Cosmological Brane Systems in Warped Spacetime

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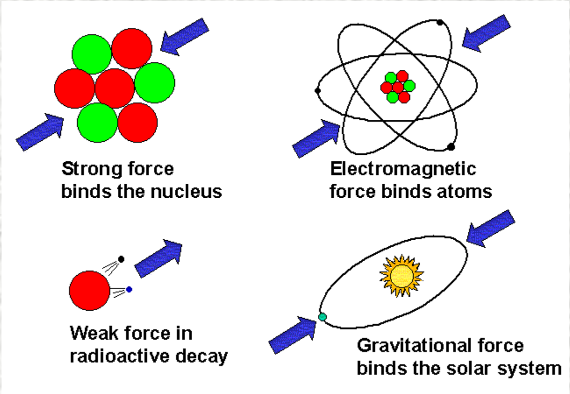
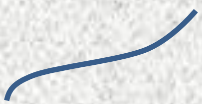
Introduction

String theory



✓ A promising candidate for the *unified theory* of the fundamental interactions

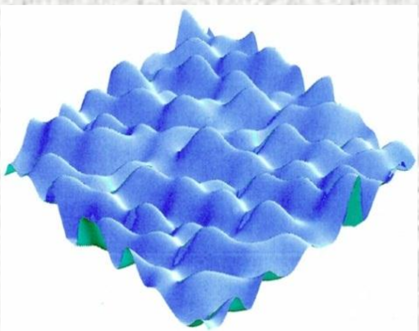

gauge interactions


gravity

✓ Higher-dimensional spacetime

Compactification ⟶ A large population of universes

Multiverse



✓ Branes

- confine the gauge interactions
- curve the surrounding spacetime

Time-dependent brane solutions in the gravity theory can lead to expansion of the Universe

After brief reviews of black holes and branes,
we introduce the time-dependent brane solutions.

Charged Black Holes in 4D

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

$$ds^2 = -f(R)dt^2 + \frac{dR^2}{f(R)} + R^2 d\Omega^2 \qquad f(R) = 1 - \frac{2M}{R} + \frac{Q^2}{R^2}$$

mass charge

Horizons $f(R_\pm) = 0$ $\quad R_+ = M + \sqrt{M^2 - Q^2}$ event horizon

$R_- = M - \sqrt{M^2 - Q^2}$ inner horizon

Increasing the mass of a point particle with a fixed charge Q.

$M < Q$ no horizon

$M = Q$ the minimal BH is formed.

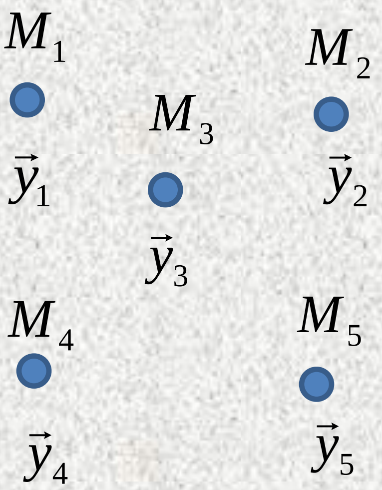
Extremal BHs ⇒ Extension to multi-centered case

- **Multi-centered extension** Majumdar-Papapetrou

$$ds^2 = -h^{-2}(y)dt^2 + h^2(y)\delta_{ij}dy^i dy^j$$

$$F_2 = d\left(h^{-1}\right)\wedge dt$$

$$\Delta_Y h(y) = 0 \longrightarrow h(y) = 1 + \sum_\ell \frac{M_\ell}{|\vec{y} - \vec{y}_\ell|}$$



Extremal black holes at $\vec{y} \to \vec{y}_\ell$

A single black hole $r^2 = \delta_{ij}y^i y^j$

$$R := r + M$$

$$ds^2 = -\left(1 - \frac{M}{R}\right)^2 dt^2 + \left(1 - \frac{M}{R}\right)^{-2} dR^2 + R^2 d\Omega^2$$

