Superstring theory and integration over the moduli space

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arXiv:1303.7299 [KO,Yuji Tachikawa]

Section 1

introduction

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- '12: "Superstring Perturbation Theory Revisited" [E.Witten]:
  Superstring perturbation theory and supergeometry

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- $\mathcal{N} = \mathbf{0} \subset \mathcal{N} = \mathbf{1}$ embedding [N. Berkovits,C. Vafa '94]
- Topological amplitudes

  [I. Antoniadis, E. Gava, K.S. Narain, T.R. Taylor '94]

  [M. Bershadsky, S. Cecotti, H. Ooguri, C. Vafa '94]

Section 2

Supergeometry and Superstring

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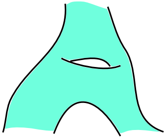

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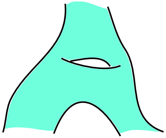

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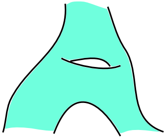

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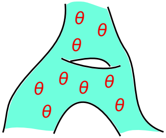

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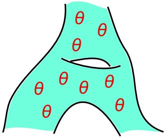

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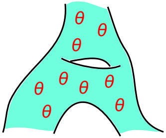

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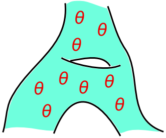

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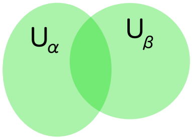

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- $\mathbf{x}'_\mathbf{i} = \mathbf{f}_\mathbf{i}(\mathbf{x}|\theta)$
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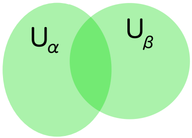
- $\mathbf{M}_{\text{red}} = \{(\mathbf{x}_\mathbf{i}, \cdots, \mathbf{x}_\mathbf{p})\} \hookrightarrow \mathbf{M}$



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- $\mathbf{D}_\theta = \partial_\theta + \theta\partial_{\mathbf{z}} = \mathbf{F}(\mathbf{z}|\theta)(\partial_{\theta'} + \theta'\partial_{\mathbf{z}'})$

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- $\theta \in \mathbf{H}^0(\mathbf{\Sigma}, \mathbf{T\Sigma}_{\text{red}}^{*1/2})$
  $\Rightarrow$ split SRS $\leftrightarrow$ Riemann surface with spin str.

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Supermoduli space of SRS with punctures

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- dim $\mathcal{M}_{\text{super}} = \boldsymbol{\Delta_e | \Delta_o}$
  $= \mathbf{3g - 3 + n_{\text{NS}} + n_{\text{R}} | 2g - 2 + n_{\text{NS}} + n_{\text{R}}/2 \quad (g \geq 2)}$

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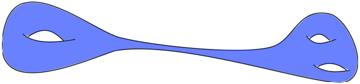
- if we fix the behavior of Γ near boundary.



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Coupling between gravitino and SCFT

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- $\mathbf{S}_{\chi\widetilde{\chi}} = \int_{\mathbf{\Sigma}_{\mathrm{red}}} \chi\widetilde{\chi}\mathbf{A}$
- $\mathbf{A} \propto \psi^\mu\widetilde{\psi}_\mu$ (Flat background)

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- c.f. $\mathbf{D}_\mu \phi^\dagger \mathbf{D}^\mu \phi \ni \mathbf{A}_\mu \mathbf{A}^\mu \phi^\dagger \phi$

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- $F_{\mathcal{V},g}(m, \eta, \bar{m}, \bar{\eta}) =$
$\Big\langle \prod_i \mathcal{V}_i \prod_{a=1}^{\Delta_e} (\int_{\Sigma_{red}} d^2 z b \mu^a) \prod_{b=1}^{\Delta_e} (\int_{\Sigma_{red}} d^2 z \widetilde{b} \widetilde{\mu}^b)$
$\prod_{\sigma=1}^{\Delta_o} \delta(\int_{\Sigma_{red}} d^2 z \beta \chi^\sigma) \prod_{\tau=1}^{\widetilde{\Delta}_o} \delta(\int_{\Sigma_{red}} d^2 z \widetilde{\beta} \widetilde{\chi}^\tau)$
$\exp(-S_\chi - S_{\widetilde{\chi}} - S_{\chi\widetilde{\chi}}) \Big\rangle_m$

