Implications of string theory for cosmology

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at KMI, Nagoya University
May 19, 2011

String theory

- Theory of quantum gravity: Important at Planck scale
  - Should also have <u>implications for physics at lower scales</u>.

  e.g., Why is our universe flat?
  - There was inflation in the early universe.
  - Quantum fluctuations during inflation is observed as temperature fluctuations of CMB.

- Difficult to make precise predictions for cosmology
  - String theory is formulated in a limited class of background; Have to rely on low energy effective theory.

Topics of this talk

- Bubble nucleation: Spectrum of fluctuations, holographic duality

  w/ B. Freivogel, L. Susskind, C.-P. Yeh, hep-th/0606204;
  w/ L. Susskind, arXiv:0908.3844 [hep-th]

- Bubble collisions, phases of eternal inflation

  w/ R. Bousso, B. Freivogel, S. Shenker, L. Susskind, I.-S. Yang,
    C.-P. Yeh, arXiv:0807.1947 [hep-th] ;
  w/ S. Shenker, L. Susskind, arXiv:1003.1347[hep-th]

- CMB fluctuations from quantum effects of many fields:

 w/ Y. Habara, H. Kawai, M. Ninomiya, arXiv:1103.0299 [hep-th]

"String landscape"

String theory has meta-stable de Sitter vacua (V>0)
   (not only stable vacua with V=0)

- To understand physics, consider one
  false vacuum and one true vacuum

  $V(\Phi_F)>0$, $V(\Phi_T)=0$
  (true vacuum: our universe)

- If we ignore gravity, first order phase transition:
  - Nucleation of bubbles of true vacuum (Callan, Coleman, …)
  - The whole space eventually turns into true vacuum.
- What happens in a theory with gravity?

V(Φ)

Φ_F Φ_T Φ
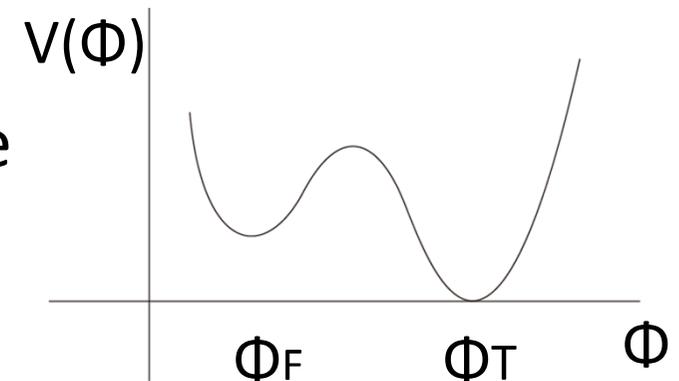
False vacuum: de Sitter space
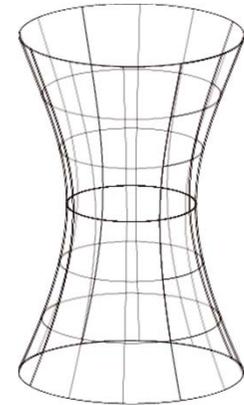
- de Sitter space: hyperboloid in $R^{4,1}$

$$ds^2 = -dt^2 + H^{-2}\cosh^2(Ht)d^2\Omega_3$$

Hubble parameter (expansion rate)

$$H^2 = \frac{8\pi G}{3}V(\phi_F)$$

- Causal structure: Two points separated by $> H^{-1}$ (horizon) are causally disconnected.

- Inflation in false vacuum leads to a completely different picture than the case without gravity.
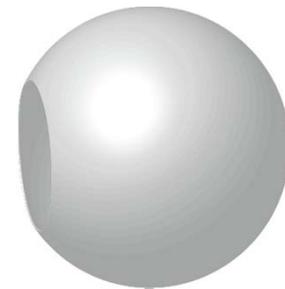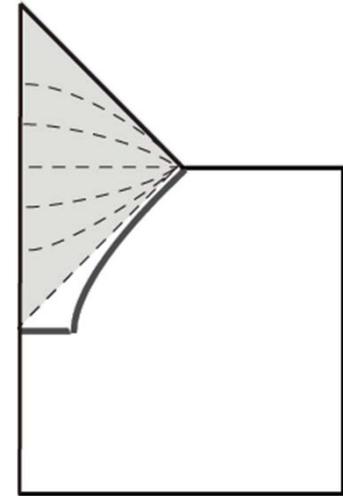
Bubble of true vacuum

Described by Coleman-De Luccia instanton (Euclidean "bounce" solution).

Euclidean geometry:

- Interpolates between true and false vacua

- Deformed S^4 (Euclidean de Sitter is S^4)

- Rotationally symmetric: SO(4)

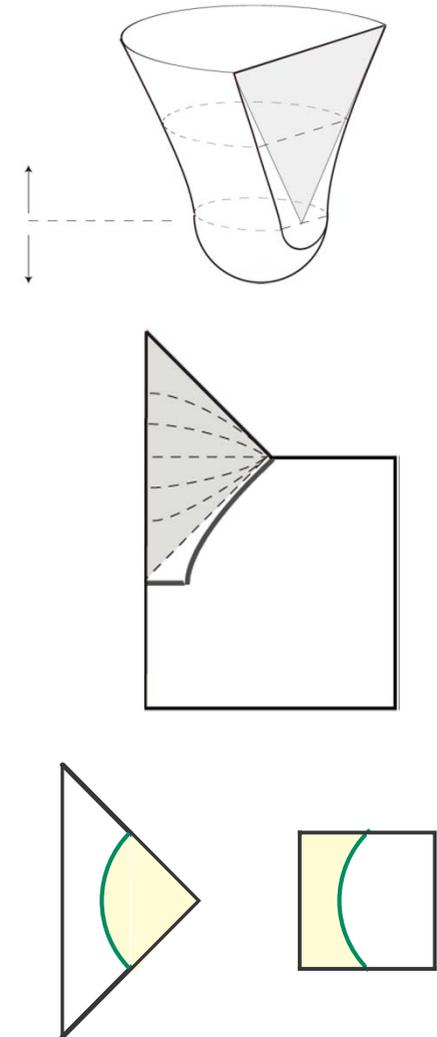- Nucleation rate: $\Gamma \sim e^{-(S_{\mathrm{cl}} - S_{\mathrm{deSitter}})}$

Lorentzian geometry: given by analytic continuation

- Flat space and de Sitter patched across a domain wall.
- SO(3,1) symmetric.

- Bubble wall is constantly accelerated.
- <u>Open FRW universe</u> (shaded region) inside the bubble. Spatial slice: H^3

$$ds^2 \sim -dt^2 + t^2(dR^2 + \sinh^2 R\, d\Omega^2)$$

- Beginning of FRW universe: non-singular (equivalent to Rindler horizon)

Eternal inflation

- A single bubble does not cover the whole space. (Fills only the horizon volume).

- Many bubbles will form in the de Sitter region with the rate Γ (per unit physical 4-volume).

  (In fact, bubble collisions are inevitable.)

