Hawking radiation in non-equilibrium SYM plasmas

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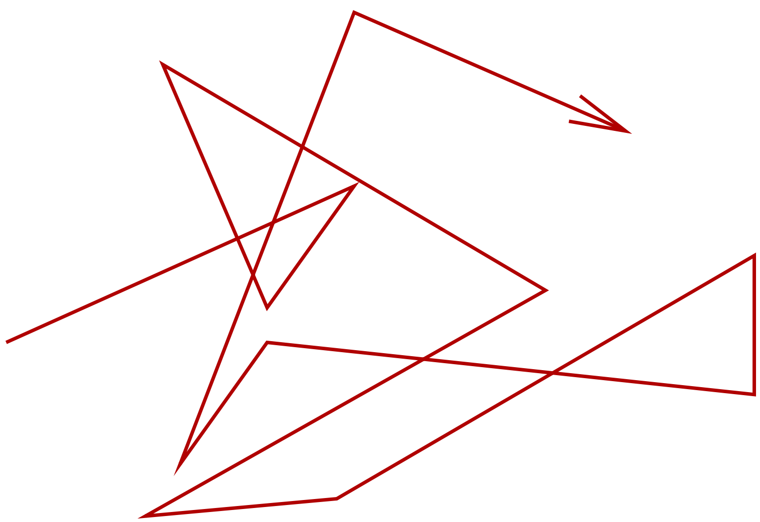
- Heavy quarks: Jorge Casalderrey-Solana, DT; hep-th/0701123
- Dam T. Son, DT; JHEP. arXiv:0901.2338
- Simon Caron-Huot, DT, Paul Chesler; PRD, arXiv:1102.1073
- Paul Chesler and DT; arXiv:1112.6196
- Paul Chesler and DT; arXiv:1211.0343

Brownian Motion and Equilibrium

$$M\frac{d^2\mathbf{x}}{dt^2} = \underbrace{-\eta}_{\text{Drag}}\,\dot{\mathbf{x}} + \underbrace{\xi}_{\text{Noise}}$$



"Artist's" conception of Brownian Motion

1. Equilibrium is a state constant fluctuations

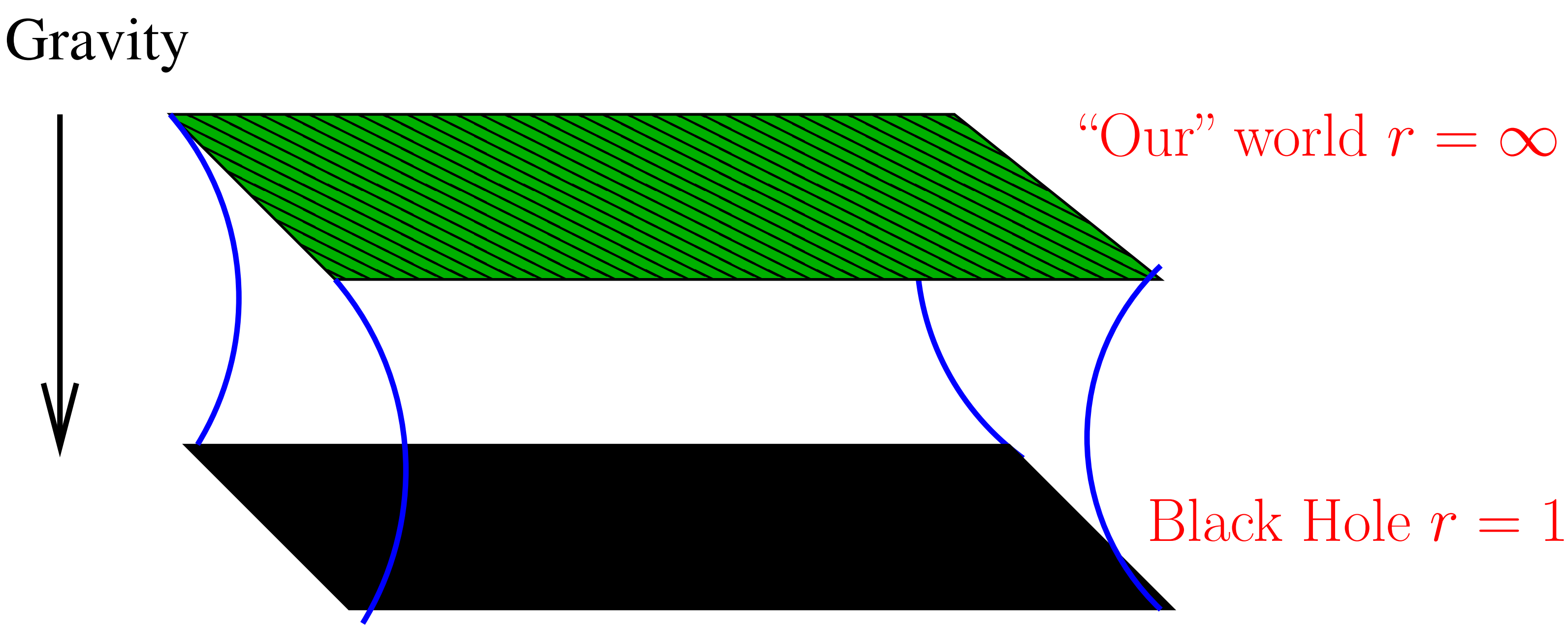
2. Equilibrium is a perpetual competition between drag and noise

$\langle \xi(t)\xi(t')\rangle = 2T\eta\,\delta(t-t')$ to reach equilibrium $P(\mathbf{p}) \propto e^{-\frac{\mathbf{p}^2}{2MT}}$

AdS/CFT

- Classical solutions in curved spacetime = CFT for nonzero temperature

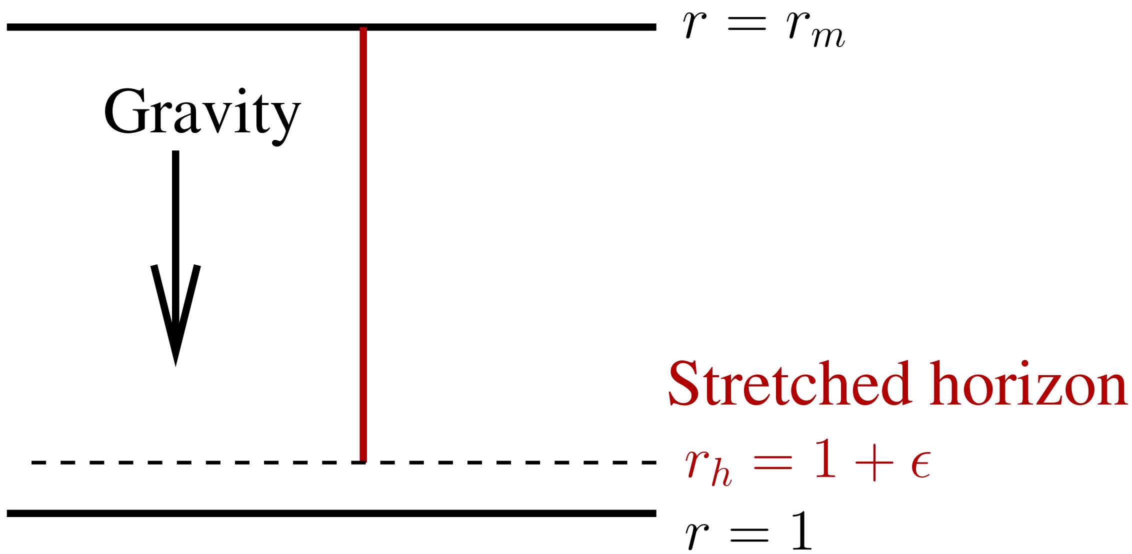
$$\mathrm{d}s^2 = (\pi T)^2 r^2 \left[-f(r)\mathrm{d}t^2 + \mathrm{d}x^2\right] + \frac{\mathrm{d}r^2}{r^2 f(r)} \qquad f(r) = 1 - \frac{1}{r^4}$$



How can a static metric be dual to equilibrium=constant fluctuations ?

A heavy quark in AdS/CFT

- Solve classical string (Nambu-Goto) EOM and find:



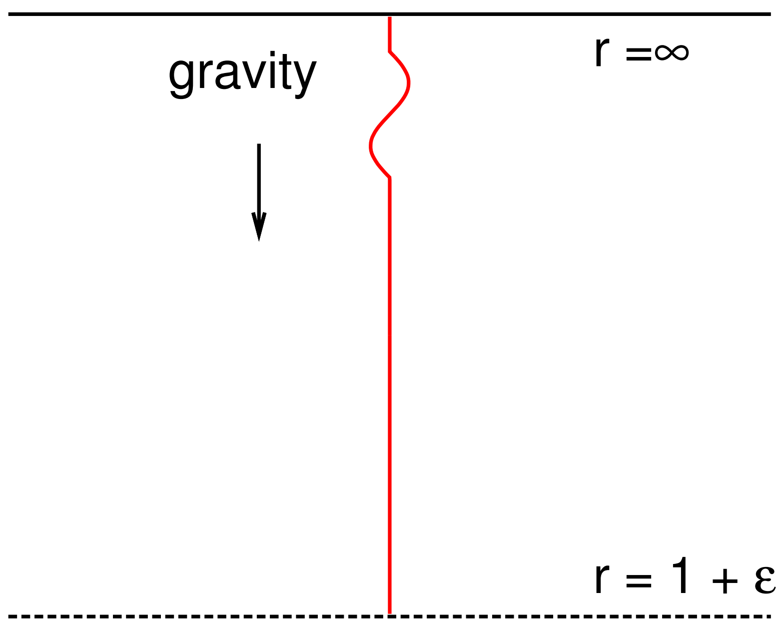
Not the dual of an equilibrated quark!

Dissipation in classical black hole dynamics

Herzog et al; DT J. Casalderrey-Solana; Gubser

$$M\frac{d^2x^o}{dt^2} = \underbrace{-\eta}_{\text{Drag}}\,\dot{x}^o$$

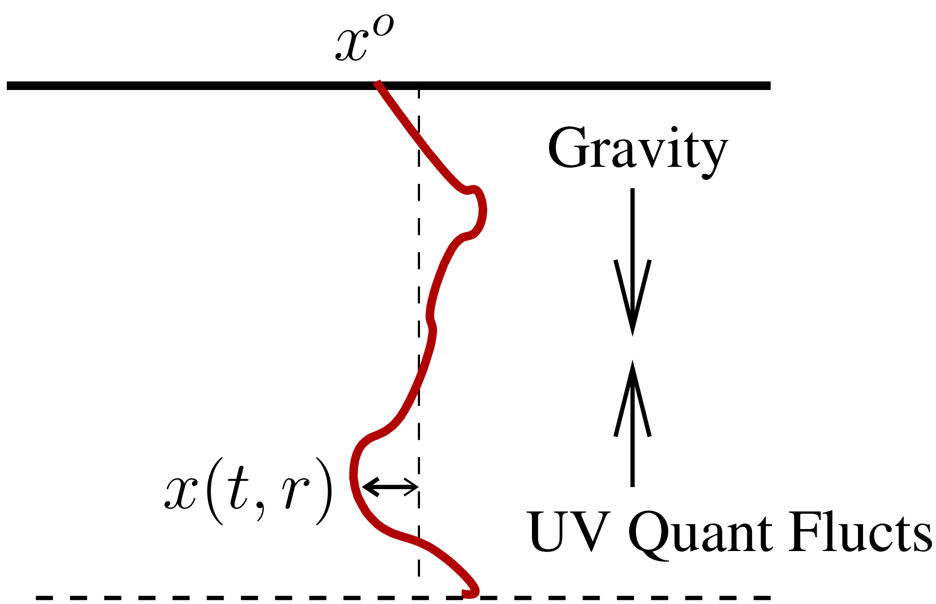
$$\underbrace{\eta = \frac{\sqrt{\lambda}}{2\pi}g_{xx}(r_h) = \frac{\sqrt{\lambda}}{2\pi}(\pi T)^2}_{\text{Coupling of string to near horizon metric}}$$



Classical dissipation determines drag

Detailed Balance and Hawking Radiation:

$$M\frac{d^2x^o}{dt^2} = \underbrace{-\eta}_{\text{Drag}}\,\dot{x}^o + \underbrace{\xi}_{\text{Noise}}$$



Evolves to Classical

Prob. Dist :

$$P[x,\pi_x] \propto e^{-\beta H[x,\pi_x]}$$

Classical Dissipation Balanced by Hawking Radiation. Find in equilibrium:

$$\langle \xi(t)\xi(t')\rangle = 2T\eta\delta(t-t')$$

How to generalize to non-equilrium?

