Isotropization in Heavy Ion Collisions at High Energy

Kobayashi-Maskawa Institute, Nagoya, March 2014

T. Epelbaum, FG :
arXiv:1307.1765
arXiv:1307.2214

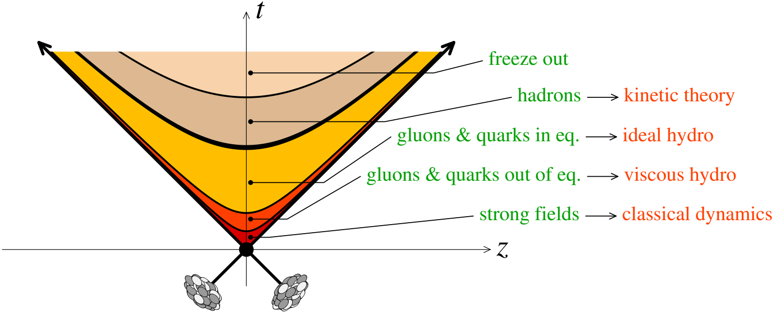
François Gelis
IPhT, Saclay

Outline

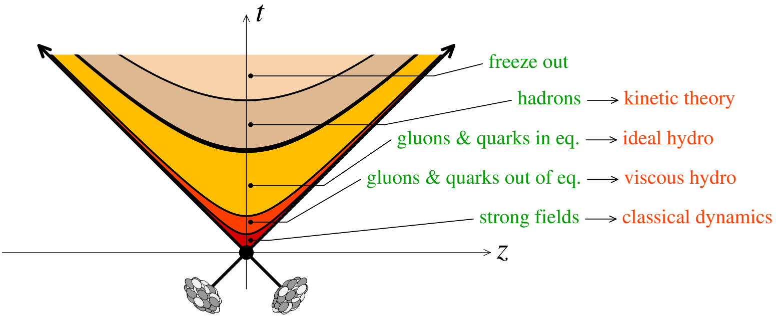
1. The surprising success of relativistic hydrodynamics
2. Modern ideas for the thermalization of quantum systems
3. CGC description of heavy ion collisions
4. Isotropization in Heavy Ion Collisions

Relativistic hydrodynamics

Stages of a nucleus-nucleus collision

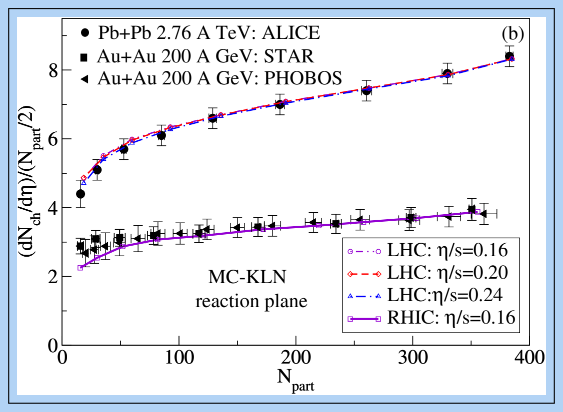


Stages of a nucleus-nucleus collision

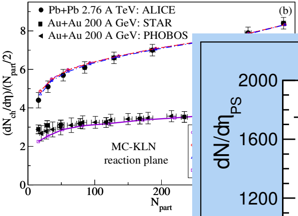


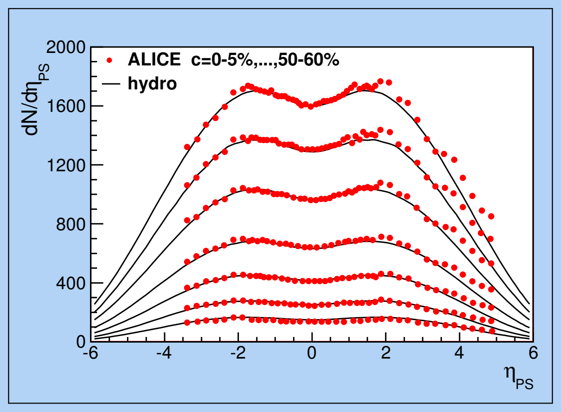
- Well described as a fluid expanding into vacuum according to relativistic hydrodynamics