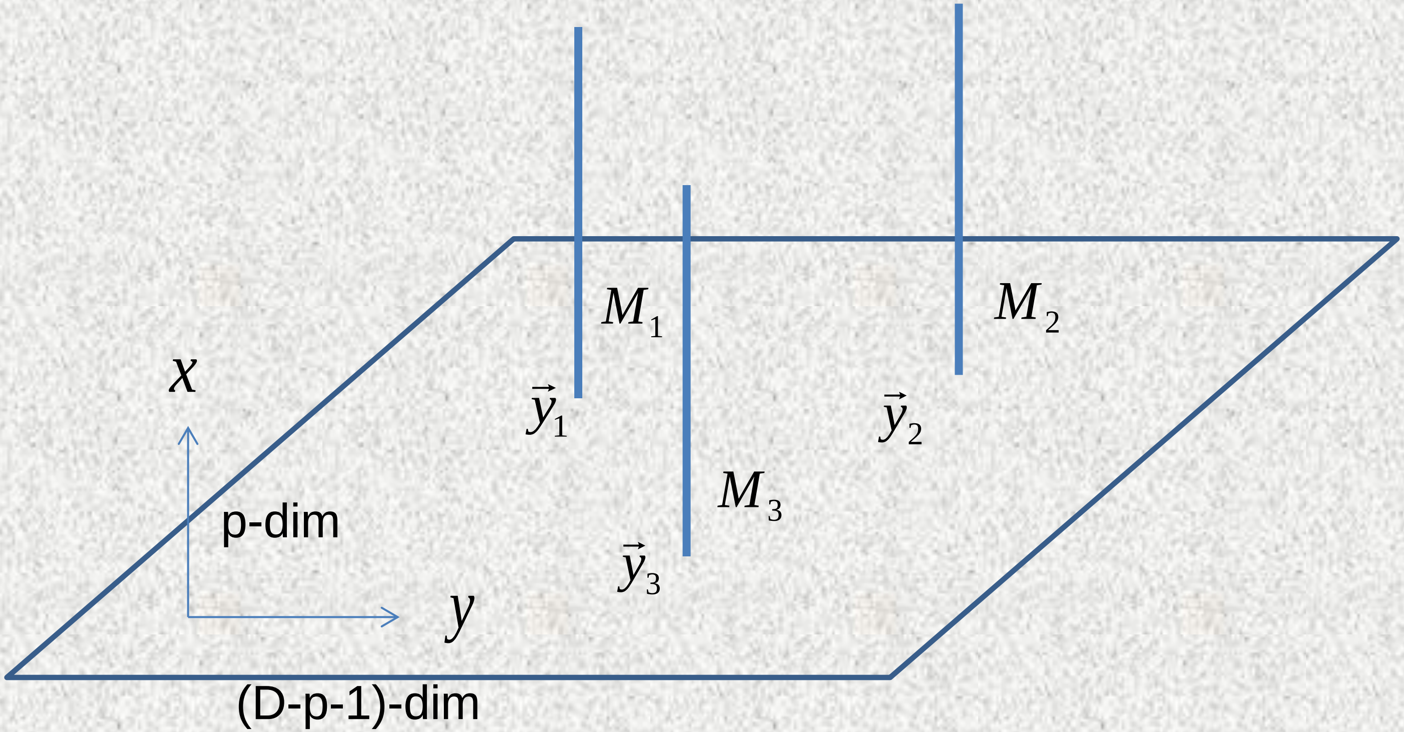
= extremal BH

✓ Black branes in higher-dimensional gravity

4D	0-brane (BH)	$\longrightarrow F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$
Higher-dim	p-branes	$\longrightarrow F_{p+2} = dA_{p+1}$

Higher-dimensional gravity coupled to a (p+1)-form gauge field
has solutions of charged black objects whose horizons are extended over p-dim space.

Black-branes



Time-dependent extension

Analogy between 4D and higher-dimensional gravity.

- 4D gravity

Charged (extremal) BH → Time-dependent BH

Majumdar-Papapetrou → Kastor-Traschen

- Higher-dimensional gravity

p-brane solution → Time-dependent p-brane

Horowitz-Strominger → Gibbons-Lu-Pope

- Time-dependent BH in 4D Kastor & Traschen 93

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R - \underline{2\Lambda} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

cosmological constant

solution $\Lambda > 0$

$$ds^2 = -h^{-2}(t,y)dt^2 + h^2(t,y)\delta_{ij}dy^i dy^j$$

$$F_2 = d\left(h^{-1}\right)\wedge dt$$

$$h(t,y) = \pm\sqrt{\frac{\Lambda}{3}}t + b + \sum_{\ell}\frac{M_\ell}{|\vec{y}-\vec{y}_\ell|}$$

$\vec{y} \to \vec{y}_\ell$ extremal black hole horizon

$|\vec{y}| \to \infty$ Asymptotically de Sitter spacetime

$$ds^2 = -\left(\sqrt{\frac{\Lambda}{3}}t + b + \frac{M}{r}\right)^{-2} dt^2 + \left(\sqrt{\frac{\Lambda}{3}}t + b + \frac{M}{r}\right)^2 \left(dr^2 + r^2 d\Omega^2\right)$$

$r \to \infty$ de Sitter space

$$ds^2 \approx -d\tau^2 + \exp\left(2\sqrt{\frac{\Lambda}{3}}t\right)\delta_{ij}dy^i dy^j \qquad \tau := \sqrt{\frac{\Lambda}{3}}^{-1} \ln\left(\sqrt{\frac{\Lambda}{3}}t + b\right)$$

$r \to 0$ (extremal) charged BH M=Q

$$ds^2 \approx -\left(1 - \frac{M}{R}\right)^2 dt^2 + \left(1 - \frac{M}{R}\right)^{-2} dR^2 + R^2 d\Omega^2$$

$$R := br + M$$

Horizon $R = M$ or $r = 0$

➢ Time-dependent p-branes

Gibbons, Lu & Pope (05),
Binetruy, Uzawa & Sasaki (09)

We consider the higher-dimensional gravity theory

$$S = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\left[R(X) - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2(p+2)!}e^{c\phi}F_{p+2}^2\right]$$

graviton scalar (p+2)-form field strength

$$c^2 = 4 - \frac{2(p+1)(D-p-3)}{D-2}$$

Coupling constant is chosen, so that in $D = 10, 11$
the theory becomes supergravity theories in Einstein frame.

➢ Time-dependent p-branes

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graviton scalar (p+2)-form field strength

$$c^2 = 4 - \frac{2(p+1)(D-p-3)}{D-2}$$

Coupling constant is chosen, so that in $D = 10, 11$
the theory becomes supergravity theories in Einstein frame.

Time-dependent p-branes

$$ds^2 = h^{-\left(\frac{D-p-3}{D-2}\right)}(x,y)\eta_{\mu\nu}dx^\mu dx^\nu + h^{\frac{p+1}{D-2}}(x,y)\delta_{ij}(Y)dy^i dy^j$$

(p+1)-dim worldvolume

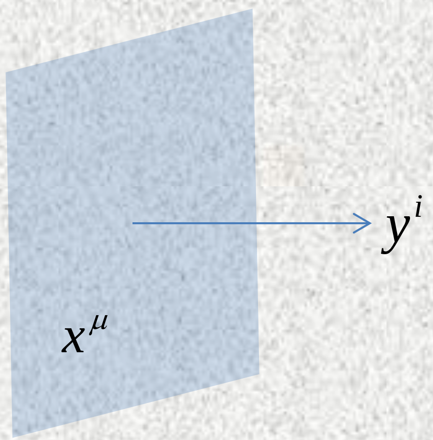
(D-p-1)-dim transverse space

$$e^\phi = h^{-\frac{c}{2}}$$

$$F_{(p+2)} = \sqrt{-q}\,d\left(h^{-1}\right)\wedge dx^0 \wedge\cdots\wedge dx^p$$

$$\Delta_Y h = 0$$

$$\longrightarrow\quad h(x,y) = A_\mu x^\mu + B + \sum_\ell \frac{M_\ell}{\left|\vec{y}-\vec{y}_\ell\right|^{D-p-3}}$$



