Entanglement and Holography

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work with Kristan Jensen and Brandon Robinson.

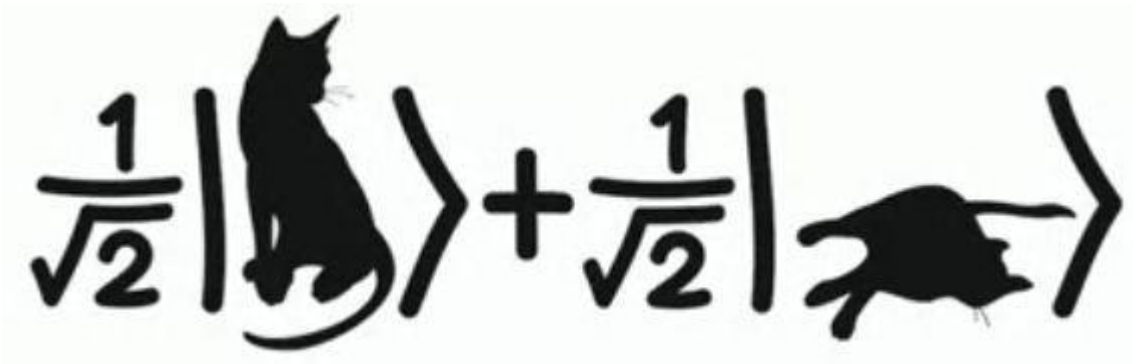
talk at KMI (Nagoya), July 2 2014

What is Entanglement?

What is an entangled state?

Single particle wavefunctions are superpositions:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|\text{cat}\rangle + \frac{1}{\sqrt{2}}|\text{mouse}\rangle$$



State for two spins: Product State

State of particle 2

$$|\psi\rangle = (|\uparrow\rangle + |\downarrow\rangle)(|\uparrow\rangle + |\downarrow\rangle)$$

State of particle 1

Particle 1 is 50/50 up or down, so is particle 2.

Probabilities are independent.

State for two spins: Entangled State

$$|\psi\rangle = |\uparrow\rangle|\uparrow\rangle + |\downarrow\rangle|\downarrow\rangle$$

Particle 1 is 50/50 up or down, so is particle 2.
But they always point in the same direction.

One particle measurement undetermined;
Two particle measurement correlated.

What is Entanglement?

It’s easy to recognize entangled wavefunction.

But wavefunction can not be directly observed.

“Entanglement = Quantum Correlation”

Entanglement as Correlation

For

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

one obtains

$$\langle \vec{S}_A \rangle = \langle \vec{S}_B \rangle = 0$$

not possible in any single particle state

Single Particle Measurement completely uncertain.

Entanglement as Correlation

For

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

one obtains

$$\langle \vec{S}_A \cdot \vec{S}_B \rangle = -\frac{3}{4}$$

perfect correlation

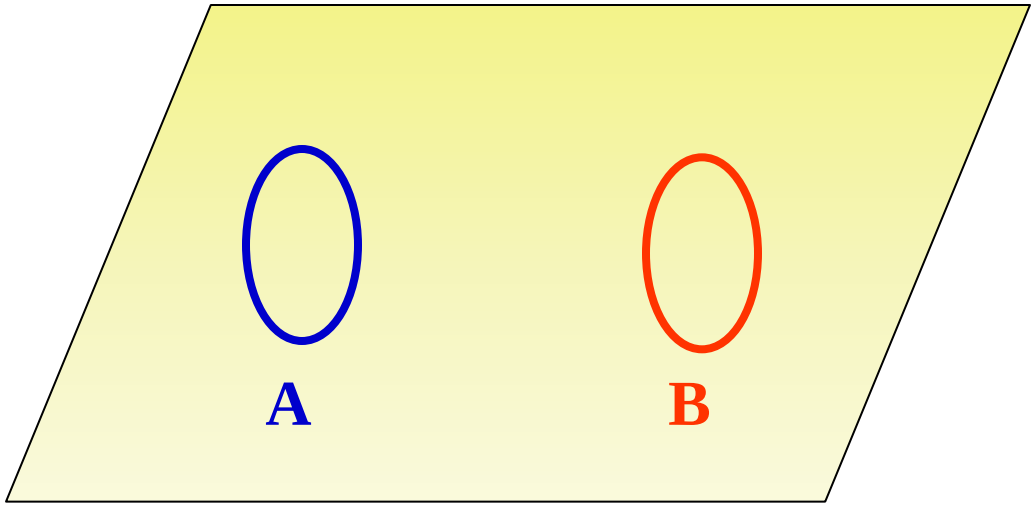
Entanglement as Correlation

$$G = \langle\psi|O_A O_B|\psi\rangle - \langle\psi|O_A|\psi\rangle\langle\psi|O_B|\psi\rangle$$

Observable measuring entanglement
are the disconnected correlation functions
for any pair of operators in subsystem A and B.

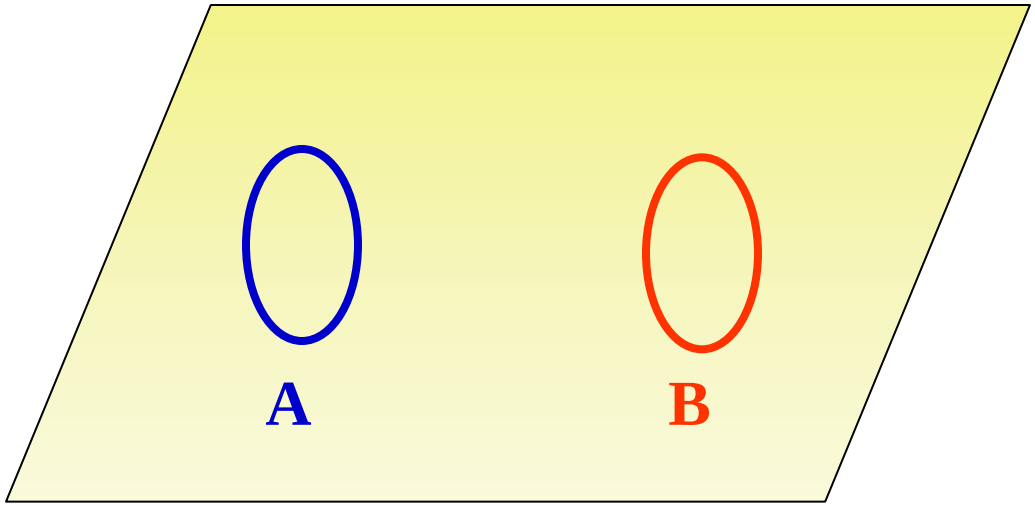
Entanglement and Spacetime

Entanglement in a local QFT



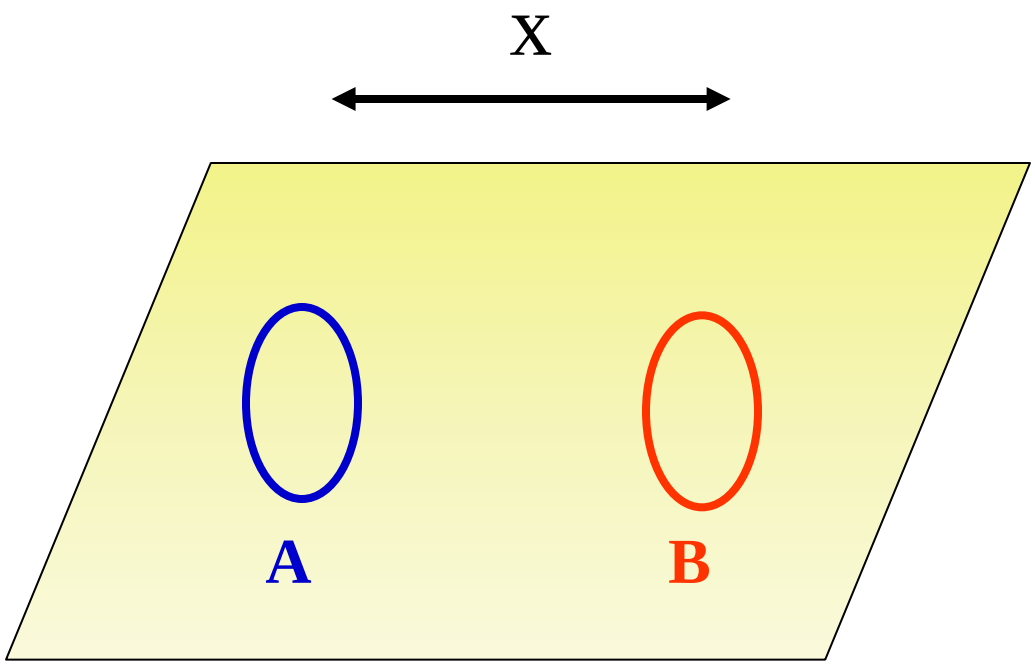
In a local quantum field theory we can identify Subsystems A and B by their location in space.

Entanglement defines distance!



In vacuum modes of the electromagnetic field in nearby regions A and B are entangled!

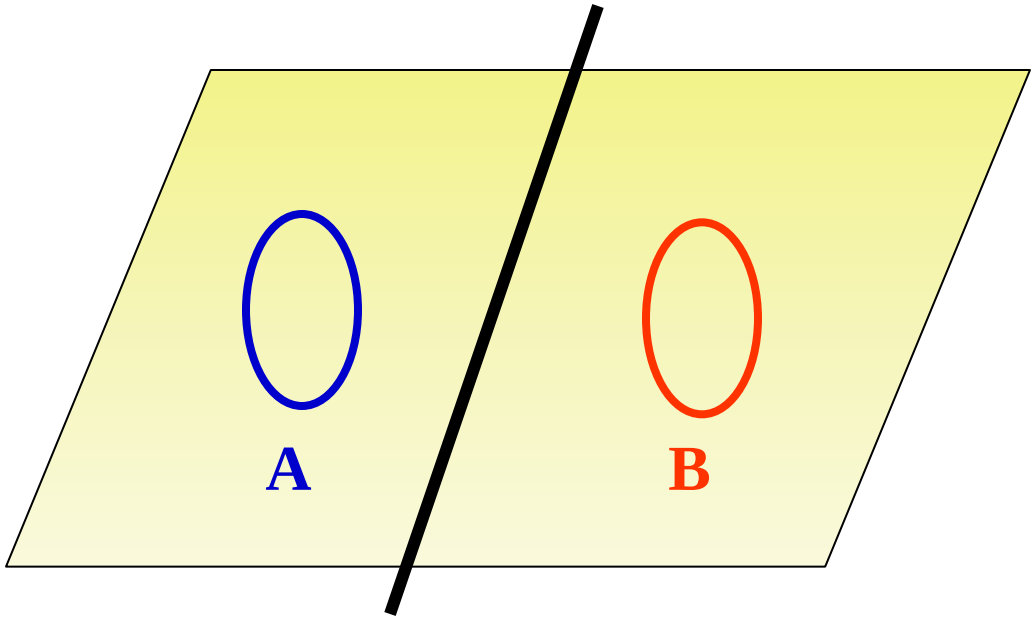
Entanglement defines distance!



Entanglement decreases with distance!

$$\langle O_A O_B \rangle \sim 1/x$$

Entanglement defines distance!



Metal plate (Dirichlet bc) disentangles the modes.

