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Entropy for curvature squared gravity using surface term and auxiliary field

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Introduction

aim

Calculate the black hole entropy for the curvature squared gravity

method

Using the surface term of the action

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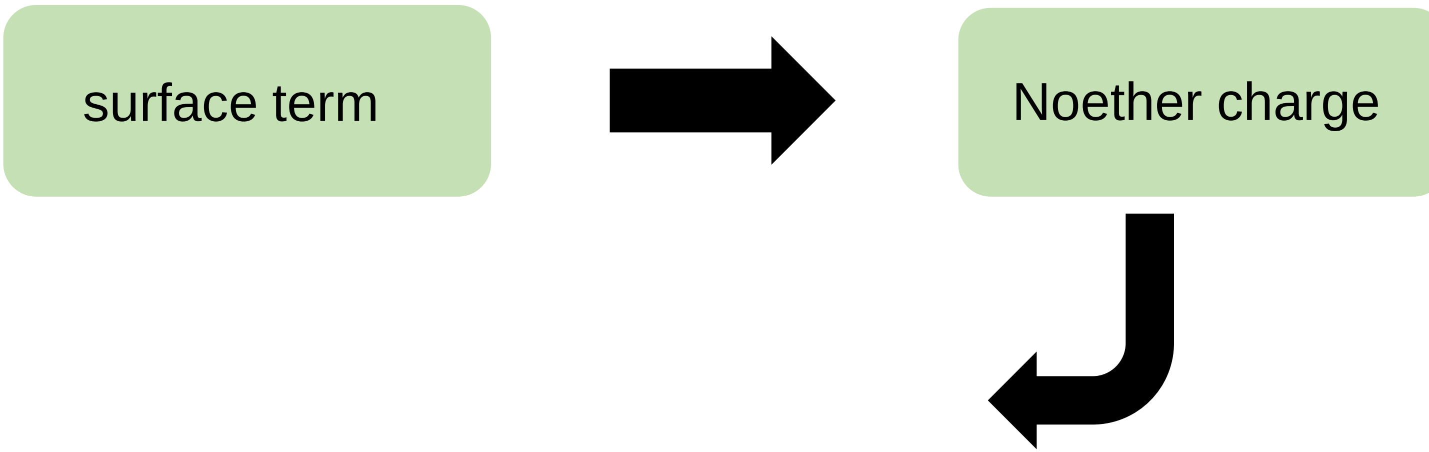

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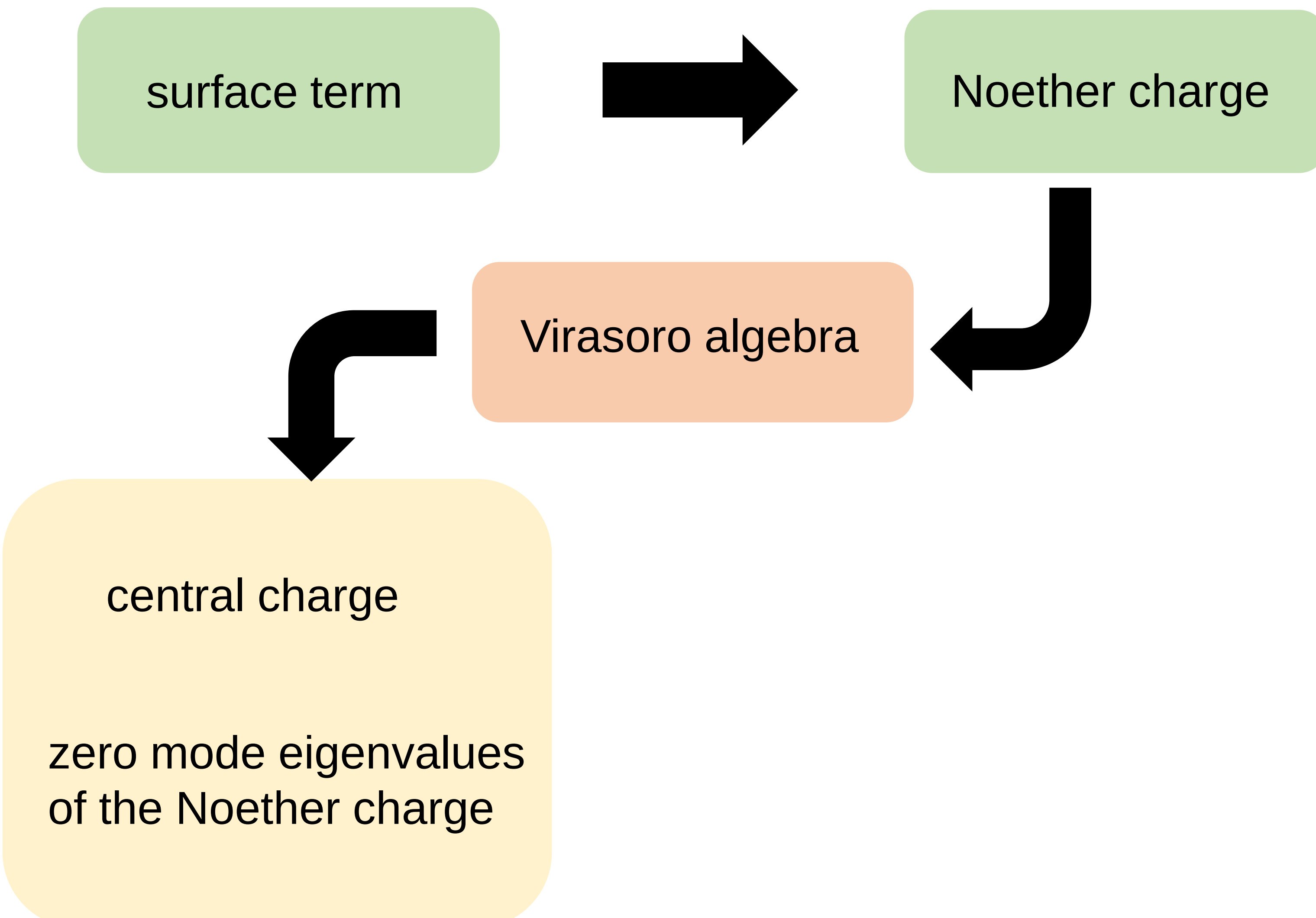

