# Nf=12 fundamental rep and Nf=2 sextet rep SU(3) fermions and the conformal window

*Lattice Higgs Collaboration with*
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KMI seminar, Nagoya University, November 2, 2011

# Outline

- Probing the Conformal Window
  - lattice BSM goals in Theory Space
  - cut-off, volume, fermion mass
  - RG flow and lattice continuum physics

- Finite size scaling theory
  - BSM specific $\chi$ PT
  - m=0 chiral limit and finite volume issues
  - conformal finite size scaling

- Nf=12 fundamental fermion rep

- Nf=2 sextet fermion rep

- Inside the conformal window
  - running coupling and tunneling
  - Nf=16 case study

# Large Hadron Collider - CERN

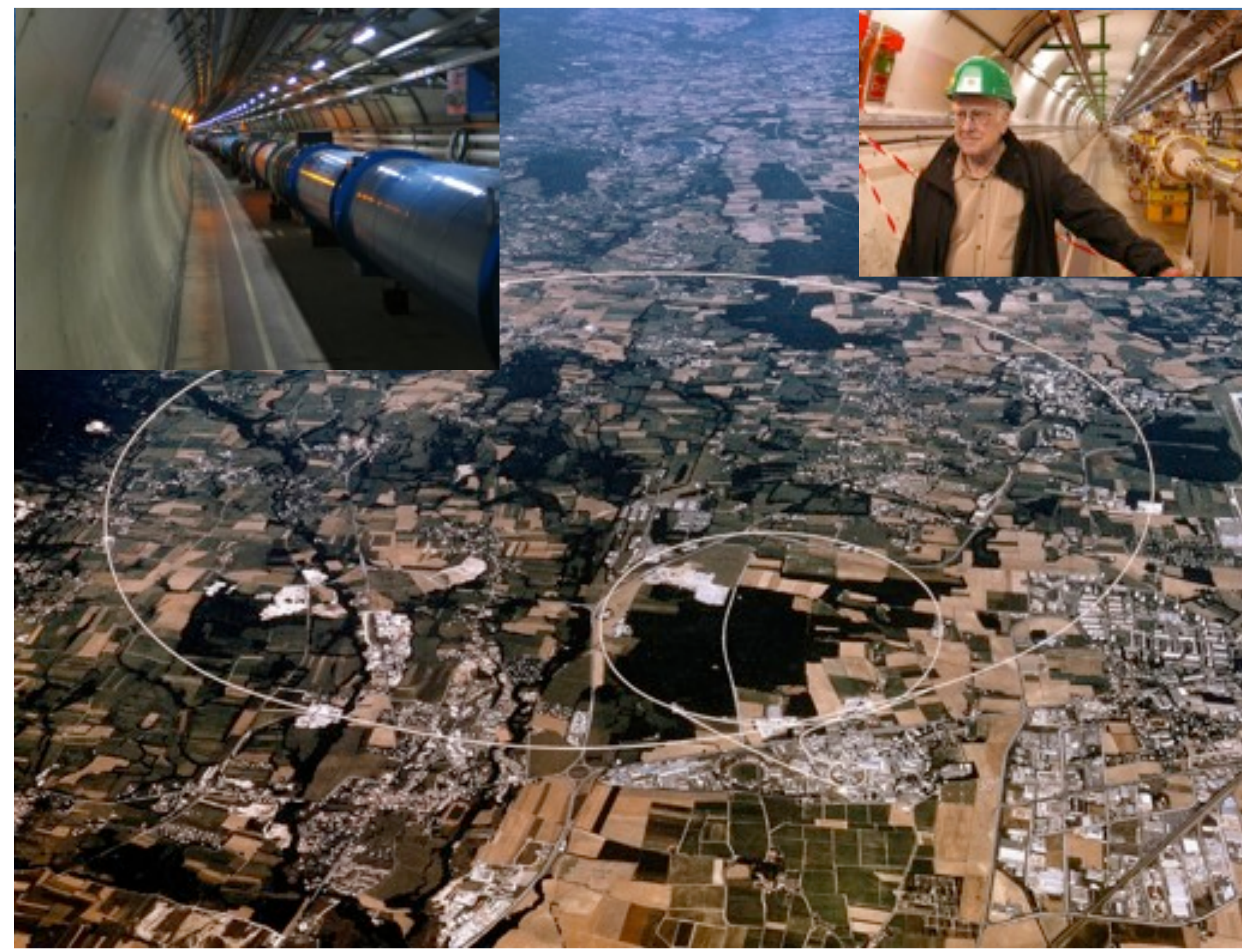
***primary mission:***

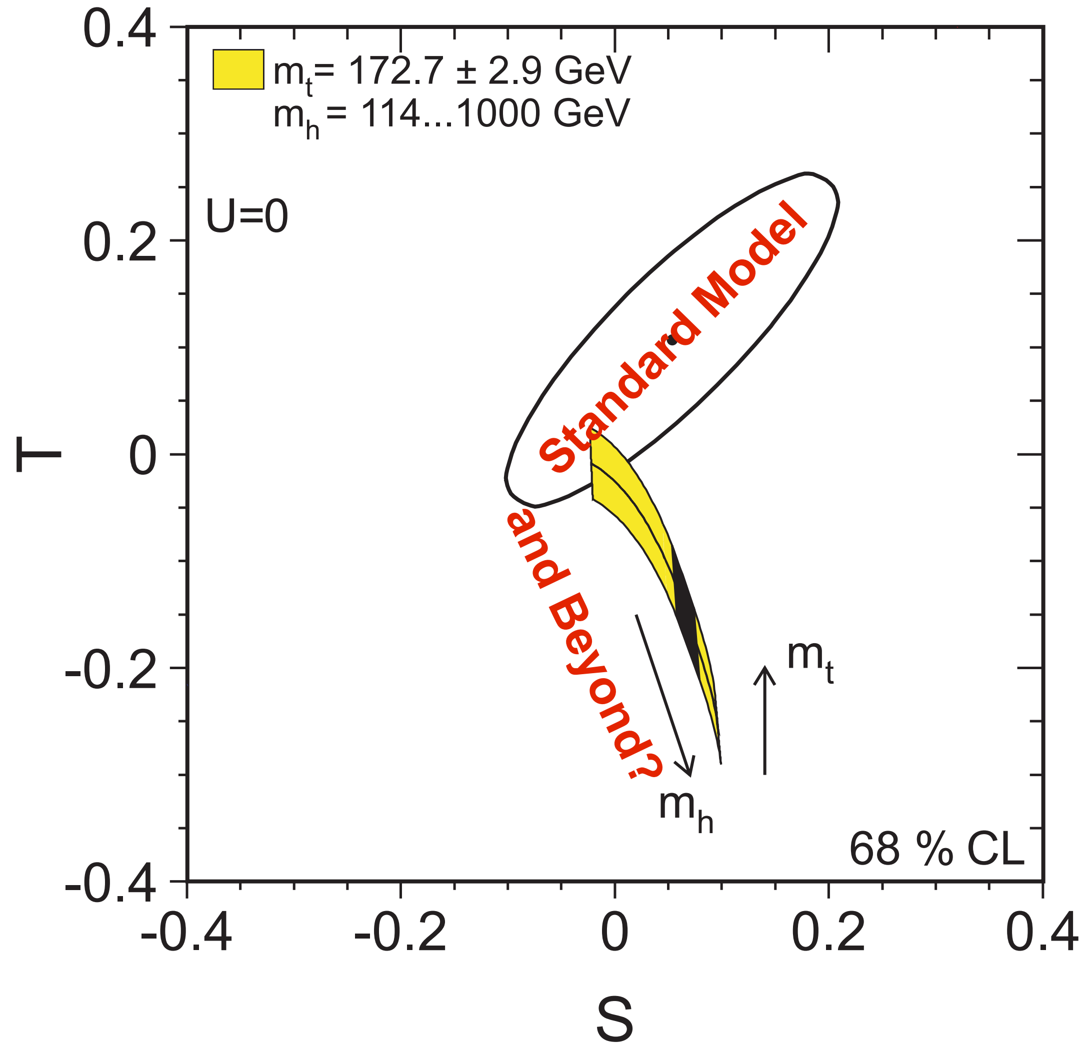
***- Search for Higgs particle***

***- Origin of Electroweak symmetry breaking***

- Is there a Standard Model Higgs particle?
- If not, what generates the masses of the weak bosons and fermions?
- New strong dynamics?
- Composite Higgs mechanism?

Primary focus of lattice BSM effort and of this talk






**Two logical choices to accommodate heavy Higgs (or no Higgs) scenario:**

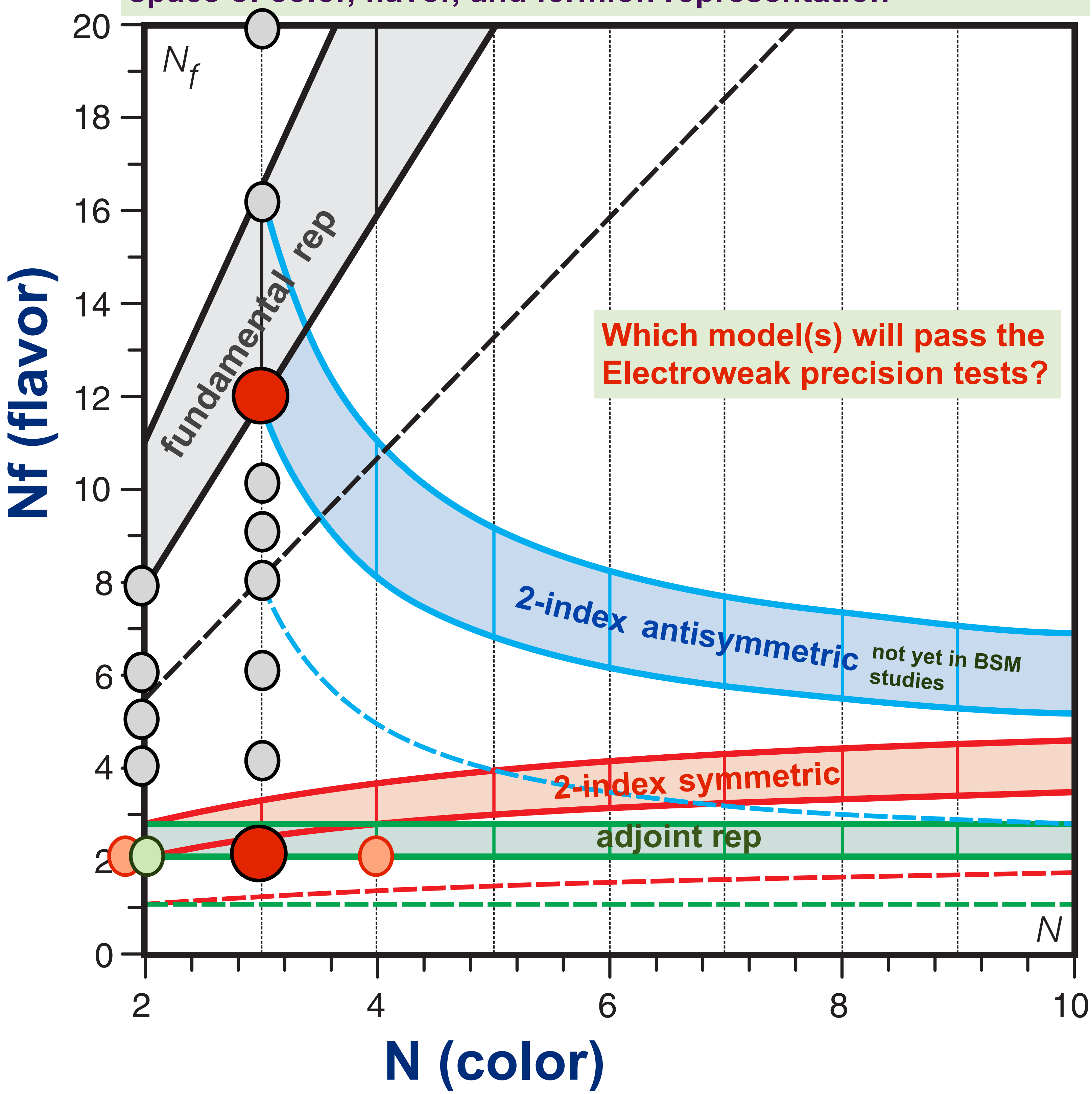
- use some effective theory with TeV scale higher dimensional operators
- new microscopic theory on TeV scale

**Composite Higgs mechanism - Technicolor 2.0 ?**

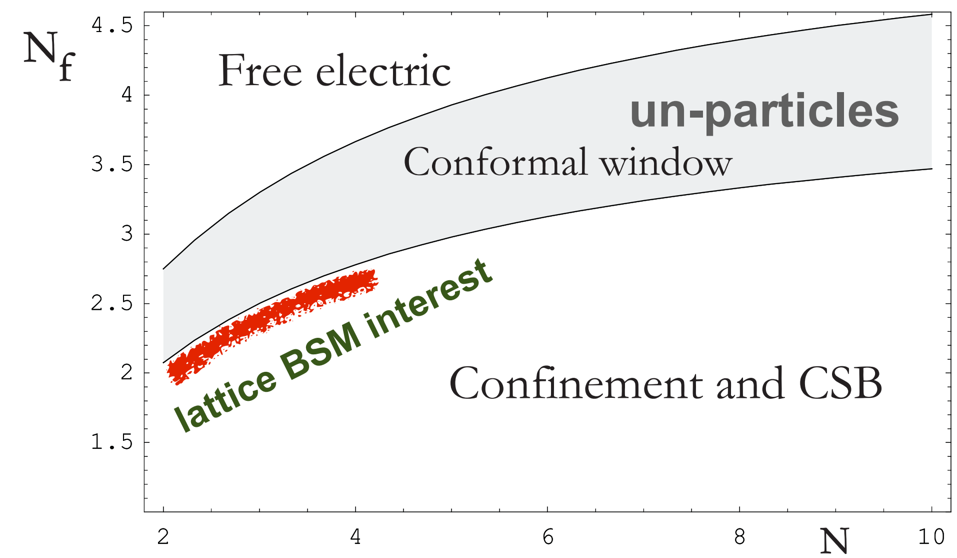
- The paradigm is interesting again
- Requires non-perturbative lattice studies
- It is difficult, but there will be real results

**theory space and conformal window: critically important for composite Higgs and TC/ETC**

space of color, flavor, and fermion representation



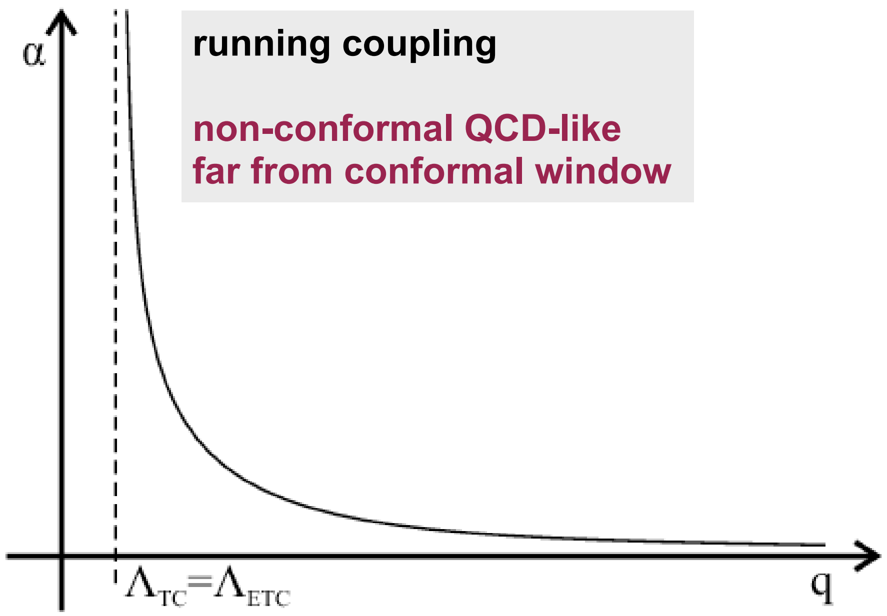

for each rep BSM interest is below conformal window but close to it:

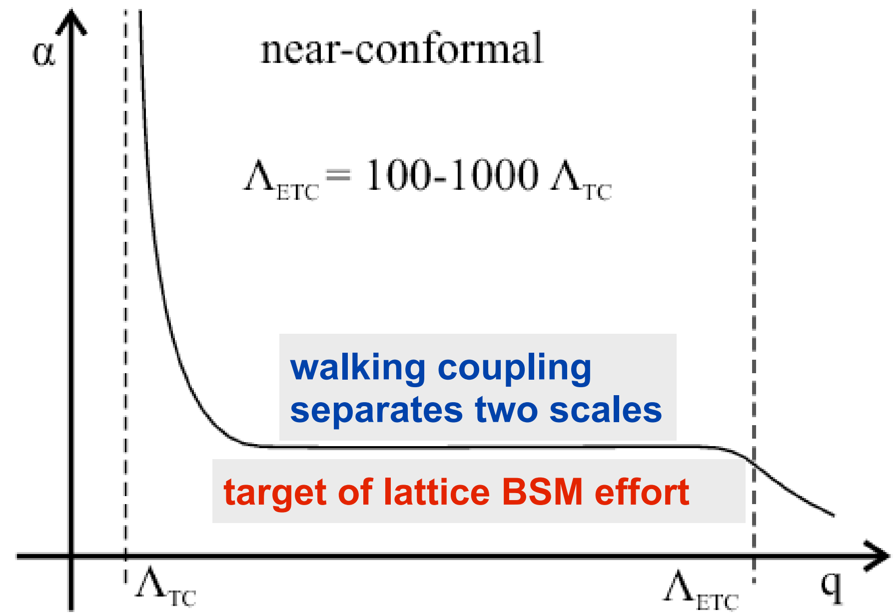



- lattice results of last 3 years in
- 3 reps including new projects
- just starting

it is stimulating to have controversial results close to the conformal window: these are the interesting candidate models

# Composite Higgs mechanism? (Technicolor and Extended Technicolor in the past)



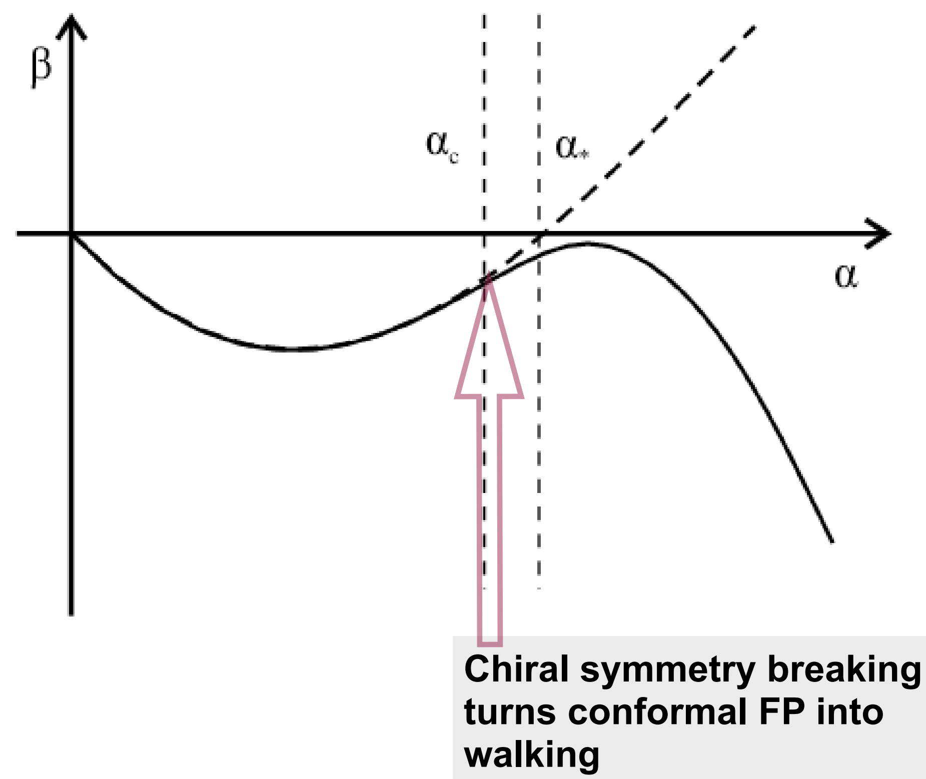




**Extended Technicolor paradigm:**

- requires walking gauge coupling chiral SB on $\Lambda_{TC} \sim TeV$ scale
- fermion mass generation from scale at $\Lambda_{ETC} \sim 100 - 1000\Lambda_{TC}$
- can solve problem of flavor changing currents
- composite Higgs mechanism
- broken Dilaton ➔ unusual composite Higgs particle in BSM ?
- can avoid conflict with EW precision constraints
- *candidate models require non-perturbative lattice studies*

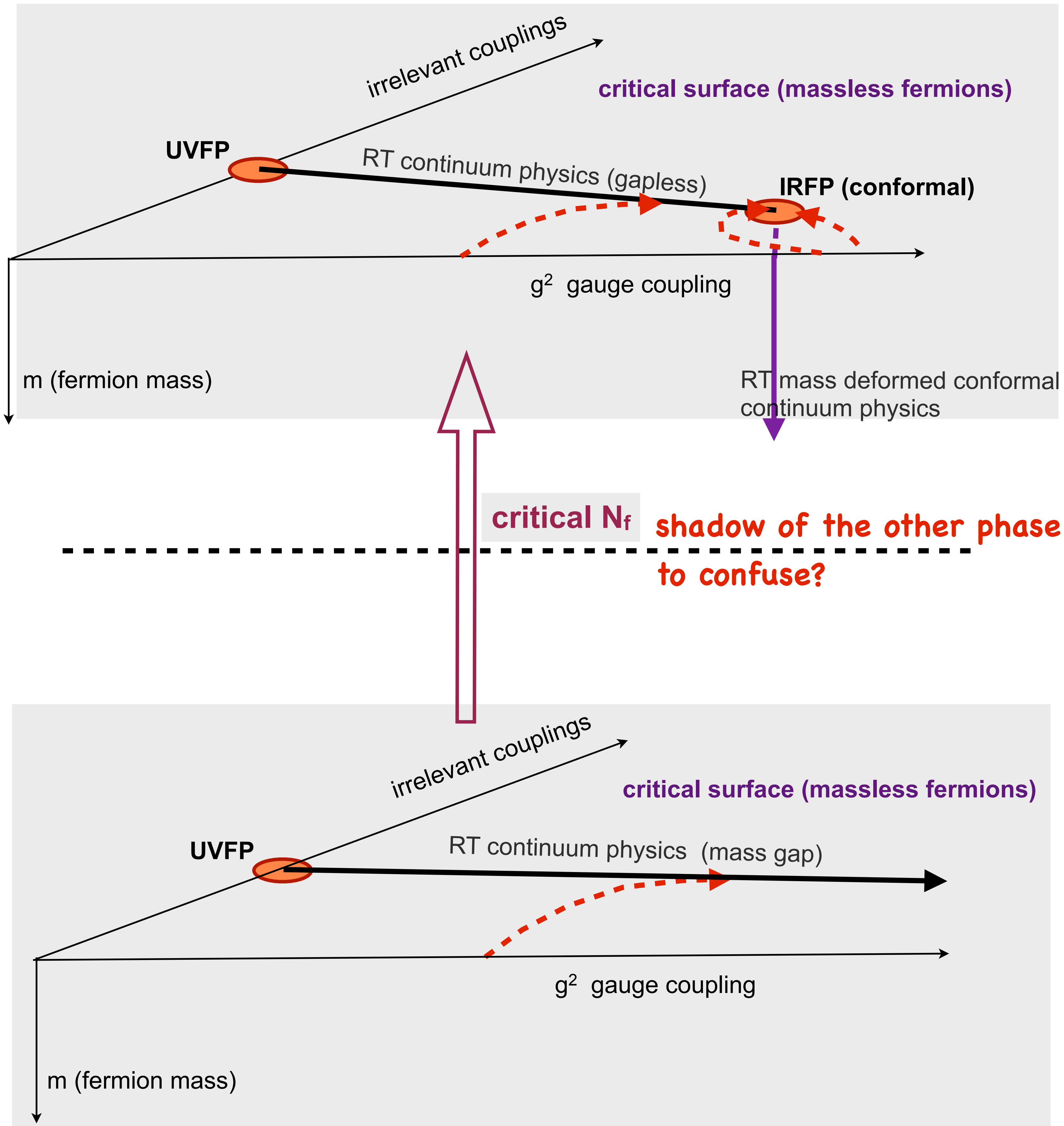
**original textbook Technicolor paradigm:**

