Quark matter in neutron stars

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Outline

I Quarks at high density
Confined, quark-gluon plasma, color superconducting

II Color superconductivity
Color-flavor locking (CFL), and beyond

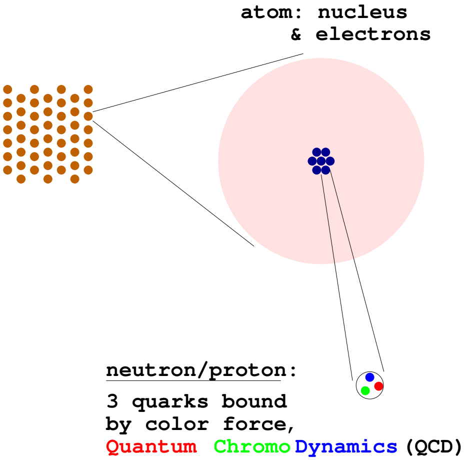
III Compact stars
Signatures of the presence of quark matter

IV Looking to the future

M. Alford, K. Rajagopal, T. Schäfer, A. Schmitt, arXiv:0709.4635 (RMP)
A. Schmitt, arXiv:1001.3294 (Springer Lecture Notes)

I. Quarks at high density

Quarks: Building blocks of matter



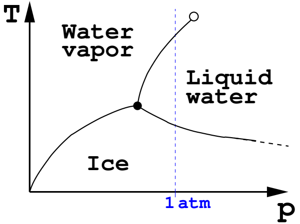
Quarks have color and flavor (“up” or “down”)

proton: uud, uud, uud

neutron: udd, udd, udd

Phase Transitions

When you heat up or compress matter, the atoms *reconfigure* themselves:
Phase transitions between solid, liquid, and gas.



At super-high temperatures or densities, when the nuclei are constantly bashed around or remorselessly crushed together, do *quarks* reconfigure themselves?

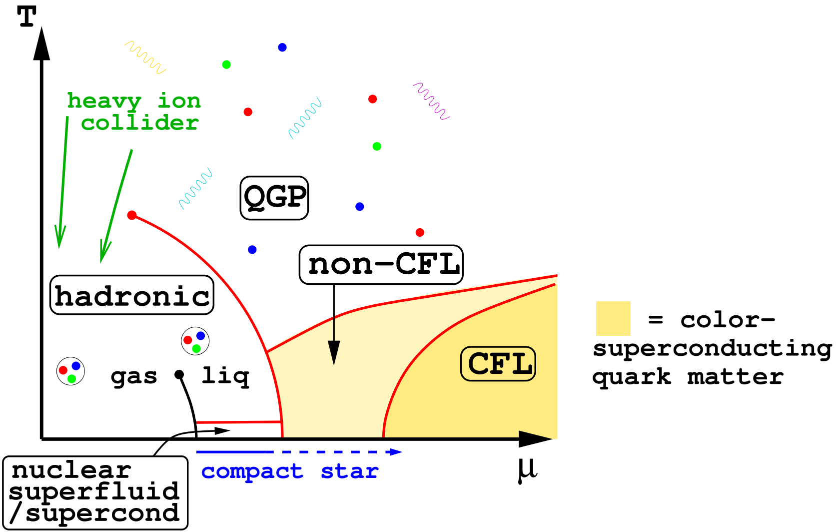
$$T \sim 150 \text{ MeV} \quad \sim 10^{12} \text{ K}$$
$$\rho \sim 300 \text{ MeV/fm}^3 \sim 10^{17} \text{ kg/m}^3$$

At such a density, a oil super-tanker is 1mm^3 in size.

Where might this occur?

