Physics of Gauge-Higgs Unification

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References

"New Ideas on
Electroweak Symmetry Breaking"
Christophe Grojean
CERN-PH-TH/2006-172

"Holographic Methods
and Gauge-Higgs Unification
in Flat Extra Dimensions"
Marco Serone
arXiv: 0909.5619 [hep-ph]

"Lecture on Gauge-Higgs Unification
in extra dimensions"
Csaba Csa'ki
Talk slides in Ringberg Pheno. Workshop

PLAN

- Introduction
- Higgs mass calculation
- Gauge-Higgs sector
- Matter content
  $ Yukawa coupling
- Flavor Mixing
- Summary

If time permits...

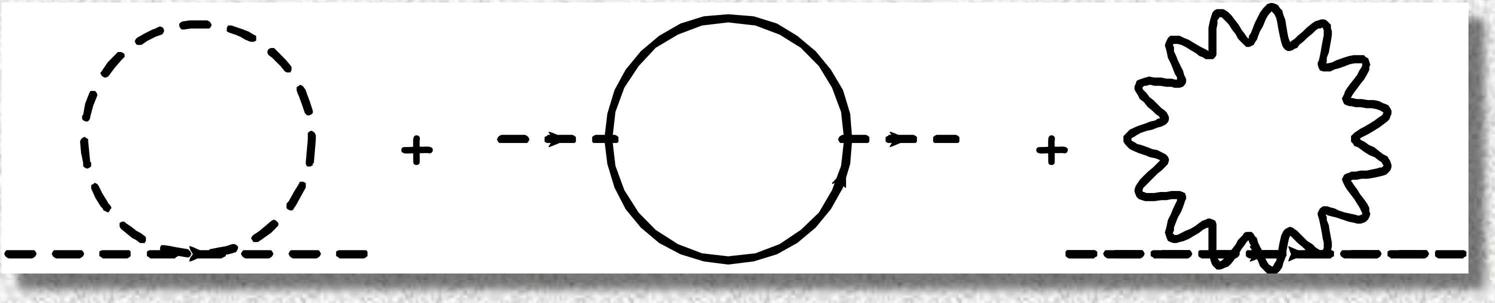
- S & T parameters
- g-2
- EDM
- GUT extension
- CP violation
  by Compactification
- Collider signals (seminar)

Introduction

One of the problems in the Standard Model:
Hierarchy Problem

Quantum corrections to the Higgs mass is sensitive to the cutoff scale of the theory

$$\delta m_H^2 =$$



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$$\delta m_H^2 \approx \frac{\Lambda^2}{16\pi^2}$$

Too large!!
(Natural cutoff scale is Planck scale or GUT scale)

To get Higgs mass of weak scale,
an unnatural fine tuning of parameters are required

$$m_H^2 = \underset{\uparrow}{m_0^2} + \underset{\uparrow}{\delta m^2} \approx \mathcal{O}\left(\left(100 GeV\right)^2\right)$$

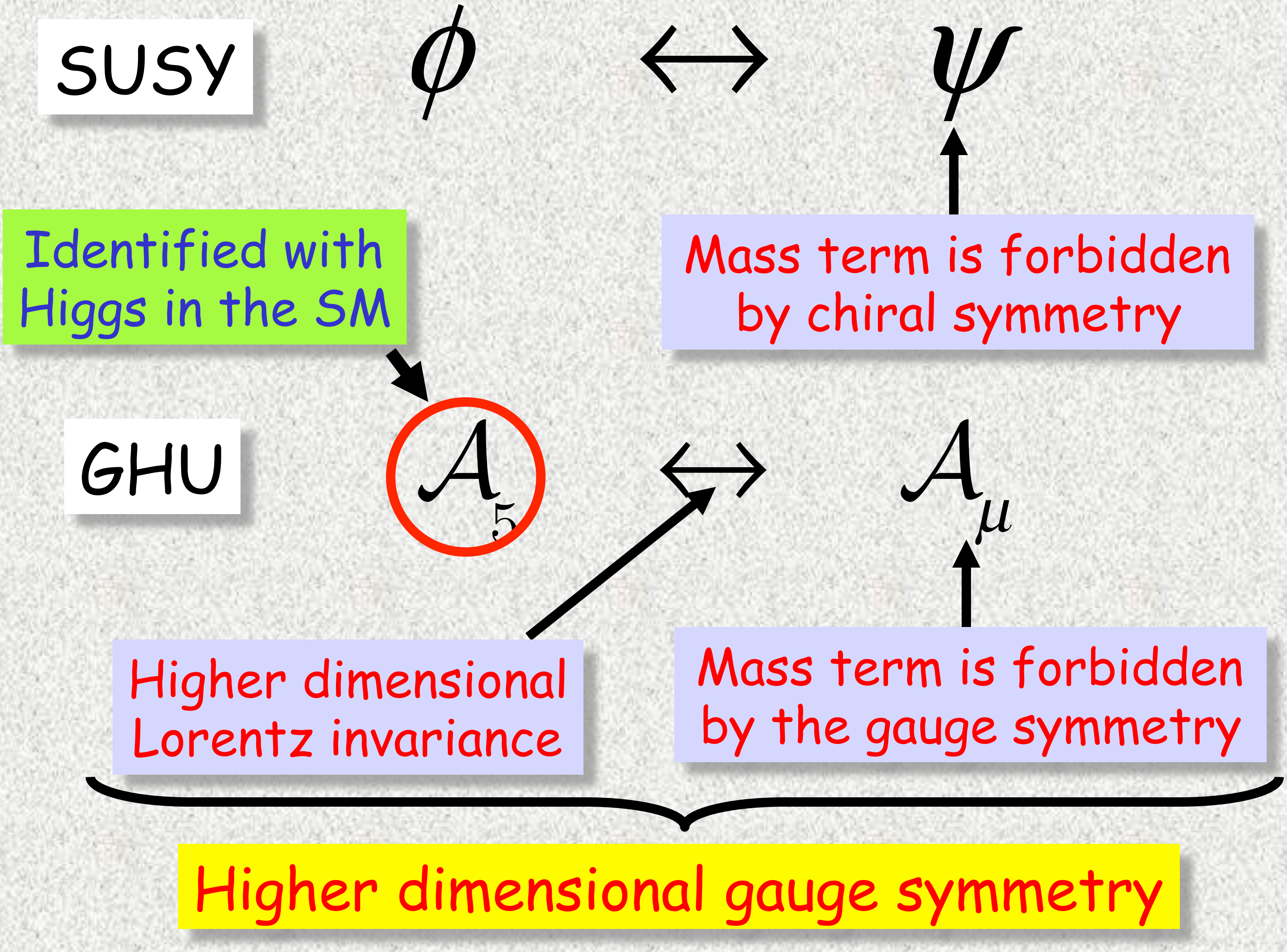
classical Quantum corrections

Naively, we have $$m_0^2 , \delta m^2 \approx \mathcal{O}\left(\left(10^{18} GeV\right)^2\right)$$

32 digits of fine tuning

1.00000000000000000000000000000001 – 1 !!

Problem: We have NO symmetry forbidding the scalar mass



SUSY $\phi \leftrightarrow \psi$

Mass term is forbidden by chiral symmetry

Identified with Higgs in the SM

GHU $\mathcal{A}_5 \leftrightarrow \mathcal{A}_\mu$

Higher dimensional Lorentz invariance

Mass term is forbidden by the gauge symmetry

Higher dimensional gauge symmetry

Indeed, the (local) mass term A_5^2 can be forbidden by the gauge symmetry for 5th component of the gauge field

$$\because A_5 \to A_5 + \partial_5 \varepsilon(x,y) + i\left[\varepsilon(x,y), A_5\right]$$

In other words, no local counter term is allowed

⇒ No quadratic divergence, finite

This symmetry is very useful in the orbifold model since it is operative even on the branes G → H

Gersdorff, Irges & Quiros (2002)

$$\because A_5 \to A_5 + \partial_5 \underbrace{\varepsilon_{G/H}(x,y)}_{\text{Z}_2\ \text{odd}} + i\left[\underbrace{\varepsilon_H(x,y)}_{\text{Z}_2\ \text{even}}, A_5\right]$$

No quadratic divergence from brane localized Higgs mass

Explicit calculations of Higgs mass

- D-dim QED on S^1@1-loop Hatanaka, Inami & Lim (1998)
- 5D Non-Abelian gauge theory on S^1/Z_2@1-loop
  Gersdorff, Irges & Quiros (2002)
- 6D Non-Abelian gauge theory on T^2@1-loop
  Antoniadis, Benakli & Quiros (2001)
- 6D Scalar QED on S^2@1-loop Lim, NM & Hasegawa (2006)
- 5D QED on S^1@2-loop
  NM & Yamashita (2006); Hosotani, NM, Takenaga & Yamashita (2007)
- 5D Gravity on S^1 (GGH) Hasegawa, Lim & NM (2004)