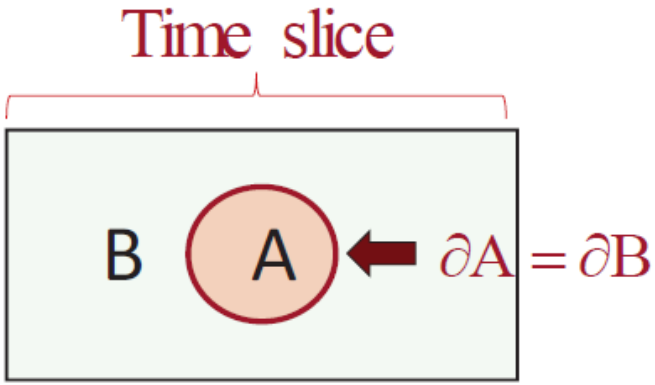
A and B are “disconnected”.

Entanglement is short range

Entanglement Entropy:

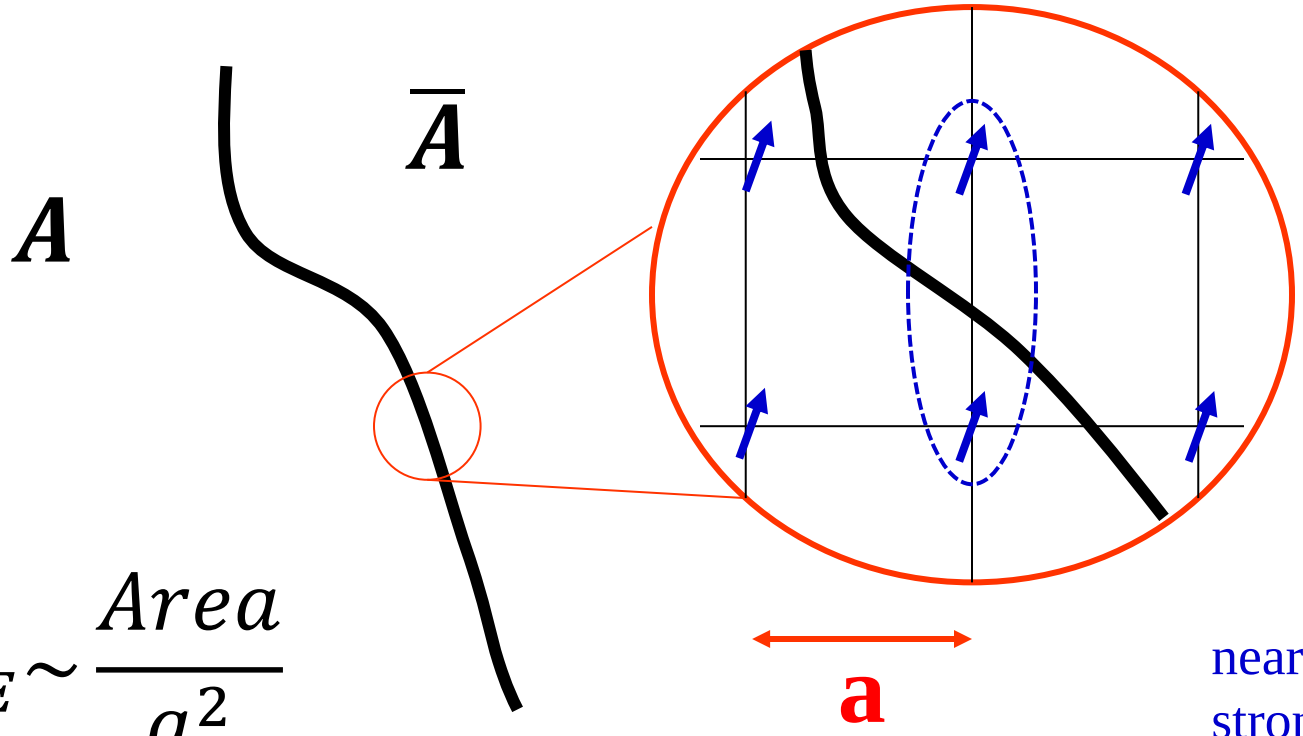
(1) Spin Chain

(2) QFT



Entanglement is short range

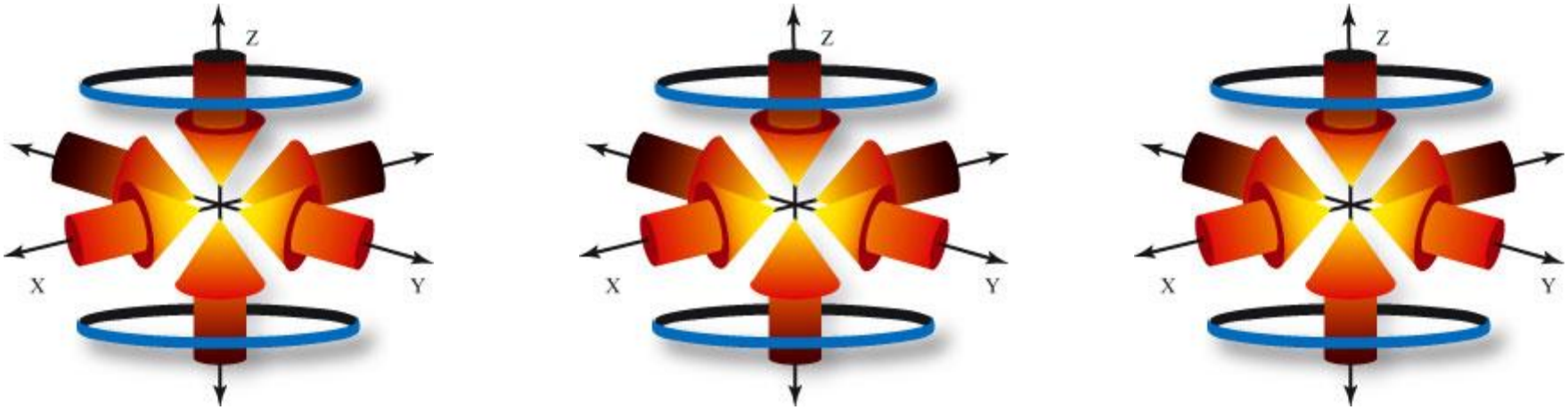
Clear Indication: Entanglement Entropy



$$S_{EE} \sim \frac{Area}{a^2}$$

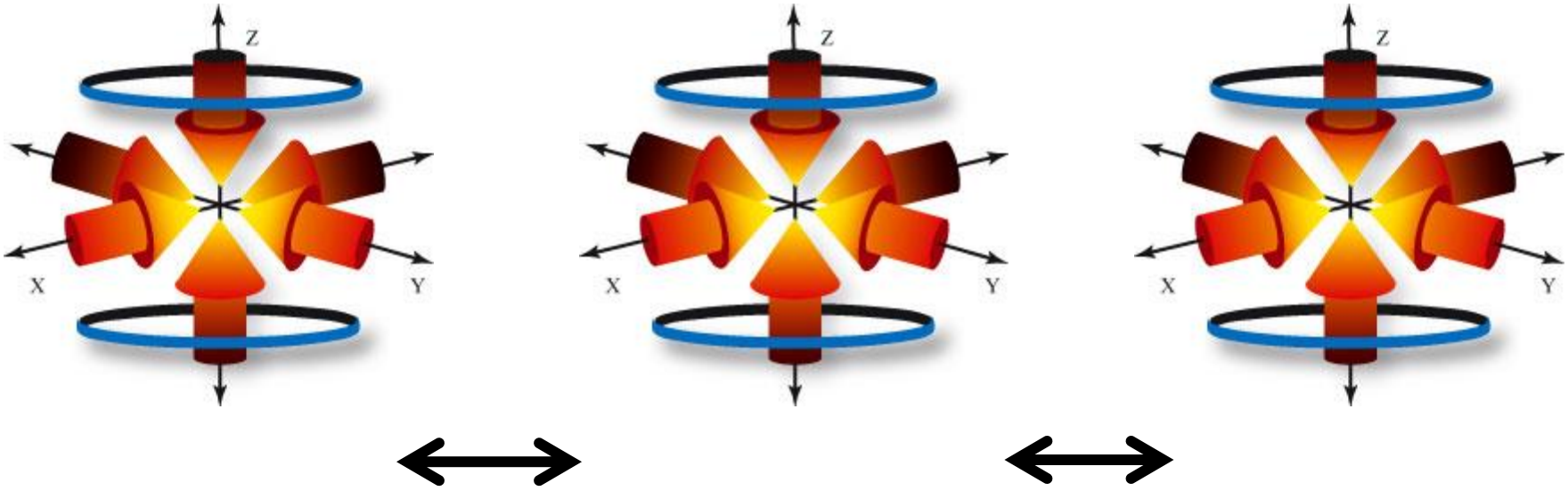
nearest neighbor pairs strongly entangled.

Entanglement = Connectedness



3 atoms in 3 traps; disconnected.

Entanglement = Connectedness



Entanglement "connects" the three into a line!

Entanglement defines notion of distance!

Entanglement and Holography

Emergent Dimensions

These ideas can be made very concrete in holography.

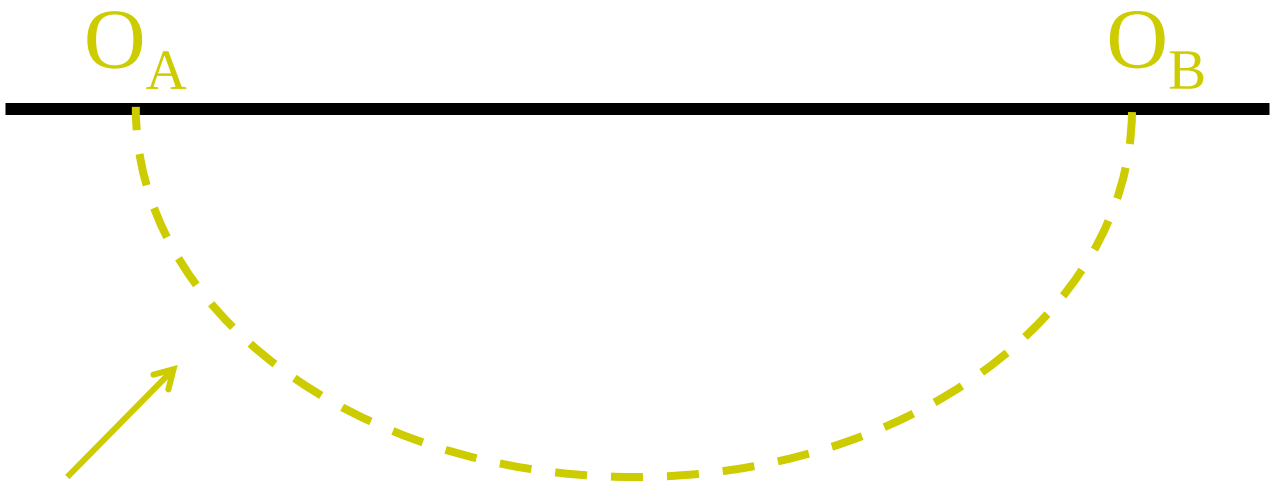
- Entanglement encodes bulk geometry

  (van Raamsdonk)

- Correlations are given by classical propagators. Finite Correlation implies finite spatial distance.

AdS/CFT = Classical Bulk

$$G = \langle\psi|O_A O_B|\psi\rangle - \langle\psi|O_A|\psi\rangle\langle\psi|O_B|\psi\rangle$$



classical bulk
Green's function

finite correlation
only possible for finite distance!