Evidence for hydrodynamical behavior

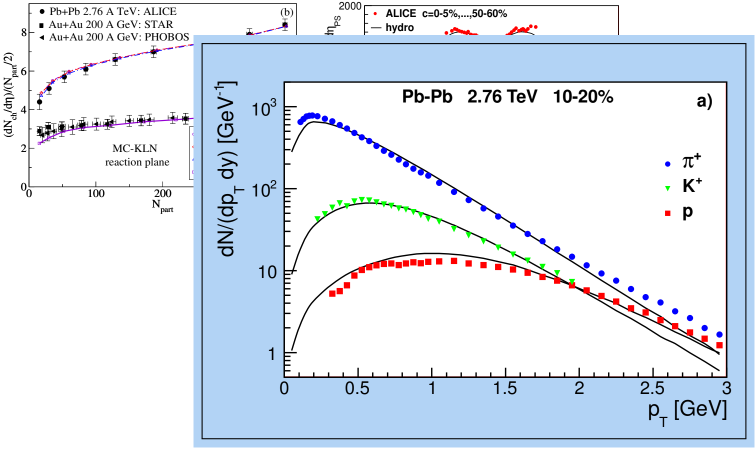


Evidence for hydrodynamical behavior

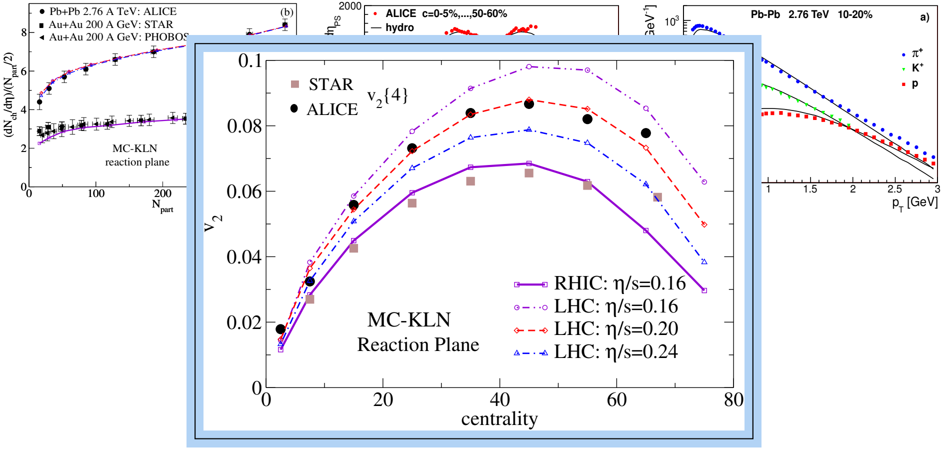




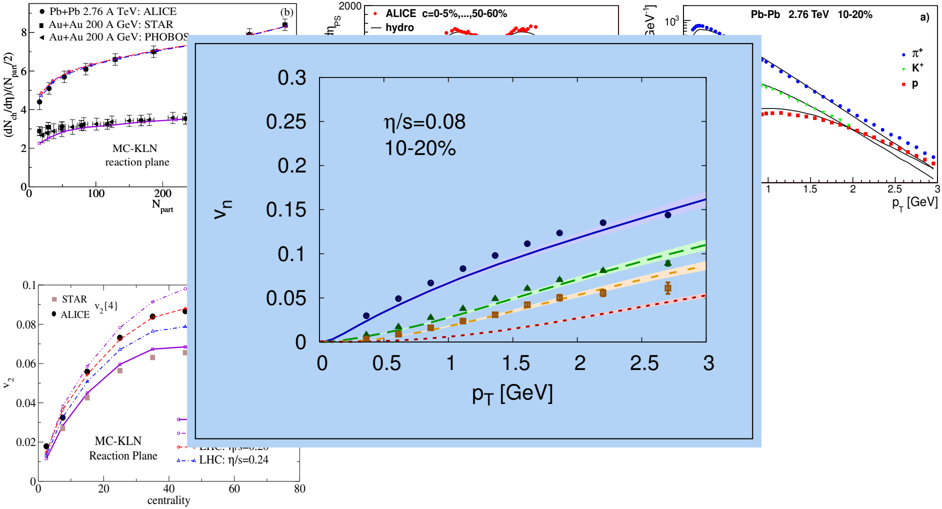
Evidence for hydrodynamical behavior



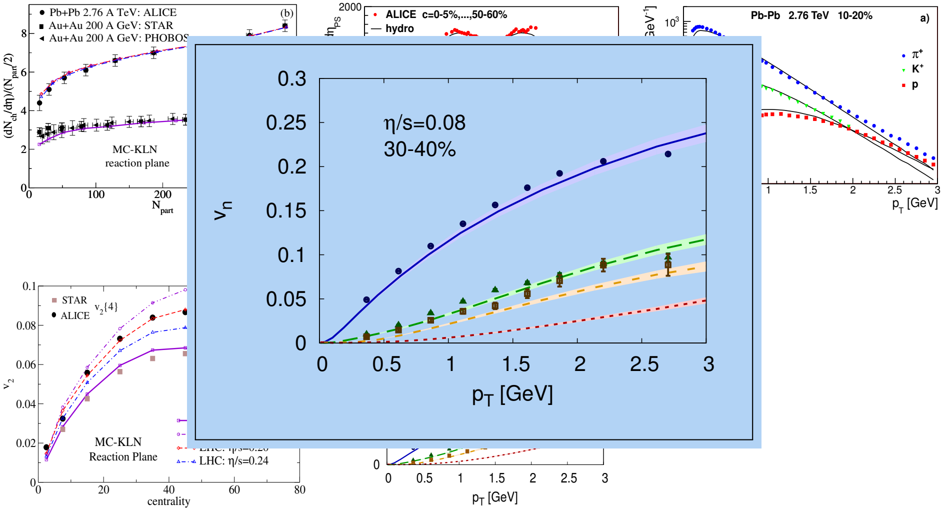
Evidence for hydrodynamical behavior



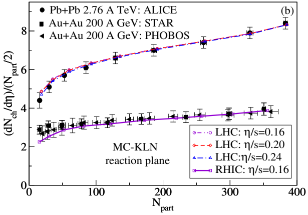
Evidence for hydrodynamical behavior

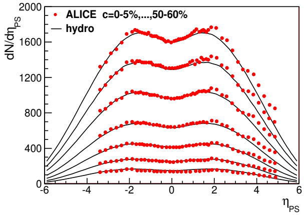


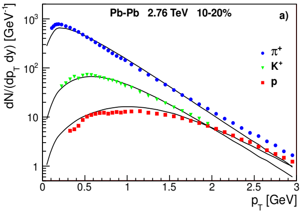
Evidence for hydrodynamical behavior

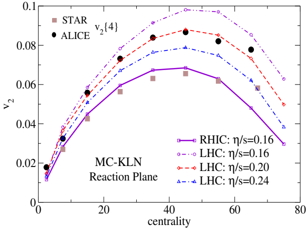


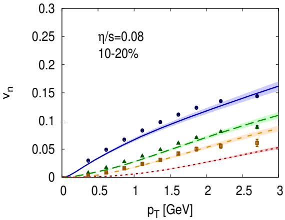
Evidence for hydrodynamical behavior

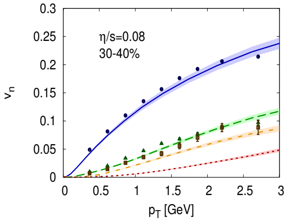




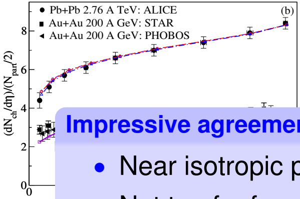


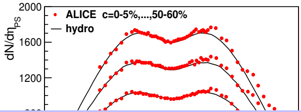


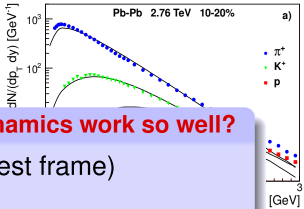




Evidence for hydrodynamical behavior



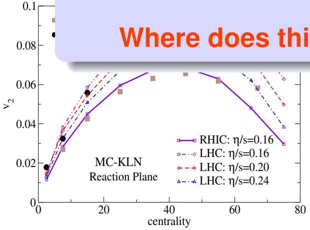


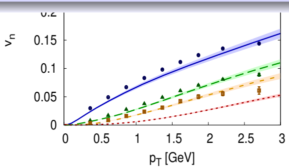


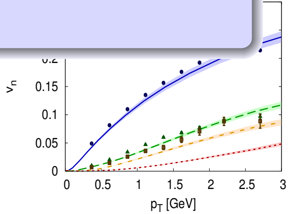
Impressive agreement, but: What makes hydrodynamics work so well?

- Near isotropic pressure tensor (in the local rest frame)
- Not too far from equilibrium
- Low viscosity

Where does this come from in pQCD...?







What is hydrodynamics?

- Hydrodynamics is a macroscopic description based on energy-momentum conservation :

$$\partial_\mu T^{\mu\nu} = 0$$

True in any quantum field theory

But does not provide a closed set of equations to fully describe the evolution of the system (4 equations, while $T^{\mu\nu}$ has 10 independent components)

What is hydrodynamics?

- Hydrodynamics is a macroscopic description based on energy-momentum conservation :

$$\partial_\mu T^{\mu\nu} = 0$$

  True in any quantum field theory

  But does not provide a closed set of equations to fully describe the evolution of the system (4 equations, while $T^{\mu\nu}$ has 10 independent components)

- Additional assumption : at macroscopic scales, $T^{\mu\nu}$ is expressible in terms of ϵ (energy density), P (pressure) and u^μ (fluid velocity field)
- For a frictionless fluid : $T^{\mu\nu}_{\text{ideal}} = (\epsilon + P)\, u^\mu u^\nu - P\, g^{\mu\nu}$

What is hydrodynamics?