- one massless fermion doublet $\begin{bmatrix} u \\ d \end{bmatrix}$ chiral SB
- three Goldstone pions
- become longitudinal components of weak bosons
- composite Higgs mechanism scale of Higgs condensate ~ F=250 GeV $\Lambda_{TC} \sim TeV$
- flavor changing currents and fermion mass generation would be problems
- conflicts with EW precision constraints

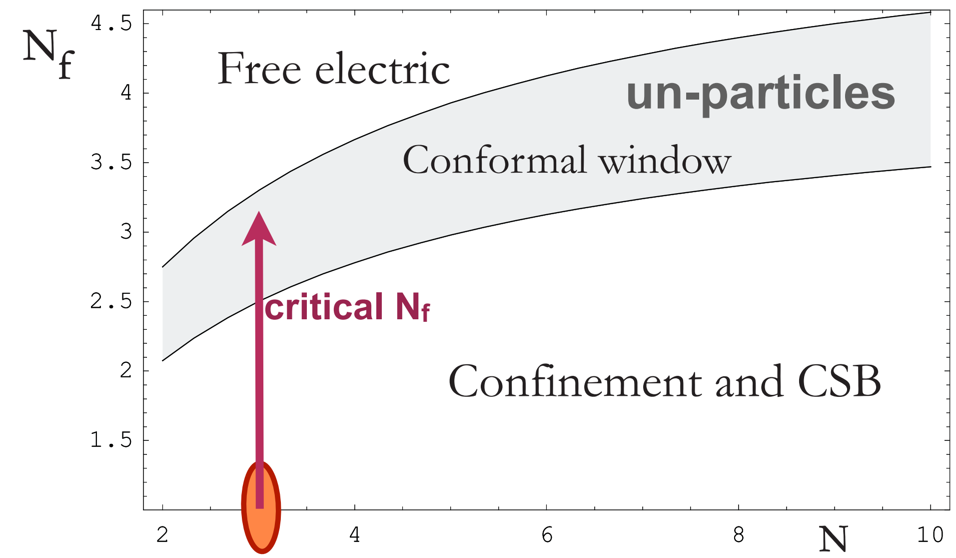



**This is what lattice studies in BSM theory space potentially could deliver**

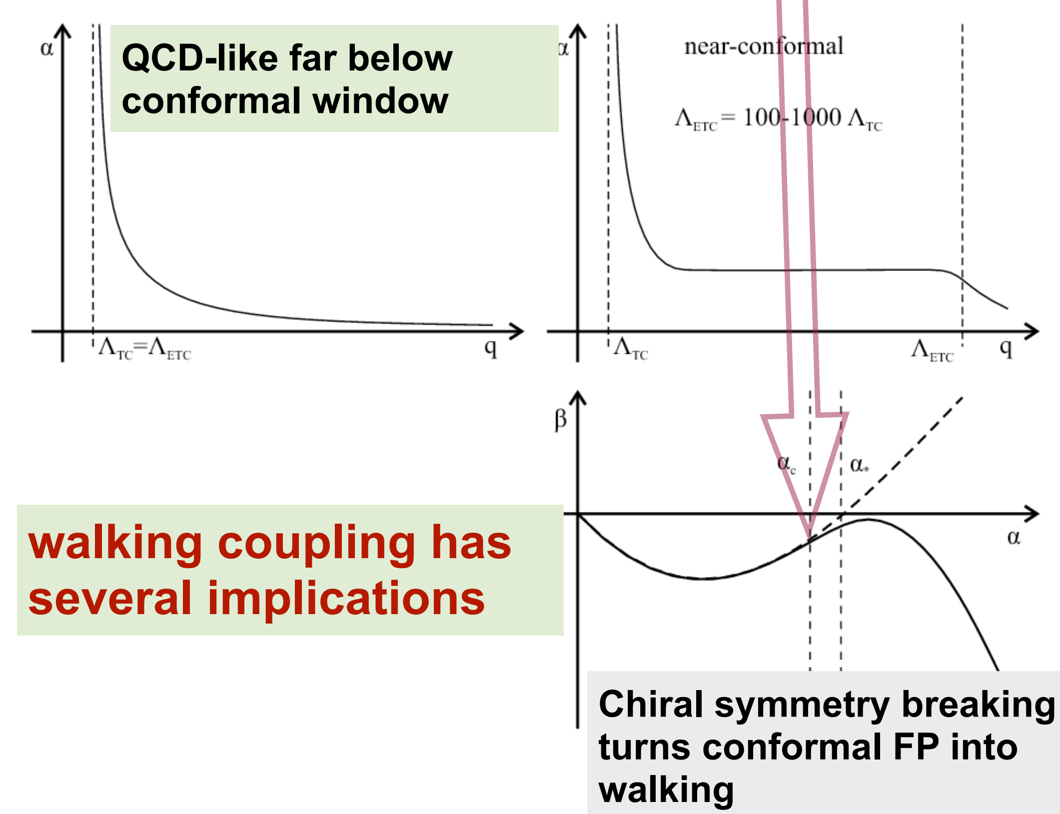
# cut-off control in non-perturbative lattice calculations from RG flow




**inside the conformal window:**

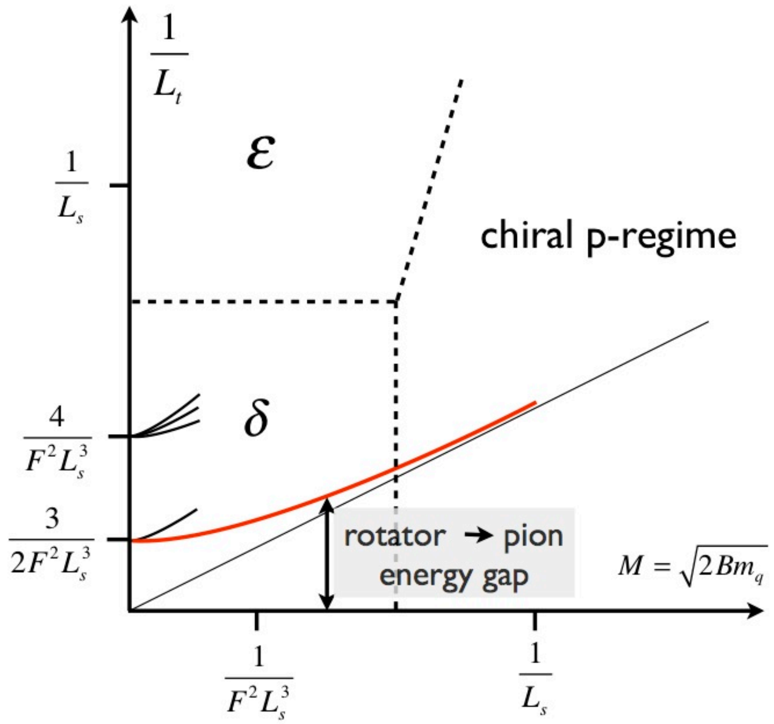



**with increasing $N_f$ walking scenario expected to arise:**



# Finite size scaling theory

# Chiral regimes to identify in theory space below conformal window:



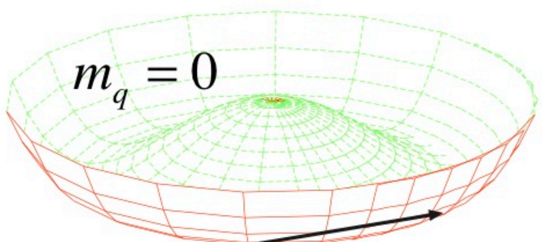

Goldstone dynamics is different in each regime

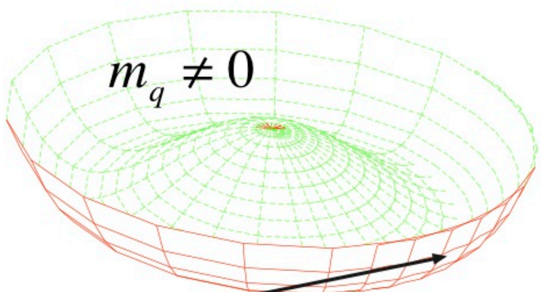
We study $\delta$ and $\varepsilon$-regimes (RMT)

and p-regime (probing chiral loops)

complement each other

interpretation of rotator levels in $m_q \to 0$ limit:

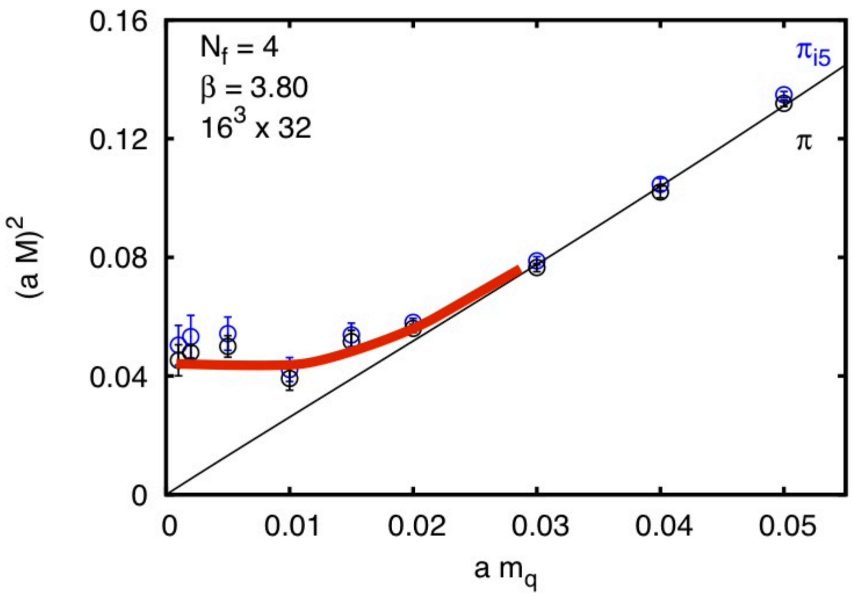


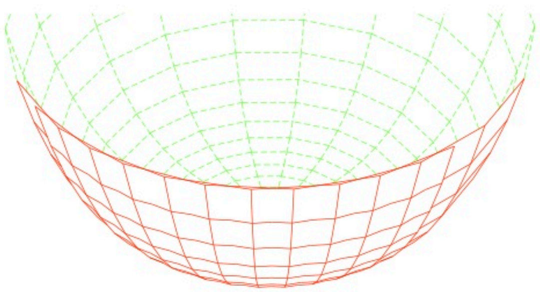




Veff: chiral condensate in flavor space

arbitrary orientation of condensate

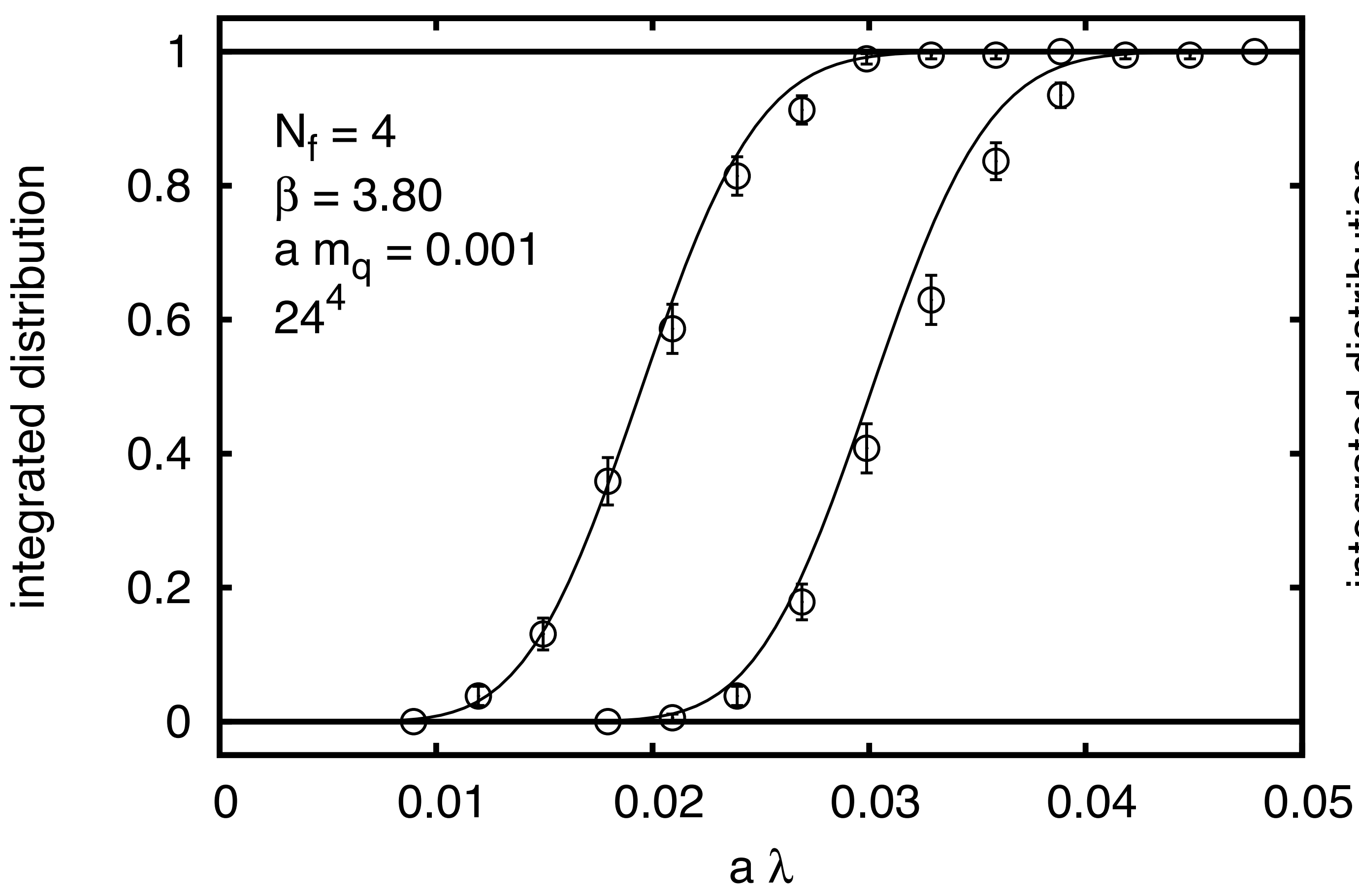
tilted condensate

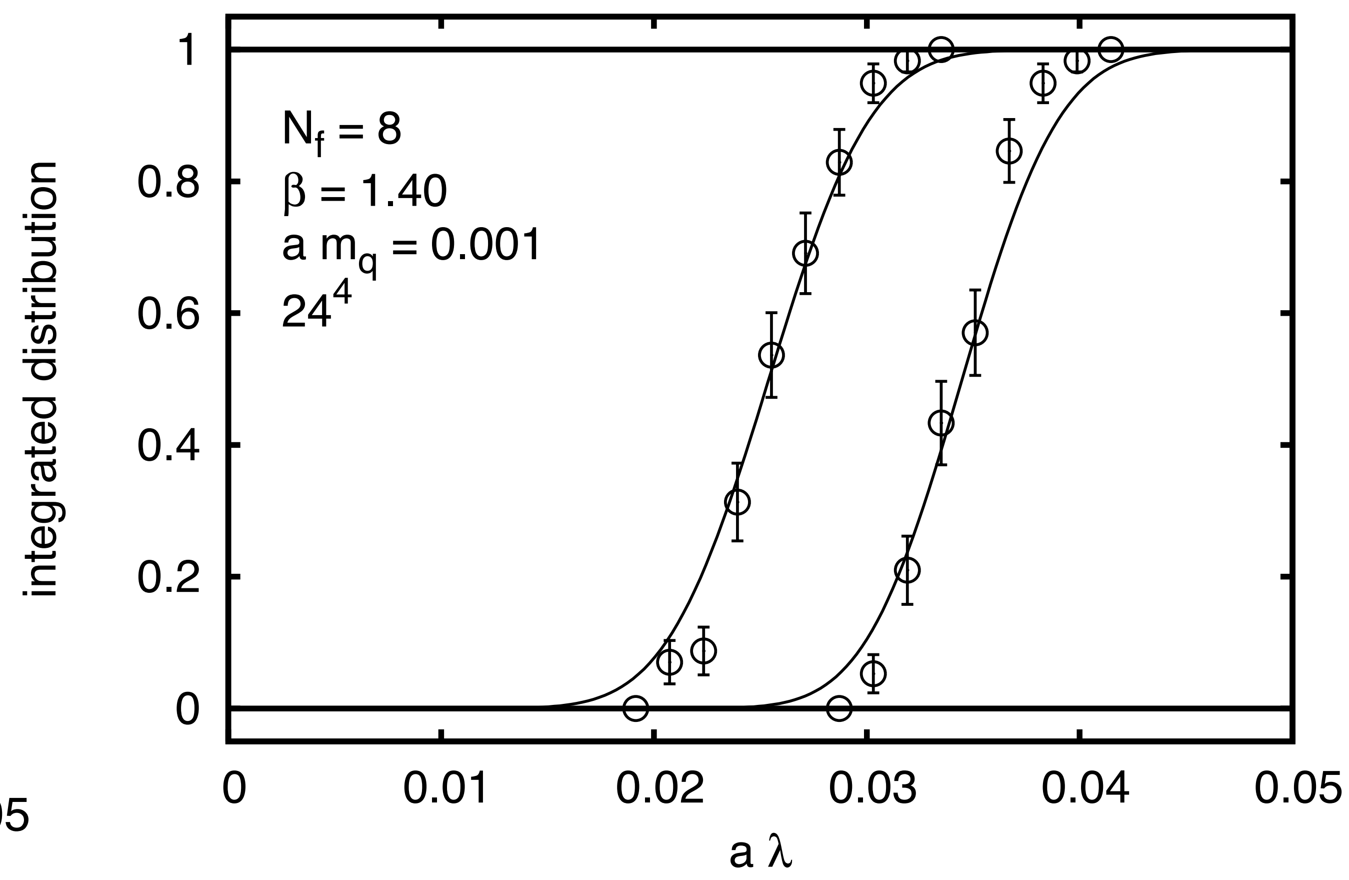


Not to misidentify rotator gaps as evidence of chirally symmetric phase !

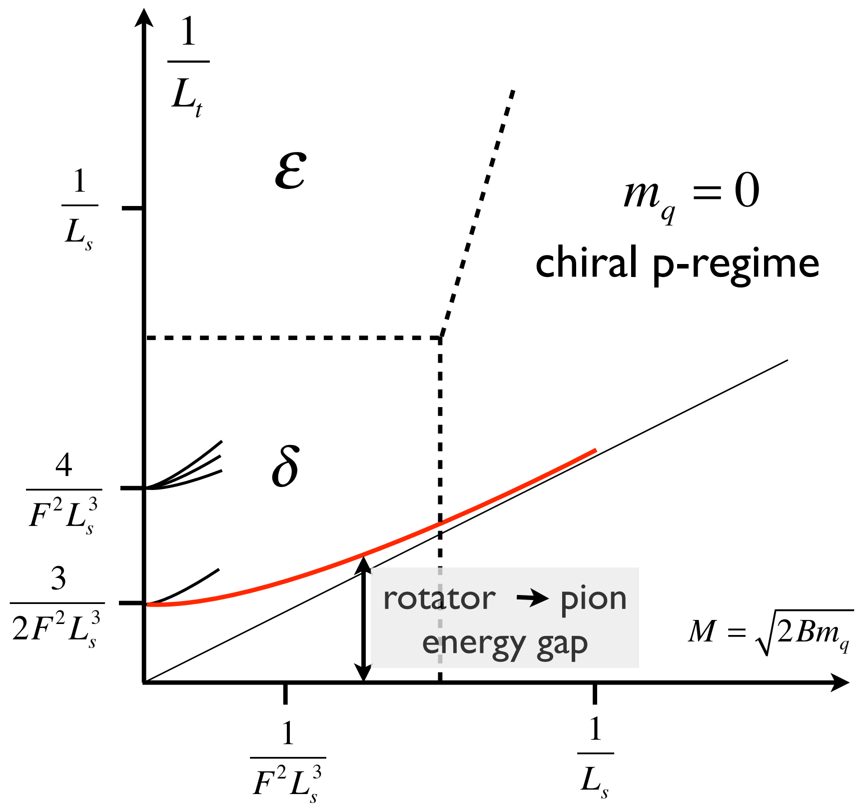
## Random Matrix Theory tests in epsilon regime:




**Dirac spectrum - integrated eigenvalue distributions of RMT**

# One-loop chiral expansion in p-regime:




**Note 1/$N_f$ scaling of pion mass!**

**warning: 2-loop ~ $(N_f)^2$ (Bijnens)**

$$M_\pi^2 = M^2\left[1 - \frac{M^2}{8\pi^2 N_f F^2} ln\left(\frac{\Lambda_3}{M}\right)\right] + O((N_f)^2) \qquad M^2 = 2Bm$$

$$F_\pi = F\left[1 + \frac{N_f M^2}{16\pi^2 F^2} ln\left(\frac{\Lambda_4}{M}\right)\right] + O((N_f)^2)$$

$$M_\pi(L_s,\eta) = M_\pi\left[1 + \frac{1}{2N_f}\frac{M^2}{16\pi^2 F^2}\cdot \widetilde{g}_1(\lambda,\eta)\right] \qquad \lambda = ML_s$$

$$F_\pi(L_s,\eta) = F_\pi\left[1 - \frac{N_f}{2}\frac{M^2}{16\pi^2 F^2}\cdot \widetilde{g}_1(\lambda,\eta)\right] \qquad \tilde{g}_1(\lambda,\eta) \approx 24K_1(\lambda)/\lambda \text{ for } \eta = \frac{L_t}{L_s} \gg 1$$

Chiral expansion parameter is $N_f \frac{M^2}{16\pi^2 F^2}$ with $\ll$ 1 condition

$N_f = 8$ fundamental rep in USQCD BSM project

set $N_f \frac{M^2}{16\pi^2 F^2}$ =0.3, with $a \cdot m_\rho = 0.25$ (to keep cut-off under control), and $m_\rho / F \approx 10$ (as expected), $a \cdot M_\pi \approx 0.10$ is needed

The $M_\pi \cdot L_s \approx 10$ condition (to control FSS) will require $L_s \approx 100$! Same scale as largest QCD projects!

