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YUKAWA INSTITUTE FOR
THEORETICAL PHYSICS

KMI theory seminar
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Open Inflation
in the String Landscape

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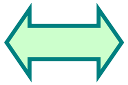
D. Yamauchi, A. Linde, A. Naruko, T. Tanaka & MS,
PRD 84, 043513 (2011) [arXiv:1105.2674 [hep-th]]

K. Sugimura, D. Yamauchi & MS, arXiv:1110.4773 [gr-qc]

1. String theory landscape

Lerche, Lust & Schellekens ('87), Bousso & Pochinski ('00),
Susskind, Douglas, KKLT ('03), ...

- There are ~ 10^{500} vacua in string theory
  - vacuum energy ρ_V may be positive or negative
  - typical energy scale ~ $M_P{}^4$
  - some of them have $\rho_V \ll M_P{}^4$

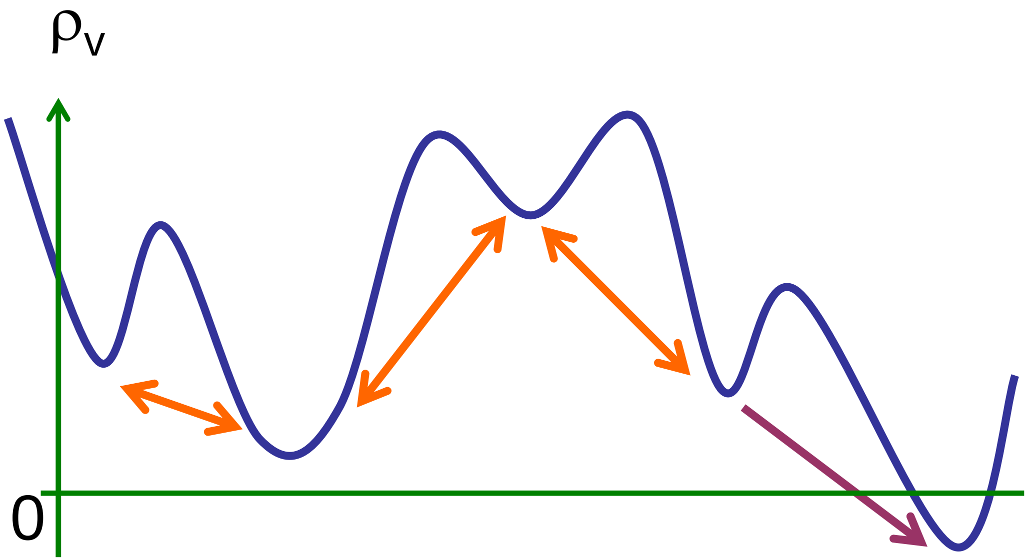


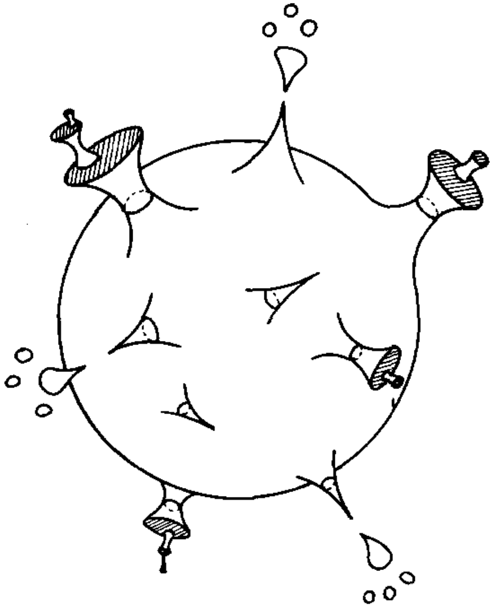
which ?

Is there any way to know what kind of landscape we live in?

Or at least to know what kind of neighborhood we live in?

- A universe jumps around in the landscape by quantum tunneling
  - it can go up to a vacuum with larger ρ_v
  - if it tunnels to a vacuum with negative ρ_v, it collapses within $t \sim M_P/|\rho_v|^{1/2}$.
  - so we may focus on vacua with positive ρ_v: dS vacua





Sato, MS, Kodama & Maeda ('81)

2. Anthropic landscape

- Not all of dS vacua are habitable.

"anthropic" landscape Susskind ('03)

- A universe jumps around in the landscape and settles down to a final vacuum with $\rho_{v,f} \sim M_P^2 H_0^2 \sim (10^{-3}\mathrm{eV})^4$.
  - $\rho_{v,f}$ must not be larger than this value in order to account for the formation of stars and galaxies.

- Just before it has arrived the final vacuum (=present universe), it must have gone through an era of (slow-roll) inflation and reheating, to create "matter and radiation."

$\rho_{vac} \to \rho_{matter}$ ~ T^4: birth of Hot Bigbang Universe

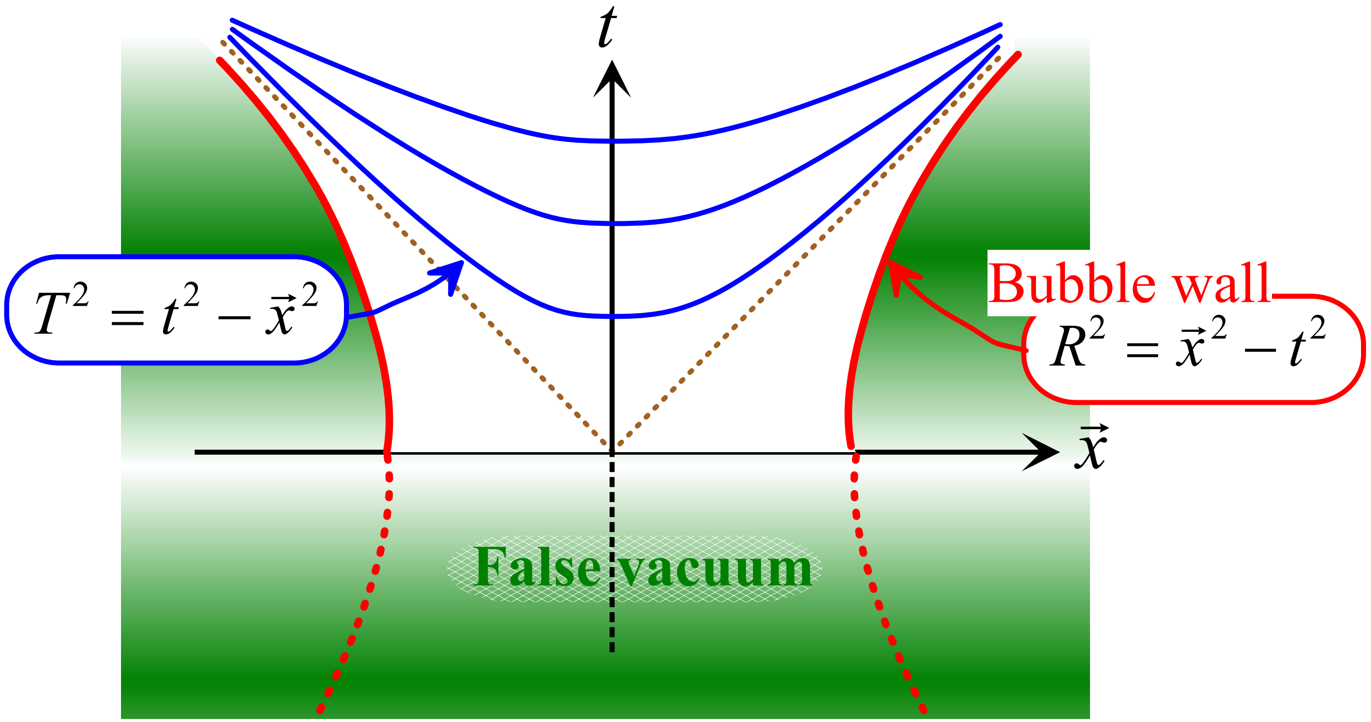
- Most plausible state of the universe before inflation is a de Sitter vacuum with $\rho_V \sim M_P^4$.