- supernovas, neutron stars;
- Brookhaven (AGS, RHIC); CERN (SPS, LHC)

Conjectured QCD phase diagram



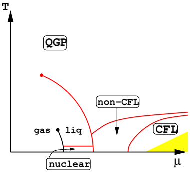
heavy ion collisions: chiral critical point and first-order line

compact stars: color superconducting quark matter core

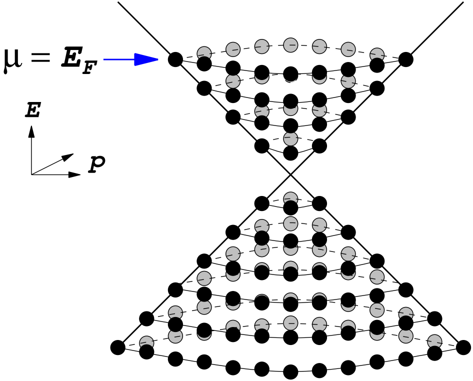
Signatures of quark matter in compact stars

Observable	←	Microphysical properties (and neutron star structure)	← Phases of dense matter
mass, radius		eqn of state	
spindown (spin freq, age)		bulk viscosity shear viscosity	
cooling (temp, age)		heat capacity neutrino emissivity thermal cond.	
glitches (superfluid, crystal)		shear modulus vortex pinning energy	

Color superconductivity



At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.



$$F = E - \mu N$$

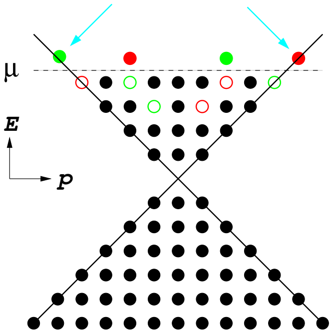
$$\frac{dF}{dN} = 0$$

But quarks have attractive QCD interactions.

Any attractive quark-quark interaction causes pairing instability of the Fermi surface: BCS mechanism of superconductivity.

BCS in quark matter: Ivanenko and Kurdgelaidze, Lett. Nuovo Cim. IIS1 13 (1969).

What is a condensate of Cooper pairs?



$$|\phi_0\rangle = \prod_{\mathbf{p}} \Big(\cos(\theta_{A\mathbf{p}}) + \sin(\theta_{A\mathbf{p}})\, a^\dagger(\mathbf{p}) a^\dagger(-\mathbf{p})\Big)$$
$$\Big(\cos(\theta_{B\mathbf{p}}) + \sin(\theta_{B\mathbf{p}})\, b^\dagger(\mathbf{p}) b^\dagger(-\mathbf{p})\Big) \quad \times \quad |\text{Fermi sea}\rangle$$

$|\phi_0\rangle$, not $|\text{Fermi sea}\rangle$, is the ground state.

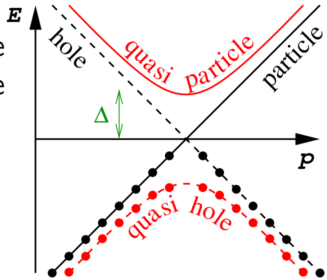
Physical consequences of Cooper pairing

Changes low energy excitations, affecting *transport properties*.

- Spontaneous breaking of global symmetries ⇒ Goldstone bosons, massless degrees of freedom that dominate low energy behavior.
  E.g.: Superfluidity
- Spontaneous breaking of local (gauged) symmetries: massive gauge bosons, exclusion of magnetic fields (Meissner effect).
  E.g.: Superconductivity
- Gap in fermion spectrum.

  Adding a fermion near the Fermi surface now costs energy because it disrupts the condensate.

$$a_p^\dagger(\cos\theta + \sin\theta\, a_p^\dagger a_{-p}^\dagger) = \cos\theta\, a_p^\dagger$$

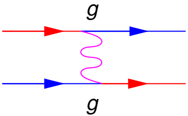


Interactions between Quarks

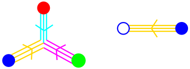
Dominant interaction between quarks is the strong interaction, mediated by exchange of gluons that couple to "color" charge (QCD).

Properties of QCD

- Short distances, $r \ll 1$ fm, asymptotically free: gauge coupling $g \ll 1$, single gluon exchange dominates, the theory is analytically tractable.



- Long distances $r > 1$ fm, QCD confines: color electric fields form flux tubes, only color-neutral states, baryons and mesons, exist.