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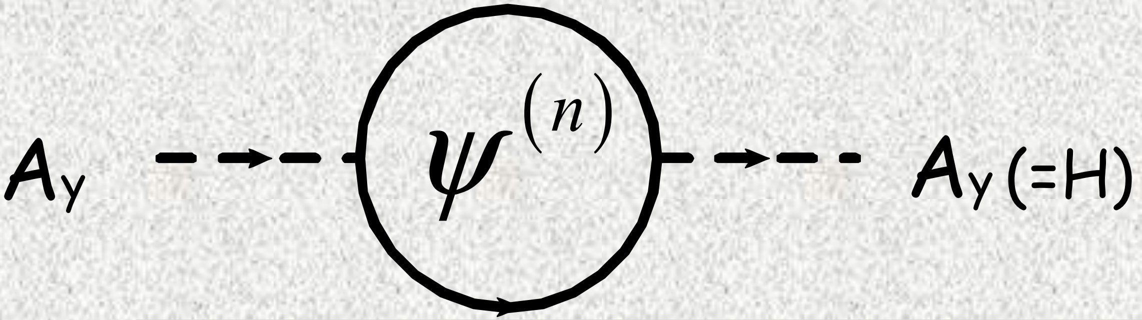
Higgs mass calculation

Consider (D+1)-dim QED on S^1

Hatanaka, Inami & Lim (1998)



$$m_H^2 = ie_D^2 \int \frac{d^Dk}{(2\pi)^D} \sum_{n=-\infty}^{\infty} Tr\left[\gamma_y \frac{1}{\not k - m}\gamma^y \frac{1}{\not k - m}\right]$$

(No sum)

L=2πR

$$\xrightarrow{L\to\infty} \frac{i}{D+1} e_{D+1}^2 \int \frac{d^{D+1}k}{(2\pi)^{D+1}} Tr\left[\gamma_M \frac{1}{\not k - m}\gamma^M \frac{1}{\not k - m}\right] \left(M = 0,1,\cdots D\right)$$

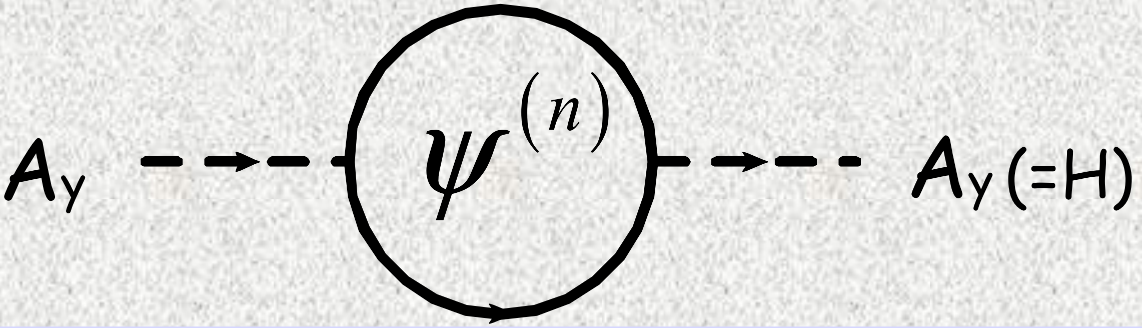
$$= \frac{i}{D+1} e_{D+1}^2 2^{[(D+1)/2]} \int \frac{d^{D+1}k}{(2\pi)^{D+1}} \left[\frac{1-D}{k^2-m^2} - \frac{2m^2}{\left(k^2-m^2\right)^2}\right]$$

$$= \frac{i}{D+1} e_{D+1}^2 2^{[(D+1)/2]} \left(1-D+2m^2\frac{\partial}{\partial m^2}\right) \int \frac{d^{D+1}k}{(2\pi)^{D+1}} \frac{1}{k^2-m^2}$$

$$= \frac{i}{D+1} e_{D+1}^2 2^{[(D+1)/2]} \frac{-i}{(4\pi)^{(D+1)/2}} \Gamma\left(\frac{1-D}{2}\right) \left(1-D+2m^2\frac{\partial}{\partial m^2}\right) \left(m^2\right)^{(D-1)/2} = 0$$

Consider (D+1)-dim QED on S^1

Hatanaka, Inami & Lim (1998)



Boundary condition

$$\psi(x_\mu, y+L) = e^{i\alpha}\psi(x_\mu, y)$$

$$m_H^2 = ie_D^2 2^{[(D+1)/2]}\int\frac{d^Dk}{(2\pi)^D}\sum_{n=-\infty}^{\infty}\left[-\frac{1}{((2\pi n+\alpha)/L)^2+\rho^2}+\frac{2\rho^2}{\left[((2\pi n+\alpha)/L)^2+\rho^2\right]^2}\right]$$

$$= -ie_D^2 2^{[(D+1)/2]}\int\frac{d^Dk}{(2\pi)^D}\left(1+\rho\frac{\partial}{\partial\rho}\right)\left(\frac{L}{2\rho}\right)\frac{\sinh(\rho L)}{\cosh(\rho L)-\cos\alpha}$$

L=2πR

$$\rho^2 = -k^2 + m^2$$

$$= \frac{e_D^2 L^{2-D}}{2^{D-[(D+1)/2]}\pi^{D/2}\Gamma(D/2)}\int_0^\infty dk\; k_E^{D-1}\frac{1-\cosh\left(\sqrt{k_E^2+m^2}\,L\right)\cos\alpha}{\left[\cosh\left(\sqrt{k_E^2+m^2}\,L\right)-\cos\alpha\right]^2} < \infty$$

Superconvergent!!

(Nonlocal mass: Wilson line phase $\alpha = g\oint dyA_y$)

Ex. take D=4 (5 dimension case) & m=0, α=π

$$m_H^2 = \frac{e_4^2}{4\pi^2}\frac{1}{(2\pi R)^2}\int_0^\infty ds s^3 \left.\frac{1-\cosh s\cos\alpha}{\left[\cosh s-\cos\alpha\right]^2}\right|_{\alpha=\pi}$$

$$= \frac{9e_4^2}{16\pi^4 R^2}\zeta(3) = \frac{9e_4^2}{16\pi^6}\underbrace{\zeta(3)}_{1.2} m_W^2$$

$m_W=\pi/R$

Higgs mass is too small

→ generic prediction of GHU

Way out to get 125-126 GeV Higgs mass

1: Realizing small Higgs VEV $a \ll 1$
by choosing appropriate matter content

$$m_H \sim m_W/(4\pi a) \quad (m_W = a/R)$$

Haba, Hosotani, Kawamura & Yamashita etc

2: D > 5 dimensions

F_{ij}^2 contains the Higgs quartic coupling $g^2[A_i, A_j]^2$ in general. Higgs mass is generated at leading order
$m_H = 2m_W$ is predicted in 6D on T^2/Z_3 model

Scrucca, Serone, Silvestrini & Wulzer (2003)

3: Warped dimension (ex. Randall-Sundrum model)

Higgs mass is enhanced by curvature scale $k\pi R \sim 30$

Contino, Nomura & Pomarol (2003)

Gauge-Higgs sector

Model building of the gauge-Higgs unification

A_5 is an SU(2) adjoint as it stands, not SU(2) doublet

⇒ need to enlarge the gauge group

$G \rightarrow SU(2)_L \times U(1)_Y$

adj → doublet + other reps

Simplest G

SU(3)

Consider 5D SU(3) model on S^1/Z_2 with Parity: P = diag (-,-,+)

$$PA_\mu(x,y_i-y)P^\dagger = A_\mu(x,y_i+y),\ PA_5(x,y_i-y)P^\dagger = -A_5(x,y_i+y)$$

$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix},\ A_5 = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}$$

Only (+,+) mode has massless mode (“0 mode”)

$$A_\mu^{(0)}=\frac{1}{2}\begin{pmatrix} W_\mu^3+B_\mu^3/\sqrt{3} & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 & 0 \\ 0 & 0 & -2B_\mu/\sqrt{3} \end{pmatrix}, A_5^{(0)}=\frac{1}{2}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix}$$

SU(2) x U(1) gauge fields

Higgs doublets

mode expansions

$$A_M^{(+,+)}(x,y)=\frac{1}{\sqrt{2\pi R}}\left[A_M^{(0)}(x)+\sqrt{2}\sum_{n=1}^{\infty}A_M^{(n)}(x)\cos\left(\frac{n}{R}y\right)\right]$$

$$A_M^{(-,-)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}A_M^{(n)}(x)\sin\left(\frac{n}{R}y\right)$$

Gauge boson spectrum

$$M_{W_n}=\frac{n+a}{R}, M_{Z_n}=\frac{n+2a}{R}, M_{\gamma_n}=\frac{n}{R}, \left\langle A_5^{(0)}\right\rangle=\frac{a}{g_5R}$$

$$M_{W_n} = \frac{n+a}{R}, M_{Z_n} = \frac{n+2a}{R}, M_{\gamma_n} = \frac{n}{R}, \left\langle A_5^{(0)} \right\rangle = \frac{a}{g_5 R}$$