false vacuum decay via O(4) symmetric (CDL) instanton

Coleman & De Luccia ('80)

$$O(4) \rightarrow O(3,1)$$

inside bubble is an open universe



- Natural outcome would be a universe with $\Omega_0 << 1$.
  - “empty” universe: no matter, no life
- Anthropic principle suggests that # of e-folds of inflation inside the bubble should be ~ 50 – 60 : just enough to make the universe habitable.

Garriga, Tanaka & Vilenkin (‘98), Freivogel et al. (‘04)

- Observational data excluded open universe with $\Omega_0 < 1$.
- Nevertheless, the universe may be slightly open:

$$1-\Omega_0 = 10^{-2} \sim 10^{-3}$$

may be confirmed by Planck+BAO

Colombo et al. (‘09)

What if $1-\Omega_0$ is actually confirmed to be non-zero:$\sim 10^{-2}$ -10^{-3}?

revisit open inflation!

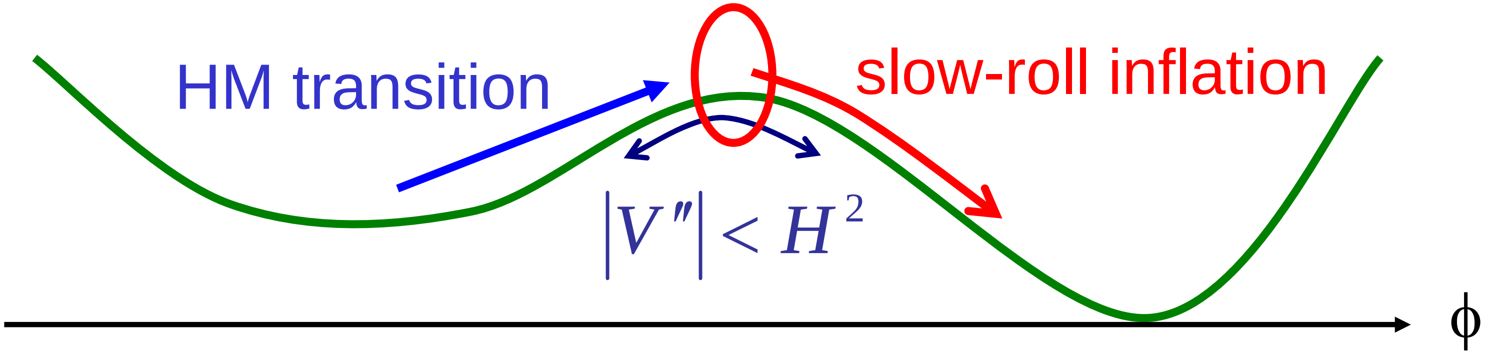
see if we can say anything about Landscape

3. Open inflation in the landscape

– constraints from scalar-type perturbations –

- Simplest polynomial potential = Hawking-Moss model
  - ϕ^4 potential: $V = \frac{m^2}{2}\phi^2 - \frac{\nu}{3}\phi^3 + \frac{\lambda}{4}\phi^4$
  - tunneling to a potential maximum ~ stochastic inflation

Hawking & Moss ('82) Starobinsky ('84)



- too large fluctuations of ϕ unless # of e-folds >> 60

Linde ('95)

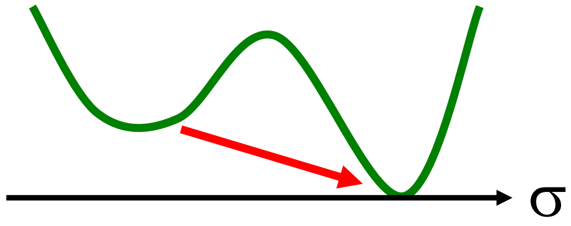
➢ Two- (multi-)field model: “quasi-open inflation”

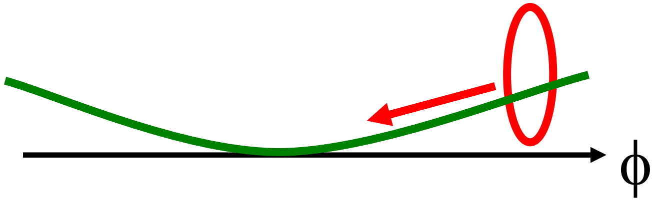
Linde, Linde & Mezhlumian (‘95)

- a “heavy” field σ undergoes false vacuum decay
- another “light” field ϕ starts rolling after fv decay

$$V\left(\phi,\sigma\right)=V_{\sigma}\left(\sigma\right)+\frac{m_{\phi}^{2}}{2}\phi^{2}$$

~ perhaps naturally/easily realized in the landscape





- If inflation is short, ⇔ $N=\frac{\kappa\phi^{2}}{4}\lesssim 60$

too large perturbations from supercurvature mode of ϕ

$$p^{2}=p_{sc}^{2}\approx-|K|;\quad\left[\overset{(3)}{\Delta}_{K}+p^{2}+|K|\right]Y_{plm}(r,\Omega)=0$$