$N_f = 2$ higher reps (like sextet) are more favorable for chiral expansion

Condition of reaching the chiral expansion regime can also be estimated from rotator spectrum $\Rightarrow$

Condition of reaching the chiral expansion regime can also be estimated from rotator spectrum $\Rightarrow$

$E_l = \frac{1}{2\theta} l(l+2)$ with $l = 0,1,2,\ldots$ rotator spectrum for $SU(2)_f \times SU(2)_f$

with $\theta = F^2 L_s^3 (1 + \frac{C(N_f = 2)}{F^2 L_s^2} + O(1/F^4 L_s^4))$ (P. Hasenfratz and F. Niedermayer)

(there is in $E_l$ an overall factor $\frac{N_f^2 - 1}{N_f}$ for arbitrary $N_f$)

$C(N_f = 2) = 0.45$, C will grow with $\sim N_f$, (P.Hasenfratz, $O(N_f)$ model)

there are similar considerations in the $\varepsilon$-regime

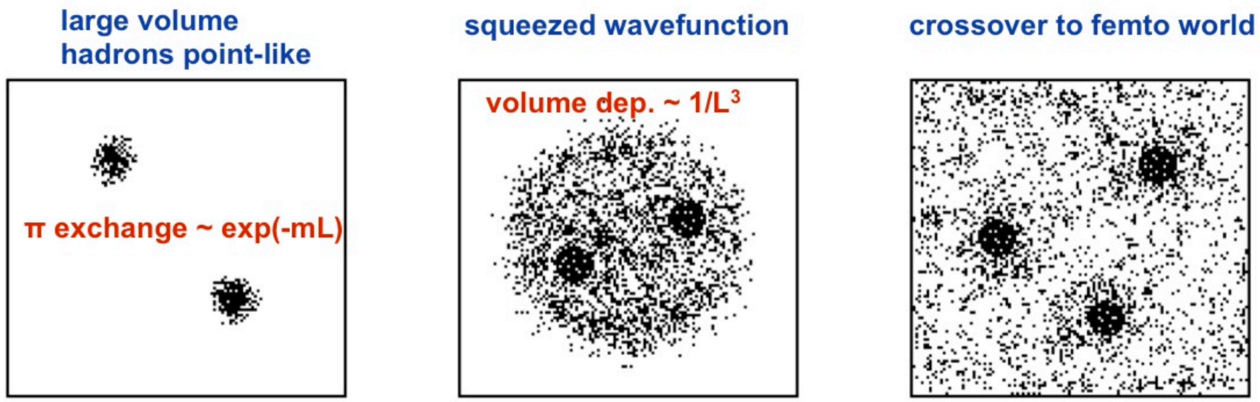
The rotator spectrum has the expansion parameter $\sim C \frac{N_f/2}{F^2 L_s^2}$ with $\ll 1$ condition

with $C \frac{N_f/2}{F^2 L_s^2} = 0.3$ $FL_s \approx 2.5$ for $N_f = 8$ (USQCD project)

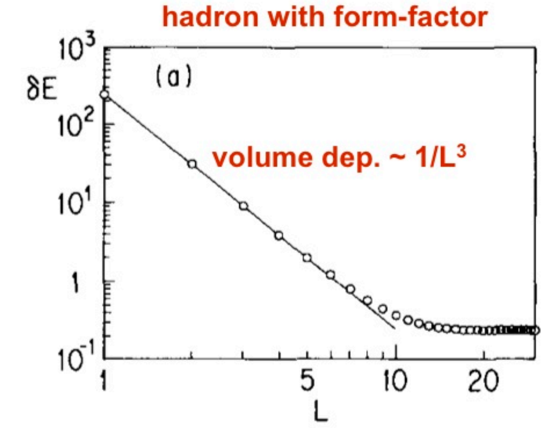
with $a \cdot m_\rho = 0.25$ (to keep cut-off under control), and $m_\rho / F \approx 10$ (as expected), $L_s \approx 100$ is needed!

When expansion breaks down in $\delta$ – regime, same is expected in the p-regime

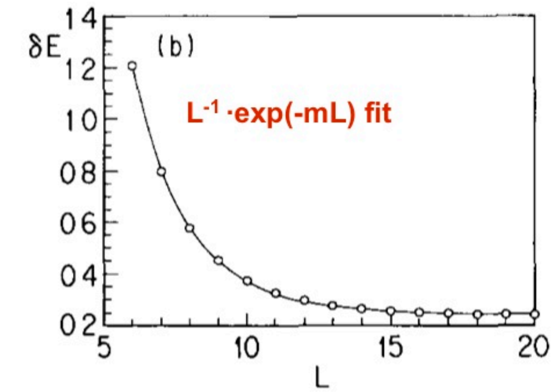
# Deceptions of finite size behavior:




$\hat{V}(\vec{k}) = \frac{F(\vec{k})^2}{\vec{k}^2 + m^2}$ extended hadron with form factor $F(\vec{k})$




$F(k) = \frac{1}{1 + c \cdot \vec{k}^2}$




$F(k) = \frac{1}{1 + c \cdot \vec{k}^2}$

$\delta E = \sum_{\vec{n}} V(\vec{n}L)$ hadrom self energy from interaction with images

$\delta E = \frac{1}{L^3} \sum_{\vec{n}} \hat{V}(\vec{n}\frac{2\pi}{L})$ Poisson resummation, $\hat{V}(\vec{k})$ is the Fourier transform

$\hat{V}(\vec{k}) = \frac{1}{\vec{k}^2 + m^2} \Rightarrow$ V(r)= $\frac{e^{-mr}}{r}$ for large r in point-like approximation

$\delta E \approx V(0) + 6V(L)$ $\quad \delta E \approx \frac{e^{-mL}}{L}$ point-like interaction for large L (non-relativistic)

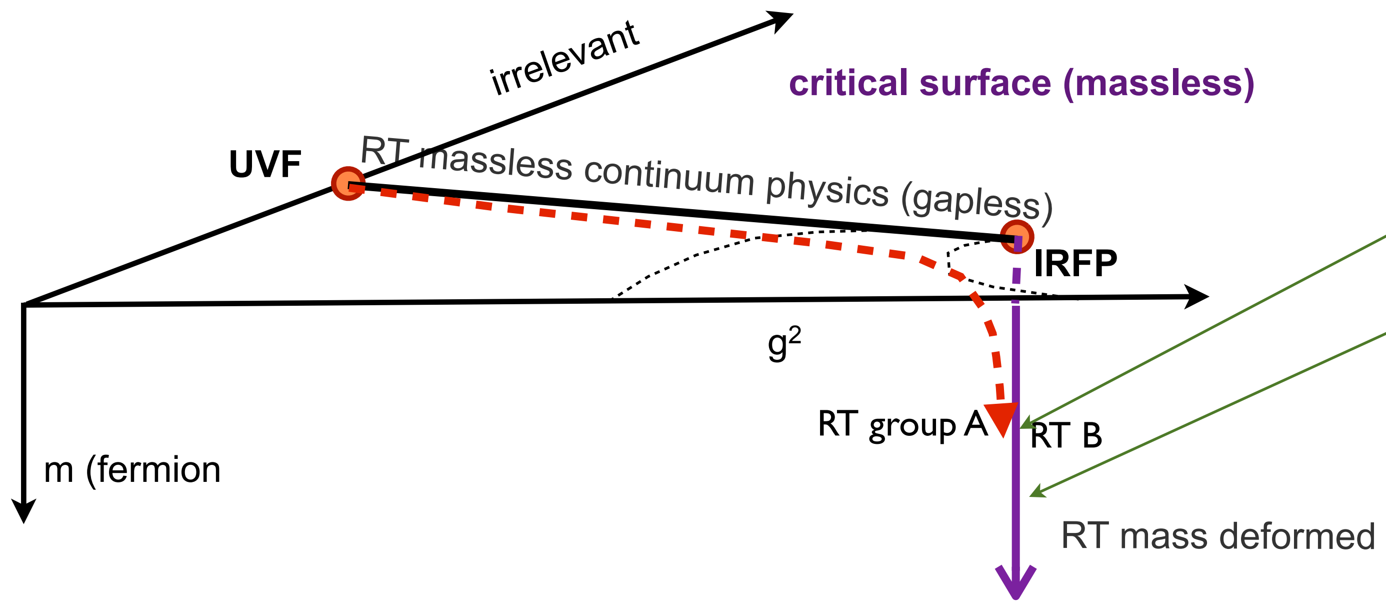
Lüscher made it relativistic using field theory

Leutwyler put in the chiral vertices, hence the $\tilde{g}(mL)$ form in chiral PT

**the size where the $1/L^3$ correction to the masses disappears and the exponential behavior sets in depends on the behavior of the hadron form factor**

**the characteristic inverse power vs. exponential behavior can frustrate at limited lattice sizes the analysis of chiral vs. conformal hypotheses**

# conformal scaling and scaling violations




if model had conformal IRFP

two interchangeable RT descriptions?

continuum mass deformed conformal theory is on RT coming out of IRFP

I worked out as an example all the details of 3D scalar theory (Ising model) with IRFP

textbook material

Del Debbio and collaborators early conform apps

free energy on RT:

$$f(u_1,u_2,\ldots) = g(u_1,u_2,\ldots) \ + \ b^{-d} f_s(b^{y_1}u_1, b^{y_2}u_2\ldots)$$

analytic　　singular

$y_1$ > 0 only relevant exponent in our case
$u_1$ = t ~ m identified, $y_1 = y_m$ in Technicolor notation

$y_2$ controls scaling violations, leading correction term

analytic function which can have terms like ~$m^k$ are typically sub-leading

**Fisher and Brezin worked out most of what we know!**

RG scaling of 2-point function:

$$G^{(2)}(r,m,u_2,\ldots) = b^{-2d} G(r/b, b^{y_m}m, b^{y_2}u_2,\ldots)$$

from $G^{(2)}(r,m,u_2,\ldots) \sim e^{-Mr}$ asymptotics with $M \sim m^{1/y_m}$ scaling follows

leading correction to the scaling term should be $\sim m^{\omega}$ where $\omega = \beta'(g^*)$

analysis would change with second relevant operator at IRFP!

- analytic terms exists, but no reason to be leading conformal scaling correction

- correlators of composite operators require inhomogeneous RG!

**similarly, in conformal finite size scaling analysis:**

$$\xi / L = f_1(x) + L^{-\omega} f_2(x) \ \text{ with } \ x = L m^{1/y_m}$$

→ This directly transcribes to hadron masses and $F_\pi$

correlation length measured in L units

finite size scaling correction terms require very accurate data

**Chiral hypothesis** incomplete analysis on each side **Conformal hypothesis**

chiral logs not reached yet in important models!
(like $N_f$=8, or $N_f$=12)

$$(M_\pi^2)_{NLO} = (M_\pi^2)_{LO} + (\delta M_\pi^2)_{1-loop} + (\delta M_\pi^2)_{m^2} + (\delta M_\pi^2)_{a^2 m} + (\delta M_\pi^2)_{a^4}$$

$\sim m^2$ kept; $\sim a^2 m$ cutoff term in B; $\sim a^4$ see LO $a^2$ term

$$(M_\pi^2)_{LO} = 2B \cdot m + a^2 \Delta_B$$

would require more data

$$(\delta M_\pi^2)_{1-loop} = [(M_\pi^2)_{LO} + a^2]^2 \ln(M_\pi^2)_{LO}$$

$$\boxed{M_\pi^2 = c_1 m + c_2 m^2 \ + \text{logs}}$$

fitted function for all Goldstones

$$\boxed{M_{nuc} = c_0 + c_1 m \ + \ \text{logs}}$$

nucleon states, rho, a1, higgs, ...

$$(F_\pi)_{LO} = F, \quad (\delta F_\pi)_{1-loop} = [(M_\pi^2)_{LO} + a^2] \ln(M_\pi^2)_{LO}$$

chiral log regime was not reached in fermion mass range

$$(\delta F_\pi)_{m^2} \sim m, \quad (\delta F_\pi)_{a^2 m} = a^2$$

kept cutoff term in F

$$\boxed{F_\pi = F + c_1 m + \text{logs}}$$

fitted function

$$\boxed{\langle\bar\psi\psi\rangle = \langle\bar\psi\psi\rangle_0 + c_1 m + c_2 m^2 + \text{logs}}$$

chiral condensate

$$\boxed{M_\pi = c_\pi \cdot m^{1/y_m}, \qquad y_m = 1 + \gamma}$$

leading conformal scaling
functional form for all hadron masses

$$\boxed{F_\pi = c_F \cdot m^{1/y_m}, \qquad y_m = 1 + \gamma}$$

same critical exponent

$$\boxed{\langle\bar\psi\psi\rangle = c_\gamma \cdot m^{(3-\gamma)/y_m} + c_1 m}$$

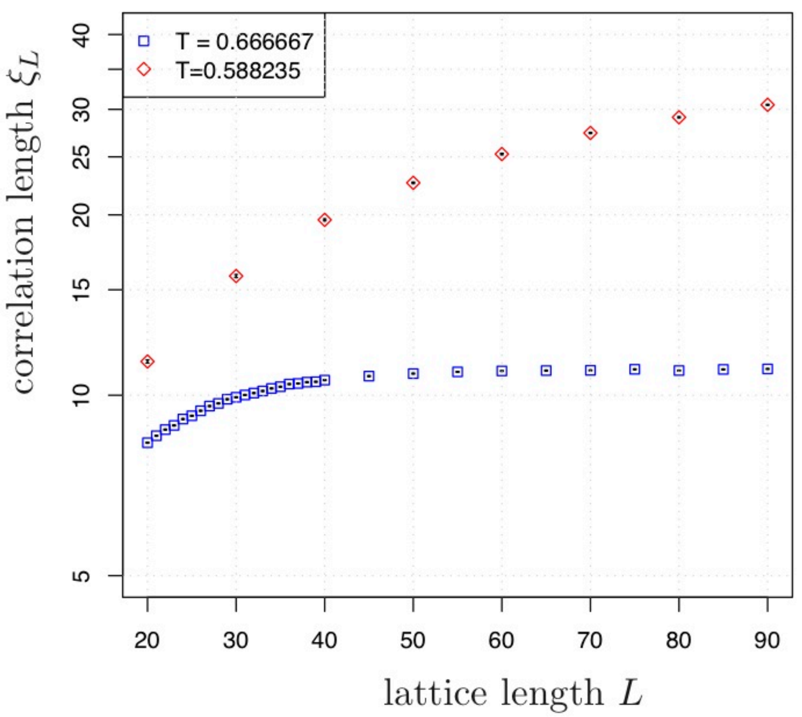
Del Debbio and Zwicky

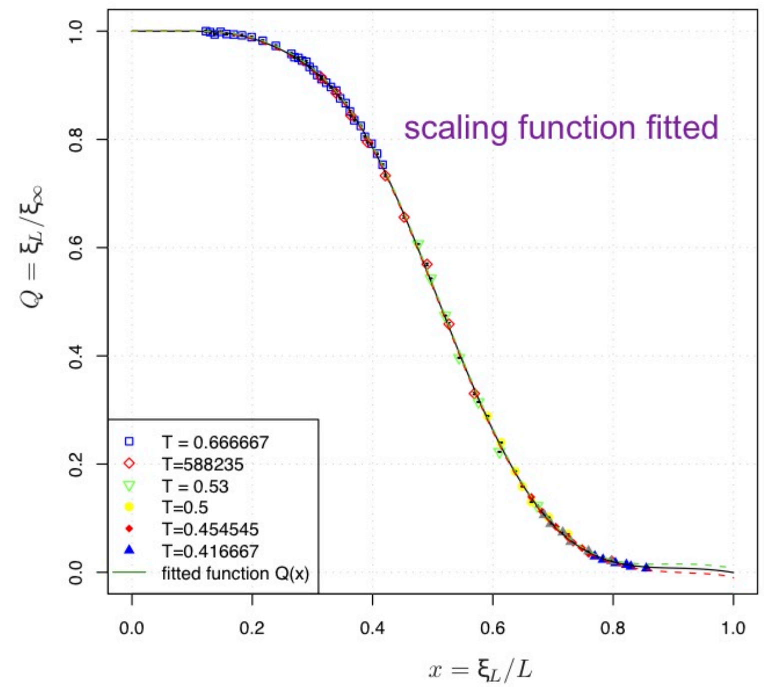
**Asymptotic infinite volume limit has not been reached yet in important candidate models for conformal window**

**infinite volume conformal scaling violation analysis ?**

**conformal finite size scaling analysis and its scaling violations ?**

# **but FSS works!** 2d O(3) model UVFP (at T=1/β=0)




$$H = -\beta \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j \qquad m = 1/\xi = C_m \cdot \Lambda_L$$

$$\Lambda_L = 2\pi\beta \cdot \exp(-2\pi\beta)[1 + a_1/\beta + ...]$$

from Bethe ansatz:

$$m/\Lambda_{MS} = 8/e \qquad \Lambda_{MS}/\Lambda_L = 2^{5/2} e^{\pi/2}$$

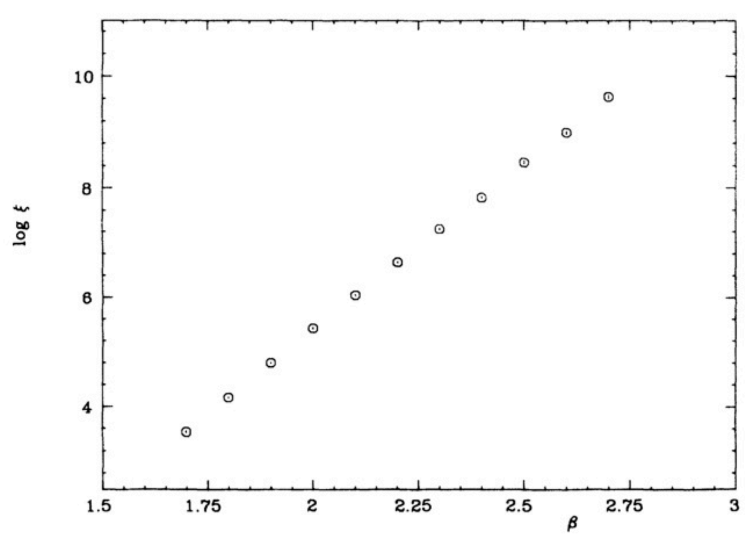
from FSS:

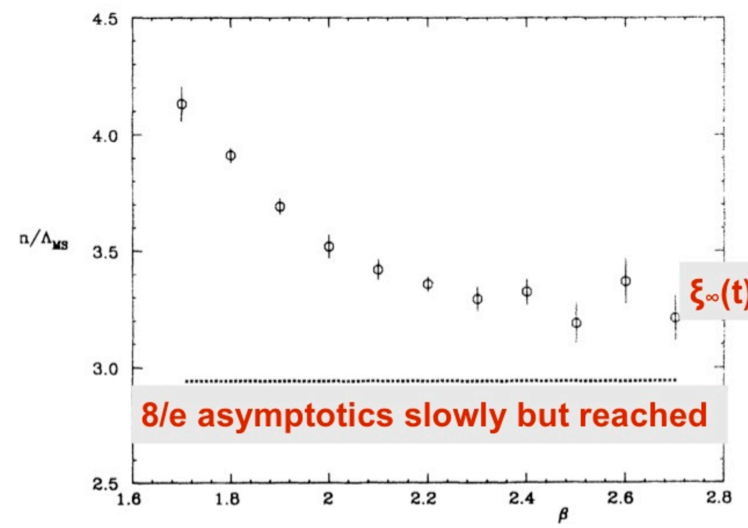
$$P_L(t) = P_\infty(t) \cdot Q_P(x(t)), \quad x(t) = \xi_L(t)/L$$

for any bulk physical quantity P(t)

**$Q_P(x(t))$ does not depend on t explicitly!**

applied to $P_\infty(t) = \xi_\infty(t)$



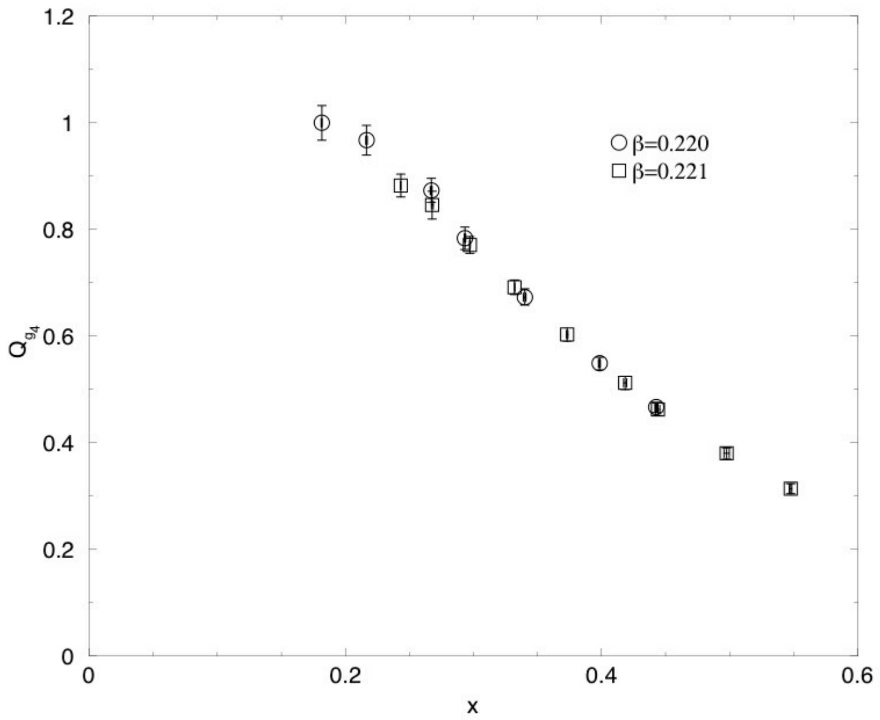




**8/e asymptotics slowly but reached**

**$\xi_\infty(t)$ would be ~ 22,000 at β=2.7 !**

**FSS is enormously powerful**

# FSS works again!  3d Ising model IRFP $(g_4)^*$ conformal




applied again from FSS:

$$P_L(t) = P_\infty(t) \cdot Q_P(x(t)), \quad x(t) = \xi_L(t)/L$$

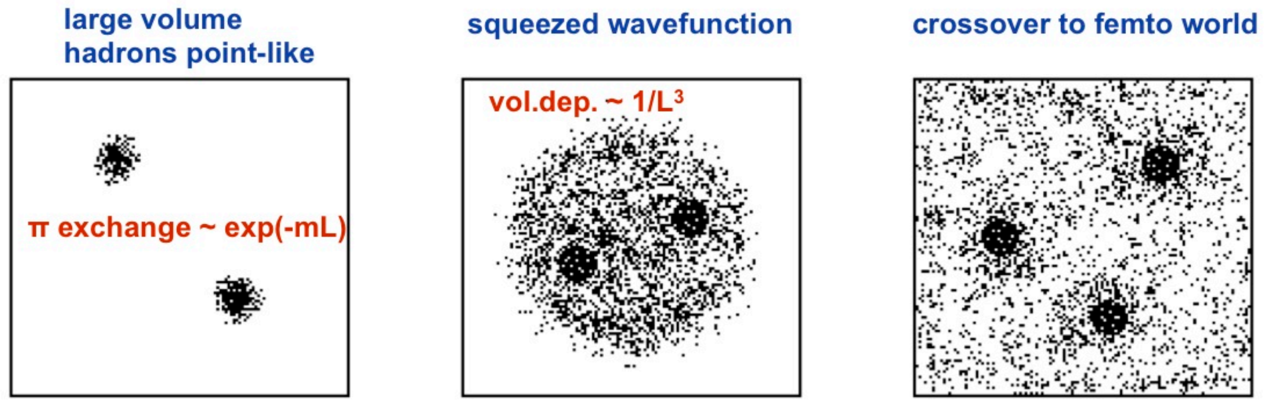
applied to $P_\infty(t)=g_4(t)$ renormalized coupling

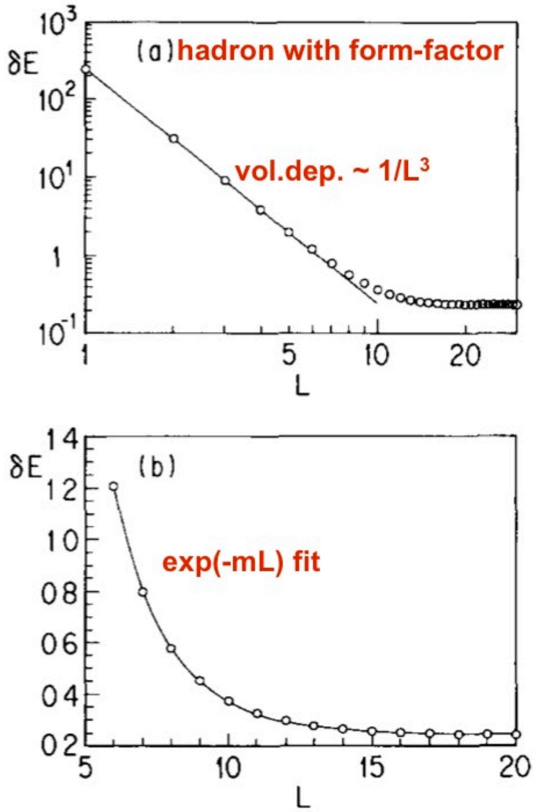
$$g_4(t) = -\frac{\chi_4(t)}{\xi^3 \cdot \chi_2(t)^2}$$

we are working on similar FSS methods in Nf=12 model under the conformal hypothesis

caviats:
composite operators and composite states make a similar analysis more difficult

can the two phases (chiral and conformal) get confused in FSS?