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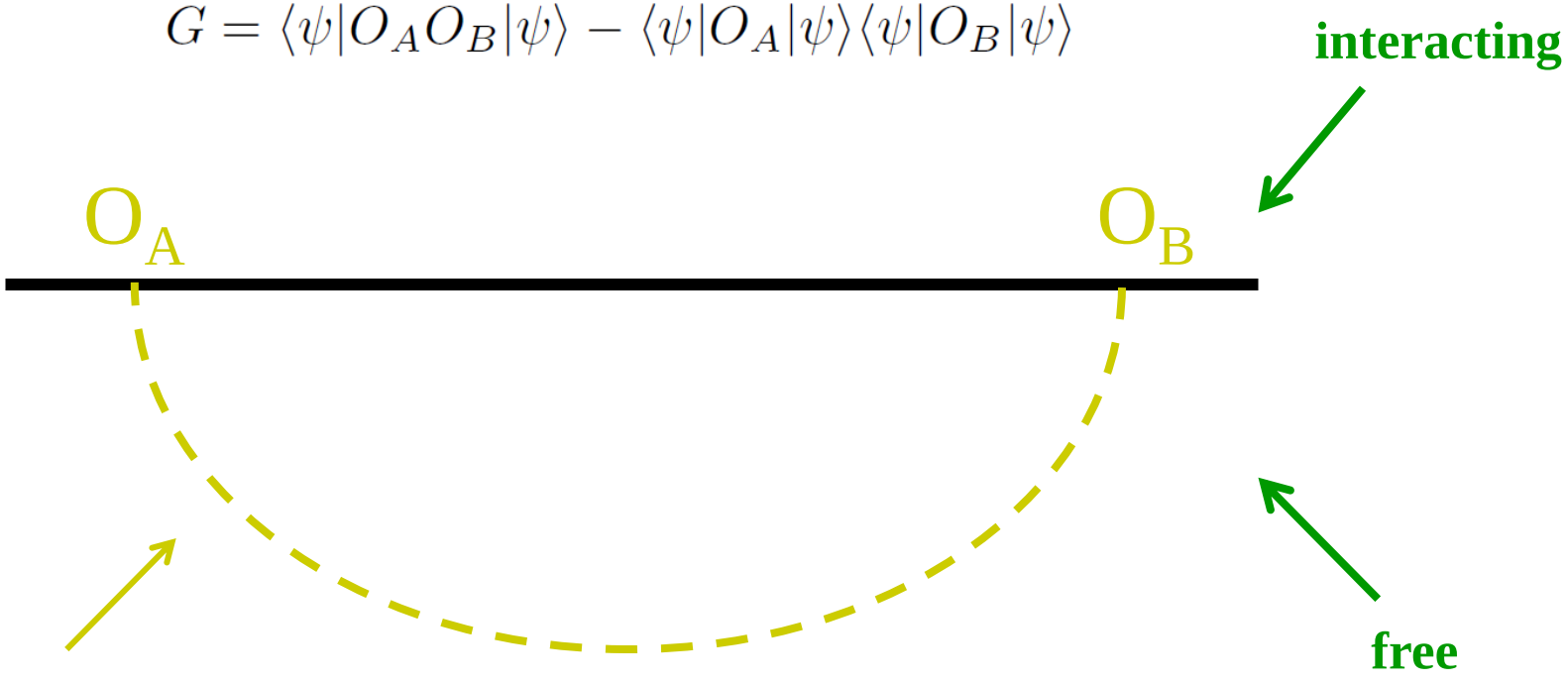

classical bulk
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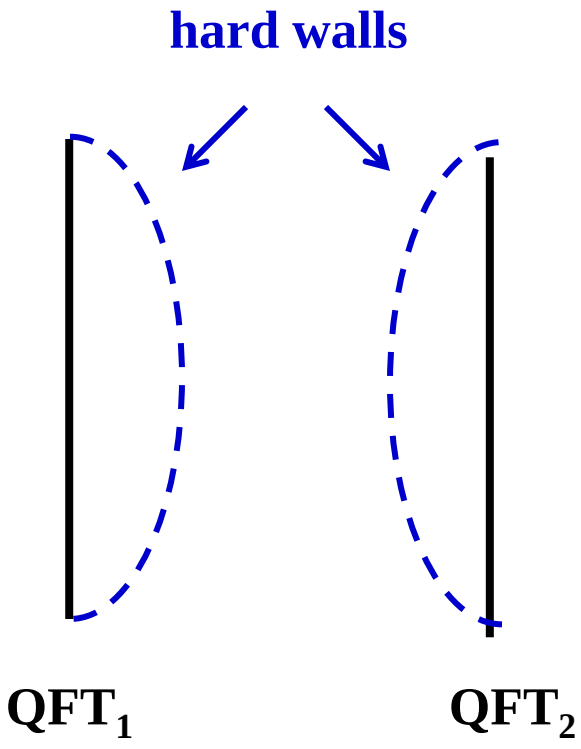
Example: 2 copies of a QFT



No Entanglement

Product State

Example: 2 copies of a QFT

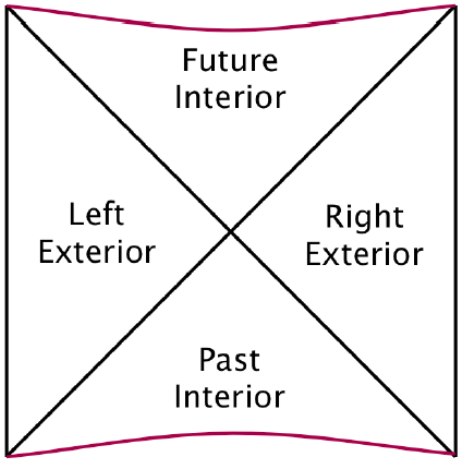
Even though the two QFTs are decoupled, we can still consider entangled states!

$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |E_n\rangle_1 \times |E_n\rangle_2$$

Entangled state
(Thermofield Double)

- Tracing over QFT_1 gives thermal density matrix for QFT_2
- Used for "real time" finite temperature calculations.

Example: 2 copies of a QFT



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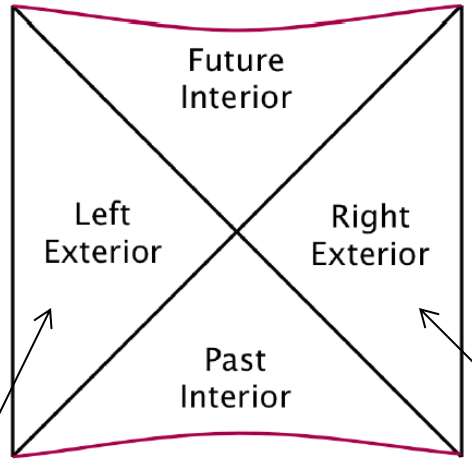
Entangled state
(Thermofield Double)

Holographic dual: Eternal AdS black hole

(Maldacena)

Example: 2 copies of a QFT



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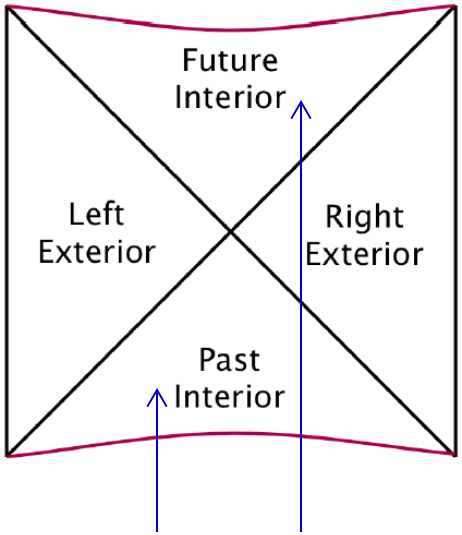
Entangled state
(Thermofield Double)

QFT_1

QFT_2

Example: 2 copies of a QFT

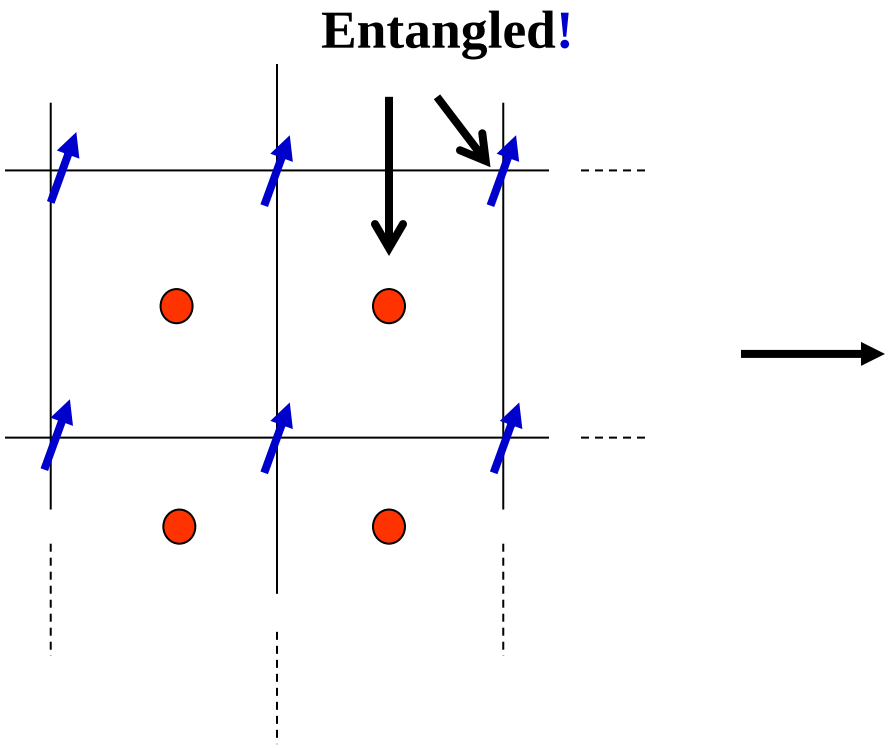


$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |E_n\rangle_1 \times |E_n\rangle_2$$

Entangled state
(Thermofield Double)

These two regions encode the correlation implied by the entanglement between the two QFTs.

Entanglement defines Distance



=earth

=moon

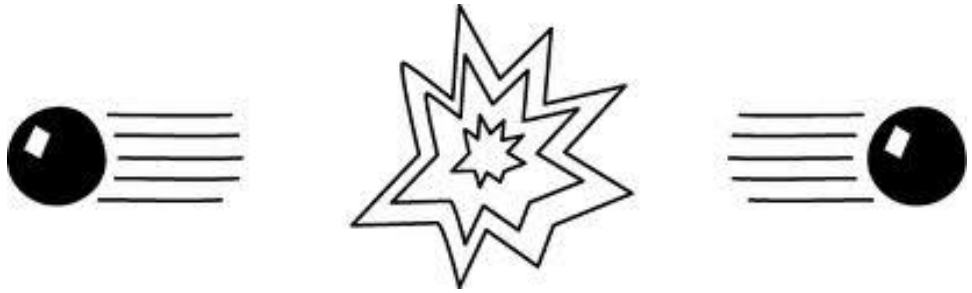
“space” static. or no space.
abstract quantum system.
here, e.g: 2d space

gravity
3d space and time are emergent concepts

The EPR pair

The EPR pair.

Extreme example of entanglement: **EPR pair**

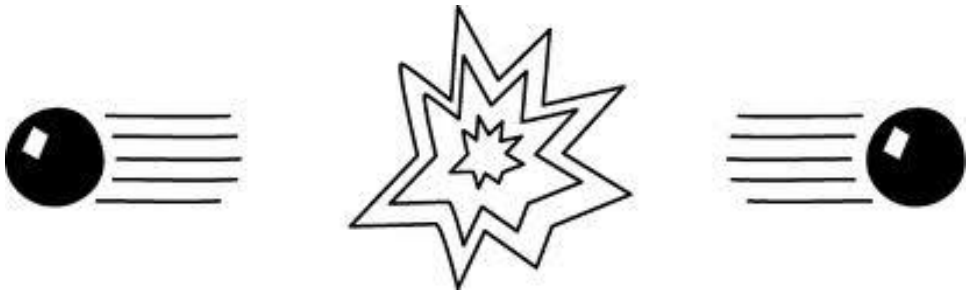


E.g.: **anti-quark** **quark**

pair created in
background electric field

The EPR pair:

spin 0 in initial state



We all know how to write the EPR wavefunction:

$$|\Psi> = \frac{1}{\sqrt{2}} (|\uparrow\downarrow> - |\downarrow\uparrow>)$$

Two particle correlation

$$< \overrightarrow{S_{(1)}} \cdot \overrightarrow{S_{(2)}} > = -3/4$$

100% likelihood for spins to point in opposite directions!