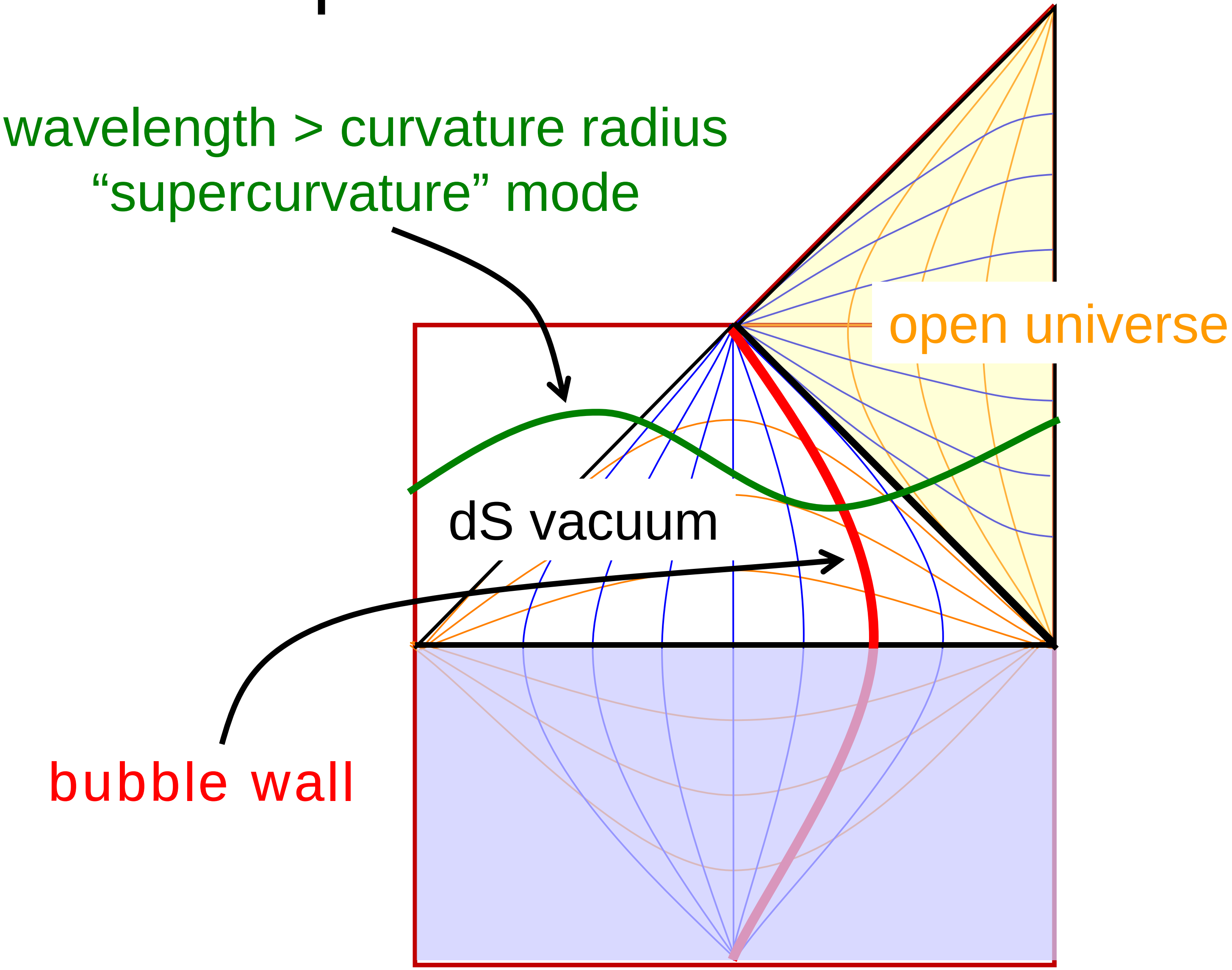
$$\delta\sigma_{sc}\sim\frac{H_{F}}{2\pi}\ ?\ \frac{H_{R}}{2\pi}$$

H_F : Hubble at false vacuum

H_R : Hubble after fv decay

MS & Tanaka (‘96)

creation of open universe & supercurvature mode



wavelength > curvature radius
“supercurvature” mode

open universe

dS vacuum

bubble wall

➢ To summarize:

The models of the tunneling in the landscape with the simplest potentials such as

$$V = \frac{m^2}{2}\phi^2 - \frac{\nu}{3}\phi^3 + \frac{\lambda}{4}\phi^4 \qquad \text{or} \qquad V(\phi,\sigma) = V_\sigma(\sigma) + \frac{m_\phi^2}{2}\phi^2$$

are ruled out by observations, assuming that inflation after the tunneling is short, N ~ 60.

(NB. a slightly more complicated two-field model may work. Sugimura, Yamauchi & MS ('11))

The same models are just fine if N >> 60 (if Ω_0=1)
This means that we are testing the models of the landscape in combination with the probability measures, which may or may not predict that the last stage of inflation is short.

➢ How about general single field models?

- if $\rho_{fv} \sim M_P^4$, the universe will most likely tunnel to a point where the energy scale is still very high unless potential is fine-tuned. Linde, MS & Tanaka ('99)

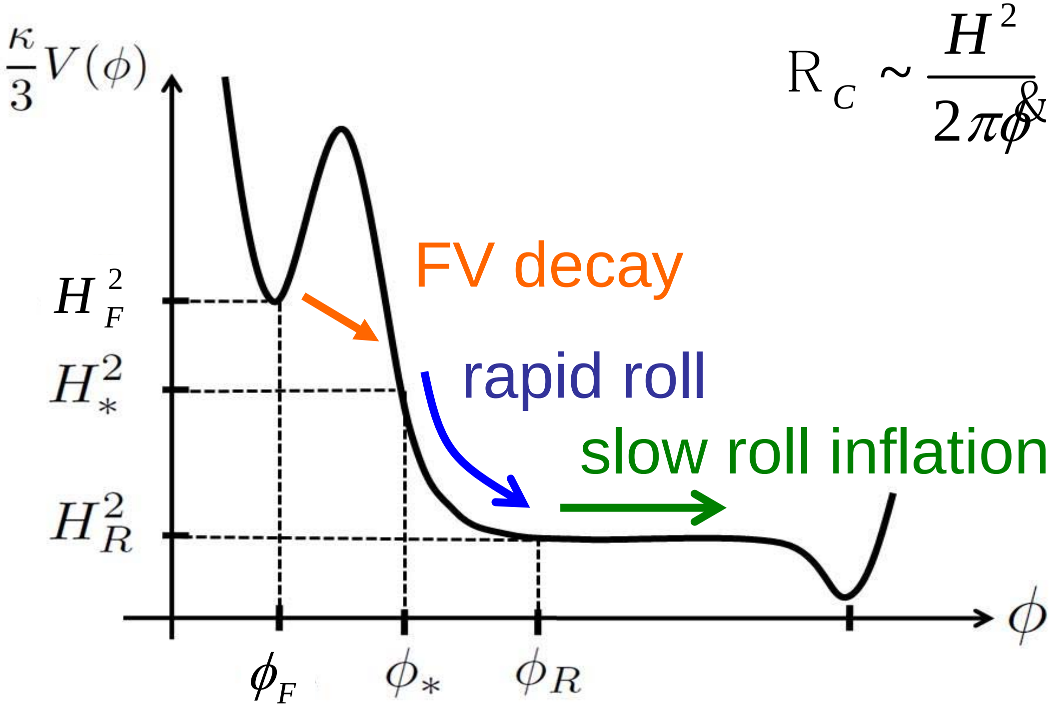
⇨ rapid-roll stage will follow right after tunneling.

- perhaps no strong effect on scalar-type pert's:

$$\mathrm{R}_c \sim \frac{H^2}{2\pi\dot{\phi}}$$

suppressed by $1/\dot{\phi}$ at rapid-roll phase

need detailed analysis
··· future issue



but tensor perturbations may not be suppressed at all.

$$h^{TT} \sim \frac{H}{M_P} \quad ?$$

Memory of H_F (Hubble rate in the false vacuum) may remain in the perturbation on the curvature scale

If $H_F \sim M_P$, we would see a huge tensor perturbation!?

