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Spontaneous supersymmetry breaking in noncritical covariant superstring theory

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collaboration with
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+ two forthcoming papers

1 Motivations

LHC $\rightarrow$ SUSY br. at (or just below) Planck scale?

$N \rightarrow \infty$ gauge th. or matrix models: promising candidates for nonpert. def.
SUSY: necessary for consistency of def. of quantum gravity

$\rightarrow$ "desirable" scenario:
SUSY: preserved for finite N, but gets spontaneously broken in the large-N limit

but very few examples (in spite of its importance!)

$\diamondsuit$ SUSY breaking/restoration in the large-N limit

[T.K.-Sugino '08 $\sim$]

SUSY DW MM

2 Review of SUSY double-well matrix model

SUSY double-well matrix model:

$$S = N\mathrm{tr}\left[\frac{1}{2}B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi)\right]$$

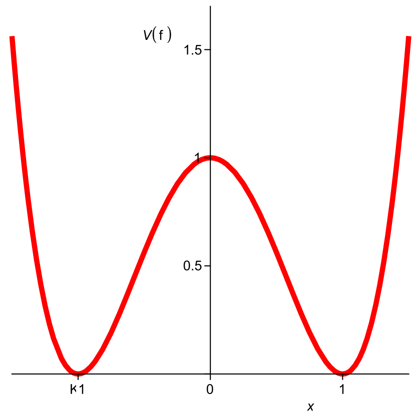
Properties:

- nilpotent SUSY:

$$Q\phi = \psi, \quad Q\psi = 0, \quad Q\bar{\psi} = -iB, \quad QB = 0,$$
$$\bar{Q}\phi = -\bar{\psi}, \quad \bar{Q}\bar{\psi} = 0, \quad \bar{Q}\psi = -iB, \quad \bar{Q}B = 0,$$

- parameters: N, μ^2; $\quad V(\phi) = \frac{1}{2}(\phi^2 - \mu^2)^2$

  finite $^\forall N$: SUSY br. $\Longleftarrow$ instanton
  ($N = 1$ case can be checked explicitly)



In terms of eigenvalues

$$Z = \int d\phi d\psi d\bar{\psi}\, e^{-N\mathrm{tr}\left(\frac{1}{2}(\phi^2-\mu^2)^2+\bar{\psi}(\phi\psi+\psi\phi)\right)}$$

$$= \int \left(\prod_i d\lambda_i\right) d\psi d\bar{\psi}\, \Delta(\lambda)^2\, e^{-N\left[\sum_i \frac{1}{2}(\lambda_i^2-\mu^2)^2+\bar{\psi}_{ij}(\lambda_j+\lambda_i)\psi_{ji}\right]}$$

$$= \int \left(\prod_i d\lambda_i\right) \prod_{i>j}(\lambda_i-\lambda_j)^2 \prod_{i,j}(\lambda_i+\lambda_j)\, e^{-N\sum_i \frac{1}{2}(\lambda_i^2-\mu^2)^2}$$

$$= \int \left(\prod_i d\lambda_i\right) \prod_i(2\lambda_i)\, \prod_{i>j}(\lambda_i^2-\lambda_j^2)^2\, e^{-N\sum_i \frac{1}{2}(\lambda_i^2-\mu^2)^2}$$

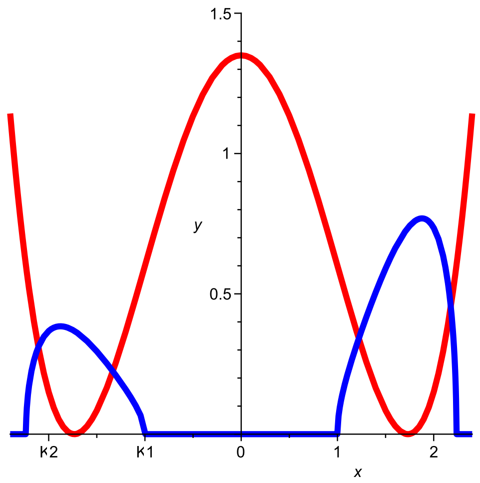
$$\mathrm{SPE:}\quad 0 = 2\sum_{j(\neq i)}\frac{1}{\lambda_i-\lambda_j} + 2\sum_j \frac{1}{\lambda_i+\lambda_j} - N(\lambda_i^2-\mu^2)\cdot 2\lambda_i$$

$$\Rightarrow \int dy\,\rho(y)\frac{P}{x-y} + \int dy\,\rho(y)\frac{P}{x+y} = x^3-\mu^2 x$$

$$\left(\rho(x) = \frac{1}{N}\mathrm{tr}\,\delta(x-\phi)\right)$$

- $N \to \infty$: two phases:

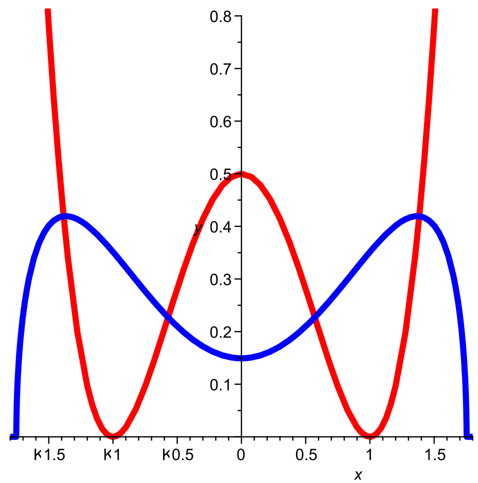
  1. $\underline{\mu^2 \geq 2\text{: two-cut phase:}}$ (ν_+, ν_-) $(\nu_+ + \nu_- = 1)$



$$\rho(x) = \begin{cases} \frac{\nu_+}{\pi} x \sqrt{(x^2 - a^2)(b^2 - x^2)} & (a < x < b) \\ \frac{\nu_-}{\pi} |x| \sqrt{(x^2 - a^2)(b^2 - x^2)} & (-b < x < -a) \end{cases}$$

$$a = \sqrt{\mu^2 - 2}, \quad b = \sqrt{\mu^2 + 2}$$

  2. $\underline{\mu^2 < 2\text{: one-cut phase:}}$



Order parameter: $(B^n = iQ(B^{n-1}\bar{\psi}) = i\bar{Q}(B^{n-1}\psi))$

$$\left\langle \frac{1}{N} \mathrm{tr}\ B^n \right\rangle \begin{aligned} &= 0 \quad \text{for } {}^\forall n \qquad &&\text{(two-cut phase)} \\ &\neq 0 \quad \text{for } n = 1 \qquad &&\text{(one-cut phase)} \end{aligned}$$

