Modified Gravity Explains Dark Matter?

Taishi Katsuragawa

Refs:

TK and S. Matsuzaki, Phys. Rev. D95 044040 (2017)

Works in progress

Brief Introduction to Modified Gravity

Background

- General Relativity
- Dark Energy and Dark Matter

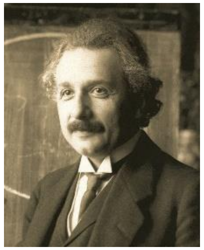
Why Modified Gravity?

General Relativity

General Relativity (GR) is simple but successful.

Einstein-Hilbert (EH) action

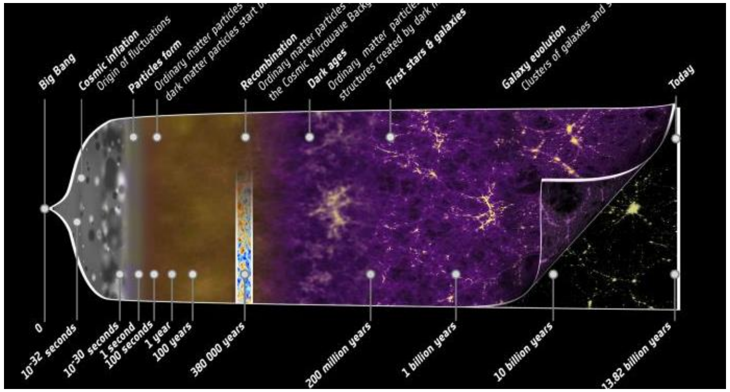
$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}R\,,\ \ \kappa^2 = 8\pi G$$



Einstein equation $$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

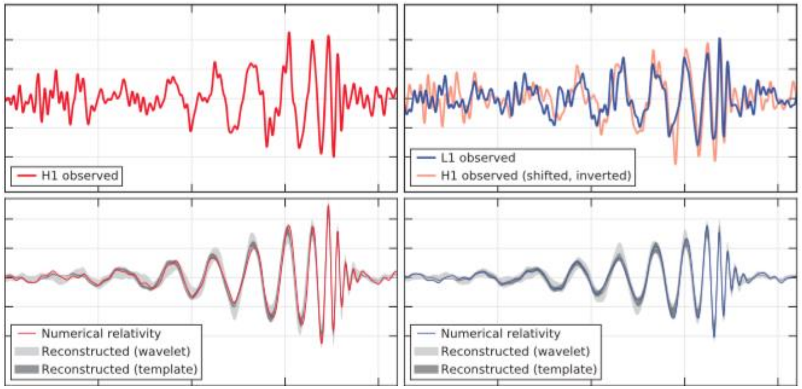
$T_{\mu\nu}$: Energy-momentum tensor

Cosmic History



[Planck (2013)]

Gravitational Waves



[LIGO (2016)]

GR万歳！

congratulations! GR!!

The Final Frontier

There are still mysteries in our Universe:

Dark Energy (DE) and Dark Matter (DM)

Dark Energy

Energy to accelerate the expansion of the current Universe.

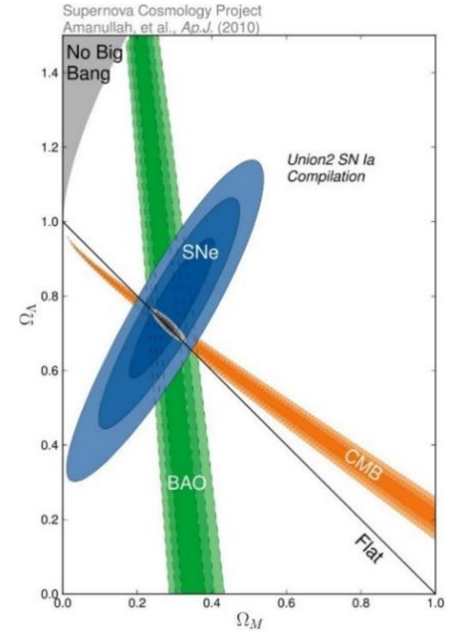
cf.) Type Ia supernova, CMB, BAO

Dark Matter

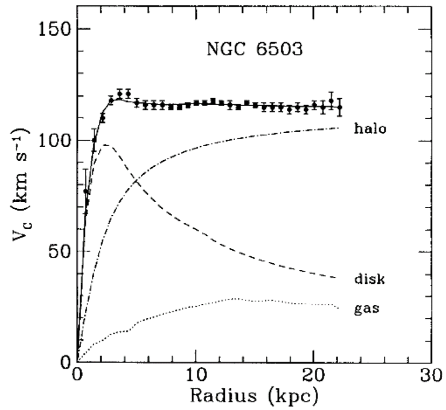
Invisible matter besides ordinary matters

cf.) Galaxy rotation curve

etc.



[Amanullah et. al (2010)]



[Begeman, Broeils, and Sanders (1991)]

Into Darkness

ΛCDM model in GR is consistent with observations.

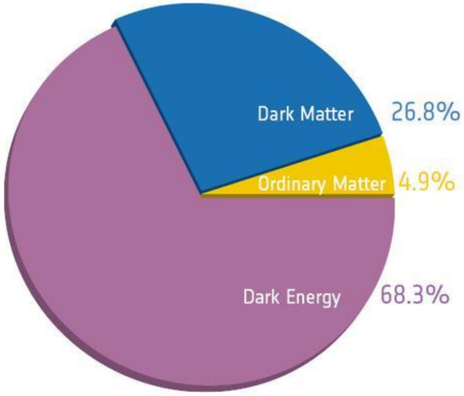
ΛCDM model

Ordinary matter

$$S = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\,[R - \underline{2\Lambda}] + S_{\underline{SM}+\underline{DM}}$$

Cosmological constant (Λ) Cold Dark Matter (CDM)

Cosmic Pie



[Planck (2013)]

Two questions remain...

- What is the cosmological constant?
- What is the origin of CDM?

Cosmological Constant (CC) problems

- Fine tuning (why so small?)
- Coincidence (why observed value?) etc.

Constant vs. Dynamical Field

Cosmological constant?

- Simple and consistent with observation
- DE “=“ Cosmological constant?