tensor perturbations
and
their effect on CMB

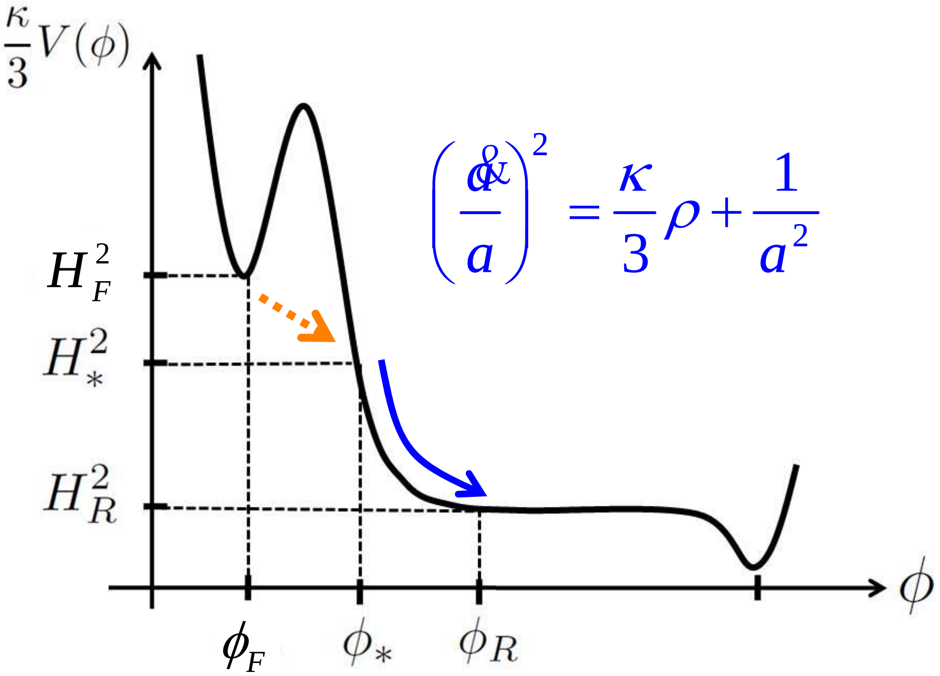
4. Single field model

- evolution after tunneling -

- **curvature dominant phase**

Right after tunneling, H is dominated by curvature:

$$a \approx t, \quad \dot{\phi} \approx -\frac{V'(\phi)}{4}t$$



$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3}\rho + \frac{1}{a^2}$$

- **rapid-roll phase**

kinetic energy grows until

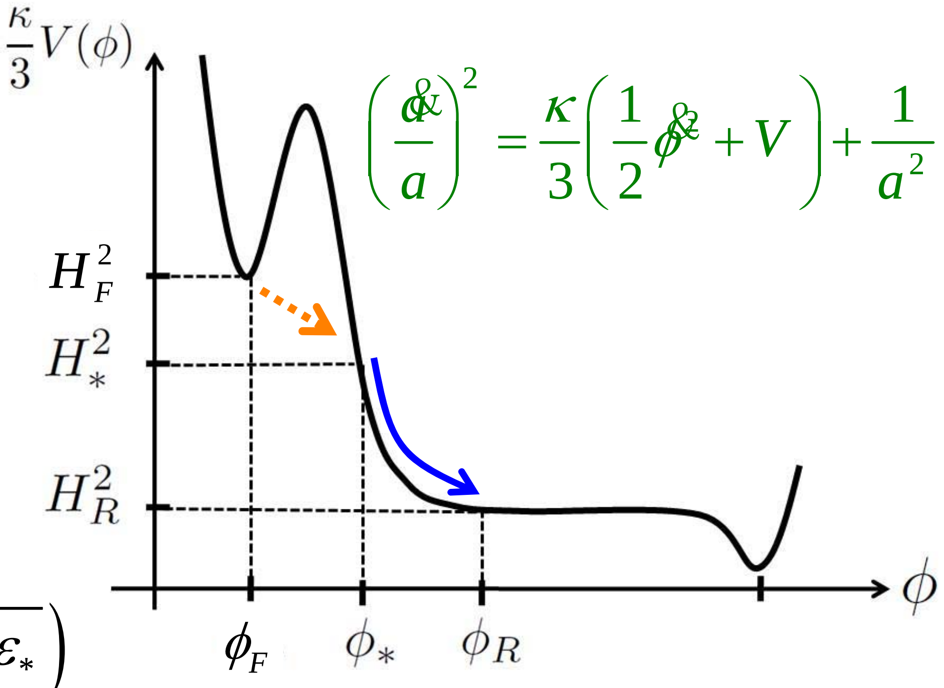
$$\dot{\phi}^2 \approx V \text{ at } t \approx \frac{H_*^{-1}}{\sqrt{\varepsilon_*}}$$

$\varepsilon \equiv \frac{1}{2\kappa}\left(\frac{V'}{V}\right)^2$: “slow-roll” parameter

➢ rapid-roll phase - continued

$$\dot{\phi}^2 \approx V \quad \text{at} \quad t \approx \frac{H_*^{-1}}{\sqrt{\varepsilon_*}}$$

$$\varepsilon \equiv \frac{1}{2\kappa}\left(\frac{V'}{V}\right)^2$$



$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right) + \frac{1}{a^2}$$

for $\varepsilon_* = 1$,

V dominates (curvature dominance ends) at

$$t \approx H_*^{-1} \left(= H_*^{-1} / \sqrt{\varepsilon_*}\right)$$

no rapid-roll phase.

slow-roll inflation starts at $t \gtrsim H_*^{-1}$

$$\left(\frac{d\ln\rho}{d\ln a} = -\frac{3\dot{\phi}^2}{\dot{\phi}^2/2 + V(\phi)}\right)$$

for $\varepsilon_* \gtrsim 1$,

ρ starts to decay at $t \gtrsim H_*^{-1}/\sqrt{\varepsilon_*} \left(< H_*^{-1}\right)$

rapid-roll continues until $\varepsilon = 1$

tracking $\left(\dot{\phi}^2 \propto V \propto 1/a^2\right)$ is realized during rapid-roll phase