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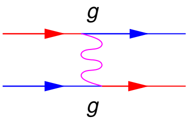

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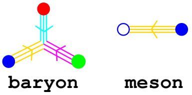
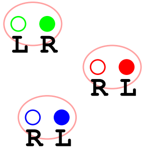


- At low temperature ($T \lesssim 170$ MeV), Chiral (left-right) symmetry is broken: color force can't turn a LH quark to RH, but our vacuum is full of $\bar{q}_L q_R$ pairs



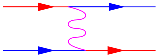
Handling QCD at high density

Lattice: "Sign problem"—negative probabilities

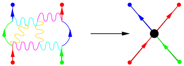
SUSY: Statistics crucial to quark Fermi surface

large N: Quarkyonic phase?

pert: Applicable far beyond nuclear density.
Neglects confinement and instantons.



NJL: Model, applicable at low density.
Follows from instanton liquid model.



EFT: Effective field theory for lightest degrees of freedom.
"Parameterization of our ignorance": assume a phase, guess coefficients of interaction terms (or match to pert theory), obtain phenomenology.

Color superconducting phases

Attractive QCD interaction $\Rightarrow$ Cooper pairing of quarks.

We expect pairing between *different flavors*.

Quark Cooper pair: $\langle q^\alpha_{ia} q^\beta_{jb} \rangle$

color $\alpha, \beta = r, g, b$
flavor $i, j = u, d, s$
spin $a, b = \uparrow, \downarrow$

Each possible BCS pairing pattern P is an 18×18 color-flavor-spin matrix

$$\langle q^\alpha_{ia} q^\beta_{jb} \rangle_{1PI} = \Delta_P \, P^{\alpha\beta}_{ij\,ab}$$

The attractive channel is:

space symmetric [s-wave pairing]
color antisymmetric [most attractive]
spin antisymmetric [isotropic]
$\Rightarrow$ flavor antisymmetric

Initially we will assume the most symmetric case, where all three flavors are massless.

High-density QCD calculations

- Guess a color-flavor-spin pairing pattern P
- to obtain gap Δ_P, minimize free energy Ω with respect to Δ_P (imposing color and electric neutrality)

$$\frac{\partial \Omega}{\partial \Delta_P} = 0 \qquad \frac{\partial \Omega}{\partial \mu_i} = 0$$

The pattern with the lowest $\Omega(\Delta_P)$ wins!

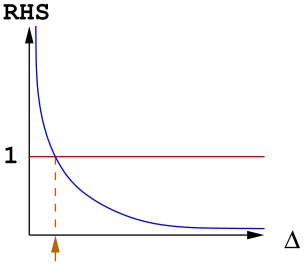
1. Weak-coupling methods. First-principles calculations direct from QCD Lagrangian, valid in the asymptotic regime, currently $\mu \gtrsim 10^6$ MeV.
2. Nambu–Jona-Lasinio models, ie quarks with four-fermion coupling based on instanton vertex, single gluon exchange, etc. This is a semi-quantitative guide to physics in the compact star regime $\mu \sim 400$ MeV, not a systematic approximation to QCD.

NJL gives $\Delta \sim 10-100$ MeV at $\mu \sim 400$ MeV.

Gap equation in a simple NJL model

Minimize free energy wrt Δ:

$$1 = \frac{8K}{\pi^2}\int_0^{\Lambda} p^2 dp \left\{ \frac{1}{\sqrt{\Delta^2 + (p-\mu)^2}} \right\}$$



Note BCS divergence as $\Delta \to 0$: there is *always* a solution, for any interaction strength K and chemical potential μ.
Roughly,

$$1 \sim K\mu^2 \ln(\Lambda/\Delta)$$
$$\Rightarrow \Delta \sim \Lambda \exp\left(-\frac{1}{K\mu^2}\right)$$

Superconducting gap is non-perturbative.

Color supercond. in 3 flavor quark matter

Color-flavor locking (CFL)

Equal number of colors and flavors gives a special pairing pattern
(Alford, Rajagopal, Wilczek, `hep-ph/9804403`)

$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta n} \epsilon_{ijn}$$

color α, β flavor i, j	This is invariant under equal and opposite rotations of color and (vector) flavor