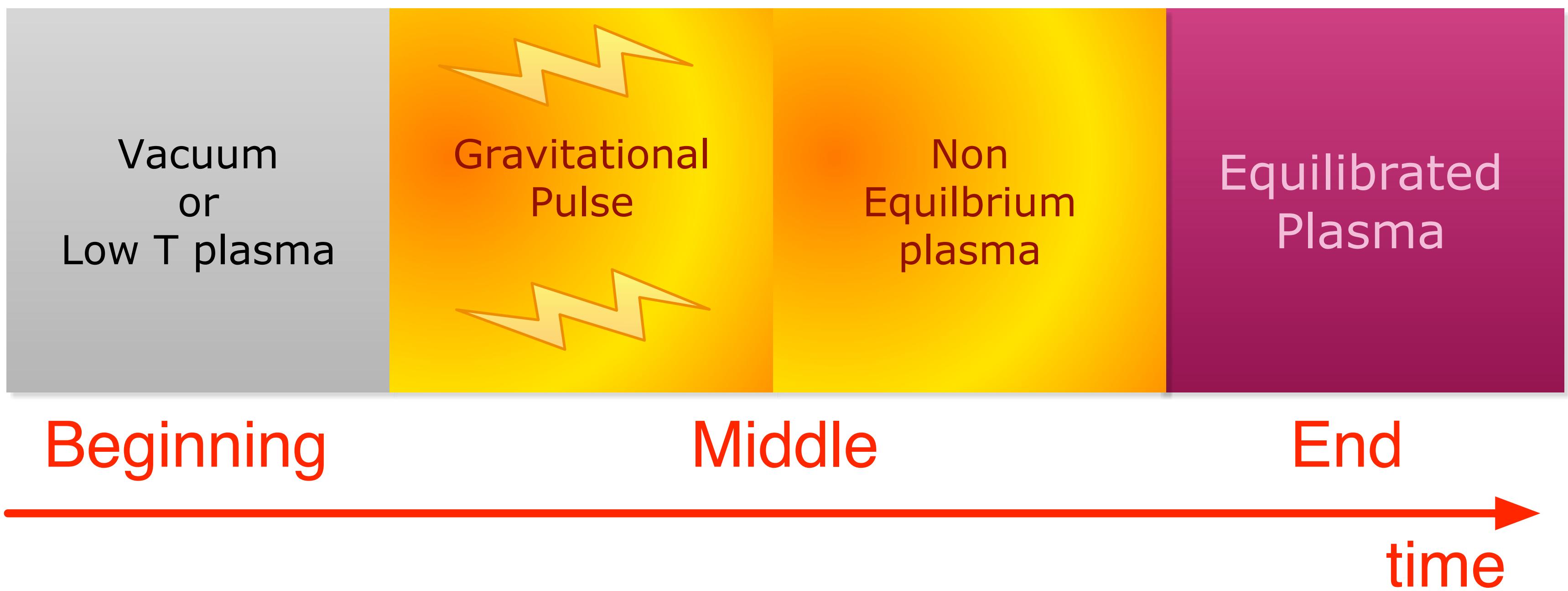
Non-equilibrium setup in 4D: (Chesler-Yaffe)

1. Chesler and Yaffe turn on a strong gravitational pulse in “our” world

$$ds^2 = -dt^2 + e^{B_o(t)} d\mathbf{x}_\perp^2 + e^{-2B_o(t)} dx_\parallel^2$$

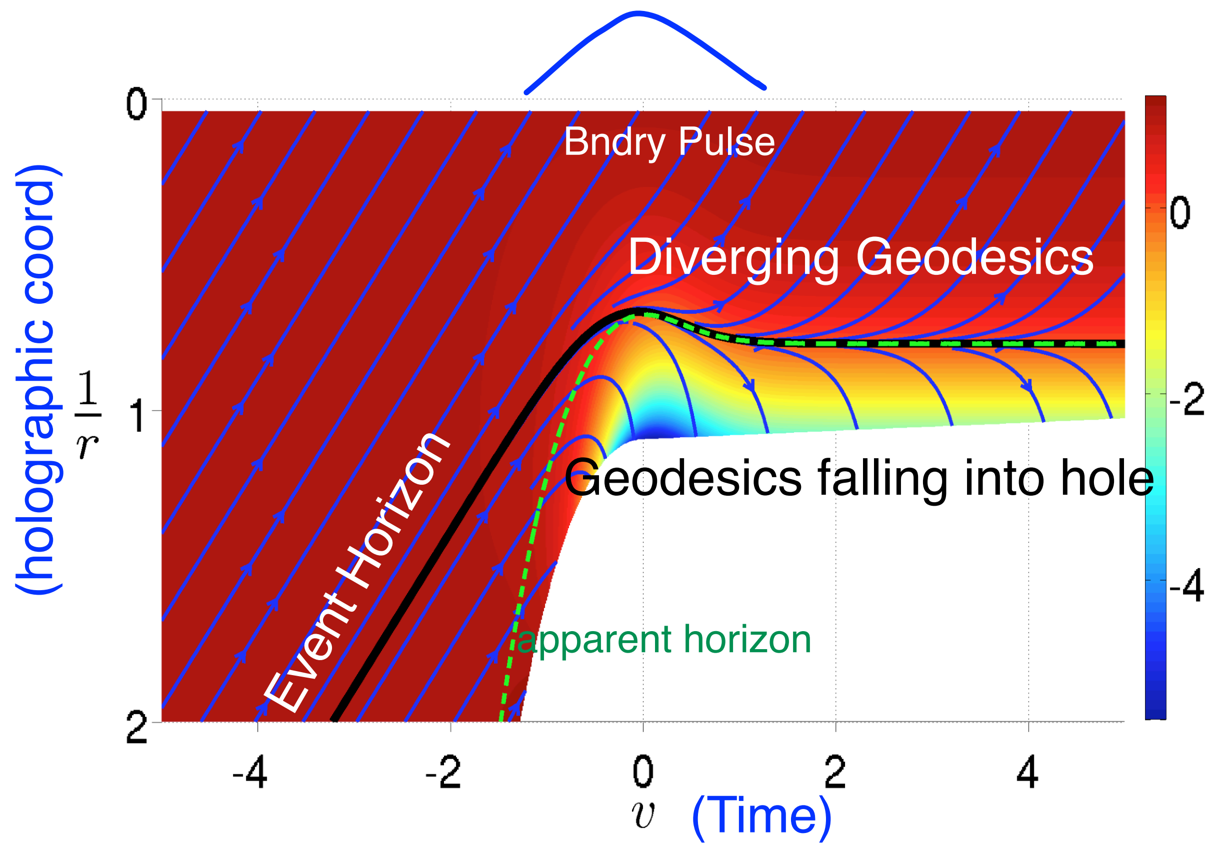
where

$$B_o(t) \propto e^{-t^2/\Delta t^2}$$



1. Corresponds to non-equilibrium geometry with BH formation in AdS_5

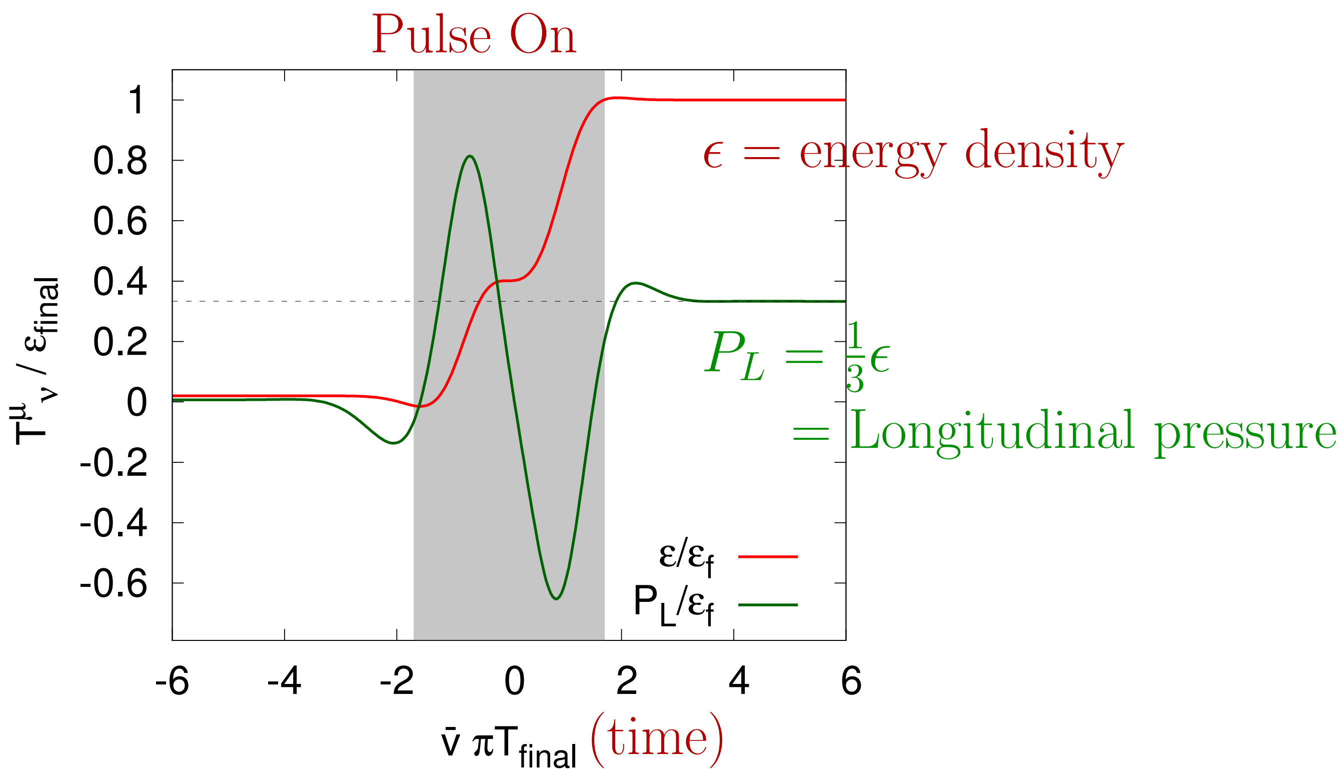
$$ds^2 = -A\,dv^2 + \Sigma^2\left[e^B d\boldsymbol{x}_\perp^2 + e^{-2B} dx_{||}^2\right] + 2dr\,dv\,,$$



Solve for $A(v,r)$, $B(v,r)$ and $\Sigma(v,r)$ with Einstein eqs with $B(v,r) \to B_o(t)$ on bndry.

The boundary stress tensor

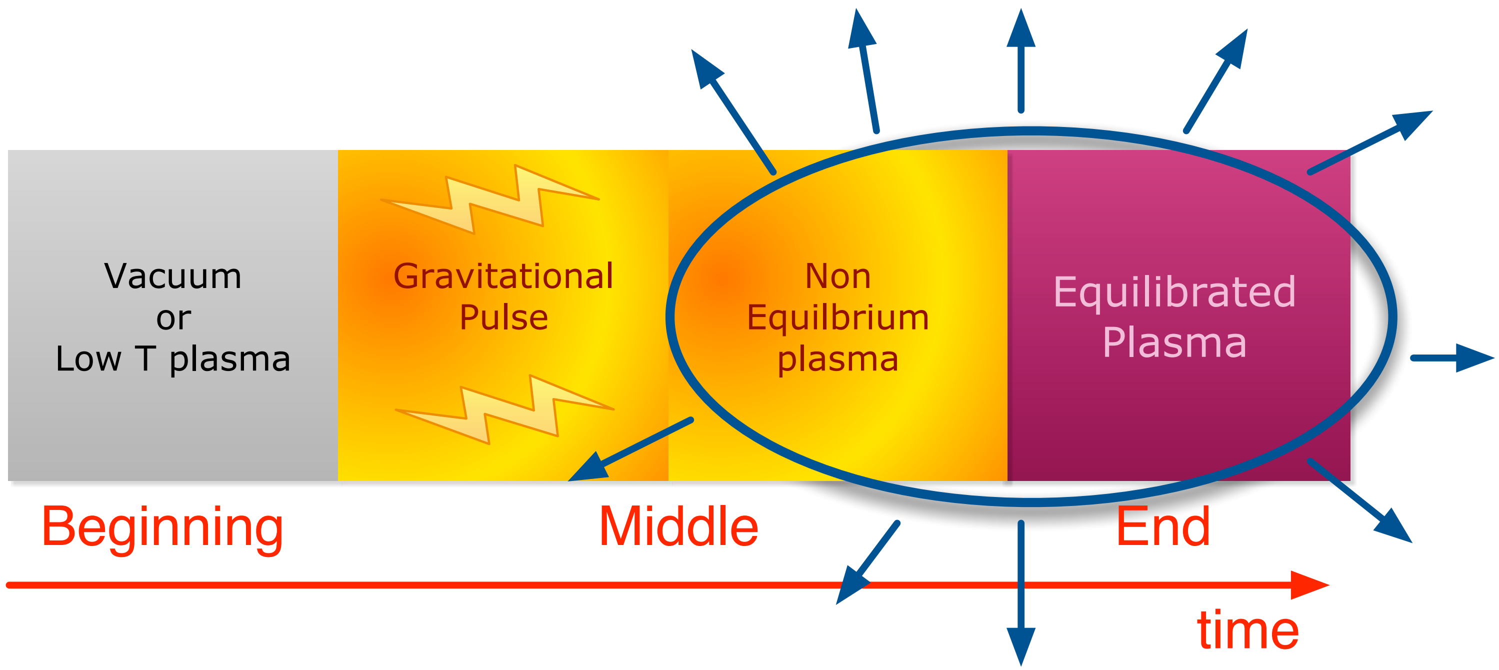
- The energy density increases by 50 times for a gaussian pulse with $\Delta t = 1/\pi T_{\mathrm{f}}$



Define an effective temperature:

$$\frac{1}{T_{\mathrm{eff}}(v)} = \beta_{\mathrm{eff}}(v) \propto \varepsilon(v)^{-1/4}$$

Hawking emission and 2pnt functions in this geometry:



I want to compute the "photon" emission rate in the non-equilibrium plasma.

1. Study the equilibration of 2pnt functions in the plasma.

2. Study the non-equilibrium *emission* of quanta from the black brane

Emission from CFT is dual to *emission* from black brane

Emission of dilatons weakly interacting with equilibrium strongly coupled SYM plasma

$$iS_{\text{int}} = i \int \mathrm{d}^4x \, \phi(x) J(x)$$

- Emission:

$$(2\pi)^3 2k \frac{d\Gamma^<}{d^3k} = G^<(K) \qquad\qquad G^<(K) = \left\langle \hat{J}(0)\hat{J}(K) \right\rangle$$

- Absorption: The absorption rate of Dilatons is

$$(2\pi)^3 2k \frac{d\Gamma^>}{d^3k} = G^>(K) \qquad\qquad G^>(K) = \left\langle \hat{J}(K)\hat{J}(0) \right\rangle$$

- FDT: The Fluctuation Dissipation Relation reads

$$\Big[\underbrace{G^<(K)}_{\text{emission}} \Big] / \Big[\underbrace{G^>(K)}_{\text{absorption}} \Big] = e^{-\omega/T}$$

We will compute the emission and absorption rates and check for detailed balance

What the classical AdS/CFT usually computes:

$n_{\mathbf{k}} =$ Dilaton occupation number

$$\partial_t n_{\mathbf{k}} = -n_{\mathbf{k}} \underbrace{\Gamma^{>}}_{\text{absorb}} + (1+n_{\mathbf{k}}) \underbrace{\Gamma^{<}}_{\text{emit}}$$

- For a classical dilaton field $n_{\mathbf{k}} \gg 1$ the damping is

$$\partial_t n_{\mathbf{k}} = -n_{\mathbf{k}} \times \underbrace{(\Gamma^{>} - \Gamma^{<})}_{\text{classical absorption rate}}$$

- The classical absorption rate

$$G^{>}(K) - G^{<}(K) = -2\,\mathrm{Im}G_R(K) = \rho(K)$$

Without assuming FDT, only the classical absorption rate is computable
with the classical black brane response.