$\therefore$ $\mu^2 \geq 2$:

- SUSY vacua continuously parametrized by ν_+
- $\mu^2 = 2$: critical pt. $\longrightarrow$ SUSY/nonSUSY phase transition! (3rd)

possible to define a superstring theory by taking a double scaling limit?:

$\mu^2 \longrightarrow 2 + 0,\ N \longrightarrow \infty$ with $(\mu^2 - 2)N^*$:fixed

3 One-point function

Nicolai mapping: [Gaiotto-Rastelli-Takayanagi '04]

$X = \phi^2 - \mu^2 \implies$ Gaussian matrix model: $c = -2$ topological gravity

loop gas $(O(-2))$ model

[Kostov-Staudacher 1992]

$$\left\langle \prod_i \frac{1}{N} \mathrm{tr}\, \phi^{2n_i} \right\rangle: \text{ regular in } \mu^2 \longrightarrow 2$$

However, this model also has

$$\frac{1}{N}\mathrm{tr}\ \phi^{2n+1},\ \frac{1}{N}\mathrm{tr}\ \psi^{2n+1},\ \frac{1}{N}\mathrm{tr}\ \bar{\psi}^{2n+1}\ (n = 0, 1, 2, \cdots) \ \longrightarrow \ \text{nontrivial}$$

One-point function $(N \to \infty)$

$$\left\langle \frac{1}{N} \mathrm{tr}\ \phi^n \right\rangle_0 = \int_\Omega dx\, x^n \rho(x) \qquad (\Omega = [-b, -a] \cup [a, b])$$

$$= (\nu_+ + (-1)^n \nu_-)\, (\mu^2 + 2)^{\frac{n}{2}}\, F\left(-\frac{n}{2}, \frac{3}{2}, 3; \frac{4}{\mu^2 + 2}\right)$$

- n: even: $(\nu_+,\ \nu_-)$-indep., poly. in μ^2
- n: odd: $(\nu_+ - \nu_-)$-dep., logarithmic singular behavior:

$\omega = \dfrac{\mu^2 - 2}{4}$: deviation from the critical pt.

$$\left\langle \frac{1}{N} \mathrm{tr}\ \phi^{2k+1} \right\rangle_0 = (\nu_+ - \nu_-)\, (\mathrm{const.}) \omega^{k+2} \ln \omega + \cdots$$

4 Multi-point functions

two-point functions for boson (leading part containing log)

- $\left\langle \frac{1}{N}\mathrm{tr}\,\phi^{2k}\frac{1}{N}\mathrm{tr}\,\phi^{2\ell}\right\rangle_C$: indep. of (ν_+,ν_-), poly. of μ^2
- $\left\langle \frac{1}{N}\mathrm{tr}\,\phi^{2k+1}\frac{1}{N}\mathrm{tr}\,\phi^{2\ell}\right\rangle_C \sim (\nu_+-\nu_-)\,(\mathrm{const.})\omega^{k+1}\ln\omega$
- $\left\langle \frac{1}{N}\mathrm{tr}\,\phi^{2k+1}\frac{1}{N}\mathrm{tr}\,\phi^{2\ell+1}\right\rangle_C \sim (\nu_+-\nu_-)^2\,(\mathrm{const.})\omega^{k+\ell+1}\,(\ln\omega)^2$

$$\therefore \quad \left\langle \frac{1}{N}\mathrm{tr}\,\phi^{2k_1+1}\cdots\frac{1}{N}\mathrm{tr}\,\phi^{2k_n+1}\right\rangle_{C,0} \sim (\nu_+-\nu_-)^n\,(\mathrm{const.})\omega^{2-\gamma+\sum_{i=1}^{n}(k_i-1)}\,(\ln\omega)^n+\cdots$$

- confirmed for general 2-pt functions, first two simplest 3-pt. functions
- new critical behavior as power of log
- $\gamma=-1$: string susceptibility of $c=-2$ topological gravity
  $\Rightarrow$ double scaling limit: $N^2\omega^3\sim 1/g_s^2$: fixed

two-point function for fermion

$$\left\langle \frac{1}{N}\mathrm{tr}\,\psi^{2k+1}\frac{1}{N}\mathrm{tr}\,\bar{\psi}^{2l+1}\right\rangle = \delta_{kl}(\nu_+ - \nu_-)^{2k+1}\omega^{2k+1}\ln\omega + \cdots$$

- confirmed up to $k, l = 0, 1$

5 Correspondence to $D = 2$ IIA superstring

logarithmic sigularity → scaling violation in bosonic string in $D = 2$

[Brezin, Kazakov Zamolodchikov 1990]

[Gross-Klebanov 1990][Polchinski 1990]

"new" MM interpretation: matrix=field on target space (cf. "old" MM)

→ $D = 2$ superstring theory with unbroken target space SUSY

→ $D = 2$ IIA superstring theory [Kutasov-Seiberg 1990][Ita-Nieder-Oz '05]

- action: $\mathcal{N} = 2$ Liouville theory

$$S = \frac{1}{2\pi}\int d^2z\Big(\partial x\bar\partial x + \partial\varphi\bar\partial\varphi + \frac{Q}{4}R\varphi + g_\pm(\psi_l \pm i\psi_x)(\bar\psi_l \pm i\bar\psi_x)e^{\frac{1}{Q}(\varphi\pm ix)}$$
$$+ \text{fermion kin. terms}\Big)$$

i.e. target sp. (x,φ): $2D$, $Q = 2$

- target sp. SUSY:

$$\text{L}: \quad q_+ = e^{-\frac{1}{2}\phi-\frac{i}{2}H-ix}, \quad \text{R}: \quad \bar q_- = e^{-\frac{1}{2}\bar\phi+\frac{i}{2}\bar H+i\bar x} \quad (\psi_l \pm i\psi_x = \sqrt{2}e^{\mp iH})$$

$$\Rightarrow \quad Q_+^2 = \bar Q_-^2 = \{Q_+, \bar Q_-\} = 0: \quad \text{nilpotent!, no spacetime translation}$$

- physical vertex ops.:
  - NS: (-1)-picture tachyon: $T_k(z) = e^{-\phi + ikx + p_l \varphi}(z)$
    with $p_l = 1 - |k|$ (Seiberg bound)
  - R: (-1/2)-picture R field: $V_{k,\epsilon}(z) = e^{-\frac{1}{2}\phi + \frac{i}{2}\epsilon H + ikx + p_l \varphi}(z)$
- both WS & TS SUSY $\to x \in S^1$ with $R = 2/Q = 1$ (self-dual radius)
- physical states: (winding background)