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- $\mathcal{A}_{\mathcal{V},g} = \int_\Gamma \mathrm{d}m\mathrm{d}\eta\mathrm{d}\bar{m}\mathrm{d}\bar{\eta}\ F_{\mathcal{V},g}(m,\eta,\bar{m},\bar{\eta})$
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$\Big\langle \prod_i \mathcal{V}_i \prod_{a=1}^{\Delta_e} (\int_{\Sigma_{red}} \mathrm{d}^2 z b \mu^a) \prod_{b=1}^{\Delta_e} (\int_{\Sigma_{red}} \mathrm{d}^2 z \tilde{b} \tilde{\mu}^b)$
$\prod_{\sigma=1}^{\Delta_o} \delta(\int_{\Sigma_{red}} \mathrm{d}^2 z \beta \chi^\sigma) \prod_{\tau=1}^{\tilde{\Delta}_o} \delta(\int_{\Sigma_{red}} \mathrm{d}^2 z \tilde{\beta} \tilde{\chi}^\tau)$
$\exp(-S_\chi - S_{\tilde{\chi}} - S_{\chi\tilde{\chi}}) \Big\rangle_m$
- $S_\chi = \sum_{\sigma=1}^{\Delta_o} \frac{\eta_\sigma}{2\pi} \int_{\Sigma_{red}} \mathrm{d}^2 z \chi^\sigma T_F$

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- Valid where $[\delta(\mathbf{z} - \mathbf{p}_\sigma)]$ spans odd deformations
  $\Rightarrow$ Coordinates given by picture changing formalism is not global!

Integration over a supermanifold

Integration over a supermanifold

- $\mathbf{M}$: dim$\mathbf{1}|\mathbf{2}$ supermanifold,
  $\mathbf{M} = \mathbf{U}_1 \cup \mathbf{U}_2, \mathbf{U}_1 \ni (\mathbf{m}|\eta_1, \eta_2),\ \mathbf{U}_2 \ni (\mathbf{m}'|\eta_1', \eta_2')$
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- non-holomorphic projection destroys holomorphic factorization.

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- Global integration on $\mathcal{M}_{\text{super}}$
  $\not\Rightarrow$ Global integration on $\mathcal{M}_{\text{spin}}$
  (with holomorphic factorization)

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Section 3

Reduction to integration over moduli

Reduction condition

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- $\omega = \mathbf{i}_* \alpha \wedge \mathbf{P.D.}[\mathcal{M}_{\text{spin}}]$
- ω does not depend on the locations of picture changing operators

Topological amplitudes in string theory

[I. Antoniadis, E. Gava, K.S. Narain, T.R. Taylor '94]

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- Type II string on CY ($\mathcal{N} = (2,2)$SCFT) $\times \mathbb{R}^{1,3}$ $\Rightarrow$ 4d $\mathcal{N} = 2$ supergravity model
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- $\mathbf{F_g}$: $\mathbf{g}$ loop vacuum amplitude

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- $\mathcal{A}_g = (\mathbf{g}!)^2 \mathbf{F}_g = \int_{\mathcal{M}_{\text{bos}}} \cdots$

Vertex operators

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- Graviphoton: $\mathbf{V_T = \Sigma \times \text{spacetime} \times (\text{ghost})}$
  $\mathbf{\Sigma = \exp(i\frac{\sqrt{3}}{2}(H(z) \mp \widetilde{H}(z)))} \quad \mathbf{H : U(1)_R}$ boson
  $\mathbf{\Sigma}$: $\mathbf{U(1)_R}$ charge $(3/2, \mp 3/2)$

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- Graviton: $\mathbf{V_R} = \int_{\mathbf{\Sigma}_{red}}$ spacetime,ghost

Calculation of amplitude

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- $\mathcal{A}_g =$
  $\int_\Gamma \left\langle \prod_{i=1}^{2g-2} \Sigma(x_i) \right.$
  $\times(\text{spacetime}) \times (\mathbf{bc}) \times$
  $\prod_{\sigma=1}^{\Delta_o} \delta(\int_{\Sigma_{red}} d^2z \beta \chi^\sigma) \prod_{\tau=1}^{\tilde{\Delta}_o} \delta(\int_{\Sigma_{red}} d^2z \tilde{\beta} \tilde{\chi}^\tau)$
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- $T_F = T_{F,\text{spacetime,ghost}} + G^+ + G^-$
- $G^\pm$: $U(1)_R$ charge ± 1
- $A = A_{++} + A_{+-} + A_{-+} + A_{--} + A_{\text{spacetime}}$
  $A_{\pm\pm}$: $U(1)_R$ charge $(\pm 1, \pm 1)$
  $\Leftarrow$ Coupling A to $\mathcal{N} = (2,2)$ supergravity:
  $\int_{\Sigma_{red}} A_{\pm\pm} \chi^\mp \tilde{\chi}^\mp \to \chi = \chi^+ = \chi^-$

