KOBAYASHI-MASKAWA INSTITUTE FOR THE ORIGIN OF PARTICLES AND THE UNIVERSE

NAGOYA UNIVERSITY

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Cosmology with broken NEC

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THIS TALK IS MOSTLY BASED ON

- ***Hidden Negative Energies in Strongly Acclerated Universes*** **arXiv:1209.2961**
- ***G-Bounce,*** **arXiv:1109.1047,** JCAP 1111:021, 2011
- ***Imperfect Dark Energy from Kinetic Gravity Braiding*** **arXiv:1008.0048,** JCAP 1010:026, 2010
- ***The Imperfect Fluid behind Kinetic Gravity Braiding*** **arXiv:1103.5360,** JHEP 1111 (2011) 156

IN COLLABORATION WITH

Damien Easson, Cédric Deffayet,
Oriol Pujolàs, Ignacy Sawicki

PLAN

- *What is the NEC--Null Energy Condition?*
- *Flying through the NEC violating medium*
- *Is it possible to violate the NEC without catastrophic instabilities?*
- *Imperfect fluid picture*
- *G-Bounce and Galilean Genesis*
- *The NEC violation caused by imperfection*
- *Open questions*

WHAT IS THE NEC?

NULL ENERGY CONDITION (NEC):

$$T_{\mu\nu} n^{\mu} n^{\nu} \geq 0$$

for *all* null vectors n^{μ}, i.e. vectors for which: $g_{\mu\nu} n^{\mu} n^{\nu} = 0$

"*observable energy*" is bounded from below

the *weakest* of all classical energy conditions

THE **NEC** FOR PERFECT FLUIDS AND IN COSMOLOGY

for a perfect fluid:

$$T_{\mu\nu} = (\varepsilon + p)\, u_\mu u_\nu - g_{\mu\nu}\, p$$

$$\textbf{NEC} \Longleftrightarrow p + \varepsilon \geq 0$$

for a positive ε, $w \equiv p/\varepsilon \geq -1$

in cosmology:

$$\dot{H} = -4\pi\,(\varepsilon + p) + \frac{k}{a^2}$$

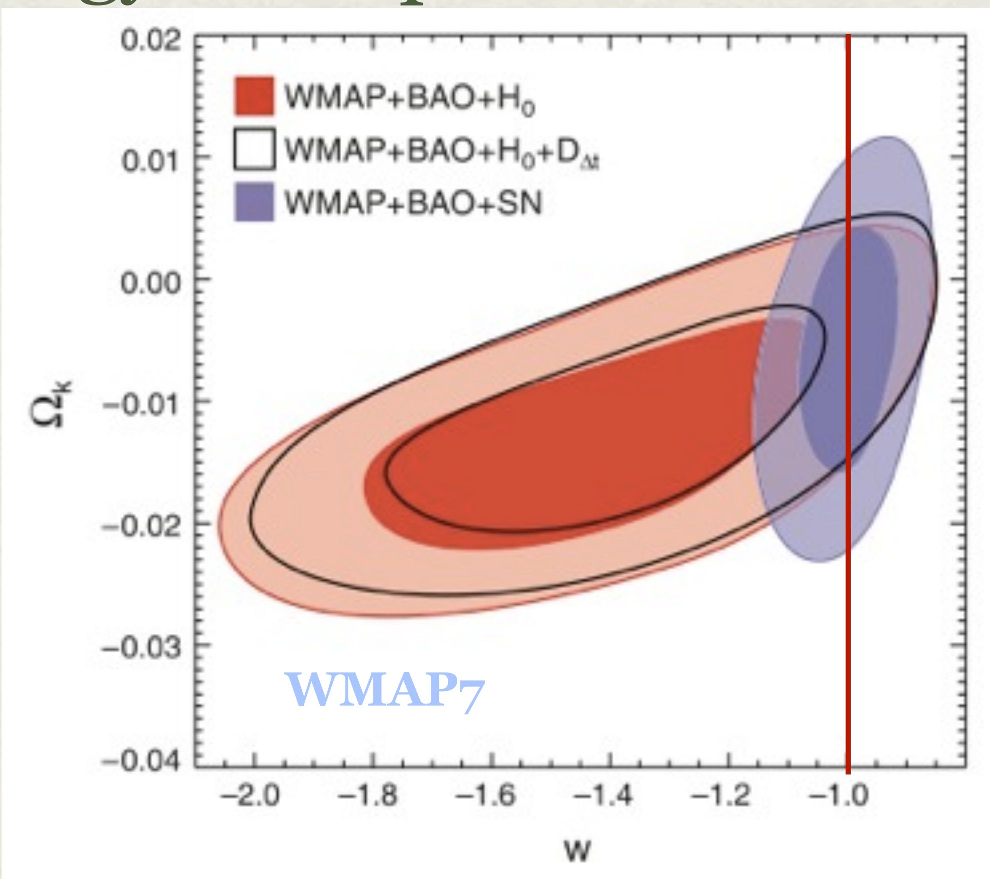
the **NEC** implies that the Hubble parameter *can never grow* in open and flat Friedmann universes