- this is an equal time correlator
- in EPR pair this is non-zero despite of **lack of causal contact**.

Operational definition EPR pair.

From now on we define an EPR pair to be an entangled pair not in causal contact.

Holographic EPR pair

EPR pair = non-vanishing correlators,
but no causal contact.

(Holography): Finite spatial distance between the two points in the bulk, but no causal contact.

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EPR pair = non-vanishing correlators,
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(Holography): Finite spatial distance between the two points in the bulk, but no causal contact.

This defines an ER bridge / wormhole!

ER=EPR

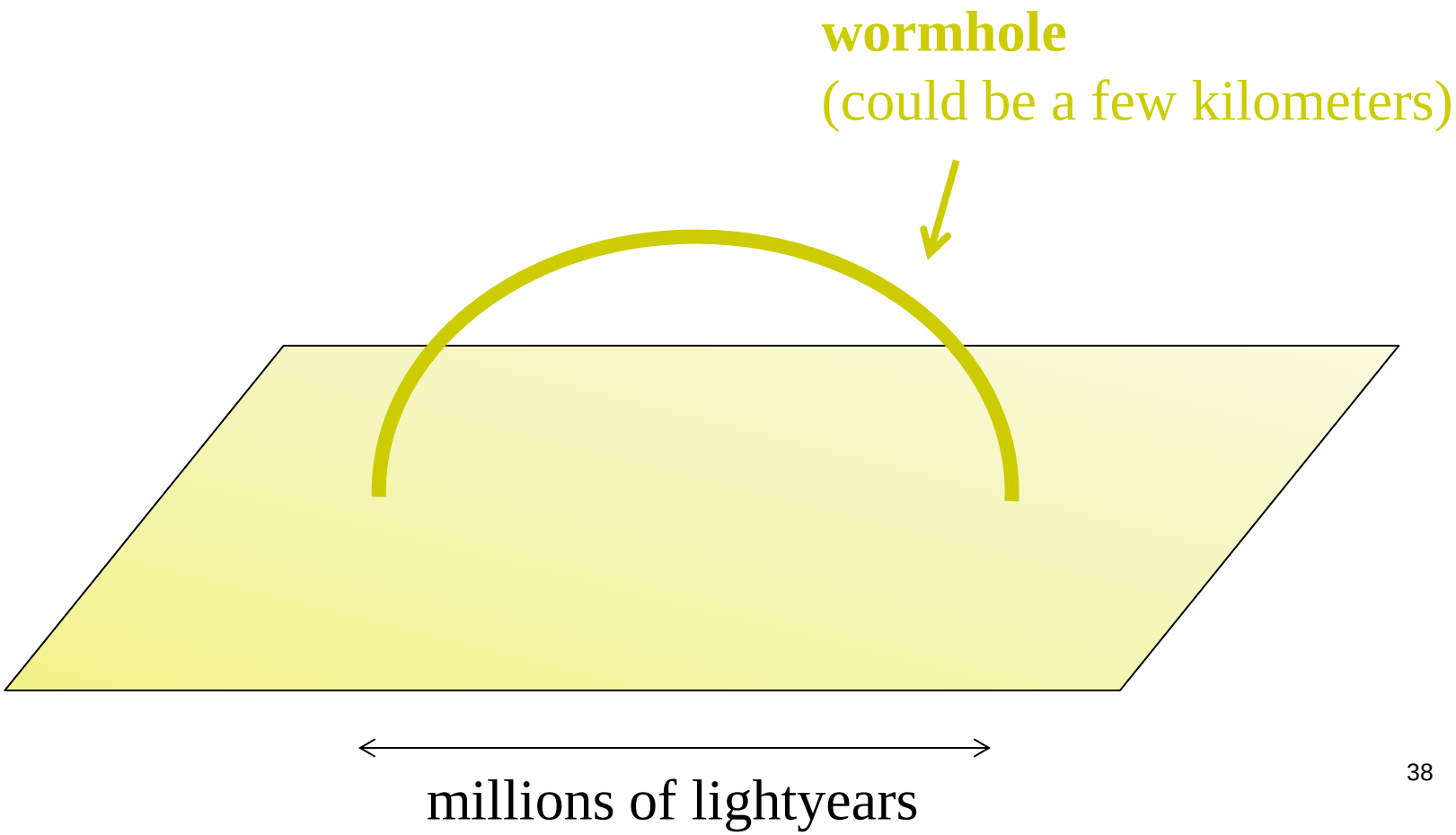
So almost by definition, the holographic dual of an EPR pair is a non-traversible ER wormhole.

(Maldacena-Susskind)

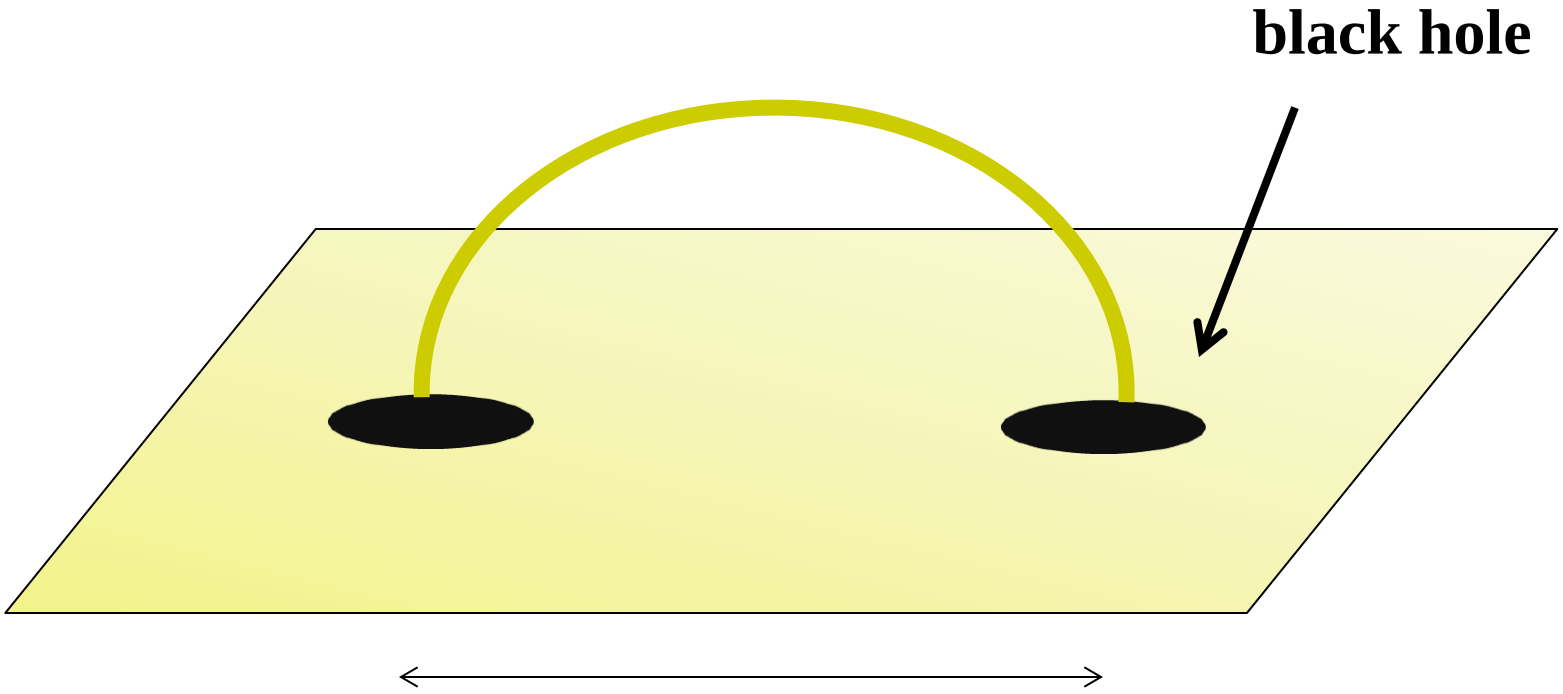
(MS make slightly stronger statement. EPR=ER. Crisp statement: ER and EPR give rise to identical physical consequences = correlators)