- Hydrodynamics is a macroscopic description based on energy-momentum conservation :

$$\partial_\mu T^{\mu\nu} = 0$$

  True in any quantum field theory

  But does not provide a closed set of equations to fully describe the evolution of the system (4 equations, while $T^{\mu\nu}$ has 10 independent components)

- Additional assumption : at macroscopic scales, $T^{\mu\nu}$ is expressible in terms of ϵ (energy density), P (pressure) and u^μ (fluid velocity field)
- For a frictionless fluid : $T^{\mu\nu}_{\rm ideal} = (\epsilon + P)\, u^\mu u^\nu - P\, g^{\mu\nu}$
- In general : $T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} \oplus \eta\, \nabla^\mu v^\nu \oplus \zeta\, g^{\mu\nu} (\nabla_\rho v^\rho) \oplus \cdots$

What is hydrodynamics?

- Hydrodynamics is a macroscopic description based on energy-momentum conservation :

$$\partial_\mu T^{\mu\nu} = 0$$

True in any quantum field theory

But does not provide a closed set of equations to fully describe the evolution of the system (4 equations, while $T^{\mu\nu}$ has 10 independent components)

- Additional assumption : at macroscopic scales, $T^{\mu\nu}$ is expressible in terms of ϵ (energy density), P (pressure) and u^μ (fluid velocity field)
- For a frictionless fluid : $T^{\mu\nu}_{\rm ideal} = (\epsilon + P)\, u^\mu u^\nu - P\, g^{\mu\nu}$
- In general : $T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} \oplus \eta\, \nabla^\mu v^\nu \oplus \zeta\, g^{\mu\nu}(\nabla_\rho v^\rho) \oplus \cdots$
- Microscopic inputs : $\epsilon = f(P)$ (EoS), $\eta, \zeta, \cdots$ (transport coeff.)

Why is it hard to justify in QCD?

Just after the collision, $T^{\mu\nu}$ is far from ideal

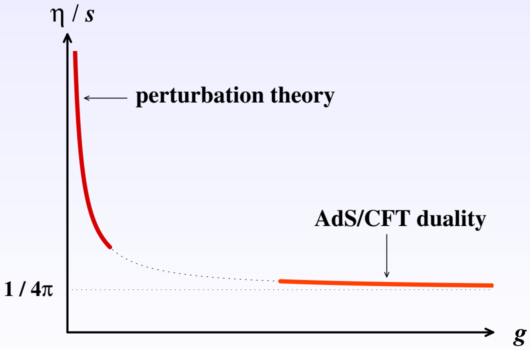
$$T^{\mu\nu}_{\substack{\textbf{QCD}\\ \text{rest frame}}} = \begin{pmatrix} \epsilon & & & \\ & \epsilon & & \\ & & \epsilon & \\ & & & -\epsilon \end{pmatrix} \qquad T^{\mu\nu}_{\substack{\textbf{ideal}\\ \text{rest frame}}} = \begin{pmatrix} \epsilon & & & \\ & \frac{\epsilon}{3} & & \\ & & \frac{\epsilon}{3} & \\ & & & \frac{\epsilon}{3} \end{pmatrix}$$

$\Rightarrow$ Very large viscous corrections

Why is it hard to justify in QCD?

Shear viscosity at weak coupling in QCD

$$\frac{\eta}{s} = \frac{5.12}{g^4 \ln\left(\frac{2.42}{g}\right)}$$



QM, Quantum chaos
Berry’s conjecture

Formulation of QM in the classical phase-space

- Quantum Mechanics introduces a natural smearing due to the uncertainty principle. To make the connection with classical mechanics, it is useful to use Moyal's formulation of QM in terms of classical variables

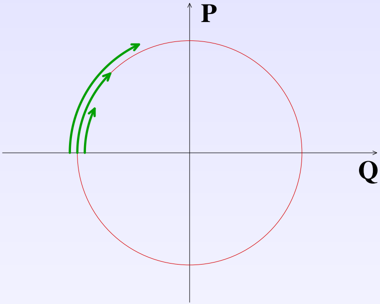
- Dual formulation of QM :

Density	$\widehat{\rho}$		$W(Q,P)$
Evolution	$\partial_t\widehat{\rho} + i[\widehat{H},\widehat{\rho}] = 0$	$\overset{\text{Weyl-Wigner}}{\underset{\text{trans.}}{\longleftrightarrow}}$	$\partial_t W + \{\{W, H\}\} = 0$
Initial condition	coherent state		Gaussian in Q, P

- Moyal bracket : $\{\{\cdot,\cdot\}\} = \underbrace{\{\cdot,\cdot\}}_{\text{Poisson bracket}} + \mathcal{O}(\hbar^2)$

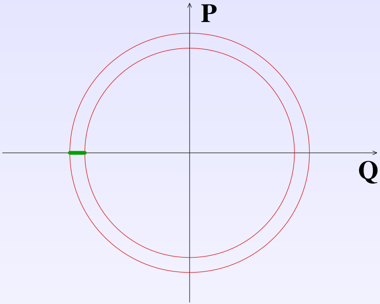
  The Moyal equation becomes the Liouville equation in the classical limit $\hbar \to 0$

Microcanonical equilibration of an anharmonic oscillator



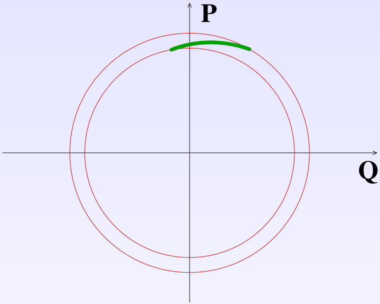
- The oscillation frequency depends on the initial condition

Microcanonical equilibration of an anharmonic oscillator



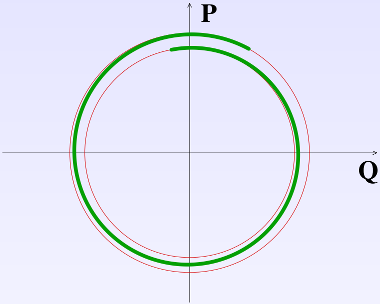
- The oscillation frequency depends on the initial condition
- Because of QM, the initial ensemble is a set of width $\sim \hbar$

Microcanonical equilibration of an anharmonic oscillator



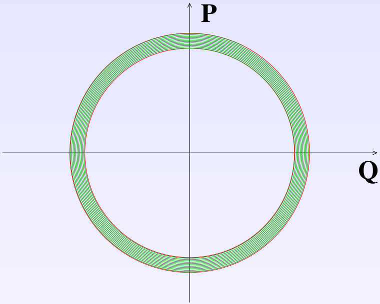
- The oscillation frequency depends on the initial condition
- Because of QM, the initial ensemble is a set of width $\sim\hbar$
- This ensemble of initial configurations spreads in time

Microcanonical equilibration of an anharmonic oscillator



- The oscillation frequency depends on the initial condition
- Because of QM, the initial ensemble is a set of width $\sim\hbar$
- This ensemble of initial configurations spreads in time

Microcanonical equilibration of an anharmonic oscillator



- The oscillation frequency depends on the initial condition
- Because of QM, the initial ensemble is a set of width $\sim\hbar$
- This ensemble of initial configurations spreads in time
- At large times, the ensemble fills densely all the region allowed by energy conservation $\Rightarrow$ microcanonical equilibrium

Quantum chaos

- Central issue : consider a Hamiltonian that leads to chaotic classical behavior; What happens when this system is quantized?