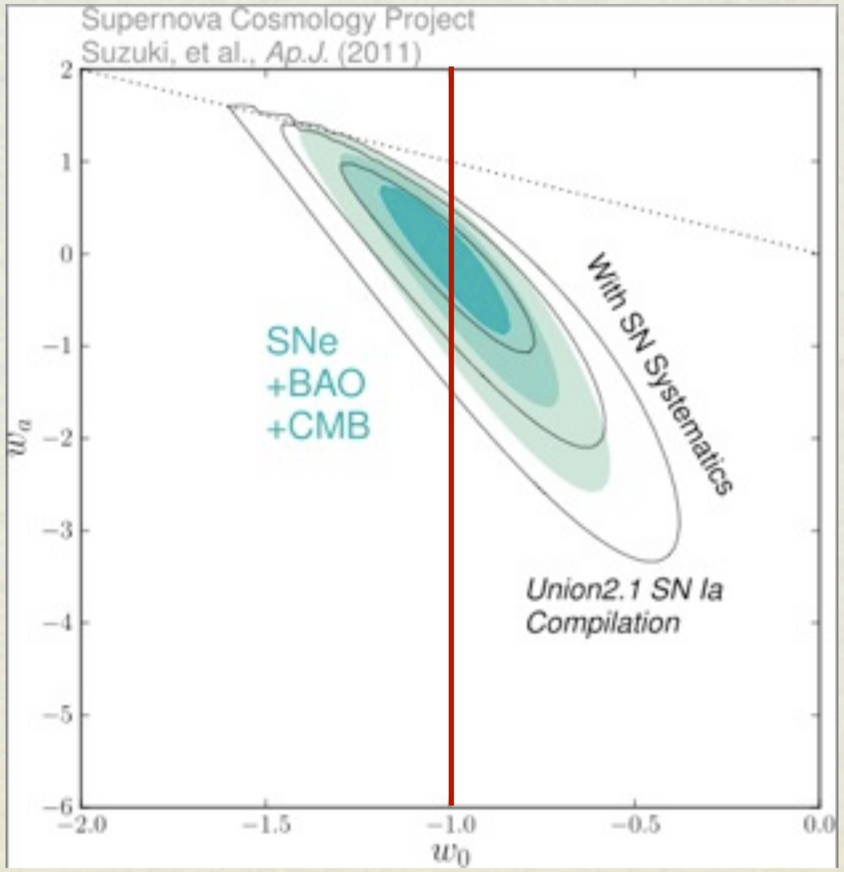
THE NEC

- prohibits Lorentzian Wormholes
- guaranties that, *classically*, Black Holes always grow
- excludes bounces in spatially flat Friedmann universes
- excludes *Phantom* Dark Energy with $w < -1$
- excludes superinflation with blue spectrum of gravity waves

CURRENT STATUS OF THE NEC

- SM fields do not violate the NEC *classically*
- Black Holes do evaporate! i.e. $\langle 0| \hat{T}^{\mu\nu} |0\rangle$ violates the NEC!
- Dark Energy has equation of state:





BROKEN NEC IMPLIES THAT OBSERVABLE ENERGIES ARE UNBOUNDED FROM BELOW

Consider an observer u^μ measuring energy density $\varepsilon_u = T_{\mu\nu} u^\mu u^\nu$

suppose there is a light ray $\mathcal{R}$ for which NEC is broken

pick n^μ from $\mathcal{R}$ such that $u^\mu n_\mu = 1$

another observer with $V^\mu = \alpha\, u^\mu + \eta\, n^\mu$, $\alpha^2 + 2\alpha\eta = 1$

i.e. moving with $v = \dfrac{1-\alpha^2}{1+\alpha^2}$, measures energy density $\varepsilon_V = T_{\mu\nu} V^\mu V^\nu$

for small α :

$$\varepsilon_V\left(\alpha^2\right) \simeq \frac{1}{4\alpha^2} T_{\mu\nu} n^\mu n^\nu < 0$$

arbitrary negative!

FLYING THROUGH A NEC-VIOLATING "PERFECT FLUID"

- Spatial part of the fluid momentum density $p_\mu = T_{\mu\nu}V^\nu$ always points in the same direction as the velocity of the observer-- it helps to boost further!
- for speed higher than $v > v_{\rm sp} \equiv 1/\left|w\right|$ momentum density of the fluid becomes *spacelike*
- for speed higher than $v > v_{\rm neg} \equiv 1/\sqrt{-w}$ energy density of the fluid becomes *negative*, thus by busting further the observer measures even more negative energy density in the fluid

- If the speed of the sound waves in this NEC-violating fluid is higher than v_{neg} , they feel negative energy around them. The sound waves definitely interact with the fluid!

- This looks like well known run away instability despite of the point that the sound waves can be ghost free, have positive energy, and real sound speed

- Similarly to the well known situation with ghosts the time scale of instability depends on details of the interaction

If cosmic rays can interact with Dark Energy (DE) and if the equation of state of the latter is as slightly (%) below -1

as it is currently allowed,

then the protons from the cosmic rays observe *negative* energy density of DE because

$$v_{\mathrm{cray}} \simeq 1 - 10^{-20} > v_{\mathrm{neg}} = 1/\sqrt{-w}$$

ORIGIN OF THE UNIVERSE?

- Was there a beginning of time i.e. of the quasiclassical universe? Was there a strong quantum gravity époque in our past?

- If there was a beginning, was the universe collapsing or expanding immediately afterwards?