- W, Z, γ are identified with zero modes:
  $M_W = a/R, \quad M_Z = 2a/R, \quad M_\gamma = 0$
- $M_Z = 2M_W \rightarrow \cos\theta_W = \frac{1}{2}$ ($\sin^2\theta_W = \frac{3}{4} \gg 0.23$)
- The spectrum is invariant under $a \Leftrightarrow -a$
  → physical range [0, 1/2]

  (this kind of spectrum is specific to GHU compared to UED case: $M_{W_n} = \sqrt{M_W^2 + (n/R)^2}$)
- Non-zero KK modes of A_5 are eaten by non-zero KK modes of A_μ (Higgs mechanism)

Hypercharge of the doublet

Check the hypercharge of Higgs doublet

$$\delta_{U(1)}A_5^{(0)} = g\left[T^8, A_5^{(0)}\right] = \frac{g}{2\sqrt{3}}\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix}\right]$$

$$= \frac{g}{2\sqrt{3}}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -2H^- & -2H^{0*} & 0 \end{pmatrix} - \frac{g}{2\sqrt{3}}\begin{pmatrix} 0 & 0 & -2H^+ \\ 0 & 0 & -2H^0 \\ H^- & H^{0*} & 0 \end{pmatrix} = \frac{g\sqrt{3}}{2}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -H^- & -H^{0*} & 0 \end{pmatrix}$$

$$\sin^2\theta_W = \frac{g_Y^2}{g^2 + g_Y^2} = \frac{\left(\sqrt{3}g\right)^2}{g^2 + \left(\sqrt{3}g\right)^2} = \frac{3}{4} \neq 0.23(\text{Exp})$$

Too Big!!

Well-known by Fairlie, Manton (6D on S^2 w/ monopole bkgd)

	G_2	SO(5)	SU(3)
$\sin^2\Theta_W$	1/4	1/2	3/4

Way out to get a correct Θ_W

1: Additional U(1) $SU(3) \times U(1)' \rightarrow SU(2)_L \times U(1)_Y \times U(1)_X$

Scrucca, Serone & Silvestrini (2003)

$$A_Y = \frac{g' A_8 + \sqrt{3} g A'}{\sqrt{3g^2 + g'^2}}, A_X = \frac{\sqrt{3} g A_8 - g' A'}{\sqrt{3g^2 + g'^2}} \Rightarrow g_Y = \frac{\sqrt{3} g g'}{\sqrt{3g^2 + g'^2}}$$

➡

$$\sin^2 \theta_W = \frac{g_Y^2}{g^2 + g_Y^2} = \frac{3}{4 + 3 g^2 / g'^2}$$

2: Localized gauge kinetic terms

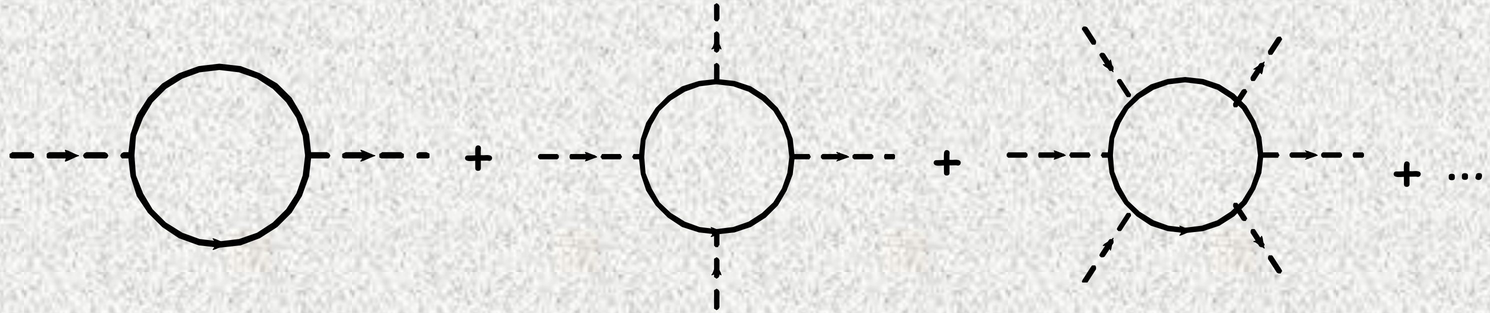
$$\mathcal{L} = \underset{\text{SU(3) invariant}}{-\frac{1}{2g_5^2} Tr F_{MN} F^{MN}} - \underbrace{\left[\frac{1}{2g_4^2} \delta(y) + \frac{1}{2g_4'^2} \delta(y - \pi R) \right] Tr F_{\mu\nu} F^{\mu\nu}}_{\text{SU(2) x U(1) invariant}}$$

Electroweak symmetry breaking

In GHU, EW symmetry is dynamically broken
by the Hosotani mechanism Hosotani (1983,1989)

Higgs potential is radiatively generated
since the tree level potential is forbidden
by the gauge invariance (Coleman-Weinberg potential)

$V(A_5) =$



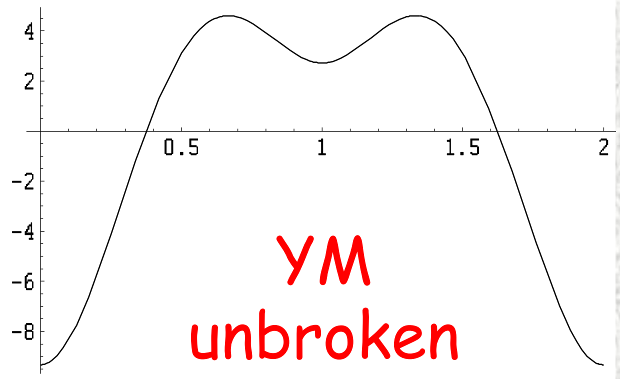
$$V(a) = (-1)^F \frac{(\text{DOF})}{2} \int \frac{d^4 p_E}{(2\pi)^4} \frac{1}{2\pi R} \sum_n \log\left(p_E^2 + \underset{\substack{\uparrow \\ \text{KK mass}}}{m_n^2}\right)$$

Ex. 5D SU(3) model on S^1/Z_2 with N_f fundamental & N_a adjoint fermions

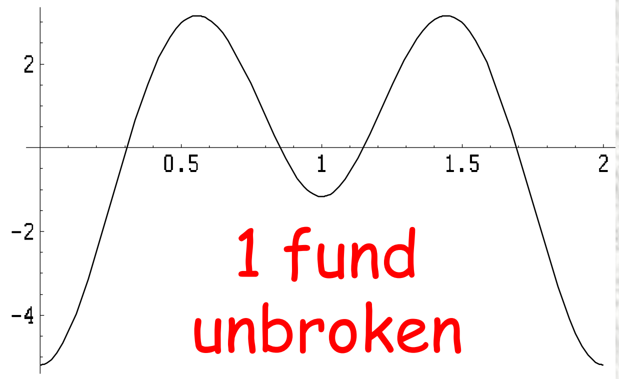
Kubo, Lim & Yamashita (2002)

$$V(a)=\frac{3}{128\pi^7R^5}\sum_{n=1}^{\infty}\frac{1}{n^5}\left[(4N_a-3)\underbrace{\left(\cos[2\pi na]+2\cos[\pi na]\right)}_{\text{adjoint}}+4N_f\underbrace{\cos[\pi na]}_{\text{fund}}\right]$$

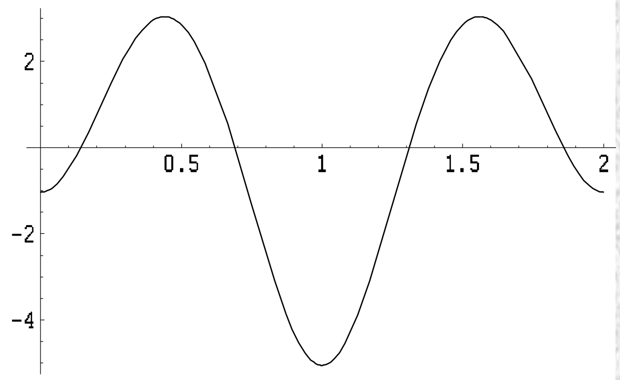
Gauge + ghost



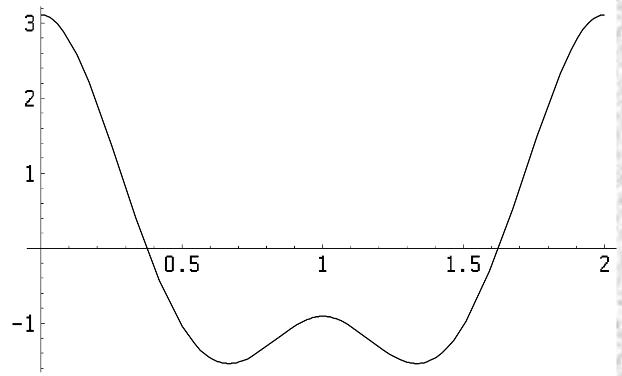
YM
unbroken



1 fund
unbroken



2 fund
SU(2)xU(1) -> U(1)xU(1)