A_μ B : integration constants

✓ $\vec{y}\to\vec{y}_\ell$ p-branes

✓ Time-dependence appears as a linear function

$$A_\mu x^\mu = A_0 t + A_i x^i$$

Dynamical D3-branes $D=10 \quad p=3$

Gibbons, Lu and Pope (05)

$$A_0 < 0 \quad A_i = B = 0$$

$$h(x,y) = -|A_0|t + \sum_{\ell} \frac{M_\ell}{|\vec{y} - \vec{y}_\ell|^4}$$

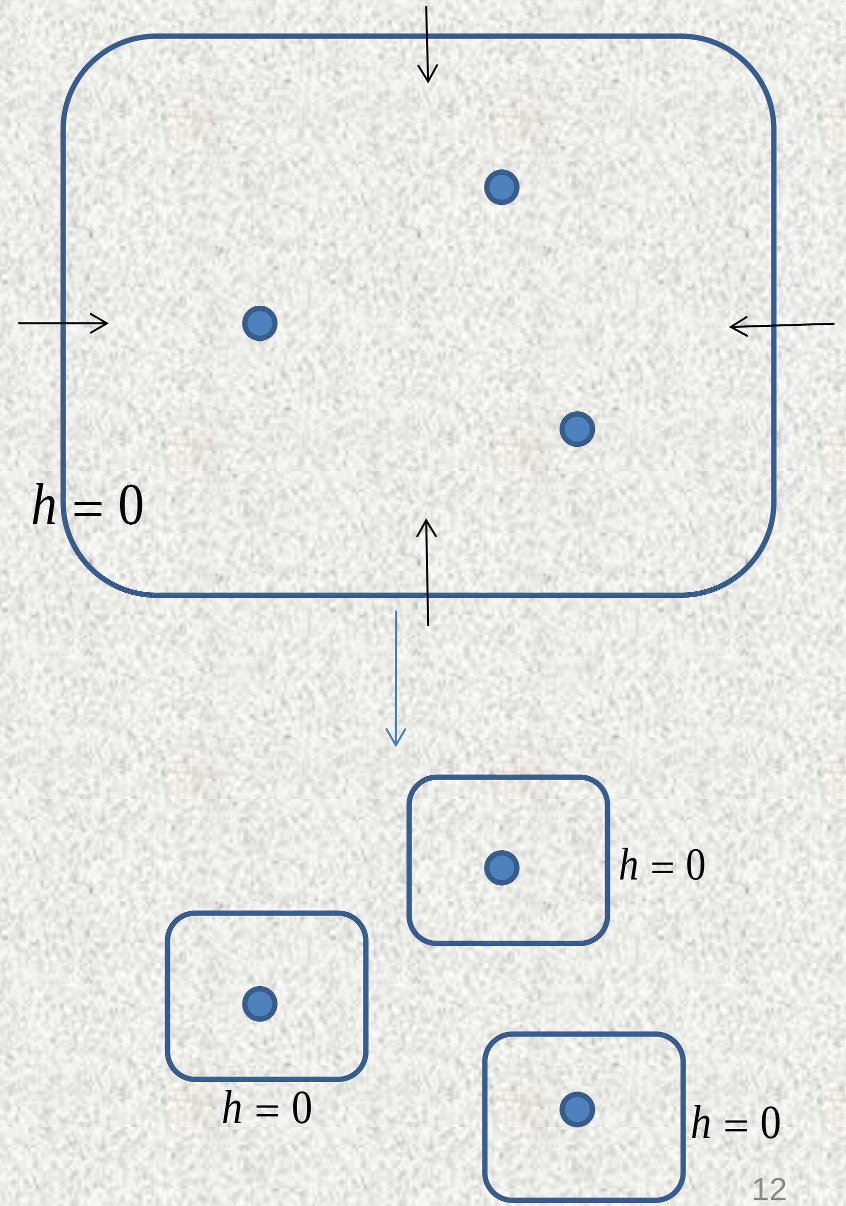
10-dim Kasner solution if all $M_\ell = 0$

Singularity appears at infinity at t=0

$h = 0$: singularity

Regular region shrinks

Separation into domains,
and each contains a single D3-brane.



Dynamical intersecting branes

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➢ Intersecting branes

Guven (92), Bergshoeff, et.al (97)

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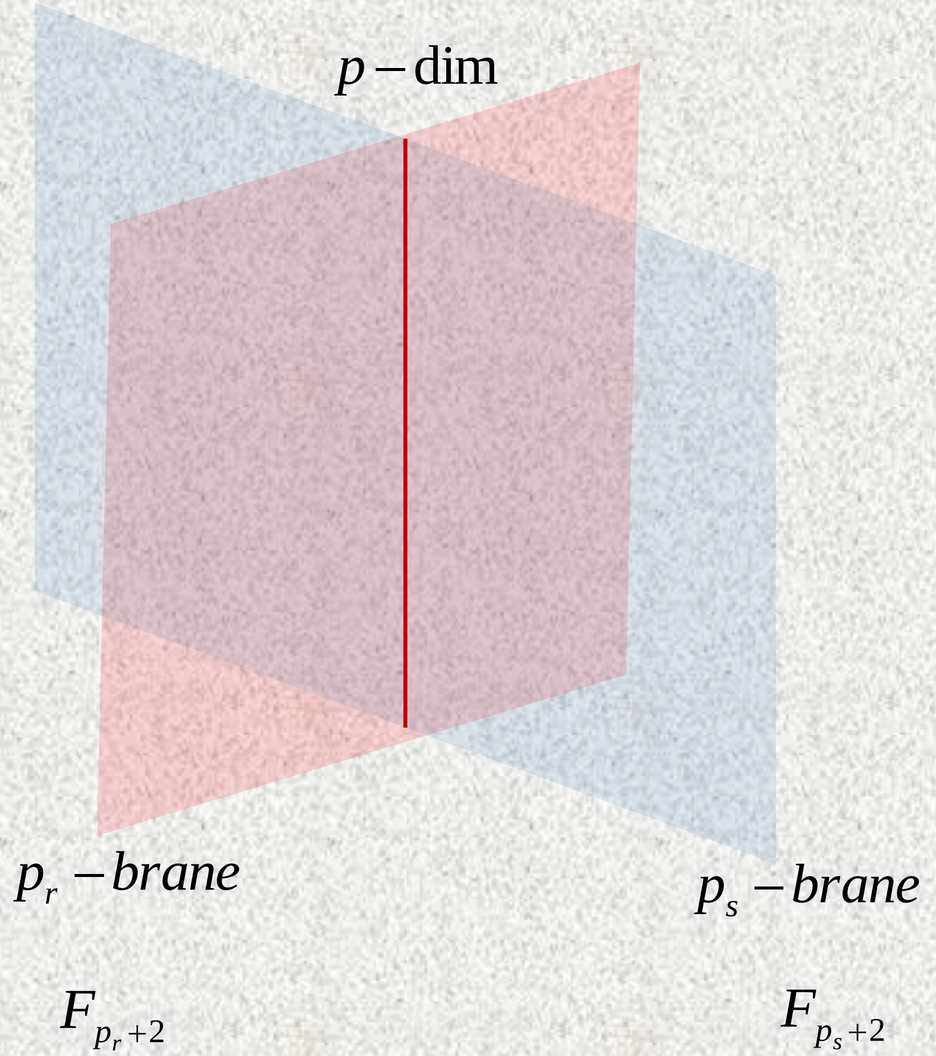