- If the universe experienced an early period of inflation, which *all* observations currently ***perfectly*** confirm, what happened ***before inflation***? Indeed, *inflation* is not past-complete - Borde, Guth, Vilenkin (2001)

CAN ONE CONSTRUCT

A **STABLE(?)**

(WITH REAL SOUND SPEED AND WITHOUT GHOSTS)

CLASSICAL MODEL

WHERE A

SPATIALLY FLAT

FRIEDMANN UNIVERSE

BOUNCES:

GOES FROM COLLAPSE TO EXPANSION **???**

OBSTACLE:

TO BOUNCE ONE HAS TO VIOLATE

NULL ENERGY CONDITION,

BUT NORMAL FIELDS AND PERFECT FLUIDS CANNOT DO IT WITHOUT GHOSTS OR / AND GRADIENT INSTABILITIES!

go to imperfect fluids i.e. to even less canonical fields!

GO *G*!

G STANDS FOR GALILEON

Nicolis, Rattazzi , Trincherini, (2008)

In current literature: ***Galileons* =** scalar-tensor theories with

***higher derivatives* in the action**

but with **equations of motion** which are ***all*** of the

second order - NO *Ostrogradsky's* ***ghosts***

the most general theory of this type was derived

by *Horndeski (1974), and rederived by Deffayet, Gao, Steer, Zahariade (2011)*

International Journal of Theoretical Physics, Vol. 10, No. 6 (1974), pp. 363–384

Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

GREGORY WALTER HORNDESKI

Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada

Received: 10 *July* 1973

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$$\mathcal{L} = K + \mathcal{G}\phi^{;\mu}_{;\mu} +$$
$$+ \mathcal{G}_2 R + \mathcal{G}_2' \left[\left(\phi^{;\mu}_{;\mu}\right)^2 - \phi^{;\nu}_{;\mu}\phi^{;\mu}_{;\nu} \right] +$$
$$+ \mathcal{G}_3 G^{\mu}_{\nu}\phi^{;\nu}_{;\mu} - \frac{1}{6}\mathcal{G}_3' \left[\left(\phi^{;\mu}_{;\mu}\right)^3 - 3\phi^{;\lambda}_{;\lambda}\phi^{;\nu}_{;\mu}\phi^{;\mu}_{;\nu} + 2\phi^{;\mu}_{;\alpha}\phi^{;\alpha}_{;\beta}\phi^{;\beta}_{;\mu} \right]$$

Kobayashi, Yamaguchi, Yokoyama (2011)

where we have 4 free functions!

$K(\phi, X)$ and $\mathcal{G}_i(\phi, X)$

$$\mathcal{G}_i' = \frac{\partial \mathcal{G}_i}{\partial X}$$

$$X \equiv \tfrac{1}{2} g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

VIOLATING THE NEC IS NOT A CRIME?

G can violate the Null Energy Condition **without** gradient instabilities (i.e. with real sound speed) and **without** ghosts in the perturbations

Pandora's box

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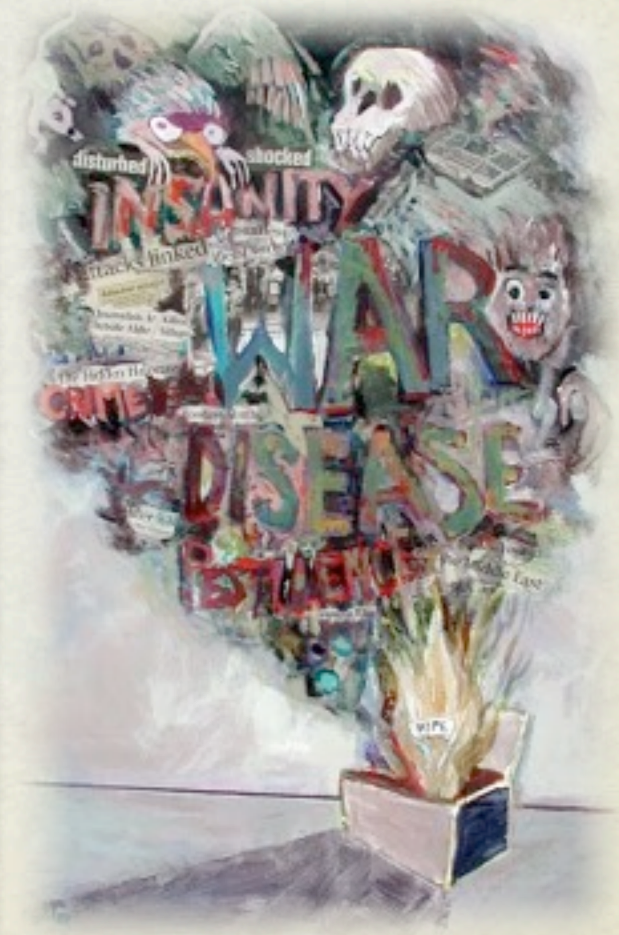

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Pandora's box

- *Healthy* and *testable* Phantom Dark Energy,

  Deffayet, Pujolas, Sawicki, AV, 2010

- *Bouncing* Cosmology for a *spatially flat* Friedmann universe

  Creminelli, Nicolis, Trincherini 2010; *Easson, Sawicki, AV;*
  Taotao Qiu, Evslin, Cai, Li, Zhang 2011

- *Super*inflation with *blue* spectra of gravity waves

  Kobayashi, Yamaguchi, Yokoyama 2010

SIMPLEST INTERESTING SUBSECTOR OF GALILEONS /HORNDESKI'S THEORIES -

Kinetic Gravity Braiding

kinetic mixing / braiding

$$\overset{\partial\phi \qquad \partial g}{}$$

$$S_{\phi} = \int \mathrm{d}^4 x \sqrt{-g}\left[K\left(\phi, X\right) + G\left(\phi, X\right)\Box\phi\right]$$