1 adj
SU(2)xU(1) -> U(1)

Wilson line phase

$$W = \mathcal{P}\exp\left(ig\oint_{S^1} dy A_5\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi a) & i\sin(\pi a) \\ 0 & i\sin(\pi a) & \cos(\pi a) \end{pmatrix} \quad (a \bmod 2) = \begin{cases} SU(2)\times U(1) & \text{for a = 0} \\ U(1)' \times U(1) & \text{for a = 1} \\ U(1)_{\text{em}} & \text{for other cases} \end{cases}$$

$$\langle A_5 \rangle = \frac{a}{gR}\frac{T^6}{2} \equiv A_5^{6(0)}\frac{T^6}{2}$$

$$T^3 = diag(1,-1,0)$$

$$T^8 = diag(1,1,-2)/\sqrt{3}$$

$$a = 1 : W = diag(1,-1,-1) \Rightarrow \left[W, T^3\right] = \left[W, T^8\right] = 0$$

U(1) x U(1)’ unbroken

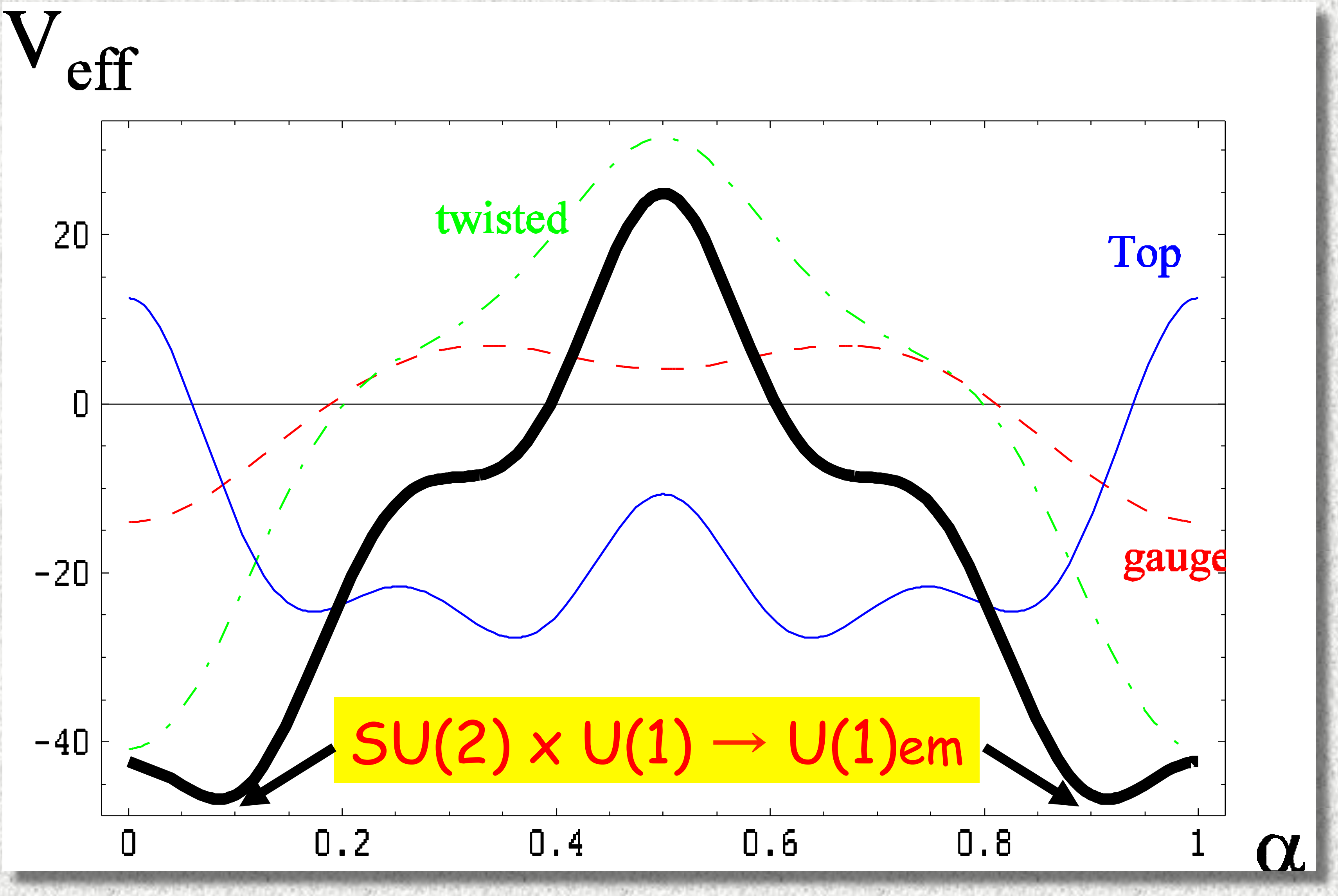
$$0 < a < 1 : \left[W, \sqrt{3}T^3 + T^8\right] = \left[W, \sin\theta_W T^3 + \cos\theta_W T^8\right] = 0$$

U(1)$_{\text{em}}$ unbroken

Higgs potential (top (15*) + bottom (3) + tau (10))

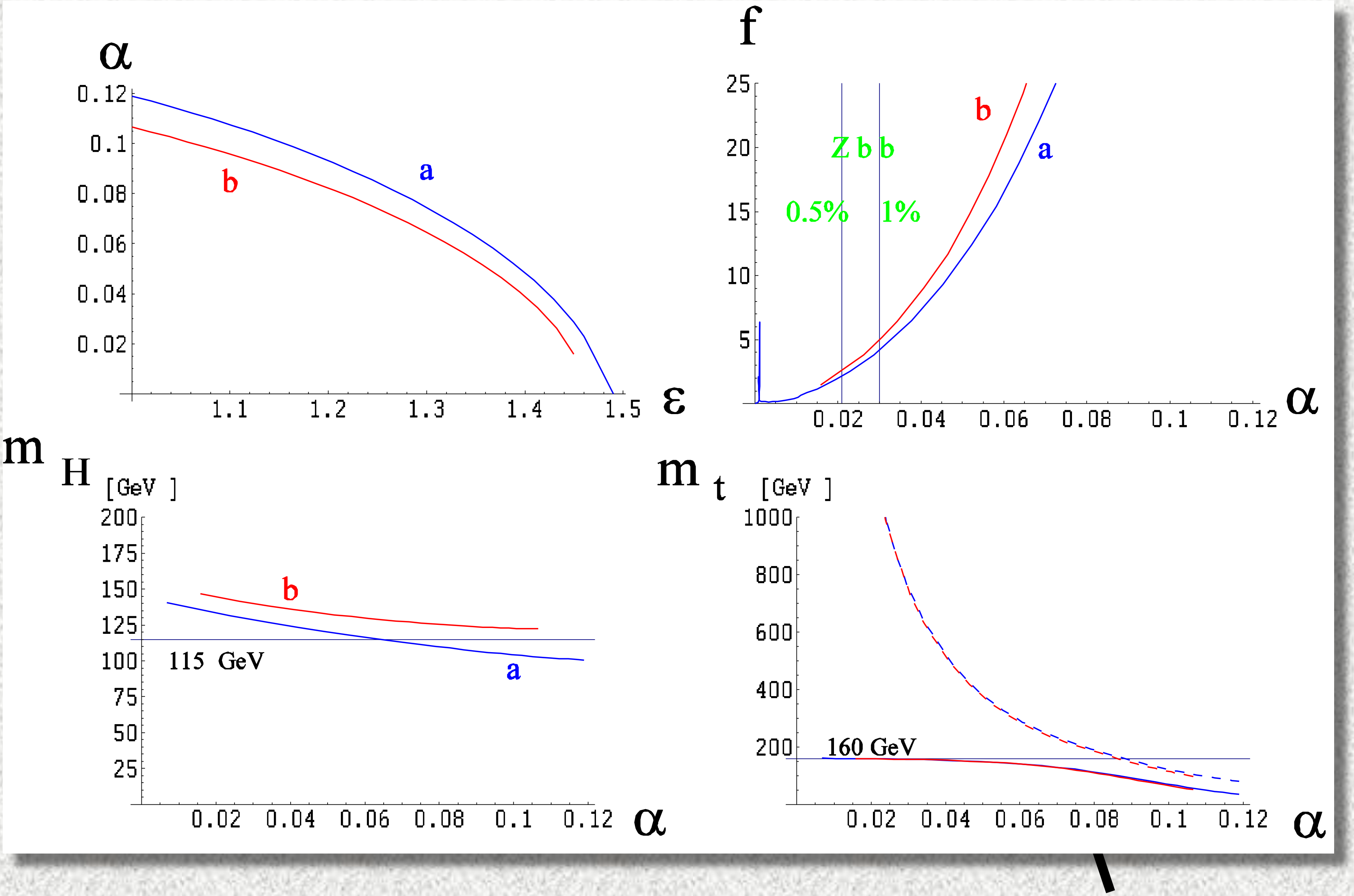
Cacciapaglia, Csaki & Park (2005)