Equation of state of DE: p=wρ (p: pressure ρ:energy density)

- If w<-1/3, we can explain late-time acceleration.
- DE is not necessarily cosmological constant (w=-1).
- Dynamical Dark Energy

Value of w	Category
w=1/3	Radiation (relativistic matter)
w=0	Dust (non-relativistic matter)
-1<w<-1/3	Quintessence
w=-1	Cosmological Constant
w<-1	Phantom

Dark Energy

Beyond General Relativity for Dark Energy

How to introduce new dynamical field DOF?

→ New Matter or Modified Gravity

Dynamical Dark Energy

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

Gravity — Matter

Modification of gravity sector — Modification of matter sector

= Modified gravity

- The modification leads to the emergence of new DOF.
- New DOF causes deviations from GR.
  - to explain the Dark Energy 　ヽ(ﾟ∀ﾟ)ﾉ
  - to bring undesirable deviations 　(´･ω･`)
  - Modifications are constrained by observations.

F(R) Gravity and Scalaron

F(R) Gravity

- Weyl Transformation
- Equivalence to Scalar-Tensor Theory

Scalaron

- Matter coupling to SM Particles

F(R) Gravity

Basics on F(R) gravity

Action of F(R) gravity

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} F(R)$$

[Buchdahl (1970)]

cf.) EH-action

$$\int d^4x \sqrt{-g} R$$

Replace: $R \to F(R)$

- EoM with matter field

$$F_R(R) R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} F(R) + \left(g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu \right) F_R(R) = \kappa^2 T_{\mu\nu}$$

- Trace of the EOM

$$\Box F_R(R) = \frac{1}{3} \kappa^2 T + \frac{1}{3} \left[2F(R) - F_R(R) R \right]$$

The Ricci scalar is dynamical although $R = -\kappa^2 T$ in GR.

From F(R) to Scalar-Tensor Theory

(1) Rewrite the action with an auxiliary field

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,[F_A(A)R - \{F_A(A)A - F(A)\}]$$

where A is auxiliary scalar field, and $F_R(R) = \partial_R F(R)$

- EoM of auxiliary field *A*

$$F_{AA}(A)\,(R - A) = 0 \longrightarrow \boxed{A = R} \quad \text{if } F_{RR}(R) \neq 0$$

(2) Transform the metric

Weyl Transformation

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2(x) g_{\mu\nu}$$

Jordan frame : $g_{\mu\nu}$ → Einstein frame : $\tilde{g}_{\mu\nu}$

From F(R) to Scalar-Tensor Theory

- Choose the Weyl trans. as

$$\Omega^2(x) = F_R(R) \equiv e^{2\sqrt{1/6}\kappa\varphi(x)}, \quad \varphi(x) = \frac{\sqrt{6}}{2\kappa} \ln F_R(R)$$

F(R) gravity in Einstein frame

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2\kappa^2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right]$$

where $V(\varphi) = \dfrac{1}{2\kappa^2} \dfrac{F_R(R)R - F(R)}{F_R^2(R)}$

After the Weyl trans., F(R) gravity can be expressed in terms of GR with scalar field $\varphi(x)$

– Mathematical equivalence to Scalar-Tensor theory

We call the scalar field as Scalaron

Scalaron Couplings with Matters

Short Summary

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}F(R)$$

$$\xrightarrow{(1)} S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[F_A(A)R - \{F_A(A)A - F(A)\}\right]$$

$$\xrightarrow{(2)} S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-\tilde{g}}\tilde{R} + \int d^4x\sqrt{-\tilde{g}}\left[-\frac{1}{2}\tilde{g}^{\mu\nu}\left(\partial_\mu\varphi\right)\left(\partial_\nu\varphi\right) - V\left(\varphi\right)\right]$$

Consider the matter sector

$$\begin{aligned} S_{\mathrm{Matter}} &= \int d^4x\sqrt{-g}\mathcal{L}\left(g^{\mu\nu},\Psi\right) \\ &= \int d^4x\sqrt{-\tilde{g}}\,\underline{\mathrm{e}^{-4\sqrt{1/6}\kappa\varphi(x)}\mathcal{L}\left(\mathrm{e}^{2\sqrt{1/6}\kappa\varphi(x)}\tilde{g}^{\mu\nu}\right.}\left.,\Psi\right) \end{aligned}$$

Dilatonic coupling btw. Scalaron and matter field

- Weak interaction because of gravitational origin
- Suppressed by Planck mass $\kappa = 1/M_{\mathrm{pl}}$, $M_{\mathrm{pl}} = 10^{19}[\mathrm{GeV}]$

Chameleon Mechanism

Screening Mechanism

- Solar-System Constraint

Chameleon Mechanism

- Environment Dependence

Screening Mechanism

Modifications to GR introduce additional DOF.

However, the Solar-System constraints often exclude modifications.

- The fifth force φ should act only on large scale, and it should be screened on small scale.

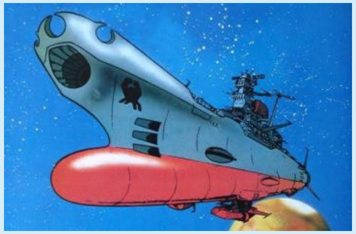
φ is responsible for modification of gravity