***Armendariz-Picon, Damour, Mukhanov, Steinhardt* 1999/2000**

Pujolàs, *Deffayet, Sawicki, AV*, 2010

where $X \equiv \frac{1}{2} g^{\mu\nu}\nabla_{\mu}\phi\nabla_{\nu}\phi$

Minimal coupling to gravity $S_{\mathrm{tot}} = S_{\phi} + S_{\mathrm{EH}}$

However, derivatives of the metric are coupled to the derivatives of the scalar, provided $G_X \neq 0$

No "Galilean symmetry"! $\cancel{\partial_{\mu}\phi \rightarrow \partial_{\mu}\phi + c_{\mu}}$

EQUATION OF MOTION I

$$L^{\mu\nu}\nabla_\mu\nabla_\nu\phi + (\nabla_\alpha\nabla_\beta\phi)\,Q^{\alpha\beta\mu\nu}\,(\nabla_\mu\nabla_\nu\phi) +$$
$$+Z - G_X R^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = 0$$

Braiding

EOM is of the second order: $L_{\mu\nu}\,, Q^{\alpha\beta\mu\nu},\ Z$

constructed from field and it's first derivatives

$Q^{\alpha\beta\mu\nu}$ **is such that EOM is a 4D Lorentzian generalization of the Monge-Ampère Equation, always *linear* in** $\ddot{\phi}$

SHOULD ϕ BE FUNDAMENTAL?
NO NOT AT ALL
ϕ CAN MODEL SOME HYDRODYNAMICS !

$K(X)$ for equation of state, $G(X)$: "transport coefficient"

EQUATION OF MOTION II

- **Shift-Charge Noether Current: - interpret as "particle" current** J_μ

- **New Equivalent Lagrangian:** $\mathcal{P}$ **pressure!**

- **Equation of motion is a "conservation law":**

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$$\boxed{\nabla_\mu J^\mu = \mathcal{P}_\phi}$$

EQUATION OF MOTION II

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$$J_\mu = (\mathcal{L}_X - 2G_\phi)\,\nabla_\mu\phi - G_X\nabla_\mu X$$

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$$\mathcal{P} = K - 2XG_\phi - G_X\nabla^\lambda\phi\nabla_\lambda X$$

- **Equation of motion is a "conservation law":**

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IMPERFECT FLUID FOR TIMELIKE GRADIENTS

- **Four velocity** : $u_\mu \equiv \dfrac{\nabla_\mu \phi}{\sqrt{2X}}$ ⇒ ϕ is an ***internal clock***

- **projector:** $\perp_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$

- **Time derivative:** $(\dot{\ }) \equiv \dfrac{\mathrm{d}}{\mathrm{d}\tau} \equiv u^\lambda \nabla_\lambda$

- **Expansion :** $\theta \equiv \perp^\lambda_\mu \nabla_\lambda u^\mu = \dot{V}/V$

comoving volume

Shift-symmetry
$\phi \to \phi + c$
violates
$\phi \to -\phi$
and introduces
arrow of time

EFFECTIVE MASS & CHEMICAL POTENTIAL

- charge density: $n \equiv J^\mu u_\mu = n_0 + \kappa\theta$

"Braiding" $\kappa \equiv 2XG_X$

- energy density: $\mathcal{E} \equiv T^{\mu\nu} u_\mu u_\nu = \mathcal{E}_0 + \theta\dot{\phi}\kappa$

- effective mass per shift-charge / chemical potential:

$$m \equiv \left(\frac{\partial \mathcal{E}}{\partial n}\right)_{V,\phi} = \sqrt{2X} = \dot{\phi}$$

c.f. Schutz 1970

$$\mathrm{d}E = -\mathcal{P}\mathrm{d}V + m\mathrm{d}\mathcal{N}_{\mathrm{dif}}$$

SHIFT-CURRENT AND "DIFFUSION"

"Diffusion"

$$J_\mu = n u_\mu - \frac{\kappa}{m} \perp_\mu^\lambda \nabla_\lambda m$$

§ 59, L&L, vol. 6

$$\kappa \equiv 2 X G_X$$

Is a "diffusivity"/ transport coefficient

Particle / charge current is not parallel to energy flow!

IMPERFECT FLUID ENERGY-MOMENTUM TENSOR

- **Pressure**

$$\mathcal{P} = P_0 - \kappa \dot{m}$$

- **Energy Flow**

No Heat Flux!

$$q_\mu = -\kappa \perp_\mu^\lambda \nabla_\lambda m = m \perp_\mu^\lambda J_\lambda$$

- **Energy Momentum Tensor**

$$T_{\mu\nu} = \mathcal{E} u_\mu u_\nu - \perp_{\mu\nu} \mathcal{P} + 2u_{(\mu} q_{\nu)}$$

Solving for $\dot{m}$ *for small gradients or small* κ *one obtains "bulk viscosity" but change of frame results in a* ***prefect fluid*** *with vorticity up to* $\mathcal{O}\left(\kappa^2\right)$