ε=1.25



SU(2) x U(1) → U(1)em

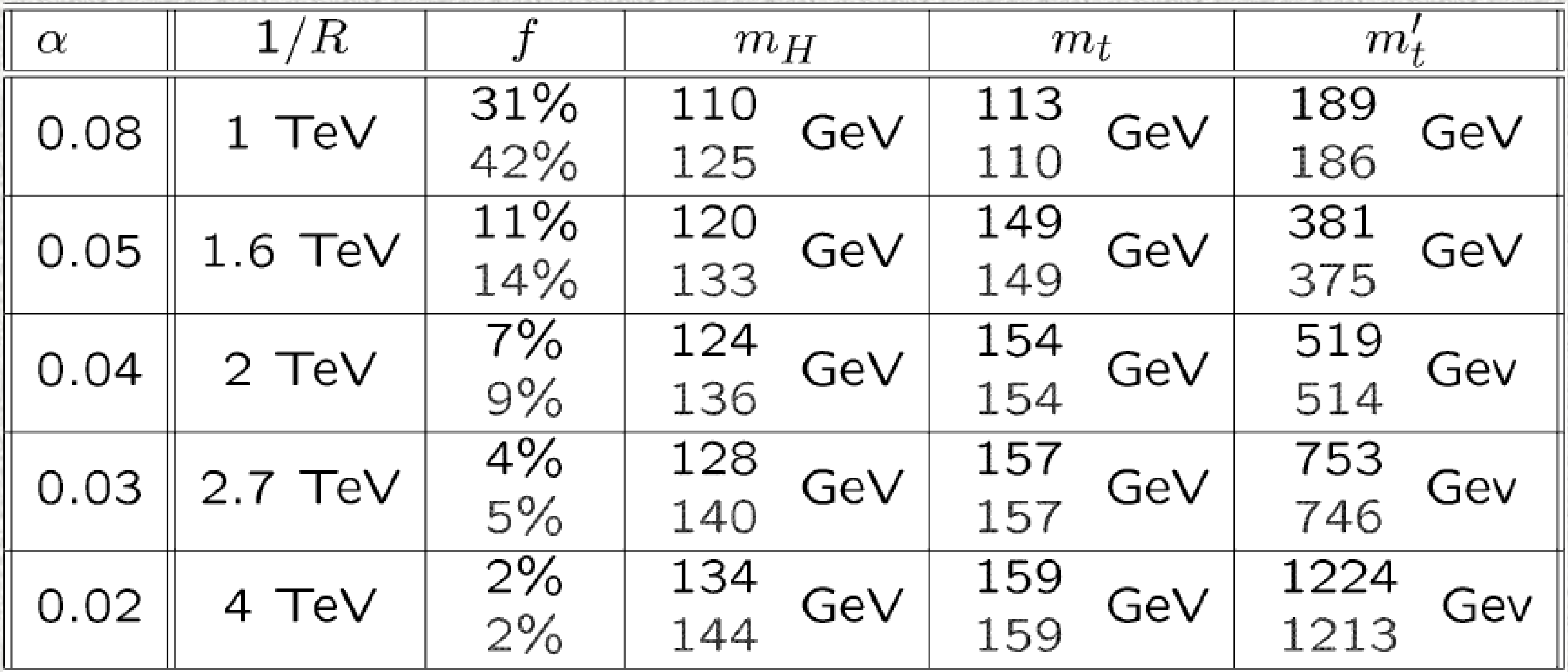
Higgs mass, top mass,…etc



top

Model a: b(3), τ(10)   Model b: b(6), τ(3)

Sample points



α	$1/R$	f	m_H	m_t	m_t'
0.08	1 TeV	31% 42%	110 125 GeV	113 110 GeV	189 186 GeV
0.05	1.6 TeV	11% 14%	120 133 GeV	149 149 GeV	381 375 GeV
0.04	2 TeV	7% 9%	124 136 GeV	154 154 GeV	519 514 Gev
0.03	2.7 TeV	4% 5%	128 140 GeV	157 157 GeV	753 746 Gev
0.02	4 TeV	2% 2%	134 144 GeV	159 159 GeV	1224 1213 Gev

Fine-tuning required to
obtain the potential minimum

Model A (top low)
Model B (bottom low)

Matter Content
$
Yukawa Coupling

Quark & Lepton embedding

Consider a fundamental rep of SU(3)

$\mathbf{3} = (q, q-1, 1-2q)^T$ (q: electric charge)

Putting q=2/3, we get

$$\mathbf{3} = \mathbf{2}_{1/6} + \mathbf{1}_{-1/3} = (2/3, -1/3, -1/3)^T = (\overset{+}{u_L}, \overset{+}{d_L}, \overset{-}{d_R})^T$$

$(+, +, -)_L$

Only fundamental reps cannot incorporate right-handed up-type quarks as well as leptons

2-rank sym: $\mathbf{6}^* = \begin{cases} \mathbf{3}_{L-1/3} + \boxed{\mathbf{2}_{L1/6}\,(Q)} + \mathbf{1}_{L2/3} \\ \boxed{\mathbf{3}_{R-1/3}} + \mathbf{2}_{R1/6} + \boxed{\mathbf{1}_{R2/3}\,(u_R)} \end{cases}$

3-rank sym: $\mathbf{10} = \begin{cases} \boxed{\mathbf{4}_{L1/2}} + \mathbf{3}_{L0} + \boxed{\mathbf{2}_{L-1/2}\,(L)} + \mathbf{1}_{L-1} \\ \mathbf{4}_{R1/2} + \boxed{\mathbf{3}_{R0}} + \mathbf{2}_{R-1/2} + \boxed{\mathbf{1}_{R-1}\,(e_R)} \end{cases}$

Many massless exotics ⇒ brane localized mass term

Big
Hurdle

In the gauge-Higgs unification,
Yukawa coupling = gauge coupling

How can we get fermion mass hierarchy???

As will be shown below,
fermion masses except for top quark are relatively easy

1: Localizing fermions@different point in 5th direction

Yukawa ~ exponentially suppressed overlap integral

Arkani-Hamed & Schmaltz (1999)

2: Bulk fermions mixed with localized fermions
@the fixed points

Non-local Yukawa coupling

Csaki, Grojean & Murayama (2002)

1: Yukawa coupling from localizing fermions @different points

1: To localize fermions at different points along the 5th direction, bulk masses are introduced

2: To be consistent with Z2 orbifold,
Z2 parity of bulk mass must be odd ⇒ kink mass

Consider a 5D fermion satisfying the following Dirac equation

$$0=\left[i\Gamma^M D_M - M\varepsilon(y)\right]\psi(x,y)$$

$$D_M=\partial_M - igA_M, \Gamma^M=\left(\gamma^\mu, i\gamma^5\right), (M=0,1,2,3,5), \varepsilon(y)=\begin{cases}1\,(y>0)\\-1\,(y<0)\end{cases}$$

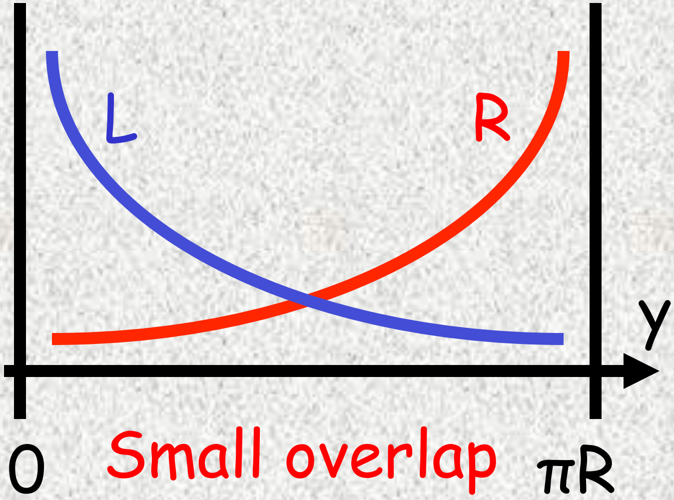
Focusing zero modes

$$\psi(x,y):\ \psi^{(0)}_{L(R)}(x)f^{(0)}_{L(R)}(y),\ \gamma^5\psi_{L(R)}=(-)\psi_{L(R)}$$

Zero mode wave functions

$$0=\left[\partial_y+M\varepsilon(y)\right]f_L^{(0)}(y)\rightarrow f_L^{(0)}(y)=\sqrt{\frac{M}{1-e^{-2\pi MR}}}e^{-M|y|}$$

$$0=\left[\partial_y-M\varepsilon(y)\right]f_R^{(0)}(y)\rightarrow f_R^{(0)}(y)=\sqrt{\frac{M}{e^{2\pi MR}-1}}e^{M|y|}$$