$(\mathrm{NS}, \mathrm{NS})$	$T_k \bar{T}_{-k}$	$k \in Z + 1/2$
$(\mathrm{R}+, \mathrm{R}-)$	$V_{k,+1} \bar{V}_{-k,-1}$	$k \in Z_{\geq 0} + 1/2$
$(\mathrm{R}-, \mathrm{R}+)$	$V_{k,-1} \bar{V}_{-k,+1}$	$k \in Z_{\leq 0}$
$(\mathrm{NS}, \mathrm{R}-)$	$T_k \bar{V}_{k,-1}$	$k \in Z_{\leq 0} - 1/2$
$(\mathrm{R}+, \mathrm{NS})$	$V_{k,+1} \bar{T}_k$	$k \in Z_{\geq 0} + 1/2$

"new" matrix model interpretation $\Rightarrow$ natural to identify

$$\frac{1}{N} \mathrm{tr}\, \psi \quad \longleftrightarrow \quad (\mathrm{NS}, \mathrm{R}) \qquad\qquad \frac{1}{N} \mathrm{tr}\, \bar{\psi} \quad \longleftrightarrow \quad (\mathrm{R}, \mathrm{NS})$$

$\Rightarrow$ how about boson?

SUSY multiplet: identify $(Q, \bar{Q})$ in MM $\Longleftrightarrow$ $(Q_+, \bar{Q}_-)$ in IIA

lowest momentum $(k = \pm\frac{1}{2})$ sector:

$(\mathrm{R}+, \mathrm{R}-)$		$V_{\frac{1}{2},+1}\bar{V}_{-\frac{1}{2},-1}$		$\mathrm{tr}\ \phi$
	$Q_+ \swarrow$		$\searrow \bar{Q}_-$	$Q \swarrow \quad \searrow \bar{Q}$
$(\mathrm{NS}, \mathrm{R}), (\mathrm{R}, \mathrm{NS})$	$T_{-\frac{1}{2}}\bar{V}_{-\frac{1}{2},-1}$		$V_{\frac{1}{2},+1}\bar{T}_{\frac{1}{2}}$	$\mathrm{tr}\ \psi \qquad \mathrm{tr}\ \bar{\psi}$
	$\bar{Q}_- \searrow$		$\swarrow Q_+$	$\bar{Q} \searrow \quad \swarrow Q$
$(\mathrm{NS}, \mathrm{NS})$		$T_{-\frac{1}{2}}\bar{T}_{\frac{1}{2}}$		$\mathrm{tr}\ B$

i.e.

$$\frac{1}{N}\mathrm{tr}\,\psi \longleftrightarrow (\mathrm{NS}, \mathrm{R}) \quad T_{-\frac{1}{2}}\bar{V}_{-\frac{1}{2},-1} \qquad \frac{1}{N}\mathrm{tr}\,\bar{\psi} \longleftrightarrow (\mathrm{R}, \mathrm{NS}) \quad V_{\frac{1}{2},+1}\bar{T}_{\frac{1}{2}}$$

$$\frac{1}{N}\mathrm{tr}\,\phi \longleftrightarrow (\mathrm{R}+, \mathrm{R}-) \quad V_{\frac{1}{2},+1}\bar{V}_{-\frac{1}{2},-1} \qquad \frac{1}{N}\mathrm{tr}\,B \longleftrightarrow (\mathrm{NS}, \mathrm{NS}) \quad T_{-\frac{1}{2}}\bar{T}_{\frac{1}{2}}$$

Then

Z_2-symmetry in MM: $\bar{\psi} \longrightarrow -\bar{\psi},\ \phi \longrightarrow -\phi$

$$S = N\mathrm{tr}\left[\frac{1}{2}B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi)\right]$$

automatically realized as $(-1)^{F_L}$ symmetry in IIA: $V_{\frac{1}{2},+1} \longrightarrow -V_{\frac{1}{2},+1}$

observations:

- where's information of momentum?

  cf. Penner model [Distler-Vafa '90][Mukhi '03]

$$Z(t,\bar{t}) = \int dM\, e^{\mathrm{tr}\left(-\nu M + (\nu - N)\log M - \sum_{k=1}^{\infty} \bar{t}_k (MA^{-1})^k\right)}, \qquad t_k = \frac{1}{\nu k}\mathrm{tr}\, A^{-k}$$

$$\rightarrow \quad \langle T_{k_1} \cdots T_{k_m} T_{-l_1} \cdots T_{-l_n} \rangle_{c=1,R=1} = \frac{\partial}{\partial t_{k_1}} \cdots \frac{\partial}{\partial t_{k_m}} \frac{\partial}{\partial \bar{t}_{k_1}} \cdots \frac{\partial}{\partial \bar{t}_{k_n}} F(t,\bar{t}) \Big|_{t,\bar{t}=0}$$

  $\rightarrow$ power of matrices

- $\left\langle \frac{1}{N}\mathrm{tr}\, \phi^{2k+1} \right\rangle_0 \neq 0$ (logarithmic behavior)?

  (R+, R–) one-point function$\neq 0 \rightarrow$ RR background!

- missing (R–, R+) sector?

  not in MM (i.e. (asymptotic) target sp. fields),

  but this must be a background in IIA!

  In fact, (R–,R+)-sector: Q_+, $\bar{Q}_-$-singlet. $\rightarrow$ taget sp. SUSY inv.!

Claim

SUSY DW MM = $2D$ type IIA at the level of correlation functions under:

$$\frac{1}{N}\mathrm{tr}\ \phi^{2k+1} \quad \Leftrightarrow \quad (\mathrm{R+,R-}): \quad \int d^2z\, V_{k+\frac{1}{2},+1}(z)\, \bar{V}_{-k-\frac{1}{2},-1}(\bar{z})$$

$$\frac{1}{N}\mathrm{tr}\ \psi^{2k+1} \quad \Leftrightarrow \quad (\mathrm{NS,R-}): \quad \int d^2z\, T_{-k-\frac{1}{2}}(z)\, \bar{V}_{-k-\frac{1}{2},-1}(\bar{z})$$

$$\frac{1}{N}\mathrm{tr}\ \bar{\psi}^{2k+1} \quad \Leftrightarrow \quad (\mathrm{R+,NS}): \quad \int d^2z\, V_{k+\frac{1}{2},+1}(z)\, \bar{T}_{k+\frac{1}{2}}(\bar{z})$$

$$\frac{1}{N}\mathrm{tr}\ B \quad \Leftrightarrow \quad (\mathrm{NS,NS}): \quad \int d^2z\, T_{-\frac{1}{2}}(z)\, \bar{T}_{\frac{1}{2}}(\bar{z})$$

where the IIA correlation functions are

$$\left\langle\!\!\left\langle \prod_i \int d^2z_i V_i(z_i,\bar{z}_i) \right\rangle\!\!\right\rangle$$

$$= \left\langle \prod_i \int d^2z_i V_i(z_i,\bar{z}_i)\, e^{(\nu_+-\nu_-)\sum_{k\in Z} a_k \omega^{k+1}\int d^2z\, V_{-|k|,-1}\bar{V}_{|k|,+1}} \right\rangle_{\mathcal{N}=2\,\mathrm{Liouville}}$$

with

$$\underline{\omega = g_-}, \qquad g_+ = 0$$

Note:

- RR flux term: $(\nu_+ - \nu_-) \sum_{k\in Z} a_k \omega^{k+1} \int d^2z V_{-|k|,-1} \bar{V}_{|k|,+1}$

  $(\nu_+ - \nu_-)$: RR flux source [Takayanagi '04]

  $k > 0$: wrong branch breaking Seiberg bound:

  $$V^{(\mathrm{NL})}_{-k,-1} = e^{-\frac{\phi}{2} - \frac{i}{2}H - ikx + p_l \varphi} \quad \text{with} \quad p_l = 1 + |k|$$

  : nonlocal disturbance on string WS

- MM & IIA action:

$$S_{\mathrm{MM}} = N \mathrm{tr} \left[\frac{1}{2} B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi) \right], \quad \omega = \frac{\mu^2 - 2}{4}$$

$$S_{\mathrm{IIA}} = \frac{1}{2\pi} \int d^2z \Big(\partial x \bar{\partial} x + \partial\varphi \bar{\partial}\varphi + \frac{Q}{4} \sqrt{g} R \varphi + g_- \underbrace{(\psi_l - i\psi_x)(\bar{\psi}_l - i\bar{\psi}_x) e^{\frac{1}{Q}(\varphi - ix)}}_{\propto T_{-\frac{1}{2}} \bar{T}_{\frac{1}{2}}} + \cdots \Big)$$

$$\therefore \quad \partial_\omega \propto \mathrm{tr}\ B \quad \Longleftrightarrow \quad \int d^2z\, T_{-\frac{1}{2}} \bar{T}_{\frac{1}{2}} \propto \partial_{g_-}$$

Examples:

- $$\left\langle \frac{1}{N}\mathrm{tr}\,(-iB)\frac{1}{N}\mathrm{tr}\,\phi^{2k+1}\right\rangle_{C,0} = \frac{1}{4}\partial_\omega \left\langle \frac{1}{N}\mathrm{tr}\,\phi^{2k+1}\right\rangle_0 = (\nu_+ - \nu_-)c_k\omega^{k+1}\ln\omega$$

$$\left\langle\!\!\left\langle \int d^2z_1 T_{-\frac{1}{2}}(z_1)\bar{T}_{\frac{1}{2}}(\bar{z}_1) \int d^2z_2 V_{k+\frac{1}{2},+1}(z_2)\bar{V}_{-k-\frac{1}{2},-1}(\bar{z}_2)\right\rangle\!\!\right\rangle$$

$$= \left\langle \int d^2z_1 T_{-\frac{1}{2}}(z_1)\bar{T}_{\frac{1}{2}}(\bar{z}_1) \int d^2z_2 V_{k+\frac{1}{2},+1}(z_2)\bar{V}_{-k-\frac{1}{2},-1}(\bar{z}_2) \times (\nu_+ - \nu_-)a_k\omega^{k+1}\int d^2z\, V^{(\mathrm{NL})}_{-k,-1}(z)\bar{V}^{(\mathrm{NL})}_{k,+1}(\bar{z})\right\rangle$$

$$= (\nu_+ - \nu_-)a_k\,\omega^{k+1}\cdot \underbrace{2\ln g_-}_{\text{Liouville vol.}} \qquad (a_k:\ \text{finite via } {}^{\exists}\text{reguarlization})$$

$$\bullet \left\langle \frac{1}{N}\mathrm{tr}\,\phi^{2k+1}\frac{1}{N}\mathrm{tr}\,\phi^{2\ell+1}\right\rangle_{C,0} = (\nu_+ - \nu_-)^2 c_{kl}\omega^{k+\ell+1}(\ln\omega)^2$$

$$\left\langle\!\!\left\langle \int d^2z_1 V_{k+\frac{1}{2},+1}(z_1)\bar{V}_{-k-\frac{1}{2},-1}(\bar{z}_1)\int d^2z_2 V_{\ell+\frac{1}{2},+1}(z_2)\bar{V}_{-\ell-\frac{1}{2},-1}(\bar{z}_2)\right\rangle\!\!\right\rangle$$

$$= \Big\langle \int d^2z_1 V_{k+\frac{1}{2},+1}(z_1)\bar{V}_{-k-\frac{1}{2},-1}(\bar{z}_1)\int d^2z_2 V_{\ell+\frac{1}{2},+1}(z_2)\bar{V}_{-\ell-\frac{1}{2},-1}(\bar{z}_2)$$

$$\times (\nu_+ - \nu_-)a_{-1}\omega^{-1+1}\int d^2z\, V_{-1,-1}(z)\bar{V}_{1,+1}(\bar{z})$$

$$\times (\nu_+ - \nu_-)a_{k+\ell}\omega^{k+\ell+1}\int d^2w\, V^{(\mathrm{NL})}_{-k-\ell,-1}(w)\bar{V}^{(\mathrm{NL})}_{k+\ell,+1}(\bar{w})\Big\rangle$$

$$= (\nu_+ - \nu_-)^2 a_{-1}a_{k+\ell}C_{kl}\,\omega^{k+\ell+1}(2\ln g_-)^2$$

similar for fermion 2-pt. function

strong evidence that our matrix model provides nonperturbative def. of $D=2$ IIA superstring theory in the RR-background!