- Schrodinger's equation is linear :

$$i\partial_t \Psi = \widehat{H}\,\Psi$$

- Once we know the spectrum of the Hamiltonian $\{E_n, \Psi_n\}$, any wavefunction evolves as :

$$\Psi(t) = \sum_n c_n \, e^{iE_n t}\, \Psi_n$$

$E_n \in \mathbb{R} \;\Rightarrow\;$ nothing is unstable. Where is the chaos?

Berry's conjecture (1977)

- The complexity of the classical dynamics translates in the complexity of the high lying eigenfunctions

- Berry's conjecture : for most practical purposes, high lying eigenfunctions of classically chaotic systems behave as Gaussian random functions with 2-point correlations given by

$$\left\langle \Psi^*(\mathbf{X}-\frac{\mathbf{s}}{2})\Psi(\mathbf{X}+\frac{\mathbf{s}}{2})\right\rangle = \int d\mathbf{P}\; e^{i\mathbf{P}\cdot\mathbf{s}/\hbar}\;\delta\big[E - H(\mathbf{X},\mathbf{P})\big]$$

- Then, the Wigner distribution associated with the eigenfunction Ψ_{E} is

$$\begin{aligned} W(\mathbf{X},\mathbf{P}) &= \int d\mathbf{s}\; e^{-i\mathbf{P}\cdot\mathbf{s}/\hbar}\;\Psi^*_{E}(\mathbf{X}-\frac{\mathbf{s}}{2})\Psi_{E}(\mathbf{X}+\frac{\mathbf{s}}{2}) \\ &\sim \delta\big[E - H(\mathbf{X},\mathbf{P})\big] \end{aligned}$$

$\Rightarrow$ micro-canonical equilibrium for a single eigenstate

Eigenstate thermalization hypothesis (Srednicki, 1994)

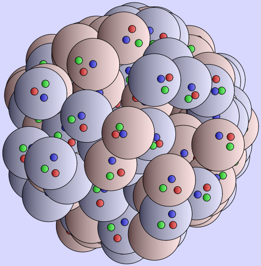
- If an energy eigenstate obeys Berry's conjecture, then a measurement performed on that state will lead to the Bose-Einstein (or Fermi-Dirac) distribution for the single particle distribution

- Generic states approach equilibrium via decoherence of their individual energy eigenstate components

CGC Description of Heavy Ion Collisions

What do we need to know about nuclei?

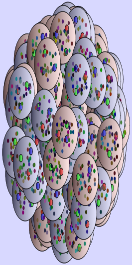
Nucleus at rest



- At low energy : valence quarks

What do we need to know about nuclei?

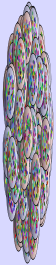
Slightly boosted nucleus



- At low energy : valence quarks
- At higher energy :
  - Lorenz contraction of longitudinal sizes
  - Time dilation ▷ slowing down of the internal dynamics
  - Gluons start becoming important

What do we need to know about nuclei?

High energy nucleus

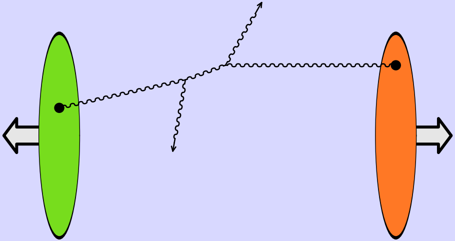


- At low energy : valence quarks
- At higher energy :
  - Lorenz contraction of longitudinal sizes
  - Time dilation ▷ slowing down of the internal dynamics
  - Gluons start becoming important
- At very high energy : gluons dominate

Multiple scatterings and gluon recombination

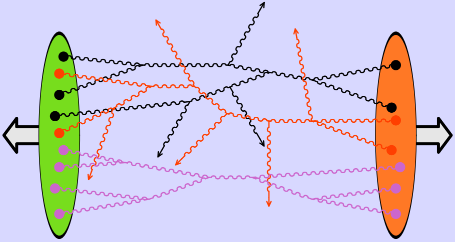
- Main difficulty: How to treat collisions involving a large number of partons?

Multiple scatterings and gluon recombination



- Dilute regime : one parton in each projectile interact
  - ▷ single parton distributions, standard perturbation theory

Multiple scatterings and gluon recombination



- Dense regime : multiparton processes become crucial

  ▷ gluon recombinations are important (saturation)

  ▷ multi-parton distributions

  ▷ alternative approach : treat the gluons in the projectiles as external currents

$$\mathcal{L} = -\frac{1}{4}F^2 + A \cdot (J_1 + J_2)$$

(gluons only, field A for $k^+ < \Lambda$, classical source J for $k^+ > \Lambda$)

Color Glass Condensate

CGC = effective theory of small x gluons

- The fast partons ($k^+ > \Lambda^+$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu +} \rho(x^-, \vec{x}_\perp) \qquad (0 < x^- < 1/\Lambda^+)$$

- The color sources ρ are random, and described by a probability distribution $W_{\Lambda^+}[\rho]$

- Slow partons ($k^+ < \Lambda^+$) cannot be considered static over the time-scales of the collision process
  ▷ must be treated as standard gauge fields
  ▷ eikonal coupling to the current J^μ : $A_\mu J^\mu$

Terminology

- Weakly coupled : $g \ll 1$

- Weakly interacting : $g\mathcal{A} \ll 1 \qquad g^2 f(\mathbf{p}) \ll 1$

  $(2 \to 2) \gg (2 \to 3), (3 \to 2), \cdots$

- Strongly interacting : $g\mathcal{A} \sim 1 \qquad g^2 f(\mathbf{p}) \sim 1$

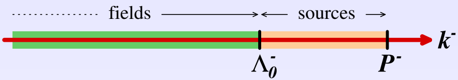
  $(2 \to 2) \sim (2 \to 3) \sim (3 \to 2) \sim \cdots$

  No well defined quasi-particles

CGC = weakly coupled, but strongly interacting effective theory

Power counting

- CGC effective theory with cutoff at the scale Λ_0 :

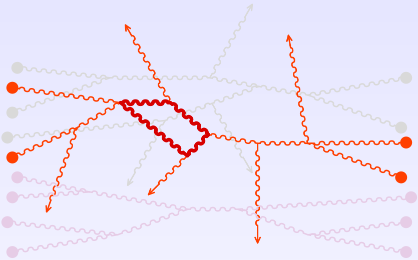


$$\mathcal{S} = \underbrace{-\frac{1}{4}\int F_{\mu\nu}F^{\mu\nu}}_{\mathcal{S}_{\rm YM}} + \int \underbrace{(J_1^\mu + J_2^\mu)}_{\text{fast partons}} A_\mu$$

- Expansion in g^2 in the saturated regime:

$$T^{\mu\nu} \sim \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

Power counting



In the saturated regime: $J^\mu \sim g^{-1}$

$$g^{-2}\; g^{\#\text{ of external legs}}\; g^{2\times(\#\text{ of loops})}$$

- No dependence on the number of sources J^μ
  - ▷ infinite number of graphs at each order