わ た し で す

Sun

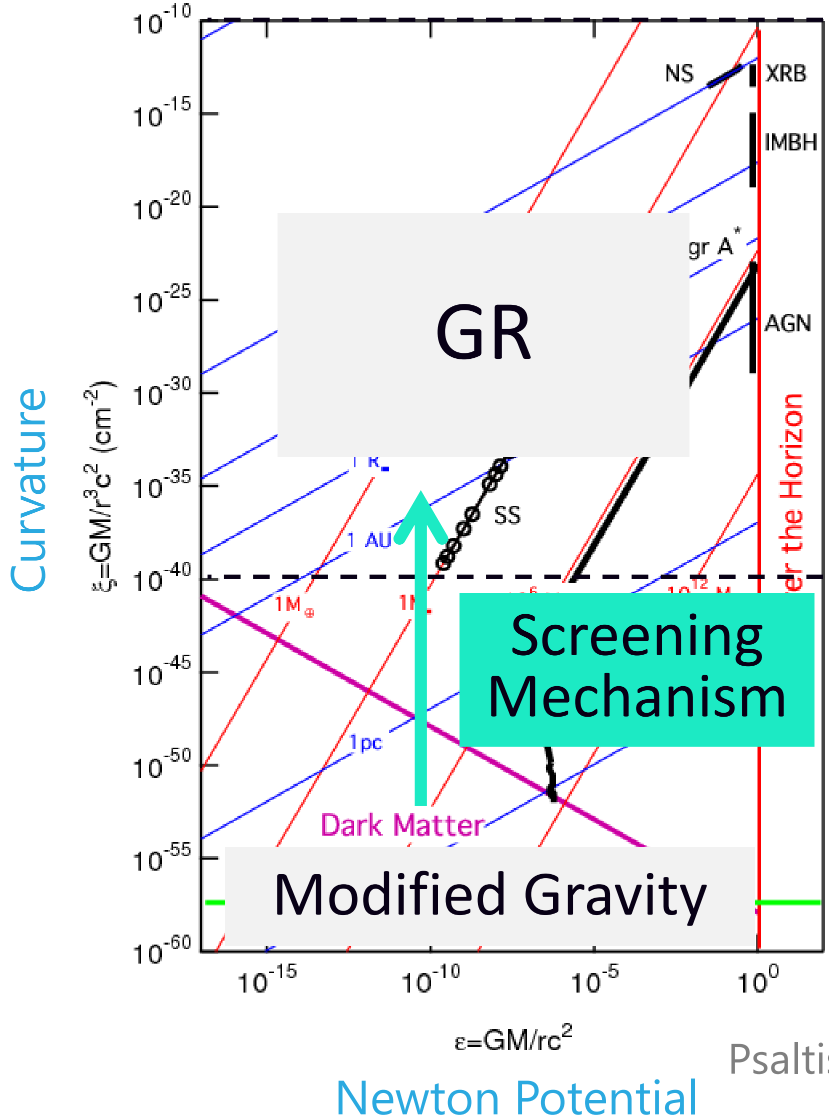
Solar system

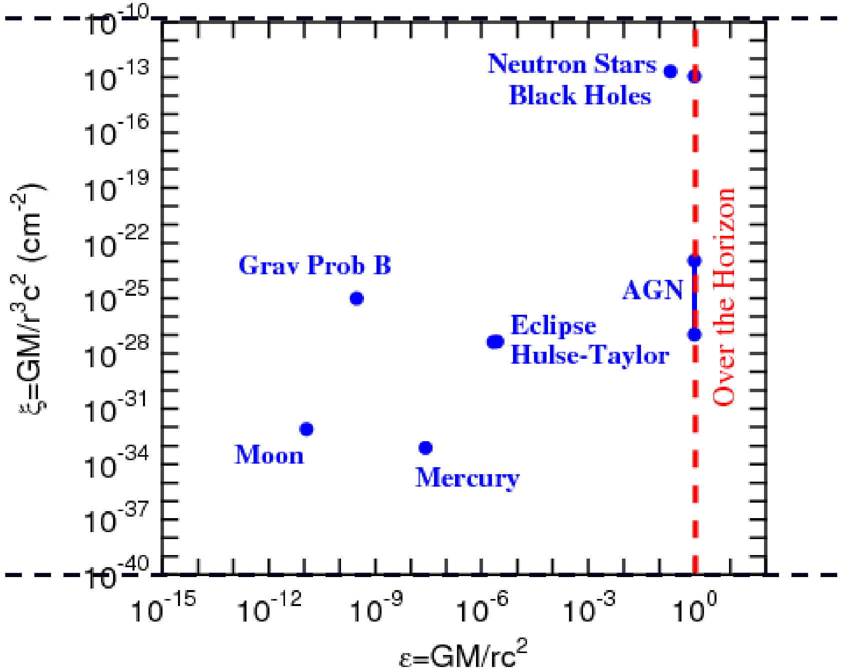
Inter-stellar
(or -galactic) space



φ is screened on small scale

Test of Gravity and Screening Mechanism





Psaltis (2008)

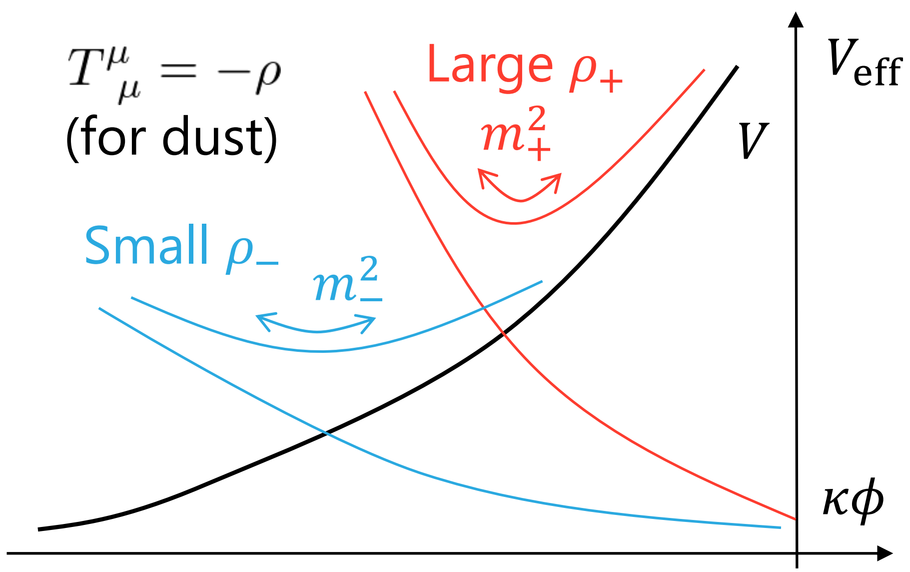
Viable F(R) gravity possesses Chameleon mechanism

[Khoury and Weltman, (2004)]

- Restrictive constraints from obs. in Solar System
- Scalaron effective potential couples to trace of $T_{\mu\nu}$

$$\tilde{\Box}\varphi = \partial_\varphi V_{\rm eff}(\varphi)\,,\ V_{\rm eff}(\varphi) = V(\varphi) - \frac{1}{4}e^{-4\sqrt{1/6}\kappa\varphi}T^\mu_{\ \mu}$$

Chameleon Mechanism



<u>Scalaron mass</u>

$$m_\varphi = V''_{\rm eff}(\varphi_{\rm min})$$

In high-density region, scalar field is heavy and suppressed.