- But if $\Gamma \ll H^4$, bubble nucleation cannot catch up the expansion of space, and false vacuum exists forever ("Eternal Inflation"; Guth, Linde, Vilenkin, …).
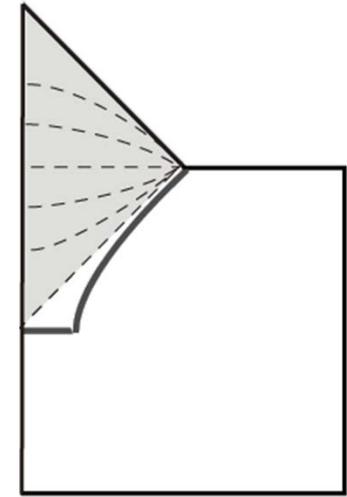
Spectrum of fluctuations and holographic duality

w/ B. Freivogel, L. Susskind, C.-P. Yeh,
"A Holographic framework for eternal inflation,"
hep-th/0606204;
w/ L. Susskind,
"Census taking in the hut: FRW/CFT duality,"
arXiv:0908.3844 [hep-th]

(perturbative calculation ignoring bubble collisions)

Correlation functions in the bubble

- Effect of the ancestor vacuum enters through the initial condition.

- Vacuum is chosen by the Euclidean prescription: We need the whole global space.

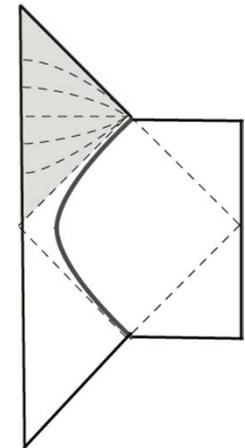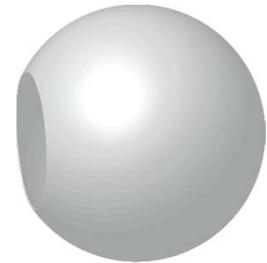- Euclidean CDL geometry: $(-\infty \leq X \leq \infty)$

$$ds_E^2 = a^2(X)\left(dX^2 + d\theta^2 + \sin^2\theta d\Omega_2^2\right)$$

$$a(X) = \tilde{H}_A^{-1}e^X \quad \text{(flat)}, \qquad a(X) = \frac{H_A^{-1}}{\cosh X} \quad \text{(de Sitter)}$$

- Analytic continuation to FRW:

$$X \rightarrow T + \frac{\pi}{2}i, \quad \theta \rightarrow iR$$
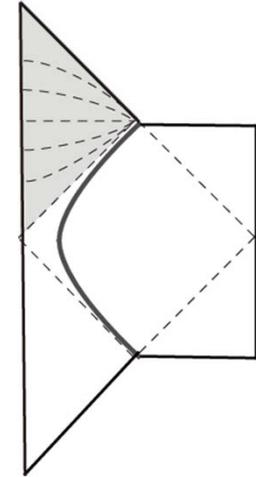
$$ds^2 = a^2(T)(-dT^2 + dR^2 + \sinh^2 R d\Omega_2^2)$$

Calculation of the correlator

- e.o.m. for a minimally coupled scalar:

$$\left[-\partial_X^2 + \frac{a''}{a} - \nabla_S^2 + m^2 a^2 \right] (a\phi) = 0$$

- <u>Calculation of the correlator is essentially a 1-dimensional scattering problem:</u>

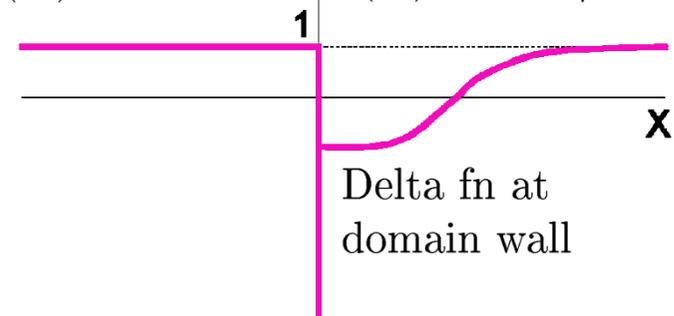$$\left[-\partial_X^2 + \frac{a''}{a} + m^2 a^2 \right] u_k(X) = (k^2 + 1) u_k(X)$$

- Correlator can be written in terms of the reflection coefficient $\mathcal{R}(k)$.
- Bound state exists when mass is small compared to H_A

$V(X) = a''/a$ (in the thin-wall limit)

Flat
$V(X) = 1$

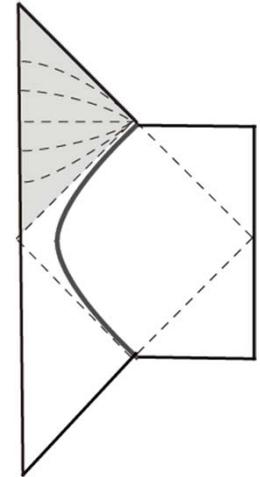de Sitter (sphere)
$V(X) = 1 - 2/\cosh^2 X$

1

X

Delta fn at domain wall
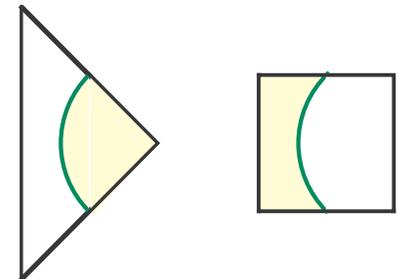
Correlator inside the bubble

- Final result:

$$\langle \phi(T,R)\phi(T',0) \rangle = \tilde{H}_A^2 e^{-(T+T')} \int_{C_1} dk \Big(e^{ik(T-T')} \cosh k\pi$$
$$+ \mathcal{R}(k) e^{-ik(T+T')} \Big) \frac{\sin kR}{\sinh k\pi \sinh R}$$

(R: Geodesic distance on H^3)

- 1st term: "Flat space piece" (Minkowski correlator written in hyperbolic slicing: Initial condition usually assumed for inflation)
- 2nd term: Effect of the ancestor vacuum.

Classification by "scaling dimensions"

- The <u>ancestor piece</u> can be expressed as

$$\langle \phi(T,R)\phi(T',0)\rangle \; = \; \sum_{\Delta} G_{\Delta}^{(1)} e^{-\Delta(R_1+R_2)} e^{-\Delta(T_1+T_2)}(1-\cos\Omega)^{-\Delta}$$

$$+ \sum_{\Delta'=2}^{\infty} G_{\Delta'}^{(2)} e^{-\Delta'(R_1+R_2)} e^{(\Delta'-2)(T_1+T_2)}(1-\cos\Omega)^{-\Delta'}$$

(Ω: angular separation on S^2)

Spectrum of the Dimensions: Δ = 2,3,4,…, and additional lower values (massless case: Δ=0, 1+α (0<α<1)).

