\nabla\pi)^2\Box\pi$$

$$L^{(4)} = -\frac{1}{2}(\nabla\pi)^2\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu} + \pi^{;\mu}\pi^{;\mu}G_{\mu\nu}\right] + (\Box\pi)\pi_{;\mu}\pi_{;\nu}\pi^{;\mu\nu} - \pi_{;\mu}\pi^{;\mu\nu}\pi_{;\nu\rho}\pi^{;\rho}$$

$$L^{(5)} = -\frac{1}{2}(\nabla\pi)^2\left[(\Box\pi)^3 - 3\,(\Box\pi)\,(\pi_{;\mu\nu}\,\pi^{;\mu\nu}) + 2\left(\pi_{;\mu}{}^{\nu}\,\pi_{;\nu}{}^{\rho}\,\pi_{;\rho}{}^{\mu}\right) - 3\left(\pi_{;\mu}\,\pi_{;\nu}\,\pi_{;\rho\sigma}\,R^{\mu\rho\nu\sigma}\right)\right] + \ldots$$

$$c_i\mathcal{E}^{(i)} = -\beta T^{(m)}$$

$$G_{\mu\nu} = T^{(m)}_{\mu\nu} + c_iT^{(i)}_{\mu\nu}$$

$$\pi(x) \to \pi(x) + b_\mu x^\mu + c$$

A. Nicolis,R. Rattazi, E. Trincherini, hep-th/0811.2197

Vainstein Effect- dynamical suppression of fifth force

$$T^{(m)} = -M\delta(r); \qquad \frac{1}{r^2}\frac{d}{dr}\left(r^3\left[c_2(\pi'/r) + 2c_3(\pi'/r)^2\right]\right) = \beta M\delta(r)$$

$$c_2\left(\frac{\pi'(r)}{r}\right) + 2c_3\left(\frac{\pi'(r)}{r}\right)^2 = \beta\frac{r_s}{r^3}$$

Short distances: $\pi' = \left(\frac{r_S\beta}{2c_3}\right)^{1/2}\frac{1}{r^{1/2}}$

$$\frac{F_\pi}{F_{\rm grav}} = \left(\frac{r}{r_V}\right)^{3/2}, \quad r << r_V, \quad r_V = \left(\frac{2c_3 r_S}{\beta}\right)^{1/3}$$

Large distances: $\pi' = \frac{r_S\beta}{c_2}\frac{1}{r^2} \quad \Rightarrow \quad \frac{F_\pi}{F_{\rm grav}} \sim \frac{\beta}{c_2}$

COSMOLOGICAL DYNAMICS- Background evolution

$$3H^2 = \rho_m + \rho_r + \frac{c_2}{2}\dot{\pi}^2 - 3c_3 H\dot{\pi}^3 + \frac{45}{2}c_4 H^2\dot{\pi}^4$$

$$2\dot{H} + 3H^2 = -\frac{1}{3}\rho_r - \frac{c_2}{2}\dot{\pi}^2 - c_3\dot{\pi}^2\ddot{\pi} + \frac{3}{2}c_4\dot{\pi}^3\left(3H^2\dot{\pi} + 2\dot{H}\dot{\pi} + 8H\ddot{\pi}\right)$$