6 Spontaneous SUSY breaking of superstring

Let's try to compute SUSY br. order parameter exactly in the DSL

order parameter:

$$\left\langle \frac{1}{N}\mathrm{tr}\, B \right\rangle \left(= -\frac{i}{4N^2}\partial_\omega F \right) \quad (\text{recall } iQ\mathrm{tr}\,(\bar{\psi}) = \mathrm{tr}\, B)$$

$$\left\langle \frac{1}{N}\mathrm{tr}\, B \right\rangle = -i\underline{\left\langle \frac{1}{N}\mathrm{tr}\,(\phi^2 - \mu^2) \right\rangle} = \left\langle\!\!\left\langle \int d^2z\, T_{-\frac{1}{2}}(z)\bar{T}_{\frac{1}{2}}(\bar{z}) \right\rangle\!\!\right\rangle$$

Nicolai mapping

$$X = \phi^2 - \mu^2 \quad \text{or} \quad x_i = \lambda_i^2 - \mu^2$$

$$Z = \int dBdX\, e^{-N\mathrm{tr}\,\left(\frac{1}{2}B^2 + iBX\right)} = \prod_i \left(\int_{-\mu^2}^{\infty} dx_i \right) \prod_{i>j} (x_i - x_j)^2 e^{-N\sum_i \frac{1}{2}x_i^2}$$

if we can ignore effect of boundary

$$\left\langle \frac{1}{N}\mathrm{tr}\, B^n \right\rangle = \frac{1}{Z}\int dB \int dX \frac{1}{N}\mathrm{tr}\, B^n e^{-N\mathrm{tr}\,\left(\frac{1}{2}B^2 + iBX\right)} = 0 \quad \text{for } {}^\forall n \in N$$

$$\to \left\langle \frac{1}{N}\mathrm{tr}\, B^n \right\rangle = 0 \quad \text{i.e.} \quad \text{SUSY for all order of } 1/N\text{-expansion}$$

exact calculation: evaluation of boundary effect

$$Z = \int_{-\mu^2}^{\infty} \left(\prod_i dx_i \right) \Delta(x)^2 e^{-\frac{N}{2}\sum_i x_i^2}$$

orthogonal polynomial:

$$P_n(x) = x^n + \mathcal{O}(x^{n-1}),$$

$$(P_n, P_m) \equiv \int_{-\mu^2}^{\infty} dx\, e^{-\frac{N}{2}x^2} P_n(x) P_m(x) = h_n \delta_{nm}$$

$$\to \quad x P_n(x) = P_{n+1}(x) + s_n P_n(x) + r_n P_{n-1}(x) \quad \text{e.g.} \quad r_n = \frac{h_n}{h_{n-1}}$$

$$\left\langle \frac{1}{N} \mathrm{tr}\,(\phi^2 - \mu^2) \right\rangle = \frac{1}{N} \sum_{k=0}^{N-1} s_k$$

without boundary, $\quad P_n(x) = H_n(\sqrt{N}x)$

$$x H_n(x) = H_{n+1}(x) + n H_{n-1}(x) \quad \longrightarrow \quad s_n = 0, \quad r_n = \tfrac{n}{N}$$

However, taking account of the boundary,

$$P_1(x) = x + c,$$

$$0 = (P_0, P_1) = \int_{-\mu^2}^{\infty} dx\, e^{-\frac{N}{2}x^2} 1 \cdot (x + c) = \int_{-\mu^2}^{\infty} dx\, e^{-\frac{N}{2}x^2} x + ch_0$$

$$= \frac{1}{N} e^{-\frac{N}{2}\mu^4} + ch_0, \qquad h_0 = (P_0, P_0) = \int_{-\mu^2}^{\infty} dx\, e^{-\frac{N}{2}x^2}$$

$$\therefore \quad c = -\frac{1}{N}\frac{1}{h_0} e^{-\frac{N}{2}\mu^4} = s_0 \neq 0$$

In general, $\quad s_k = \dfrac{1}{N}\dfrac{1}{h_n} P_k(-\mu^2)^2 e^{-\frac{N}{2}\mu^4}$

nonperturbative effect: $\exp(-NC)$ makes s_n nonvanishing!!

boundary effect $\Longleftrightarrow$ nonperturbative effect

[Gaiotto-Rastelli-Takayanagi '04]

$$\left\langle \frac{1}{N}\mathrm{tr}\, B \right\rangle = \frac{1}{32\pi N^2 \omega} e^{-\frac{32}{3}N\omega^{\frac{3}{2}}} + \mathcal{O}(e^{-\frac{64}{3}N\omega^{\frac{3}{2}}}) \quad \text{for} \quad N\omega^{\frac{3}{2}} \sim \frac{1}{g_s} : \ \text{fixed, large}$$

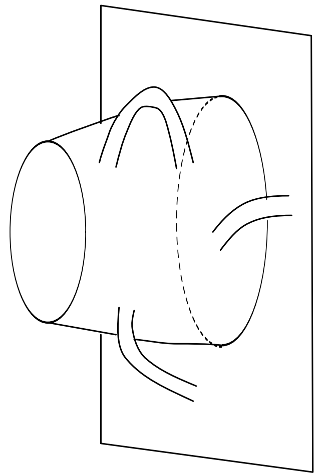
$$\longrightarrow \quad F = \frac{1}{128\pi}\frac{1}{N\omega^{\frac{3}{2}}} e^{-\frac{32}{3}N\omega^{\frac{3}{2}}} + \mathcal{O}(e^{-\frac{64}{3}N\omega^{\frac{3}{2}}})$$

Note:

- zero in all orders in $1/N$-expansion, but nonperturbatively nonzero due to boundary effect $\longrightarrow$ spontaneous breaking of SUSY in SUSY DW MM
- finite in the double scaling limit (cf. correlation functions)
- TS SUSY can be broken in nonperturbative superstring theory (we DO NOT put a D-brane by hand!! RR flux DOES NOT break SUSY) "D-brane superposition" triggers SU$\!\!\!/$SY
- exact result in the one-instanton sector by $\mathrm{Ai}(t)$:

$$\mathrm{Ai}'(4/g_s)^2 - \frac{4}{g_s}\mathrm{Ai}(4/g_s)^2$$

(∴ disk amp. with arbitrary holes and handles)



Physical interpretation

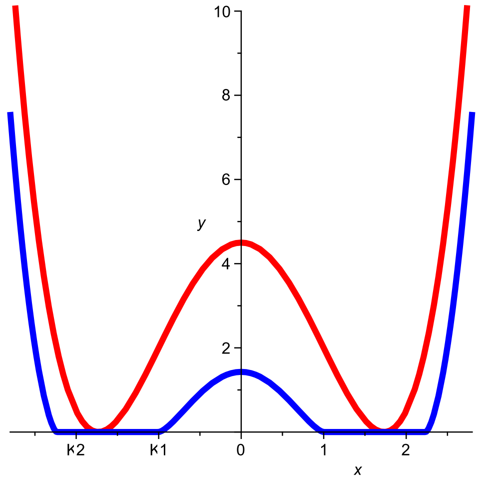
MM instanton action

$$V_{\rm eff}^{(0)}(0) - V_{\rm eff}^{(0)}(a) = \int_0^{a^2} dy \sqrt{(y-\mu^2)^2 - 4}$$

$$= \frac{1}{2}\mu^2\sqrt{\mu^4 - 4} + 2\log\left(\frac{\mu^2 - \sqrt{\mu^4 - 4}}{2}\right)$$