# Nf=12 fundamental representation

Nf=12 flavors with fermions in the fundamental rep of SU(3) color gauge group
just below the conformal window?
fermion condensate, $F_{ps}$ and hadron spectrum were determined

published data set (condensate in separate table):

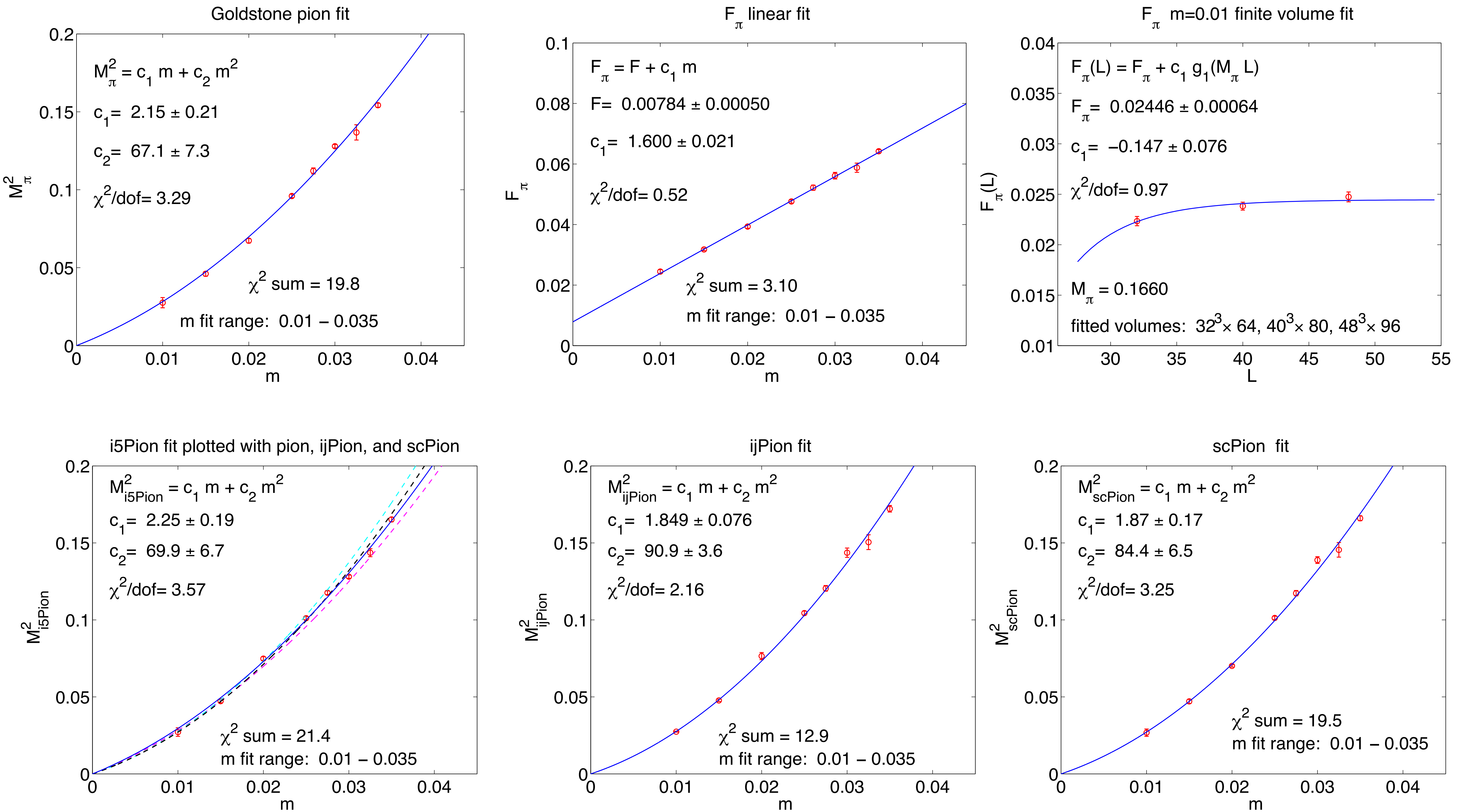
**<u>Twelve massless flavors and three colors below the conformal window.</u>**
**Phys.Lett. B703 (2011) 348-358**
e-Print: **arXiv:1104.3124 [hep-lat]** **Lattice Higgs Collaboration**

| mass | lattice | $M_\pi$ | $F_\pi$ | $M_{i5}$ | $M_{sc}$ | $M_{ij}$ | $M_{nuc}$ | $M_{pnuc}$ | $M_{Higgs}$ | $M_{rho}$ | $M_{A1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0100 | $32^3 \times 64$ | 0.2195(35) | 0.02234(46) | 0.2171(31) | 0.194(10) | 0.195(11) | 0.386(16) | 0.387(22) | 0.2162(53) | 0.239(19) | 0.246(21) |
| 0.0100 | $40^3 \times 80$ | 0.1819(28) | 0.02382(39) | 0.1842(29) | 0.1835(35) | 0.1844(44) | 0.3553(93) | 0.352(16) | 0.2143(81) | 0.2166(73) | 0.237(12) |
| 0.0100 | $48^3 \times 96$ | 0.1647(23) | 0.02474(49) | 0.1650(13) | 0.16437(95) | 0.1657(10) | 0.3066(69) | 0.3051(81) | 0.247(13) | 0.1992(28) | 0.2569(83) |
| 0.0150 | $32^3 \times 64$ | 0.2322(34) | 0.03168(64) | 0.2319(11) | 0.2318(17) | 0.2341(16) | 0.4387(60) | 0.4333(84) | 0.2847(33) | 0.2699(41) | 0.324(16) |
| 0.0150 | $40^3 \times 80$ | 0.2200(23) | 0.03167(53) | 0.2210(21) | 0.2218(30) | 0.2239(34) | 0.4095(84) | 0.411(10) | 0.291(11) | 0.2574(36) | 0.327(14) |
| 0.0150 | $48^3 \times 96$ | 0.2140(14) | 0.03153(51) | 0.2167(16) | 0.2165(17) | 0.2185(18) | 0.3902(67) | 0.3881(84) | 0.296(13) | 0.2506(33) | 0.3245(87) |
| 0.0200 | $40^3 \times 80$ | 0.2615(17) | 0.03934(56) | 0.2736(22)* | 0.2651(8) | 0.2766(42)* | 0.4673(62) | 0.4699(66) | 0.330(17) | 0.3049(28) | 0.361(32) |
| 0.0250 | $32^3 \times 64$ | 0.3098(18) | 0.04762(53) | 0.3179(17) | 0.3183(18) | 0.3231(20) | 0.563(12) | 0.563(14) | 0.4137(88) | 0.3683(19) | 0.469(14) |
| 0.0275 | $24^3 \times 48$ | 0.3348(29) | 0.05218(85) | 0.3430(18) | 0.3425(25) | 0.3471(26) | 0.609(21) | 0.628(23) | 0.460(16) | 0.4050(69) | 0.523(34) |
| 0.0300 | $24^3 \times 48$ | 0.3576(15) | 0.0561(11) | 0.3578(15)* | 0.3726(29) | 0.3790(40) | 0.640(12)* | 0.633(16)* | 0.470(15) | 0.4160(26)* | 0.5222(90)* |
| 0.0325 | $24^3 \times 48$ | 0.3699(66) | 0.0588(15) | 0.3790(34) | 0.3814(62) | 0.3879(62) | 0.680(18) | 0.686(26) | 0.500(21) | 0.4481(39) | 0.548(31) |
| 0.0350 | $24^3 \times 48$ | 0.3927(17) | 0.06422(57) | 0.4065(18) | 0.4074(19) | 0.4149(26) | 0.703(28) | 0.741(20) | 0.538(30) | 0.4725(64) | 0.669(65) |

tested with two opposite hypotheses (chiSB vs. conformal symmetry)

assumptions:
- with exception of condensate only minimal leading functions are applied in both hypotheses
- global analysis is used in different channel combinations and linear term is added to condensate to account for UV effects
- continuum fitting at fixed gauge coupling without further tests of cutoff effects (will be addressed)

# Nf=12 Goldstone spectrum and $F_{ps}$ (Lattice Higgs Collaboration)

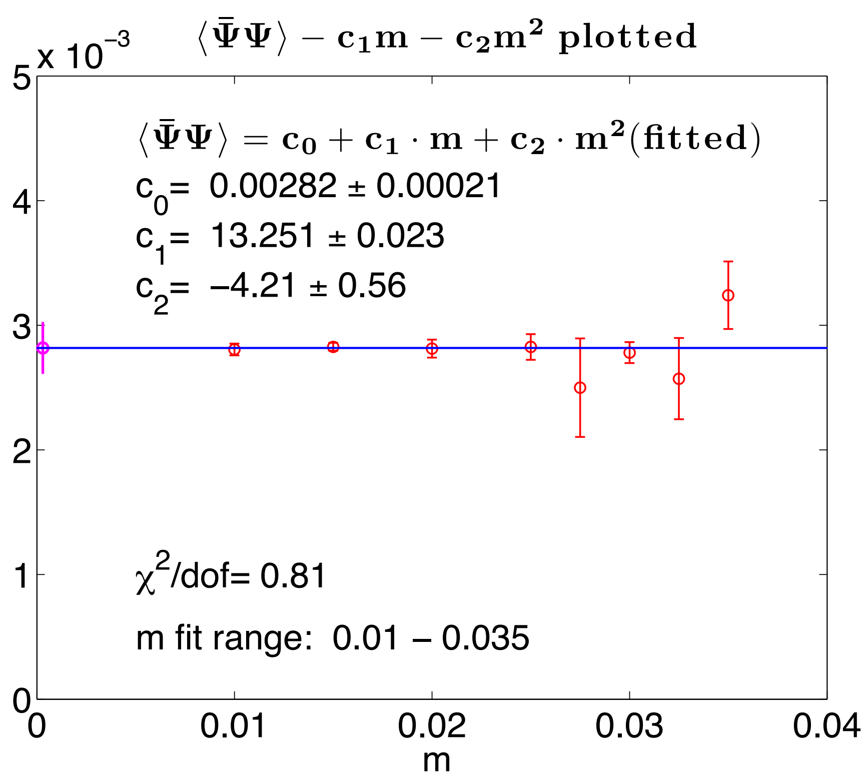


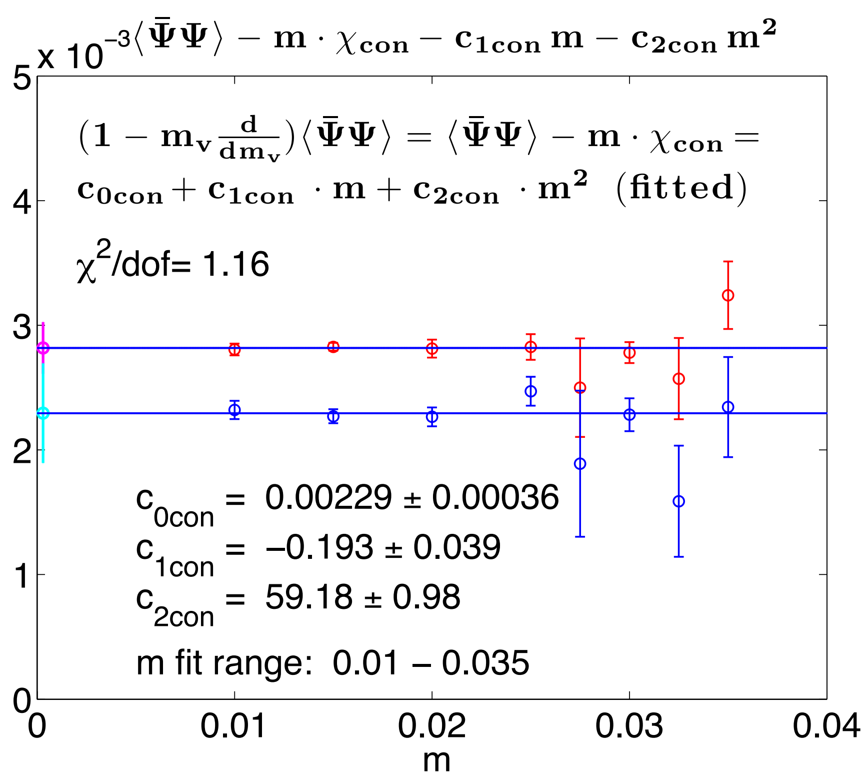
# Chiral condensate (LHC)

| mass | lattice | $\langle\bar\psi\psi\rangle$ | $\langle\bar\psi\psi\rangle - m\cdot\chi_{con}$ |
|---|---|---|---|
| 0.0100 | $48^3\times96$ | 0.134896(47) | 0.006305(73) |
| 0.0150 | $48^3\times96$ | 0.200647(31) | 0.012685(56) |
| 0.0200 | $40^3\times80$ | 0.266151(72) | 0.022069(76) |
| 0.0250 | $32^3\times64$ | 0.33147(10) | 0.03462(12) |
| 0.0275 | $24^3\times48$ | 0.36372(40) | 0.04133(59) |
| 0.0300 | $32^3\times32$ | 0.396526(84) | 0.04974(13) |
| 0.0325 | $24^3\times48$ | 0.42879(33) | 0.05781(45) |
| 0.0350 | $24^3\times48$ | 0.46187(27) | 0.06807(40) |

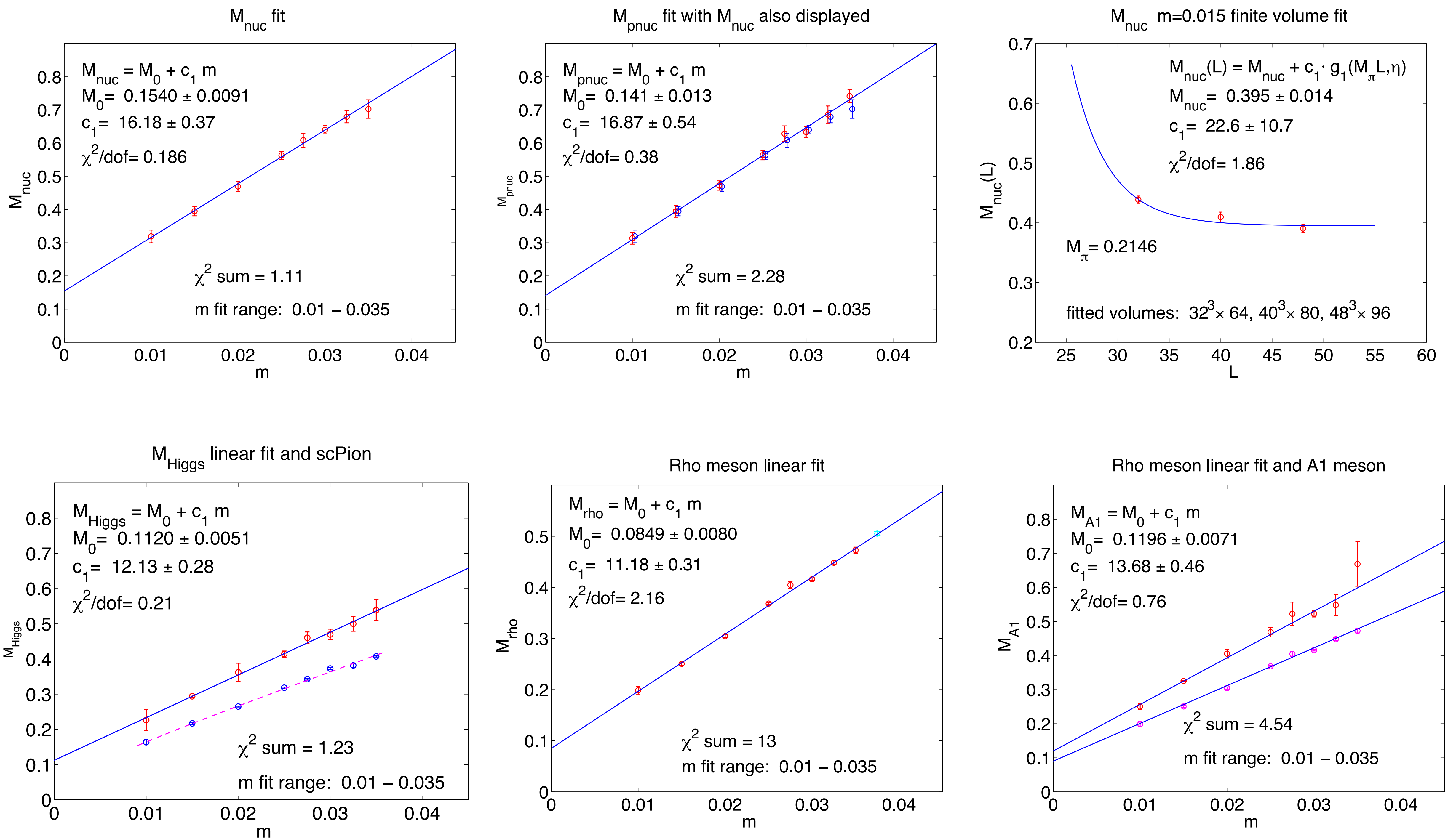
$$\langle\bar\psi\psi\rangle = -2m\cdot\int_0^{\mu}\frac{d\lambda\rho(\lambda)}{m^2+\lambda^2}$$
$$= -2m^5\cdot\int_\mu^{\infty}\frac{d\lambda}{\lambda^4}\frac{\rho(\lambda)}{m^2+\lambda^2} + c_1\cdot m + c_3\cdot m^3$$

$$(1-m_v\frac{d}{dm_v})\langle\bar\psi\psi\rangle\,|_{m_v=m} = \langle\bar\psi\psi\rangle - m\cdot\chi_{con}\,,$$
$$\chi = \frac{d}{dm}\langle\bar\psi\psi\rangle = \chi_{con}+\chi_{disc}\,,$$
$$\chi_{con} = \frac{d}{dm_v}\langle\bar\psi\psi\rangle_{pq}\,|_{m_v=m}\,.$$