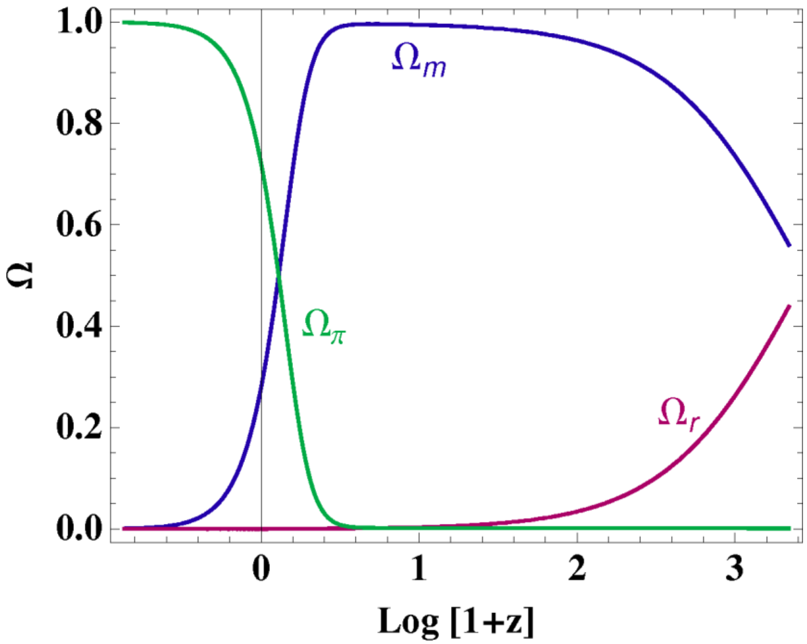
$$\beta\rho_m = -c_2\left(3H\dot{\pi} + \ddot{\pi}\right) + 3c_3\dot{\pi}\left(3H^2\dot{\pi} + \dot{H}\dot{\pi} + 2H\ddot{\pi}\right) -18c_4 H\dot{\pi}^2\left(3H^2\dot{\pi} + 2\dot{H}\dot{\pi} + 3H\ddot{\pi}\right),$$

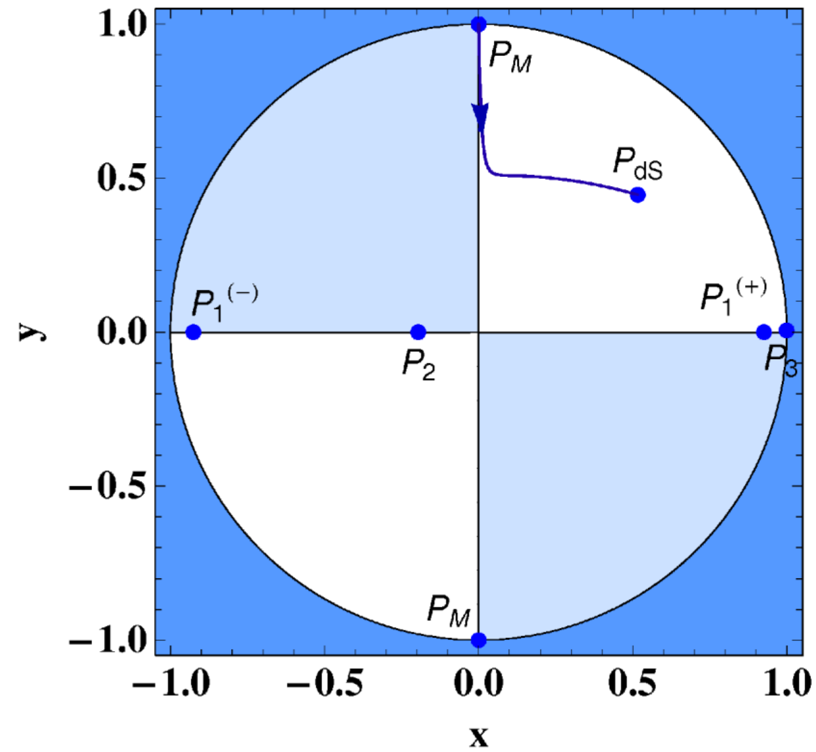
Self accelerating solution:

$$c_3^2 - 8c_2c_4 > 0,\ \ A > 0\ ; \quad A = \left(c_3^2 - 12c_2c_4 \pm c_3\sqrt{c_3^2 - 8c_2c_4}\right)/c_4$$