(complete agreement with OP)

$$\rightarrow \quad \frac{32}{3}\omega^{\frac{3}{2}} \qquad \left(\omega = \frac{\mu^2 - 2}{4}\right)$$

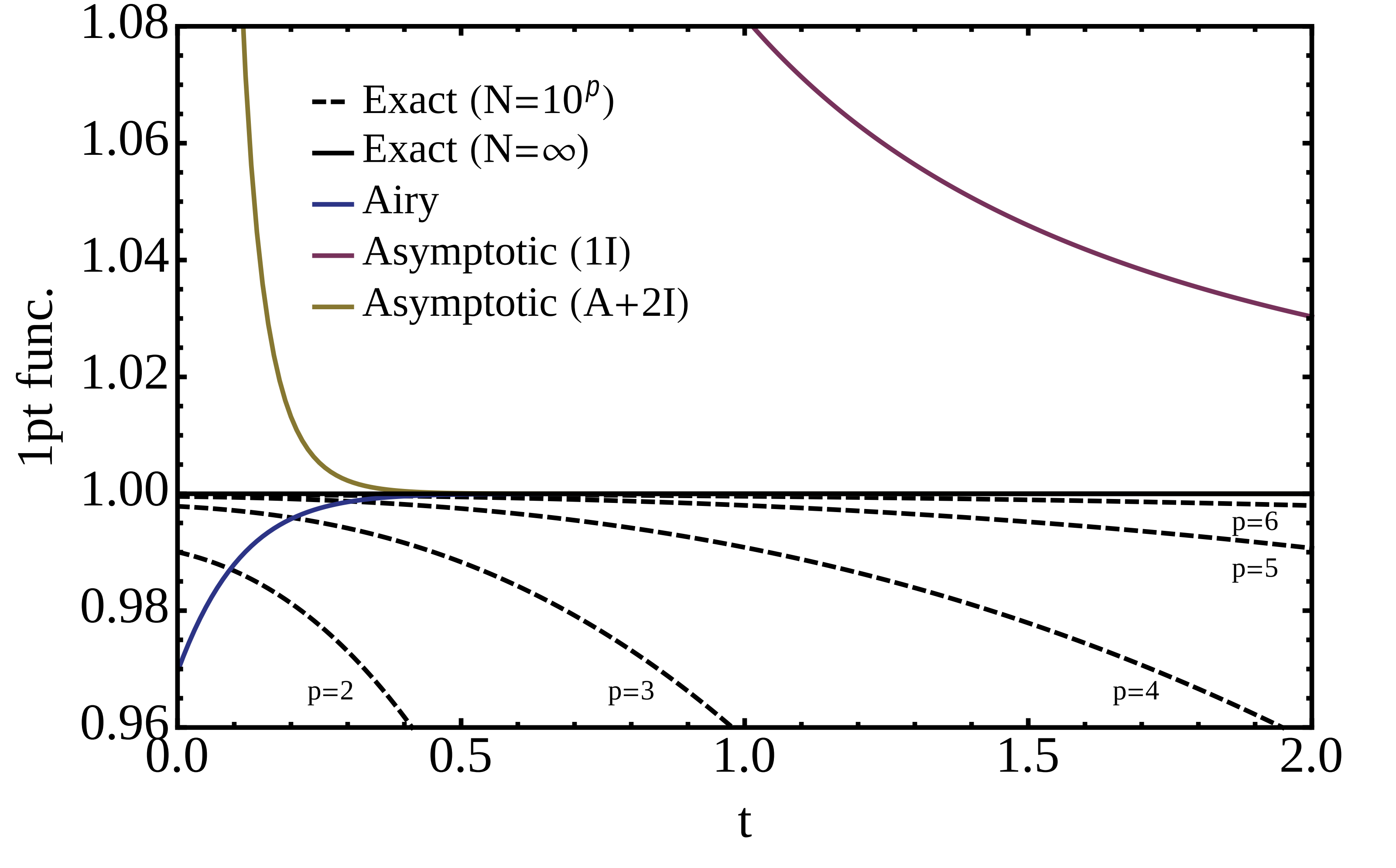


→ eigenvalue tunneling, condensation of D-brane? [Hanada et. al. '04]