- Term with lowest dimension is most important
  - Decays most slowly after bubble nucleation.
  - Amplitude: $H_I \left(\dfrac{H_I}{H_A}\right)^{\Delta-1}$ (H_I : Hubble of inflation after tunneing)
  - Terms with $0 \leq \Delta \leq 1$ gives large effect (in the IR)

Holographic duality

Conjecture (FSSY '06):

The open FRW created by bubble nucleation is dual to a conformal field theory on S^2 (at the boundary of the 3-hyperboloid)
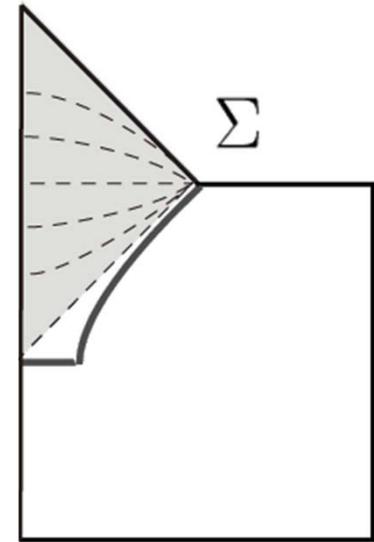
- SO(3,1): conformal sym in 2D (as in AdS/CFT).
- The dual has 2 less dimensions than the bulk.
  - The dual theory contains gravity (Liouville field). (It is a 2D gravity coupled to matter with c>25).
  - Liouville field plays the role of time.
- (Different from dS/CFT correspondence.)

One bulk field corresponds to a tower of operators

$$\phi \leftrightarrow \sum_{\Delta} e^{-\Delta T - \Delta R} \tilde{q}_{\Delta} \tilde{\mathcal{O}}_{\Delta} + \sum_{\Delta'=2}^{\infty} e^{(\Delta'-2)T - \Delta' R} q_{\Delta'} \mathcal{O}_{\Delta'}$$

(roughly speaking, KK reduction in the time direction)

- Operators which scale like $e^{-\Delta(T+R)}$:
  "RG-invariant" operators (defined at the UV scale)
- Operators which scale like $e^{(\Delta-2)T - \Delta R}$:
  "RG-covariant" operators (defined at reference scale, such as effective action, energy momentum tensor)

Graviton correlator

- There is a non-normalizable mode (Δ=0).
  - Boundary (2D) curvature correlator is finite:

  $$\langle R^{(2)} R^{(2)} \rangle = \frac{1}{(1 - \cos \Omega)^2}$$

  Gravity is not decoupled at the boundary (because of the compactness of the Euclidean geometry).

- Dimension 2 piece of graviton is transverse (conserved)-traceless in 2D.
  - Identified as energy-momentum tensor of 2D CFT
  - Evidence for the existence of local 2D theory.

Bubble collisions and phases of eternal inflation

w/ R. Bousso, B. Freivogel, S. Shenker,
L. Susskind, I.-S. Yang, C.-P. Yeh,
"Future foam: Non-trivial topology from bubble collisions
in eternal inflation"
arXiv:0807.1947 [hep-th] ;

w/ S. Shenker, L. Susskind,
"Topological Phases of eternal inflation,"
arXiv:1003.1347[hep-th]

Outline

- There are three phases of eternal inflation, depending on the nucleation rate.

- Phases are characterized by the existence of percolating structures (lines, sheets) of bubbles in global de Sitter. (First proposed by Winitzki, '01)

- The cosmology of the true vacuum region is qualitatively different in each phase.

View from the future infinity

- Consider conformal future of de Sitter.
  (future infinity in comoving coordinates)

  $$ds^2 \sim \frac{-d\eta^2 + d\vec{x}^2}{H^2\eta^2} \qquad (-\infty < \eta < 0)$$

- A bubble: represented as a sphere cut out
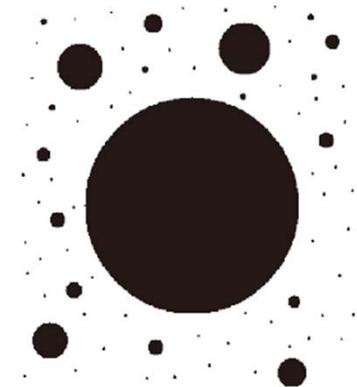 from de Sitter.
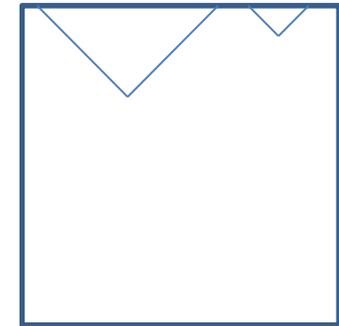
- "Scale invariant" distribution of bubbles

  Bubbles nucleated earlier:
  appear larger: radius $\sim H^{-3}|\eta|^3$
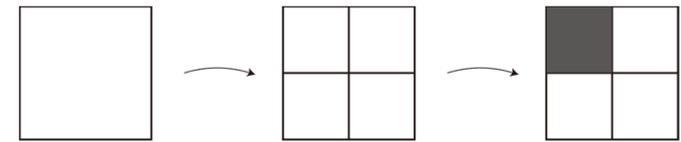 rarer: volume of nucleation sites $\sim |\eta|^{-3}$

Model for eternal inflation

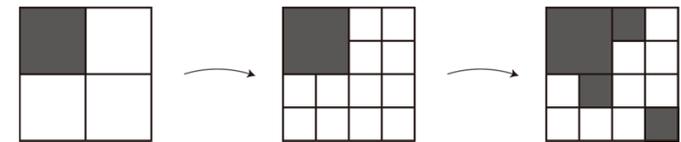- **Mandelbrot model (Fractal percolation)**
  - Start from a white cell.
    (White: inflating, Black: non-inflating
    Cell: One horizon volume)

  - Divide the cell into cells
    with half its linear size.
    (The space grows by a factor of 2.
    Time step: $\Delta t = H^{-1} \ln 2$)

    Picture of the 2D version

  - Paint each cell in black with probability P.
    (P ~ $\Gamma V_{\mathrm{hor}} \Delta t$ = nucleation rate per horizon volume: constant)
  - Subdivide the surviving (white) cells, and paint cells
    in black w/ probability P. Repeat this infinite times.

Mandelbrot model defines a fractal

- If $P > 1 - (1/2)^3 = 7/8$, the whole space turns black, since (the rate of turning black) > (the rate of branching).
  (No eternal inflation)

- If $P < 7/8$, white region is a fractal. Non-zero fractal dimension d_F (rate of growth of the cells):

$$N_{\text{cells}} = 2^{n d_F}, \quad d_F = 3 - |\log(1 - P)| / \log 2$$
$$(n : \# \text{ of steps})$$

  Physical volume of de Sitter region grows.
  (Eternal inflation)

- Fractals in eternal inflation: noted by Vilenkin, Winitzki, [21]..

Three phases of eternal inflation

From the result on the 3D Mandelbrot model

[Chayes et al, Probability Theory and Related Fields 90 (1991) 291]

In order of increasing P (or Γ), there are
(white = inflating, black = non-inflating)

- Black island phase: Black regions form isolated clusters;
  $\exists$ percolating white sheets.

- Tubular phase: Both regions form tubular network;
  $\exists$ percolating black and white lines.

