First results for SU(2) Yang-Mills with one adjoint Dirac Fermion

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from research with
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Outline

Introduction

- Motivation
- Dirac $\rightarrow$ Majorana decomposition
- Lattice formulation
- Quantum numbers
- Lattice topology

Results

- Phase diagram
- Spectrum
- Mass anomalous dimension
- Topological observables [arXiv:1209.5579]

What and why?

$\mathrm{SU}(2)$ + 1 adjoint Dirac flavour

- $\equiv \mathrm{SU}(2)$ + 2 adjoint Majorana flavours

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- What about 3 Majorana flavours ($\equiv$ 1.5 Dirac dof)?

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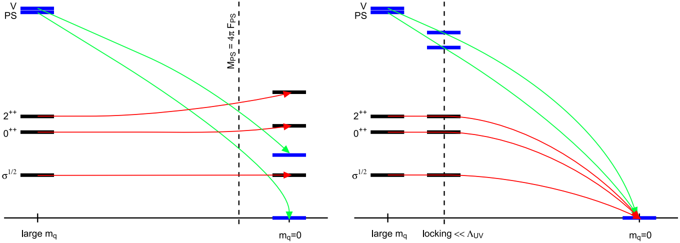

(Figure: Agostino Patella, from arXiv:0911.0020)

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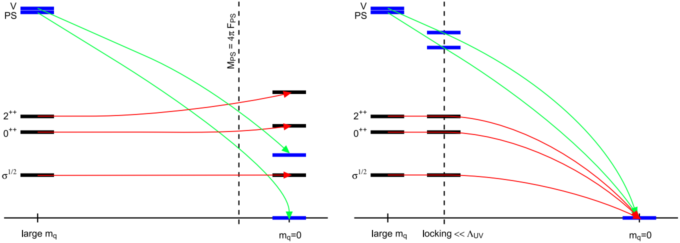

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  - Dynamical quenching in semiclassical dynamics—fermions decouple from e.g. topology

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- Translate quantum numbers

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Chiral representation of Dirac algebra:

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Charge conjugation: $\psi_{\mathsf{C}} = C\overline{\psi}^{\mathrm{T}}$,

$$C = -i\gamma^2\gamma^0 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$$

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See also e.g. Montvay, hep-lat/9510042

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Majorana constraint $\psi_{\mathsf{M}\pm\mathsf{C}} \equiv C\overline{\psi}_{\mathsf{M}\pm}^{\mathrm{T}} = \psi_{\mathsf{M}\pm}$ satisfied.
Now we can reexpress the action.

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  - $\mathrm{U}(1)$ in Weyl basis $\leftrightarrow$ baryon number B in Dirac basis

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- Then take correlation functions; e.g. for $\Gamma \in \{\mathbb{1}, \gamma_5\gamma_\mu, \gamma_5\}$,

$$\begin{aligned} &\left\langle O^{\dagger}_{+-}(x)\, O_{+-}(0)\right\rangle \\ &= -\mathrm{tr}\overline{\Gamma}\, C D^{-1\mathrm{T}}(0;x)\, C\Gamma D^{-1}(0;x) + \mathrm{tr}(\overline{\Gamma}C)^{\mathrm{T}} D^{-1\mathrm{T}}(0;x)\, C\Gamma D^{-1}(0;x) \\ &\qquad - \mathrm{tr}\, C\overline{\Gamma} D^{-1}(x;0)\Gamma C D^{-1\mathrm{T}}(x;0) + \mathrm{tr}(C\overline{\Gamma})^{\mathrm{T}} D^{-1}(x;0)\Gamma C D^{-1\mathrm{T}}(x;0) \\ &= -\frac{1}{4}\mathrm{tr}\overline{\Gamma} D^{-1}(x;0)\Gamma D^{-1}(0;x) \end{aligned}$$