In low-density region, scalar field is light and acts as DE.

Scalaron as Dark Matter Candidate

Objectives

Stability of Scalaron

- Coupling with SM Particles
- Decay width and Lifetime

Applications of Modified Gravity

How can we use the modifications for unanswered questions?

= Application of modified gravity

- Cosmology (DE etc.)
- Astrophysics (massive NS, BH, GW etc.)
- Particle Physics?

Objective. 1

Quantization of new DOF = New particle?

- Beyond Standard Model (SM) particle is introduced from the “beyond GR” sector.
- New constraints from the viewpoint of particle physics.

Dark Matter in Modified Gravity?

Can the new particle be a DM candidate?

- The origin is gravitational sector
- New particle has very weak interactions with matter
- New particle can be massive

Objective. 2

DM candidate in modified gravity?

- New constraints on modified gravity by converting the existing constraints on DM.
- Unified treatment of DM and DE in one theory

Can Scalaron be a DM?

Properties of Scalaron

- Heavy in the Solar-System (or around the Earth) by the Chameleon Mechanism
- Interaction to SM particle is suppressed by the Planck mass ($e^{\kappa\varphi} \sim 1 + \kappa\varphi$)

They suggest the Scalaron could be a CDM.

- Can F(R) gravity explain DM problem?

[Nojiri and Odintsov (2008), Choudhury et al. (2015)]

To study the Scalaron as DM candidate

- Stability = Decay process and Lifetime [TK and S. Matsuzaki (2017)
- Relic abundance
- Direct detection experiment

In progress

Coupling to Matter : Massless vector

Massless vector field $A_\mu(x)$

$$\begin{aligned}\mathcal{L}_V\left(g^{\mu\nu}, A_\mu\right) &= -\frac{1}{4} g^{\alpha\mu} g^{\beta\nu} F_{\alpha\beta} F_{\mu\nu} \\ &= -\frac{1}{4} \mathrm{e}^{4\sqrt{1/6}\kappa\varphi} \tilde{g}^{\alpha\mu} \tilde{g}^{\beta\nu} F_{\alpha\beta} F_{\mu\nu}\end{aligned}$$

Field strength is invariant under the Weyl trans.

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$$

No Coupling to Scalaron through field strength

$$\begin{aligned}S &= \int d^4x \sqrt{-\tilde{g}}\, \mathrm{e}^{-4\sqrt{1/6}\kappa\varphi} \mathcal{L}_V\left(g^{\mu\nu}, A_\mu\right) \\ &= \int d^4x \sqrt{-\tilde{g}} \mathcal{L}_V\left(\tilde{g}^{\mu\nu}, A_\mu\right)\end{aligned}$$

Coupling to Matter : Massless fermion

Massless fermion field $\psi(x)$

$$\mathcal{L}_F\left(\gamma^\mu, \psi\right) = i\bar{\psi}(x)\gamma^\mu\nabla_\mu\psi(x)$$

where
$$\gamma^\mu(x) = e_a{}^\mu(x)\gamma^a\,,\ \{\gamma^\mu,\, \gamma^\nu\} = 2g^{\mu\nu}$$
$$\nabla_\mu\psi(x) = \partial_\mu\psi(x) + \frac{1}{8}\omega_{\mu ab}(x)[\gamma^a,\, \gamma^b]\psi(x)$$
$$w_{\mu ab}(x) = e_{a\nu}\left(\partial_\mu e_b{}^\nu + \Gamma^\nu_{\mu\rho}e_b{}^\rho\right)$$

Action in the Einstein frame

$$S = \int d^4x\sqrt{-g}\,\mathcal{L}_F\left(\gamma^\mu, \psi\right)$$
$$= \int d^4x\sqrt{-\tilde{g}}\left[\mathrm{e}^{-3\sqrt{1/6}\kappa\varphi}i\bar{\psi}\tilde{\gamma}^\mu\tilde{\nabla}_\mu\psi - \frac{3i}{2}\sqrt{\frac{1}{6}}\kappa\mathrm{e}^{-3\sqrt{1/6}\kappa\varphi}\left(\partial_\mu\varphi\right)\bar{\psi}\tilde{\gamma}^\mu\psi\right]$$

No coupling after field redefinition

$$\psi \to \psi' = \mathrm{e}^{-3/2\sqrt{1/6}\kappa\varphi}\psi \qquad S = \int d^4x\sqrt{-\tilde{g}}i\bar{\psi}'\tilde{\gamma}^\mu\tilde{\nabla}_\mu\psi'$$

Coupling to Matter : Massless vector again

The scalaron would affect the quantum dynamics of fermion field although the scalaron coupling can be eliminated by field redefinition in classical dynamics.

- Path integral measure induces the anomaly

$$\psi(x) = \sum_n a_n \psi_n(x)\,, \quad \bar{\psi}(x) = \sum_n \hat{a}_n \hat{\bar{\psi}}_n$$

$$\psi'(x) = (1 + \phi(x))\psi(x) \qquad \phi(x) \equiv \frac{3}{2}\sqrt{\frac{1}{6}}\kappa\varphi(x)$$

$$\Pi_n da_n d\hat{a}_n \to \Pi_n da'_n d\hat{a}'_n \cdot \mathcal{J}^{-2}$$

$$\mathcal{J} = \exp\left[i \int d^4x \phi(x) \cdot \frac{g^2}{4(4\pi)^2}\mathrm{tr}[F_{\mu\nu}^2]\right]$$

The couplings with massless vector fields show up.

$$\mathcal{L}_{\rm anomaly} = -\frac{g^2}{2(4\pi)^2}\phi\,\mathrm{tr}[F_{\mu\nu}^2]$$

Coupling to Matter : Massive vector

Massive vector field $A_\mu(x)$

$$\mathcal{L}_{V-\mathrm{mass}}\left(g^{\mu\nu}, A_\mu\right) = -\frac{1}{2} m_V^2 \mathrm{e}^{2\sqrt{1/6}\kappa\varphi} \tilde{g}^{\mu\nu} A_\mu A_\nu$$