COSMOLOGY

$$q_\mu = 0 \quad \textbf{and} \quad \theta = 3H$$

Friedmann Equation:

$$H^2 = \kappa m H + \frac{1}{3}\left(\mathcal{E}_0 + \rho_{\text{ext}}\right)$$

$$r_{\text{c}}^{-1} = \kappa m$$ **"crossover" scale in DGP**

EQUATION OF MOTION IN COSMOLOGY

(CHARGE CONSERVATION)

$$\dot{n} + 3Hn = \mathcal{P}_\phi$$

If there is shift-symmetry then

$$\mathcal{P}_\phi = 0$$

$$n \propto a^{-3}$$

ACTION FOR THE COSMOLOGICAL PERTURBATIONS

$$S_2 = \int \mathrm{d}^3x\,\mathrm{d}t\,A\left[\dot{\zeta}^2 - \frac{c_\mathrm{s}^2}{a^2}\left(\partial_i\zeta\right)^2\right]$$

Pujolàs, *Deffayet, Sawicki, AV*, 2010

where $A = \dfrac{Xa^3}{(H - m\kappa/2)^2} D$

$$D = \frac{\mathcal{E}_m - 3H\kappa}{m} + \frac{3}{2}\kappa^2$$

Controls "ghosts" $D > 0$ ➜ No ghosts!

SOUND SPEED

Pujolàs, *Deffayet, Sawicki, AV,* 2010

$$c_s^2 = \frac{\mathcal{P}_m + 2\dot{\kappa} + \kappa\left(4H - \kappa m/2\right)}{\mathcal{E}_m - 3\kappa\left(H - \kappa m/2\right)} \neq \frac{\dot{\mathcal{P}}}{\dot{\mathcal{E}}}$$

The relation between the equation of state, the sound speed and the presence of ghosts is different from the k-*essence* & perfect fluid.

A manifestly stable *Phantom* ($w_X < -1$) is possible even with a *single* degree of freedom and *minimal* coupling to gravity

G BOUNCE IDEA I

- **Consider matter with constant equation of state - radiation, dust, spatial curvature etc...** $p_{\text{ext}} = w\rho_{\text{ext}}$ **with** $w = \text{const}$
- **Shift-symmetric Lagrangian for the scalar field** $K\left(\cancel{\phi}, X\right)$ **and** $G\left(\cancel{\phi}, X\right)$
- **Phase space is two dimensional** $\left(m, \rho_{\text{ext}}\right)$
- **Go to new coordinates** $\left(m, H\right)$ **by solving the Friedmann equation**
- **Integral of motion i.e. 1st integral**

$$I\left(m, H\right) = \frac{n^{1+w}}{\rho_{\text{ext}}}$$

G BOUNCE IDEA II

- **In new coordinates** (m, H) **at** $H = 0$ **pose conditions**

$$\rho_{\rm ext} > 0,\ D > 0,\ \dot{H} > 0,\ c_{\rm s}^2 > 0$$

- **These conditions are on the range of chemical potential** Δm **and on the equation of state** $K(m)$ **along with the transport coefficient** $\kappa(m)$ **or on** $K(X)$ **and** $G(X)$

"Healthy" Bounce!

IS IT POSSIBLE TO SATISFY ALL THESE CONDITIONS ?


We Can Do It!
WAR PRODUCTION CO-ORDINATING COMMITTEE

SIMPLE HIERARCHY

if $K(m)$ and $\kappa(m)$ satisfy the hierarchy:

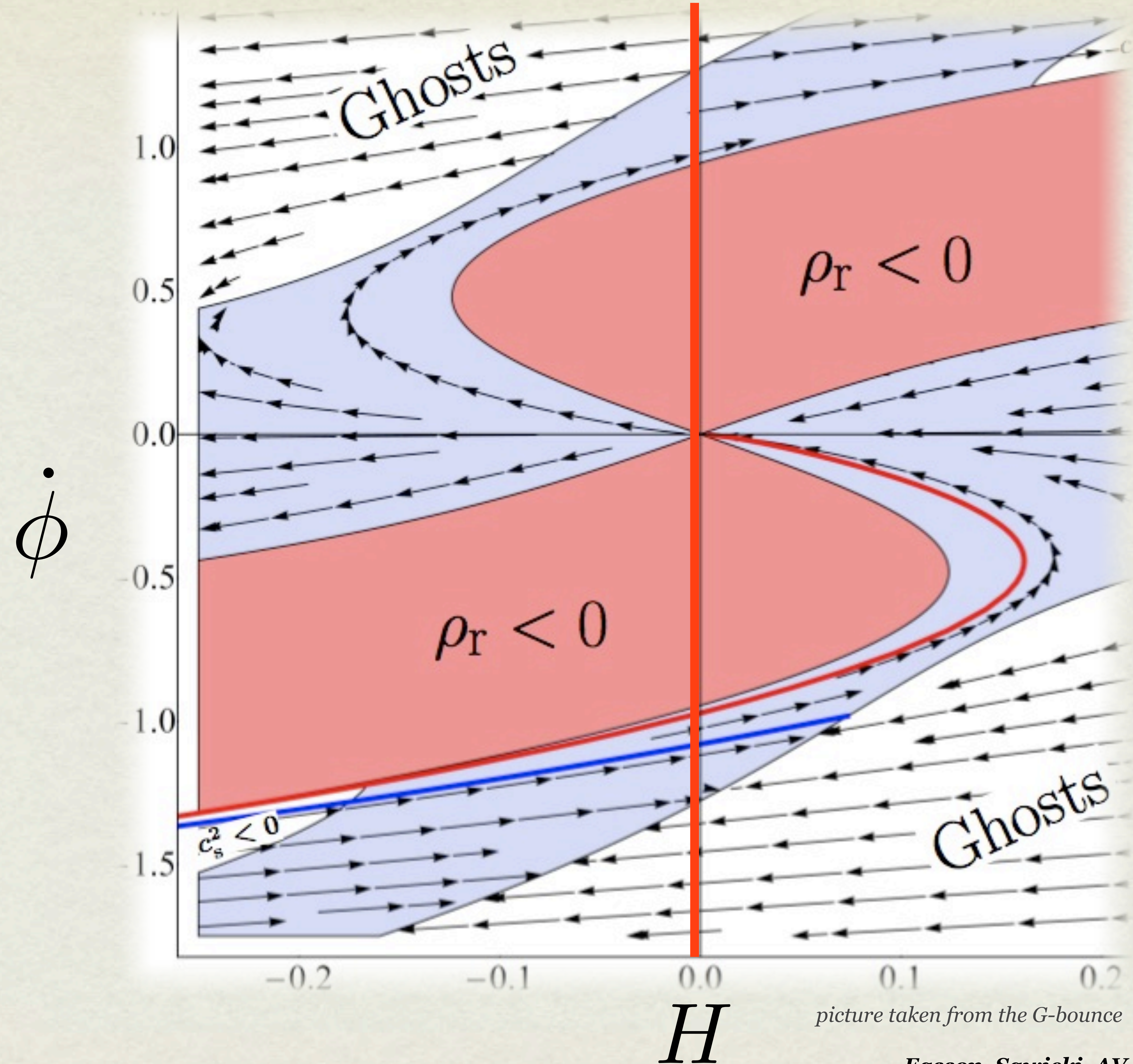
$$K > mK_m > \frac{1}{2}(m\kappa)^2 > 0 > \frac{1}{2}m^2 K_{mm} > -\frac{3}{4}(m\kappa)^2$$