Leading Order in g^2 : tree diagrams

- The Leading Order is the sum of all the tree diagrams

  Observables can be expressed in terms of classical solutions of Yang-Mills equations :

$$\mathcal{D}_\mu \mathcal{F}^{\mu\nu} = J_1^\nu + J_2^\nu$$

- Boundary conditions for inclusive observables :

$$\lim_{x^0 \to -\infty} \mathcal{A}^\mu(x) = 0$$

Example : 00 component of the energy-momentum tensor

$$T^{00}_{\rm LO} = \frac{1}{2}\Big[\underbrace{\mathcal{E}^2 + \mathcal{B}^2}_{\text{class. fields}}\Big]$$

Next to Leading Order in g^2 : 1-loop diagrams

Getting the NLO from tree graphs...

$$\mathcal{O}_{\rm NLO} = \left[\frac{1}{2}\int_{\mathbf{u},\mathbf{v}} \boldsymbol{\Gamma}_2(\mathbf{u},\mathbf{v})\,\mathbb{T}_{\mathbf{u}}\mathbb{T}_{\mathbf{v}} + \int_{\mathbf{u}} \boldsymbol{\alpha}(\mathbf{u})\,\mathbb{T}_{\mathbf{u}}\right]\,\mathcal{O}_{\rm LO}$$

- $\mathbb{T}$ is the generator of the shifts of the initial value of the field :

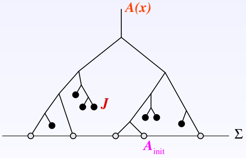
$$\mathbb{T}_{\mathbf{u}} \sim \frac{\partial}{\partial \mathcal{A}_{\rm init}}$$

$$\exp\left[\int_{\mathbf{u}} \boldsymbol{\alpha}_{\mathbf{u}}\,\mathbb{T}_{\mathbf{u}}\right]\;\mathcal{O}\big[\overbrace{\mathcal{A}_\tau(\underbrace{\mathcal{A}_{\rm init}}_{\text{init. value}})}^{\text{class. field at }\tau}\big] = \mathcal{O}\big[\mathcal{A}_\tau(\underbrace{\mathcal{A}_{\rm init}+\boldsymbol{\alpha}}_{\text{shifted init. value}})\big]$$

Shift operator $\mathbb{T}$ – Definition

Equations of motion for a field $\mathcal{A}$ and a small perturbation α

$$\begin{aligned} \square\mathcal{A} + V'(\mathcal{A}) &= J \\ [\square + V''(\mathcal{A})]\ \alpha &= 0 \end{aligned}$$



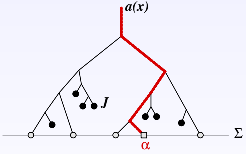
- Getting the perturbation by shifting the initial condition of $\mathcal{A}$ at one point :

$$\alpha(x) = \int_u \alpha_u\, \mathbb{T}_u\ \mathcal{A}(x)$$

Shift operator $\mathbb{T}$ – Definition

Equations of motion for a field $\mathcal{A}$ and a small perturbation α

$$\begin{aligned} \square\mathcal{A} + V'(\mathcal{A}) &= J \\ [\square + V''(\mathcal{A})]\ \alpha &= 0 \end{aligned}$$



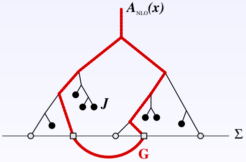
- Getting the perturbation by shifting the initial condition of $\mathcal{A}$ at one point :

$$\alpha(x) = \int_u \alpha_u\, \mathbb{T}_u\ \mathcal{A}(x)$$

Shift operator $\mathbb{T}$ – Definition

Equations of motion for a field $\mathcal{A}$ and a small perturbation α

$$\begin{aligned} \square\mathcal{A} + V'(\mathcal{A}) &= J \\ [\square + V''(\mathcal{A})]\ \alpha &= 0 \end{aligned}$$



- Getting the perturbation by shifting the initial condition of $\mathcal{A}$ at one point :

$$\alpha(x) = \int_u \alpha_u\, \mathbb{T}_u\ \mathcal{A}(x)$$

- A loop is obtained by shifting the initial condition of $\mathcal{A}$ at two points

Initial state logarithms

- In the CGC, upper cutoff on the loop momentum : $k^{\pm} < \Lambda$, to avoid double counting with the sources $J^{\nu}_{1,2}$
  $\triangleright$ logarithms of the cutoff

Central result for factorization at Leading Log

$$\frac{1}{2}\int_{u,v} \Gamma_2(u,v)\, \mathbb{T}_u \mathbb{T}_v + \int_u \alpha(u)\, \mathbb{T}_u =$$

$$= \log\left(\Lambda^+\right)\, \mathcal{H}_1 + \log\left(\Lambda^-\right)\, \mathcal{H}_2 + \text{terms w/o logs}$$

$\mathcal{H}_{1,2}$ = JIMWLK Hamiltonians of the two nuclei

- No mixing between the logs of the two nuclei
- Since the LO $\leftrightarrow$ NLO relationship is the same for all inclusive observables, these logs have a universal structure

Factorization of the logarithms

Inclusive observables at Leading Log accuracy

$$\langle \mathcal{O} \rangle_{\text{Leading Log}} = \int [D\rho_1 \, D\rho_2] \, W_1[\rho_1] \, W_2[\rho_2] \, \underbrace{\mathcal{O}_{\text{LO}}[\rho_1, \rho_2]}_{\text{fixed } \rho_{1,2}}$$

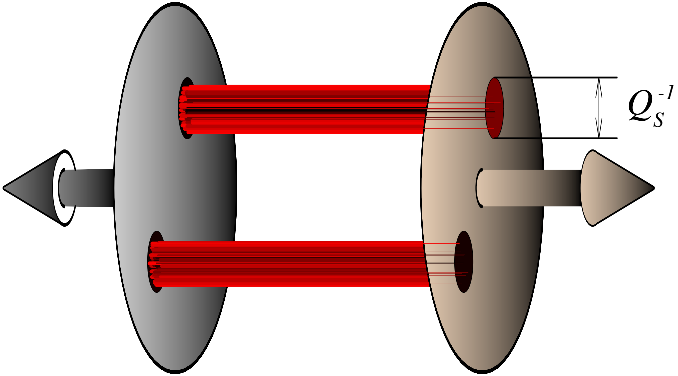
- Logs absorbed into the scale evolution of $W_{1,2}$

$$\Lambda \frac{\partial W}{\partial \Lambda} = \mathcal{H} \, W \qquad \text{(JIMWLK equation)}$$

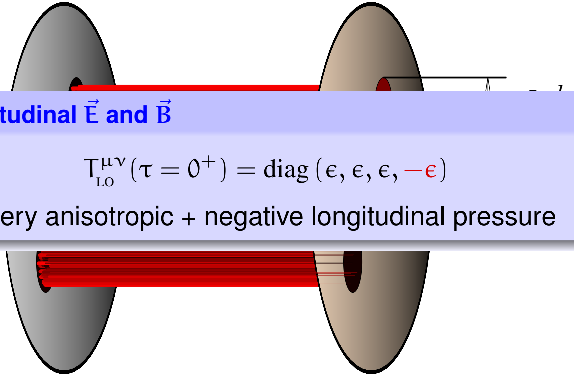
- Universality : the same W's for all inclusive observables