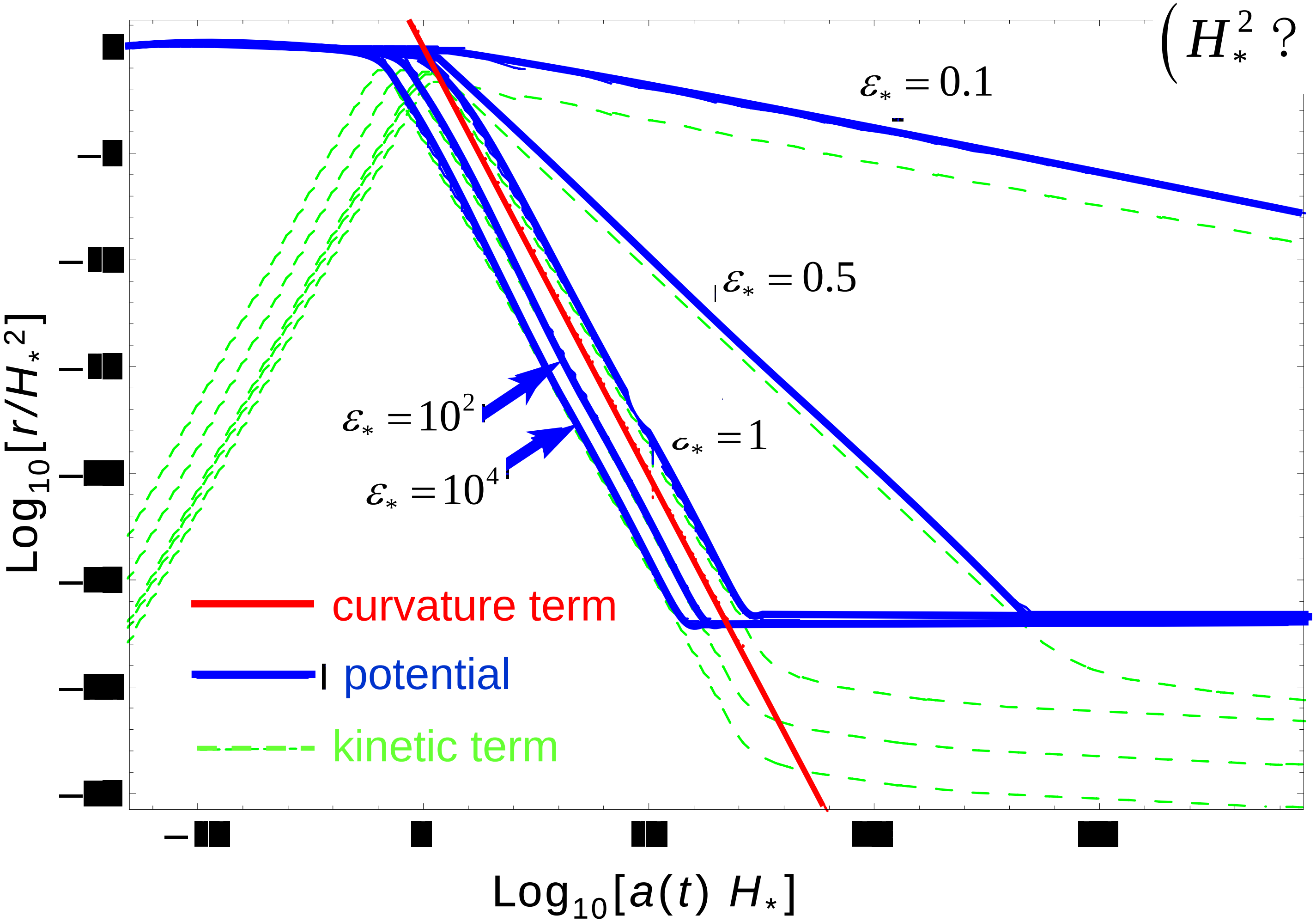
➢ exponential potential model

$$V \propto \exp\left(\sqrt{2\kappa\varepsilon}\,\phi\right) \Rightarrow V'/V = \mathrm{const.} \Rightarrow \varepsilon = \mathrm{const.}$$

$$V = \left(H_*^2 - H_R^2\right)\exp\left(\sqrt{2\kappa\varepsilon_*}\,(\phi - \phi_*)\right) + \boxed{H_R^2} \leftarrow$$

added to realize slow-roll inflation

$$\left(H_*^2\ ?\ \ H_R^2\right)$$



5. Tensor perturbations

- some technical details...

- action

$$L = \sqrt{-g}\left[\frac{1}{2\kappa}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right]$$

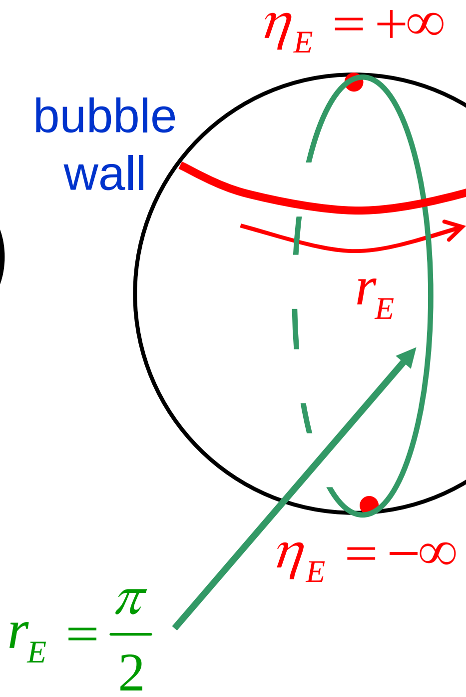
- CDL instanton

$$ds^2 = dt_E^2 + a_E^2(t_E)\left(dr_E^2 + \sin^2 r_E d\Omega^2\right)$$

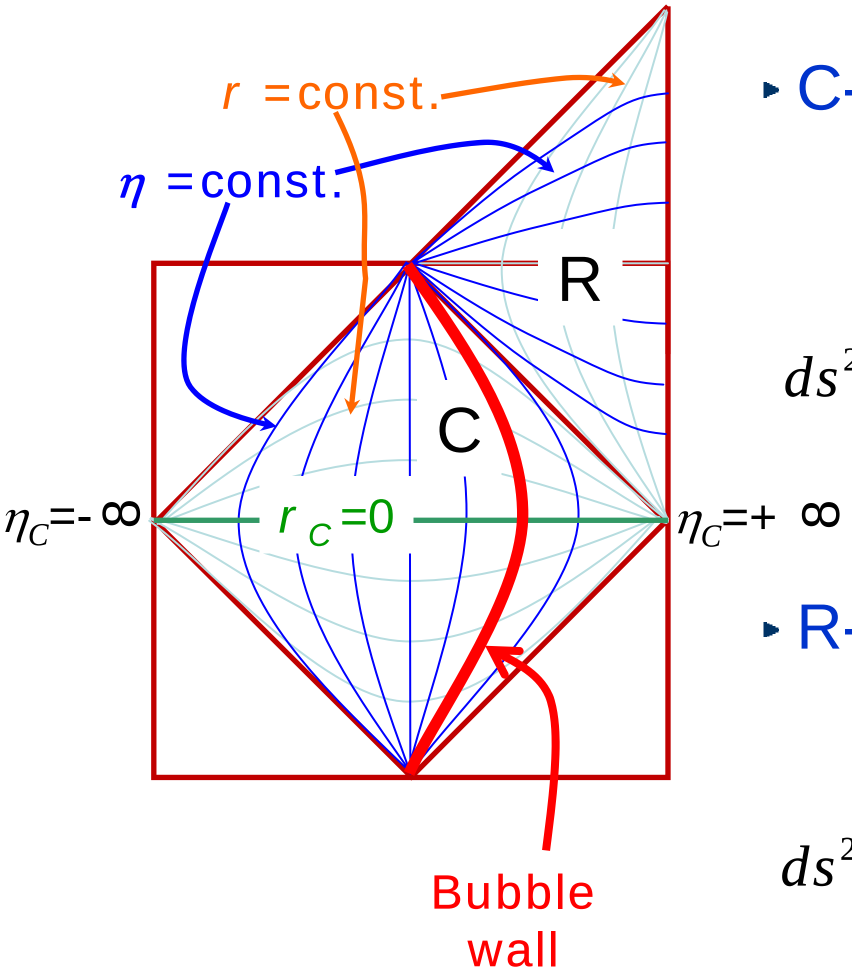
$$= a_C^2(\eta_E)\left(d\eta_E^2 + dr_E^2 + \sin^2 r_E d\Omega^2\right)$$

$$\phi = \phi(\eta_E)$$

$$-\infty < \eta_E < \infty$$



- analytic continuation to Lorentzian space (through $r_E=\pi/2$)