does not depend on external equation of state w !

then at $H = 0$:

$$\rho_{\mathrm{ext}} > 0,\ D > 0,\ \dot{H} > 0,\ c_{\mathrm{s}}^2 > 0$$

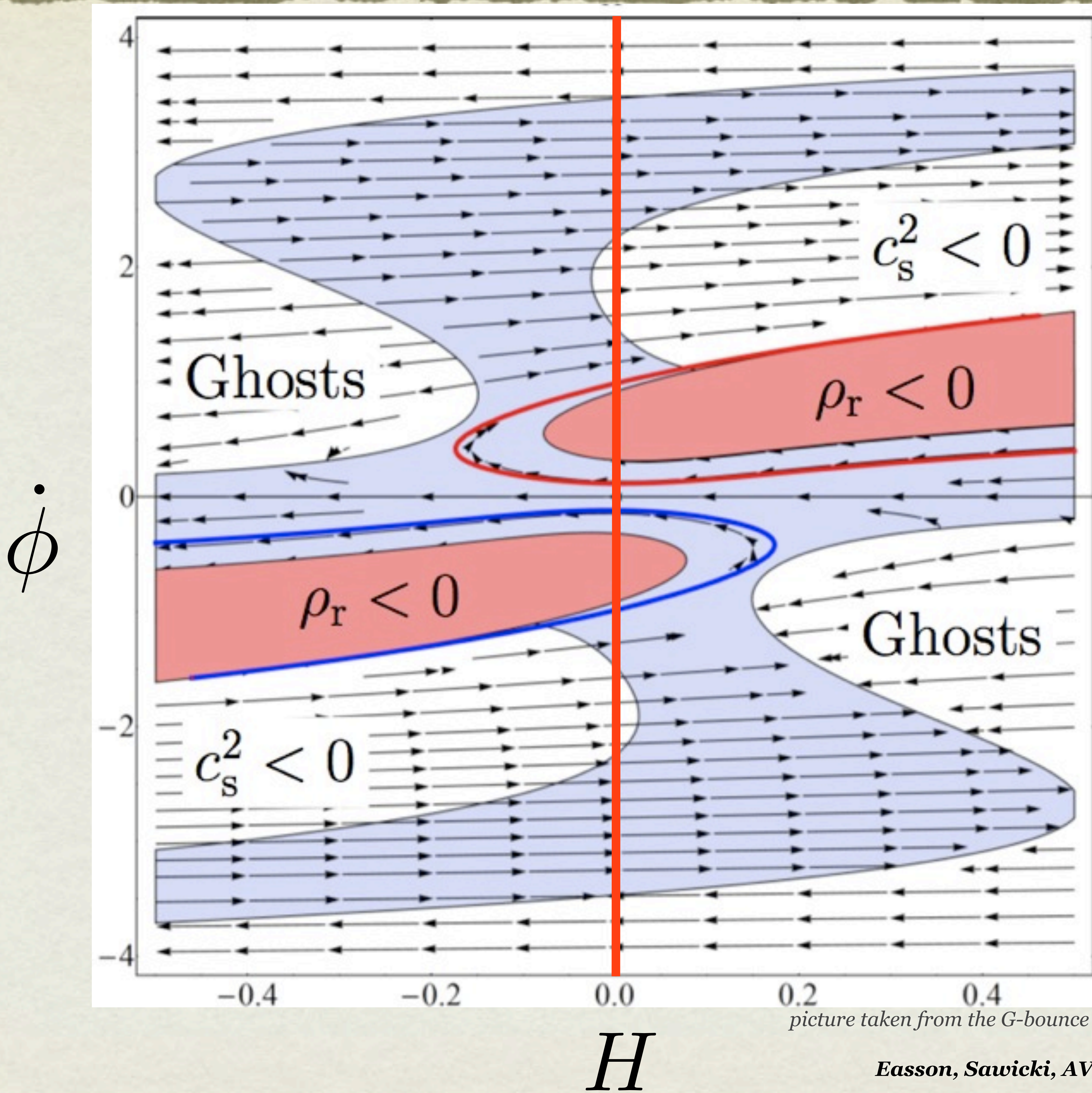
"Healthy" Bounce!