- White island phase: White regions are isolated;
  $\exists$ percolating black sheets.
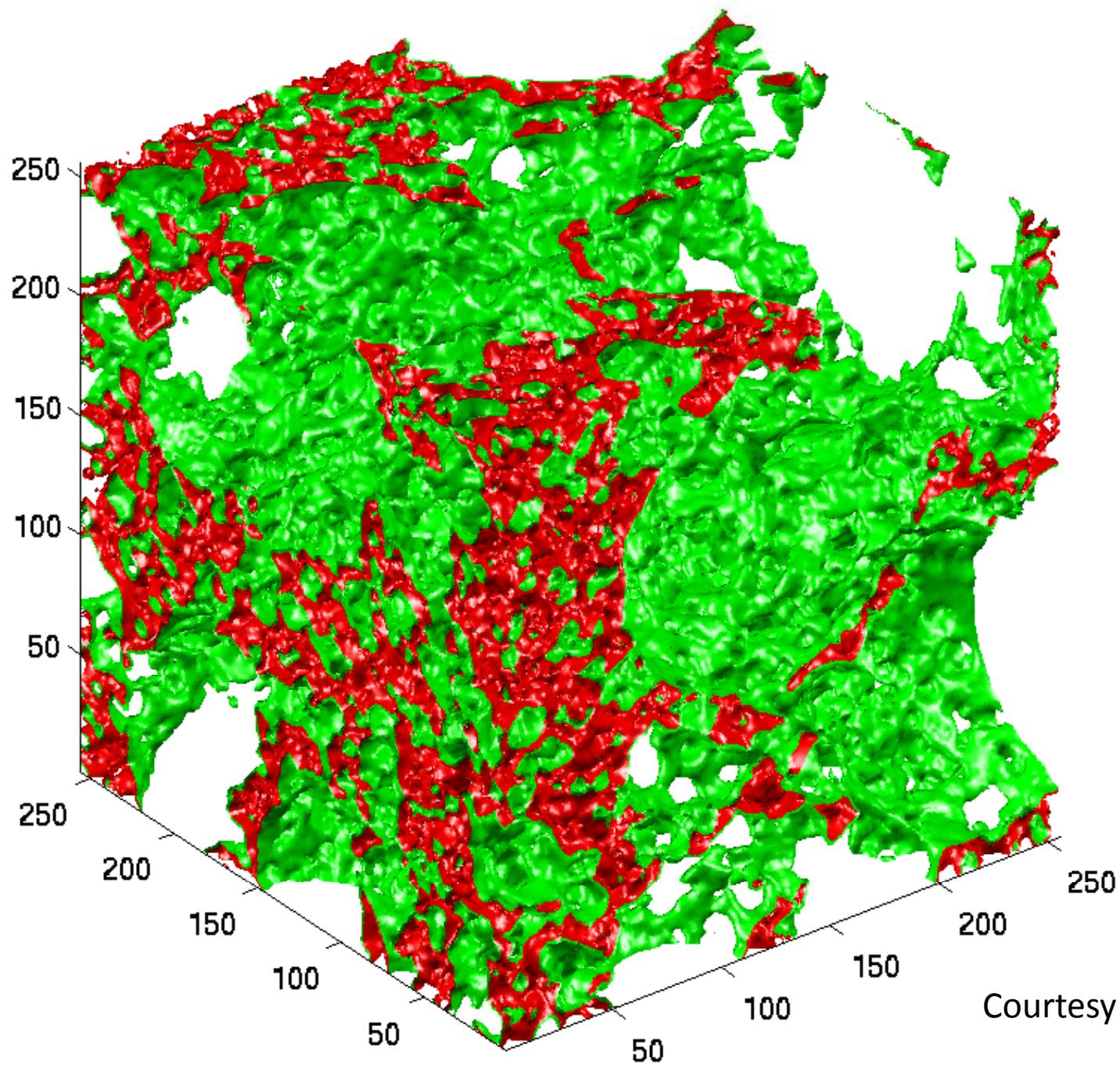
Courtesy of Vitaly Vanchurin

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Geometry of the true vacuum region

- Mandelbrot model: the picture of the de Sitter side. (de Sitter region outside the light cone of the nucleation site is not affected by the bubble.)

- To find the spacetime in the non-inflating region inside (the cluster of) bubbles, we need to understand the dynamics of bubble collisions.

- In the following, we study this using the intuition gained from simple examples of bubble collisions.

Black island phase (isolated cluster of bubbles)

Small deformations of open FRW universe.

- Basic fact: A collision of two bubbles (of the same vacuum) does not destroy the bubble
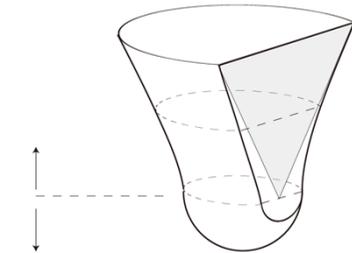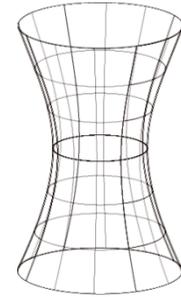 [c.f. Bousso, Freivogel, Yang, '07]
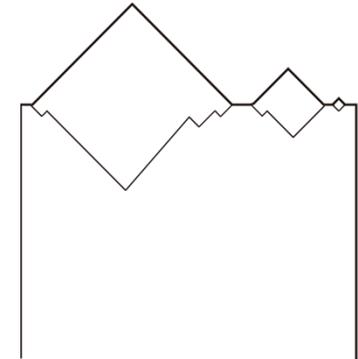
  - Spatial geometry approaches smooth H^3 at late time.
  - Residual symmetry SO(2,1): spatial slice has H^2 factor
  - Negative curvature makes the space expand.

Collision of two bubbles

- De Sitter space: hyperboloid in $R^{4,1}$

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = \ell^2$$

- One bubble: plane at $X_4 = \text{const.} = \sqrt{\ell^2 - r_0^2}$

- Second bubble: plane at $X_3 = \text{const.} = \sqrt{\ell^2 - r_0^2}$

  Residual sym: SO(2,1)

- Parametrization of de Sitter w/ manifest H^2 factor :

$$ds^2 = -f^{-1}(t)dt^2 + f(t)dz^2 + t^2 dH_2^2$$
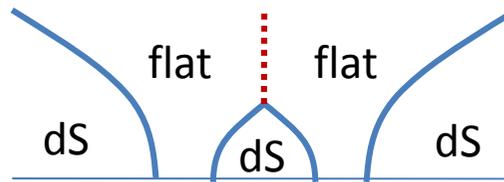$$f(t) = 1 + t^2/\ell^2, \quad (0 \le z \le 2\pi\ell)$$
$$\left(X_a = tH_a \ (a = 0,1,2), \quad X_3 = \sqrt{t^2 + \ell^2}\cos(z/\ell), \quad X_4 = \sqrt{t^2 + \ell^2}\sin(z/\ell) \right)$$