The ER Bridge

Einstein-Rosen Bridge

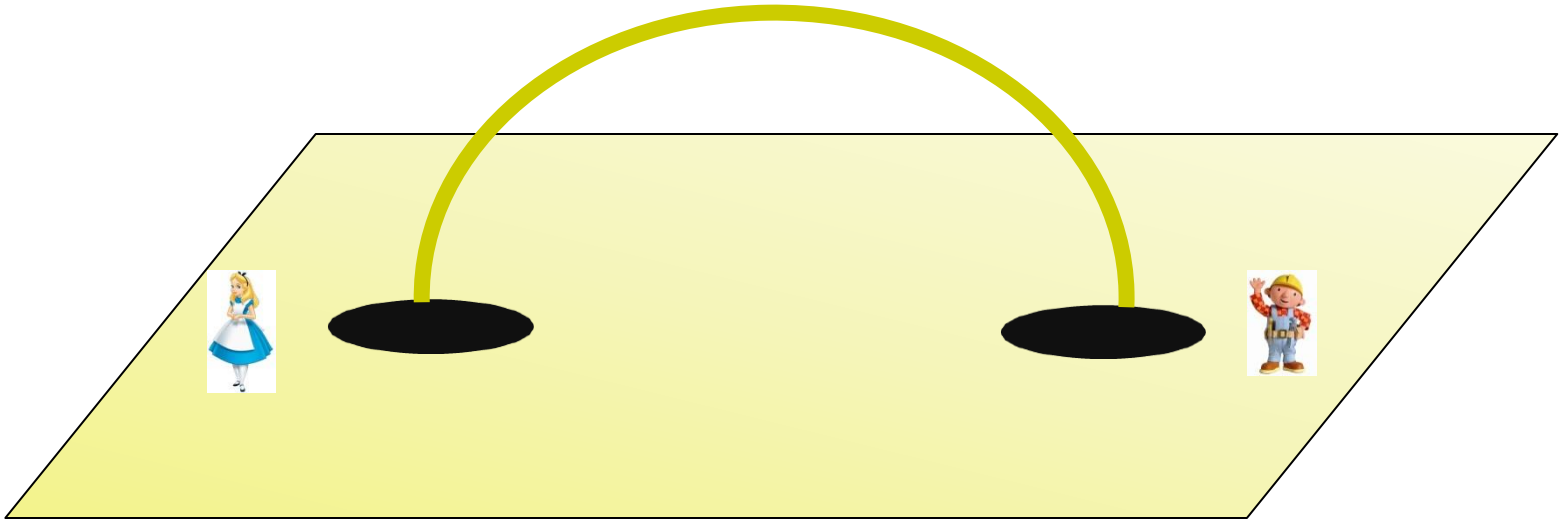


Einstein-Rosen Bridge



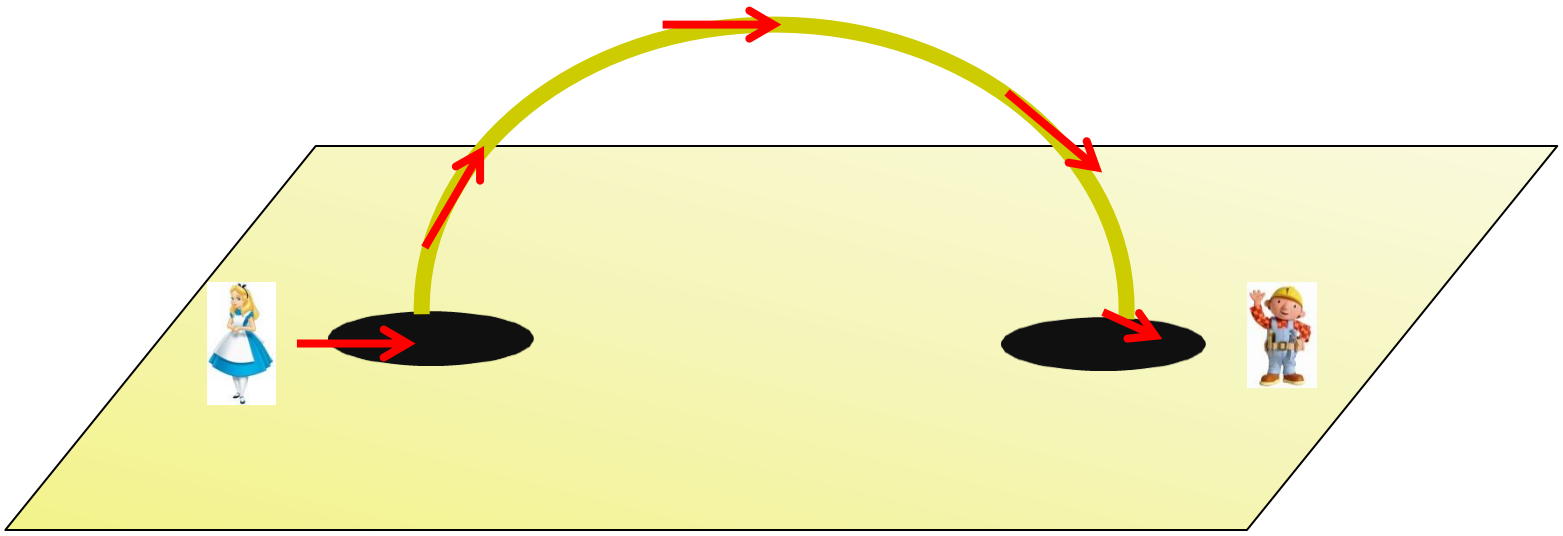
Einstein-Rosen Bridge

distance (through wormhole) just
a couple of kilometers
(finite spatial distance)



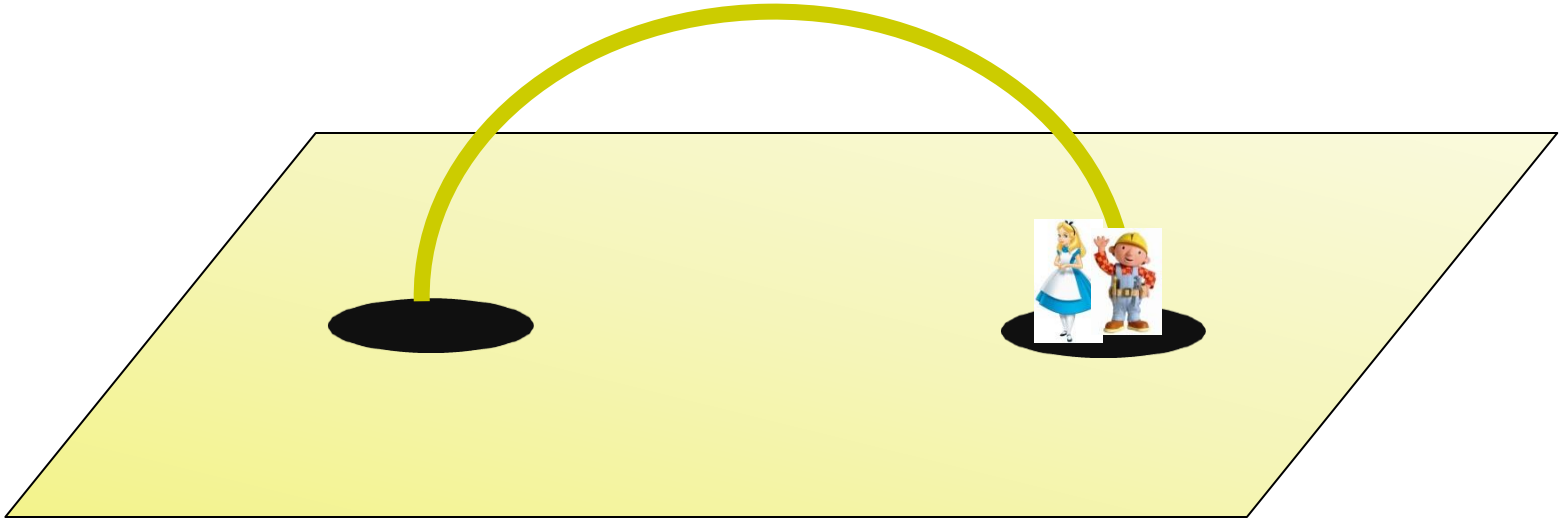
Einstein-Rosen Bridge

Alice can send signal into wormhole,
but signal can't reach Bob.
(Causally disconnected).

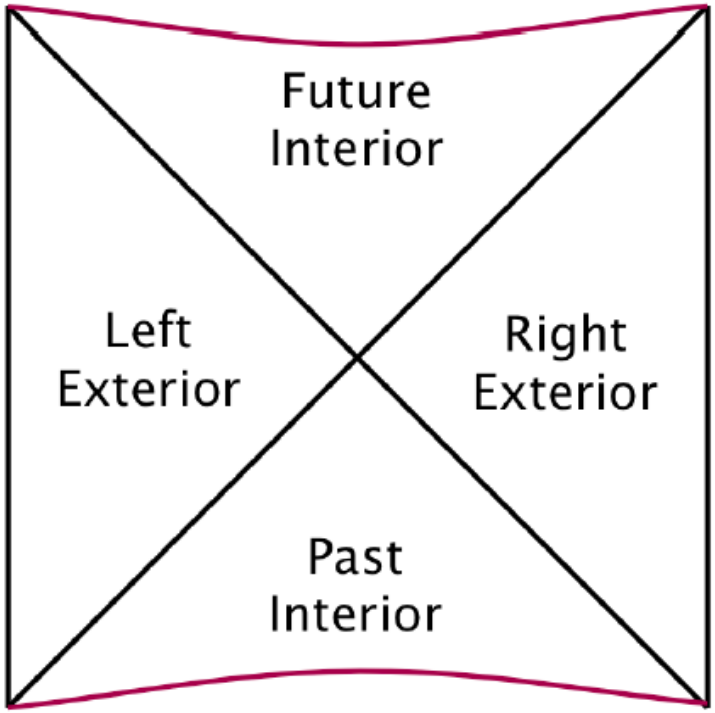


Einstein-Rosen Bridge

But if **both** jump into their respective black hole, they can easily meet (but never come back out)

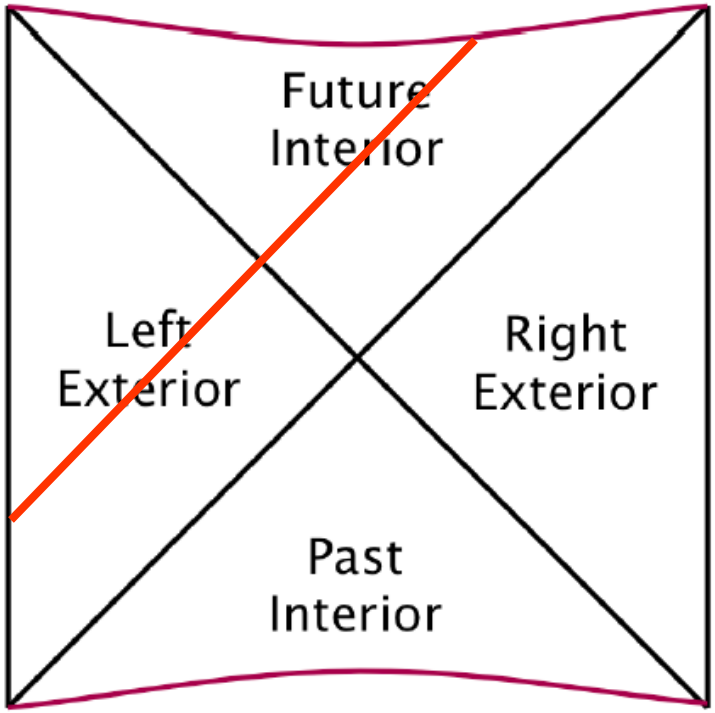


Example: Eternal Black Hole



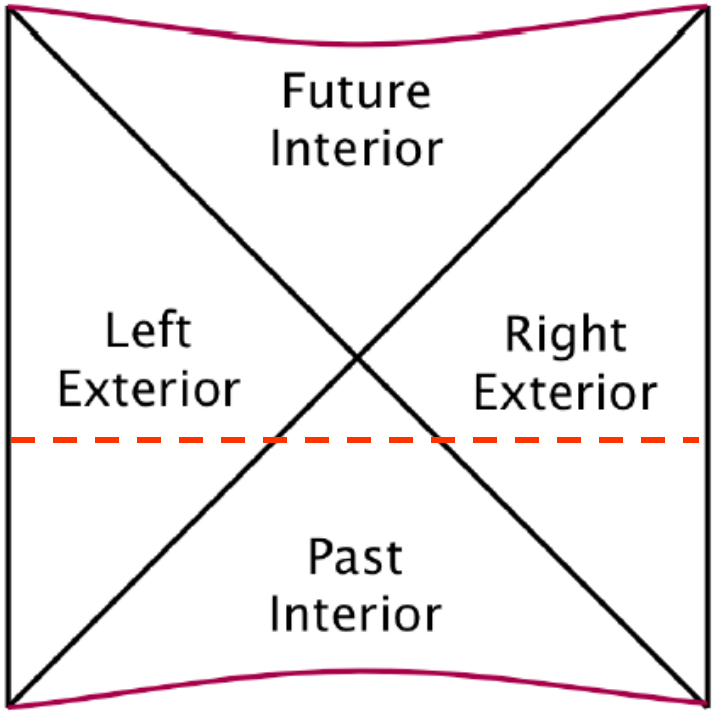
Think of this is describing two far away entangled black holes

Example: Eternal Black Hole



no causal contact between the two asymptotic regions…

Example: Eternal Black Hole

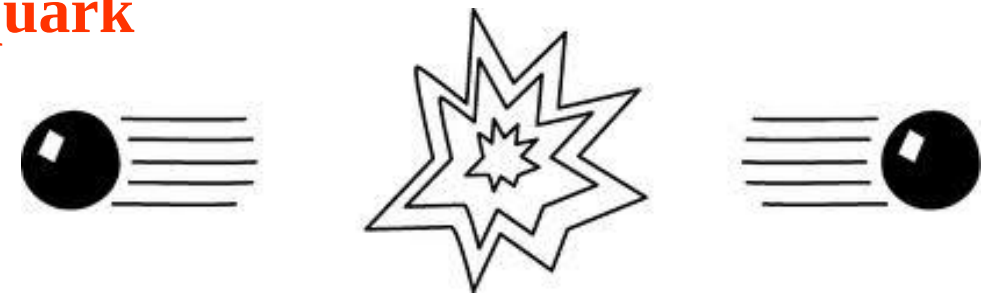


… but finite spatial distance.