The metric of the intersecting brane systems

$$ds^2 = h_r^{\alpha} h_s^{\beta} \Big[\overbrace{(h_r h_s)^{-1} q_{\mu\nu}(X)}^{(p+1)\text{-dim world volume}} dx^{\mu} dx^{\nu} + \overbrace{h_s^{-1} \gamma_{ij}(Y_1) dy^i dy^j}^{(p_s-p)\text{-dim relative transverse}}$$

$$+ \underbrace{h_r^{-1} w_{mn}(Y_2) dv^m dv^n}_{(p_r-p)\text{-dim relative transverse}} + \underbrace{u_{ab}(Z) dz^a dz^b}_{(D-p-p_r-p_s-1)\text{-dim transverse}} \Big]$$

$$\alpha = \frac{p_r + 1}{D - 2} \qquad \beta = \frac{p_s + 1}{D - 2}$$

The metric of the intersecting brane systems

$$ds^2 = h_r^{\alpha} h_s^{\beta} \Big[\boxed{\underbrace{(h_r h_s)^{-1} q_{\mu\nu}(X) dx^{\mu} dx^{\nu}}_{\text{(p+1)-dim world volume}} + \underbrace{h_s^{-1} \gamma_{ij}(Y_1) dy^i dy^j}_{(p_s\text{-p})\text{-dim relative transverse}}} \quad p_s - brane$$

$$+ \underbrace{h_r^{-1} w_{mn}(Y_2) dv^m dv^n}_{(p_r\text{-p})\text{-dim relative transverse}} + \underbrace{u_{ab}(Z) dz^a dz^b}_{(D\text{-p-}p_r\text{-}p_s\text{-1})\text{-dim transverse}} \Big]$$

$$\alpha = \frac{p_r + 1}{D - 2} \qquad \beta = \frac{p_s + 1}{D - 2}$$

The metric of the intersecting brane systems

$p_r - brane$

$$
\begin{aligned}
ds^2 = h_r^{\alpha} h_s^{\beta} \Big[& \overbrace{(h_r h_s)^{-1} q_{\mu\nu}(X) dx^{\mu} dx^{\nu}}^{\text{(p+1)-dim world volume}} + \overbrace{h_s^{-1} \gamma_{ij}(Y_1) dy^i dy^j}^{\text{(}p_s\text{-p)-dim relative transverse}} \\
& + \underbrace{h_r^{-1} w_{mn}(Y_2) dv^m dv^n}_{\text{(}p_r\text{-p)-dim relative transverse}} + \underbrace{u_{ab}(Z) dz^a dz^b}_{\text{(D-p-}p_r\text{-}p_s\text{-1)-dim transverse}} \Big]
\end{aligned}
$$

$p_s - brane$

$$
\alpha = \frac{p_r + 1}{D - 2} \qquad \beta = \frac{p_s + 1}{D - 2}
$$

The metric of the intersecting brane systems

$p_r - brane$

$$ds^2 = h_r^{\alpha} h_s^{\beta} \Big[\overbrace{(h_r h_s)^{-1} q_{\mu\nu}(X) dx^{\mu} dx^{\nu}}^{\text{(p+1)-dim world volume}} + \overbrace{h_s^{-1} \gamma_{ij}(Y_1) dy^i dy^j}^{(\text{p}_\text{s}\text{-p})\text{-dim relative transverse}}$$

$p_s - brane$

$$+ \underbrace{h_r^{-1} w_{mn}(Y_2) dv^m dv^n}_{(\text{p}_\text{r}\text{-p})\text{-dim relative transverse}} + \underbrace{u_{ab}(Z) dz^a dz^b}_{(\text{D-p-p}_\text{r}\text{-p}_\text{s}\text{-1})\text{-dim transverse}} \Big]$$

$$\alpha = \frac{p_r + 1}{D - 2} \qquad \beta = \frac{p_s + 1}{D - 2}$$

In general, $h_r = h_r(x, y, z) \qquad h_s = h_s(x, v, z)$

The metric of the intersecting brane systems

$p_r - brane$

$$ds^2 = h_r^{\alpha} h_s^{\beta} \Big[\overbrace{(h_r h_s)^{-1} q_{\mu\nu}(X) dx^{\mu} dx^{\nu}}^{\text{(p+1)-dim world volume}} + \overbrace{h_s^{-1} \gamma_{ij}(Y_1) dy^i dy^j}^{(p_s\text{-p})\text{-dim relative transverse}} \qquad p_s - brane$$

$$+ \underbrace{h_r^{-1} w_{mn}(Y_2) dv^m dv^n}_{(p_r\text{-p})\text{-dim relative transverse}} + \underbrace{u_{ab}(Z) dz^a dz^b}_{(\text{D-p-}p_r\text{-}p_s\text{-1})\text{-dim transverse}} \Big]$$

$$\alpha = \frac{p_r + 1}{D-2} \qquad \beta = \frac{p_s + 1}{D-2}$$

In general, $h_r = h_r(x, y, z)$ $\quad h_s = h_s(x, v, z)$

→ Here we focus on

Case I	$h_r = h_r(x, z)$	$h_s = h_s(x, z)$
Case II	$h_r = h_r(x, y)$	$h_s = h_s(x, z)$
Case III	$h_r = h_r(x, y)$	$h_s = h_s(x, v)$

Classification of the intersecting branes

Behrndt, Bergshoeff & Janssen (96)