- Parametrization of flat space:

$$ds^2 = -dt^2 + dz^2 + t^2 dH_2^2$$

- Profile of domain wall in (t, z) space (H^2 is attached)
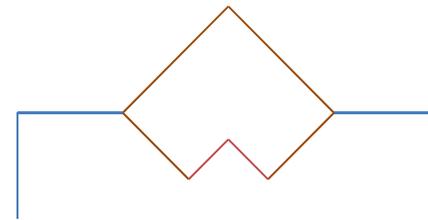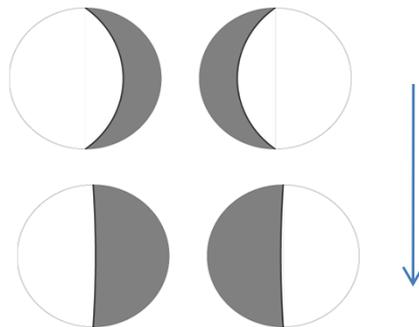

$$ds_{\mathrm{DW}}^2 = -d\tau^2 + R^2(\tau)dH_2^2 \qquad (R(\tau) = t(\tau))$$

- Energy on the domain wall decays at late time

$$\rho = \rho_0/t^2 \quad \text{(for dust wall)}$$

- The spatial geometry approach smooth H^3

Tubular phase (tube-like structure of bubbles)

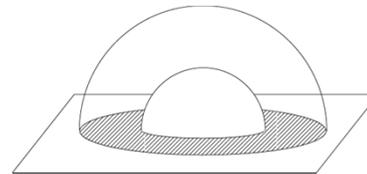In the late time limit: spatial slice is a negatively curved space whose boundary has infinite genus.

Late time geometry: $ds^2 = -dt^2 + t^2 ds^2_{H/\Gamma}$

- Negatively curved space with non-trivial boundary topology: H^3 modded out by discrete elements of isometry
- Boundary genus = # of elements

$$ds^2_{H^3} = \frac{dx^2 + dy^2 + dz^2}{z^2}$$
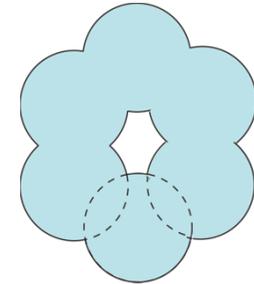
$$(x, y, z) \sim \lambda(x, y, z)$$

Genus 1 case (universe: bulk of a torus)

- Simpler example: true vacuum with toroidal boundary

[Bousso, Freivogel, YS, Shenker, Susskind, Yang, Yeh, '08]

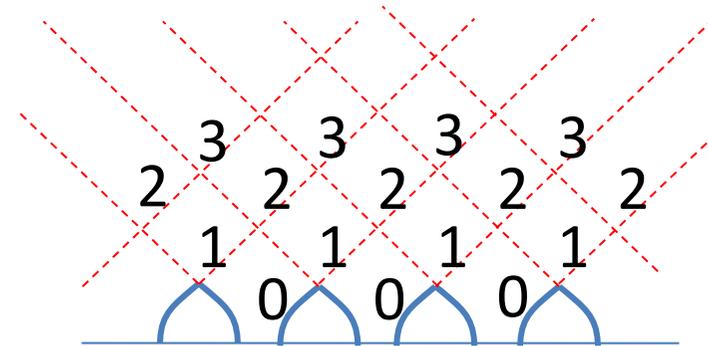- Ring-like initial configuration of bubbles (with the hole larger than horizon size)

- Solve a sequence of junction conditions

$$ds^2 = -f(t)dt^2 + f^{-1}(t)dz^2 + t^2 dH_2^2$$
$$f(t) = 1 + t^2/\ell^2 \quad \text{(de Sitter)}$$
$$f(t) = 1 - t_n/t \quad \text{(in region } n; \ t_n: \text{ const.)}$$

- Approaches flat spacetime at late time. (Negatively curved spatial slice with toroidal boundary)

White island phase (isolated inflating region)

An observer in the black region is "surrounded" by the white region (contrary to the intuition from Mandelbrot model).

- Simple case: two white islands (with S^2 symmetry)
  [Kodama et al '82, BFSSSYY '08]
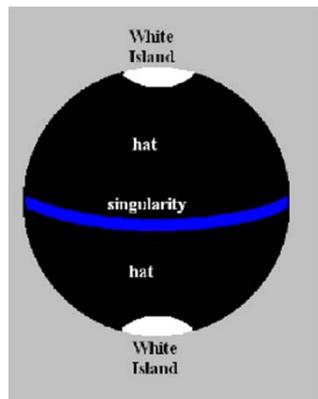
Global slicing (S^3) of de Sitter
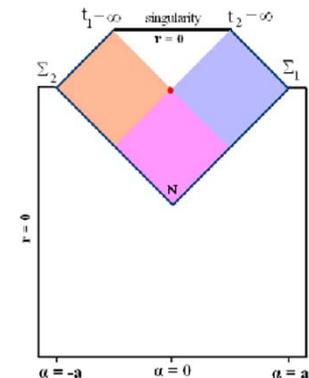
Penrose diagram

  – An observer can see only one boundary; the other boundary is behind the black hole horizon. [c.f. "non-traversability of a wormhole", "topological censorship"]

- In the white island phase, a white region will split.
  - Late time geometry for the three white island case:

    [Kodama et al '82]
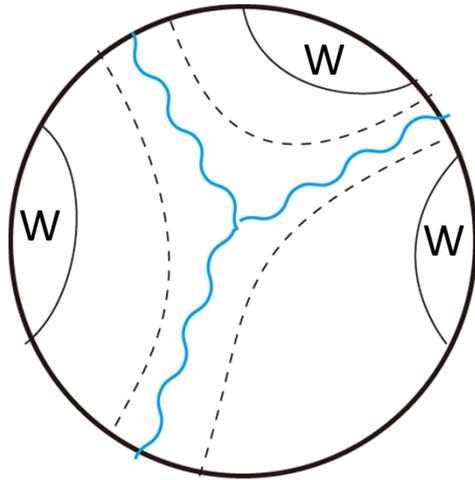
  - Singularity and horizons will form so that the boundaries are causally disconnected from each other.
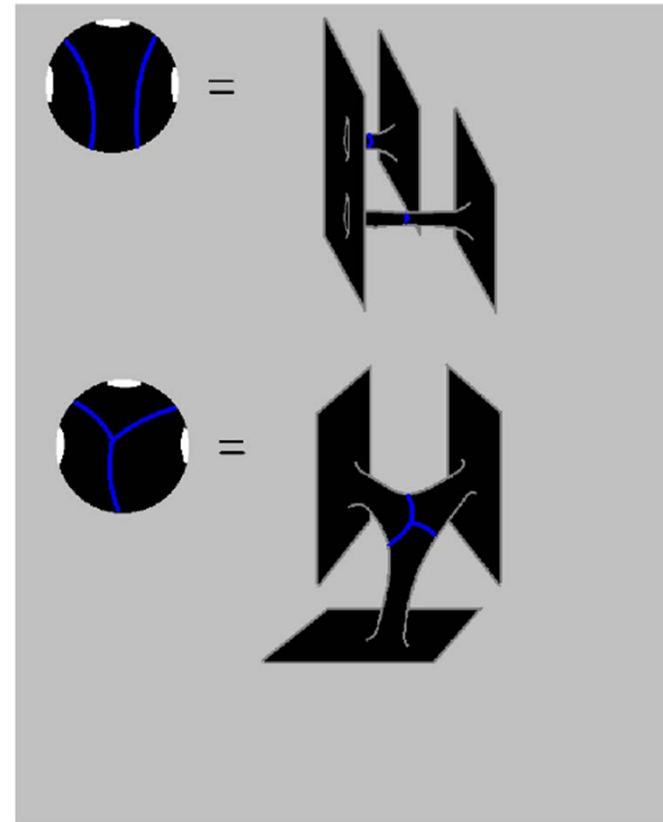
- From the Mandelbrot model: A single white island is of order Hubble size (due to frequent bubble nucleation)

- The boundary moves away from a given observer, but its area remains finite. (Effectively a closed universe)

- Black hole in the bulk.