Quantum numbers

Dirac bilinears	Majorana bilinears	$U(1)^P$	correlators
$\bar{\psi}\gamma_0\gamma_5\psi$	$O_{++}(\gamma_0\gamma_5) + O_{--}(\gamma_0\gamma_5)$	0^-	singlet γ_5, $\gamma_0\gamma_5$
$\bar{\psi}\gamma_5\psi$	$O_{++}(\gamma_5) + O_{--}(\gamma_5)$		
$\psi^{\mathrm{T}}C\gamma_5\psi$	$-i(O_{++}(1) - O_{--}(1) + 2iO_{+-}(1))$	2^-	triplet 1
$\psi^{\dagger}C\gamma_5\psi^*$	$-i(O_{++}(1) - O_{--}(1) - 2iO_{+-}(1))$	-2^-	
$\bar{\psi}\psi$	$O_{++}(1) + O_{--}(1)$	0^+	singlet 1, γ_0
$\bar{\psi}\gamma_0\psi$	$O_{+-}(\gamma_0)$		
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$\bar{\psi}\gamma_5\boldsymbol{\gamma}\psi$	$O_{++}(\gamma_5\boldsymbol{\gamma}) + O_{--}(\gamma_5\boldsymbol{\gamma})$	0^+	singlet $\gamma_5\boldsymbol{\gamma}$, $\gamma_0\gamma_5\boldsymbol{\gamma}$
$\bar{\psi}\gamma_0\gamma_5\boldsymbol{\gamma}\psi$	$O_{+-}(\gamma_0\gamma_5\boldsymbol{\gamma})$		
$\bar{\psi}\gamma_0\boldsymbol{\gamma}\psi$	$O_{+-}(\gamma_0\boldsymbol{\gamma})$	0^-	singlet $\boldsymbol{\gamma}$, $\gamma_0\boldsymbol{\gamma}$
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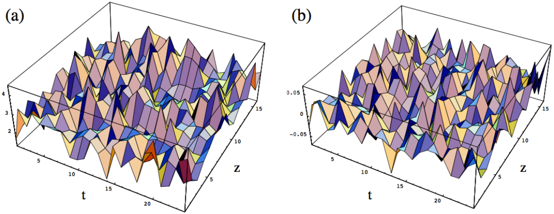
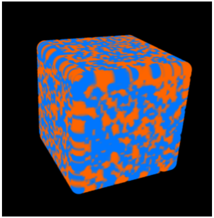
Gauge noise





(left: from Schäfer & Shuryak arXiv:hep-ph/9610451 §III.B.2)

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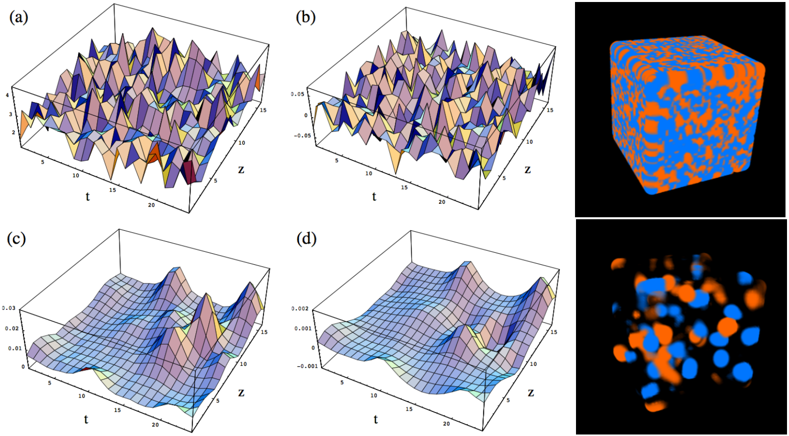

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Lattice results

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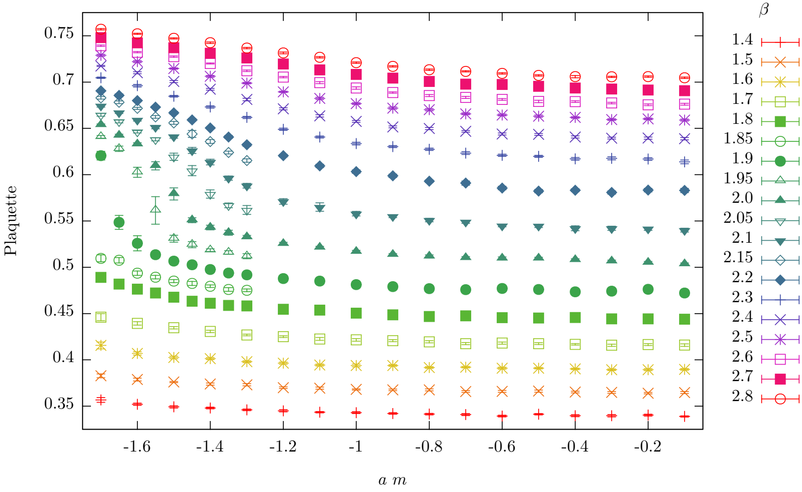
Phase diagram