Isotropization in Heavy Ion Collisions

Energy momentum tensor of the initial classical field



Energy momentum tensor of the initial classical field

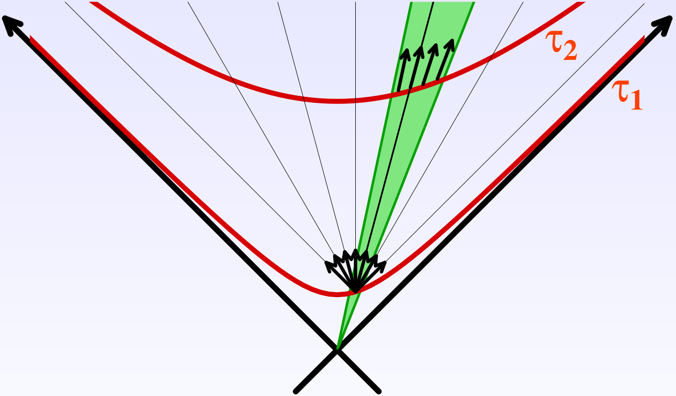


$T^{\mu\nu}$ for longitudinal $\vec{E}$ and $\vec{B}$

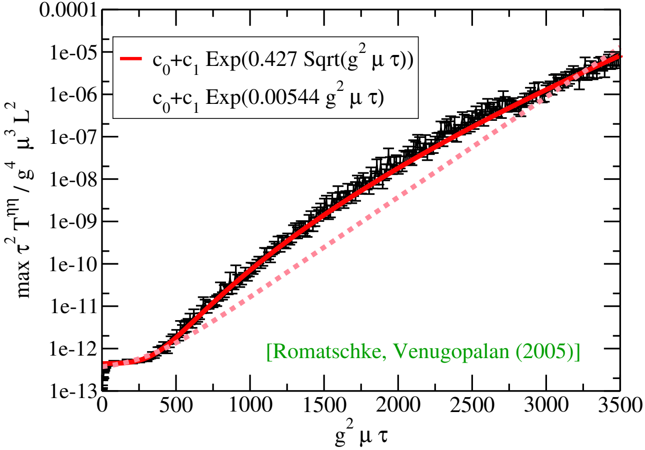
$$T^{\mu\nu}_{LO}(\tau = 0^+) = \mathrm{diag}\,(\epsilon, \epsilon, \epsilon, -\epsilon)$$

▷ very anisotropic + negative longitudinal pressure

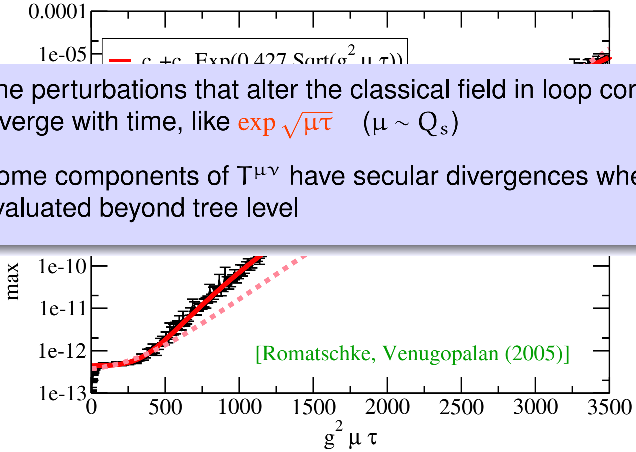
Competition between Expansion and Isotropization



Weibel instabilities for small perturbations



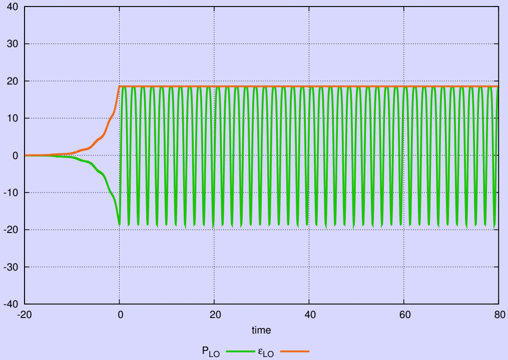
Weibel instabilities for small perturbations



- The perturbations that alter the classical field in loop corrections diverge with time, like $\exp\sqrt{\mu\tau}$ $\quad(\mu\sim Q_s)$
- Some components of $T^{\mu\nu}$ have secular divergences when evaluated beyond tree level

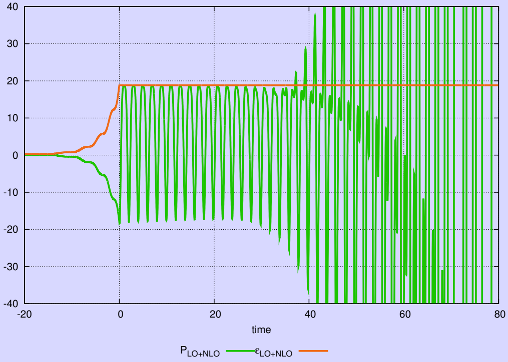
Example of pathologies in fixed order calculations (scalar theory)

LO



Example of pathologies in fixed order calculations (scalar theory)

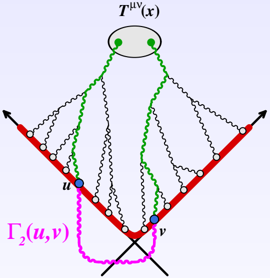
LO + NLO



- Small correction to the energy density (protected by energy conservation)
- Secular divergence in the pressure

Improved power counting and resummation

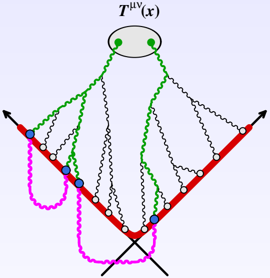
$$\text{Loop} \sim g^2 \qquad , \qquad \mathbb{T} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop : $\left(g e^{\sqrt{\mu\tau}}\right)^2$

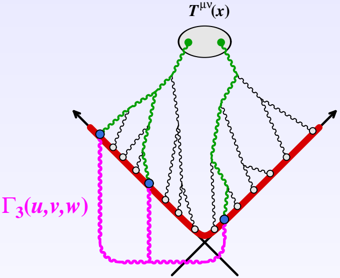
Improved power counting and resummation

$$\text{Loop} \sim g^2 \qquad , \qquad \mathbb{T} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop : $(ge^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops : $(ge^{\sqrt{\mu\tau}})^4$

$$\text{Loop} \sim g^2 \qquad , \qquad \mathbb{T} \sim e^{\sqrt{\mu\tau}}$$



- 1 loop : $(ge^{\sqrt{\mu\tau}})^2$
- 2 disconnected loops : $(ge^{\sqrt{\mu\tau}})^4$
- 2 entangled loops : $g(ge^{\sqrt{\mu\tau}})^3$ ▷ subleading