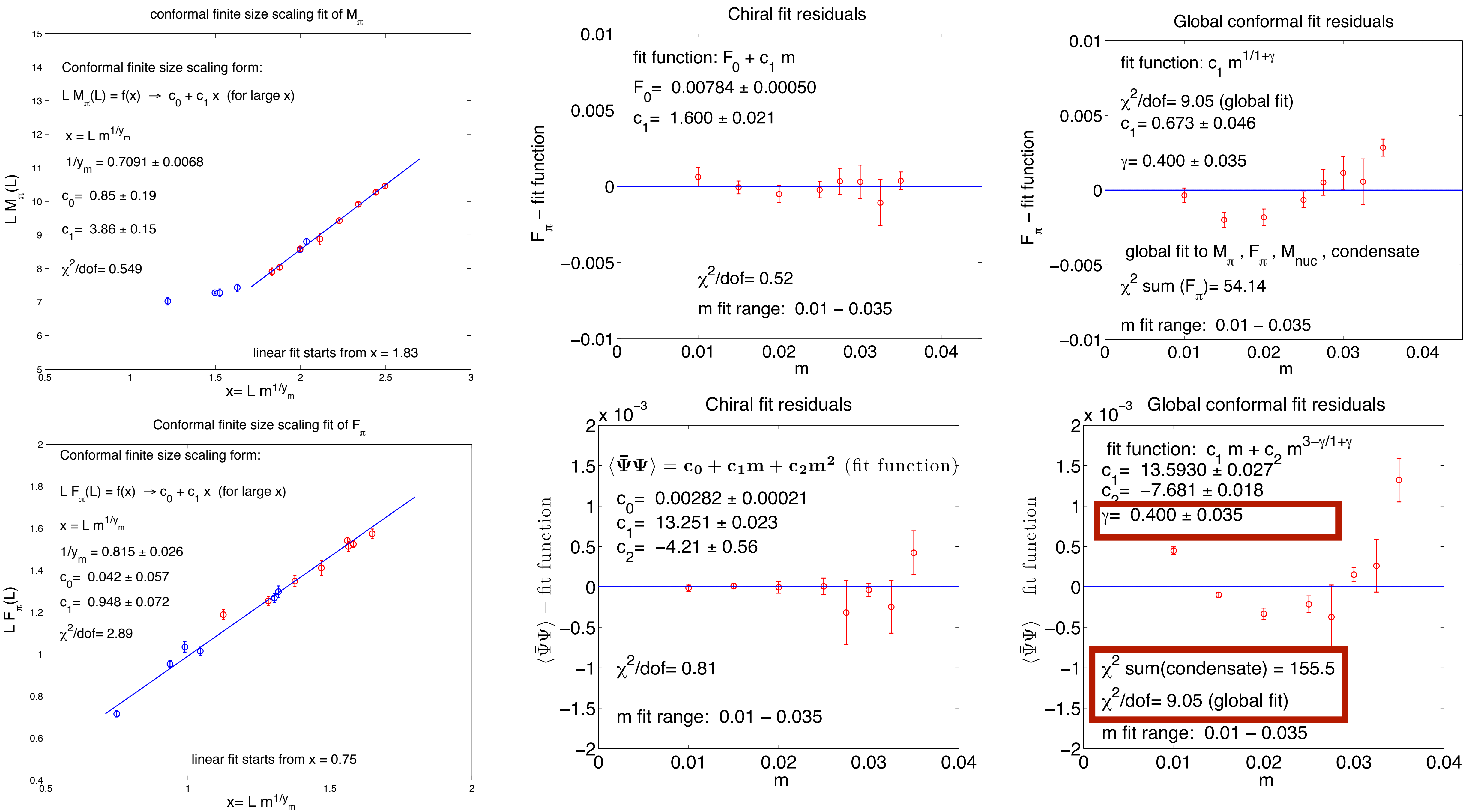


# Nf=12 Hadron spectrum (LHC)



# Limited comparison of the two Nf=12 hypotheses (LHC)




**re-analysis of Appelquist et al. adds analytic non-leading terms: conformal OK**

**new FSS analysis from Lattice Higgs Collaboration: conformal not OK ... to continue**

**DeGrand's objection (ignoring Lattice 2011 LHC analysis)**

# Summary of fits without the condensate using minimal fitting functions and Appelquist et al. added terms

## 10 channels, global fit, condensate not included

| m-range | chi2 | chi2/dof | Mpi | Fpi | Mnuc | Mrho | $Ma_1$ | Mhiggs | M'nuc | Msc | Mi5 | Mij |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 masses | 14.56 | 0.73 | 1.21 | 0.81 | 0.45 | 3.23 | 0.33 | 0.16 | 0.20 | 2.26 | 4.18 | 1.72 |
| 0.01-0.025 | 88.52 | 3.05 | 4.94 | 7.79 | 7.80 | 19.01 | 3.70 | 7.70 | 4.27 | 17.26 | 7.67 | 8.38 |
| | 87.52 | 3.13 | 4.86 | 6.82 | 7.69 | 8.89 | 3.66 | 7.64 | 4.20 | 17.15 | 7.87 | 8.74 |
| 5 masses | 18.99 | 0.63 | 1.51 | 1.64 | 0.56 | 4.56 | 0.44 | 0.79 | 1.11 | 2.26 | 4.19 | 1.93 |
| 0.01-0.0275 | 112.09 | 2.87 | 7.35 | 11.90 | 9.13 | 23.34 | 4.33 | 8.79 | 6.34 | 25.79 | 8.02 | 7.10 |
| | 109.69 | 2.89 | 7.23 | 9.57 | 8.97 | 23.16 | 4.27 | 8.72 | 6.26 | 25.67 | 8.28 | 7.55 |
| 6 masses | 49.82 | 1.25 | 1.53 | 1.94 | 0.61 | 12.58 | 3.10 | 1.13 | 1.89 | 3.97 | 19.45 | 3.62 |
| 0.01-0.030 | 164.88 | 3.36 | 11.74 | 15.46 | 9.61 | 25.31 | 7.09 | 9.31 | 6.84 | 47.08 | 18.35 | 14.08 |
| | 160.60 | 3.35 | 11.94 | 11.30 | 9.42 | 24.95 | 6.93 | 9.22 | 6.72 | 47.44 | 17.89 | 14.79 |
| 7 masses | 62.42 | 1.25 | 4.31 | 2.16 | 0.64 | 12.61 | 3.36 | 1.23 | 1.89 | 8.44 | 20.89 | 6.89 |
| 0.01-0.0325 | 170.03 | 2.88 | 12.74 | 16.44 | 9.57 | 26.46 | 7.33 | 9.36 | 6.89 | 47.15 | 19.73 | 14.36 |
| | 164.80 | 2.84 | 12.92 | 11.35 | 9.37 | 26.18 | 7.14 | 9.26 | 6.77 | 47.56 | 19.13 | 15.11 |
| 8 masses | 98.88 | 1.65 | 19.76 | 3.10 | 1.11 | 12.98 | 4.54 | 1.23 | 2.28 | 19.48 | 21.43 | 12.95 |
| 0.01-0.035 | 214.30 | 3.11 | 18.96 | 43.07 | 10.62 | 27.88 | 9.22 | 9.74 | 8.49 | 50.43 | 22.97 | 12.91 |
| | 188.33 | 2.77 | 17.94 | 18.05 | 10.11 | 27.19 | 8.83 | 9.52 | 8.34 | 52.21 | 21.62 | 1 4.52 |

Red chi2 values are based on chiSB hypothesis based on fits to posted PLB paper including finite volume corrections for low mass values

There are two blue chi2 fits:

(a) original minimal conformal fit
(b) Appelquist et al. fits to posted PLB Table including the $D_F\, m$ "correction term" they introduced

chiSB hypotheses (analytic form) good confidence level chi2/dof~1 in low mass range
conformal fit shows lower confidence level: chi2/dof ~ 3 in low mass range
it drops to chi2/dof ~ 2 if 3 pseudo-Goldstones are left out and condensate included with added extra fit term

## 6 channels, global fit, condensate not included

| m-range | chi2 | chi2/dof | Mpi | Fpi | Mrho | Msc | Mi5 | Mij | conform exponent |
|---|---|---|---|---|---|---|---|---|---|
| 4 masses | 13.41 | 1.12 | 1.21 | 0.81 | 3.23 | 2.26 | 4.18 | 1.72 | |
| 0.010-0.025 | 63.76 | 3.99 | 5.24 | 6.85 | 19.41 | 17.71 | 7.17 | 7.37 | 0.398(19) |
| 5 masses | 16.09 | 0.89 | 1.51 | 1.64 | 4.56 | 2.26 | 4.19 | 1.93 | |
| 0.01-0.0275 | 81.05 | 3.68 | 7.52 | 9.60 | 23.59 | 26.01 | 7.74 | 6.58 | 0.383(15) |
| 6 masses | 43.09 | 1.80 | 1.53 | 1.94 | 12.58 | 3.97 | 19.45 | 3.62 | |
| 0.01-0.030 | 127.69 | 4.56 | 11.54 | 11.33 | 25.79 | 46.72 | 19.02 | 13.28 | 0.377(15) |
| 7 masses | 55.30 | 1.84 | 4.31 | 2.16 | 12.61 | 8.44 | 20.89 | 6.89 | |
| 0.01-0.0325 | 131.68 | 3.87 | 12.61 | 11.38 | 26.76 | 46.84 | 20.37 | 13.72 | 0.378(15) |
| 8 masses | 89.70 | 2.49 | 19.76 | 3.10 | 12.98 | 19.48 | 21.43 | 12.95 | |
| 0.01-0.035 | 151.14 | 3.78 | 18.56 | 18.09 | 27.62 | 51.00 | 22.44 | 13.44 | 0.374(11) |

Red chi2 values are based on chiSB hypothesis based on fits to posted PLB paper including finite volume corrections for low mass values

blue chi2 fits: Appelquist et al. type fits to posted PLB Table including the $D_F\, m$ "correction term" they introduced

- chiSB hypotheses (analytic form) good confidence level chi2/dof~1 in low mass range
- conformal fit shows significantly lower confidence level chi2/dof ~ 4 in low mass range

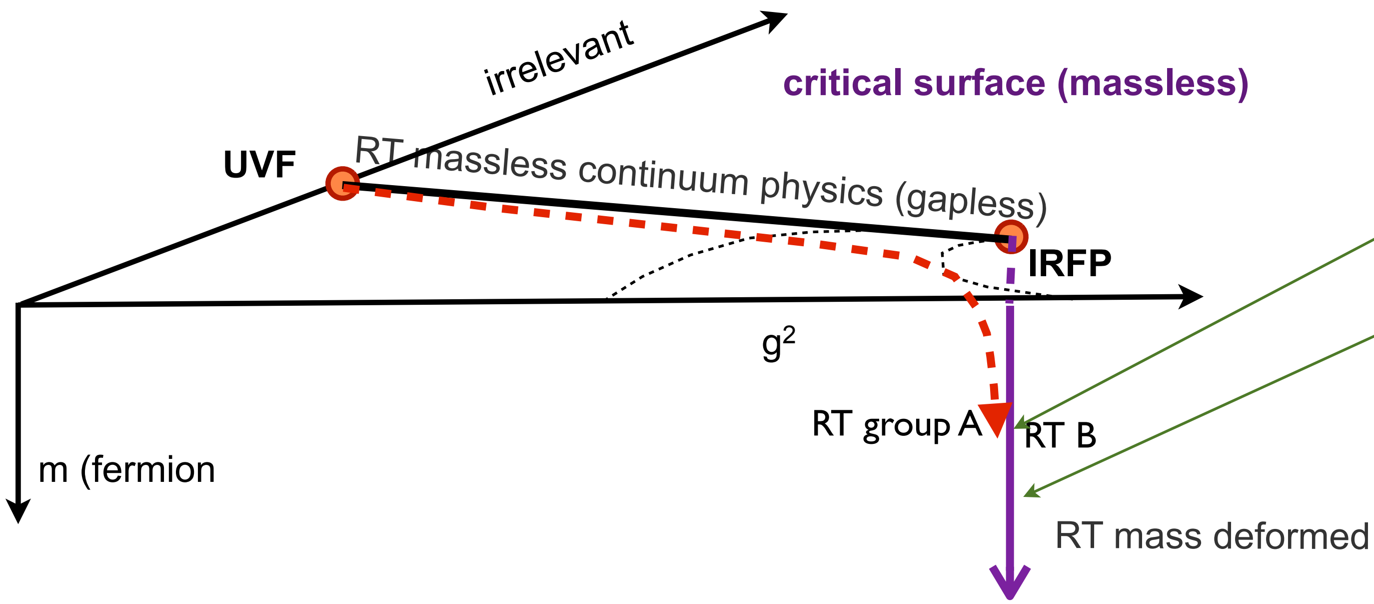
# Onto more tests with 6 channels using conformal finite size scaling

# Conformal finite size scaling analysis

## based on our extended subset of data:

**This table will not be posted before our new paper is submitted**

# Conformal scaling and scaling violation




if Nf=12 had conformal IRFP

two interchangeable RT descriptions?

continuum mass deformed conformal theory is on RT coming out of IRFP

we worked out as an example all the details of 3D scalar theory with IRFP

textbook material

free energy on RT:

$$f(u_1,u_2,...) = g(u_1,u_2,...) \ + \ b^{-d} f_s(b^{y_1}u_1,b^{y_2}u_2...)$$

analytic singular

$y_1 > 0$ only relevant exponent in our case
$u_1 = t \sim m$ identified, $y_1 = y_m$ in Technicolor notation

$y_2$ controls scaling violations, leading correction term
analytic function which can have terms like ~m are typically sub-leading
like ~ $D_F$ correction term of Appelquist et al.

RG scaling of 2-point function:

$$G^{(2)}(r,m,u_2,...) = b^{-2d} G(r/b, b^{y_m} m, b^{y_2} u_2,...)$$

from $G^{(2)}(r,m,u_2,...) \sim e^{-Mr}$ asymptotics $M \sim m^{1/y_m}$ scaling follows

leading correction to the scaling term should be $\sim m^{\omega}$ where $\omega = \beta'(g^*)$

Appelquist et al. assumed $\omega = 1$ with the $D_F m$ term added to $F_\pi$

and similarly for hadron masses

the term exists, but no reason to be leading conformal scaling correction

the correction term $\sim m^{3/1+\gamma}$ added to $\langle \bar{\psi}\psi \rangle$ is even more ad hoc and may not exist

**similarly, in conformal finite size scaling analysis:**

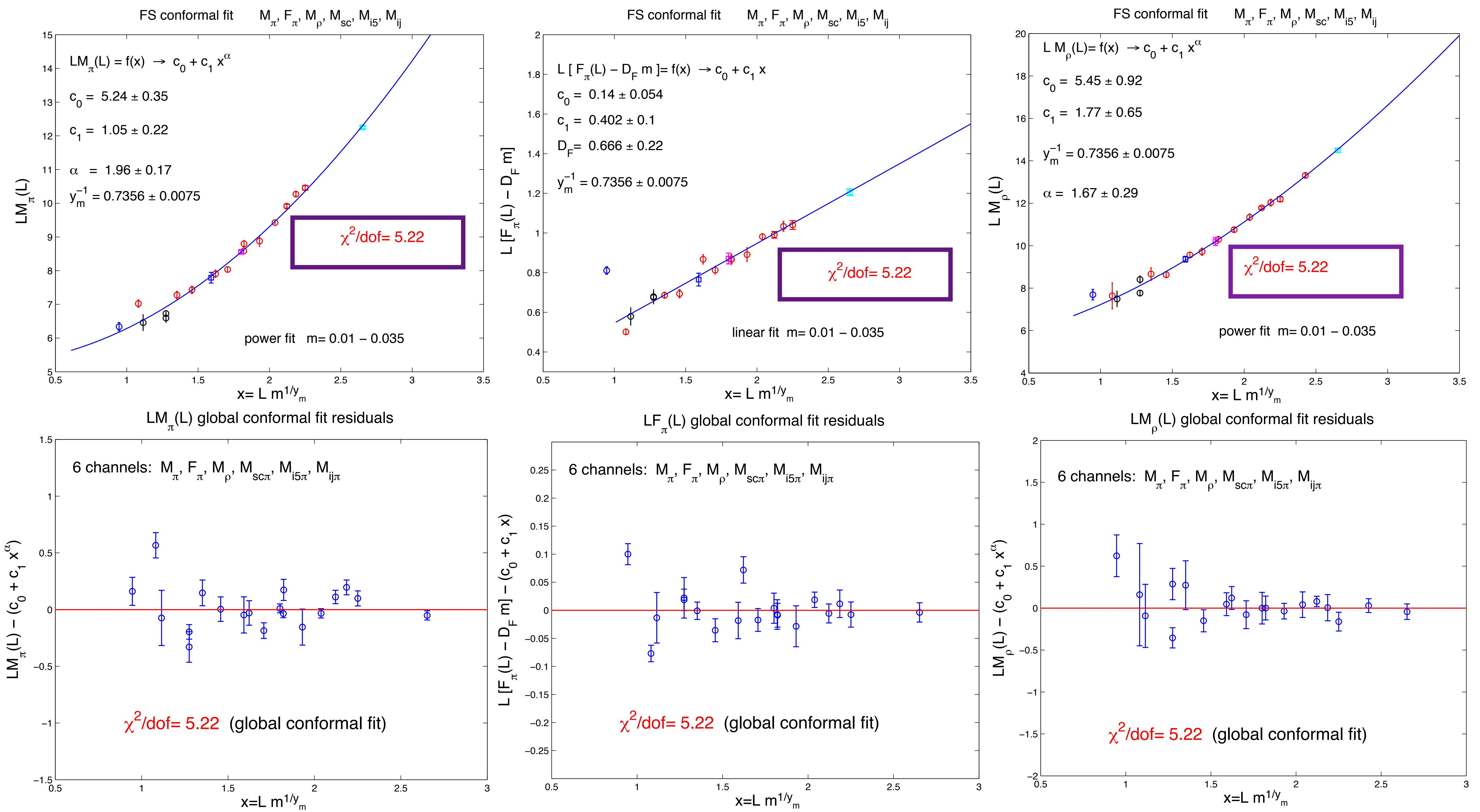
$$\xi / L = f_1(x) + L^{-\omega} f_2(x) \ \text{ with } \ x = Lm^{1/y_m}$$ →

correlation length measured in L units

This directly transcribes to hadron masses and $F_\pi$

finite size scaling correction term requires very accurate data

# Conformal finite size scaling analysis with 6 channels in m=0.01-0.035 range with 9 mass values

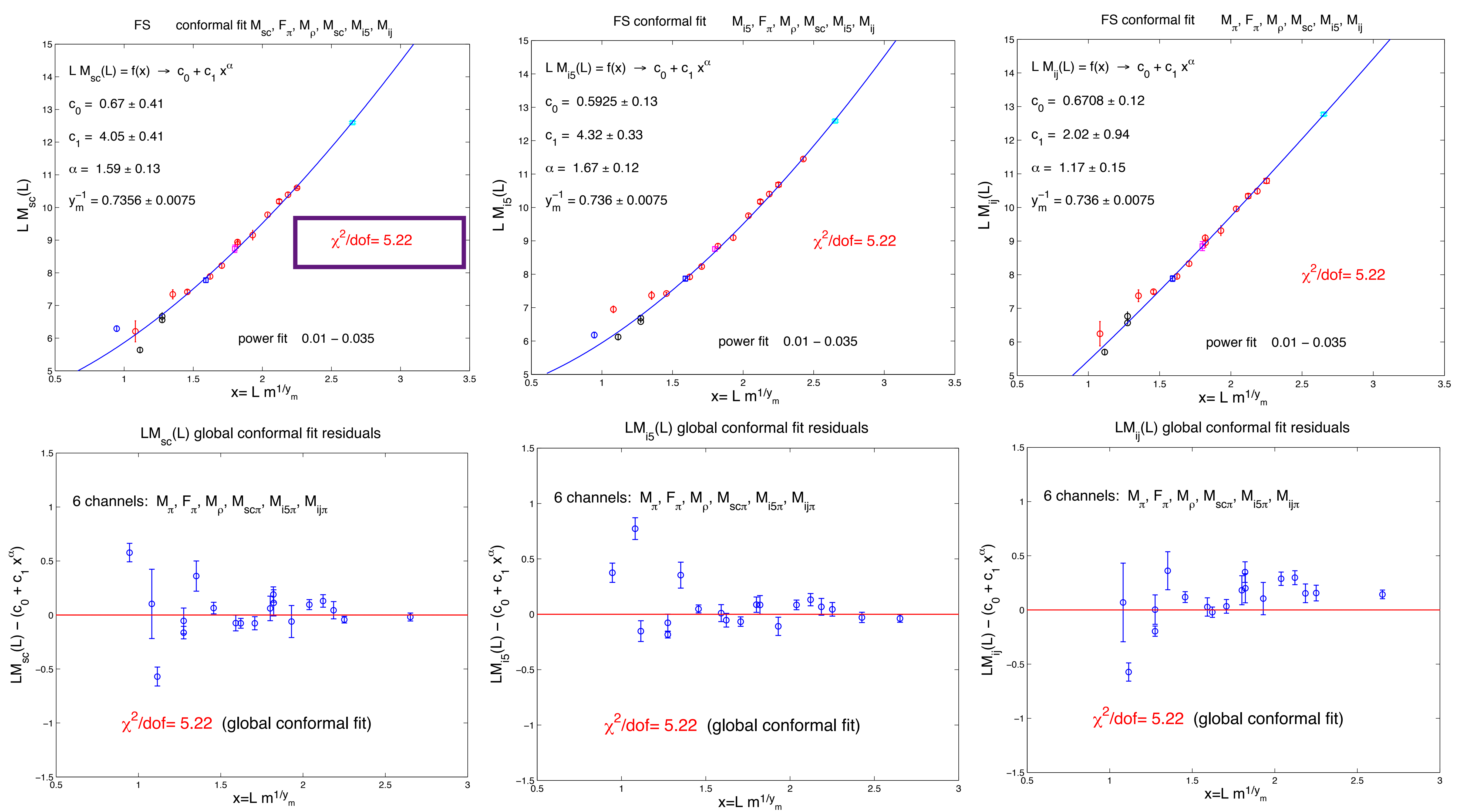



(a) the power fit to $F_\pi$ is consistent with $\alpha = 1$ and does not improve the fit

(b) concern about barely detectable taste breaking in pseudo-Goldstones?
removing them is still a bad conformal fit!

(c) lowering the mass range to m=0.01-0.025, or m=0.01-0.02 will make the fits worse

# Conformal finite size scaling analysis with 6 channels in m=0.01-0.035 range with 9 mass values

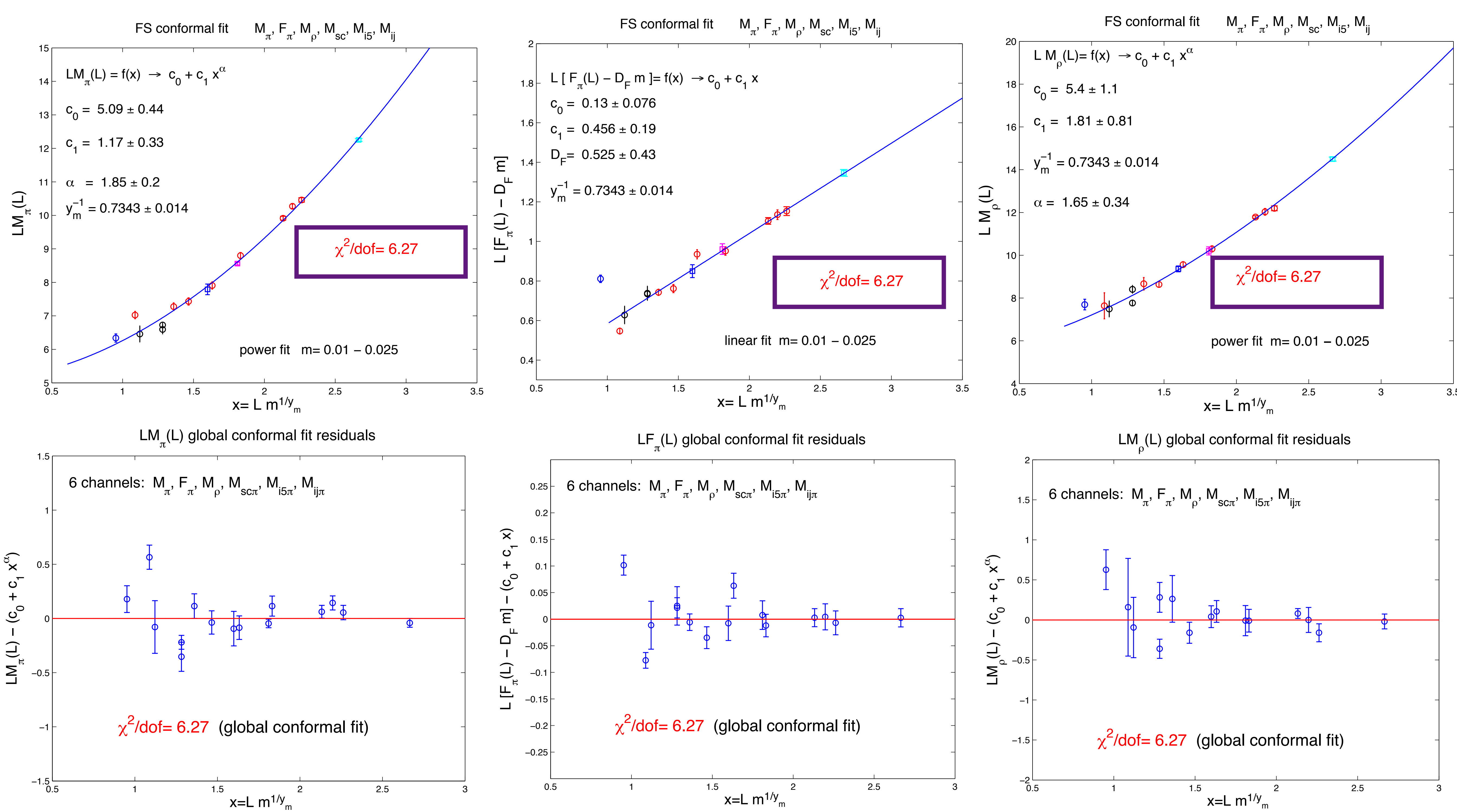



(a) the power fit to $F_\pi$ is consistent with $\alpha = 1$ and does not improve the fit

(b) concern about barely detectable taste breaking in pseudo-Goldstones?
removing them is still a bad conformal fit!