Holographic EPR pair

Example: holographic EPR (Jensen-AK)

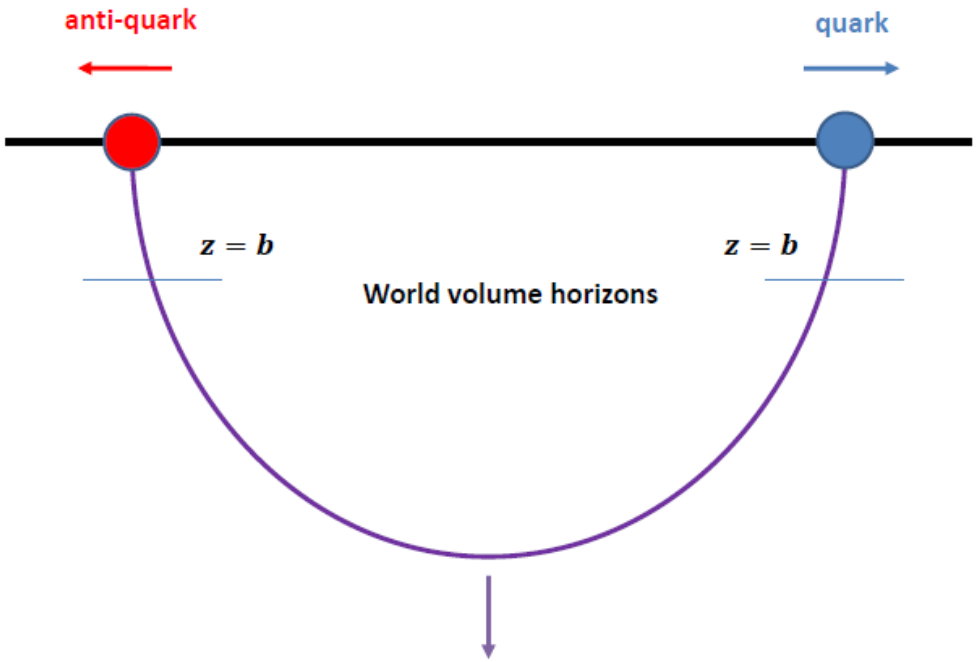
anti-quark **quark**



pair created in background electric field

We can do this in N=4 SYM.
What is the holographic dual?

Holographic EPR

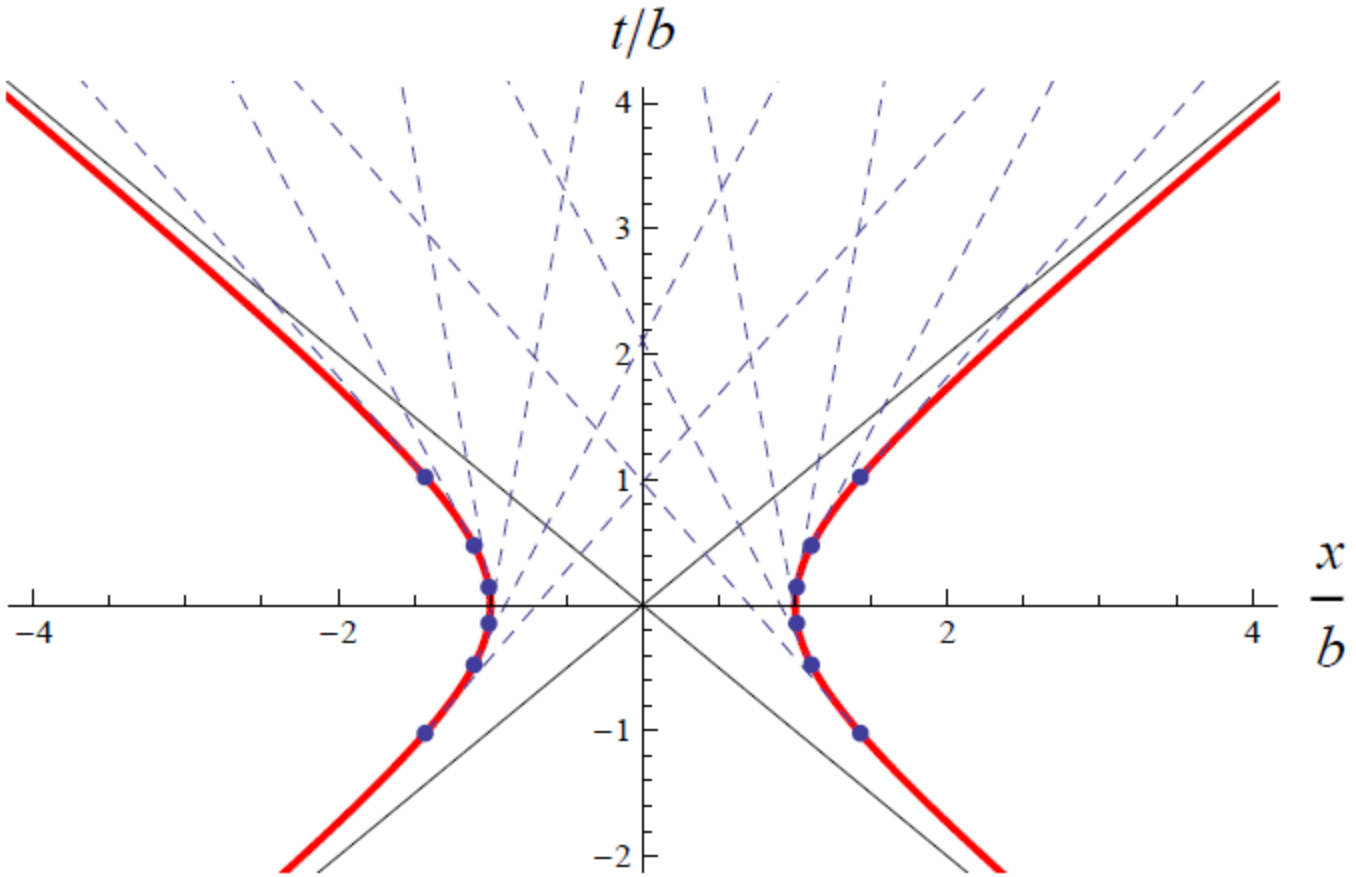


$$x^2 = t^2 + b^2 - z^2 \,.$$

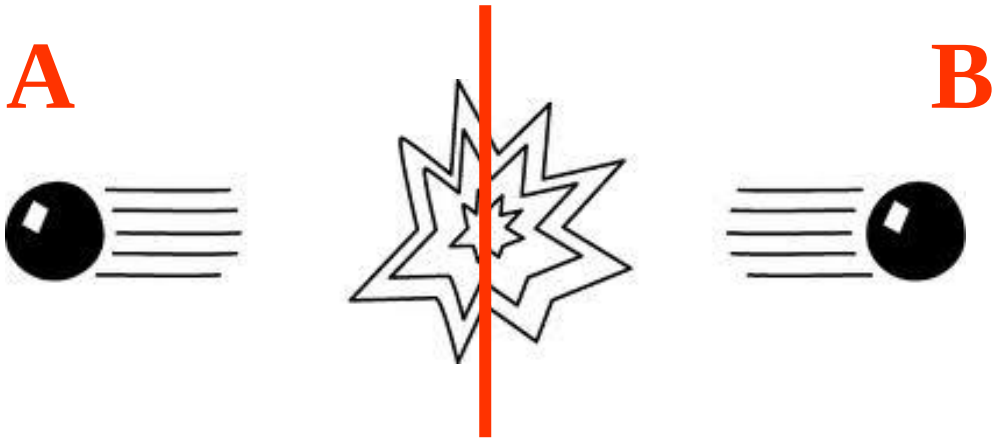
(Xiao)

Worldsheet = ER bridge (finite distance, no causal connection)

Holographic EPR: geodesics



S_{EE} for the holographic EPR pair



Finally: calculate EE for a probe brane!

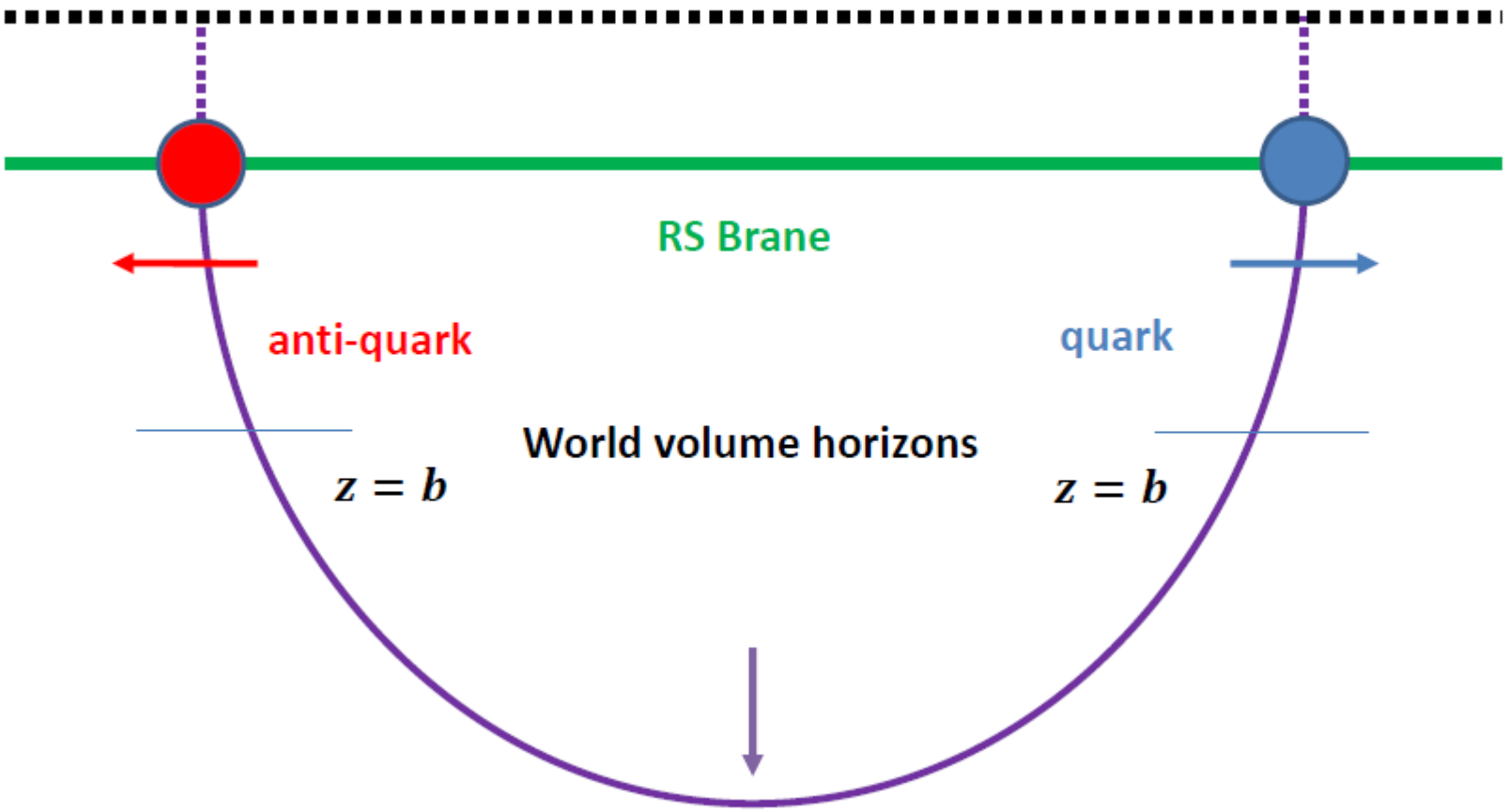
$$S_{EE} = \sqrt{\lambda}/3$$

quark not just a single parton, $\sqrt{\lambda}$ gluons part of quasi-particle

Generalizations

- Including Dynamical Gravity
- Holographic Hawking Pairs

1) Dynamical Gravity via RS



Bottom Line: No Change

RS-holography:

CFT with UV cutoff a
+ dynamical gravity with
G_N induced by matter loops

=

Brane located at fixed radial position; 5d fluctuations induce a localized 4d mode.