Leading terms

- All disconnected loops to all orders
  - ▷ exponentiation of the 1-loop result

Resummation of the leading secular terms

$$T^{\mu\nu}_{\text{resummed}} = \exp\left[\frac{1}{2}\int_{u,v} \Gamma_2(u,v)\,\mathbb{T}_u\mathbb{T}_v\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}]$$

$$= \underbrace{T^{\mu\nu}_{\text{LO}} + T^{\mu\nu}_{\text{NLO}}}_{\text{in full}} + \underbrace{T^{\mu\nu}_{\text{NNLO}} + \cdots}_{\text{partially}}$$

- The exponentiation of the 1-loop result collects all the terms with the worst time behavior

Resummation of the leading secular terms

$$T^{\mu\nu}_{\text{resummed}} = \exp\left[\frac{1}{2}\int_{u,v} \Gamma_2(u,v)\mathbb{T}_u\mathbb{T}_v\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}}]$$

$$= \int[Da]\,\exp\left[-\frac{1}{2}\int_{u,v} a(u)\Gamma_2^{-1}(u,v)a(v)\right] T^{\mu\nu}_{\text{LO}}[\mathcal{A}_{\text{init}} + a]$$

- The exponentiation of the 1-loop result collects all the terms with the worst time behavior
- Equivalent to Gaussian fluctuations of the initial field + classical time evolution
- At $Q_s\tau_0 \ll 1$: $\mathcal{A}_{\text{init}} \sim Q_s/g$, $a \sim Q_s$

$$e^{\frac{\alpha}{2}\partial_x^2}\, f(x) = \int_{-\infty}^{+\infty} dz\, \frac{e^{-z^2/2\alpha}}{\sqrt{2\pi\alpha}}\, f(x+z)$$

Note : Classical field + Fluctuations = Coherent state

- This Gaussian distribution of initial fields is the Wigner distribution of a **coherent state** $\left|\mathcal{A}\right\rangle$

  Coherent states are the "most classical quantum states"

  Their Wigner distribution has the minimal support permitted by the uncertainty principle ($\mathcal{O}(\hbar)$ for each mode)

- $\left|\mathcal{A}\right\rangle$ is not an eigenstate of the full Hamiltonian
  $\triangleright$ decoherence via interactions

What needs to be done?

Main steps

1. Determine the 2-point function $\Gamma_2(u, v)$ that defines the Gaussian fluctuations, for the initial time $Q_s\tau_0$ of interest

   Note : this is an initial value problem, whose outcome is uniquely determined by the state of the system at $x^0 = -\infty$, and depends on the history of the system from $x^0 = -\infty$ to $\tau = \tau_0$

   Problem solvable only if the fluctuations are weak, $a^\mu \ll Q_s/g$

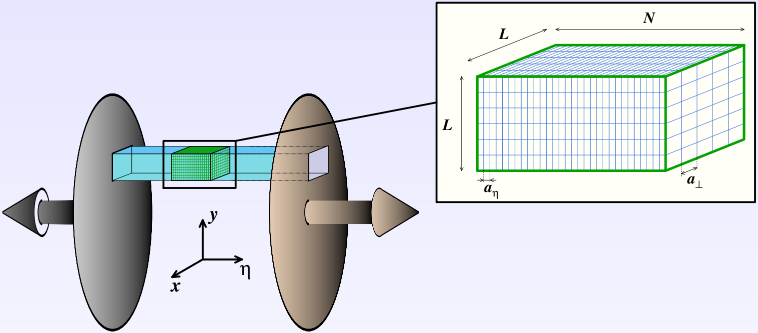
   $Q_s\tau_0 \ll 1$ necessary for the fluctuations to be Gaussian

2. Solve the classical Yang-Mills equations from τ_0 to τ_f

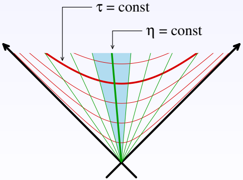
   Note : the problem as a whole is boost invariant, but individual field configurations are not $\implies$ 3+1 dimensions necessary

3. Do a Monte-Carlo sampling of the fluctuating initial conditions

Discretization of the expanding volume



- Comoving coordinates : $\tau, \eta, x_\perp$
- Only a sub-volume is simulated + periodic boundary conditions
- $L^2 \times N$ lattice

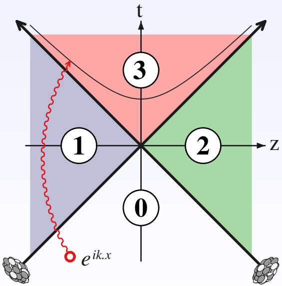


Gaussian spectrum of fluctuations

Expression of the variance (from 1-loop considerations)

$$\Gamma_2(u,v) = \int_{\text{modes }\mathbf{k}} a_{\mathbf{k}}(u)\,a^*_{\mathbf{k}}(v)$$

$$\Big[\mathcal{D}_\rho\mathcal{D}^\rho\delta^\nu_\mu - \mathcal{D}_\mu\mathcal{D}^\nu + ig\,\mathcal{F}_\mu{}^\nu\Big]a^\mu_{\mathbf{k}} = 0 \quad , \quad \lim_{x^0\to-\infty} a_{\mathbf{k}}(x) \sim e^{ik\cdot x}$$



0. $\mathcal{A}^\mu = 0$, trivial

1,2. $\mathcal{A}^\mu =$ pure gauge, analytical solution

3. $\mathcal{A}^\mu$ non-perturbative
$\Rightarrow$ expansion in $Q_s\tau$

- We need the fluctuations in Fock-Schwinger gauge $x^+a^- + x^-a^+ = 0$
- Delicate light-cone crossings, since $\mathcal{F}^{\mu\nu} = \infty$ there

Mode functions for given quantum numbers : $\nu, k_\perp, \lambda, c$

$$a^i = \beta^{+i} + \beta^{-i} \qquad a^\eta = \mathcal{D}^i\left(\frac{\beta^{+i}}{2+i\nu} - \frac{\beta^{-i}}{2-i\nu}\right)$$

$$e^i = -i\nu\left(\beta^{+i} - \beta^{-i}\right) \qquad e^\eta = -\mathcal{D}^i\left(\beta^{+i} - \beta^{-i}\right)$$

$$\beta^{+i} \equiv e^{\frac{\pi\nu}{2}}\Gamma(-i\nu)e^{i\nu\eta}\,\mathcal{U}_1^\dagger(\mathbf{x}_\perp)\int_{\mathbf{p}_\perp} e^{i\mathbf{p}_\perp\cdot\mathbf{x}_\perp}\,\widetilde{\mathcal{U}}_1(\mathbf{p}_\perp+\mathbf{k}_\perp)\left(\frac{p_\perp^2\tau}{2k_\perp}\right)^{i\nu}\left(\delta^{ij}-2\frac{p_\perp^i p_\perp^j}{p_\perp^2}\right)\epsilon_\lambda^j$$

$$\beta^{-i} \equiv e^{-\frac{\pi\nu}{2}}\Gamma(i\nu)e^{i\nu\eta}\,\mathcal{U}_2^\dagger(\mathbf{x}_\perp)\int_{\mathbf{p}_\perp} e^{i\mathbf{p}_\perp\cdot\mathbf{x}_\perp}\,\widetilde{\mathcal{U}}_2(\mathbf{p}_\perp+\mathbf{k}_\perp)\left(\frac{p_\perp^2\tau}{2k_\perp}\right)^{-i\nu}\left(\delta^{ij}-2\frac{p_\perp^i p_\perp^j}{p_\perp^2}\right)\epsilon_\lambda^j$$