Small overlap

4D effective Yukawa coupling

$$Y=g_4\int_{-\pi R}^{\pi R}dyf_L^{(0)}(y)f_R^{(0)}(y)=g_4\int_{-\pi R}^{\pi R}dy\sqrt{\frac{M^2}{\left(1-e^{-2\pi MR}\right)\left(e^{2\pi MR}-1\right)}}$$

$$\approx 2\pi MRg_4e^{-\pi MR}\leq g_4\Leftrightarrow m_f\leq m_W$$

$\pi MR \gg 1$

Fermion masses except top is easy, but top is hard
No need of unnatural fine-tuning for 5D parameters M,R

2: Mixing between bulk and boundary localized fermions

Csaki, Grojean & Murayama (2002)

Consider the massive bulk fermion
coupling with SM fermions on the branes

$$\mathcal{L}_{Bulk} = \bar{\Psi}(x,y)\left(i\Gamma^M D_M - M(\varepsilon)\right)\Psi(x,y), \Psi = \left(\psi^d, \chi^s\right)^T$$

$$\mathcal{L}_{Brane} = \delta(y-y_L)\left[i\bar{Q}_L\bar{\sigma}^\mu\partial_\mu Q_L + \frac{\varepsilon_L}{\sqrt{\pi R}}\bar{\psi}^d Q_L + h.c.\right] + \delta(y-y_R)\left[i\bar{q}_L\bar{\sigma}^\mu\partial_\mu q_L + \frac{\varepsilon_R}{\sqrt{\pi R}}\bar{q}_R\chi^s + h.c.\right]$$

Mixing mass term between bulk & brane fermions

Integrating out massive fermion generates mass term as

$$\varepsilon_L\varepsilon_R\pi MRe^{-\pi MR}\bar{q}_R e^{ig\int_0^{\pi R} dyA_y} Q_L \Rightarrow m_f \propto \varepsilon_L\varepsilon_R\pi MRe^{-\pi MR}M_W$$

Exponentially suppressed coupling

⇒ easy to generate fermion masses except for top

How do we obtain top mass???

Top mass generation

Cacciapaglia, Csaki & Park (2005)

Consider large dimensional reps,
then an upper bound on fermion mass is modified as follows

$$m_t \leq \sqrt{n}\, m_W$$

(n: # of indices of rep)

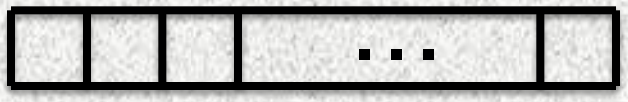
For $m_t = 2m_W$ ⇒ need a 4-index rep top is embedded
To saturate this bound, bulk mass should be zero

Simplest example: 

$$(\mathbf{15}^*)_{-2/3} \longrightarrow (\mathbf{1},\, 2/3)(t_R) + (\mathbf{2},\, 1/6)(t_L)$$
$$+ (\mathbf{3},\, -1/3) + (\mathbf{4},\, -5/6) + (\mathbf{5},\, -4/3)$$

$\sqrt{N}$ enhancement

Consider a rank N symmetric tensor of SU(3)



N boxes

Decompose it into SU(2) reps as 3 = 2 + 1
and make a singlet & a doublet

singlet

unique

doublet

etc N patterns

Canonical kinetic term ⇒ $1/\sqrt{N}$

$$\text{Yukawa} = 1_R\, 2_L\, 2_H \Rightarrow N \times 1/\sqrt{N} = \sqrt{N}$$

Fermion matter content

$3 = 2_{L1/6}(Q) + 1_{L-1/3}$

$2_{R1/6} + 1_{R-1/3}(d_R)$

Down quark sector

$6^* = 3_{L-1/3} + 2_{L1/6}(Q) + 1_{L2/3}$

$3_{R-1/3} + 2_{R1/6} + 1_{R2/3}(u_R)$

Up quark sector (except for top)

$10 = 4_{L1/2} + 3_{L0} + 2_{L-1/2}(L) + 1_{L-1}$

$4_{R1/2} + 3_{R0} + 2_{R-1/2} + 1_{R-1}(e_R)$

Charged lepton sector

$15^* = 5_{L-4/3} + 4_{L-5/6} + 3_{L-1/3} + 2_{L1/6}(Q) + 1_{L2/3}$

Top quark

$5_{R-4/3} + 4_{R-5/6} + 3_{R-1/3} + 2_{R1/6} + 1_{R2/3}(t_R)$

Unwanted massless exotics (blue reps) & two extra Qs must be massive by brane localized mass terms

Flavor Mixing

"Flavor Mixing in Gauge-Higgs Unification"
Adachi, Kurahashi, Lim and NM, JHEP1011 (2010) 015

"D^0-D^0bar Mixing in Gauge-Higgs Unification"
Adachi, Kurahashi, Lim and NM, JHEP1201 (2012) 047

"B^0-B^0bar Mixing in Gauge-Higgs Unification"
Adachi, Kurahashi, NM and Tanabe, PRD85 (2012) 096001

$$\mathcal{L} = -\frac{1}{4}F^{MN}F_{MN} - \frac{1}{4}B^{MN}B_{MN} - \frac{1}{4}G^{MN}G_{MN}$$

$$+\bar{\psi}_3^{1,2}\left(i\not{D} - M^{1,2}\varepsilon(y)\right)\psi_3^{1,2} + \bar{\psi}_{\bar{6}}^{1,2}\left(i\not{D} - M^{1,2}\varepsilon(y)\right)\psi_{\bar{6}}^{1,2}$$

$$+\bar{\psi}_3 i\not{D}\psi_3 + \bar{\psi}_{\overline{15}} i\not{D}\psi_{\overline{15}}$$

$$+\delta(y)\sqrt{2\pi R}\bar{Q}_R^i(x)\left[\eta_{ij}Q_{3L}^j(x,y) + \lambda_{ij}Q_L^j(x,y)\right](i,j=1,2,3)$$

$+$ brane mass terms for exotics $\quad Q_L = \left(Q_{6L}^1, Q_{6L}^2, Q_{15L}\right)^T$

- Brane mass matrices η, λ :

  off-diagonal elements → Flavor mixing
- Brane localized fields Q_R
- $M^3 = 0$ to avoid $m_\psi \sim M_W \exp[-\pi MR]$

$$\mathcal{L}^{Q}_{BM} \sim \delta(y)\bar{Q}_R\left[\eta \quad \lambda\right]\begin{bmatrix} Q_3 \\ Q \end{bmatrix}_L = \delta(y)\bar{Q}'_R\left[m_{diag} \quad 0_{3\times3}\right]\begin{bmatrix} Q_H \\ Q_{SM} \end{bmatrix}_L$$

$$\begin{bmatrix} Q_3 \\ Q \end{bmatrix}_L = \begin{bmatrix} U_1 & U_3 \\ U_2 & U_4 \end{bmatrix}\begin{bmatrix} Q_H \\ Q_{SM} \end{bmatrix}_L , \quad U^{\bar{Q}} Q_R = Q'_R$$

$$\mathcal{L}_{Yukawa} = g_5 A^6_y \bar{d}^{\,i} Q^i_3 + g_5 A^6_y \bar{u}^{\,i} Q^i \quad \leftarrow \text{Gauge interaction}$$

$$\to g_5 \left\langle A^6_y \right\rangle \left(\bar{d}^{\,i(0)}_R Y^{ii}_d U^{ij}_3 Q^{j(0)}_{SM} + \bar{u}^{\,i(0)}_R Y^{ii}_u U^{ij}_4 Q^{j(0)}_{SM} \right)$$

$$Y^{ii} = \int_{-\pi R}^{\pi R} dy f^{i(0)}_L f^{i(0)}_R \approx 2\pi R M^i e^{-\pi R M^i}$$

Diagonalization

$$\begin{cases} \hat{Y}_d = V^{\dagger}_{dR} Y_d U_3 V_{dL} \\ \hat{Y}_u = V^{\dagger}_{uR} Y_u U_4 V_{uL} \end{cases} \qquad V_{CKM} = V^{\dagger}_{uL} V_{dL} \quad \left(U^{\dagger}_3 U_3 + U^{\dagger}_4 U_4 = 1_{3\times3} \right)$$

$M_{3,6} \propto 1$ ($Y_{u,d} \propto 1$) case (flavor symmetry restored)

$$\begin{cases} \hat{Y}_d = V_{dR}^\dagger Y_d U_3 V_{dL} \to V_{dR}^\dagger U_3 V_{dL} \Rightarrow \hat{Y}_d^\dagger \hat{Y}_d = V_{dL}^\dagger U_3^\dagger U_3 V_{dL} \\ \hat{Y}_u = V_{uR}^\dagger Y_u U_4 V_{uL} \to V_{uR}^\dagger U_4 V_{uL} \Rightarrow \hat{Y}_u^\dagger \hat{Y}_u = V_{uL}^\dagger U_4^\dagger U_4 V_{uL} \end{cases}$$

$$\xrightarrow{U_3^\dagger U_3 + U_4^\dagger U_4 = 1} V_{uL} \propto V_{dL}$$

$$\Rightarrow V_{CKM} = V_{uL}^\dagger V_{dL} \propto V_{dL}^\dagger V_{dL} = 1 \left(\text{No mixing}\right)$$