Time-dependent generalization

Case I : Both h_r and h_s depend on the overall transverse coordinates

Maeda, Ohta & Uzawa (09)

$$h_r = h_r(x,z) \quad h_s = h_s(z) \quad \text{or} \quad h_r = h_r(z) \quad h_s = h_s(x,z)$$

$$p+1-\frac{(p_r+1)(p_s+1)}{D-2}+\frac{1}{2}\varepsilon_r\varepsilon_s c_r c_s = 0$$ Argurio, et. al (97), Ohta (97)

$$c_I^2 = 4-\frac{2(p_I+1)(D-p_I-3)}{D-2} \qquad \begin{aligned}\varepsilon_I &= +1: \text{ electric} \\ \varepsilon_I &= -1: \text{magnetic}\end{aligned}$$

Case II : h_s depends on the overall transverse space z^a

h_r depends on the relative transverse space y^i

$$h_r = h_r(x,y) \qquad h_s = h_s(z)$$

no $h_r = h_r(y) \qquad h_s = h_s(x,z)$

Case III :

$$h_r = h_r(x, y) \qquad h_s = h_s(v) \qquad \text{or} \qquad h_r = h_r(y) \qquad h_s = h_s(x, v)$$

Intersecting rule

$$p + 1 - \frac{(p_r + 1)(p_s + 1)}{D - 2} + \frac{1}{2}\varepsilon_r \varepsilon_s c_r c_s = -2$$

Common features for all cases

- Linear dependence $h_r(x, z) = A_\mu x^\mu + g_r(z) = A_0 t + A_i x^i + g_r(z)$
- $g_r(z)$ is by the harmonic functions, $\Delta_Z g_r(z) = 0$
- Transverse spaces are Ricci-flat.

$$R_{\mu\nu}(X) = 0 \qquad R_{ij}(Y_1) = 0$$
$$R_{mn}(Y_2) = 0 \qquad R_{ab}(Z) = 0$$

Partially localized branes

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- Localized branes

  - ✓ If there is solution such as $h_s = h_s(x, v, z)$ $h_r = h_r(x, y, z)$

    each brane is *localized* in both the relative and overall transverse space.

    $\longrightarrow$ Difficult to solve analytically

    We could find the partially localized branes.

    p_r -branes are localized on a single p_s-brane

$$h_s = \underline{h_s(z)} \qquad h_r = h_r(x, y, z)$$

~single brane

  - ✓ Static solutions Youm (97)
  - ✓ Time-dependence appears as a linear function

$$\longrightarrow \quad h_r(x, y, z) = A_\mu x^\mu + g_r(y, z)$$

$$h_s(z)\Delta_{Y_1} g_r(y, z) + \Delta_Z g_r(y, z) = 0$$

$$S=\frac{1}{2\kappa^2}\int d^Dx\sqrt{-g}\left[R(X)-\frac{1}{2}(\partial\phi)^2-\frac{1}{2(p_r+2)!}e^{c_r\phi}F^2_{p_r+2}-\frac{1}{2(p_s+2)!}e^{c_s\phi}F^2_{p_s+2}\right]$$

$$c_I^2=N_I-\frac{2(p_I+1)(D-p_I-3)}{D-2}$$

$$\varepsilon_I=+1:\ \text{electric}$$

$$\varepsilon_I=-1:\ \text{magnetic}$$

Case		0	1	$\cdots$	p	$p+1$	$\cdots$	p_s	p_s+1	$\cdots$	p_s+p_r-p	p_s+p_r-p+1	$\cdots$	$D-1$
p_r-p_s	p_r	$\circ$	$\circ$	$\circ$	$\circ$				$\circ$	$\circ$	$\circ$			
	p_s	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						
	x^N	t	x^1	$\cdots$	x^p	y^1	$\cdots$	y^{p_s-p-1}	v^1	$\cdots$	v^{p_r-p-1}	z^1	$\cdots$	$z^{D+p-p_r-p_s-1}$

Ansatz
$$e^{\phi} = h_r^{2\varepsilon_r c_r/N_r} h_s^{2\varepsilon_s c_s/N_s}$$

$$F_{(p_r+2)} = \frac{2}{\sqrt{N_r}} d\left[h_r^{-1}(x, y, z)\right] \wedge \Omega(X) \wedge \Omega(Y_2)$$

$$F_{(p_s+2)} = \frac{2}{\sqrt{N_s}} d\left[h_s^{-1}(x, v, z)\right] \wedge \Omega(X) \wedge \Omega(Y_2)$$

Intersecting rule
$$p + 1 - \frac{(p_r+1)(p_s+1)}{D-2} + \frac{1}{2}\varepsilon_r \varepsilon_s c_r c_s = 0$$

Off-diagonal Einstein equations

$$\frac{2}{N_r} h_r^{-1} \left(\partial_\mu \partial_i h_r + \frac{4}{N_s} \partial_\mu \ln h_s \partial_i h_r \right) = 0,$$
$$\frac{2}{N_s} h_s^{-1} \left(\partial_\mu \partial_m h_s + \frac{4}{N_r} \partial_\mu \ln h_r \partial_m h_s \right) = 0,$$
$$\frac{2}{N_r} h_r^{-1} \partial_\mu \partial_a h_r + \frac{2}{N_s} h_s^{-1} \partial_\mu \partial_a h_s = 0,$$

$$\partial_i \ln h_r \partial_m \ln h_s = 0$$

Ansatz $$e^{\phi} = h_r^{2\varepsilon_r c_r/N_r} h_s^{2\varepsilon_s c_s/N_s}$$

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$$F_{(p_s+2)} = \frac{2}{\sqrt{N_s}} d\left[h_s^{-1}(x,v,z)\right] \wedge \Omega(X) \wedge \Omega(Y_2)$$

Intersecting rule $$p+1-\frac{(p_r+1)(p_s+1)}{D-2}+\frac{1}{2}\varepsilon_r\varepsilon_s c_r c_s = 0$$

Off-diagonal Einstein equations

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$$\frac{2}{N_s} h_s^{-1}\left(\cancel{\partial_\mu \partial_m h_s} + \frac{4}{N_r}\partial_\mu \ln h_r \partial_m h_s\right) = 0,$$
$$\frac{2}{N_r} h_r^{-1}\cancel{\partial_\mu \partial_a h_r} + \frac{2}{N_s} h_s^{-1}\cancel{\partial_\mu \partial_a h_s} = 0,$$

$$\partial_i \ln h_r \partial_m \ln h_s = 0$$

For $\partial_\mu h_s = 0$, $\quad h_r = h_0(x) + h_1(y,z) \quad h_s = h_s(v,z)$

Ansatz $$e^{\phi} = h_r^{2\varepsilon_r c_r/N_r} h_s^{2\varepsilon_s c_s/N_s}$$

$$F_{(p_r+2)} = \frac{2}{\sqrt{N_r}} d\left[h_r^{-1}(x,y,z)\right] \wedge \Omega(X) \wedge \Omega(Y_2)$$

$$F_{(p_s+2)} = \frac{2}{\sqrt{N_s}} d\left[h_s^{-1}(x,v,z)\right] \wedge \Omega(X) \wedge \Omega(Y_2)$$