- C-region: ~ outside the bubble

$$r_C = i\left(r_E - \frac{\pi}{2}\right),\ \eta_C = \eta_E$$

$$ds^2 = a_C^2(\eta_C)\left(d\eta_C^2 - dr_C^2 + \cosh^2 r_C d\Omega^2\right)$$

⇧
time

- R-region: inside the bubble

$$r_R = r_C + \frac{\pi}{2}i,\ \eta_R = -\eta_C - \frac{\pi}{2}i,\ a_R = i\,a_C$$

$$ds^2 = a_R^2(\eta_R)\left(-d\eta_R^2 + dr_R^2 + \sinh^2 r_R d\Omega^2\right)$$

⇧
time

Euclidean vacuum ➡ C-region ➡ R-region

- tensor mode function

Euclidean vacuum

$$h_{ij}^{TT}=a^2(\eta_C)X_p(\eta_C)Y_{ij}^{p\,l\,m}(r_C,\Omega)$$

Y_{ij}: regular at r_c=0

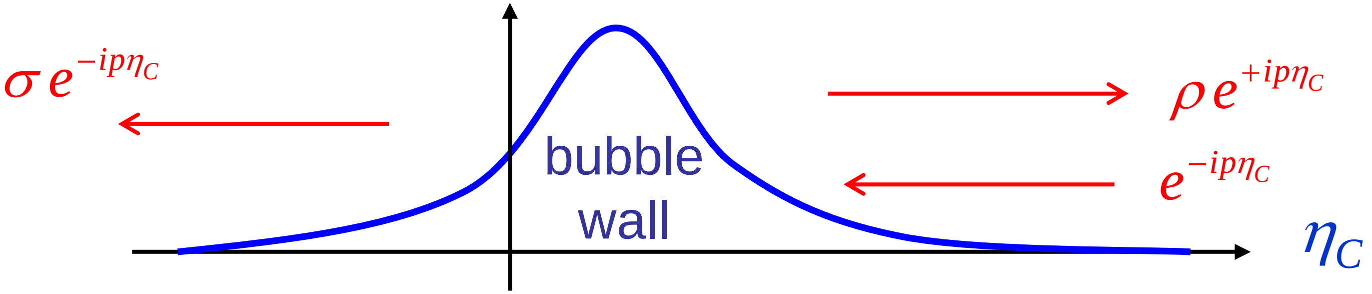
$$\left[\frac{d^2}{d\eta_C^2}+2\frac{a'(\eta_C)}{a(\eta_C)}\frac{d}{d\eta_C}+(p^2+1)\right]X_p=0;\quad K=-1$$

- new variable w^p : $a^2X_p\equiv\frac{d}{d\eta_C}\left(a\,w^p\right)$

$$\left[-\frac{d^2}{d\eta_C^2}+U_T(\eta_C)\right]w^p=p^2w^p\ ;\qquad U_T(\eta_C)=\frac{\kappa}{2}\phi'^2(\eta_C)$$

$$=\frac{\kappa}{2}a_C^2(t_C)\dot{\phi}^2(t_C)$$

$\boldsymbol{U_T}$ $\quad|\rho|^2+|\sigma|^2=1$



- analytic continuation from C-region to open univ. (=R-region)

$$\eta_C \to \eta_R = -\eta_C - \frac{\pi}{2} i$$

effect of wall

⇩

$$w_C^p = \rho e^{+ip\eta_C} + e^{-ip\eta_C} \quad \to \quad w_R^p = e^{p\pi/2} \rho e^{-ip\eta_R} + e^{-p\pi/2} e^{ip\eta_R}$$

epoch of bubble nucleation: $\eta_R \to -\infty$

- there will be time evolution of w^p in R-region:

$$\left[\frac{d^2}{d\eta_R^2} - U_T(\eta_R) + p^2 \right] w^p = 0 \; ; \quad U_T(\eta_R) = \frac{\kappa}{2} \phi'^2$$

or $$\left[\frac{d^2}{d\eta_R^2} + 2\mathrm{H} \frac{d}{d\eta_R} + (p^2 + 1) \right] X_p = 0 \; ; \quad \mathrm{H} \equiv \frac{a'(\eta_R)}{a(\eta_R)}$$

$$X_p = \frac{1}{a^2} \frac{d}{d\eta} \left[a w^p \right] \quad \left(\propto \; h^{TT} \right)$$

final amplitude of X_p depends both
on the effect of wall & on the evolution after tunneling

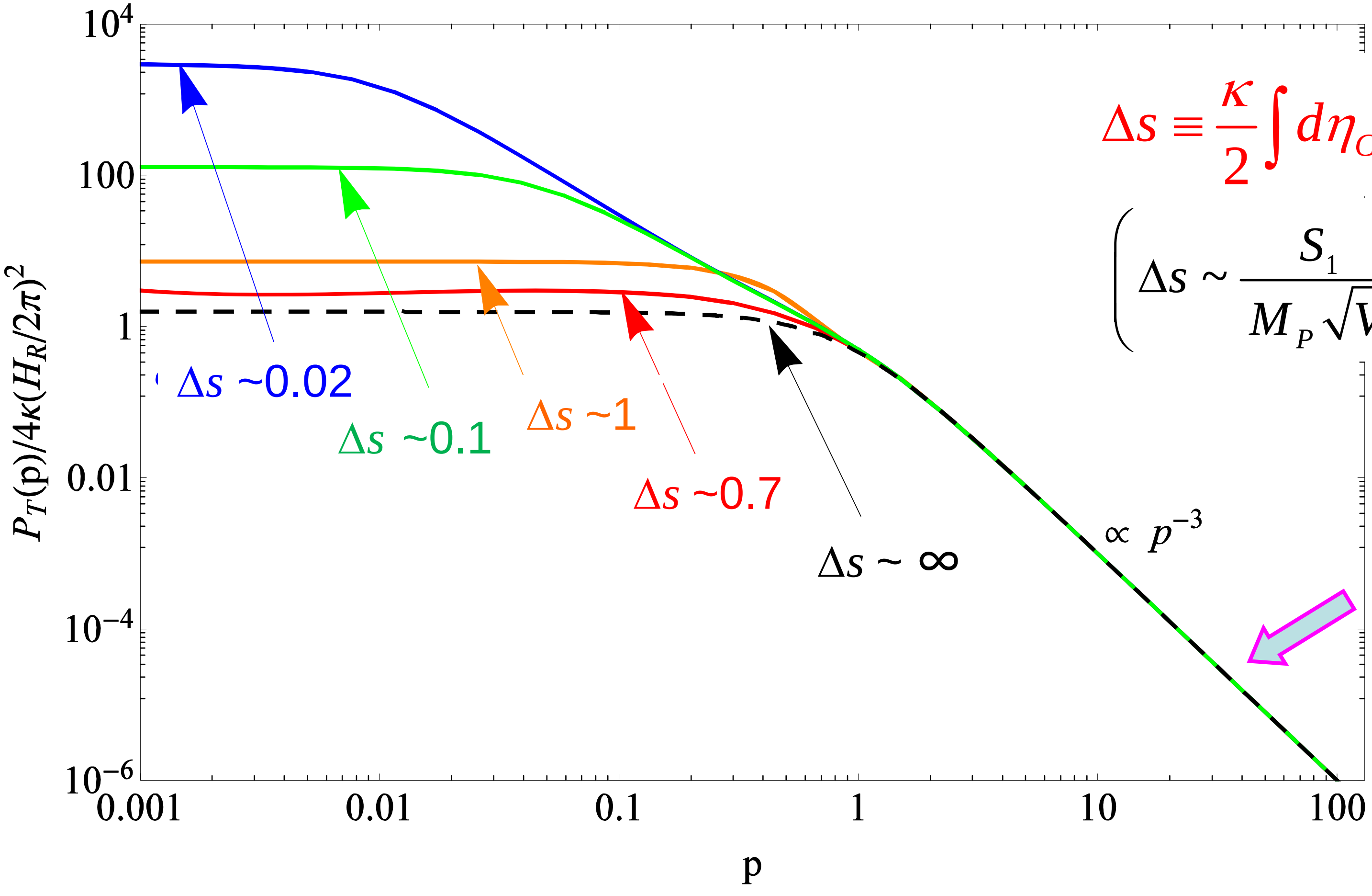
➢ Effect of tunneling/bubble wall on $P_T(p)$ $\left(\propto \left|X_p\right|^2\right)$