Lesson

To get flavor mixing,
we need non-degenerate bulk masses
as well as the off-diagonal brane masses
(specific to gauge-Higgs unification)

Parametrization of Unitary matrices $U_{3,4}$ (CP violation ignored)

$$U_4 = R_u \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix}, U_3 = R_d \begin{pmatrix} \sqrt{1-a_1^2} & 0 & 0 \\ 0 & \sqrt{1-a_2^2} & 0 \\ 0 & 0 & \sqrt{1-a_3^2} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2' & \sin\theta_2' \\ 0 & \sin\theta_2' & \cos\theta_2' \end{pmatrix} \begin{pmatrix} \cos\theta_3' & 0 & \sin\theta_3' \\ 0 & 1 & 0 \\ -\sin\theta_3' & 0 & \cos\theta_3' \end{pmatrix} \begin{pmatrix} \cos\theta_1' & -\sin\theta_1' & 0 \\ \sin\theta_1' & \cos\theta_1' & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2 & \sin\theta_2 \\ 0 & \sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} \cos\theta_3 & 0 & \sin\theta_3 \\ 0 & 1 & 0 \\ -\sin\theta_3 & 0 & \cos\theta_3 \end{pmatrix} \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Physical observables: 6 quark masses + 3 CKM angles

# of parameters: $a_{1,2,3}$, $b_{1,2,3}=I_{RL}^{1,2(00)}$, $\theta_{1,2,3}$, $\theta'_{1,2,3}$

⇒ 11-9=2 free parameters (6-5=1 for 2 generations)

Parameter fitting

Numerical results reproducing quark masses & mixings
(2 parameter scan technically hard
⇒ 3 angles fixed & m_t unfixed)

(i) No up-type mixing case

$$\begin{aligned}
R_u = 1_{3\times 3} : a_1^2 \approx 0.1023,\, b_1^2 \approx 4.335\times 10^{-9}, \sin\theta_1 \approx -2.587\times 10^{-2} \\
a_2^2 \approx 0.9887,\, b_2^2 \approx 1.302\times 10^{-4}, \sin\theta_2 \approx 2.224\times 10^{-2} \\
a_3^2 \approx 0.9966, \qquad\qquad\qquad\qquad\quad , \sin\theta_3 \approx 2.112\times 10^{-4}
\end{aligned}$$

(ii) No down-type mixing case

$$\begin{aligned}
R_d = 1_{3\times 3} : a_1^2 \approx 0.0650,\, b_1^2 \approx 3.973\times 10^{-9}, \sin\theta_1' \approx 0.6704 \\
a_2^2 \approx 0.9931,\, b_2^2 \approx 2.235\times 10^{-4}, \sin\theta_2' \approx -3.936\times 10^{-2} \\
a_3^2 \approx 0.9966, \qquad\qquad\qquad\qquad\quad , \sin\theta_3' \approx 1.773\times 10^{-2}
\end{aligned}$$

FCNC @tree level

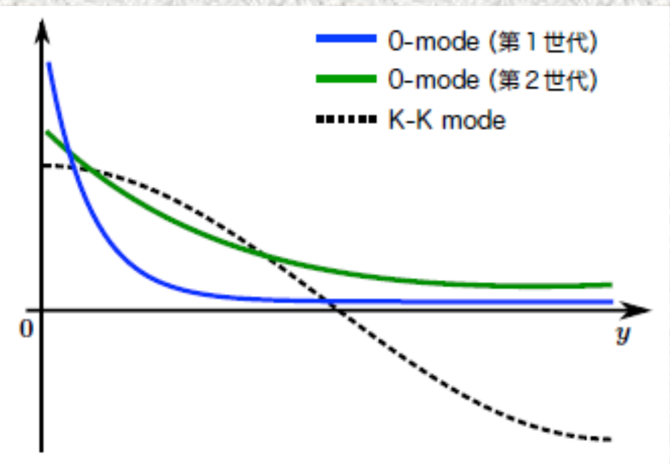
FCNC@tree level even in QCD sector

$$\begin{aligned}\mathcal{L}_{strong} \supset\ & \frac{g_s}{\sqrt{2\pi R}} G_\mu^{(0)} \left(\bar{\psi}_R^{i(0)}\gamma^\mu\psi_R^{i(0)} + \bar{\psi}_L^{i(0)}\gamma^\mu\psi_L^{i(0)}\right) \\ & + g_s G_\mu^{(n)}\bar{\psi}_R^{i(0)}\gamma^\mu\psi_R^{j(0)}\left(V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR}\right)_{ij} \\ & + g_s G_\mu^{(n)}\bar{\psi}_L^{i(0)}\gamma^\mu\psi_L^{j(0)}\left[V_{dL}^\dagger\left(U_3^\dagger I_{LL}^{(0n0)} U_3 + U_4^\dagger I_{LL}^{(0n0)} U_4\right)V_{dL}\right]_{ij}\end{aligned}$$

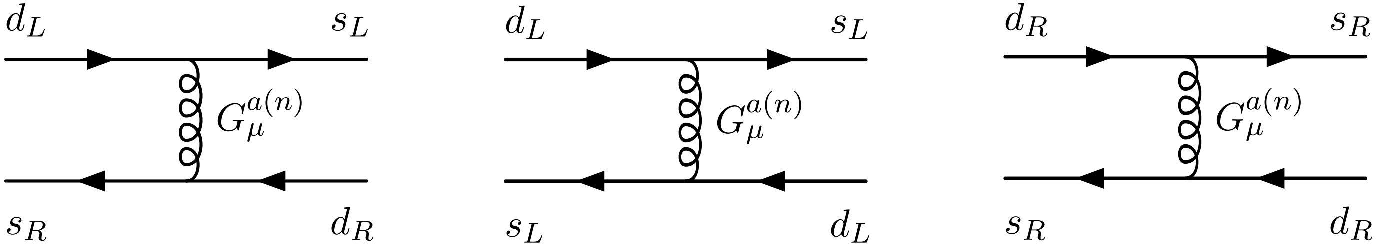
0 mode sector: No mixing O.K.

Nonzero KK gluon couplings induce nontrivial flavor mixing

⇒ flavor mixing@tree level

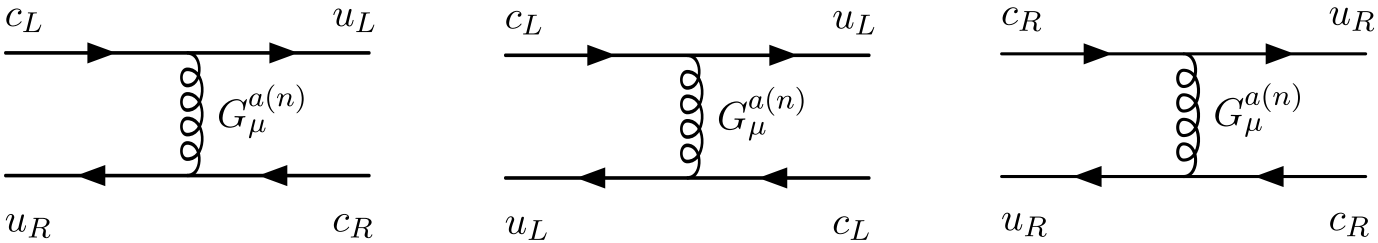


$K^0 - \bar{K}^0$



(i) LR type (ii) LL type (iii) RR type

$D^0 - \bar{D}^0$

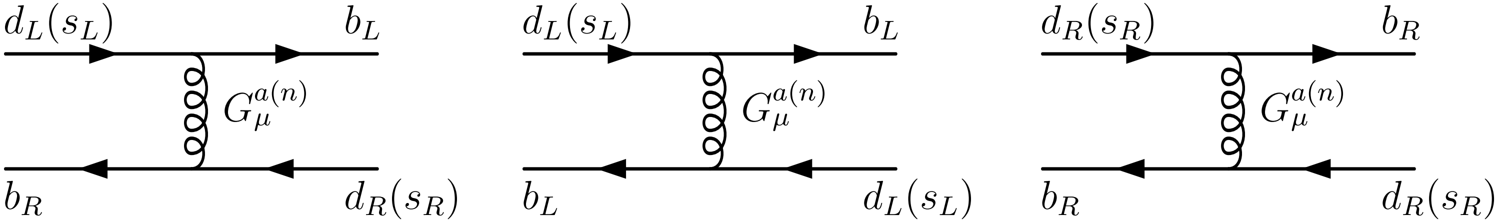


(i) LR type (ii) LL type (iii) RR type

$B_d^0 - \bar{B}_d^0$

&

$B_s^0 - \bar{B}_s^0$



(i) LR type (ii) LL type (iii) RR type

K_L-K_S mass difference

$$\Delta m_K(\mathrm{KK}) = 2\langle \bar{K} | \mathcal{L}_{eff}^{\Delta S=2} | K \rangle \approx \alpha_S C B_1 R^2 f_K^2 m_K \sum_n \frac{1}{n^2}\left[I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right]^2$$

$$I_{RR}^{i(0n0)} = \frac{1}{\sqrt{\pi R}} \int_{-\pi R}^{\pi R} \left(f_R^{(0)}(y) \right)^2 \cos\left(\frac{n}{R} y \right)$$