Summary: spectral density and statistical fluctuations

1. Spectral Density (commutator or $G^{>} - G^{<}$)

$$\rho(t_1|t_2) = \langle [\phi(t_1), \phi(t_2)] \rangle$$

- Records the dissipation of classical waves

2. Statistical fluctuations (anti-commutator or $\frac{1}{2}(G^{>} + G^{<})$)

$$G_{rr}(t_1|t_2) = \frac{1}{2} \langle \{\phi(t_1), \phi(t_2)\} \rangle$$

- Invariably suppressed at large N and only due to Hawking radiation.

In non-equilibrium systems these correlators determine the emission/abs rates

A non-equilibrium definition of the Emission and Absorption Rates

Want to know the rate to emit and absorb in a frequency band ω at time t

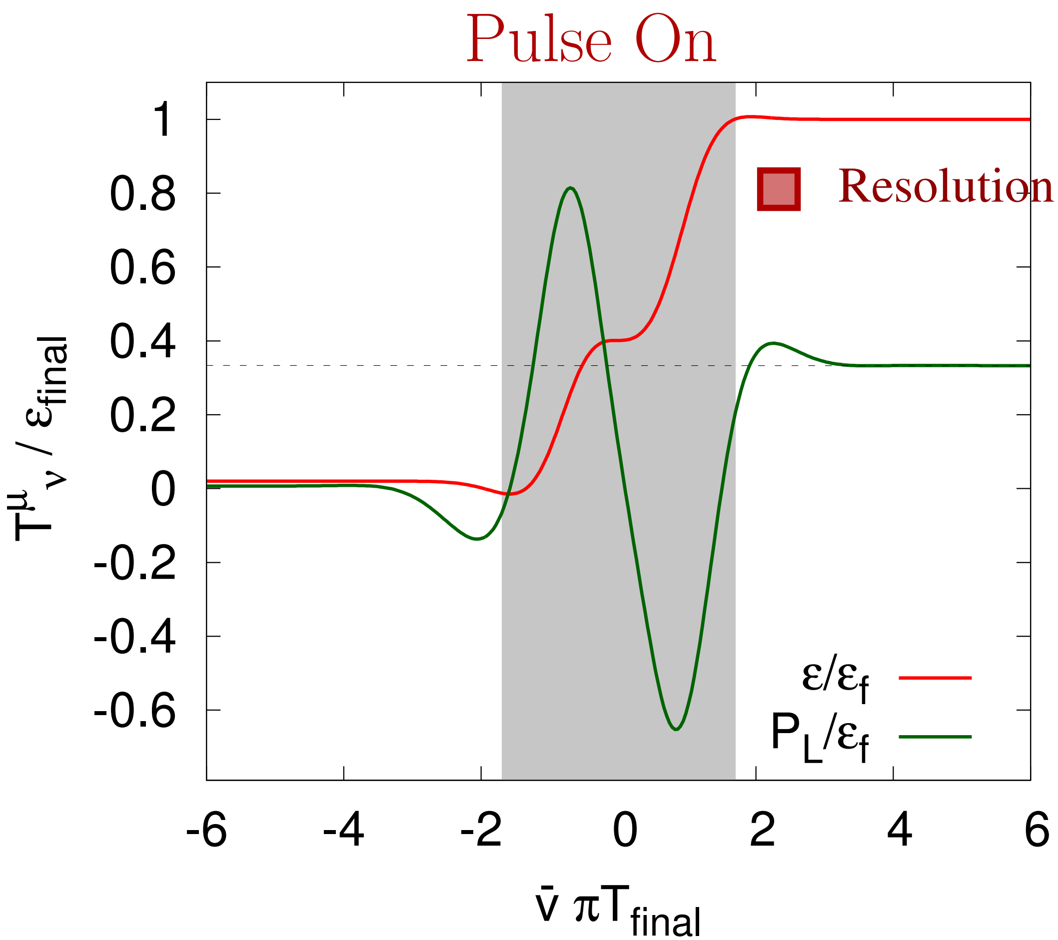
1. Wigner Transforms – perfect frequency resolution, but no time resolution

$$G^{<}(\bar{t},\omega) = \int_{-\infty}^{\infty} \mathrm{d}\Delta t e^{+i\omega\Delta t} \left\langle J(\bar{t}-\Delta t)J(\bar{t}+\Delta t)\right\rangle$$

2. Gabor Transform – Wigner smeared with a minimum uncertainty wave packet

$$\underbrace{\bar{G}^{<}(\bar{t}_o,\bar{\omega}_o)}_{\text{Gabor}} = \int \frac{dtd\omega}{2\pi} \underbrace{2e^{-(\omega-\omega_o)^2\sigma^2} e^{-(\bar{t}-\bar{t}_o)^2/\sigma^2}}_{\text{minimum wave packet}} \underbrace{G^{<}(\bar{t},\omega)}_{\text{Wigner Trans}}$$

$\bar{G}^{<}(\bar{t}_o,\bar{\omega}_o)$ determines for the local emission rate for a given temporal resolution



- Temporal Resolution

$$\sigma_v \pi T_{\mathrm{f}} = \frac{1}{\sqrt{2}} \simeq 0.7$$

- Frequency Resolution

$$\frac{\sigma_\omega}{\omega} \simeq \frac{1/\sqrt{2}}{8} \simeq 10\%$$

Equilibration and the coarse-grained FDT

1. If the FDT is satisfied

$$G^{<}(K) = e^{-\omega/T} G^{>}(K)$$

then, the coarse-grained quantities satisfy

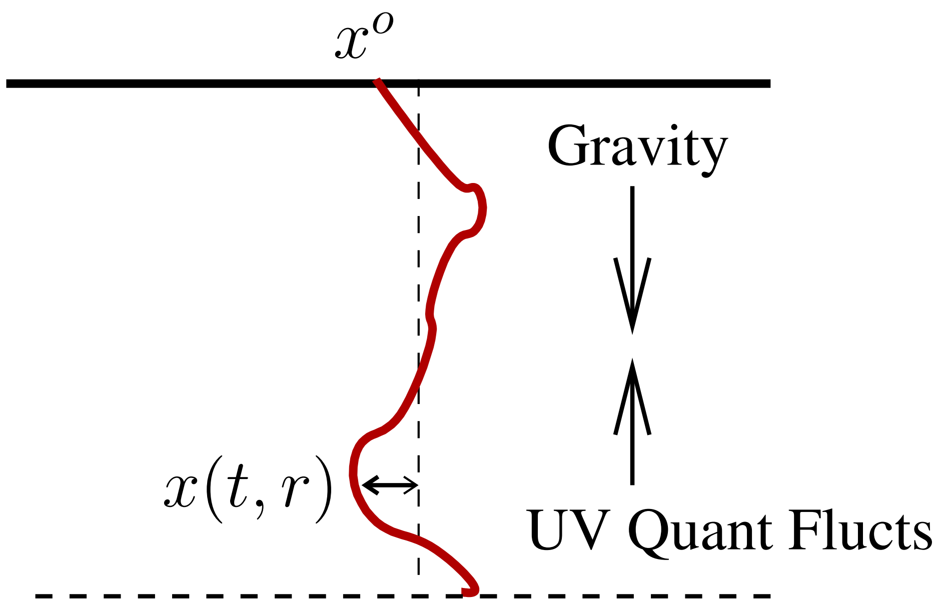
$$\underbrace{\bar{G}^{<}(\bar{t}_o, \bar{\omega}_o, \boldsymbol{q})}_{\text{emission}} = e^{-\omega_o \beta_{\rm eff}} \underbrace{\left[e^{\beta_{\rm eff}^2/4\sigma^2} \bar{G}^{>}(\bar{t}_o, \bar{\omega}_o - \beta_{\rm eff}/2\sigma^2, \boldsymbol{q}) \right]}_{\text{absorption}}$$

We will monitor this “FDT” as a function of time to quantify equilibrium

Hawking Radiation in and out of equilibrium

Equilbrium:

$$M\frac{d^2x^o}{dt^2} = \underbrace{-\eta}_{\text{Drag}}\,\dot{x}^o + \underbrace{\xi}_{\text{Noise}}$$

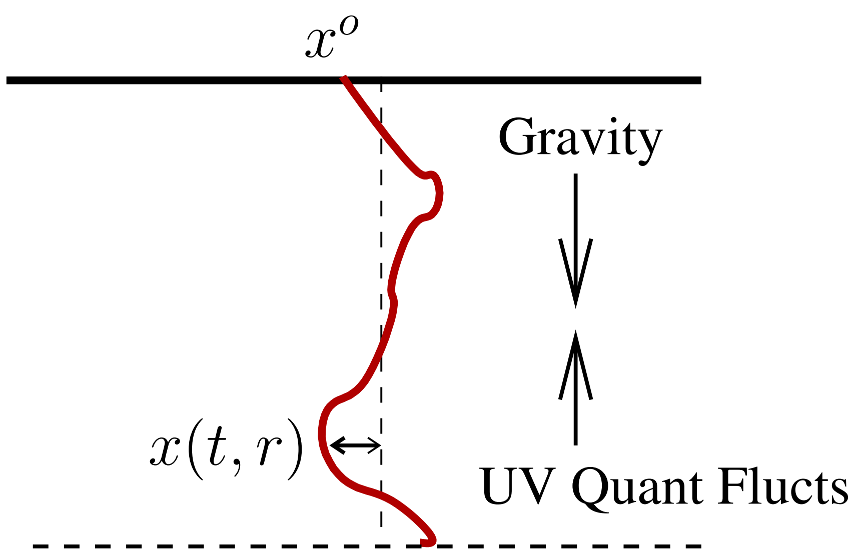


Goals:

1. Will show that Hawking radiation is balanced by gravity in equilbrium

2. Generalize to non-equilibrium

Detailed Balance and Hawking Radiation (Technical Discussion)



1. Fluctuations:

$$G_{rr} \equiv \frac{1}{2} \langle \{ \hat{x}(t_1, r_1), \hat{x}(t_2, r_2) \} \rangle \ ,$$

2. Dissipation (Spectral Density)

$$\rho \equiv \langle [\hat{x}(t_1, r_1), \hat{x}(t_2, r_2)] \rangle \ .$$

- Equilibrium $\equiv$ Fluctuation Dissipation Theorem

$$G_{rr}(\omega, r_1, r_2) = \left(\frac{1}{2} + n_B(\omega) \right) \rho(\omega, r_1, r_2) \qquad n(\omega) \equiv \frac{1}{e^{\omega/T} - 1}$$

Formulas

- Action for string fluctuations, $h^{\mu\nu}$ = string metric

$$S = \frac{\sqrt{\lambda}}{2\pi} \int \mathrm{d}t\mathrm{d}r\, g_{xx} \left[-\sqrt{h} h^{\mu\nu} \partial_\mu x \partial_\nu x \right] \,,$$

- $h^{\mu\nu}$ is the string metric

$$h_{\mu\nu} \mathrm{d}\sigma^\mu \mathrm{d}\sigma^\nu = -(\pi T)^2 r^2 f(r) dt^2 + \frac{dr^2}{f(r) r^2} \,,$$

- Retarded Green Function

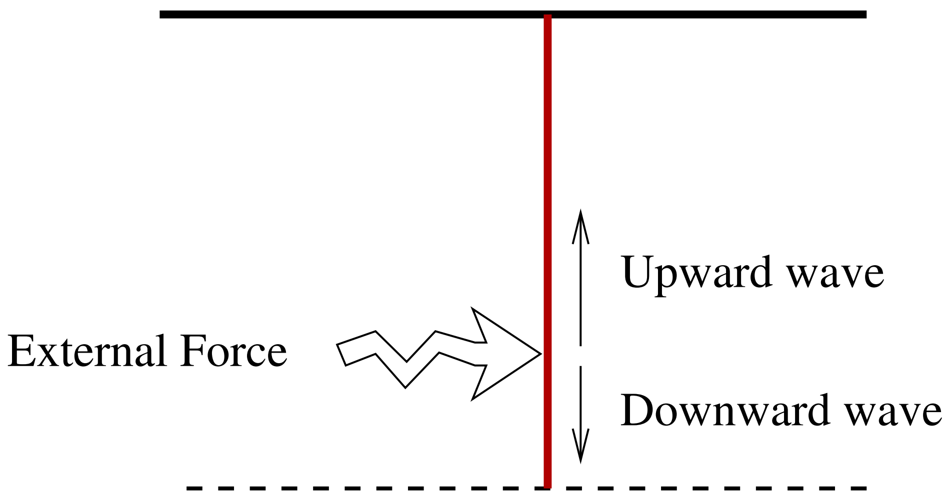
$$iG_R(t_1 r_1 | t_2 r_2) \equiv \theta(t - t') \left\langle [\hat{x}(t_1, r_1), \hat{x}(t_2, r_2)] \right\rangle \,,$$

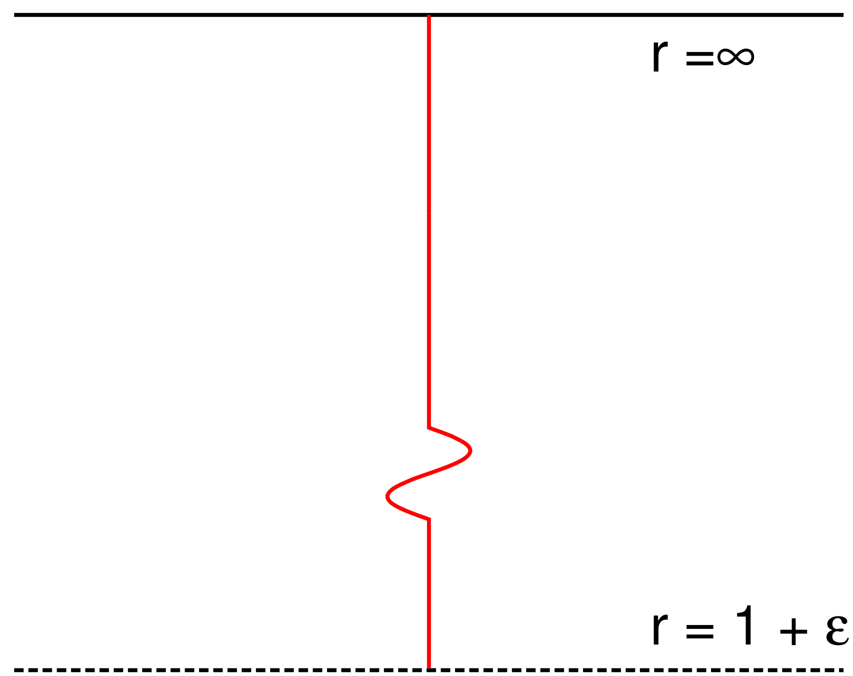
$G_R(t_1 r_1 | t_2 r_2)$ is the classical response to a force at $t_2 r_2$