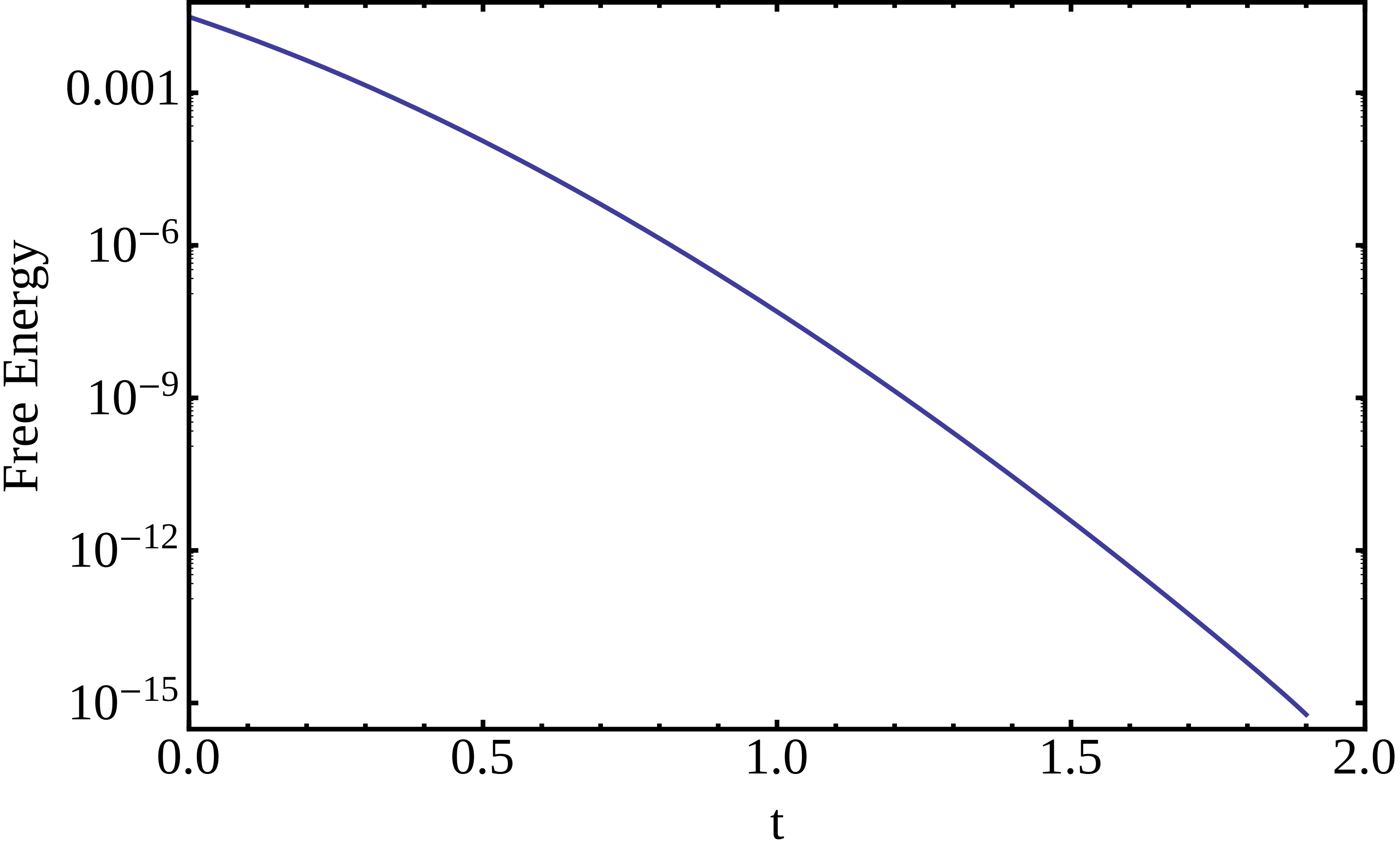
♠ SUSY br. nonpert. effect ⟵ boundary of Nicolai mapping $x = -\mu^2$
$\Longleftrightarrow$ $\lambda = 0$ the instanton is located

"dramatic" story!!

finite N: SU$\not{S}$Y (MM instanton), $N \rightarrow \infty$: SUSY (by $\exp(-NC)$),
double scaling lim.: SU$\not{S}$Y (MM instanton with finite action)



1.08
1.06
1.04
1.02
1.00
0.98
0.96
0.0
0.5
1.0
1.5
2.0
t
1pt func.
Exact (N=10^p)
Exact (N=∞)
Airy
Asymptotic (1I)
Asymptotic (A+2I)
p=2
p=3
p=4
p=5
p=6



Free Energy
0.001
10^{-6}
10^{-9}
10^{-12}
10^{-15}
0.0
0.5
1.0
1.5
2.0
t

7 Conclusions & Discussions

- at last we would get nonperturbative formulation of covariant superstring theory with (perturbatively) unbroken target space SUSY!
  (target sp. interpretation)
  ※ agreement in fundamental correlation functions cf. Kaku-Kikkawa
  not in D-brane decay rate [Takayanagi '04]
- But nonperturbatively, target space SUSY is broken spontaneously without introducing source for it by hand.
  ※ Even quite difficult in field theory case
- noncritical (restricted to $R = 1$), nilpotent SUSY
  SUSY version of Penner model
- " matrix reloaded " interpretation:
  origin of MM: effective aciton on IIA D-particle?
  (power=winding or momentum $\rightarrow$ large-N reduced model?)
- origin of breakdown of Seiberg bound? (D-brane?)
- identification of missing states (positive winding tachyon, discrete states, $\cdots$), more general correlation functions, $s = 1$ correlation functions, $\cdots$

- **usefulness of orthogonal polynomial with boundary, or Nicolai mapping**
  $\longrightarrow$ application of Yang-Mills type?
  (essentially Gaussian, but taking account of boundaries)
- **SUSY is not for using it, but (may be) for breaking it!**

A Sectors for finite N

define the sector with (ν_+, ν_-) for finite N:

decomposition of integration region of eigenvalues:

divide the integration region for each λ_i:

$$\int_{-\infty}^{\infty} d\lambda_i = \int_{-\infty}^{0} d\lambda_i + \int_{0}^{\infty} d\lambda_i$$

$\rightarrow$ (ν_+, ν_-)-sector:

$\nu_+ N$ eigenvalues integrated over $R_{\geq 0}$, $\nu_- N$ ones over $R_{\leq 0}$

$$Z = \sum_{\nu_+ N = 0}^{N} {}_N C_{\nu_+ N} Z_{(\nu_+,\nu_-)}$$

$$Z_{(\nu_+,\nu_-)} = \prod_{i=1}^{\nu_+ N} \int_0^{\infty} (d\lambda_i 2\lambda_i) \prod_{j=\nu_+ N+1}^{N} \int_{-\infty}^{0} (d\lambda_j 2\lambda_j) \times \prod_{i>j} (\lambda_i^2 - \lambda_j^2)^2 \, e^{-N \sum_i \frac{1}{2}(\lambda_i^2 - \mu^2)^2}$$

flipping sign: $\lambda_j \rightarrow -\lambda_j$ $\longrightarrow$ $Z_{(\nu_+,\nu_-)} = (-1)^{\nu_- N} Z_{(1,0)}$

Note:

$Z = 0$: **corresponding to "Witten index" (SUSY breaking case)**

this argument can be applied to correlation func. (confirmed up to 3-pt.):

$$\frac{1}{N}\mathrm{tr}\,\phi^{2n} \propto (\nu_+ + \nu_-) = 1, \quad \frac{1}{N}\mathrm{tr}\,\phi^{2n+1} \propto (\nu_+ - \nu_-)$$

simple (ν_+, ν_-)-dep. $\longrightarrow$ calculations can be reduced to $(1, 0)$-sector

B $\mathcal{N}=2$ Liouville theory

$\mathcal{N}=(2,2)$ WS SUSY, flat Euclidean:

$$S = \frac{1}{8\pi}\int d^2z d\theta^+\theta^- d\bar{\theta}^+ d\bar{\theta}^- \Phi\bar{\Phi} + \frac{g}{2\pi}\int d^2z d\theta^+ d\bar{\theta}^+ e^{-\frac{1}{Q}\Phi} + \frac{\bar{g}}{2\pi}\int d^2z d\theta^- d\bar{\theta}^- e^{-\frac{1}{Q}\bar{\Phi}}$$