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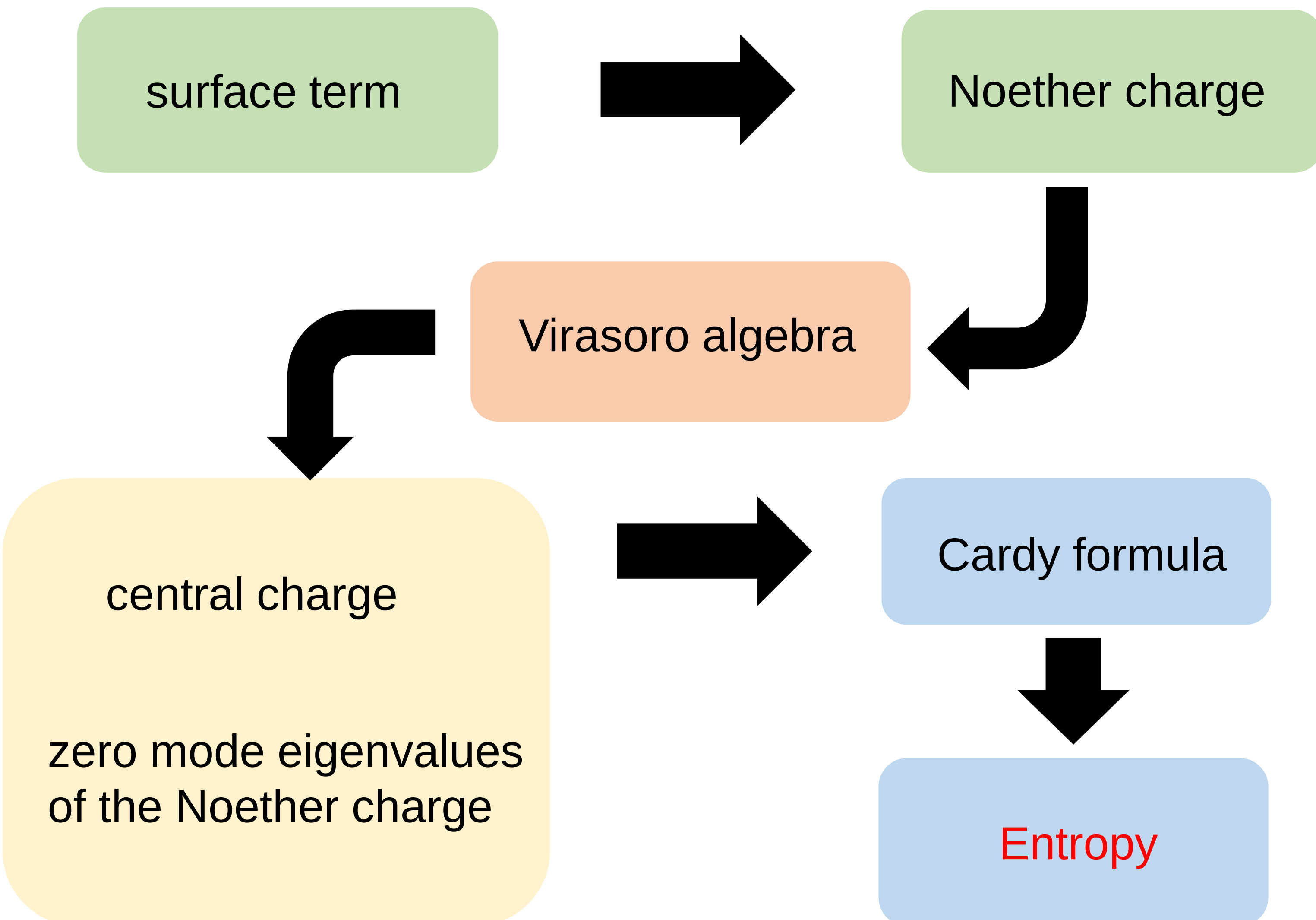