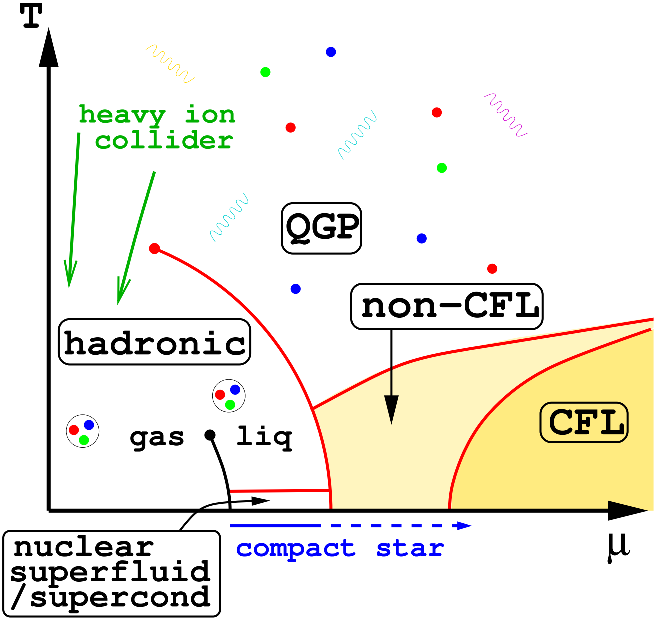
$$SU(3)_{\text{color}} \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset\, U(1)_Q} \times U(1)_B \to \underbrace{SU(3)_{C+L+R}}_{\supset\, U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

- Breaks chiral symmetry, but *not* by a $\langle \bar{q} q \rangle$ condensate.
- There need be no phase transition between the low and high density phases: ("quark-hadron continuity")
- Unbroken "rotated" electromagnetism, $\tilde{Q}$, photon-gluon mixture.

Color-flavor-locked ("CFL") quark pairing

$\tilde{Q}$	0	0	0	-1	$+1$	-1	$+1$	0	0
	u	d	s	d	u	s	u	s	d
u		Δ	Δ						
d	Δ		Δ						
s	Δ	Δ							
d					$-\Delta$				
u				$-\Delta$					
s							$-\Delta$		
u						$-\Delta$			
s									$-\Delta$
d								$-\Delta$	

Conjectured QCD phase diagram



III. Quark matter in compact stars

Where in the universe is color-superconducting quark matter most likely to exist? In compact stars.

A quick history of a compact star.

A star of mass $M \gtrsim 10 M_\odot$ burns Hydrogen by fusion, ending up with an Iron core. Core grows to Chandrasekhar mass, collapses $\Rightarrow$ supernova. Remnant is a compact star:

mass	radius	density	initial temp
$\sim 1.4 M_\odot$	$\mathcal{O}(10 \text{ km})$	$\gtrsim \rho_{\text{nuclear}}$	~ 30 MeV

The star cools by neutrino emission for the first million years.

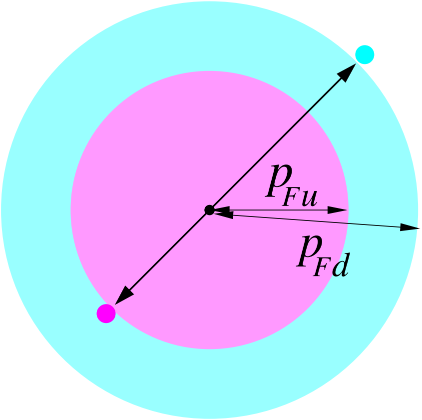
The real world: M_s and neutrality

In the real world there are three factors that combine to oppose pairing between different flavors.

1. **Strange quark mass** is not infinite nor zero, but intermediate. It depends on density, and ranges between about 500 MeV in the vacuum and about 100 MeV at high density.
2. **Neutrality requirement.** Bulk quark matter must be neutral with respect to all gauge charges: color and electromagnetism.
3. **Weak interaction equilibration.** In a compact star there is time for weak interactions to proceed: neutrinos escape and flavor is not conserved.

These factors favor **different Fermi momenta for different flavors** which *obstructs* pairing between different flavors.

Mismatched Fermi surfaces oppose Cooper pairing



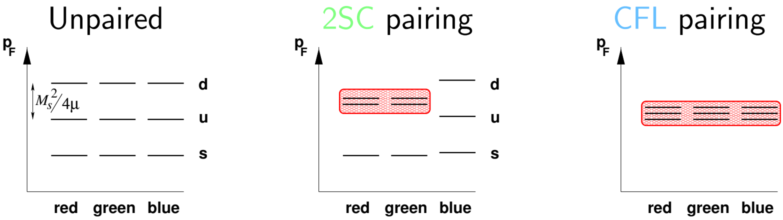
u and d quarks near their Fermi surfaces cannot have equal and opposite momenta.