Intersecting rule $$p+1-\frac{(p_r+1)(p_s+1)}{D-2}+\frac{1}{2}\varepsilon_r\varepsilon_s c_r c_s = 0$$

Off-diagonal Einstein equations

$$\frac{2}{N_r} h_r^{-1}\left(\cancel{\partial_\mu \partial_i h_r} + \frac{4}{N_s}\cancel{\partial_\mu \ln h_s \partial_i h_r}\right) = 0,$$
$$\frac{2}{N_s} h_s^{-1}\left(\cancel{\partial_\mu \partial_m h_s} + \frac{4}{N_r}\cancel{\partial_\mu \ln h_r \partial_m h_s}\right) = 0,$$
$$\frac{2}{N_r} h_r^{-1}\cancel{\partial_\mu \partial_a h_r} + \frac{2}{N_s} h_s^{-1}\cancel{\partial_\mu \partial_a h_s} = 0,$$
$$\cancel{\partial_i \ln h_r \partial_m \ln h_s} = 0$$

For $\partial_\mu h_s = 0$, $$h_r = h_0(x) + h_1(y,z) \quad h_s = h_s(z)$$

$$R_{\mu\nu}(\mathrm{X}) = 0, \quad R_{ij}(\mathrm{Y}_1) = 0, \quad R_{mn}(\mathrm{Y}_2) = 0, \quad R_{ab}(\mathrm{Z}) = 0,$$

$$h_r = h_0(x) + h_1(y, z), \quad h_s = h_s(v, z), \quad \partial_i h_r \partial_m h_s = 0\,,$$

$$D_\mu D_\nu h_0 = 0, \quad \left(1 - \frac{4}{N_r}\right) \partial_\mu h_0 \partial_\nu h_0 = 0, \quad h_s^{4/N_s} \triangle_{\mathrm{Y}_1} h_1 + \triangle_{\mathrm{Z}} h_1 = 0,$$

$$h_r^{4/N_r} \triangle_{\mathrm{Y}_2} h_s + \triangle_{\mathrm{Z}} h_s = 0.$$

$$R_{\mu\nu}(\mathrm{X}) = 0, \quad R_{ij}(\mathrm{Y}_1) = 0, \quad R_{mn}(\mathrm{Y}_2) = 0, \quad R_{ab}(\mathrm{Z}) = 0,$$

$$h_r = h_0(x) + h_1(y, z), \quad h_s = h_s(v, z), \quad \partial_i h_r \partial_m h_s = 0\,,$$

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$$h_r^{4/N_r} \triangle_{\mathrm{Y}_2} h_s + \triangle_{\mathrm{Z}} h_s = 0.$$

$$h_s = h_s(z)$$

linear function of x

$$R_{\mu\nu}(\mathrm{X}) = 0, \quad R_{ij}(\mathrm{Y}_1) = 0, \quad R_{mn}(\mathrm{Y}_2) = 0, \quad R_{ab}(\mathrm{Z}) = 0,$$

$$h_r = h_0(x) + h_1(y, z), \quad h_s = h_s(v, z), \quad \partial_i h_r \partial_m h_s = 0\,,$$

$$D_\mu D_\nu h_0 = 0, \quad \left(1 - \frac{4}{N_r}\right) \partial_\mu h_0 \partial_\nu h_0 = 0, \quad h_s^{4/N_s} \triangle_{\mathrm{Y}_1} h_1 + \triangle_{\mathrm{Z}} h_1 = 0,$$

$$h_r^{4/N_r} \triangle_{\mathrm{Y}_2} h_s + \triangle_{\mathrm{Z}} h_s = 0.$$

$$N_r = 4 \qquad h_s = h_s(z)$$

$$R_{\mu\nu}(\mathrm{X}) = 0, \quad R_{ij}(\mathrm{Y}_1) = 0, \quad R_{mn}(\mathrm{Y}_2) = 0, \quad R_{ab}(\mathrm{Z}) = 0,$$

$$h_r = h_0(x) + h_1(y,z), \quad h_s = h_s(v,z), \quad \partial_i h_r \partial_m h_s = 0\,,$$

linear function of x

$$D_\mu D_\nu h_0 = 0, \quad \left(1 - \frac{4}{N_r}\right) \partial_\mu h_0 \partial_\nu h_0 = 0, \quad h_s^{4/N_s} \triangle_{\mathrm{Y}_1} h_1 + \triangle_{\mathrm{Z}} h_1 = 0,$$

$$h_r^{4/N_r} \triangle_{\mathrm{Y}_2} h_s + \triangle_{\mathrm{Z}} h_s = 0.$$

$$N_r = 4 \qquad\qquad h_s = h_s(z)$$

$$\boxed{h_s \triangle_{\mathrm{Y}_1} h_1 + \triangle_{\mathrm{Z}} h_1 = 0, \quad \triangle_{\mathrm{Z}} h_s = 0}$$

$$R_{\mu\nu}(\mathrm{X})=0, \quad R_{ij}(\mathrm{Y}_1)=0, \quad R_{mn}(\mathrm{Y}_2)=0, \quad R_{ab}(\mathrm{Z})=0,$$

$$h_r=h_0(x)+h_1(y,z), \quad h_s=h_s(v,z), \quad \partial_i h_r \partial_m h_s=0\,,$$

$$D_\mu D_\nu h_0=0, \quad \left(1-\frac{4}{N_r}\right)\partial_\mu h_0 \partial_\nu h_0=0, \quad h_s^{4/N_s}\triangle_{\mathrm{Y}_1}h_1+\triangle_{\mathrm{Z}}h_1=0,$$

$$h_r^{4/N_r}\triangle_{\mathrm{Y}_2}h_s+\triangle_{\mathrm{Z}}h_s=0.$$

linear function of x

$$N_r=4 \qquad h_s=h_s(z)$$

$$\boxed{h_s\triangle_{\mathrm{Y}_1}h_1+\triangle_{\mathrm{Z}}h_1=0, \quad \triangle_{\mathrm{Z}}h_s=0}$$