Gravitational action (with curvature-squared term)

$$S = \frac{1}{16\pi G}\int d^D x\sqrt{-g}\left(\underbrace{\sigma R - 2\Lambda_0}_{\text{E-H term}} + \underbrace{a_1 R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} + a_2 R^{\mu\nu}R_{\mu\nu} + a_3 R^2}_{\text{curvature-squared term}}\right)$$

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E-H term   curvature-squared term

the surface term

for the Einstein-Hilbert term

$$\mathrm{S}_{\mathrm{GH}} = \frac{1}{16\pi G}\int d^{D-1}x\sqrt{-\gamma}\,(-2\sigma K)$$

"Gibbons-Hawking term"

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For theories including higher curvature term, there is no corresponding surface term

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alternative method

construct the action including up to second-derivative terms by introducing an auxiliary field which is equivalent with the curvature squared action (on-shell)

Action (including second-order derivative terms)

$$S_\phi = \frac{1}{16\pi G}\int d^D x\sqrt{-g}\left(\phi^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} + b_1\phi^{\mu\nu\rho\sigma}\phi_{\mu\nu\rho\sigma} + b_2\phi^{\mu\nu}\phi_{\mu\nu} + b_3\phi^2\right)$$

$\phi_{\mu\nu\rho\sigma}$ … auxiliary field (same symmetry properties with the Riemann tensor)

$$\phi_{\mu\nu} \equiv g^{\rho\sigma}\phi_{\rho\mu\sigma\nu}, \quad \phi \equiv g^{\mu\nu}\phi_{\mu\nu}$$

Equation of Motion for ϕ

$$R_{\mu\nu\rho\sigma} + 2b_1\phi_{\mu\nu\rho\sigma} + 2b_2\phi_{\langle\mu\rho}g_{\nu\sigma\rangle} + 2b_3\phi g_{\langle\mu\rho}g_{\nu\sigma\rangle} = 0$$

$$\phi_{\langle\mu\rho}g_{\nu\sigma\rangle} = \frac{1}{4}\left(\phi_{\mu\rho}g_{\nu\sigma} - \phi_{\nu\rho}g_{\mu\sigma} - \phi_{\mu\sigma}g_{\nu\rho} + \phi_{\nu\sigma}g_{\mu\rho}\right)$$

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parameter choice

$$a_1 = -\frac{1}{4b_1} \qquad a_2 = \frac{b_2}{b_1\left\{4b_1 + (D-2)\,b_2\right\}}$$

$$a_3 = -\frac{b_2^2 - 4b_1b_3 + Db_2b_3}{2b_1\left\{4b_1 + (D-2)\,b_2\right\}\left\{2b_1 + (D-1)\,b_2 + D\,(D-1)\,b_3\right\}}$$

$$S\left(R^2_{\mu\nu\rho\sigma}, \ldots\right) \qquad S_\phi$$

equivalence

surface term for S_ϕ

$$\mathrm{S}_\phi|_{\mathrm{surface}} = -\frac{1}{16\pi G}\int d^{D-1}x\sqrt{-\gamma}\left(\cdot\, 4\phi^{rirj}K_{ij}\right)$$

$$\phi_{\mu\nu\rho\sigma} = -\frac{1}{2b_1}\left[R_{\mu\nu\rho\sigma} - \frac{4b_2 R_{\langle\mu\rho}g_{\nu\sigma\rangle}}{4b_1+(D-2)\,b_2} + \frac{2\left(b_2^2-4b_1b_3+Db_2b_3\right)Rg_{\langle\mu\rho}g_{\nu\sigma\rangle}}{\{4b_1+(D-2)\,b_2\}\{2b_1+(D-1)\,b_2+D\,(D-1)\,b_3\}}\right]$$

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surface term for the curvature squared action

$$S_{\mathrm{GGH}} = -\frac{1}{16\pi G}\int d^{D-1}x\sqrt{-\gamma}\left(2\sigma K + 4\phi^{rirj}K_{ij}\right)$$