$\langle u(k) d(-k) \rangle$ condensate is energetically penalized.

The strange quark mass is the cause of the mismatch:

$$p_{Fd} - p_{Fu} \approx p_{Fu} - p_{Fs} \approx \frac{M_s^2}{4\mu}$$

Cooper pairing vs. the strange quark mass



CFL: Color-flavor-locked phase, favored at the highest densities.

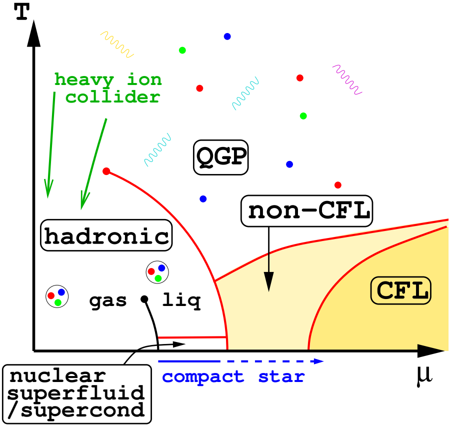
$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta N} \epsilon_{ijN}$$

2SC: Two-flavor pairing phase. May occur at intermediate densities.

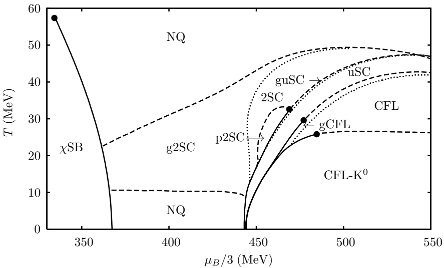
$$\langle q_i^\alpha q_j^\beta \rangle \sim \epsilon^{\alpha\beta 3} \epsilon_{ij3} \sim (rg - gr)(ud - du)$$

or: CFL with kaon condensation (CFL-K^0),
crystalline phase (LOFF), p-wave "meson" condensates,
single-flavor pairing (color-spin locking, ∼liq ^{3}He-B).

Phases of quark matter, again



NJL model, uniform phases only

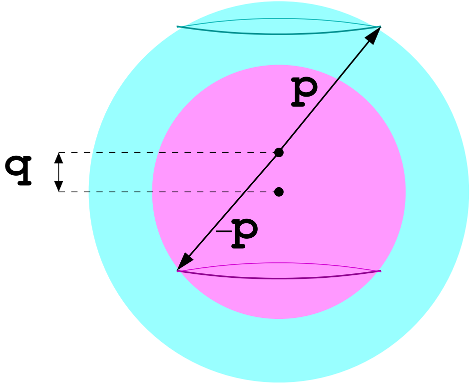


Warringa, hep-ph/0606063

But there are also non-uniform phases, such as the crystalline ("LOFF"/"FFLO") phase. (Alford, Bowers, Rajagopal, hep-ph/0008208)

Crystalline (LOFF) superconductivity

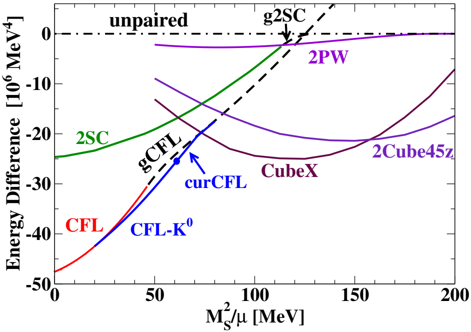
When the Fermi momenta are such that one flavor of quark is just barely excluded from pairing with another, it may be favorable to make pairs with a net momentum, so each flavor can be close to its Fermi surface.



Every quark pair in the condensate has the same nonzero total momentum $2\mathbf{q}$ (single plane wave LOFF).

Free energy comparison of phases

Assuming $\Delta_{\rm CFL} = 25$ MeV.