We further assume

$$q_{\mu\nu}=\eta_{\mu\nu}\,, \quad \gamma_{ij}=\delta_{ij}\,, \quad w_{mn}=\delta_{mn}\,, \quad u_{ab}=\delta_{ab}\,, \quad N_r=N_s=4,$$

Partially localized brane solutions

$$h_r(x,y,z) = A_\mu x^\mu + B + \sum_\ell \frac{M_\ell}{\left[|\vec{y}-\vec{y}_\ell|^2 + \frac{4M}{(4-d_z)^2} |\vec{z}-\vec{z}_0|^{4-d_z} \right]^{\frac{1}{2}\left(p_s - p - 1 + \frac{d_z}{4-d_z}\right)}}$$

$$h_s(z) = \frac{M}{|\vec{z}-\vec{z}_0|^{d_z-2}}$$

Localized brane

For $d_z \neq 2$

$$h_r(x,y,z) = A_\mu x^\mu + B + \sum_\ell \frac{M_\ell}{\left[|\vec{y}-\vec{y}_\ell|^2 + M |\vec{z}-\vec{z}_0|^2 \right]^{\frac{1}{2}(p_s - p + 1)}}$$

$$h_s(z) = M \ln |\vec{z}-\vec{z}_0|$$

Cosmological properties are the same as the delocalized cases.

Cosmology

(1) Cosmology from intersection of two branes

1. Higher-dimensional picture $\frac{\tau}{\tau_0} = (At)^{\frac{a_r+2}{2}}$ $\tau_0 = \frac{2}{(a_r+2)A}$

$$
\begin{aligned}
ds^2 = \; & h_s^{a_s}\left[1+\left(\frac{\tau}{\tau_0}\right)^{-2/(a_r+2)} h_1\right]^{a_r}\left[-d\tau^2+\left(\frac{\tau}{\tau_0}\right)^{2a_r/(a_r+2)} \delta_{PQ}(\tilde{\mathrm{X}})d\theta^P d\theta^Q\right. \\
& +\left\{1+\left(\frac{\tau}{\tau_0}\right)^{-2/(a_r+2)} h_1\right\}\left(\frac{\tau}{\tau_0}\right)^{2b_r/(a_r+2)} \gamma_{ij}(\mathrm{Y}_1)dy^i dy^j \\
& +h_s\left(\frac{\tau}{\tau_0}\right)^{2a_r/(a_r+2)} w_{mn}(\mathrm{Y}_2)dv^m dv^n \\
& \left.+h_s\left\{1+\left(\frac{\tau}{\tau_0}\right)^{-2/(a_r+2)} h_1\right\}\left(\frac{\tau}{\tau_0}\right)^{2b_r/(a_r+2)} u_{ab}(\mathrm{Z})dz^a dz^b\right],
\end{aligned}
$$

Expansion law of each space can be obtained on a slice in the other spaces.

= Brane world picture

2. Lower-dimensional effective theory

$$ds^2 = ds^2(\mathrm{M}) + ds^2(\mathrm{N}) \,,$$

(D-d)-dim d-dim

↓

compactified

$$d = d_1 + d_2 + d_3 + d_4$$

X Y$_1$ Y$_2$ Z

$$ds^2(\mathrm{M}) = h_r^B h_s^C ds^2(\bar{\mathrm{M}}) \,,$$

Einstein frame

$$B = \frac{-(a_r+1)d + d_1 + d_3}{D-d-2}, \qquad C = \frac{-(a_s+1)d + d_2 + d_4}{D-d-2}.$$

$$ds^2(\bar{\mathrm{M}}) = h_r^{B'} h_s^{C'} \Big[-dt^2 + \delta_{P'Q'}(\tilde{\mathrm{X}}')d\theta^{P'}d\theta^{Q'} + h_r \gamma_{k'l'}(\mathrm{Y_1}')dy^{k'}dy^{l'}$$
$$+ h_s w_{m'n'}(\mathrm{Y_2}')dv^{m'}dv^{n'} + h_r h_s u_{a'b'}(\mathrm{Z}')dz^{a'}dz^{b'} \Big] \,,$$

$$B' = -B + a_r \qquad C' = -C + a_s$$

Cosmic expansion law of each space can be read.

p=3 : higher-dimensional picture

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{\mathrm{M}}$	$\lambda(\tilde{\mathrm{M}})$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{M}})$	BW
	D3	$\circ$	$\circ$	$\circ$	$\circ$							$\surd$	$\tilde{\mathrm{X}}$	$\lambda(\tilde{\mathrm{X}}) = -1/3$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_2+d_4-4}{12-2d_1-d_2-d_4}$	
D3-D7	D7	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$				or	$\lambda(\mathrm{Y}_1) = 1/3$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{4-d_1}{12-2d_1-d_2-d_4}$	
	x^N	t	x^1	x^1	x^3	y^1	y^2	y^3	y^4	z^1	z^2		Y_1 & Z	$\lambda(\mathrm{Z})=1/3$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{4-d_1}{12-2d_1-d_2-d_4}$	
	D3	$\circ$	$\circ$	$\circ$	$\circ$								$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = 0$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_4}{16-2d_1-2d_3-d_4}$	
D3-D7	D7	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$			$\surd$		$\lambda(\mathrm{Y}_2) = 0$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_4}{16-2d_1-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	v^1	v^2	v^3	v^4	z^1	z^2					
	D4	$\circ$	$\circ$	$\circ$	$\circ$				$\circ$			$\surd$	$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = \lambda(\mathrm{Y}_2) = \frac{-3}{13}$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-3}{13-2d_1-d_2-2d_3-d_4}$	
D4-D6	D6	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					or	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{5}{13}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{5-d_1-d_3}{13-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x	x^2	x^3	y^1	y^2	y^3	v	z^1	z^2		Y_1 & Z			
	D4	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$							$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = \lambda(\mathrm{Y}_2) = \frac{-1}{15}$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-1}{15-2d_1-d_2-2d_3-d_4}$	
D4-D6	D6	$\circ$	$\circ$	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$			$\surd$	or	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{7}{15}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{7-d_1-d_3}{15-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	y	v^1	v^2	v^3	z^1	z^2		Y_1 & Z			
	D5	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$			$\surd$	$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = \lambda(\mathrm{Y}_2) = \frac{-1}{7}$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
D5-D5	D5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						or	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{3}{7}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	y^1	y^2	v^1	v^2	z^1	z^2		Y_1 & Z			