Action in the Einstein frame

$$\begin{aligned} S &= \int d^4x \sqrt{-g}\, \mathcal{L}_{V-\mathrm{mass}}\left(g^{\mu\nu}, A_\mu\right) \\ &= \int d^4x \sqrt{-\tilde{g}} \left[\mathcal{L}_{V-\mathrm{mass}}\left(\tilde{g}^{\mu\nu}, A_\mu\right) + \mathcal{L}_{V-\varphi}\left(\tilde{g}^{\mu\nu}, A_\mu, \varphi\right)\right] \end{aligned}$$

$$\mathcal{L}_{V-\varphi}\left(\tilde{g}^{\mu\nu}, A_\mu, \varphi\right) = -\frac{1}{2} m_V^2 \left(\mathrm{e}^{-2\sqrt{1/6}\kappa\varphi} - 1\right) \tilde{g}^{\mu\nu} A_\mu A_\nu$$

Expand the interacting Lagrangian w.r.t. $|\kappa\varphi| \ll 1$

Coupling to Scalaron through the mass term.

$$\mathcal{L}_{V-\varphi}\left(\tilde{g}^{\mu\nu}, A_\mu, \varphi\right) = \frac{2\kappa\varphi}{\sqrt{6}} \cdot \frac{1}{2} m_V^2 \tilde{g}^{\mu\nu} A_\mu A_\nu + \mathcal{O}(\kappa^2\varphi^2)$$

Coupling to Matter : Massive fermion

After field redefinition, massive fermion field $\psi'(x)$

$$\mathcal{L}_{F-\mathrm{mass}}\left(\psi\right) = -m_F \mathrm{e}^{3\sqrt{1/6}\kappa\varphi}\bar{\psi}'\psi'$$

Action in the Einstein frame

$$\begin{aligned} S &= \int d^4x \sqrt{-g}\,\mathcal{L}_{V-\mathrm{mass}}\left(g^{\mu\nu},\psi\right) \\ &= \int d^4x \sqrt{-\tilde{g}}\left[\mathcal{L}_{F-\mathrm{mass}}\left(\psi'\right) + \mathcal{L}_{F-\varphi}\left(\psi',\varphi\right)\right] \end{aligned}$$

$$\mathcal{L}_{F-\varphi}\left(\psi',\varphi\right) = -m_F\left(\mathrm{e}^{-\sqrt{1/6}\kappa\varphi} - 1\right)\bar{\psi}'\psi'$$

Coupling to Scalaron through the mass term.

$$\mathcal{L}_{F-\varphi}\left(\psi',\varphi\right) = \frac{\kappa\varphi}{\sqrt{6}} \cdot m_F\bar{\psi}'\psi' + \mathcal{O}(\kappa^2\varphi^2)$$

Coupling to SM Particles

For massless vector field (Photon, Gluon)

$$\mathcal{L} = -\frac{3g^2}{4(4\pi)^2}\left(\frac{3}{2}\sqrt{\frac{1}{6}}\kappa\varphi\right) \mathrm{tr}\left[F_{\mu\nu}^2\right] + \mathcal{O}(\kappa^2\varphi^2)$$

For massive vector field (Weak bosons)

$$\mathcal{L} = \frac{2\kappa\varphi}{\sqrt{6}} \cdot \frac{1}{2} m_V^2 \tilde{g}^{\mu\nu} A_\mu A_\nu + \mathcal{O}(\kappa^2\varphi^2)$$

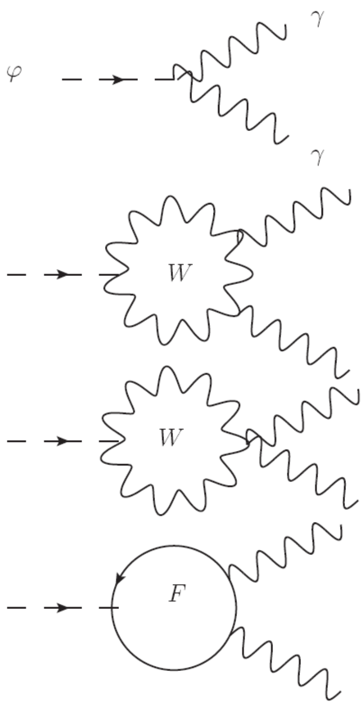
For massive fermion field (Quarks, Leptons)

$$\mathcal{L} = \frac{\kappa\varphi}{\sqrt{6}} \cdot m_F \bar{\psi}'\psi' + \mathcal{O}(\kappa^2\varphi^2) \qquad \psi \to \psi' = \mathrm{e}^{-3/2\sqrt{1/6}\kappa\varphi}\psi$$

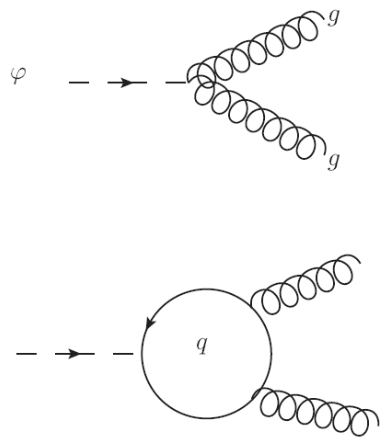
More on Decay to Massless Bosons

As to the couplings to diphoton and digluon, the scalaron couplings are generated at one-loop level of the SM perturbation.