- This universe will eventually collapse.
  - Simpler model: Shells of bubbles constantly colliding to a given bubble
  - Any given observer in the true vacuum will end up at singularity.
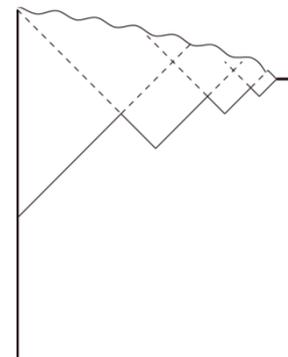
Summary of this part

Three phases of eternal inflation and their cosmology:

- Black island phase:
  Small deformation of an open FRW

- Tubular phase:
  Negatively curved space with an infinite genus boundary

- White island:
  Observer sees one boundary and one or more black hole horizons (behind which there are other boundaries).

CMB fluctuations from quantum effects of many fields

w/ Y. Habara, H. Kawai, M. Ninomiya,
"CMB Fluctuations and String Compactification Scales,"
arXiv:1103.0299 [hep-th]

(This is about inflation after our universe was created)

Outline

- We propose a new mechanism for generating temperature anisotropy of CMB during inflation.

- An aspect of fundamental theory: presence of a large number of fields (KK modes, string excitations).

- If the size L of extra dimension is large $L > H^{-1}$, there are KK modes with $m_{KK} \lesssim H$.

- We consider quantum effect of these light fields.

"Usual" mechanism:

- Expansion is driven by potential energy of a scalar field (inflaton): $H^2 = \dfrac{8\pi G}{3} V(\phi)$

  - Classical value of inflaton: defines time

  - Fluctuation of inflaton translates: difference in local expansion (or curvature perturbation); $\zeta \sim -aH \dfrac{\delta\phi}{\dot{\phi}_{\mathrm{cl}}}$ proportional to δT/T in CMB.

  - Tensor mode: Amplitude is universally H/m_{pl}
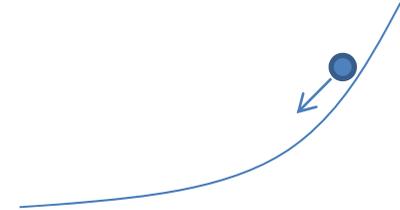
- Observation:

  $$\delta_{\mathrm{T}} \sim 2.6 \times 10^{-5}, \quad r_{t/s} \lesssim 0.22 \quad \left(\text{or } \frac{H}{m_{pl}} \lesssim 0.81 \times 10^{-4} \right)$$

- "Scalar mode is large because of slow rolling."

A model considered in this work

- Free massive scalars in de Sitter background

$$S = \sum_{A=1}^{N} \int d^4x \sqrt{-g} \left\{ \partial_\mu \phi_A \partial^\mu \phi_A - m_A^2 \phi_A^2 \right\}.$$

$$ds^2 = dt^2 - a^2(t) d\vec{x}^2, \quad a(t) = H^{-1} e^{Ht}$$

ϕ_A : <u>classically at the bottom of potential</u> $\phi_A = 0$

ϕ_A can be KK modes or string excitations.

(Later, comment on the time-dependent Hubble)

- This is different from "N-flation" or "curvaton."

Quantization in de Sitter space

- Equation of motion for a free scalar:

$$\left[\partial_t^2 + 3H\partial_t + m^2 - a^{-2}(t)\triangle\right]\phi = 0$$

  – At late times, time- and space- dependences factorize.
  – Fields with mass $m < \dfrac{3}{2}H$ do not oscillate
    (because the "friction" overdumps the oscillation).

$$\phi \sim e^{-\frac{\gamma}{2}Ht} = (Ha)^{-\frac{\gamma}{2}}$$

$$\gamma = 3\left(1 - \sqrt{1 - \frac{4}{9}m^2H^{-2}}\right) \sim \frac{2}{3}m^2H^{-2}$$

- Correlation function:

$$\langle\phi(\tau,\vec{x})\phi(\tau,\vec{x}')\rangle = \frac{H^2}{4\pi^2\gamma}(Ha|\vec{x} - \vec{x}'|)^{-\gamma}$$

Einstein equations (scalar mode)

- LHS: Einstein tensor linearized in metric fluctuations

$(0, i)$ component: $(\Psi' + \mathcal{H}\Phi), _i = 4\pi G \delta T_{0i}^{(S)},$

(i, j) component: $\left[\Psi'' + \mathcal{H}(2\Psi + \Phi)' + (2\mathcal{H}' + \mathcal{H}^2)\Phi + \dfrac{\triangle}{2}(\Phi - \Psi) \right] \delta_{ij}$

$$- \frac{1}{2}(\Phi - \Psi), _{ij} = 4\pi G \delta T_{ij}^{(S)}$$

where $\quad \mathcal{H} = \dfrac{a'}{a} = -\dfrac{1}{\tau}$

$(\tau : \text{ conformal time }, \quad a = -\dfrac{1}{H\tau}, \quad -\infty \leq \tau \leq 0)$

- $\Phi, \ \Psi$: gauge invariant variables. In longitudinal gauge,

$$g_{00} = a^2(1 + 2\Phi), \quad g_{ij} = -a^2(1 - 2\Psi)\delta_{ij}$$

- Energy momentum tensor: quadratic in matter fields

$$\delta T_{\mu\nu} = \sum \left\{ \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial^\rho \phi \partial_\rho \phi - m^2 \phi^2) \right\}$$

- $\delta T_{ij}^{(S)}$: scalar part

$$\delta T_{ij} = \partial_i \partial_j s - \frac{1}{3} \delta_{ij} \triangle s + \partial_i v_j + \partial_j v_i + t_{ij} + \underline{f \delta_{ij}}$$

- Solving for Φ,

$$\Phi' + \mathcal{H}\Phi = 4\pi G \sum \left\{ -\frac{3}{\triangle^2} \partial_i \partial_j (\partial_i \phi \partial_j \phi)' + \frac{1}{\triangle} (\partial_i \phi \partial_i \phi)' + \frac{1}{\triangle} \partial_i (\phi' \partial_i \phi) \right\}$$
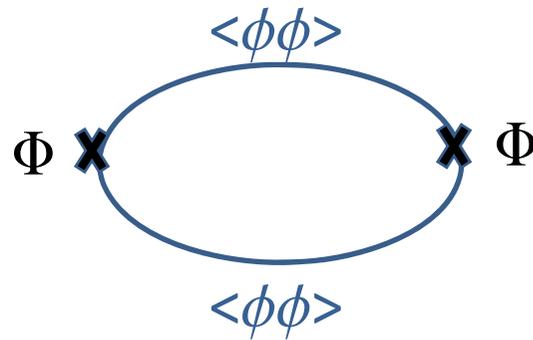
- At late times, $\phi \sim (-\tau)^{\gamma/2} \Rightarrow \Phi \sim (-\tau)^\gamma$