p=3 : Lower-dimensional effective theory

Branes	TD	dim(M)	$\tilde{\mathrm{M}}$	(d_1, d_2, d_3, d_4)	$\lambda_{\mathrm{E}}(\tilde{\mathrm{M}})$	Case
D3-D7	D3	7	$\tilde{\mathrm{X}}$ & Y_1 & Z	(0, 3, 0, 0)	4/9	I
	D3	9	$\tilde{\mathrm{X}}$ & Y_1 & Z	(1, 0, 0, 0)	3/10	II
	D7	$10-d$	$\tilde{\mathrm{X}}$ & Y_2 & Z or $\tilde{\mathrm{X}}$ & Z or Y_2 & Z	$(d_1, 0, d_3, 0)$	0	I & II
D4-D6	D4	8	$\tilde{\mathrm{X}}$ & Y_1 & Y_2 & Z	(0, 2, 0, 0)	5/11	I
	D4	9	$\tilde{\mathrm{X}}$ & Y_1 & Y_2 &Z	(1, 0, 0, 0)	4/11	II
	D4	9	$\tilde{\mathrm{X}}$ & Y_1 & Z	(0, 0, 1, 0)	4/11	II
	D6	9	$\tilde{\mathrm{X}}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	6/13	I & II
	D6	9	$\tilde{\mathrm{X}}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	6/13	I & II
D5-D5	D5	9	$\tilde{\mathrm{X}}$ & Y_1 & Y_2 & Z	(0, 1, 0, 0)	6/13	I
	D5	9	$\tilde{\mathrm{X}}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	5/12	II
	D5	9	$\tilde{\mathrm{X}}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	5/12	II

(2) Cosmology in triple brane intersection systems

1. Higher-dimensional picture

Branes		0	1	2	3	4	5	6	7	8	9	TD	$\tilde{M}$	$\lambda(\tilde{M})$	$\lambda_E(\tilde{M})$
D3-D5-NS5	D3	∘	∘	∘				∘				√	$\tilde{X}$ & W	$\lambda(\tilde{X}) = -1/3$	$\lambda_E(\tilde{X}) = \frac{d_Y+d_Z-4}{12-2d_{\tilde{X}}-d_Y-2d_W-d_Z}$
	D5	∘	∘	∘	∘	∘	∘							$\lambda(W) = -1/3$	$\lambda_E(W) = \frac{d_Y+d_Z-4}{12-2d_{\tilde{X}}-d_Y-2d_W-d_Z}$
	NS5	∘	∘	∘					∘	∘	∘		Y & Z	$\lambda(Y)=1/3$	$\lambda_E(Y) = \frac{4-d_{\tilde{X}}-d_W}{12-2d_{\tilde{X}}-d_Y-2d_W-d_Z}$
	x^N	t	x^1	x^2	y^1	y^2	y^3	w	z^1	z^2	z^3			$\lambda(Z)=1/3$	$\lambda_E(Z) = \frac{4-d_{\tilde{X}}-d_W}{12-2d_{\tilde{X}}-d_Y-2d_W-d_Z}$
D5-D7-NS5	D5	∘	∘	∘	∘	∘				∘		√	$\tilde{X}$ & W	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \frac{d_Y+d_Z-2}{14-2d_{\tilde{X}}-d_Y-2d_W-d_Z}$
	D7	∘	∘	∘	∘	∘	∘	∘	∘					$\lambda(W) = -1/7$	$\lambda_E(W) = \frac{d_Y+d_Z-2}{14-2d_{\tilde{X}}-d_Y-2d_W-d_Z}$
	NS5	∘	∘	∘	∘	∘					∘		Y & Z	$\lambda(Y)=3/7$	$\lambda_E(Y) = \frac{6-d_{\tilde{X}}-d_W}{14-2d_{\tilde{X}}-d_Y-2d_W-d_Z}$
	x^N	t	x^1	x^2	x^3	x^4	y^1	y^2	y^3	w	z			$\lambda(Z)=3/7$	$\lambda_E(Z) = \frac{6-d_{\tilde{X}}-d_W}{14-2d_{\tilde{X}}-d_Y-2d_W-d_Z}$

2. Lower-dimensional effective theory

Branes	TD	dim(M)	$\bar{M}$	d	$\lambda_E(\bar{M})$
D3-D5-NS5	D3	7	$\tilde{X}$ & Y & W & Z	$(d_{\tilde{X}}, d_Y, d_W, d_Z) = (0,1,0,2)$	4/9
	D3	7	$\tilde{X}$ & Y & W & Z	$(d_{\tilde{X}}, d_Y, d_W, d_Z) = (0,2,0,1)$	4/9
D5-D7-NS5	D5	9	$\tilde{X}$ & Y & W & Z	$(d_{\tilde{X}}, d_Y, d_W, d_Z) = (0,1,0,0)$	6/13
M2-M2-KK	M2	6	$\tilde{X}$ & Y & U & V & Z	$(d_{\tilde{X}}, d_Y, d_U, d_V, d_Z) = (0,1,2,0,2)$	3/7
D2-D6-KK	D2	6	$\tilde{X}$ & Y & V & U	$(d_{\tilde{X}}, d_Y, d_V, d_Z) = (0,2,0,2)$	3/7

Summary of cosmic expansion

- ✓ All spaces X,Y,Z can provide expanding homogeneous and isotropic 3-space, after compactification.

- ✓ In both pictures, expansion cannot be faster than that in the radiation-dominated universe.

Summary of cosmic expansion

- ✓ All spaces X,Y,Z can provide expanding homogeneous and isotropic 3-space, after compactification.

- ✓ In both pictures, expansion cannot be faster than that in the radiation-dominated universe.

We need a potential term to get accelerated expansion.

Summary

- ✓ Charged BH solutions can be extended to p-branes in higher-dimensional theory coupled to (p+2)-form anti-symmetric form field strength
- ✓ Time-dependence can be introduced by adding a linear function of time into the harmonic function.
- ✓ For intersecting brane systems, only the particular can be time-dependent.
- ✓ Time-dependent warp factor gives a cosmic expansion, which cannot be faster than the radiation universe.

Thank you.