$\underline{\varphi \to \gamma\gamma}$

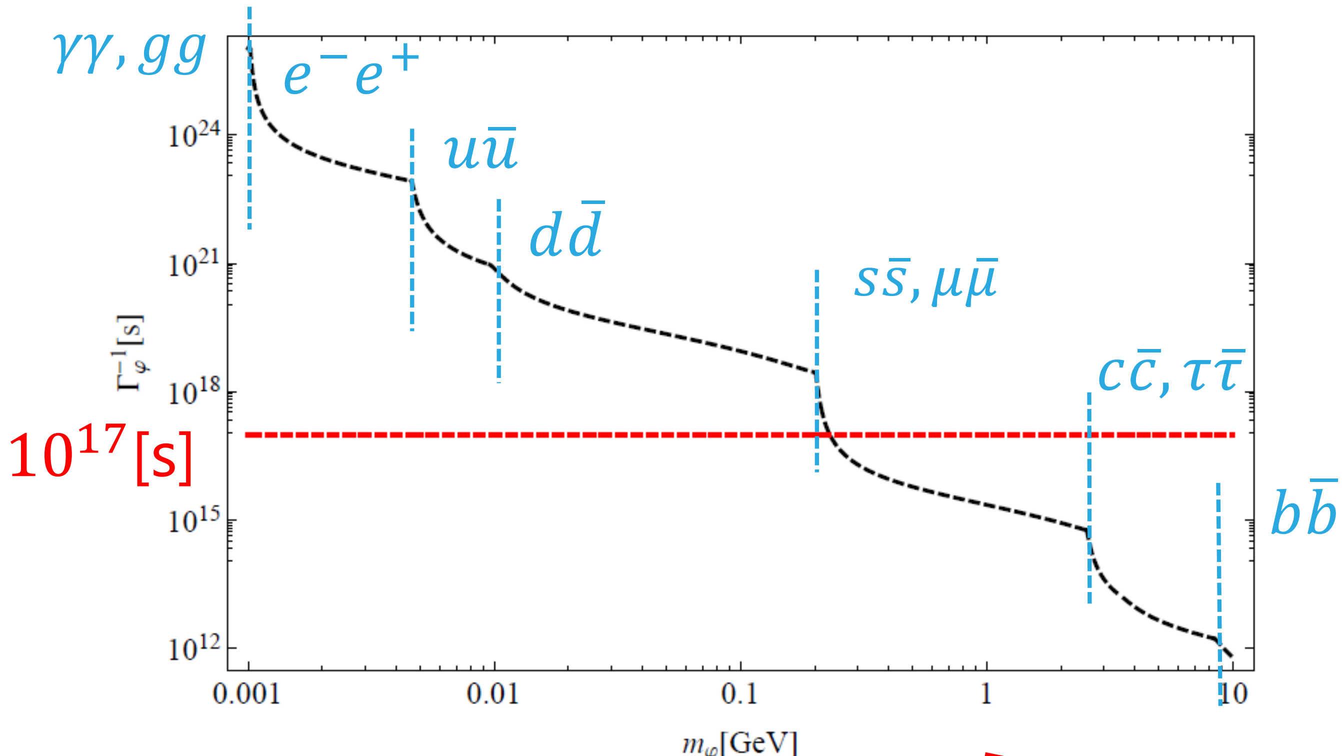


$\underline{\varphi \to gg}$



corresponding to leading order contribution in original fermion field $\psi(x)$

Consider the early Universe after EW phase transition but before QCD phase transition.



Lifetime $\Gamma_\varphi^{-1} \geq 10^{17}$[s] (age of Universe) $\cancel{m_\varphi \lesssim 0.23[\mathrm{GeV}]}$

→ Lifetime changes in the cosmic history

Scalaron Mass in Cosmic History

Scalaron mass depends on the environment in the Universe.

- We need to construct the time evolution of T_μ^μ
- For perfect fluid, $T_\mu^\mu = -(\rho - 3p)$

$$V_{\rm eff}(\varphi) = V(\varphi) + \frac{1}{4} e^{-4\sqrt{1/6}\kappa\varphi}(\rho - 3p)$$

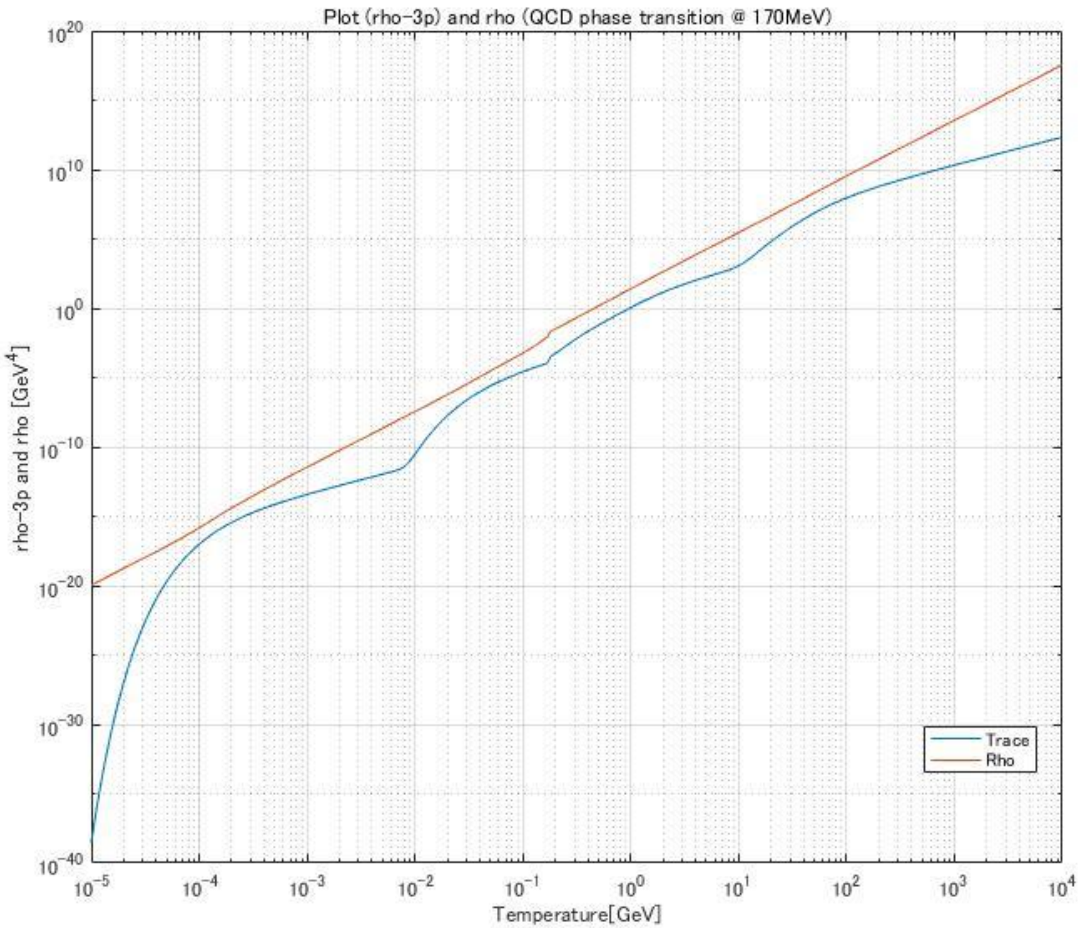
We find

$$\rho - 3p = \frac{gT^4}{2\pi^2} x^2 \int_0^\infty d\xi \frac{\xi^2}{\sqrt{\xi^2 + x^2}} \frac{1}{\mathrm{e}^{\sqrt{\xi^2+x^2}} \pm 1} \qquad x = \frac{m}{T},\ \xi = \frac{p}{T}$$