(c) lowering the mass range to m=0.01-0.025, or m=0.01-0.02 will make the fits worse

# Conformal finite size scaling analysis with 6 channels in m=0.01-0.025 range with 5 mass values

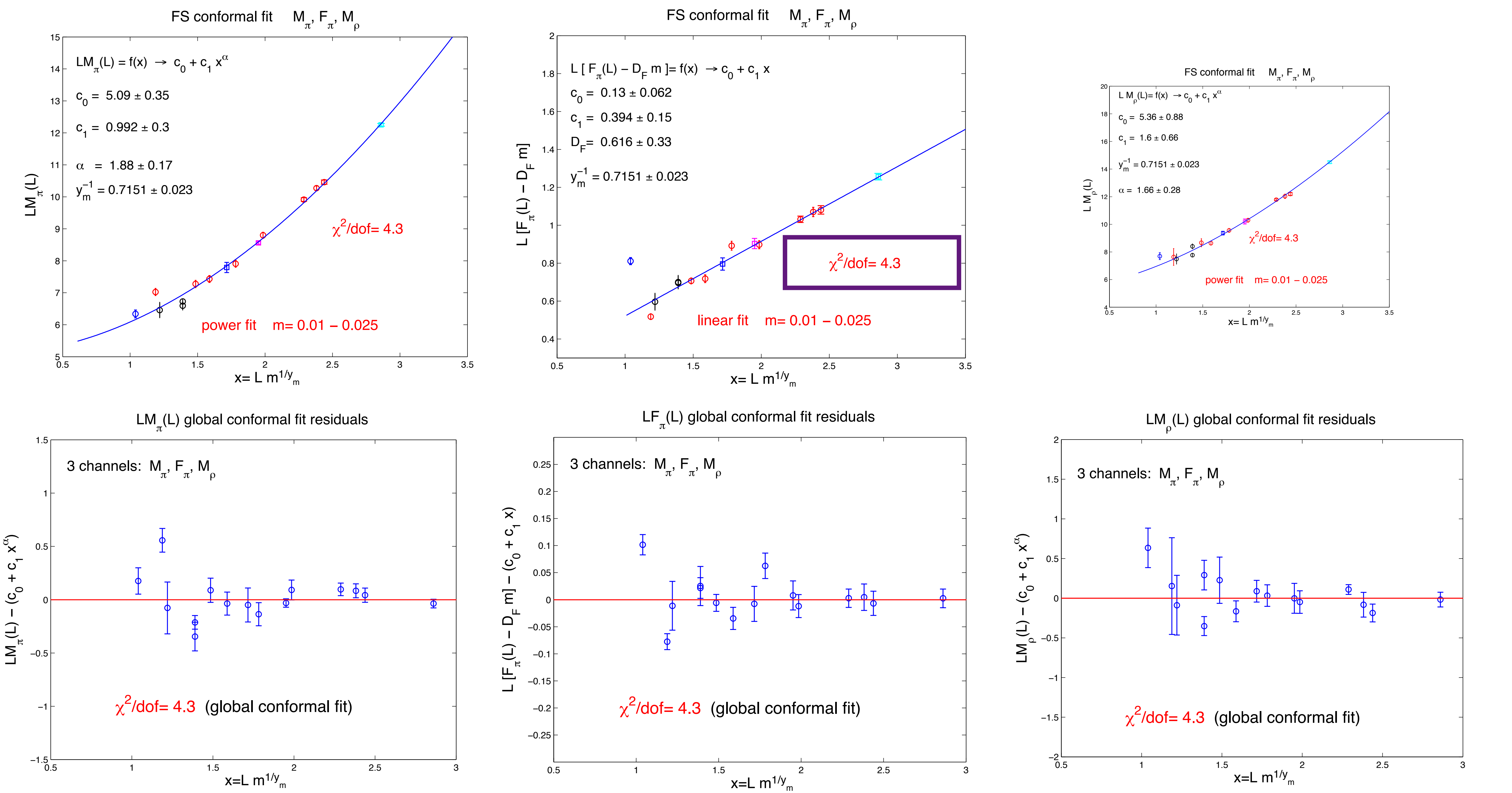



(a) the power fit to $F_\pi$ is consistent with $\alpha = 1$ and does not improve the fit

(b) concern about barely detectable taste breaking in pseudo-Goldstones?
removing them is still a bad conformal fit!

(c) lowering the mass range to m=0.01-0.025 does make the fits worse!

# Conformal finite size scaling analysis with 3 channels in m=0.01-0.025 range with 5 mass values

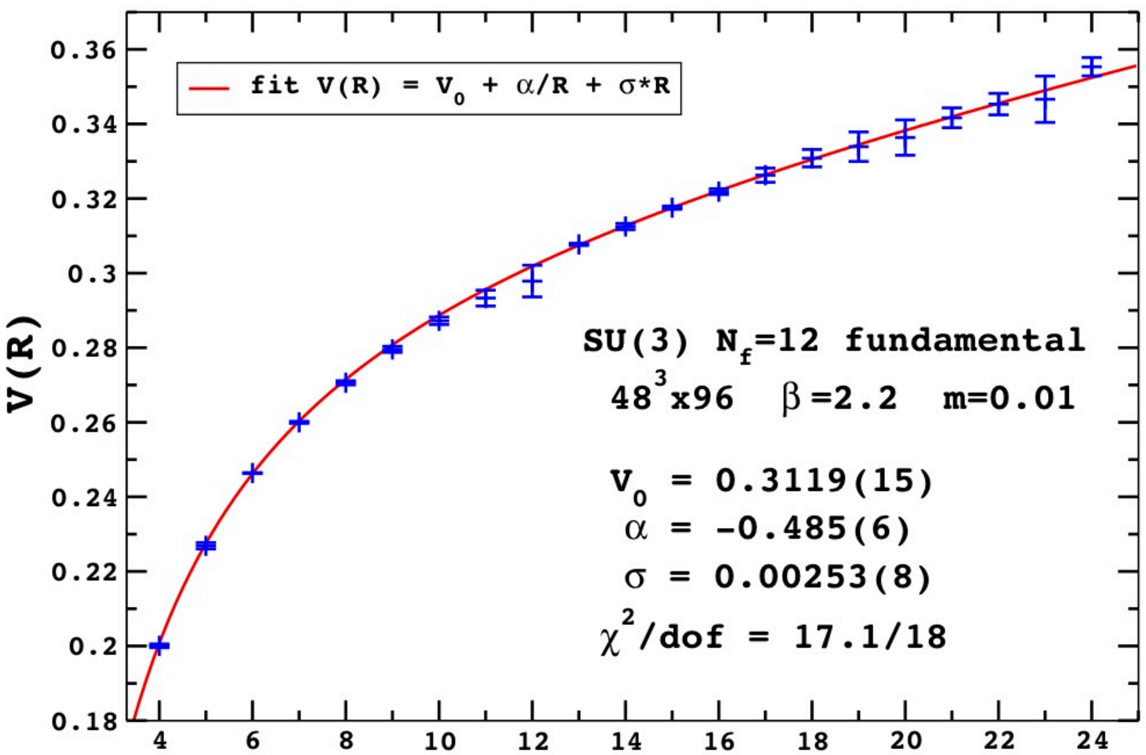


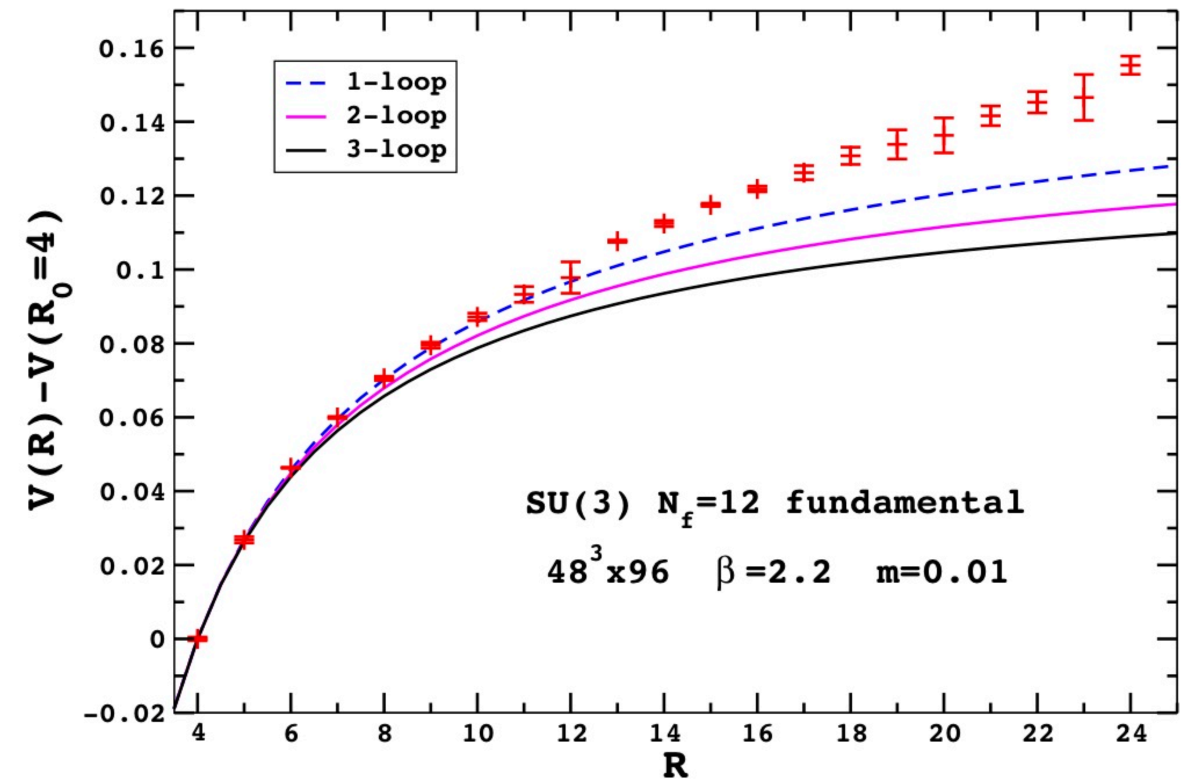

(a) the power fit to $F_\pi$ is consistent with $\alpha = 1$ and does not improve the fit

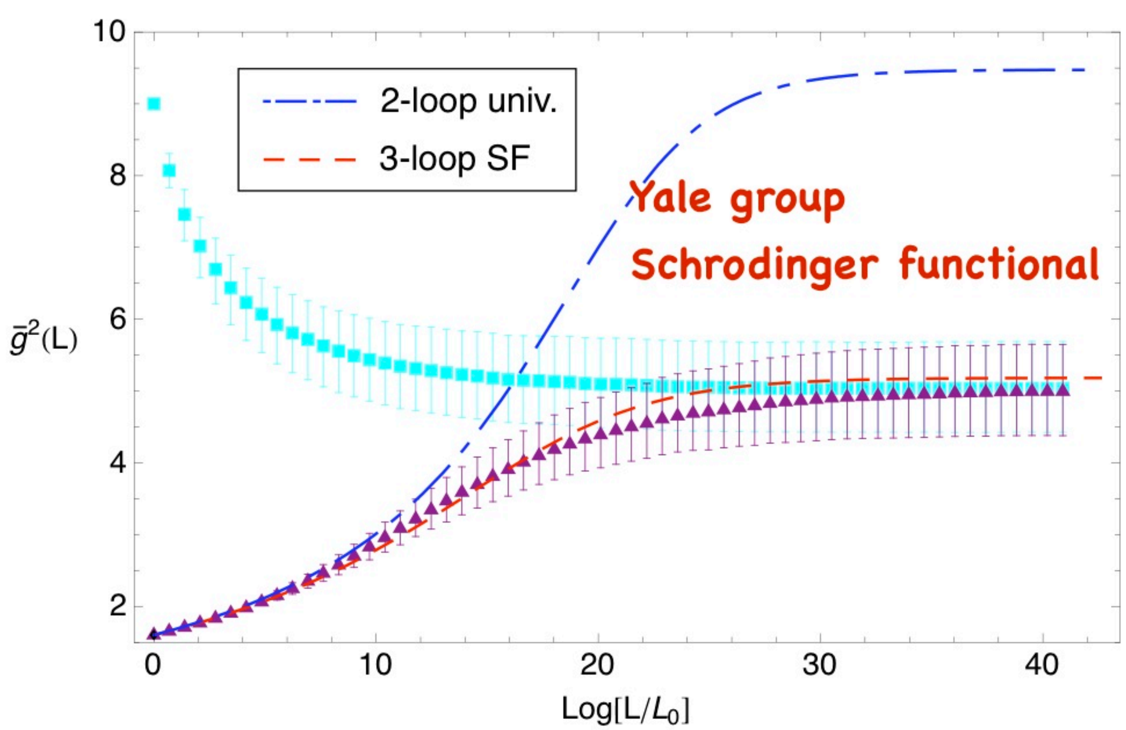
(b) concern about barely detectable taste breaking in pseudo-Goldstones?
removing them is still a bad conformal fit!

(c) lowering the mass range to m=0.01-0.025 does make the fits worse!

# Nf=12 running coupling from static force



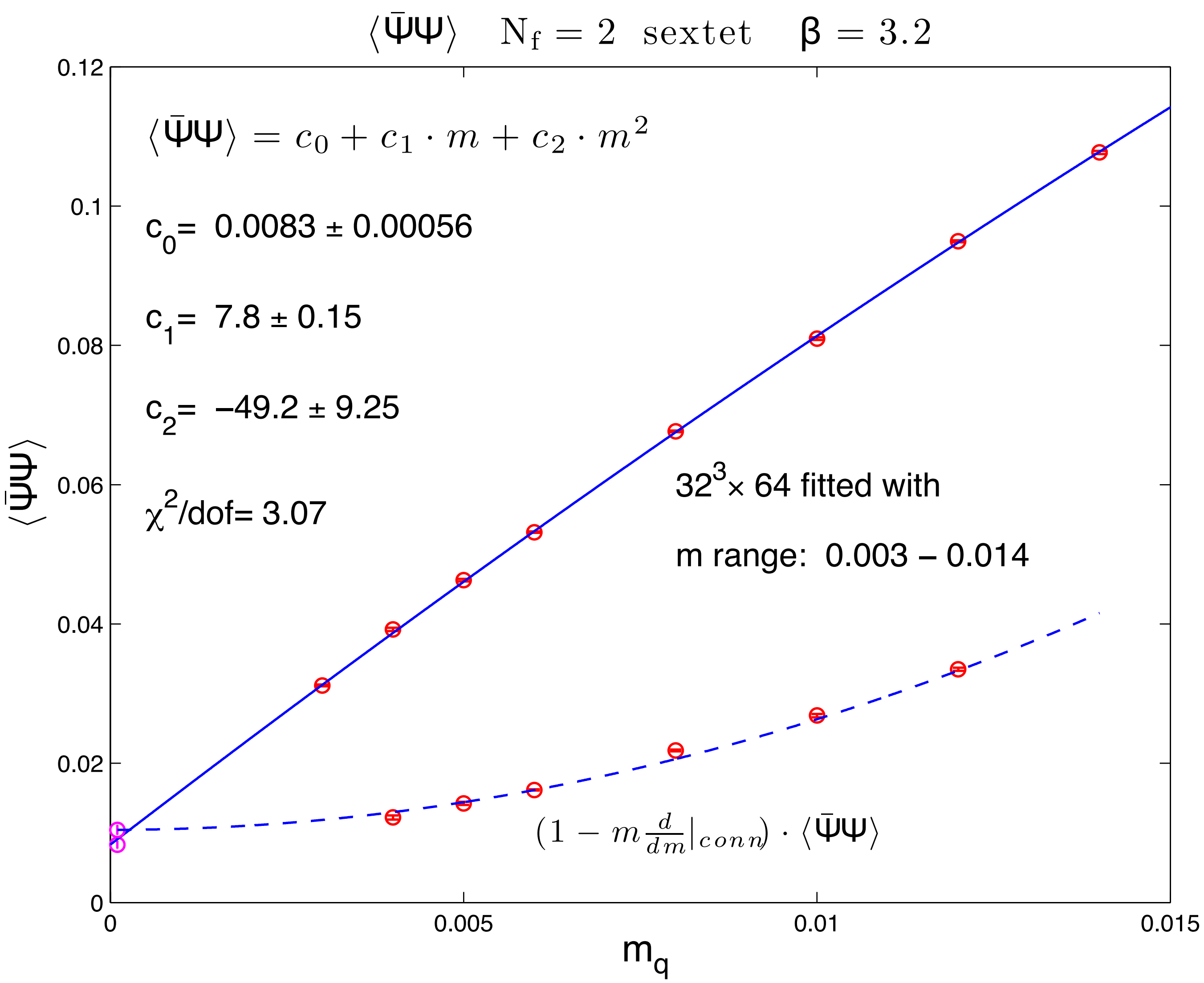



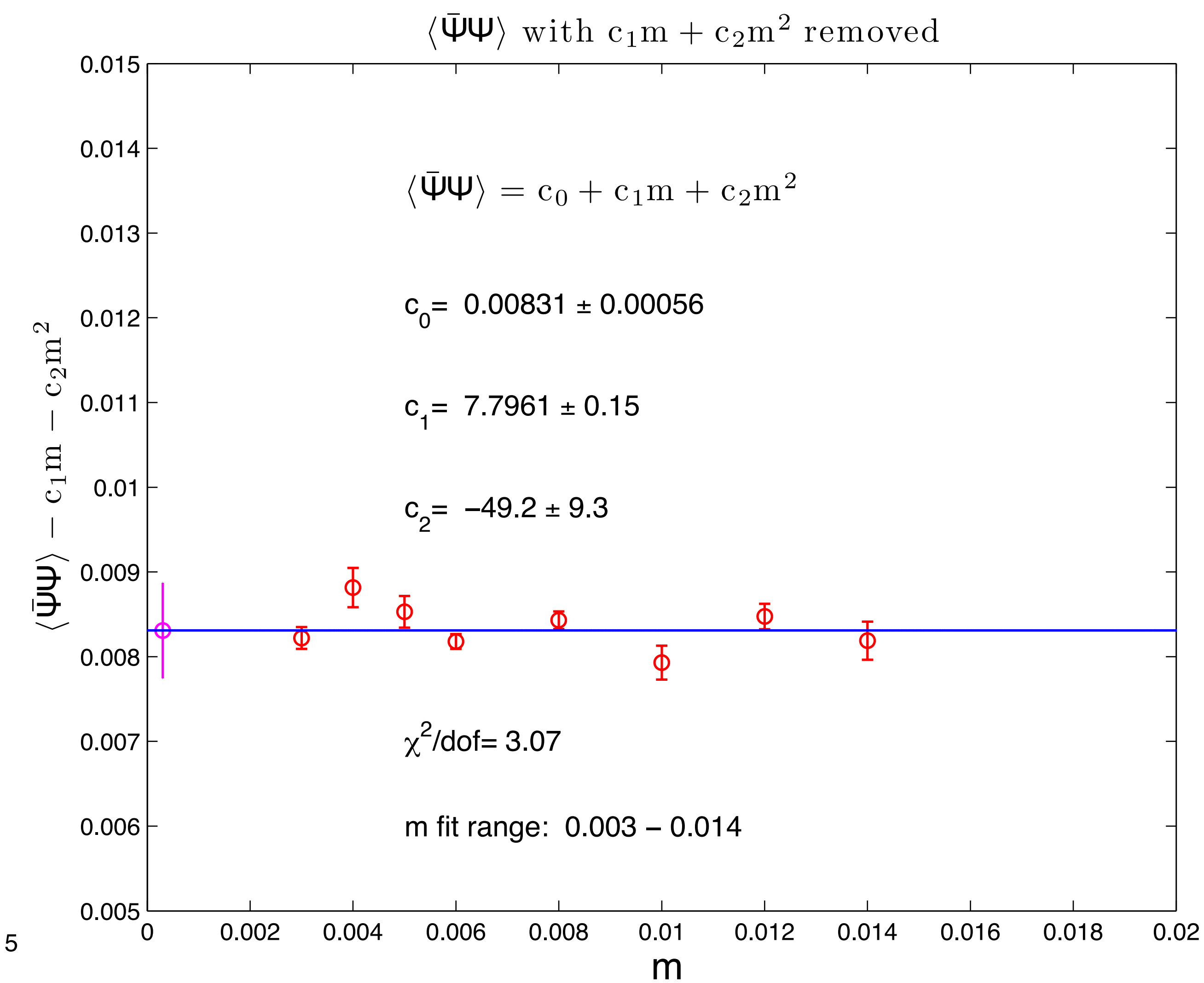

m --> 0 and a --> 0 limits ?

finite volume effects ?