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Saturation of η's

- $\mathcal{A}_g =$
  $\int_\Gamma \left\langle \prod_{i=1}^{2g-2} \Sigma(x_i) \leftarrow U(1)_R : (3g-3, \mp 3g-3) \right.$
  $\times$(spacetime) $\times$ $(\mathbf{bc})\times$
  $\prod_{\sigma=1}^{\Delta_o} \delta(\int_{\Sigma_{red}} d^2z \beta \chi^\sigma) \prod_{\tau=1}^{\tilde{\Delta}_o} \delta(\int_{\Sigma_{red}} d^2z \tilde{\beta} \tilde{\chi}^\tau)$
  $\left. \exp(-\int_{\Sigma_{red}} (T_F \chi + \tilde{T}_F \tilde{\chi} + A \tilde{\chi} \chi)) \right\rangle_m$
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- Another application of supergeometry

End.

Berkovits and Vafa embedding

[N.Berkovits, C.Vafa '94]

- Worldsheet supersymmetric model equivalent to bosonic string theory
  $\Rightarrow$ Bosonic string : An (unstable) vacuum of superstring theory
- $\langle \mathbf{V}_1 \cdots \mathbf{V}_n \rangle_{\text{bos}} = \langle \mathbf{V}'_1 \cdots \mathbf{V}'_n \rangle_{\text{super}}$
- $\Leftrightarrow \int_{\mathcal{M}_{\text{bos}}} \cdots = \int_{\mathcal{M}_{\text{super}}} \cdots$

Supermoduli space of SRS with punctures

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- dim $\mathcal{M}_{\text{super}} = \mathbf{\Delta_e} | \mathbf{\Delta_o}$
  $= \mathbf{3g - 3 + n_{\text{NS}} + n_{\text{R}} | 2g - 2 + n_{\text{NS}} + n_{\text{R}}/2 \quad (g \geq 2)}$

Superconformal transformation

- $\nu_f = f(z)(\partial_\theta - \theta\partial_z)$
- $V_g = g(z)\partial_z + 1/2\partial_z g\theta\partial_\theta$
- $G_n = \nu_{z^{n+1/2}}$
- $L_m = -V_{z^{m+1}}$

Superconformal transformation near R vertex

- $\nu_f = f(z)(\partial_\theta - z\theta\partial_z)$
- $V_g = z(g(z)\partial_z + 1/2\partial_z g\theta\partial_\theta)$
- $G_r = \nu_{z^r}$
- $L_m = -V_{z^m}$

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- $\theta' = \sqrt{\mathbf{z} - \mathbf{z_1}}\theta \Rightarrow \mathbf{D}^*_\theta = \sqrt{\mathbf{z}}(\partial_{\theta'} + \theta'\partial_\mathbf{z})$
  θ':antiperiodic around $\mathbf{z_1}$

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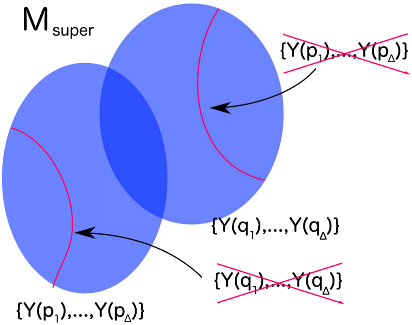

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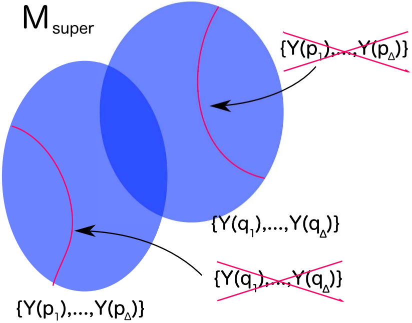

- movement of PCOs $\Rightarrow$ BRST exact term $\Leftrightarrow$ exact form on patches