- At high temp. (relativistic) $\quad \rho - 3p \approx \frac{g}{12} m^2 T^2$
- At low temp. (non-relativistic) $\quad \rho - 3p \approx \rho \approx mg \left(\frac{mT}{2\pi}\right)^{3/2} \mathrm{e}^{-m/T}$

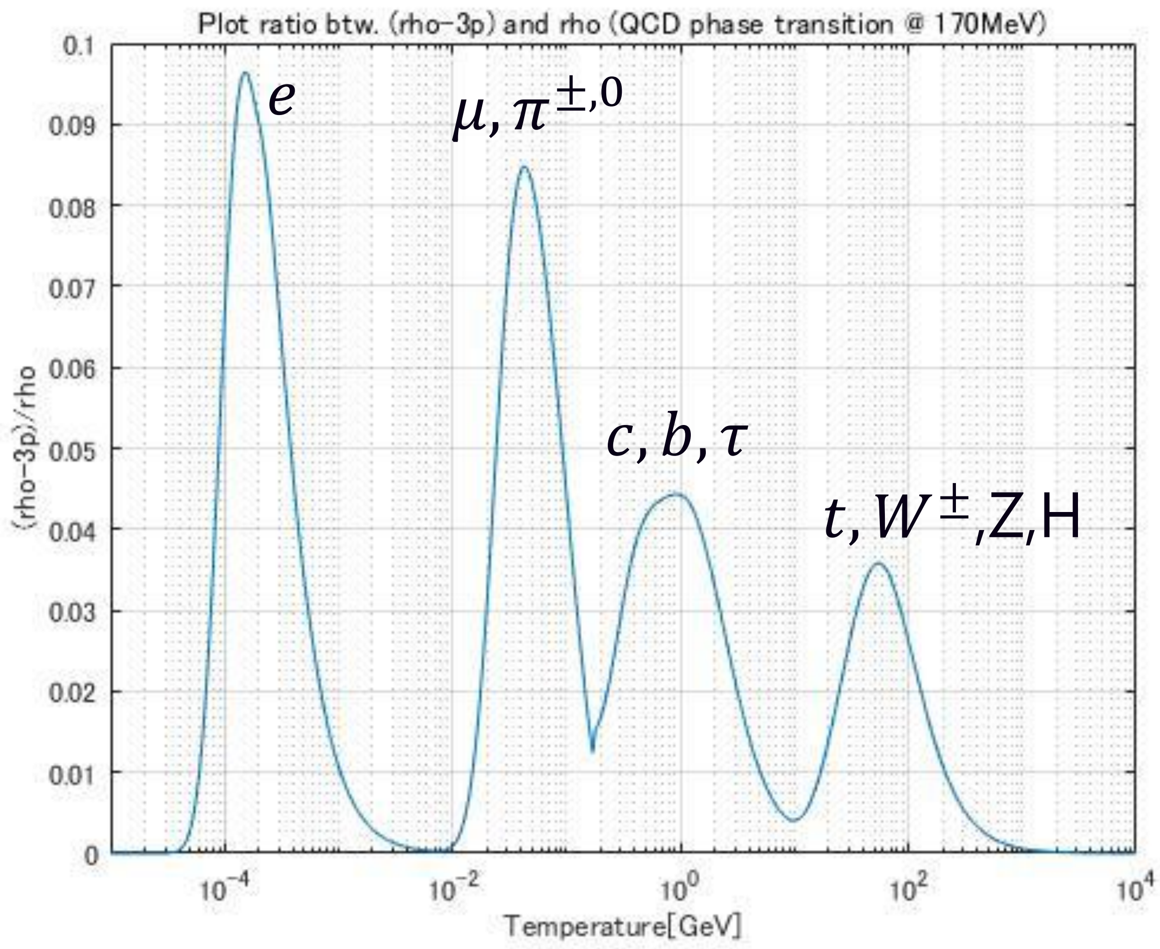
Time evolution of $T^{\mu}{}_{\mu}$

To compare ρ and $T^{\mu}{}_{\mu} = \rho - 3p$



Time evolution of $T^{\mu}{}_{\mu}$

To compare ρ and $T^{\mu}{}_{\mu} = \rho - 3p$



Starobinsky model

Particular model of F(R) gravity

Starobinsky model

Starobinsky model for late-time acceleration

$$F(R) = R - \beta R_c \left[1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right]$$

[Starobinsky (2007)]