high freq continuum + low freq resonance

$p > 1$

$p \sim 0$

wall fluctuation mode



$$\Delta s \equiv \frac{\kappa}{2}\int d\eta_C\, \phi'^2(\eta_C)$$

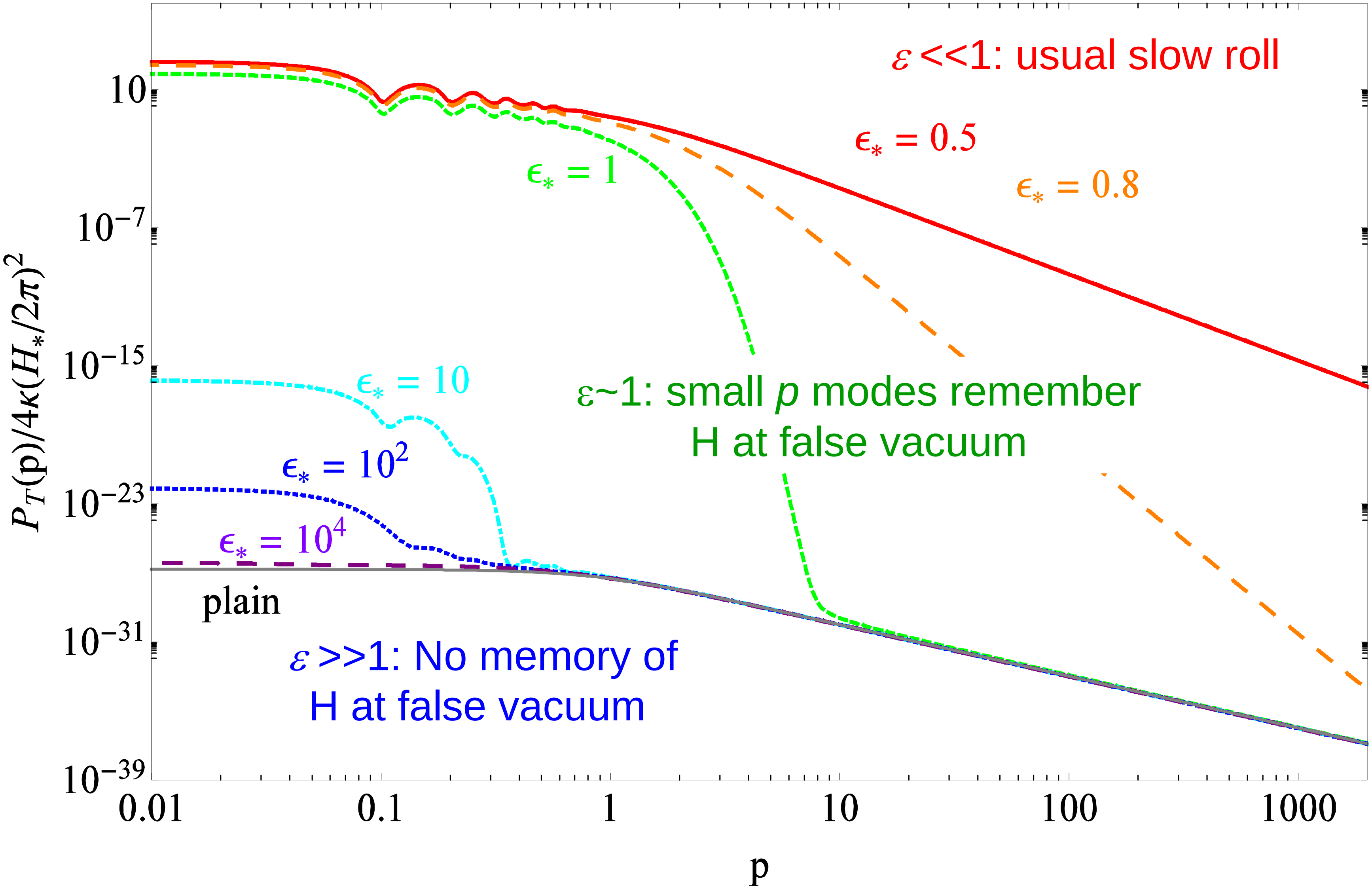
$$\left(\Delta s \sim \frac{S_1}{M_P\sqrt{V}};\ S_1 = \int dt_C \dot{\phi}^2\right)$$