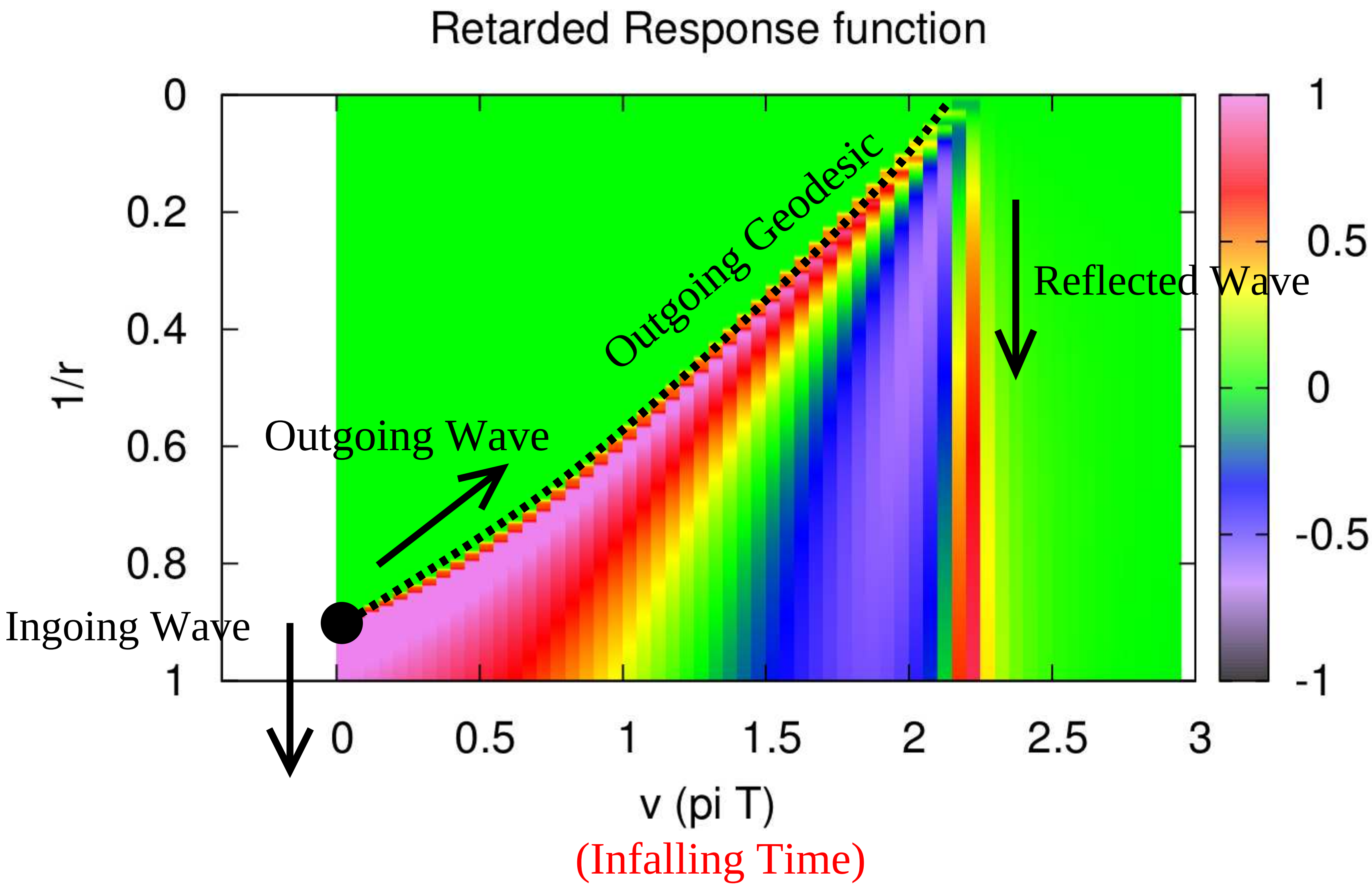
$$\frac{\sqrt{\lambda}}{2\pi} \left[\partial_\mu \, g_{xx} \sqrt{h} h^{\mu\nu} \partial_\nu \right] G_R(t_1 r_1 | t_2 r_2) = \delta(t_1 - t_2) \delta(r_1 - r_2) \,,$$

The classical Green Function or response to a force:

$$\frac{\sqrt{\lambda}}{2\pi}\left[\partial_\mu\, g_{xx}\sqrt{h}h^{\mu\nu}\partial_\nu\right] G_R = \mathcal{F}\,\delta(t_1-t_2)\delta(r_1-r_2)\,,$$

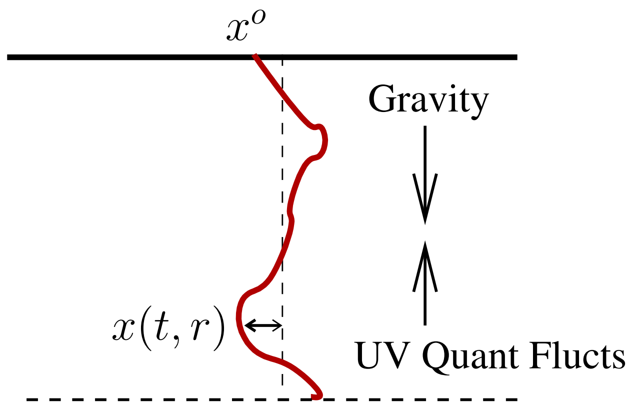






v =Eddington time

Statistical Fluctuations



$$G_{rr} = \frac{1}{2} \langle \{ x(t_1, r_1), x(t_2, r_2) \} \rangle$$

- The statistical correlator obeys the homogeneous EOM

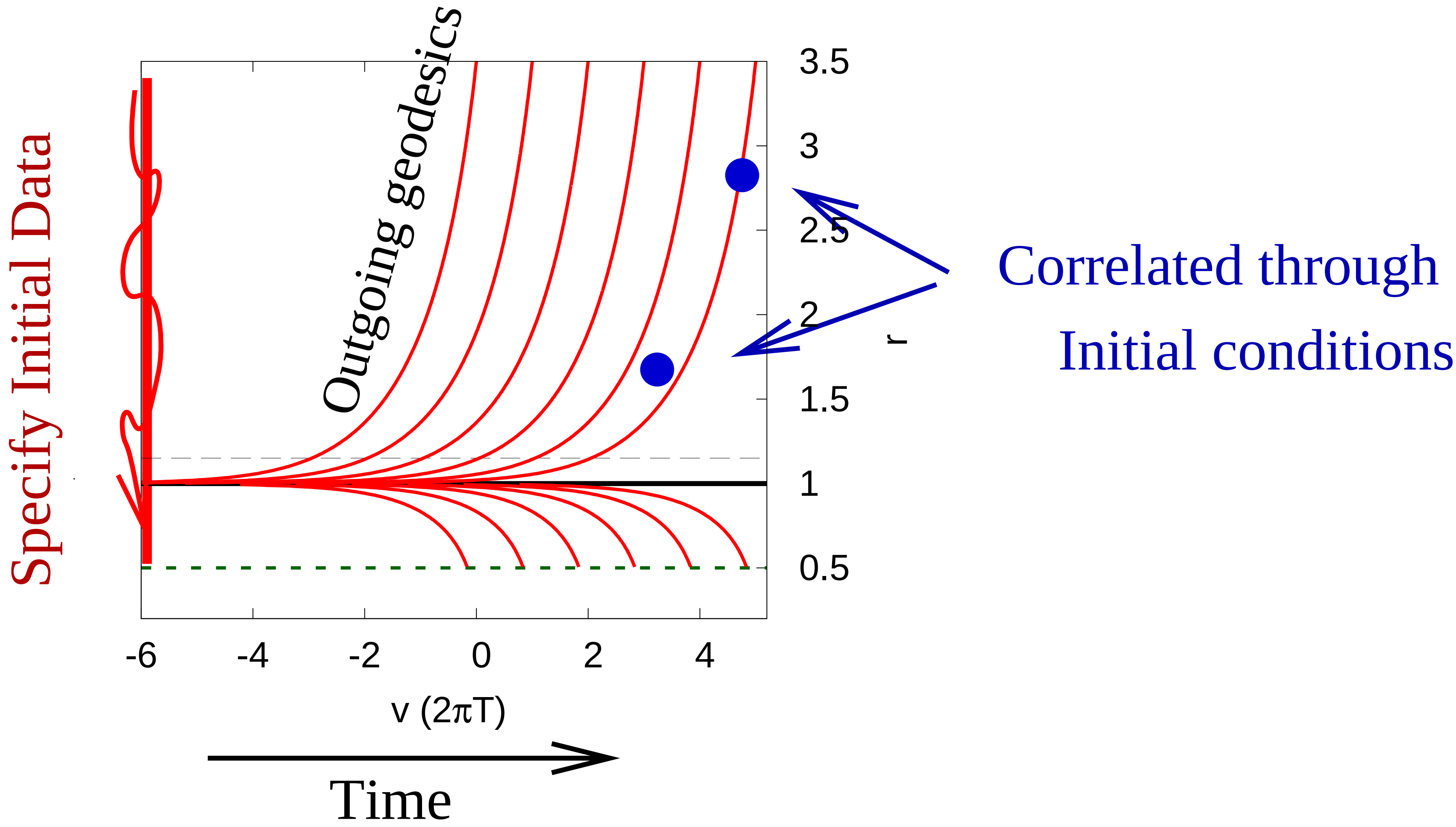
$$\frac{\sqrt{\lambda}}{2\pi} \left[\partial_\mu \, g_{xx} \sqrt{h} h^{\mu\nu} \partial_\nu \right] G_{rr}(t_1 r_1 | t_2 r_2) = 0$$

- So:

  1. Specify the correlations (or density matrix) in the past

  2. Final state fluctuations are correlated only through initial conditions

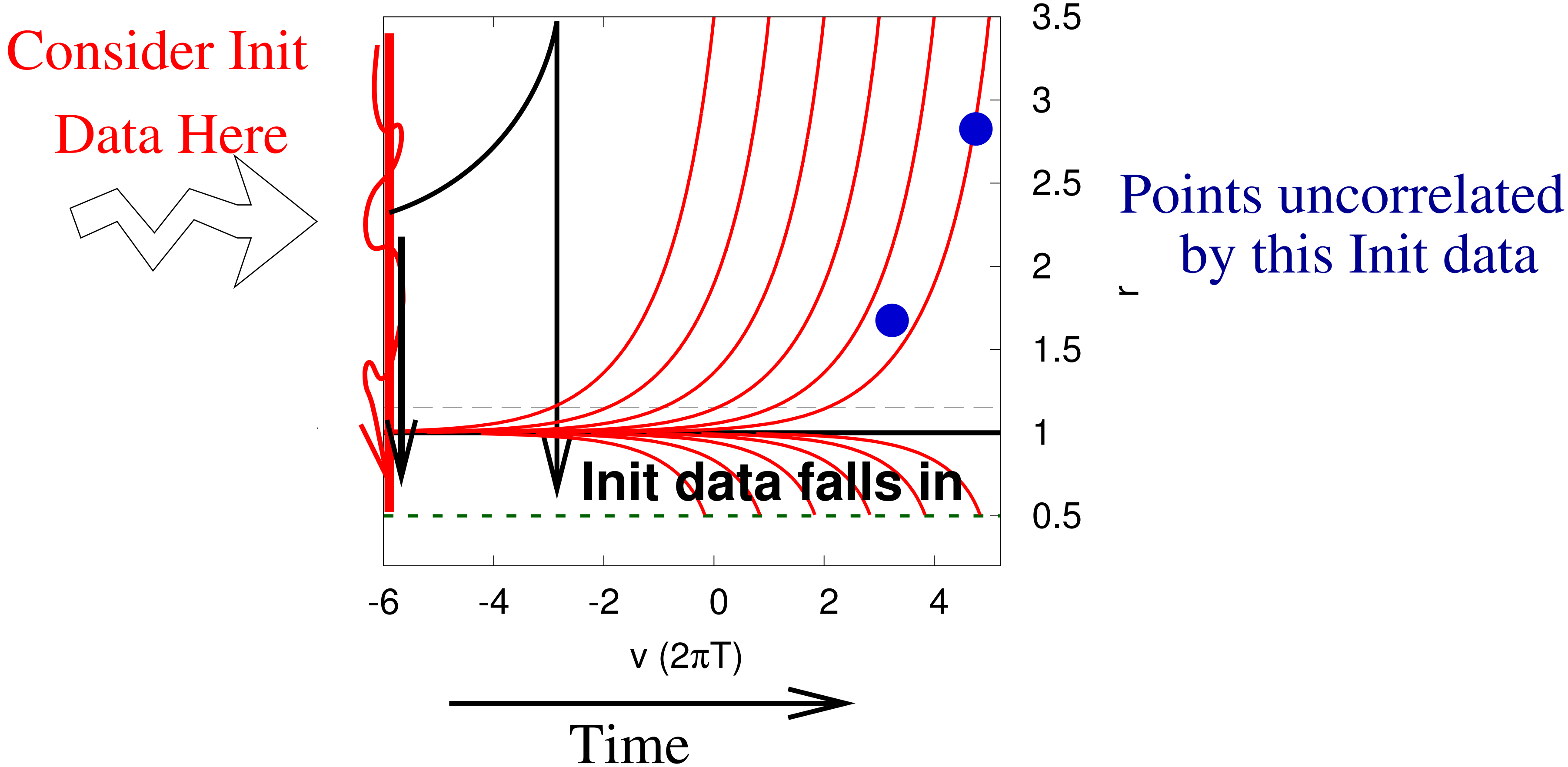
Correlations through Initial conditions



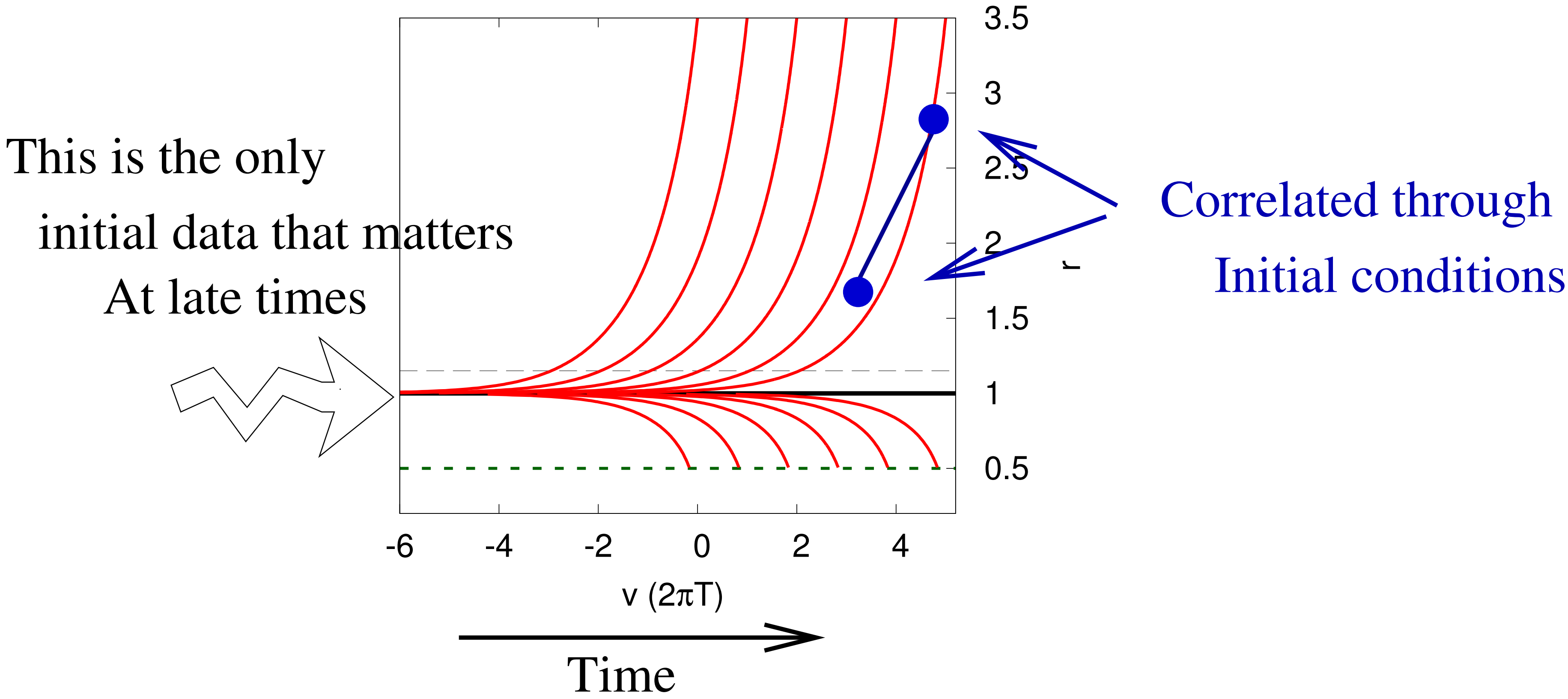
Horizon characterized by inflating outgoing geodesics:

$$r(v) - 1 = (r_o - 1)\, e^{\kappa\,(v - v_o)} \qquad \text{with} \qquad \kappa \equiv 2\pi T$$

Correlations through Initial conditions



Correlations through Initial conditions



1. Final correlation come from correlated initial data very near horizon
   - Short Wavelength
2. Initial data is inflated by near horizon geometry

Initial Data from Quantum Fluctuations:

1. Initial data is determined at short distance = Flat Space Physics
2. Scalar Field in 1+1D vacuum flat space

$$\frac{1}{2}\langle\{\phi(X_1),\phi(X_2)\}\rangle = -\frac{1}{4\pi K}\log|\mu\, \overbrace{\eta_{\mu\nu}\Delta X^\mu \Delta X^\nu}^{\Delta s^2}|$$

K=norm of action

3. String flucts in near horizon geometry

$$S^{\mathrm{near-horizon}} = \eta \int \mathrm{d}t\mathrm{d}r\,\left[-\frac{1}{2}\sqrt{h}h^{\mu\nu}\partial_\mu x \partial_\nu x\right]$$

$$\underbrace{\eta = \frac{\sqrt{\lambda}}{2\pi} g_{xx}(r_h)}_{\text{norm of near horizon-action}}$$

The near horizon initial condition is:

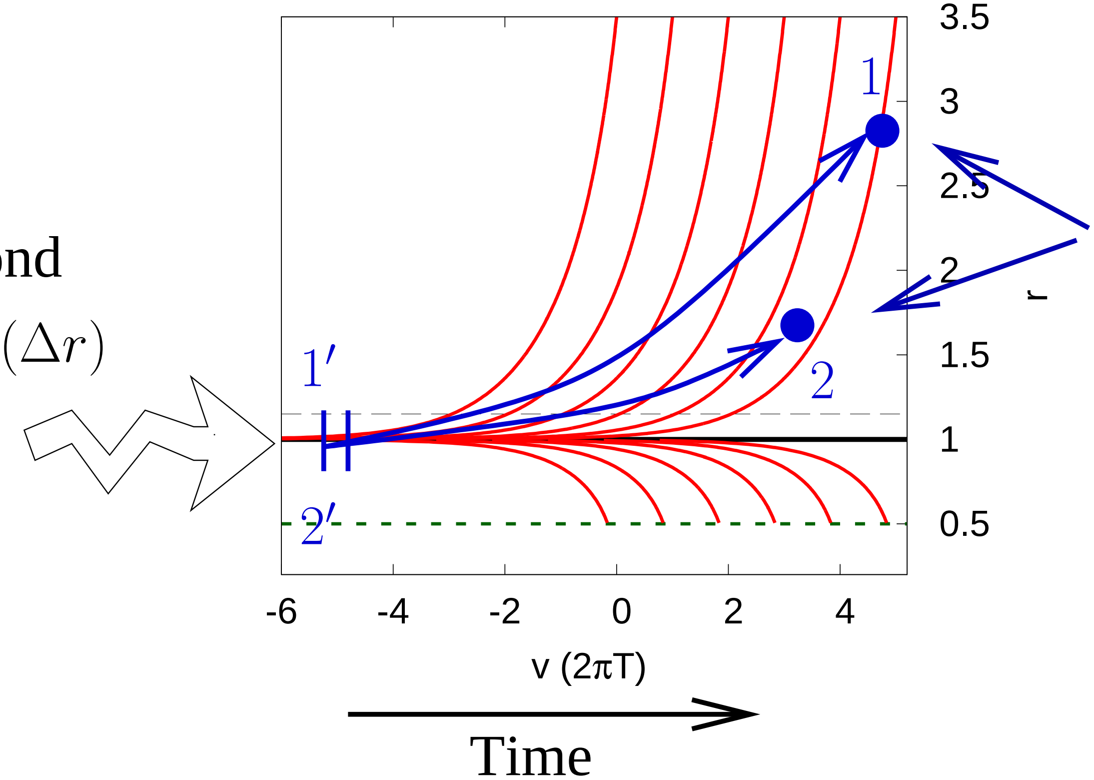
$$G_{rr}(v_1 r_1|v_2 r_2) \to -\frac{1}{4\pi\eta}\log\left|\mu\, \overbrace{2\Delta v\,\Delta r}^{\text{local } \Delta s^2}\right|$$

Summary: Specify IC and Solve Equations of Motion

$$\frac{\sqrt{\lambda}}{2\pi}\left[\partial_\mu\, g_{xx}\sqrt{h}h^{\mu\nu}\partial_\nu\right] G_{rr}(t_1 r_1|t_2 r_2) = 0$$

Init. Cond

$\propto \log(\Delta r)$



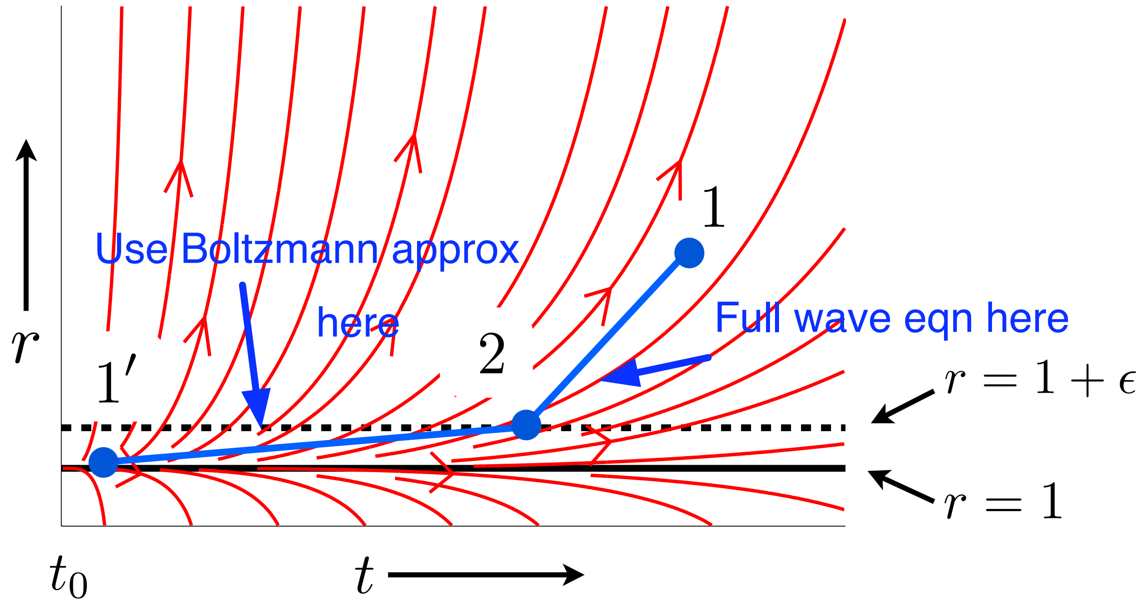
Correlated via

log Init Cond

Inflationary near horizon geometry

$$(r-1) \Longrightarrow (r-1)e^{\kappa t}$$

From initial data to final correlations in two steps:



$$G_R(1|1') = \int \mathrm{d}t_2\, G_R(1|2) \left[\eta\sqrt{h}h^{rr}(r_2)\overleftrightarrow{\partial}_{r_2}\right]_{r_2=1+\epsilon} G_R(2|1'),$$

(a) From horizon to stretched horizon – Waves are very short wavelength

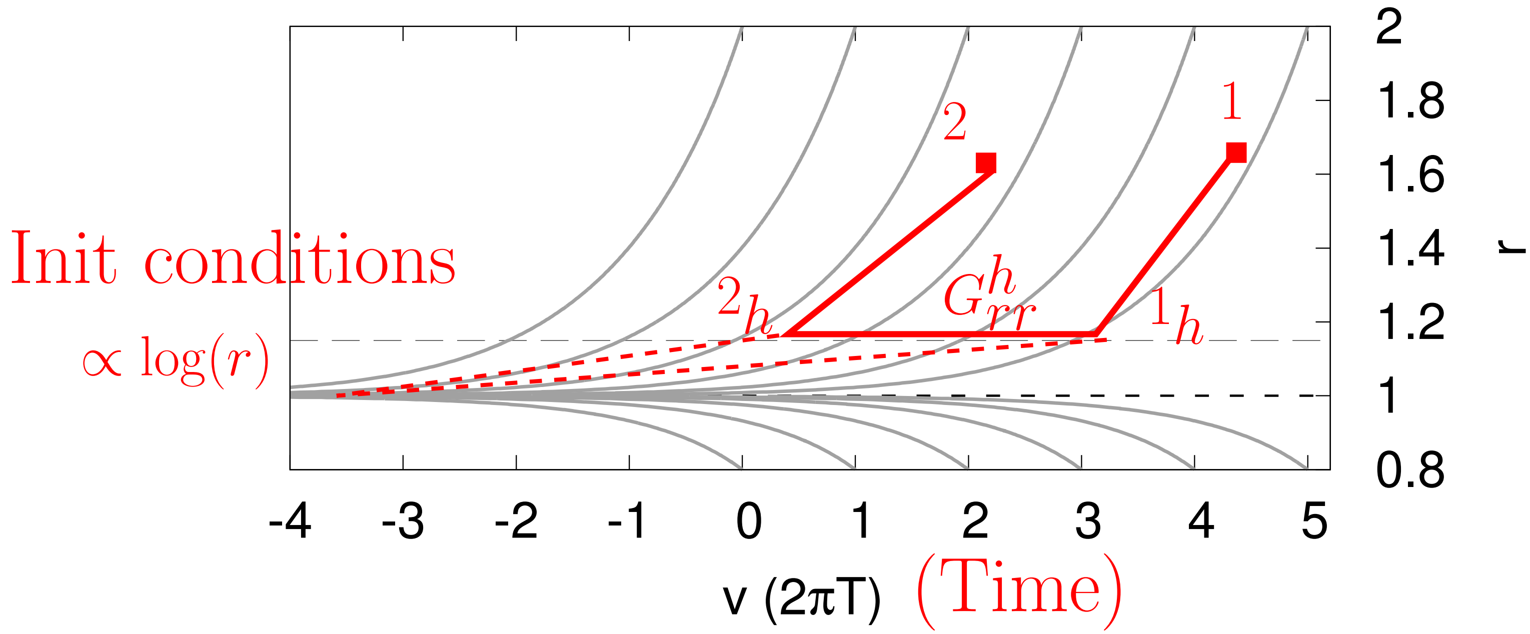
- Use collisionless Boltzmann approximation (geodesic/WKB/eikonal approx)