Only change: Horizon disappears when q-qbar separation is less than a.

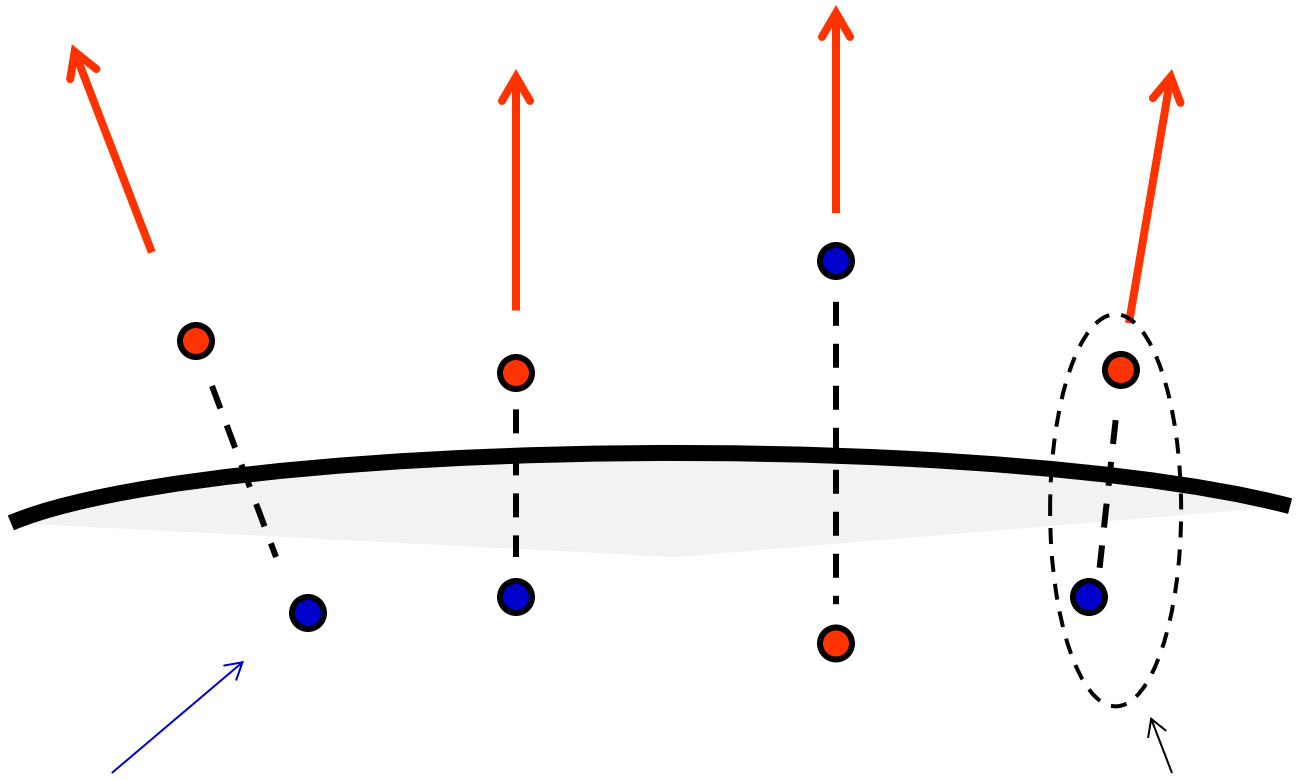
2) Hawking Pairs

AdS/CFT allows us to study the CFT
on any background metric g_0

g_0 sets the near boundary behavior of the bulk metric. Need to solve Einstein equations with that boundary behavior (Fefferman-Graham).

If we chose g_0 to be a black hole, we can study a Hawking pair (= EPR pair separated by horizon).

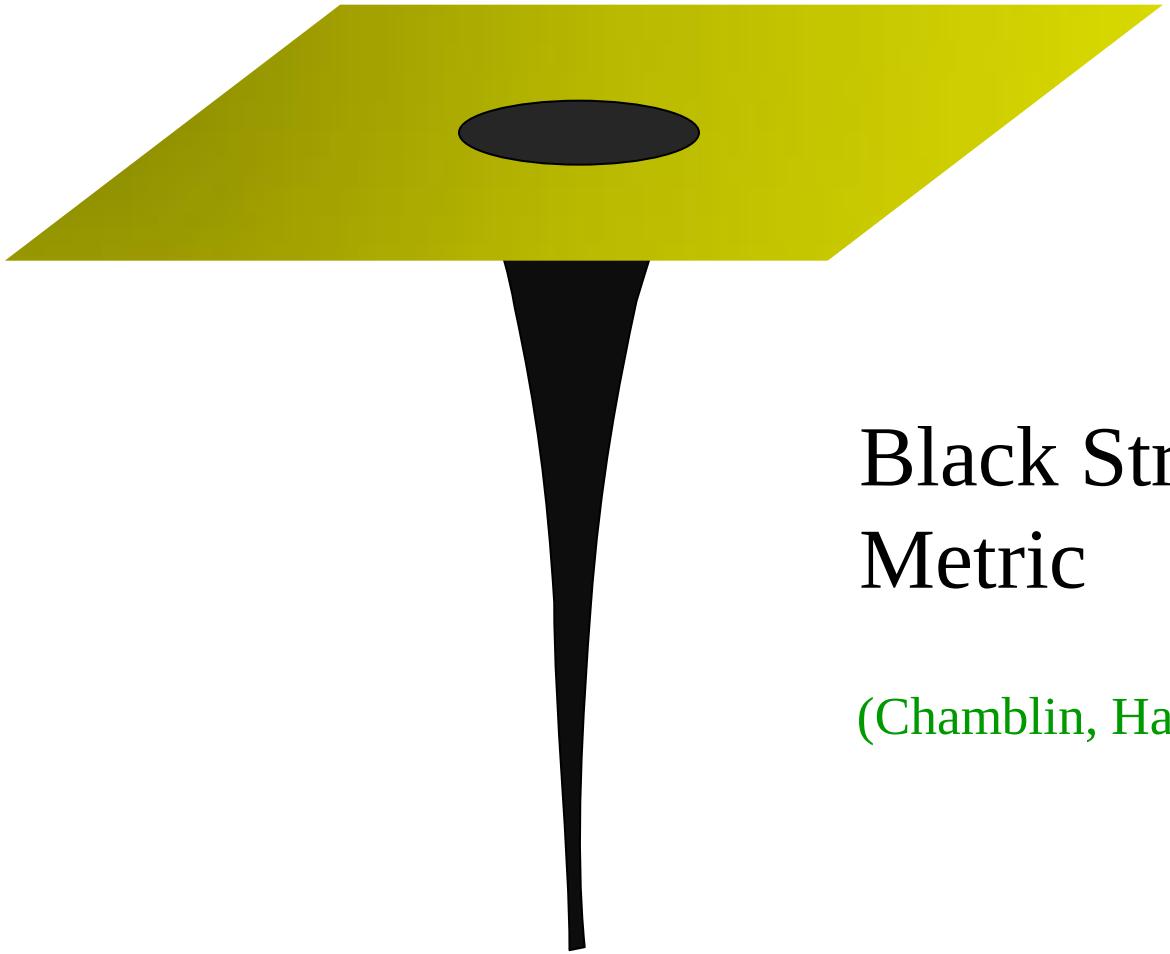
Hawking Radiation:



Quark in flavored N=4 SYM.

What's the holographic dual to a Hawking pair?

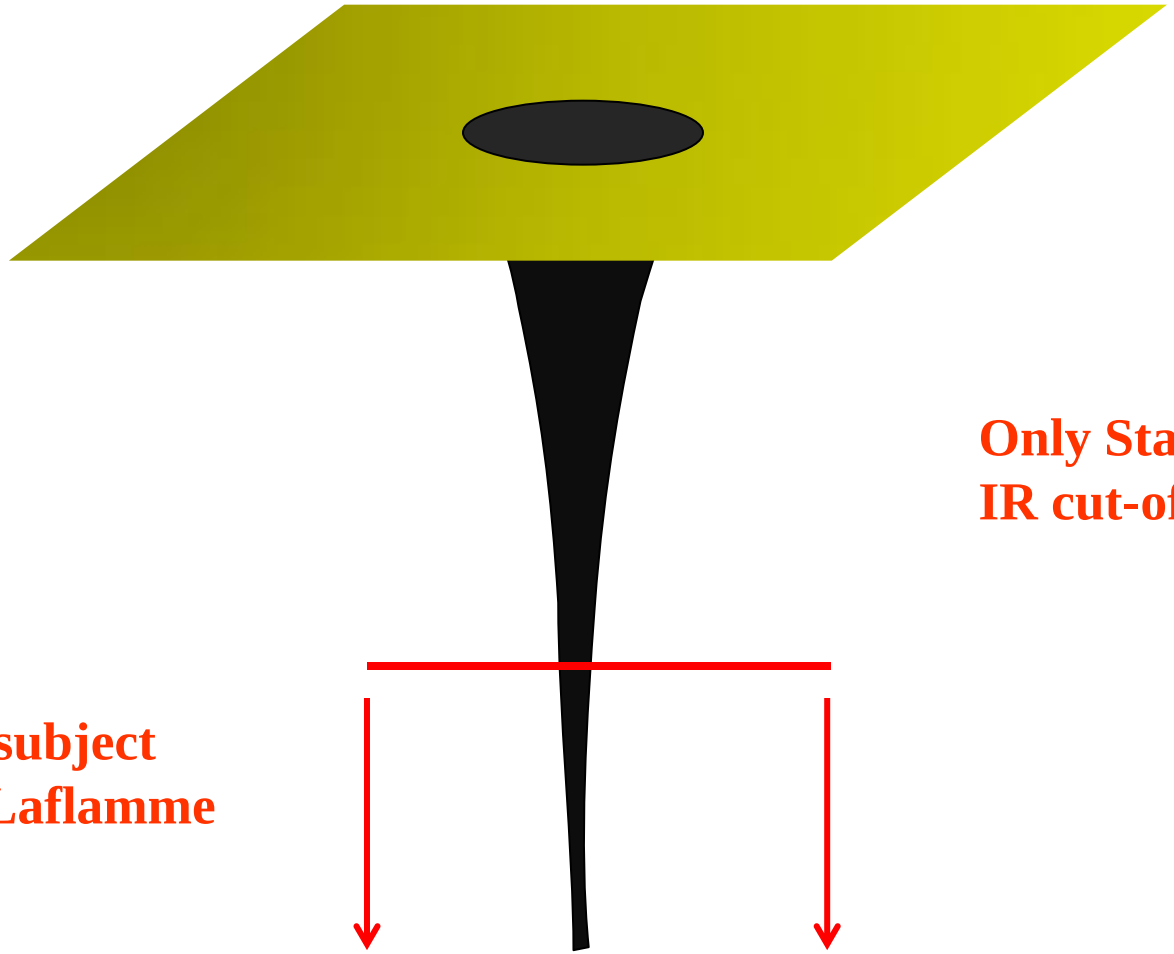
N=4 on Schwarzschild Black Hole



Black String Metric

(Chamblin, Hawking, Reall)

N=4 on Schwarzschild Black Hole



Only Stable with IR cut-off (hard wall).