- When $\gamma \ll 1$ (which is important in later applications),

$$\Phi = -\pi G \sum \gamma \phi^2 \qquad (\text{and } \Phi = \Psi)$$

- Correlation function of Φ: sum over the species

$$\langle \Phi(\tau, x)\Phi(\tau, x')\rangle = 2\sum (\pi G\gamma)^2 \langle \phi(\tau, x)\phi(\tau, x')\rangle^2$$

$$= \sum \frac{H^4 G^2}{8\pi^2}(Ha|\vec{x} - \vec{x'}|)^{-2\gamma}$$

Tensor mode

- Equation of motion:

$$h_{ij}'' + 2\mathcal{H}h_{ij}' - \triangle h_{ij} = 8\pi G \delta T_{ij}^{(T)}$$

  RHS: transverse traceless part of δT_{ij} $(\sim (-\tau)^\gamma)$

- h_{ij} consists of
  $h_{ij}^{(0)}$: solution of the homogeneous eq. $\sim (-\tau)^0$
        (usual gravitational wave)
  $h_{ij}^{(1)}$: depends on δT_{ij} $\sim (-\tau)^{2+\gamma}$

- $h_{ij}^{(1)}$ (effect of matter) is negligible at late times.

KK modes and string states

- Assume D dims. are compactified on T^D with period L (and the size of other internal space is string scale)

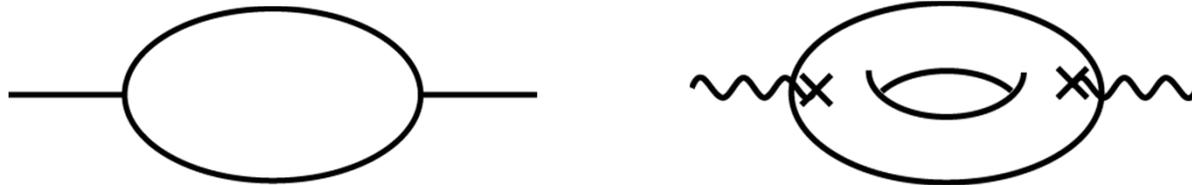- Mass of KK modes: $\quad m^2 = \displaystyle\sum_{a=1}^{D} (2\pi n_a)^2 / L^2$

- Number of states: $\quad N(m)dm = S_{D-1}(L/2\pi)^D m^{D-1} dm$

- The formula for Φ becomes

$$\langle \Phi\Phi \rangle = c_D L^D \left(\frac{H}{m_p}\right)^4 \int_0^{m_c} dm\, m^{D-1} (Ha|\vec{x} - \vec{x'}|)^{-2\beta},$$

- The upper limit should be $m_c \sim 3H/2$ in general relativity, but…

- If m_s (string scale) < H, the integration will be cut off at $m_c \sim m_s$ for the following reason:

- The fields ϕ are intermediate states in the loop.

- In string theory, Schwinger proper time is cut off at string scale. Intermediate state with $E > m_s$ do not contribute to physical processes.
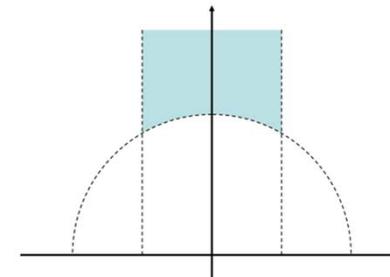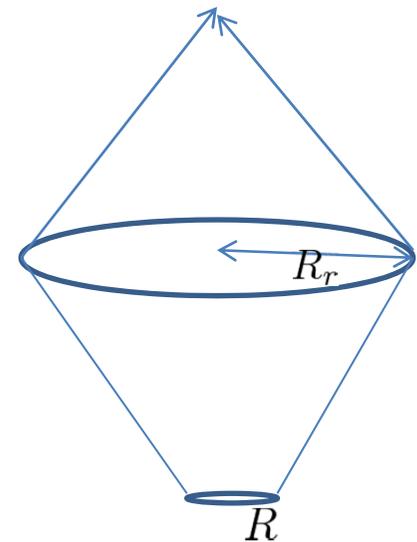
CMB fluctuations

- CMB temperature fluctuations: $\dfrac{\delta T}{T} = -\dfrac{1}{3}\Phi$

- Perturbation is frozen outside the horizon ($\theta \geq 3^o$)

- Angle θ on the sky corresponds to the distance d at the end of inflation:

$$d = 2R\sin(\theta/2) \left(= a_e|\vec{x} - \vec{x'}|\right)$$

$$R = \frac{a_e}{a_r}R_r \quad (R_r : \text{ radius of the last scattering surface})$$

- We assume $RH \sim 10^{29} \sim e^{67}$

Amplitude

- Angular power spectrum C_l is defined by

$$\langle \frac{\delta T}{T}(\theta) \frac{\delta T}{T}(0) \rangle = \sum_{l=1}^{\infty} (2l+1) C_l P_l(\cos\theta)$$

- From $\quad \langle \Phi\Phi \rangle = c_D L^D \left(\frac{H}{m_p}\right)^4 \int_0^{m_c} dm \, m^{D-1} (Ha|\vec{x} - \vec{x'}|)^{-2\gamma}$

  and expanding $(Ha|\vec{x} - \vec{x'}|)^{-2\gamma} \sim (2RH)^{-2\gamma}(1 - 2\gamma \log(\sin(\theta/2)),$

$$\delta_T^2 \equiv l(l+1) C_l = \frac{2}{3} c_D \frac{L^D}{H^2} \left(\frac{H}{m_{pl}}\right)^4 \int_0^{m_c} dm \, m^{D+1} (2RH)^{-\frac{4}{3}m^2 H^{-2}}$$
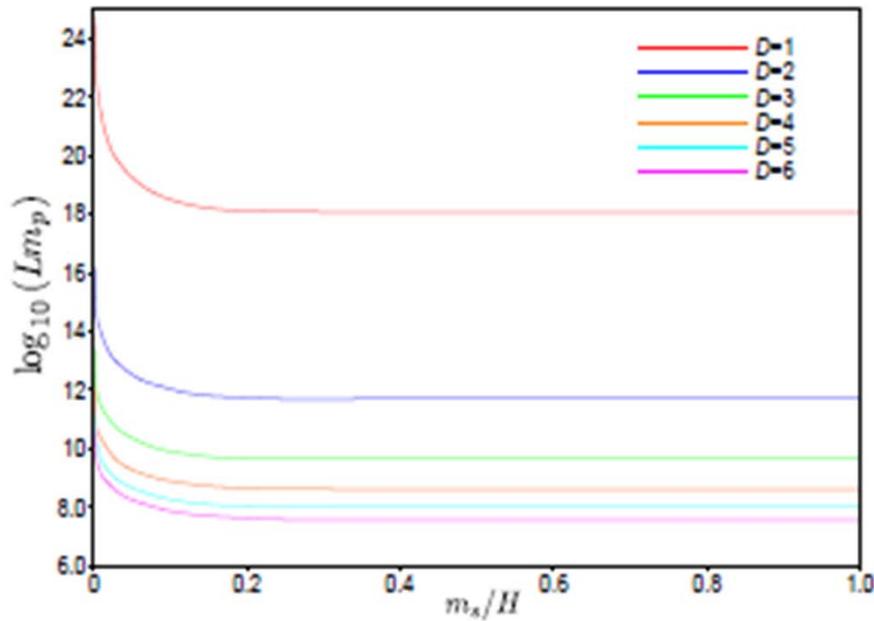
Preferred values of the parameters