where $R_c \sim \Lambda$ is constant curvature, and $\beta, n > 0$

Starobinsky model in large-curvature limit $R \gg R_c$

- Chameleon mechanism works in dense regime

$$F(R) \simeq R - \beta R_c \left[1 - \left(\frac{R_c}{R}\right)^{2n}\right]$$

where $\beta R_c \approx 2\Lambda$

- Scalaron mass

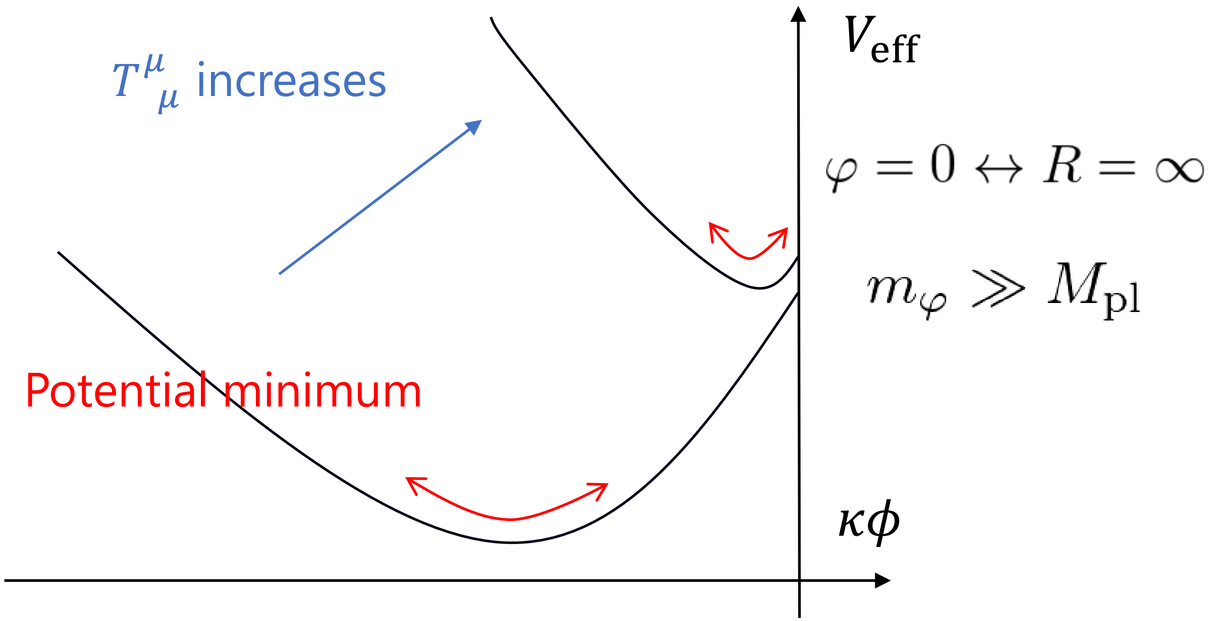
$$m_{\tilde{\varphi}}^2 = \frac{2\Lambda}{6n(2n+1)\beta} \left(\frac{\kappa^2(\rho - 3p)}{2\Lambda}\right)^{2(n+1)}$$

increasing function of $\rho - 3p$

R^2 correction

- Singularity problem in F(R) Gravity

[Frolov (2008)]
[Kobayashi and Maeda (2008)]



- In order to prevent the scalaron mass from reaching the Planck mass, we add the R^2 term

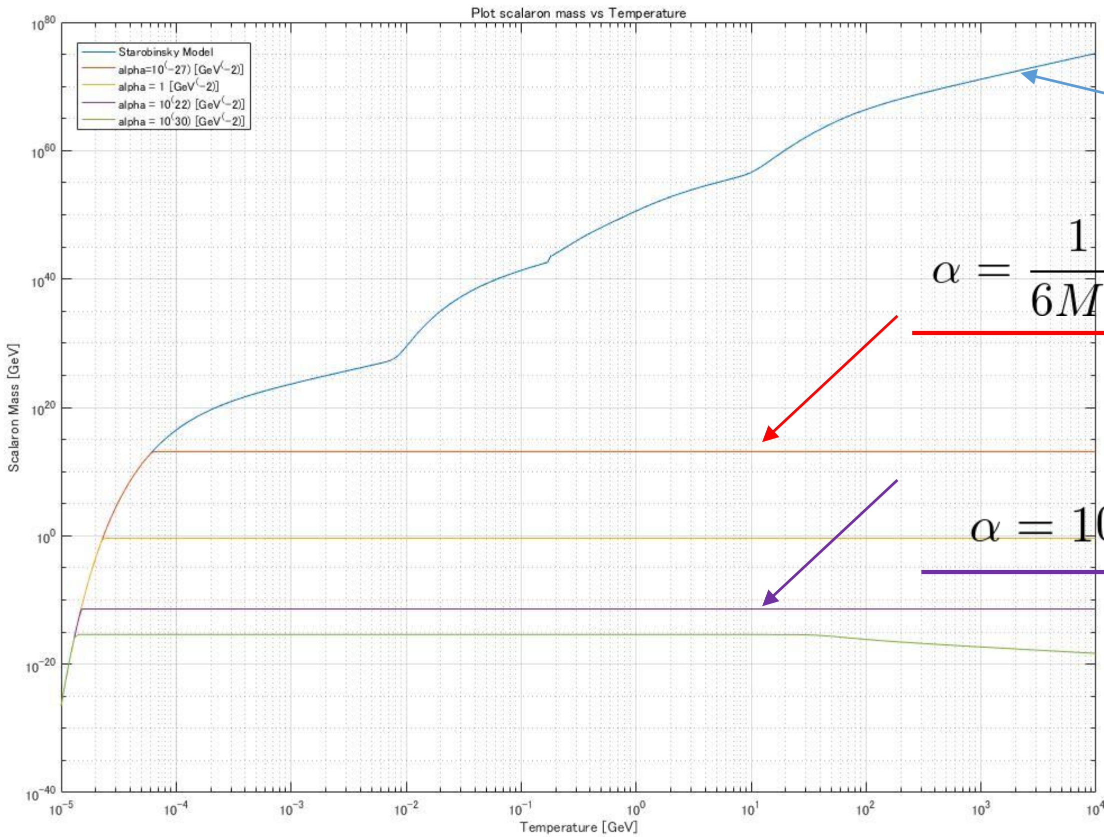
$$F(R) = R - \beta R_c \left[1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right] + \alpha R^2$$

[Dev, Jain, Jhingan, Nojiri, Sami, Thongkool(2008)]

History of Scalaron Mass

Large curvature limit,

$$m_\varphi \approx \frac{1}{6\alpha(1+2\kappa^2\alpha(\rho-3p))}$$



Starobinsky model

$$\alpha = \frac{1}{6M^2}\ ,\ \ M = 10^{13}[\mathrm{GeV}]$$

As inflaton

$$\alpha = 10^{22}[\mathrm{GeV}^{-2}]$$

Upper bound from
Eot-Wash experiments

Conclusion

Summary and Conclusion

- Modified gravity has been investigated so far to explain the dark energy.
- We are studying if the modified gravity can explain the dark matter.
- F(R) gravity predicts the new scalar field, and we study it as new dark matter candidate.

- We studied…
  - Interactions btw. scalaron and SM particles
  - Time-evolution of the scalaron mass

- Future works
  - Relic density and direct detection etc.

Thank you for your attention.
(and sorry for my bad talk)