CFL-K^0	K^0 condensate
curCFL	K^0 cond current
2PW	LOFF, 2-plane-wave
CubeX	LOFF crystal, G-L approx
2Cube45z	LOFF crystal, G-L approx

(Alford, Rajagopal, Schäfer, Schmitt, `arXiv:0709.4635`)

Curves for CubeX and 2Cube45z use G-L approx far from its area of validity: favored phase at $M_s^2 \sim 4\mu\Delta$ remains uncertain.

Signatures of quark matter in compact stars

Observable ← Microphysical properties (and neutron star structure) ← Phases of dense matter

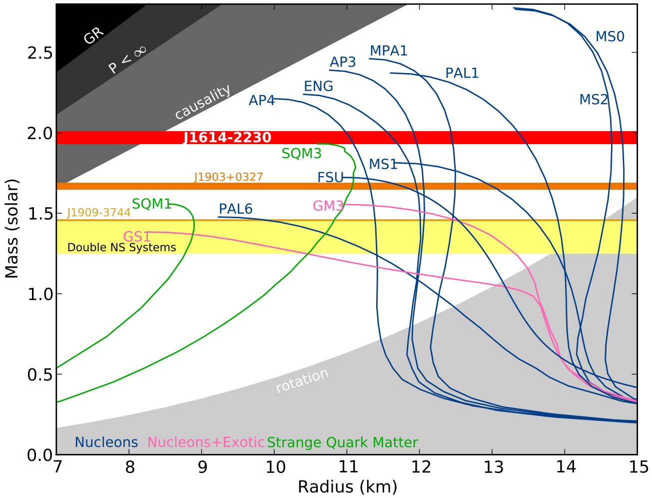
	Property	Nuclear phase	Quark phase
mass, radius	eqn of state	known up to n_{sat}	unknown, can be parameterized

Signatures of quark matter in compact stars

Observable ← Microphysical properties (and neutron star structure) ← Phases of dense matter

	Property	Nuclear phase	Quark phase
mass, radius	eqn of state	known up to n_{sat}	unknown, can be parameterized
spindown (spin freq, age)	bulk viscosity shear viscosity	Depends on phase:	Depends on phase:
		$n\,p\,e$	unpaired
		$n\,p\,e,\mu$	CFL
cooling (temp, age)	heat capacity neutrino emissivity thermal cond.	$n\,p\,e,\Lambda,\Sigma^-$	CFL-K^0
		n superfluid	2SC
		p supercond	CSL
glitches (superfluid, crystal)	shear modulus vortex pinning energy	π condensate	LOFF
		K condensate	1SC
			...

Discovery of a $2M_\odot$ mass neutron star



$M = 1.97 \pm 0.04\, M_\odot$

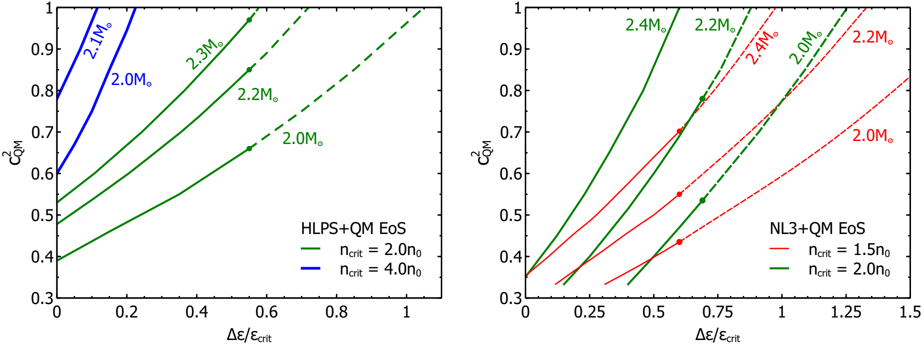
Demorest et al, Nature 467, 1081 (2010).

Can quark matter be the favored phase at high density?