Thin String subject to Gregory-Laflamme Instability!

Unstable vacuum on BH

The unstable state can still be studied.

Puzzle: No Hawking radiation!

$$\langle T_{\mu\nu} \rangle = 0$$

Potential Resolution:

at strong coupling, only color neutral Hawking radiation!

(Fitzpatrick, Randall, Wiseman)

Sharpening the Puzzle

Or maybe we just shouldn't over-interpret the unstable state?

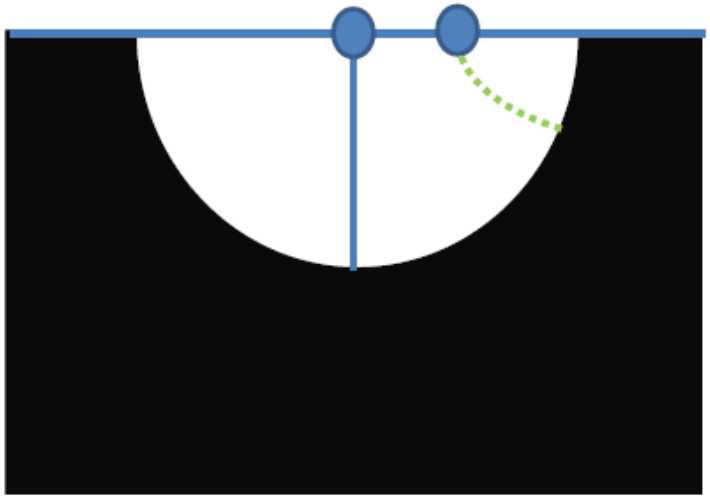
Puzzle can be sharpened:

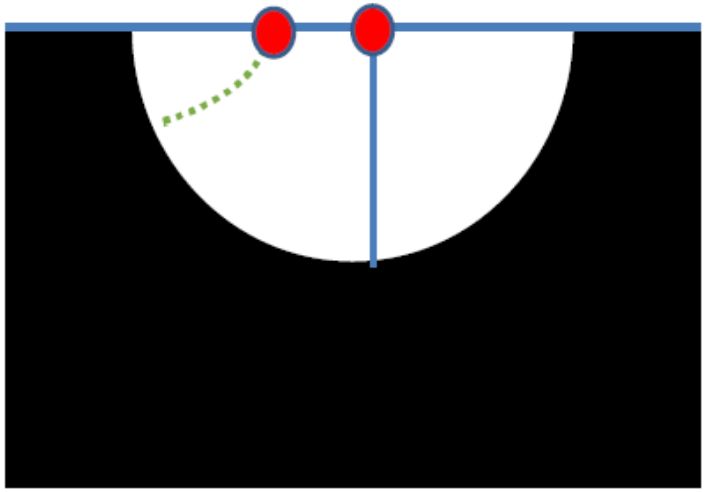
Black String for AdS_4-black hole is stable!

Still no Hawking Radiation.

(Chamblin, AK; Gregory, Ross, Zegers)

AdS_4 Black String Metric





mSYM on two joined copies of AdS_4 black holes

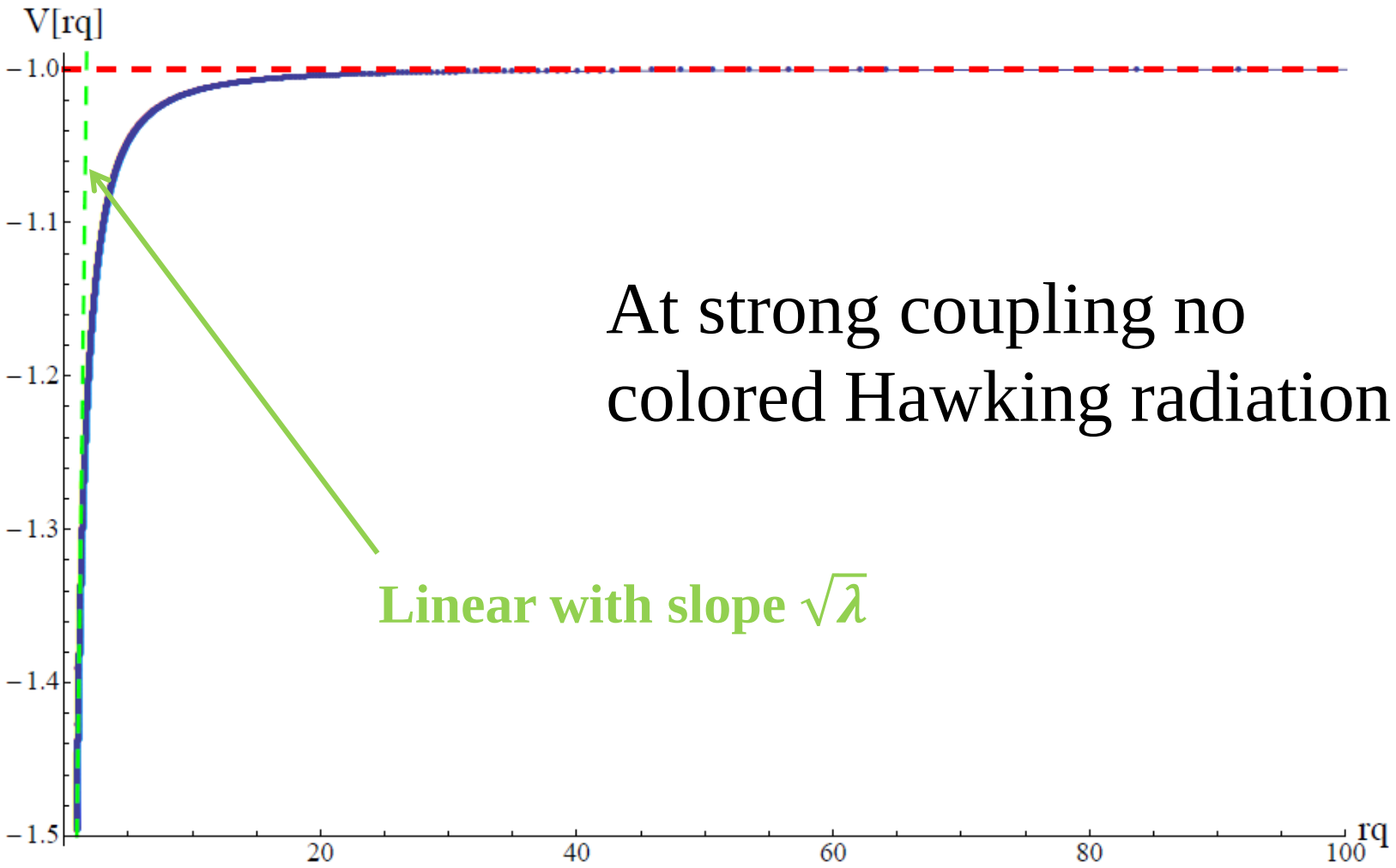
thermofield double

Hawking pair dual to ER

All our strings for all time
cross the bulk horizon through the
bifurcation point, they inherit
the causal structure of the bulk ER bridge.

Worldsheet metric = ER bridge.

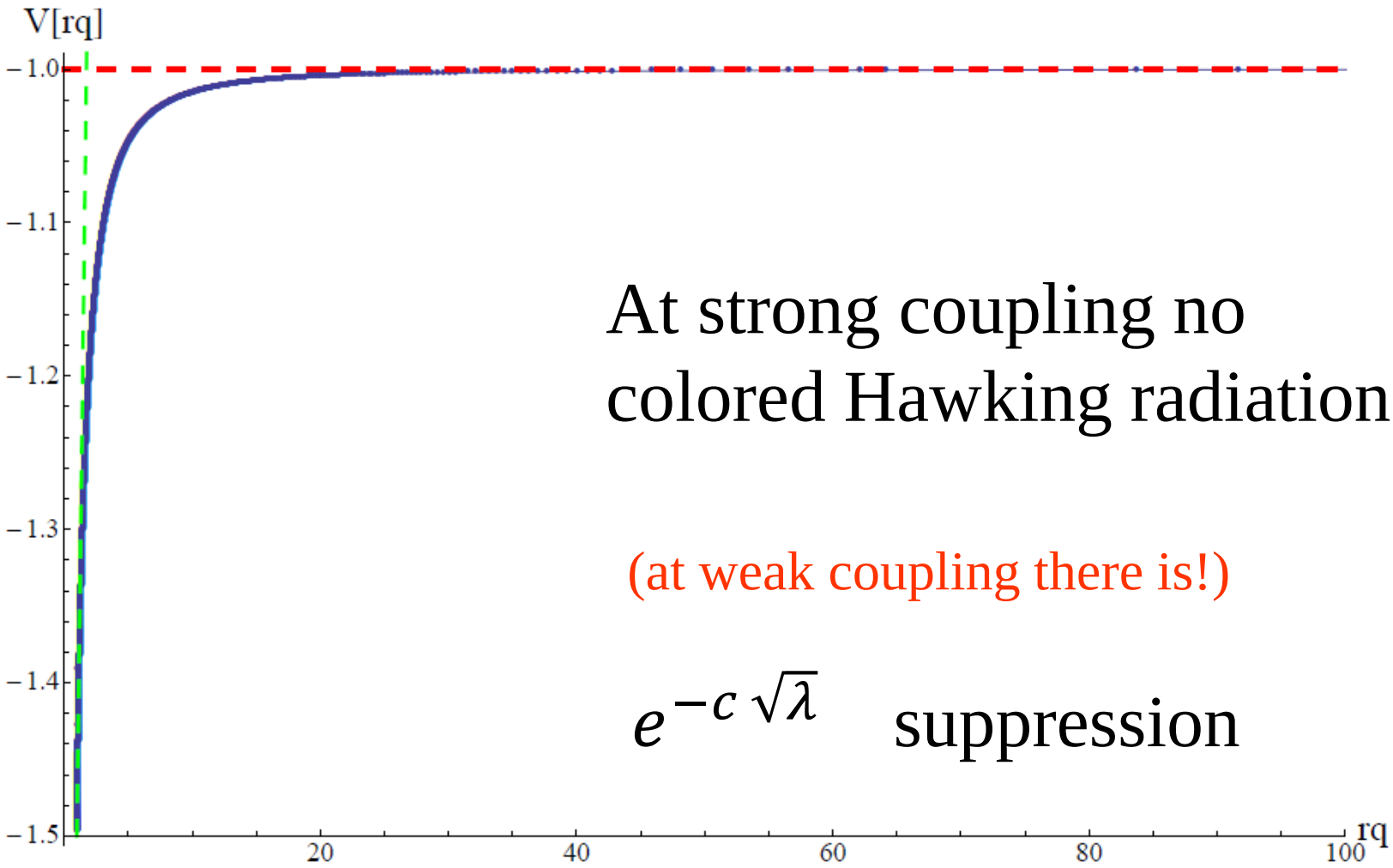
And Confinement!



At strong coupling no colored Hawking radiation!

Linear with slope $\sqrt{\lambda}$

And Confinement!



At strong coupling no colored Hawking radiation!

(at weak coupling there is!)

$e^{-c\sqrt{\lambda}}$ suppression

Conclusions

The holographic dual of an EPR pair has to have the geometric structure of an ER bridge.

ER=EPR true almost by definition.

Conclusions

Bigger insight: Entanglement encodes Geometry holographically.

Conclusions

Can study consequences of Entanglement in strongly coupled systems.

E.g: Is the ridge in heavy ion collisions related to entanglement?

Conclusions

Side Benefit:

Absence of Hawking radiation in strongly coupled mSYM indeed due to confinement!

Fitzpatrick, Randall, Wiseman were right!