(b) The stretched horizon to boundary – Waves are longer wavelength

- Use full wave equation

Fluctuations from Equations of Motion

$$\underbrace{G_{rr}(1|2)}_{\text{bulk flucts}} = \int \mathrm{d}t_{1h}\mathrm{d}t_{2h}\ \underbrace{G_R(1|1_h)\ G_R(2|2_h)}_{\text{outgoing Green fcns}}\ \underbrace{G^h_{rr}(1_h|2_h)}_{\text{horizon flucts}}\,,$$

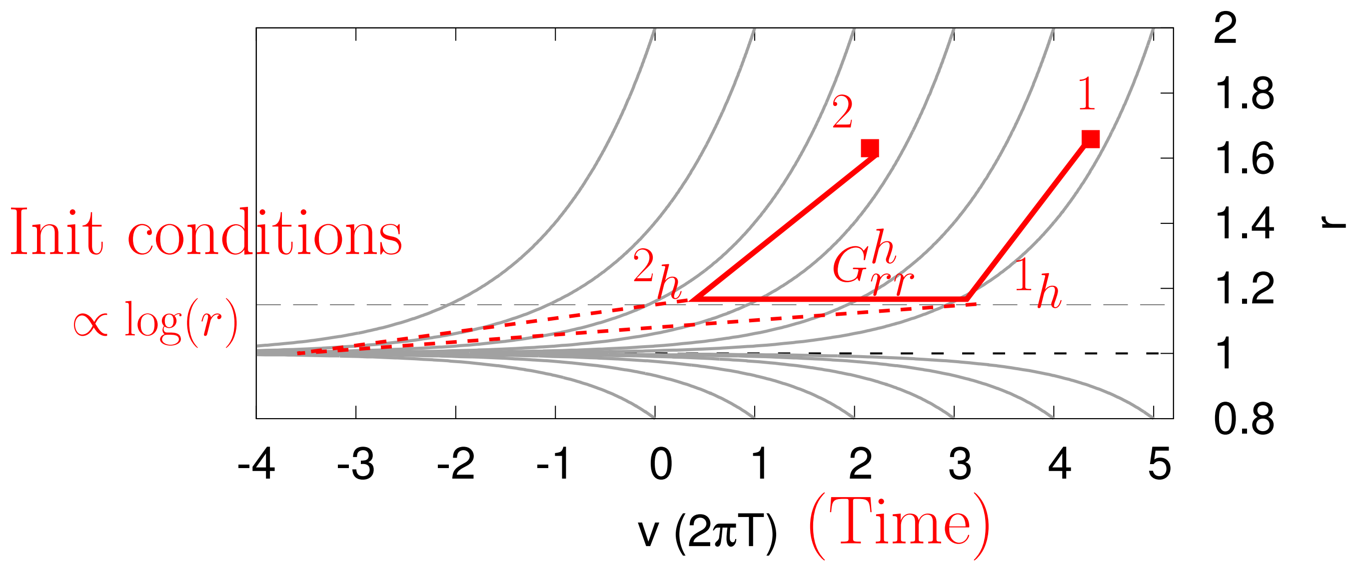


The fluctuations on the stretched horizon are from UV vacuum flucts in past

$$G^h_{rr}(t_1|t_2) = \text{Blow-up of initial data} \propto \log(r)$$

$$= -\frac{\eta}{\pi}\partial_{t_1}\partial_{t_2}\log\left|e^{\kappa\, t_1} - e^{\kappa\, t_2}\right|\,.$$

The horizon fluctuations and the Lyapunov exponent



1. Thermal looking:

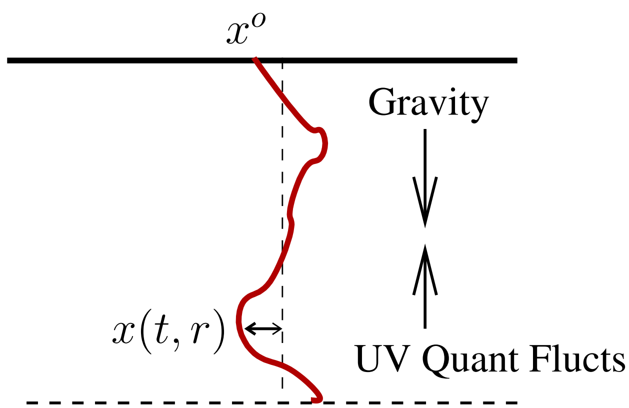
$$G^h_{rr}(\omega) = \text{Fourier-Trans of } -\frac{\eta}{\pi}\partial_{t_1}\partial_{t_2}\log\left|e^{\kappa\, t_1} - e^{\kappa\, t_2}\right|$$

$$= \left(\tfrac{1}{2} + n(\omega)\right) 2\omega\eta \qquad\qquad n(\omega) \equiv \frac{1}{e^{2\pi\omega/\kappa} - 1}$$

2. Temperature $\propto$ inflation rate

$$\kappa = 2\pi T = \text{Lyapunov exponent of diverging geodesics}$$

Dissipation - Spectral Density



$$\rho = \langle [\hat{x}(t_1, r_1), \hat{x}(t_2, r_2)] \rangle$$

- The spectral density <u>also</u> obeys the EOM

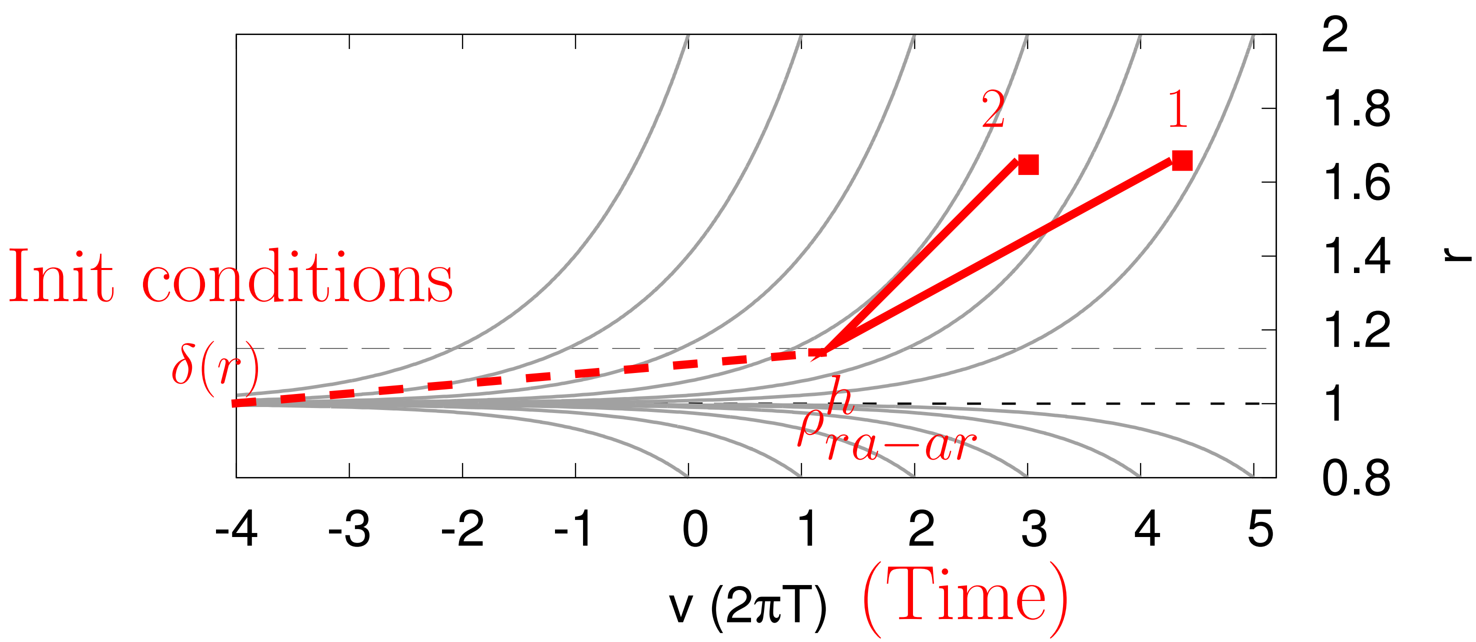
$$\frac{\sqrt{\lambda}}{2\pi} \left[\partial_\mu \, g_{xx} \sqrt{h} h^{\mu\nu} \partial_\nu \right] \rho(t_1 r_1 | t_2 r_2) = 0$$

- <u>But</u> initial conditions are given by the canonical commutation relations

$$\eta \sqrt{h} h^{tt}(r_1) \lim_{\mathrm{t}_2 \to \mathrm{t}_1} \partial_{t_1} \rho(t_1 r_1 | t_2 r_2) = i\delta(r_1 - r_2) \, .$$

Spectral Density

$$\underbrace{\rho(1|2)}_{\text{bulk spectral fcn}} = \int \mathrm{d}t_{1h}\mathrm{d}t_{2h}\ \underbrace{G_R(1|1_h)\ G_R(2|2_h)}_{\text{outgoing Green fcns}}\ \underbrace{\rho^h(1_h|2_h)}_{\text{horizon spectral fcn}} ,$$



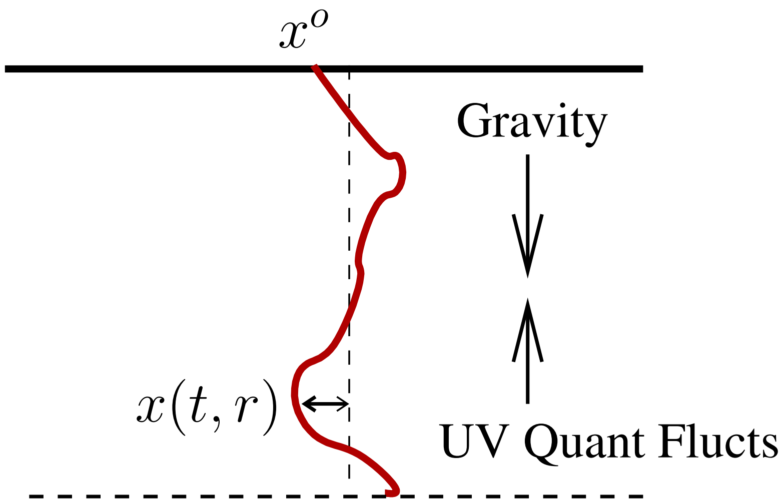
Where the horizon spectral density

$$\rho^h(t_1, t_2) = \text{local due to canonical commutation relations}$$

$$= 2\eta \left[-i\delta'(t_1 - t_2)\right] \qquad \text{(}2\omega\eta\text{ in Fourier space)}$$

Detailed Balance

$$G_{rr}(\omega, r_1, r_2) = \left(\tfrac{1}{2} + n(\omega)\right) \rho(\omega, r_1, r_2)$$



1. Fluctuations (Anti-commutator)

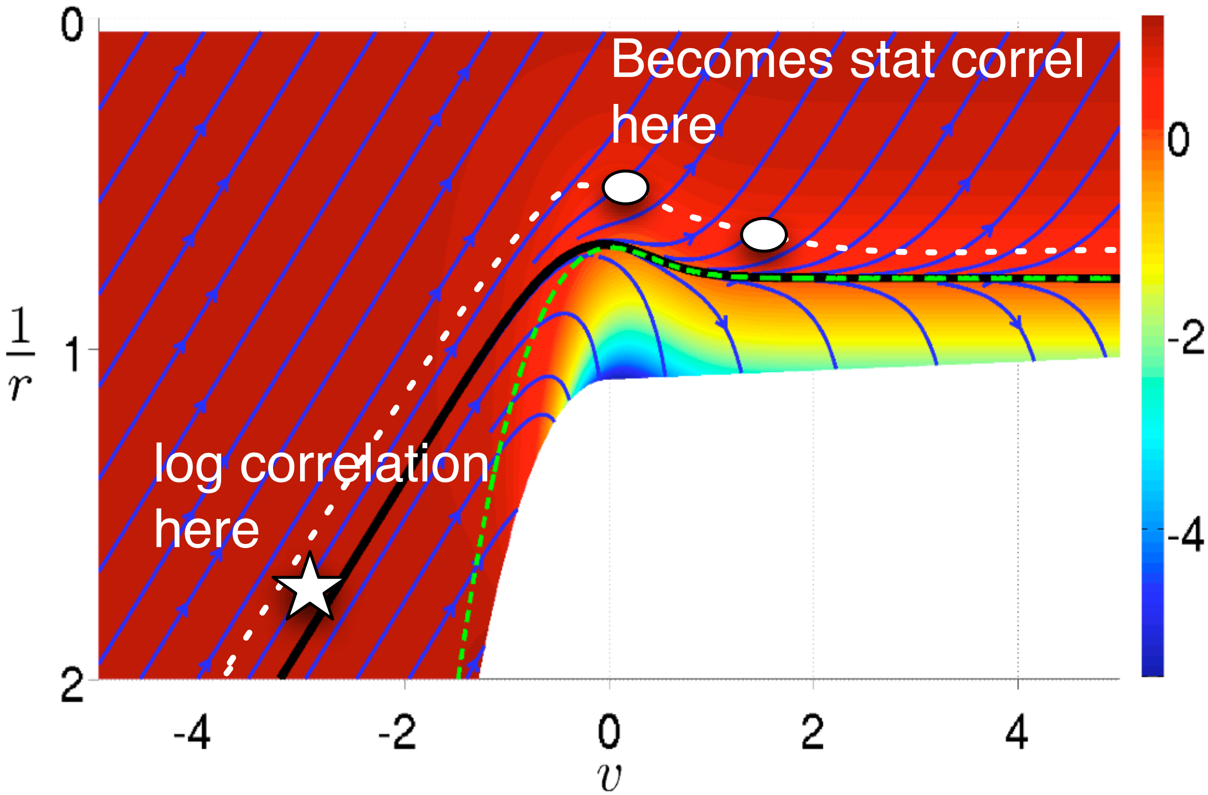
$$\underbrace{G_{rr}(\omega, r_1, r_2)}_{\text{bulk flucts}} = \underbrace{G_R(\omega, r_1|r_h)\, G_R(\omega, r_2|r_h)}_{\text{outgoing Green fcns}} \underbrace{\left(\tfrac{1}{2} + n(\omega)\right) 2\omega\eta}_{\text{Horizon-flucts}}$$