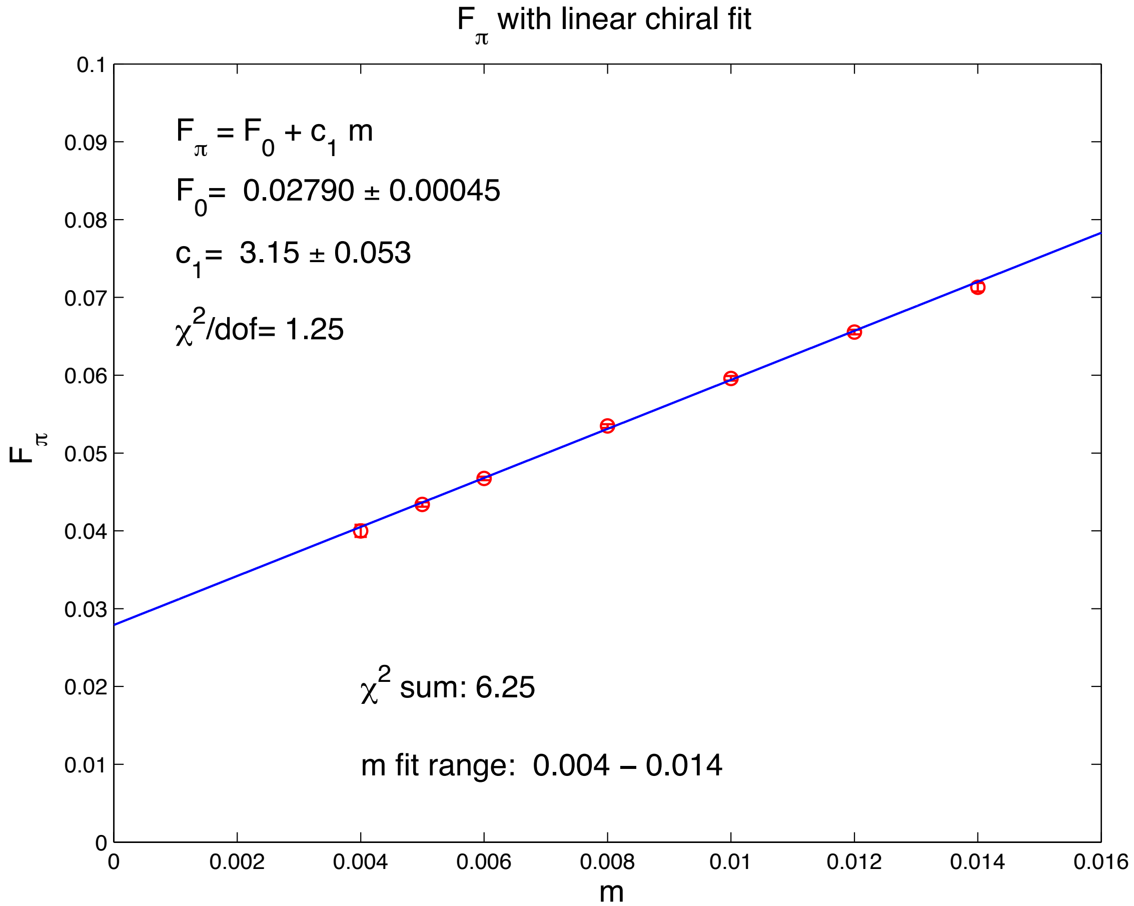
# Nf=2 sextet representation

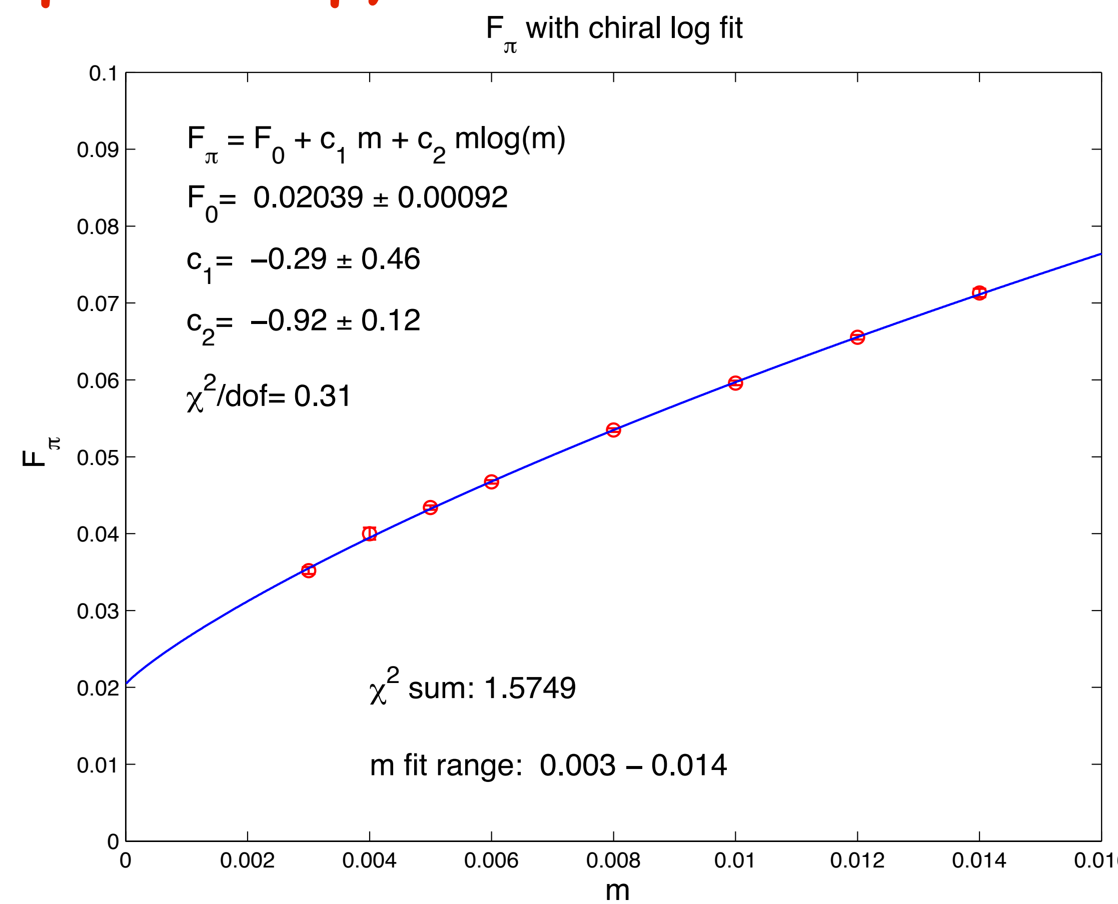
# Nf=2 sextet chiral condensate

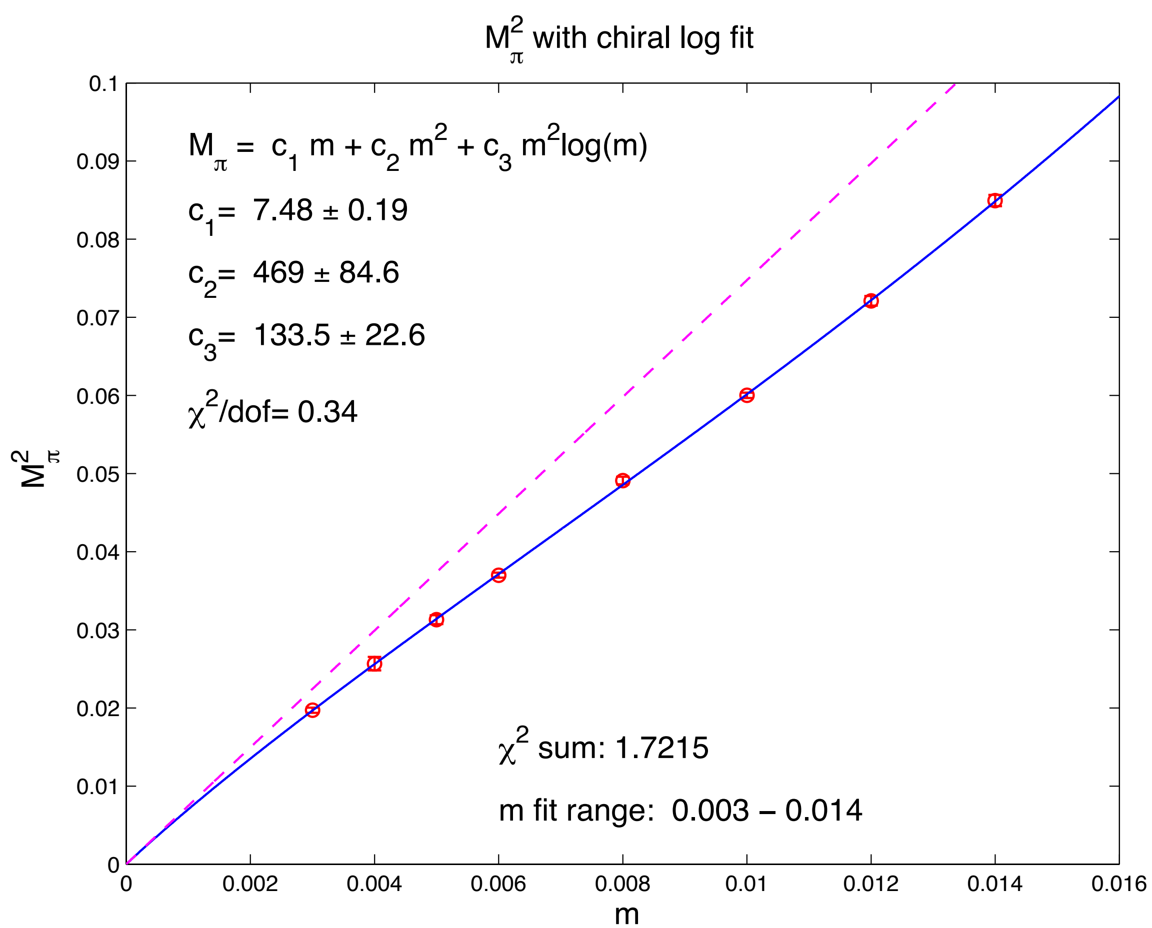


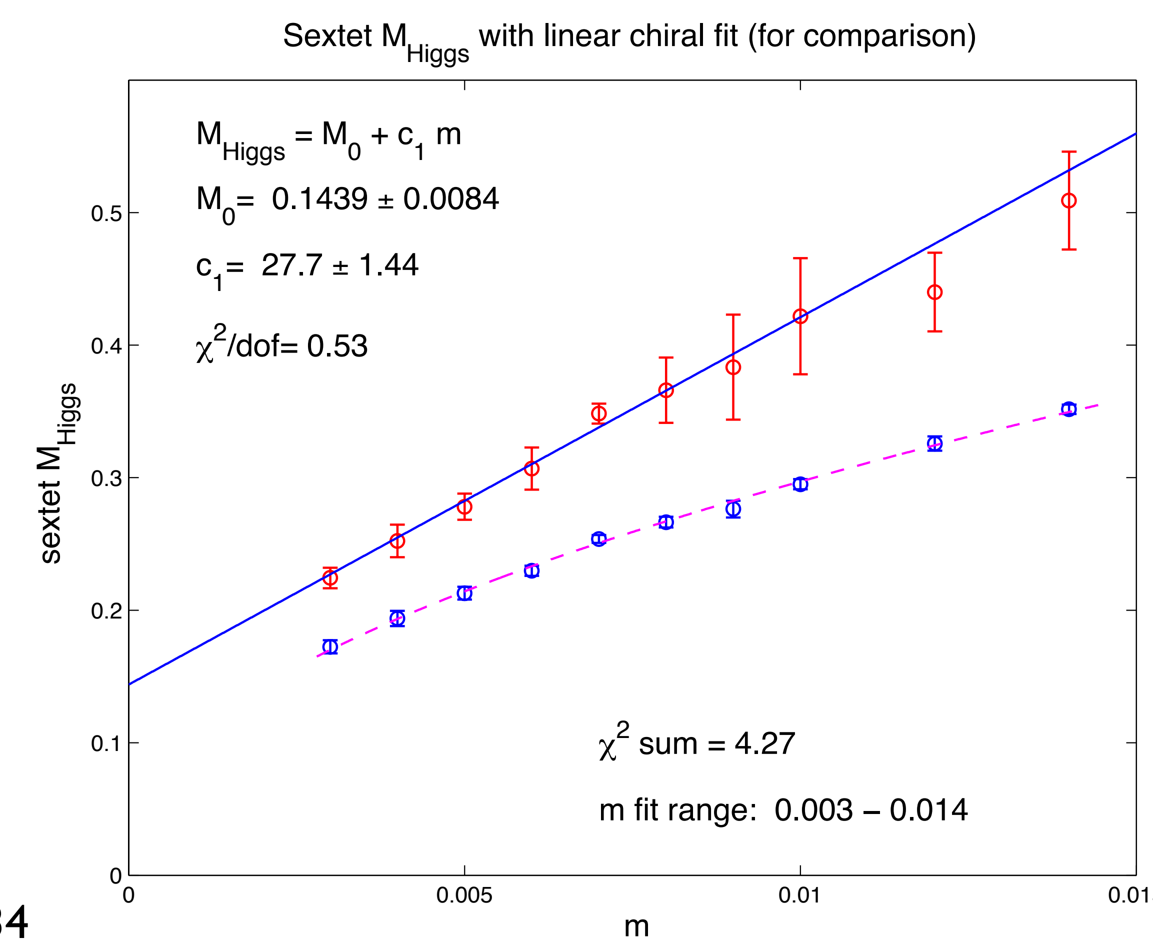



# Nf=2 sextet spectroscopy

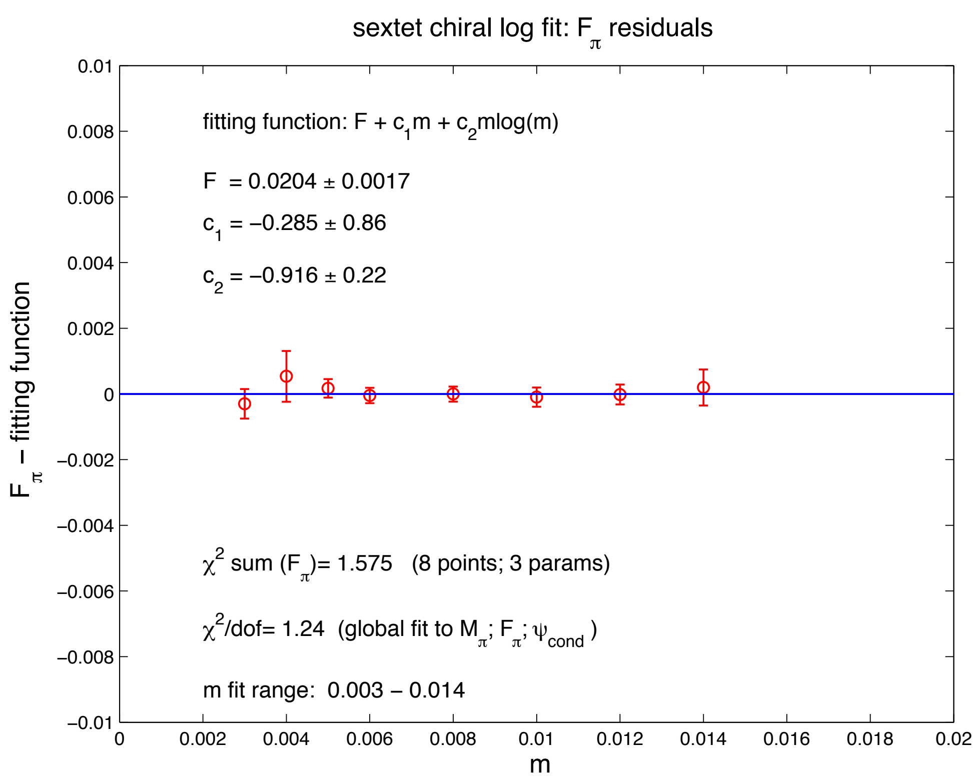


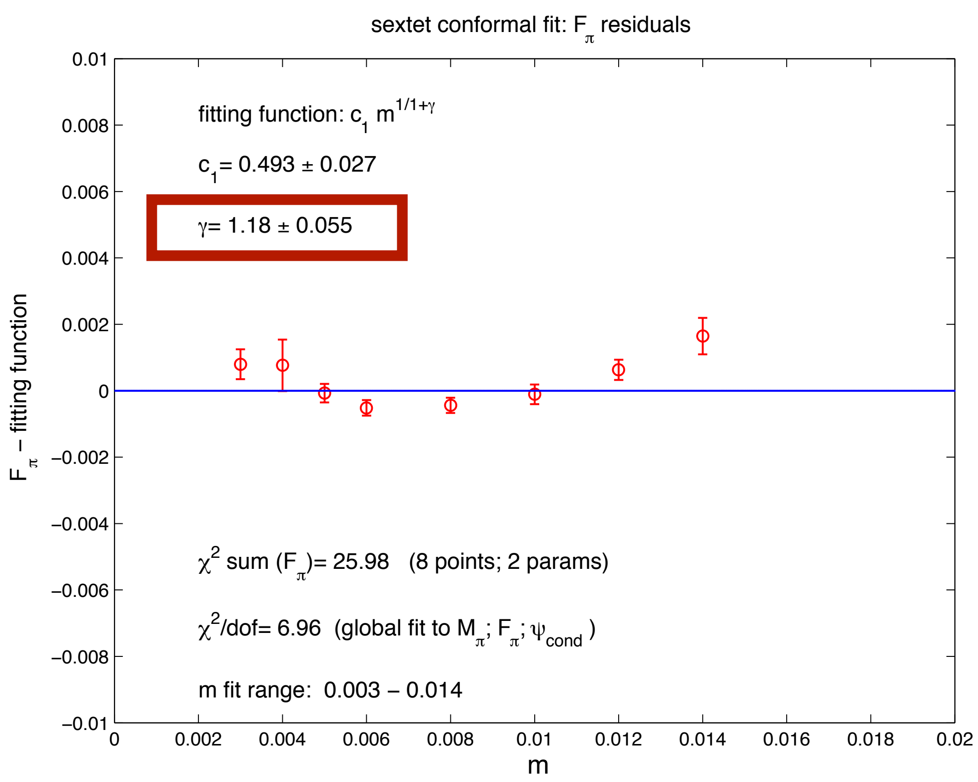



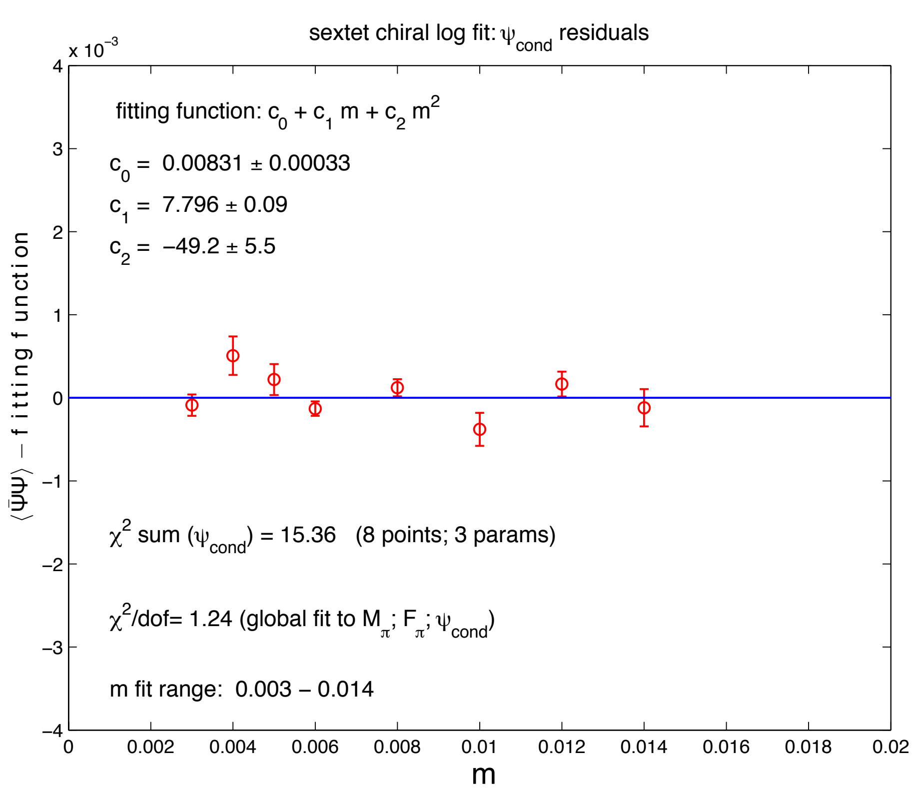



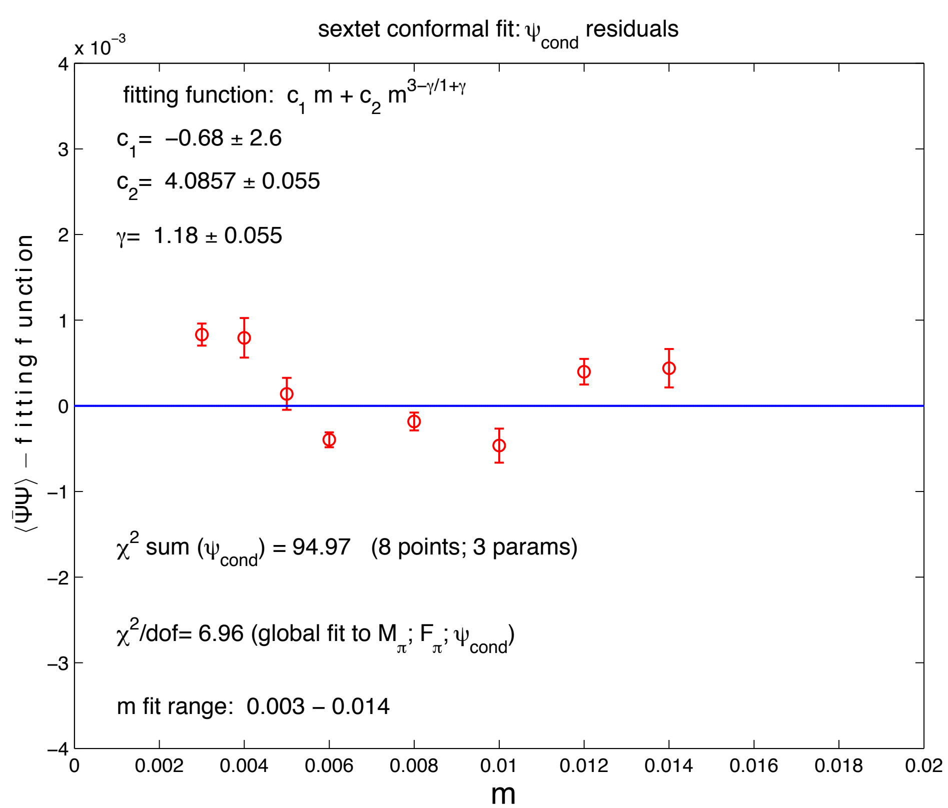



# Limited comparison of Nf=2 sextet hypotheses (LHC)



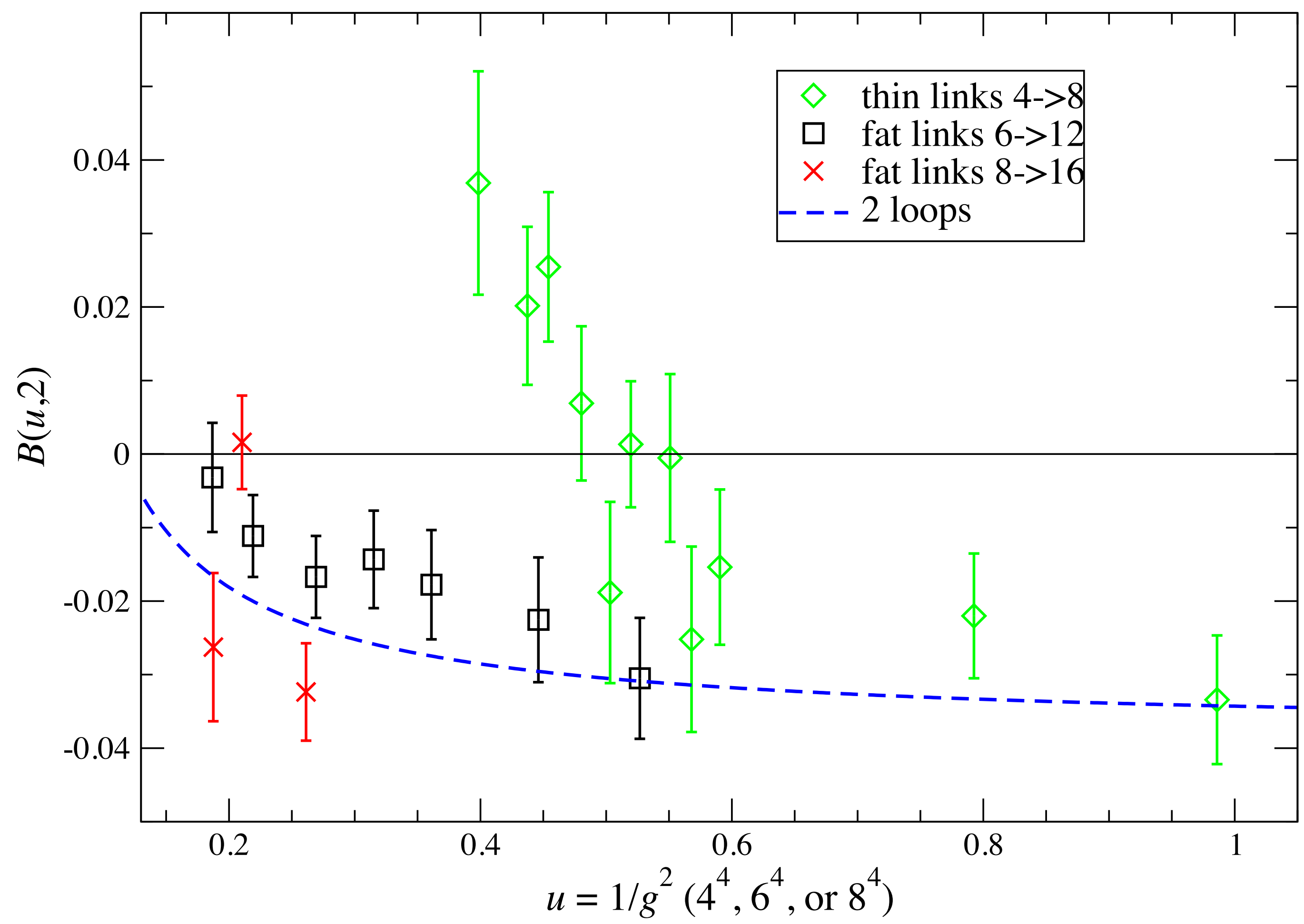



DeGrand and collaborators claim:

Nf=2 sextet beta function has an IRFP zero




But from this calculation γ ~ 0.4 is almost three times smaller than the Lattice Higgs Collaboration value

# Tunneling vacua and the conformal window

# Inside the conformal window: Nf=16 fundamental rep $SU(3)_c$ case study

Nf=16 important test of lattice technologies
From 2-loop beta function Banks-Zaks IRFP at $g^{*2} \approx 0.5$

Heller early work SF
A. Hasenfratz MCRG
Lattice Higgs Collab. FSS and $g^2(L)$

| $\alpha_{2l}$ | $\alpha_{3l}$ | $\alpha_{4l}$ |
|---|---|---|
| 0.0416 | 0.0397 | 0.0398 |

Ryttov and Shrock
$\alpha = g^2 / 4\pi$

| $\gamma_{2l}$ | $\gamma_{3l}$ | $\gamma_{4l}$ |
|---|---|---|
| 0.0272 | 0.0258 | 0.0259 |

Running coupling $g^2(L)$ evolving with L $g^2(L) \to g^{*2}$, as $L \to \infty$ infrared limit
(evolution of finite volume spectrum?)

At small $g^2(L)$ the zero momentum components of the gauge field dominate the dynamics: Born-Oppenheimer approximation

Originally it was applied to pure-gauge system Luscher, van Baal

**Small volume dynamics of QCD has spectrum which adiabatically evolves into hadron spectrum with rapid crossover around L ~ 0.7 fm**

**Method turns into important large volume dynamics around weak coupling fixed point inside conformal window**

# SU(3) $3^3=27$ gauge vacua (electric fluxes) --> $2^3=8$ massless fermion vacua (pbc)

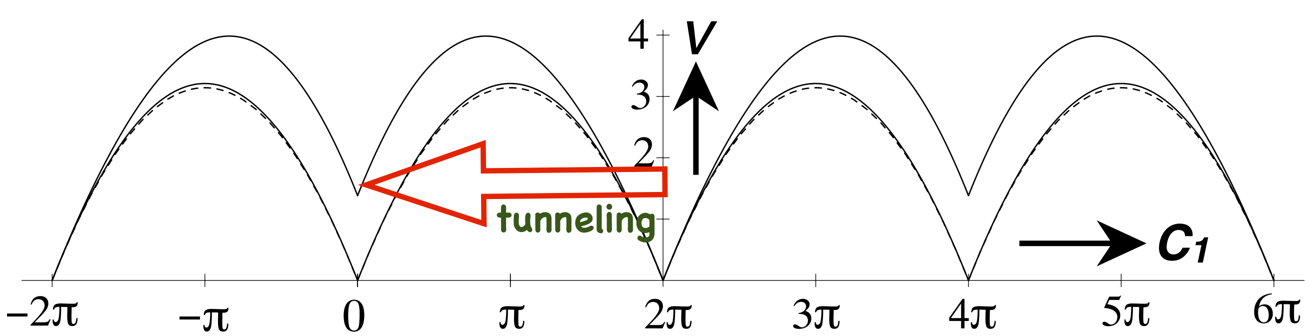
recent renewed interest:
Yaffe, Unsal
DeGrand, Hoffmann
others ...

$A_i(\mathbf{x}) = T^a C_i^a / L$ <-- zero momentum mode of gauge field

For $SU(3)$, $T_1 = \lambda_3/2$ and $T_2 = \lambda_8/2$

$$V_{\rm eff}^{\mathbf{k}}(\mathbf{C}^b) = \sum_{i>j} V(\mathbf{C}^b[\mu_b^{(i)} - \mu_b^{(j)}]) - N_f \sum_i V(\mathbf{C}^b \mu_b^{(i)} + \pi\mathbf{k}) \qquad \mu^{(1)} = (1,1,-2)/\sqrt{12} \text{ and } \mu^{(2)} = \tfrac{1}{2}(1,-1,0)$$

SU(2) $V_{\rm eff}$ shown for simplification:



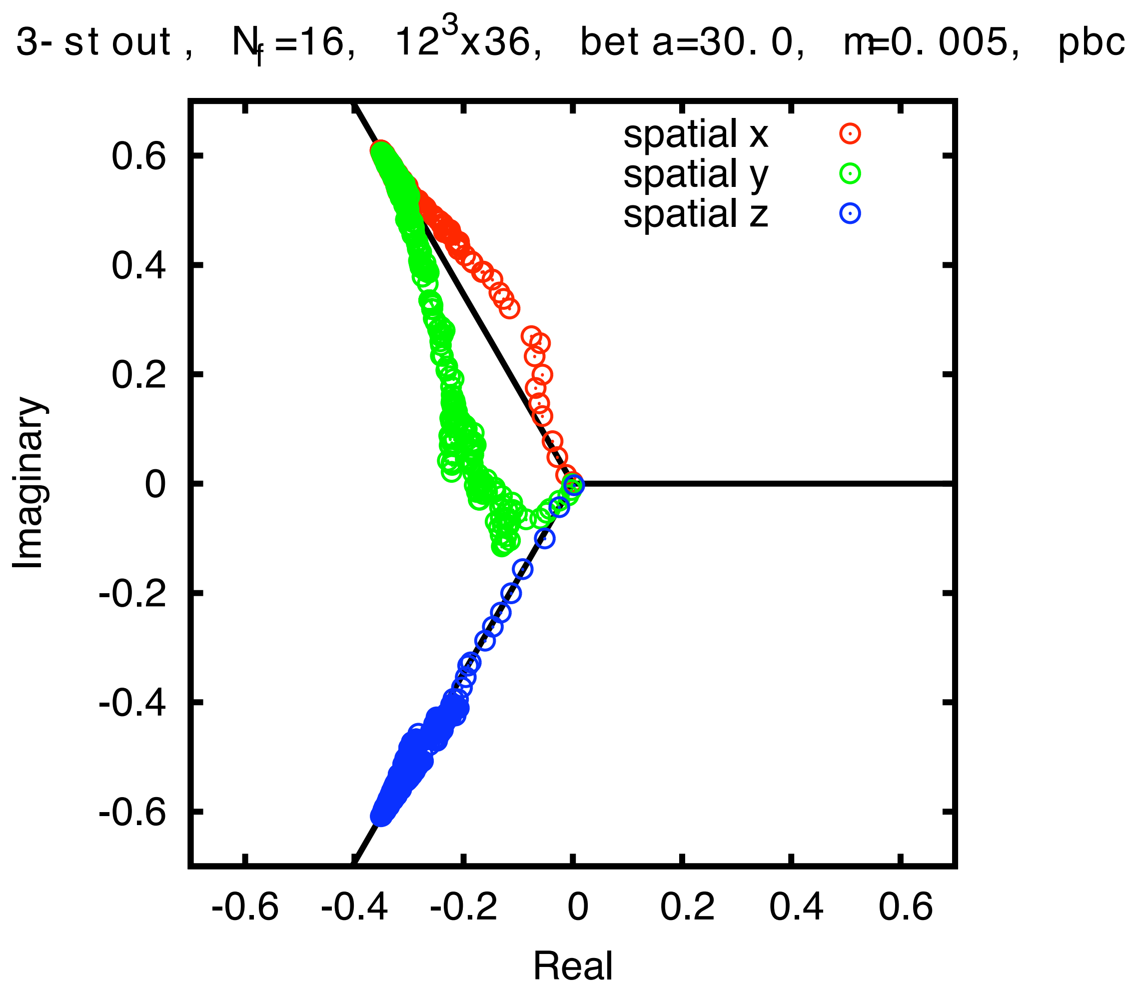

Effective potential shows the effects of massless fermions van Baal

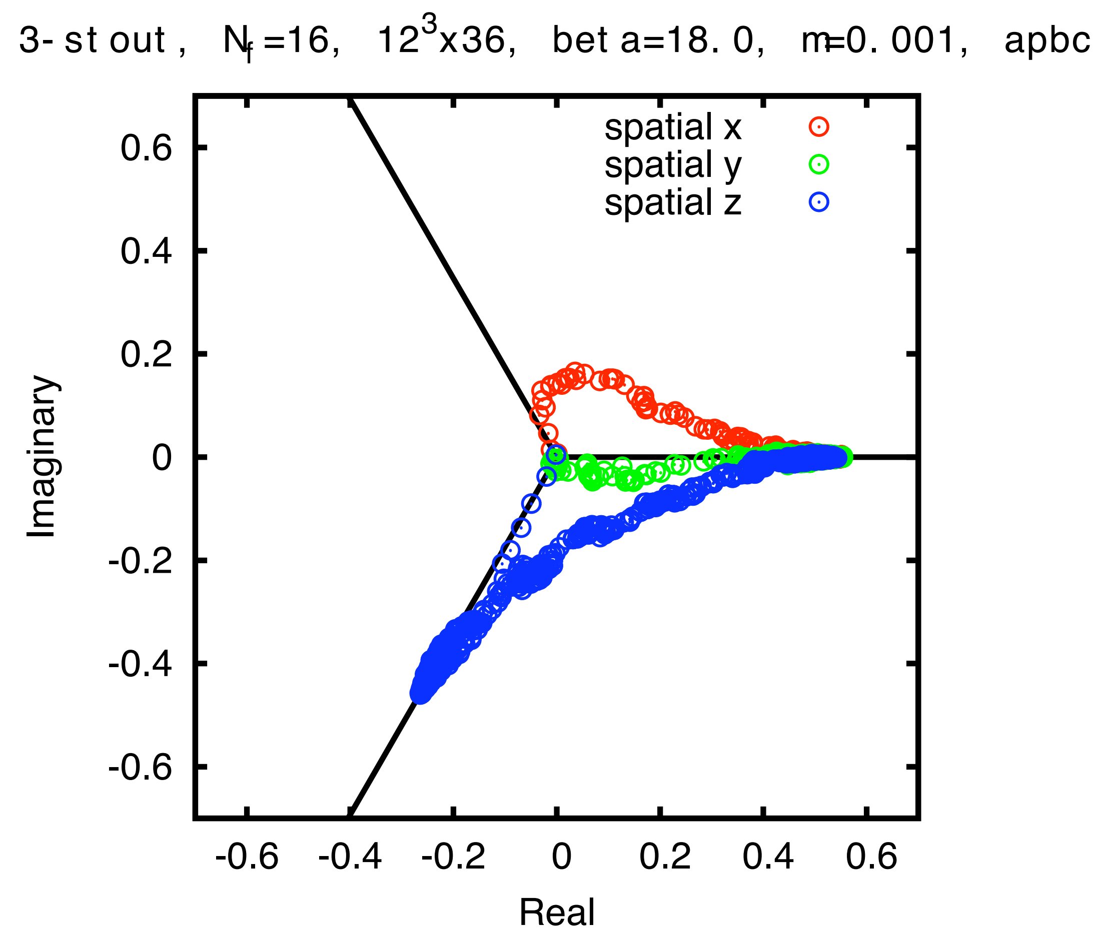
Fermions develop a gap in the spectrum $\sim \pi / L$
k=(0,0,0) periodic
k=(1,1,1) antiperiodic

Low excitations of Hamiltonian (Transfer Matrix) scale with $\sim g^{2/3}(L)/L$
will evolve into glueball states for large L

Three scales of dynamics
scale 1: on smallest scale WF is localized on one vacuum
scale 2: tunneling sets in across vacua
scale 3: spill over the barrier - confinement scale

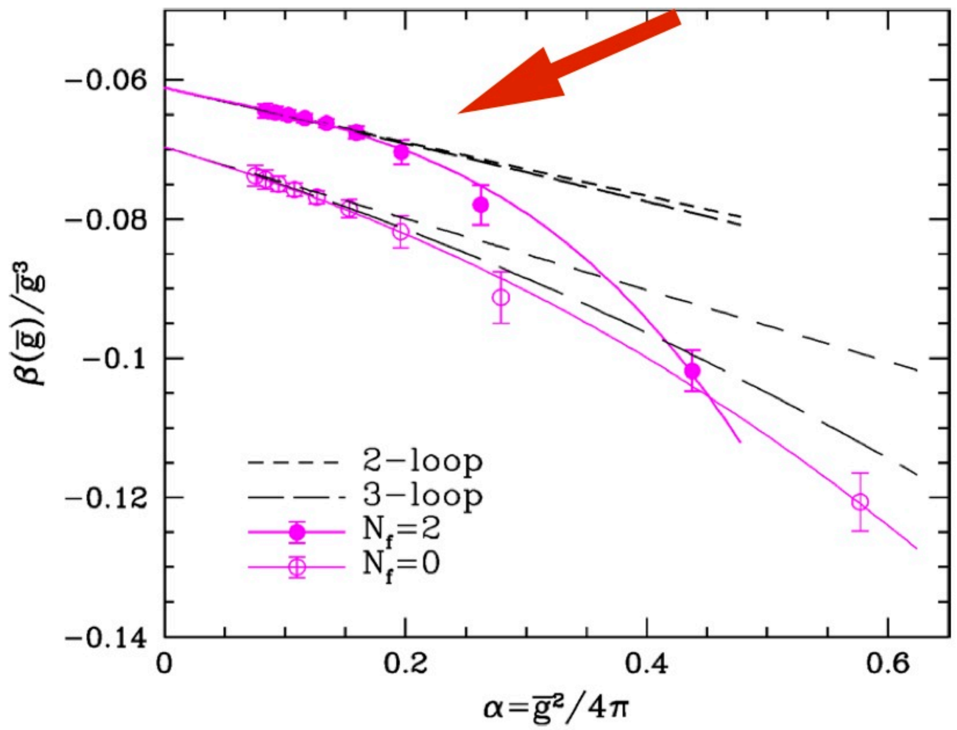
## Nf=16 inside conformal window femto volume with tunneling

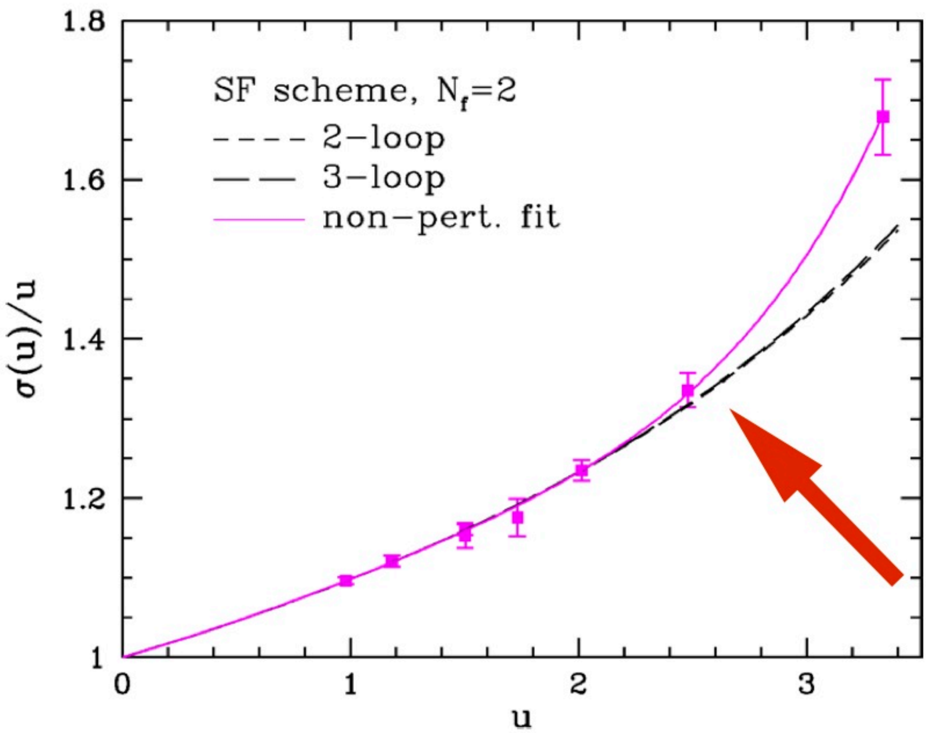



**How is this effecting running coupling calculations?**

# running coupling and tunneling




Schrödinger Functional $N_f=0$ and $N_f=2$ massless fermions
Alpha collaboration

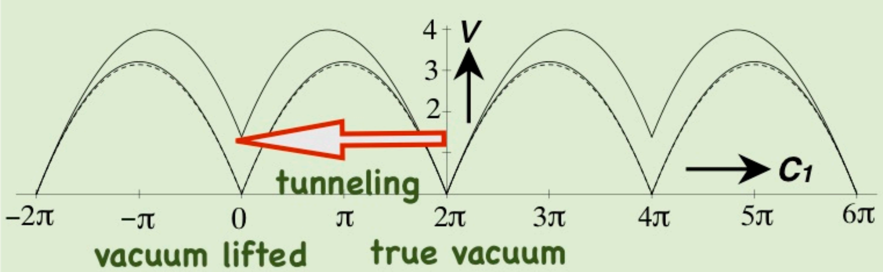
around $g^2 \sim 2.5$ the $N_f=2$ β-function breaks away from perturbative form where 2-loop and 3-loop still run closely together

$g^2 \sim 2.5$ is the onset of tunneling
(most likely to a metastable local minimum)

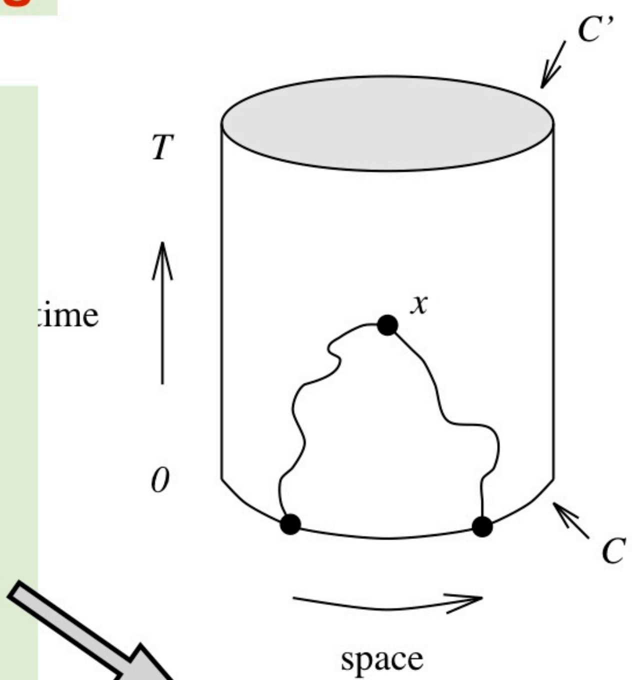
running becomes non-perturbative in very small box where $L_{max} < 0.4$ fm

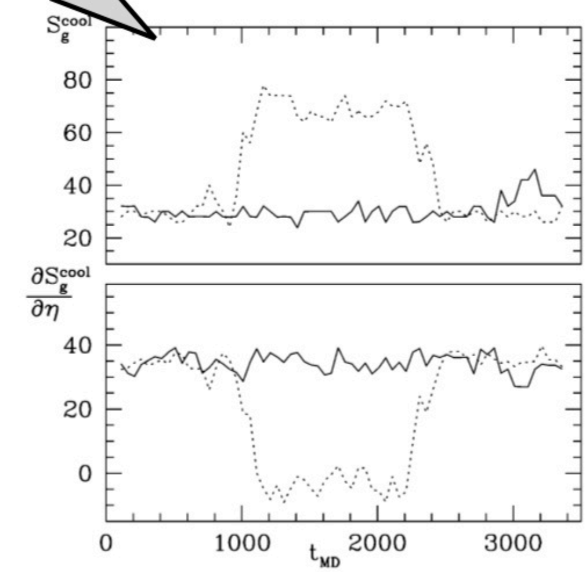
Why, and what is the underlying physics?

We need to understand femto physics better for the interpretation of the running coupling $g^2(L)$ in the presence of tunneling




Nf=16 weak coupling case study inside the conformal window shows the dynamics



# Summary and outlook

- We have technology to deal with lattice specific issues: cut-off, volume, fermion mass
  RG flow and lattice continuum physics
  BSM specific $\chi$PT
  m=0 chiral limit and finite volume issues
  Two model studies

- Inside the conformal window
  RG flow and lattice continuum physics
  importance of finite size scaling
  running coupling and tunneling
  Nf=16 case study

- Outlook
  we have only seen so far the tip of the iceberg of what lattice BSM can do
  for example: FSS analysis of current correlators in m->0 limit Lattice Higgs Collaboration
  phenomenology Strong Lattice Dynamics Collaboration
  discussions: new input into lattice projects?