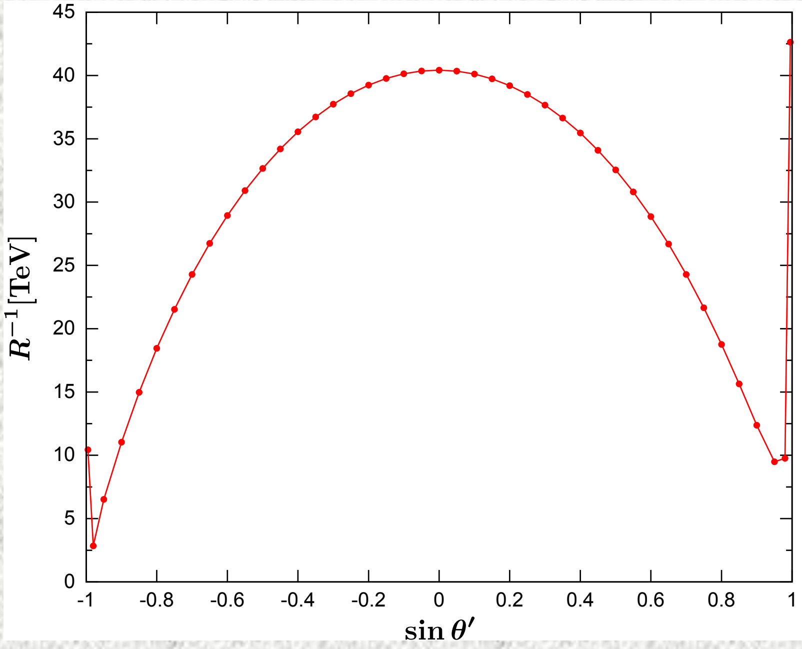
Bag parameter: B_1=0.57, $f_K \sim 1.23 f_\pi$, $m_K \sim 497$MeV

Mode sum is finite

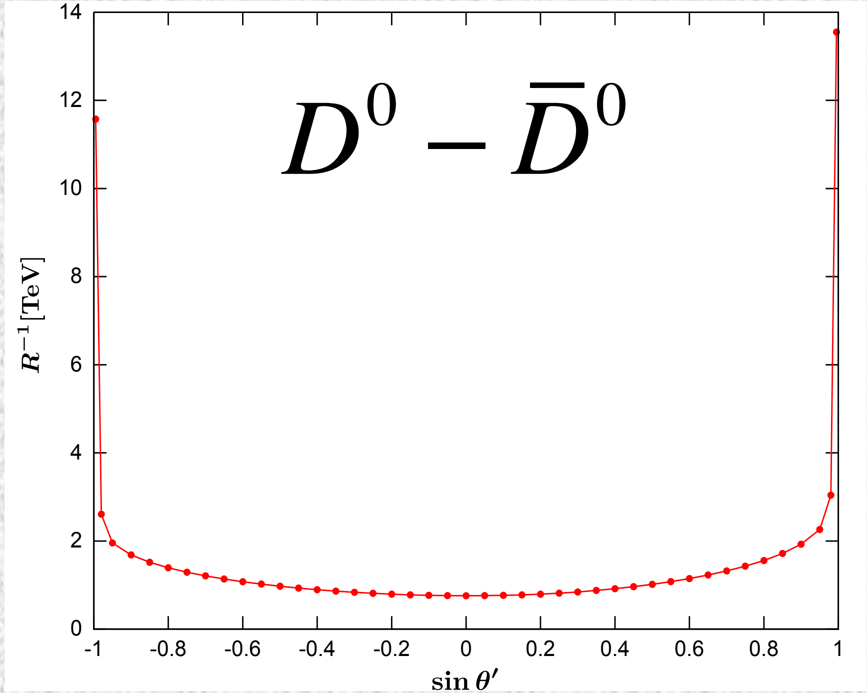
Exp. constraint:

$$\left| \Delta m_K(NP) \right| < 3.48 \times 10^{-12}\, MeV$$

$$R^{-1} \geq 2.8TeV \sim 43TeV$$



Similar analysis applied to D & B systems



$$B^0 - \bar{B}^0$$

$$R_u = 1_{3\times 3} : R^{-1} \geq 1.71 TeV \left(B_d^0 - \bar{B}_d^0 \right)$$

$$R^{-1} \geq 2.54 TeV \left(B_s^0 - \bar{B}_s^0 \right)$$

$$R_d = 1_{3\times 3} : R^{-1} \geq 0.92 TeV \left(B_d^0 - \bar{B}_d^0 \right)$$

$$R^{-1} \geq 1.79 TeV \left(B_s^0 - \bar{B}_s^0 \right)$$

Lower bounds for compactification scale

$$R^{-1} \geq 0.8 TeV \sim 14 TeV$$

Results

$$K^0-\bar{K}^0:\mathcal{O}\left(10\right)TeV$$

$$D^0-\bar{D}^0:\mathcal{O}\left(1\right)TeV$$

$$B_d^0-\bar{B}_d^0,B_s^0-\bar{B}_s^0:\mathcal{O}\left(1\right)TeV$$

"GIM-like" mechanism

Above results is smaller than naïve order estimate

$$\frac{1}{M_{KK}^2}\bar{\psi}\psi\bar{\psi}\psi \Rightarrow \begin{cases} M_{KK} \geq 1000TeV\left(K^0-\bar{K}^0, D^0-\bar{D}^0\right) \\ M_{KK} \geq 400TeV\left(B_d^0-\bar{B}_d^0\right) \\ M_{KK} \geq 70TeV\left(B_s^0-\bar{B}_s^0\right) \end{cases}$$

This apparent discrepancy can be understood since the "GIM-like" mechanism works in GHU

i.e. FCNC processes are automatically suppressed for 1st & 2nd generation of quarks

In the large bulk mass limit, the KK mode sum can be approximated as follows

$$S_{KK}^{LR}=\pi R\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\left(I_{RR}^{1(0n0)}-I_{RR}^{2(0n0)}\right)^2$$

$$\simeq-\frac{\pi^2}{2}\left(e^{-2\pi RM^1}+e^{-2\pi RM^2}\right)$$

$$-\frac{\pi}{2R}\frac{\left(M^1\right)^2-M^1M^2+\left(M^2\right)^2}{M^1M^2\left(M^1-M^2\right)}\left(e^{-2\pi RM^1}-e^{-2\pi RM^2}\right)\left(\pi RM^i\gg 1\right)$$

exponential suppression!!

$$e^{-2\pi RM^i}\Leftrightarrow\frac{m_{q^i}^2}{m_W^2}$$

similar to GIM suppression

$$\frac{m_c^2-m_u^2}{m_W^2}$$

$$S_{KK}^{LL(RR)}=\pi R\sum_{n=1}^{\infty}\frac{1}{n^2}\left(I_{RR}^{1(0n0)}-I_{RR}^{2(0n0)}\right)^2\simeq\frac{\pi}{8R}\frac{\left(M^1-M^2\right)^2}{M^1M^2\left(M^1+M^2\right)}$$

Power suppression

More intuitive understanding of “GIM-like” suppression

FCNC is controlled by the factor

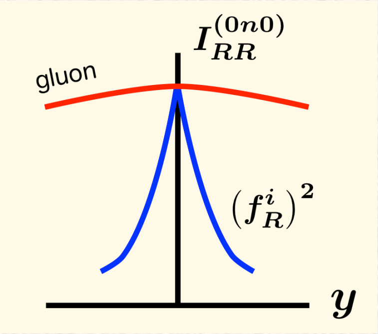
$$\left(I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)}\right)^2$$

$$I_{RR}^{i(0n0)} = \frac{1}{\sqrt{\pi R}} \int_{-\pi R}^{\pi R} dy \frac{M^i}{e^{2\pi R M^i} - 1} e^{2M^i y} \cos\left(\frac{n}{R} y\right)$$

In πMR >> 1 limit & for small mode index n

Width “1/M” of 0 mode function = Period 2πR/n of KK gluon mode function

Almost flat KK gluon mode function for fast exponential dumping 0 mode fermions



Almost flavor universal
(similar to 0 mode sector)

As for the 3rd generation,
GIM-like mechanism does not work

∵ $M^3=0$ for top mass

Suppressed FCNC due to small mixing between 1-3 & 2-3 generations