- From observations,

$$\delta_{\mathrm{T}} \sim 2.6 \times 10^{-5}, \quad r_{t/s} \lesssim 0.22 \quad \left(\text{or} \ \frac{H}{m_{pl}} \lesssim 0.81 \times 10^{-4} \right)$$

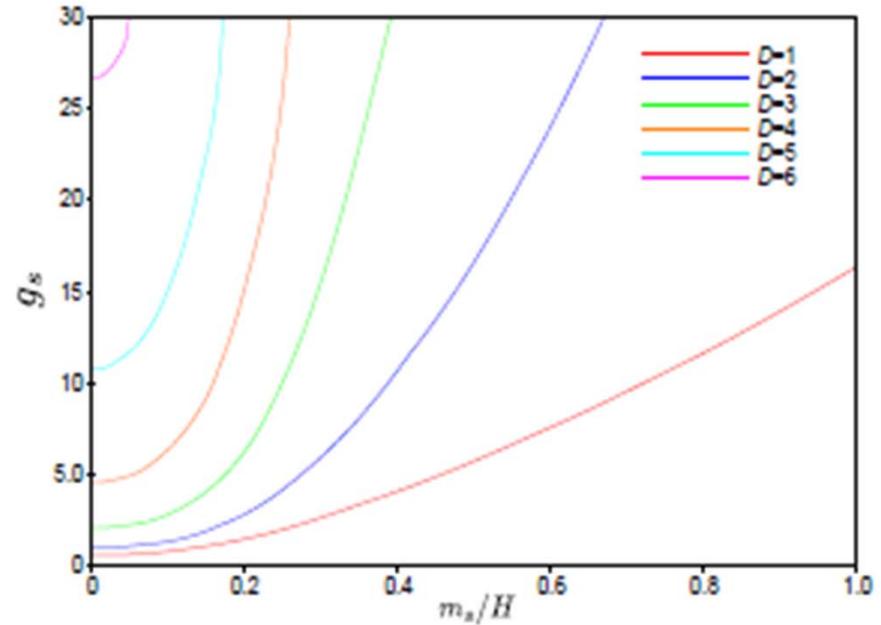- For the moment, assume the inequality is saturated.

- Then, δ_T gives a relation between m_s and L ,
  or between m_s and g_s using

$$(Lm_s)^D = 8\pi^6 g_s^2 (m_{pl}^2 / m_s^2)$$

L vs. m_s g_s vs. m_s

- We prefer weak coupling (easier with small D), and L not too large (easier with large D).
- Typical values: $\{D = 2,\ m_s/H = 0.2,\ Lm_{pl} = 10^{12},\ g_s = 3\}$,
  $\{D = 3,\ m_s/H = 0.2,\ Lm_{pl} = 10^{10},\ g_s = 5\}$,
  $\{D = 4,\ m_s/H = 0.1,\ Lm_{pl} = 10^{9},\ g_s = 7\}$.

- The number of fields $N \sim (Lm_s)^D$: $10^{14} \lesssim N \lesssim 10^{16}$

Time-dependent Hubble

- In our simple model, spectrum is tilted toward blue

$$1 \lesssim n_s \lesssim 1.02 \quad (\text{for } D = 2), \qquad 1 \lesssim n_s \lesssim 1.05 \quad (\text{for } D \leq 6)$$

  Not favored by observation. But n_s can be lowered by decreasing H with time.

- Time-dependence of H is needed at the end of inflation.

- By including inflaton in our analysis,

$$\Phi \sim -\left(\frac{V_{,\varphi}}{V}\right)^2 \left\{ H_* \left(\frac{V}{V_{,\varphi}}\right)_* + \frac{1}{4} \sum (\phi_*^2 - \phi^2) \right\} - \sum \pi G \gamma \phi^2$$
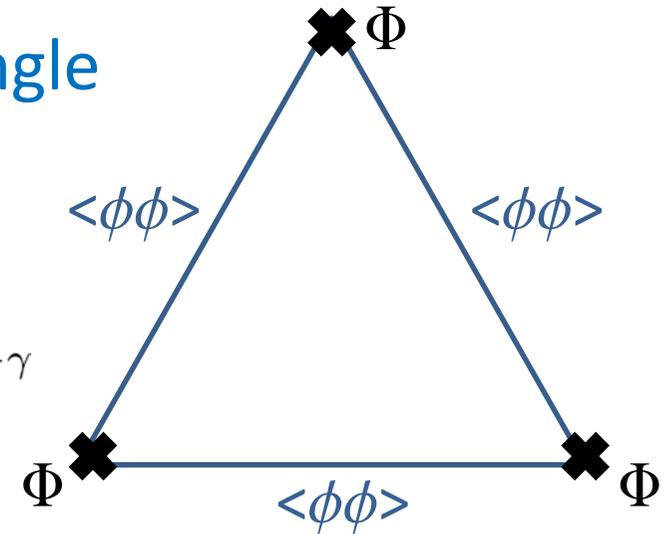
  ($*$: quantities at horizon crossing).

Non-gaussianities

- Three-point function is given by triangle diagram summed over species.

$$\langle \Phi(\tau, \vec{x}) \Phi(\tau, \vec{y}) \Phi(\tau, \vec{z}) \rangle$$

$$= \frac{1}{8\pi^3} \left(\frac{H}{m_{pl}} \right)^6 \sum \left(H^3 a^3 |\vec{x} - \vec{y}| |\vec{y} - \vec{z}| |\vec{x} - \vec{z}| \right)^{-\gamma}$$

- If we define non-linearity parameter f_{NL} by

$$\Phi \to \Phi_g + f_{NL} \Phi_g^2 \quad (\Phi_g : \text{gaussian field})$$

and evaluate it at generic separation, we get small f_{NL}

$$f_{NL} < 10^{-4}$$

Summary

- String theory suggests that our universe was created by bubble nucleation.

- The presence of the ancestor vacuum affects the spectrum of fluctuations.

- Proposed a holographic description in terms of CFT on the boundary.

- There are phases of eternal inflation characterized by the way bubbles percolate in the false vacuum.

- Found a mechanism for generating CMB temperature fluctuations from quantum effect of many fields.