picture taken from the G-bounce

Easson, Sawicki, AV (2011)

G-Bounce: $\mathcal{L} = X - \alpha X^3 + \varkappa X \Box \phi$ **+ Radiation** $\rho_{\rm r}$

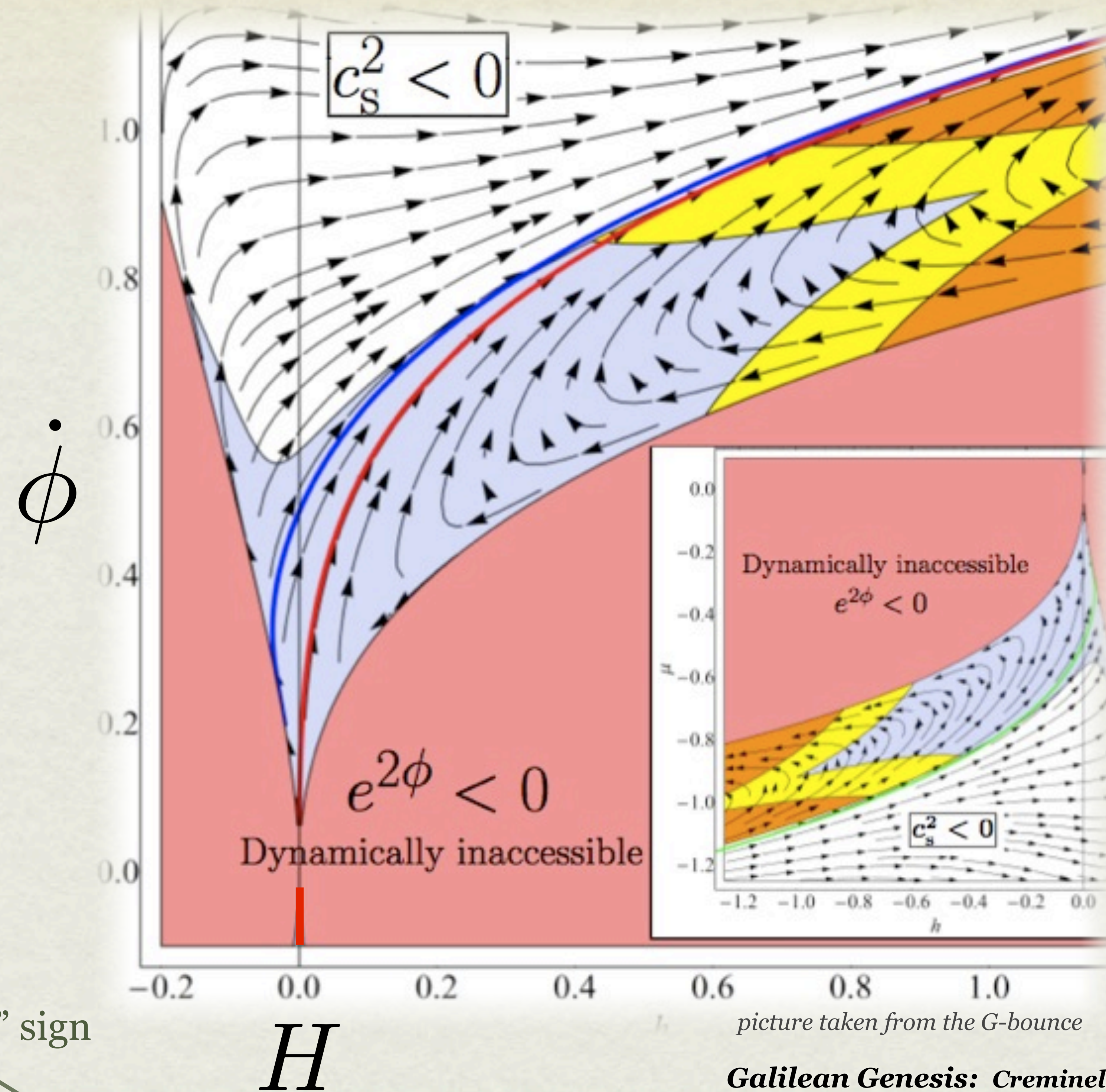


picture taken from the G-bounce

Easson, Sawicki, AV (2011)

$\kappa = 2\beta X + \gamma X^2 \quad K = -\Lambda + X - X^3$ **+ Radiation** ρ_r

CAN ONE GENERALIZE THIS APPROACH FOR SYSTEMS WITHOUT SHIFT-SYMMETRY ?



picture taken from the G-bounce

***Galilean Genesis:** Creminelli, Nicolis, Trincherini (2010)*

wrong “ghosty” sign

“Conformal Galileon”:

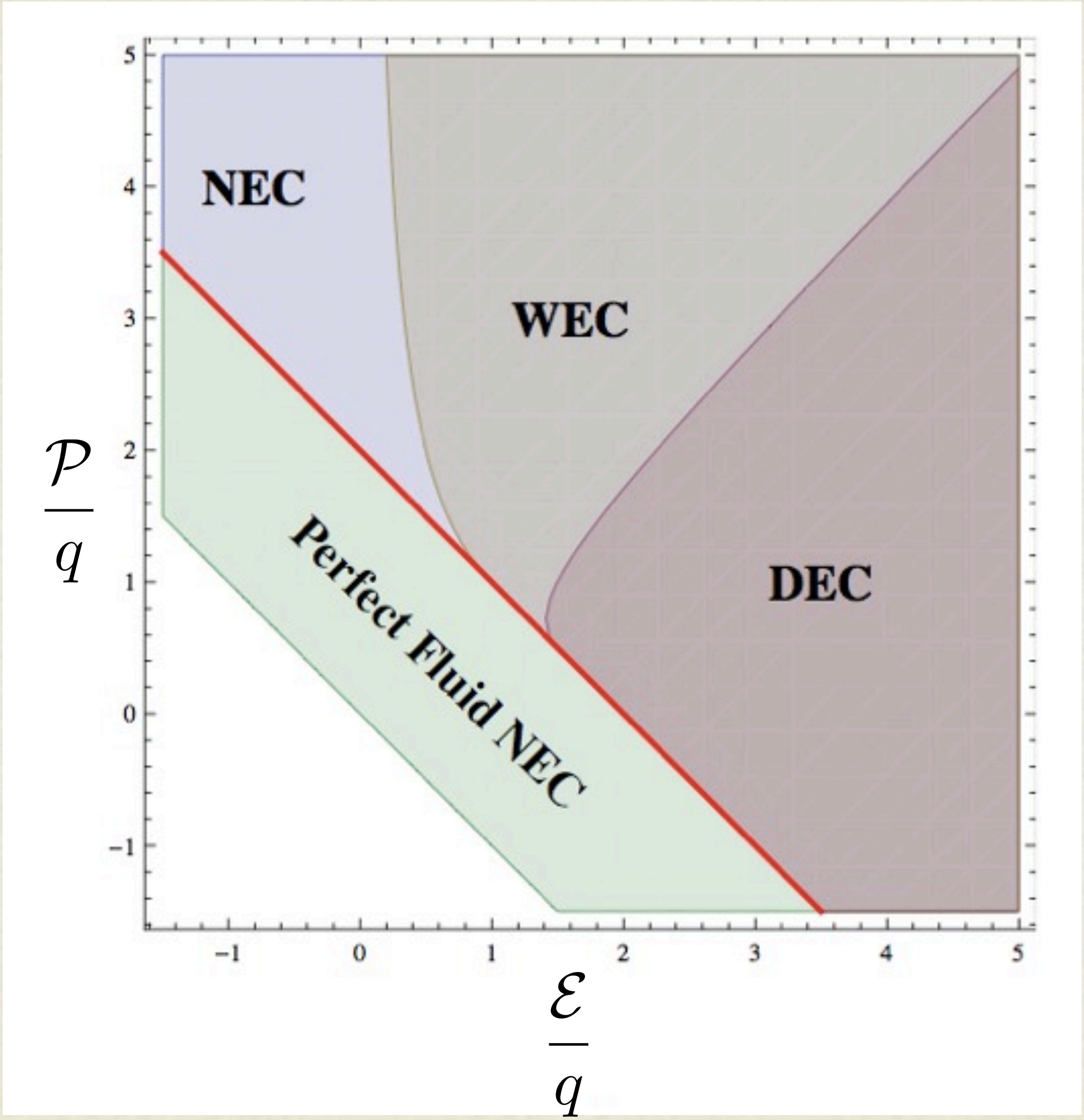
$$\mathcal{L} = -2f^2 e^{2\phi} X + \frac{2f^3}{\Lambda^3} X^2 + \frac{2f^3}{\Lambda^3} X \Box \phi$$

where: $X \equiv \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$ and Λ, f are constants

VIOLATING THE NEC SECRETLY, WITH **W**>-1?

- All models known to violate the NEC without ghosts and gradient instabilities do not have EMT of the perfect fluid for arbitrary *realistic* cosmological solutions which *include* perturbations. Instead they have at least energy fluxes q_μ so that $T_{\mu\nu} = \mathcal{E} u_\mu u_\nu - \perp_{\mu\nu} \mathcal{P} + 2u_{(\mu} q_{\nu)}$
- $\langle q_\mu \rangle = 0$ on average only! But it is present in perturbations!
- the NEC $\Longleftrightarrow$ $\mathcal{E} + \mathcal{P} \geq 2q$
- the NEC is violated for $\dfrac{2q}{\mathcal{E}} > 1 + w > 0$

OTHER ENERGY CONDITIONS



OPEN QUESTIONS

- Is there an instability for the “healthy” sound waves in NEC-violating fluids ?
- Strong coupling in bouncing models?
- Possible anisotropy? Too strong tachyonic / Jeans instabilities around bounces?
- Can one avoid all singularities and troubles for the past? Can one arrange a cyclic or oscillating evolution?
- Perturbations?
- Any *realistic* scenarios? Smooth transition to standard cosmology and inflation?

THANKS A LOT FOR YOUR ATTENTION!