2. Dissipation: (Commutator)

$$\underbrace{\rho(\omega, r_1, r_2)}_{\text{bulk spec dense}} = \underbrace{G_R(\omega, r_1|r_h)\, G_R(\omega, r_2|r_h)}_{\text{outgoing Green fcns}} \underbrace{2\omega\eta}_{\text{Horizon spec dense}}$$

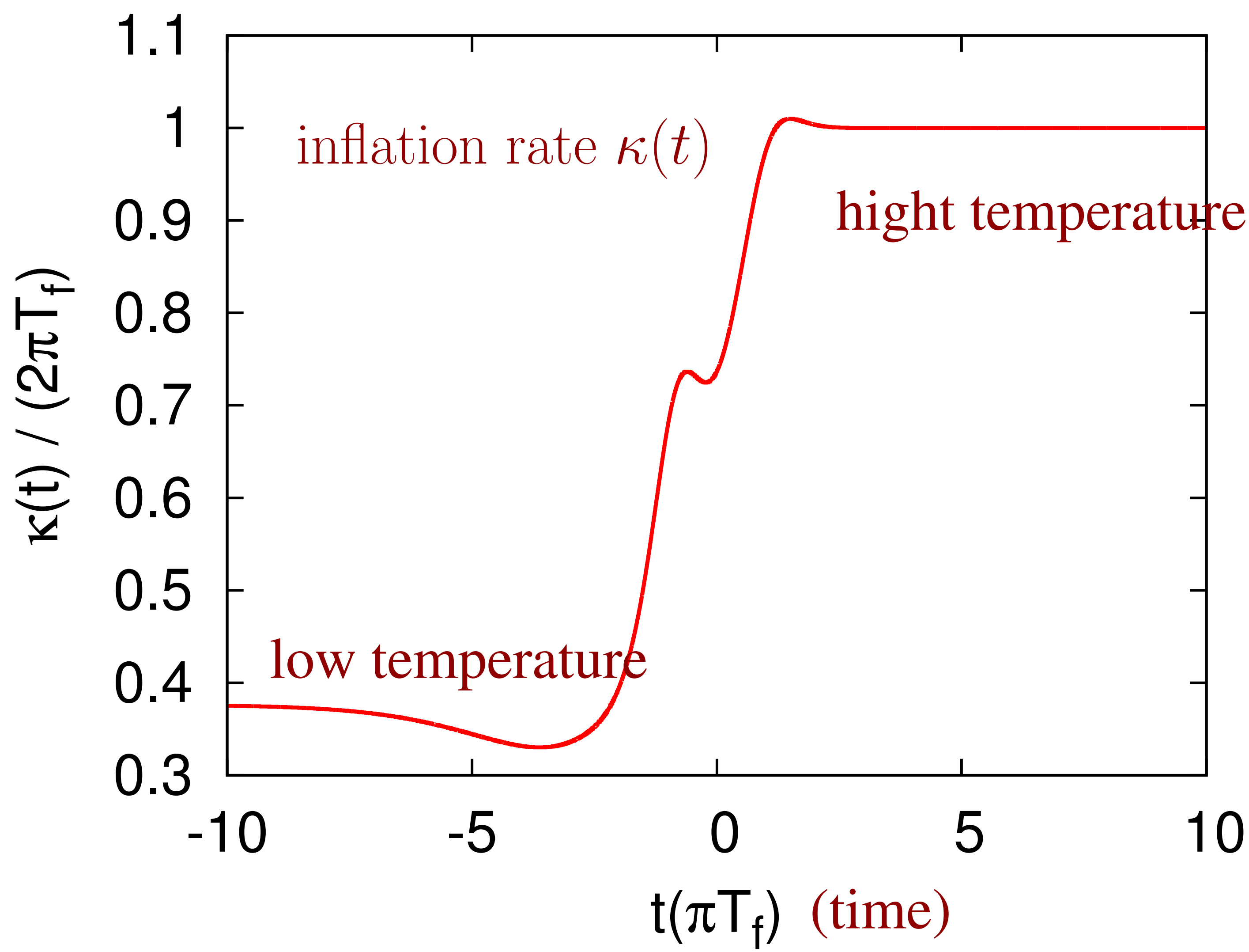
Non-equilibrium

Fluctuations in non-equilibrium



- Surface Properties – on event horizon

$$\underbrace{\kappa(v)}_{\text{time dep. Lyapunov exponent}} \equiv \overbrace{\frac{1}{2}\frac{\partial A(r,v)}{\partial r}}^{\text{Metric-coeff}}\Bigg|_{r=r_h(v)}$$



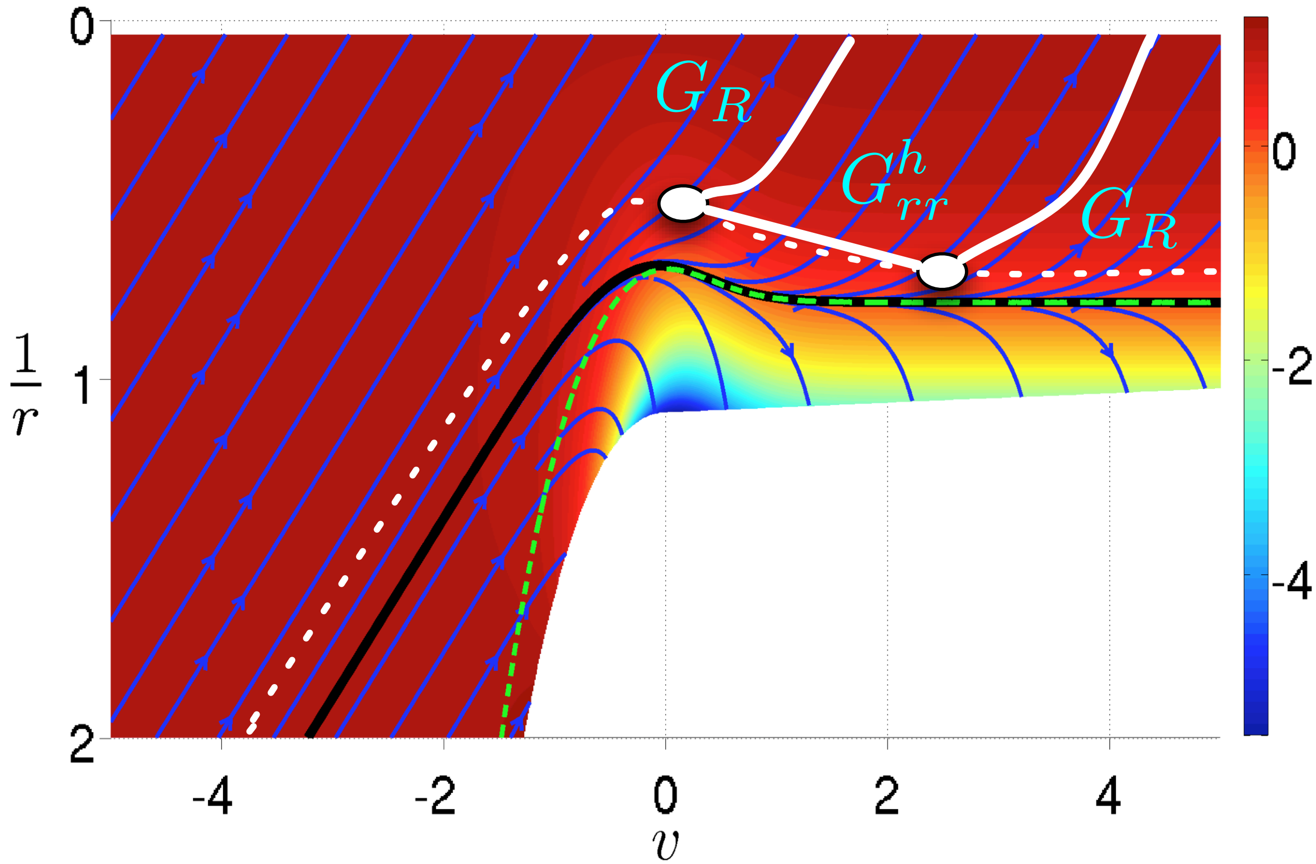
inflation rate $\kappa(t)$
hight temperature
low temperature
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
-10
-5
0
5
10
κ(t) / (2πT_f)
t(πT_f) (time)

Result:

- General form of near horizon fluctuations in non-equilibrium

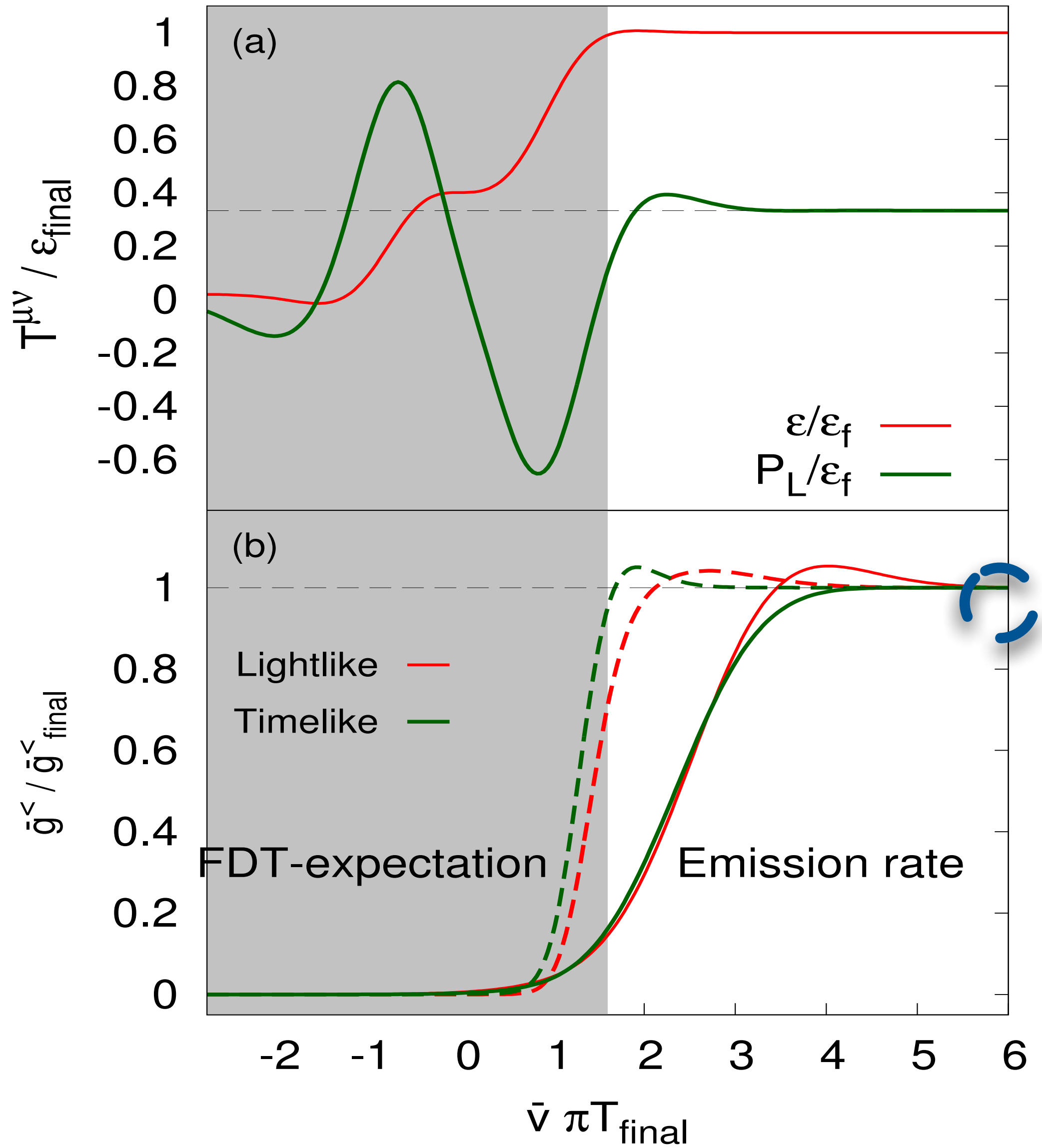
$$G^h_{rr}(v_1|v_2) = -\frac{\sqrt{\eta(v_1)\eta(v_2)}}{\pi}\partial_{v_1}\partial_{v_2}\log\left|e^{\int^{v_1}\kappa(v')dv'} - e^{\int^{v_2}\kappa(v')dv'}\right| .$$

- Can map the near horizon fluctuations up to boundary by finding G_R numerically



Results for non-equilbrium emission

Emission&Absorption rates and the FDT:



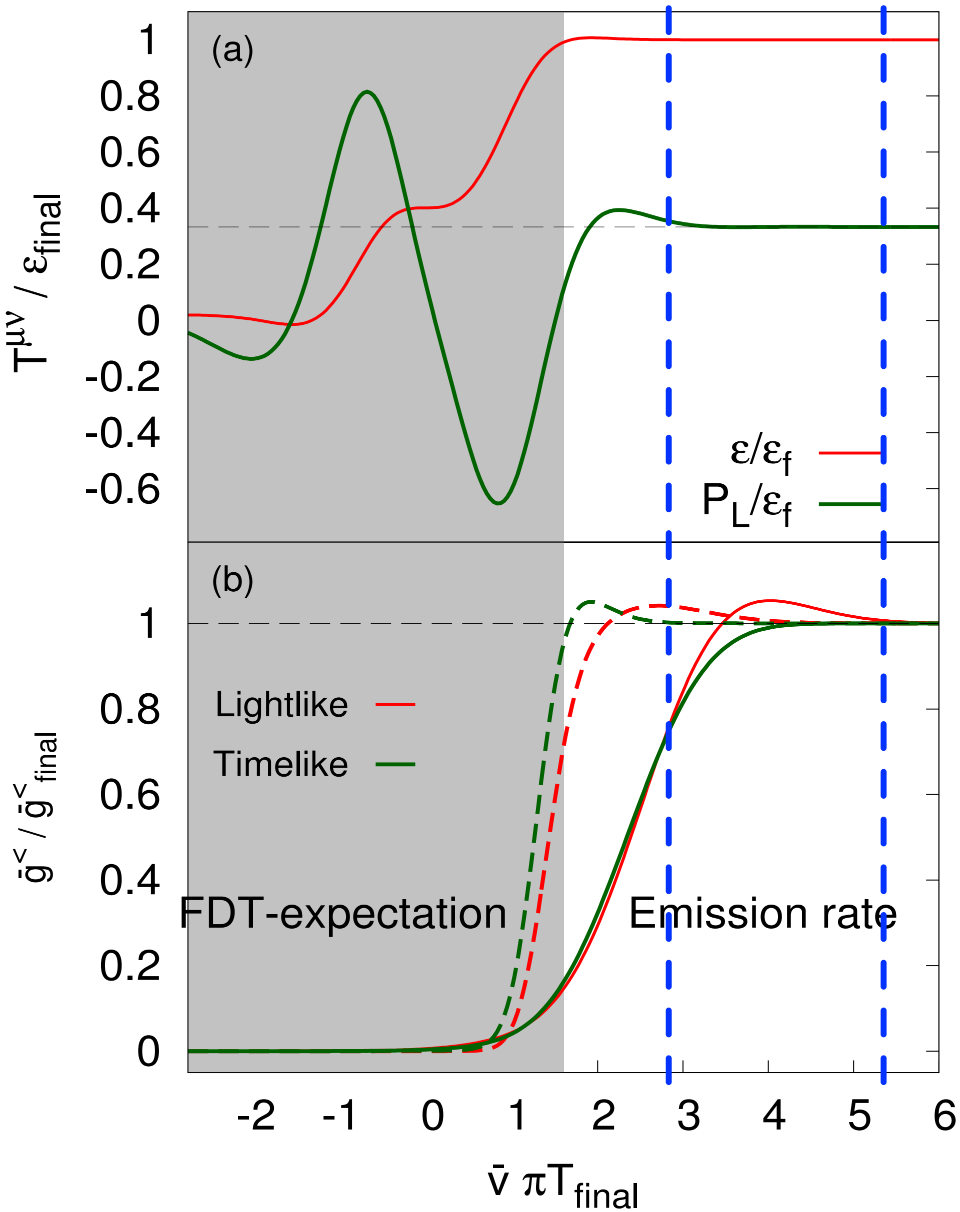
Stress Tensor

FDT satisfied

Emission & Absorption

Timelike: $\omega \simeq 8\pi T_{\mathrm{f}}$ and $\boldsymbol{q} = 0$ Lightlike: $\omega \simeq 8\pi T_{\mathrm{f}}$ and $q_T = q_L = \omega/\sqrt{2}$

Pattern of equilibration:

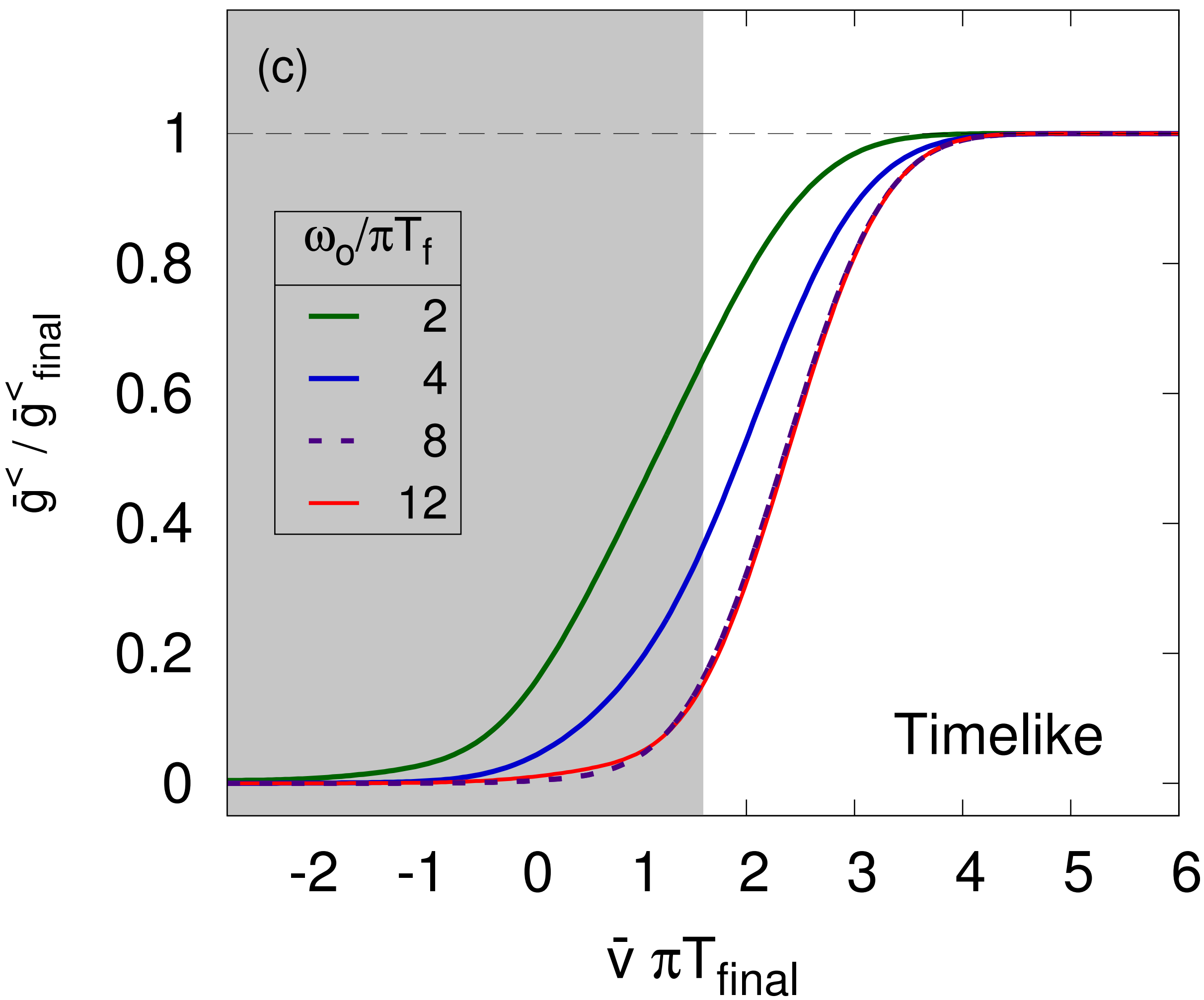


First the stress/geometry equilibrates

then

the emission rate equilibrates

Thermalization of timelike modes $\boldsymbol{q} = 0$:

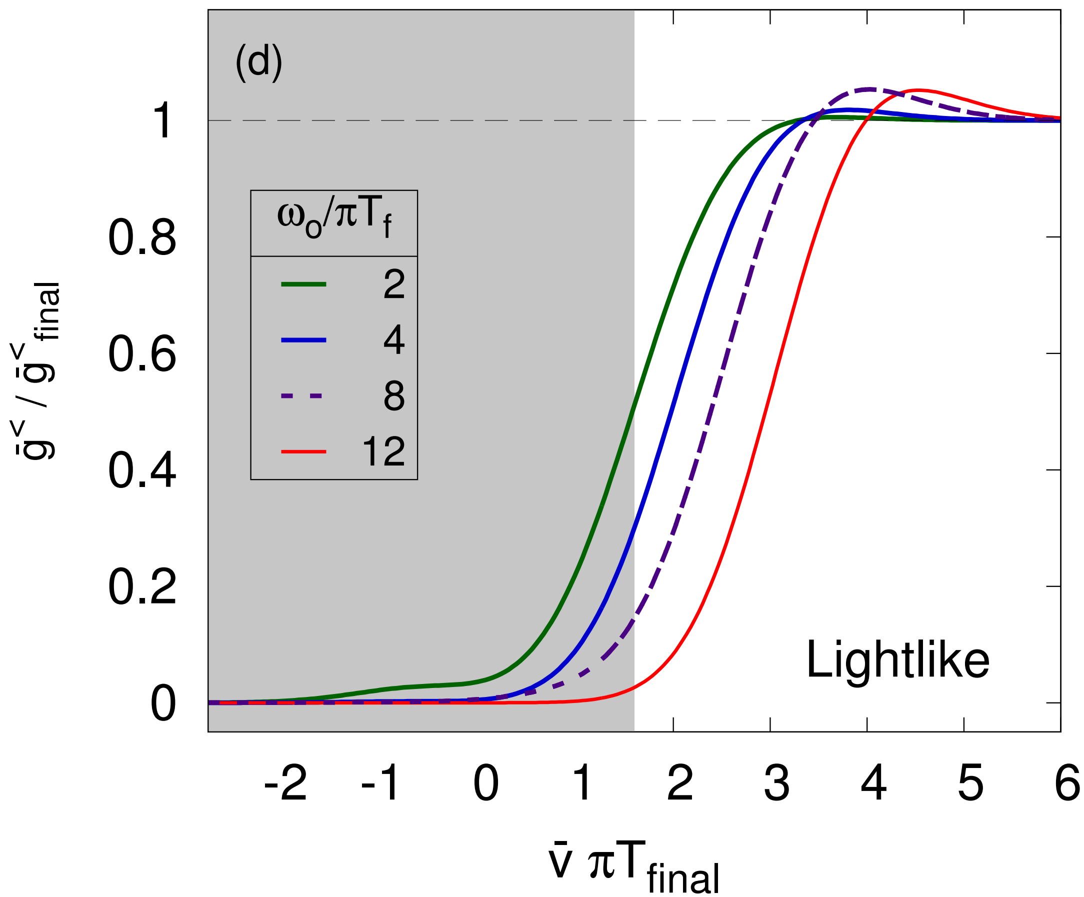


Find that massive timkelike modes thermalize in a finite time:

$$\tau_{\text{thermalize}} \sim \text{const} \qquad \omega \to \infty$$

Thermalization of approx lightlike modes ($\omega \simeq |\boldsymbol{q}|$)

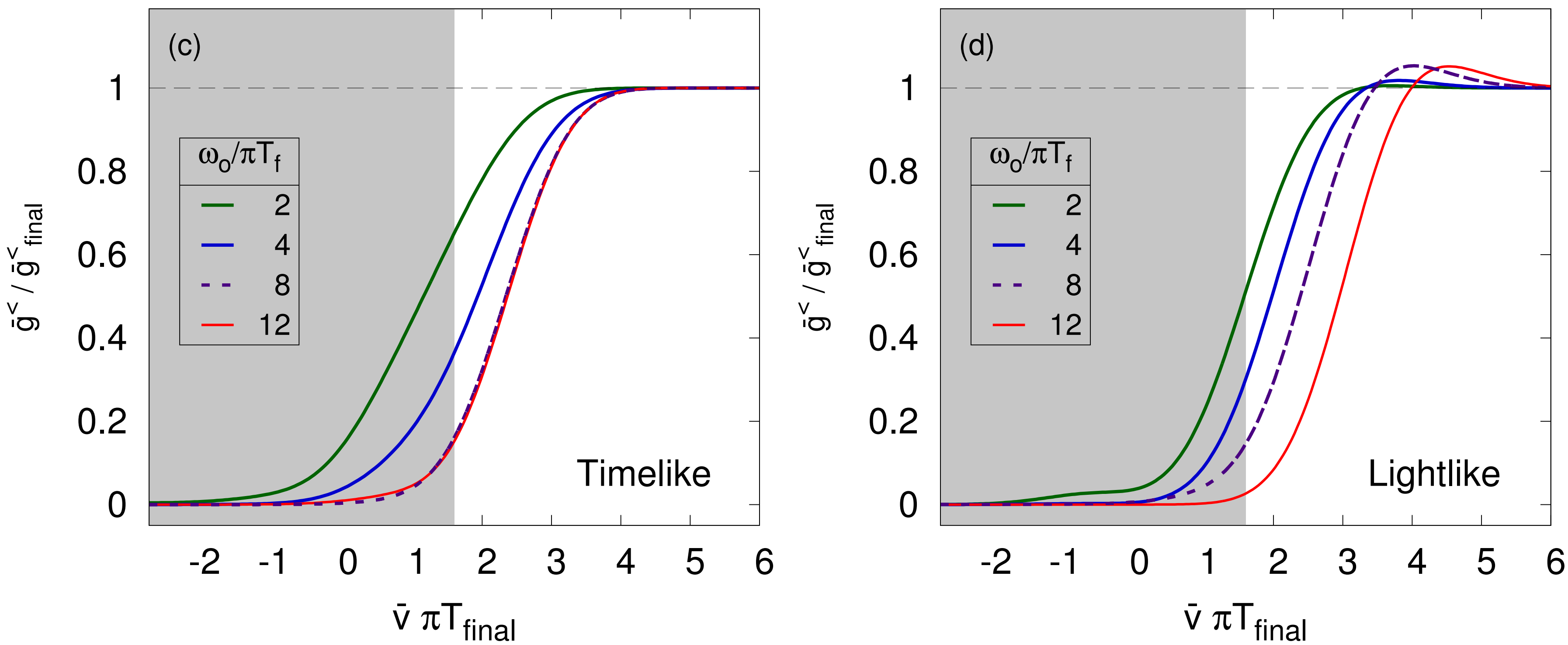
Chesler et al, Arnold&Vaman



The harder the lightlike mode, the longer it takes to equilibrate – find that

$\tau_{\rm thermalize} \sim (\omega\sigma)^{1/4}$ for $\omega \to \infty$ where $Q^2 = (\omega^2 - q^2) \sim \underbrace{\omega\sigma^{-1}}_{\text{virtuality}}$

Summary:



1. Find that massive timkelike modes thermalize in a finite time:

$$\tau_{\rm thermalize} \sim {\rm const} \qquad \omega \to \infty$$

2. The harder the lightlike mode, the longer it takes to equilibrate – expect that:

$$\tau_{\rm thermalize} \sim (\omega\sigma)^{1/4} \qquad \text{for} \qquad \omega \to \infty$$

Conclusions

- Derived Hawking Radiation for non-equilibrium geometries
  - Hawking radiation produces statistical fluctuations in strongly coupled plasma
- Used this setup to calculate emission rates in far from equilibrium plasma
- Find a distinct pattern of thermalziation (similar to weak coupling):
  1. First the stress tensor equilibrates and then the 2pnt funcs equilibrate
  2. Highly offshell modes ($\omega \to \infty$ with k fixed) thermalize first.
  3. High momentum onshell modes ($\omega \simeq k \to \infty$) thermalize last.