Stability: $(c_2, c_4, \beta) > 0, \quad c_3 > \sqrt{8c_2c_4}$

COSMOLOGICAL DYNAMICS-Attractor solution





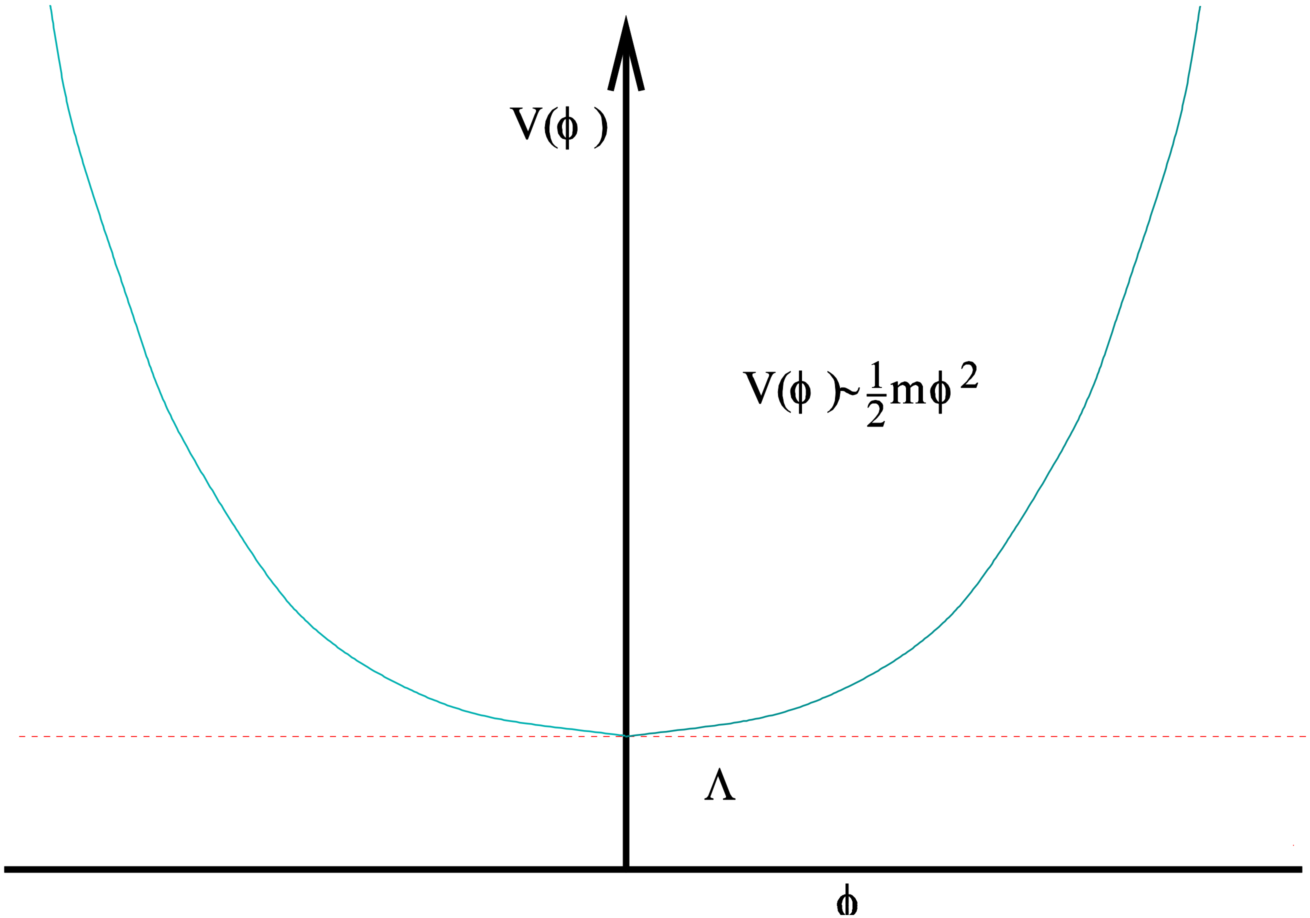
$$P_{dS} : (x, y) = \left(\sqrt{\frac{48}{A}}, \frac{\tilde{c}_3 + \sqrt{\tilde{c}_3^2 - 8c_2\tilde{c}_4}}{12\tilde{c}_4}\right)$$

x=/H, y= H/H_0^2

R. Gannouji, MS, PRD82, 024011, 2010

A. Ali, R. Gannouji, MS, Phys. Rev. D 82, 103015(2010)

BEST MODEL OF INFLATION AND DARK ENERGY?



NATURE OF DARK ENERGY

IT COULD BE ANYTHING

OR

IT COULD BE NOTHING