Constraints on the quark matter EoS

Generic ansatz: $\varepsilon(p) = \varepsilon_{\mathrm{crit}} + \Delta\varepsilon + c_{\mathrm{QM}}^{-2}(p - p_{\mathrm{crit}})$

QM + Soft Nuclear Matter | QM + Hard Nuclear Matter

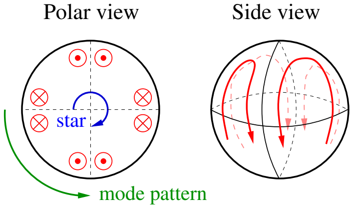


Alford, Han, Prakash, arXiv:1302.4732

1. Observations can constrain QM EoS but not rule out generic QM
2. Constraints depend on NM EoS up to transition density

r-modes: gravitational spin-down of compact stars

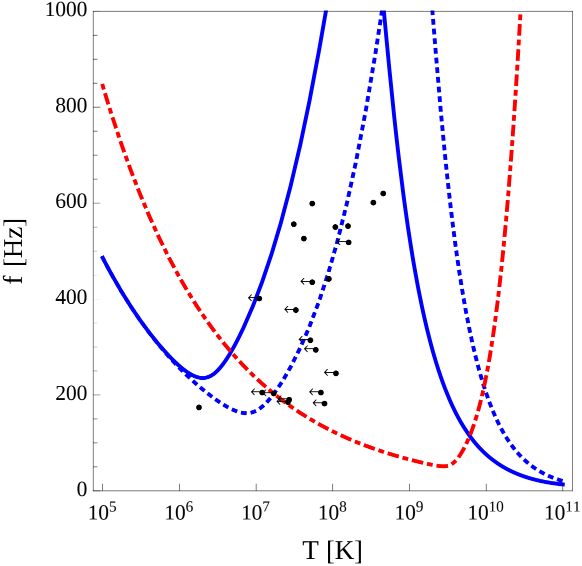
An r-mode is a quadrupole flow that emits gravitational radiation. It becomes unstable (i.e. arises spontaneously) when a star spins fast enough, and if the shear and bulk viscosity are low enough.



Andersson gr-qc/9706075

Friedman and Morsink gr-qc/9706073

Constraints from r-modes: old stars (1)



Regions above curves are "forbidden" because viscosity is too low to hold back the *r*-modes.

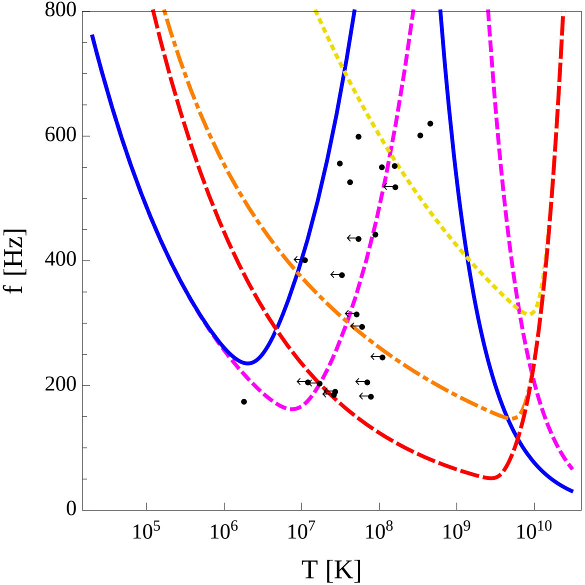
Only viscous damping included.

Data for accreting pulsars in binary systems (LMXBs) vs instability curves for nuclear and hybrid stars.

(Schwenzer, `arXiv:1212.5242`; Haskell, Degenaar, Ho, `arXiv:1201.2101`)

Need more than nuclear matter viscous damping

Constraints from r-modes: old stars (2)



Nuclear, viscous damping only

Nuclear with some core-crust friction

Nuclear with maximum core-crust friction

Free quarks

Quarks with non-Fermi corrections

(Schwenzer, unpublished)

Need something beyond the simple nuclear matter model

IV. Looking to the future

- Neutron-star phenomenology of color superconducting quark matter:
  - Are there any other r-mode damping mechanisms?
  - neutrino emissivity and cooling
  - structure: nuclear-quark interface (gravitational waves?)
  - color supercond. crystalline phase (glitches) (gravitational waves?)
  - CFL: vortices but no flux tubes; stability of vortices...
- More general questions:
  - instability of gapless phases; better treatment of LOFF
  - role of large magnetic fields
  - better weak-coupling calculations
  - better models of quark matter: Functional RG, Schwinger-Dyson
  - solve the sign problem and do lattice QCD at high density.