⇧

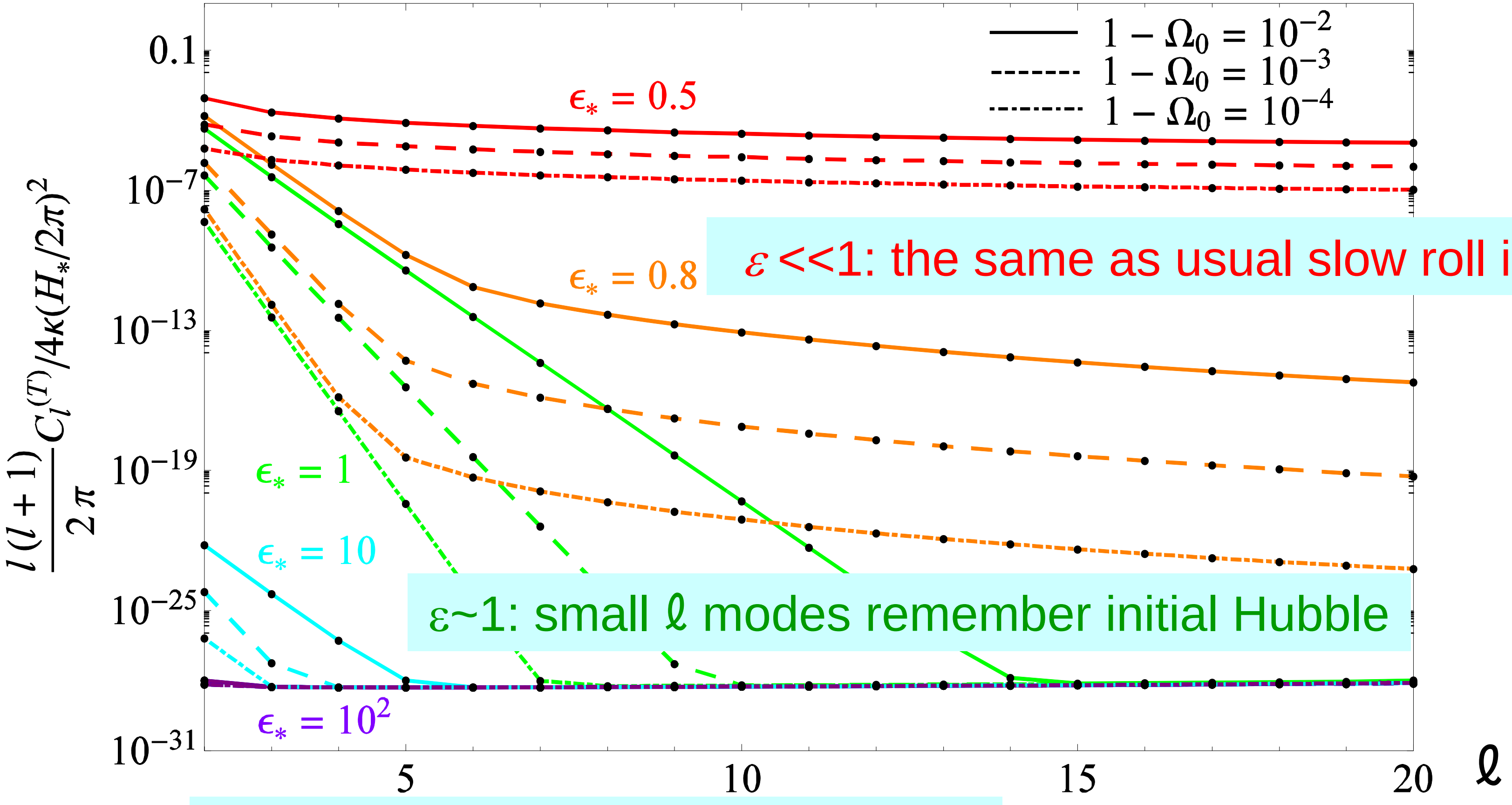
wall tension

scale-invariant

➢ rapid-roll phase (ε_* -)dependence of P_T(p)



6. CMB anisotropy



ε<<1: the same as usual slow roll inflation

ε~1: small ℓ modes remember initial Hubble

ε>>1: No memory of initial Hubble

- scales as $(1-\Omega_0)^1$ at small ℓ, scale-invariant at large ℓ

small ℓ modes enhanced for $\varepsilon_* \sim 1$

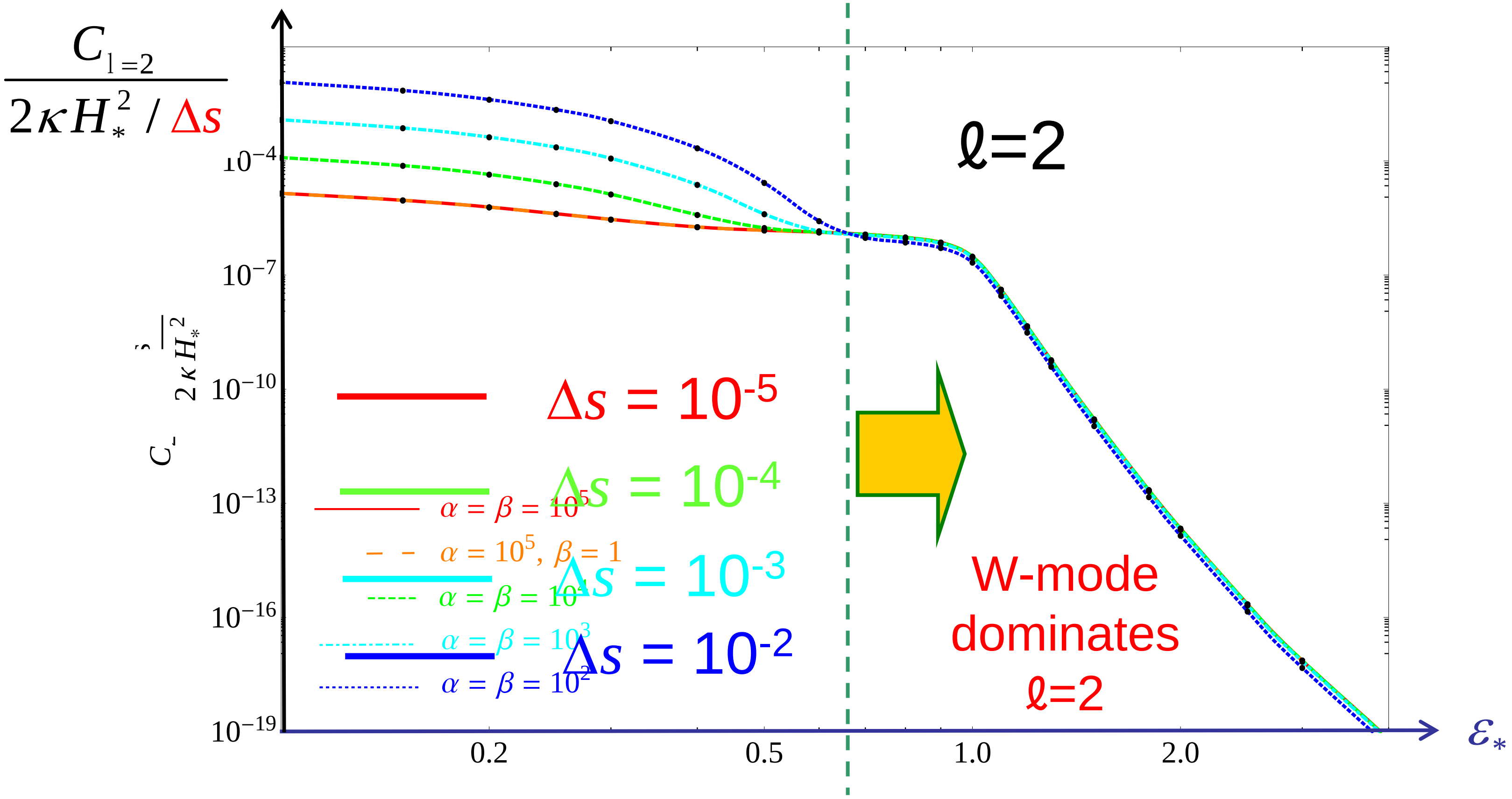
• CMB anisotropy due to wall fluctuation (W-)mode

MS, Tanaka & Yakushige ('97)

$$C_l = C_l^{(C)} + P_W \tilde{C}_l^{(W)}; \quad P_W = \int_0^\infty dp\, P_T(p) \propto \frac{1}{\Delta s}$$

scale-invariant part

$$\boxed{\tilde{C}_l^{(W)} \propto (1-\Omega_0)^l}$$



7. Summary

- Open inflation has attracted renewed interest in the context of string theory landscape

  anthropic principle + landscape ⇨ $1-\Omega_0 \sim 10^{-2} - 10^{-3}$

- Landscape is already constrained by observations

  If inflation after tunneling is short (N ~ 60):

  - simple polynomial potentials $a\phi^2 - b\phi^3 + c\phi^4$ lead to HM-transition, and are ruled out

  - simple 2-field models, naturally realized in string theory, are ruled out

  due to large scalar-type perturbations on curvature scale

- Tensor perturbations may also constrain the landscape

  "single-field models"

  - it seems difficult to implement models with short slow-roll inflation right after tunneling in the string landscape.

    if ε<<1, energy scale must have been already very low.

  - there will be a rapid-roll phase after tunneling.

    $$\varepsilon = \frac{1}{2\kappa}\left(\frac{V'}{V}\right)^2 \gtrsim 1$$ right after tunneling

  - unless ε>>1, the memory of pre-tunneling stage persists in the IR part of the tensor spectrum

    large CMB anisotropy at small ℓ $\propto (1-\Omega_0)^1$

    due to either wall fluctuation mode

    or evolution during rapid-roll phase

We are already testing the landscape!