Generalized Gibbons Hawking term

(A)dS backgrounds solutions

$$R_{\mu\nu\rho\sigma} = \frac{2\Lambda}{(D-1)(D-2)}\left(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right), \quad R_{\mu\nu} = \frac{2\Lambda}{D-2}g_{\mu\nu}, \quad R = \frac{2D\Lambda}{D-2}$$

$$\Lambda_0 = \sigma\Lambda + \frac{2(D-4)}{D-2}\left[\left\{D(a_3 - a_1) + a_2 + 4a_1\right\}\frac{1}{D-2} + a_1\left(\frac{D-3}{D-1}\right)\right]\Lambda^2$$

$$S_{\mathrm{GGH}} = -\frac{1}{8\pi G}\int d^{D-1}x\sqrt{-\gamma}\left\{(\sigma + F)K\right\}$$

$$F \equiv \frac{4\Lambda}{(D-1)(D-2)}\left\{2a_1 + (D-1)(a_2 + Da_3)\right\}$$

Noether charge

General form of the surface term

$$S_{\rm B} = \frac{1}{16\pi G}\int_{\partial\mathcal{M}} d^{D-1}x\sqrt{-\gamma}\mathcal{L}_{\rm B}$$

diffeomorphism

$x^\mu \to x^\mu + \xi^\mu$ (leaves the horizon structure invariant)

Noether charge

$$Q[\xi] = \frac{1}{2}\int_{\partial\mathcal{M}} \sqrt{-h}d\Sigma_{\mu\nu}J^{\mu\nu}$$

$$J^\mu[\xi] = \nabla_\nu J^{\mu\nu}[\xi] = \frac{1}{16\pi G}\nabla_\nu\left\{\mathcal{L}_B\left(\xi^\mu N^\nu - \xi^\nu N^\mu\right)\right\}$$

Schwarzschild type metric

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + h_{ij}dx^i dx^j$$

conserved charge

$$Q[\xi] = \frac{1}{8\pi G}\int d^{D-2}x\sqrt{-h}\left(\kappa T - \frac{1}{2}\partial_t T\right)(\sigma + F)$$

Fourier expansion

$$Q = \sum_m Q_m A_m$$

$$T = \sum_m A_m T_m, \quad A_m^* = A_{-m}$$

$$T_m = \frac{1}{\alpha}\exp\{im(\alpha t + g(\rho) + p\cdot x)\}$$

Virasoro algebra

$$[Q_m, Q_n] = i(m-n)Q_{m+n} - \frac{im^3\alpha A}{16\pi G\kappa}(\sigma + F)\delta_{m+n,0}$$

$$Q_m = \frac{\kappa A}{8\pi G\alpha}(\sigma + F)\delta_{m,0}$$

A ... surface area of black hole

Entropy (Using the Cardy formula)

Virasoro algebra

$$[Q_m, Q_n] = i(m-n)\,Q_{m+n} - \frac{im^3\alpha A}{16\pi G\kappa}(\sigma+F)\,\delta_{m+n,0}$$

$$\left(Q_m = \frac{\kappa A}{8\pi G\alpha}(\sigma+F)\,\delta_{m,0} \right)$$

central charge

$$\frac{c}{12} = \frac{\alpha}{16\pi G\kappa}A(\sigma+F)$$

zero mode eigenvalues

$$Q_0 = \frac{\kappa}{8\pi G\alpha}A(\sigma+F)$$

Cardy formula

$$S = 2\pi\sqrt{\frac{cQ_0}{6}} = \frac{A}{4}(\sigma+F)$$

entropy for the curvature squared gravity

several specific examples

Einstein+Gauss-Bonnet

$$S = \frac{A}{4}\left\{\sigma + \frac{4a\,(D-3)}{(D-1)}\Lambda\right\}$$

$$(a_1, a_2, a_3) = (1, -4, 1)$$

New Massive Gravity (3-dim)

$$S = \frac{A}{4}\left(\sigma - \frac{1}{2m^2}\Lambda\right)$$

$$(a_1, a_2, a_3) = \left(0, -1, \tfrac{3}{8}\right)$$

Critical Gravity (4-dim)

$$S = 0$$

$$(a_1, a_2, a_3) = \left(0, -\tfrac{3}{2\Lambda}, -\tfrac{1}{2\Lambda}\right)$$

consistent with the Wald entropy!

Conclusion

we have calculated the entropy for D-dimensional gravity with curvature squared term

introducing an auxiliary field, we have obtained the second-derivative formed action which is equivalent with the curvature squared action (on-shell)

we have calculated the surface action and the Black Hole entropy for the Schwarzschild type metric.

special case of the parameters

$\underline{a_1 = 0}$

$$\mathcal{L}_\phi = \phi^{\mu\nu} R_{\mu\nu} + b_2 \phi_{\mu\nu}^2 + b_3 \phi^2$$

$$a_2 = -\frac{1}{4b_2} \quad a_3 = \frac{b_3}{4b_2(b_2 + Db_3)}$$

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