- Linearized EOM and Gauss' law satisfied up to terms of order $(Q_s\tau)^2$
- Fock-Schwinger gauge condition $(a^\tau = e^\tau = 0)$
- Evolved from plane waves in the remote past

Computational cost

Initial Conditions

- Naive :

$$N \log(N) \times L^4 \log(L) \times N_{\text{confs}}$$

- Better algorithm :

$$N \log(N) \times L^4 \times \left(\log(L) + N_{\text{confs}}\right)$$

Time evolution

$$N \times L^2 \times N_{\text{confs}} \times N_{\text{time steps}}$$

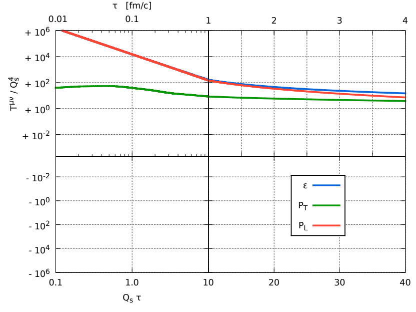
Useful statistics (at fixed volume)

$$\sqrt{N_{\text{confs}}} \sim \frac{g^2}{(a_\perp a_\eta)^2}$$

Ultraviolet subtractions

- Fixed spacing in η
  $\Longleftrightarrow \quad \Lambda_z \sim \tau^{-1}$

  Bare ϵ and P_L diverge as τ^{-2} when $\tau \to 0^+$



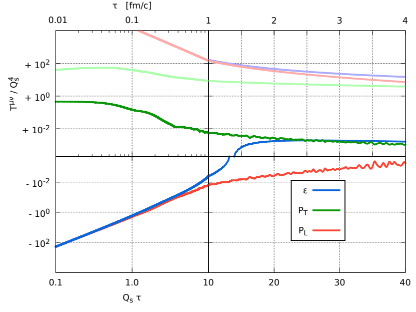
Ultraviolet subtractions

- Fixed spacing in η
  $\Longleftrightarrow \quad \Lambda_z \sim \tau^{-1}$

  Bare ϵ and P_L diverge as τ^{-2} when $\tau \to 0^+$

- Zero point energy $\sim \Lambda_\perp^2 \Lambda_z^2$:

  Subtracted by redoing the calculation with the sources turned off



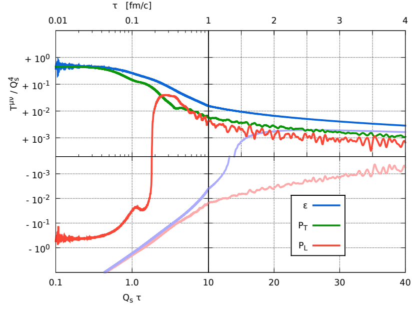
Ultraviolet subtractions

- Fixed spacing in η
  $\iff \quad \Lambda_z \sim \tau^{-1}$

  Bare ϵ and P_L diverge as τ^{-2} when $\tau \to 0^+$

- Zero point energy $\sim \Lambda_\perp^2 \Lambda_z^2$:

  Subtracted by redoing the calculation with the sources turned off



- Subleading divergences $\sim \Lambda_z^2$ in ϵ and P_L :

  Exist only at finite $\perp$ lattice spacing (not in the continuum)

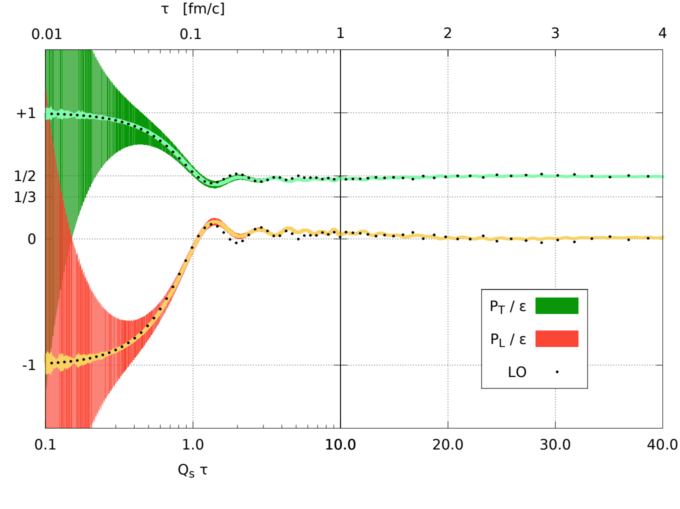
  Same counterterm in ϵ and P_L to preserve $T^\mu{}_\mu = 0$

  Must be of the form $A \times \tau^{-2}$ to preserve Bjorken's law

  At the moment, not calculated from first principles $\Rightarrow$ A fitted

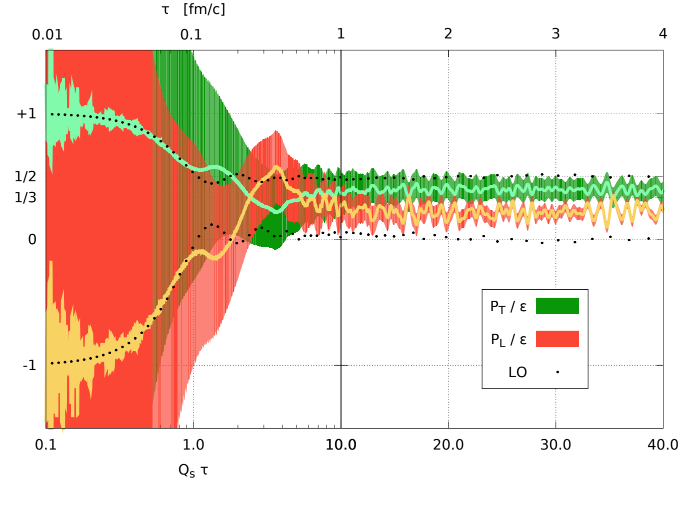
Time evolution of P_T/ϵ and P_L/ϵ ($64 \times 64 \times 128$ lattice)

$g = 0.1$ ($N_{\mathbf{confs}} = 200$)



Time evolution of P_T/ϵ and P_L/ϵ ($64 \times 64 \times 128$ lattice)

$g = 0.5$ ($N_{\mathbf{confs}} = 2000$)



Summary

Summary

- CGC calculation of the energy momentum tensor in a nucleus-nucleus collision, up to $Q_s\tau \lesssim 20$
- Method :
  - Classical statistical method
  - Initial Gaussian fluctuations : analytical, from a 1-loop calculation
  - Time evolution : numerical, 3+1d Yang-Mills equations on a lattice
- Accuracy :

  $\big\langle 0_{\rm in}\big|T^{\mu\nu}(\tau,x)\big|0_{\rm in}\big\rangle$ at LO + NLO + leading secular terms
- Results :
  - Sizable longitudinal pressure ($P_L/P_T \sim 60\%$ for $g = 0.5$)
  - Typical timescale : $Q_s\tau \sim 2-3$