Φ: chiral s.f.:

$$\left(\frac{\partial}{\partial\theta^-} - i\theta^+\partial\right)\Phi = \left(\frac{\partial}{\partial\bar{\theta}^-} - i\bar{\theta}^+\bar{\partial}\right)\Phi = 0,$$
$$\left(\frac{\partial}{\partial\theta^+} - i\theta^-\partial\right)\Phi = \left(\frac{\partial}{\partial\bar{\theta}^+} - i\bar{\theta}^-\bar{\partial}\right)\Phi = 0$$

$$\rightarrow \quad \Phi = \phi + i\sqrt{2}\theta^+\psi_+ + i\sqrt{2}\bar{\theta}^+\bar{\psi}_+ + 2\theta^+\bar{\theta}^+F + \cdots,$$
$$\bar{\Phi} = \bar{\phi} + i\sqrt{2}\theta^-\psi_- + i\sqrt{2}\bar{\theta}^-\bar{\psi}_- + 2\theta^-\bar{\theta}^-\bar{F} + \cdots$$

$$\rightarrow \quad S = \frac{1}{2\pi}\int d^2z\left(\partial x\bar{\partial}x + \partial\varphi\bar{\partial}\varphi + \psi_+\bar{\partial}\psi_- + \bar{\psi}_+\partial\bar{\psi}_-\right) + \frac{ig}{\pi Q^2}\int d^2z\psi_+\bar{\psi}_+ e^{-\frac{1}{Q}\phi} + \frac{i\bar{g}}{\pi Q^2}\int d^2z\psi_-\bar{\psi}_- e^{-\frac{1}{Q}\bar{\phi}}$$

$$\phi = -\varphi + ix, \quad \text{rescaled } \psi_\pm = -\psi_l \mp i\psi_x, \quad F = \bar{F} = 0,$$

curved sp. → linear dilation ($\mathcal{N} = 2$ WS superconf. alg.):

$$S = \frac{1}{2\pi}\int d^2z\Big(\partial x\bar{\partial}x + \partial\varphi\bar{\partial}\varphi + \frac{Q}{4}\sqrt{g}R\varphi + g_\pm(\psi_l \pm i\psi_x)(\bar{\psi}_l \pm i\bar{\psi}_x)e^{\frac{1}{Q}(\varphi \pm ix)}$$

$$+ \text{fermion kin. terms}\Big)$$

C Instanton action in SUSY double-well matrix model

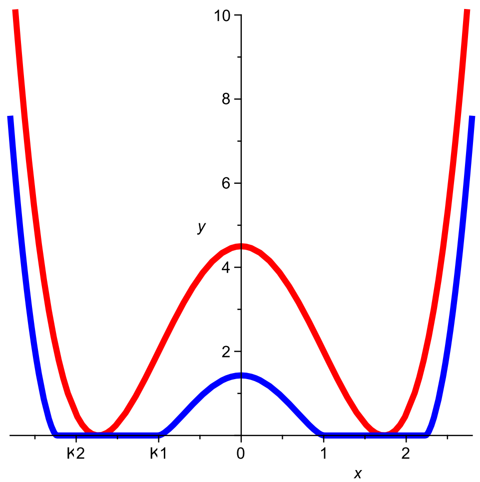
$$
\begin{aligned}
Z &= \int \left(\prod_i d\lambda_i\, 2\lambda_i\right) \prod_{i>j} (\lambda_i^2 - \lambda_j^2)^2\, e^{-\sum_i \frac{N}{2}(\lambda_i^2-\mu^2)^2} \\
&= \int dx\, 2x \int \left(\prod_i d\lambda'_i\, 2\lambda'_i\right) \prod_{i=1}^{N-1} (x^2 - \lambda_i'^2)^2 \prod_{N-1\geq i>j\geq 1} (\lambda_i'^2 - \lambda_j'^2)^2 \\
&\quad \times e^{-\sum_{i=1}^{N-1}\frac{N}{2}(\lambda_i'^2-\mu^2)^2} e^{-\frac{N}{2}(x^2-\mu^2)^2} \qquad (x = \lambda_N) \\
&= \int dx\, 2x \left\langle \det(x^2 - \phi'^2)^2 \right\rangle'^{(N-1)} e^{-\frac{N}{2}(x^2-\mu^2)} \\
&\equiv \int dx\, 2x\, e^{-N V_{\text{eff}}(x)}
\end{aligned}
$$

$$
\begin{aligned}
V_{\text{eff}}(x) &= \frac{1}{2}(x^2-\mu^2)^2 - \frac{1}{N}\log\left\langle \det(x^2-\phi'^2)^2 \right\rangle \\
&= \frac{1}{2}(x^2-\mu^2)^2 - \frac{1}{N}\log\left\langle e^{2\mathrm{Re}\,\mathrm{tr}\,\log(x^2-\phi'^2)} \right\rangle \\
&= \frac{1}{2}(x^2-\mu^2)^2 - \frac{1}{N}\log e^{\left\langle 2\mathrm{Re}\,\mathrm{tr}\,\log(x^2-\phi'^2)\right\rangle + \frac{1}{2}\left\langle \left(2\mathrm{Re}\,\mathrm{tr}\,\log(x^2-\phi'^2)\right)^2\right\rangle_c + \cdots}
\end{aligned}
$$

$$\therefore \quad V_{\rm eff}^{(0)}(x) = \frac{1}{2}(x^2-\mu^2)^2 - 2{\rm Re}\left\langle \frac{1}{N}{\rm tr}\,\log(x^2-\phi^2)\right\rangle_0$$

$$= \frac{1}{2}(x^2-\mu^2)^2 - 2{\rm Re}\int^{x^2} dy \left\langle \frac{1}{N}{\rm tr}\,\frac{1}{y-\phi^2}\right\rangle_0$$

$$= -{\rm Re}\int^{x^2} dy\,\sqrt{(y-\mu^2)^2-4}$$



$$V_{\rm eff}^{(0)}(0) - V_{\rm eff}^{(0)}(a) = \int_0^{a^2} dy\sqrt{(y-\mu^2)^2-4}$$

$$= \frac{1}{2}\mu^2\sqrt{\mu^4-4} + 2\log(\mu^2-\sqrt{\mu^4-4}) - 2\log 2$$

(complete agreement with OP)

$$\rightarrow \quad \frac{32}{3}\omega^{\frac{3}{2}} \qquad \left(\omega = \frac{\mu^2-2}{4}\right)$$