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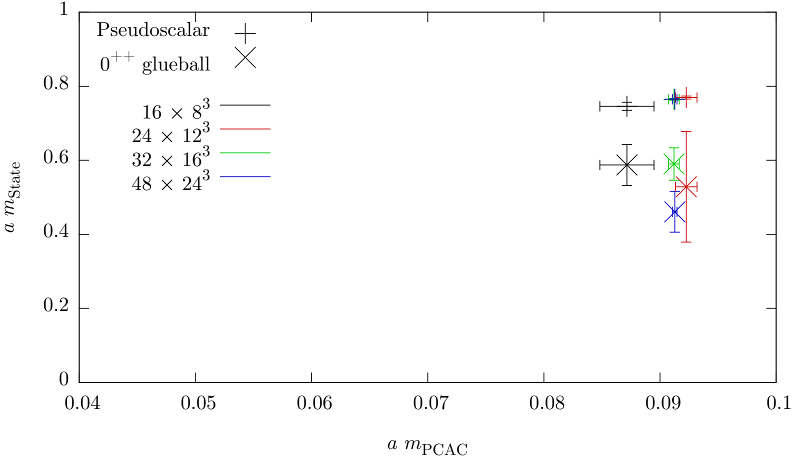
Lattice parameters

Lattice	V	$-am_0$	$N_{\rm conf}$	Acceptance	$N_{\rm pf}$	$t_{\rm len}$	$n_{\rm steps}$	Machine
A1	16×8^3	1.475	2400	91.4%	1	1.0	10	SC
A2	16×8^3	1.500	2200	90.9%	1	1.0	10	SC, UL
A3	16×8^3	1.510	2400	89.8%	1	1.0	10	SC, UL
A4	16×8^3	1.510	4000	92.4%	2	1.0	8	SC
B1	24×12^3	1.475	2400	79.9%	1	1.0	10	SC, UL
B2	24×12^3	1.500	2300	78.7%	1	1.0	10	SC, UL
B3	24×12^3	1.510	4000	88.5%	2	1.0	10	SC, UL
C1	32×16^3	1.475	2100	90.6%	1	1.0	20	SC
C2	32×16^3	1.490	2300	90.0%	1	1.0	20	SC, UL
C3	32×16^3	1.510	2200	89.4%	1	1.0	20	UL
C4	32×16^3	1.510	2300	83.2%	2	4.0	45	BGP
C5	32×16^3	1.514	2300	89.8%	1	1.0	20	UL, BGP
C6	32×16^3	1.519	2300	81.8%	1	1.0	20	UL, BGP
C7	32×16^3	1.523	2200	88.0%	1	1.0	20	SC
D1	48×24^3	1.510	1534	80.5%	2	4.0	65	BGP
D2	48×24^3	1.523	2168	91.4%	1	1.0	40	BGP

Finite-volume study



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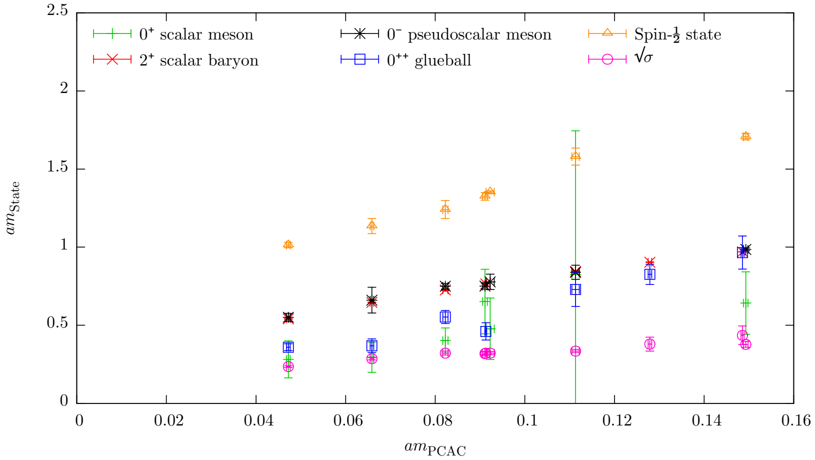
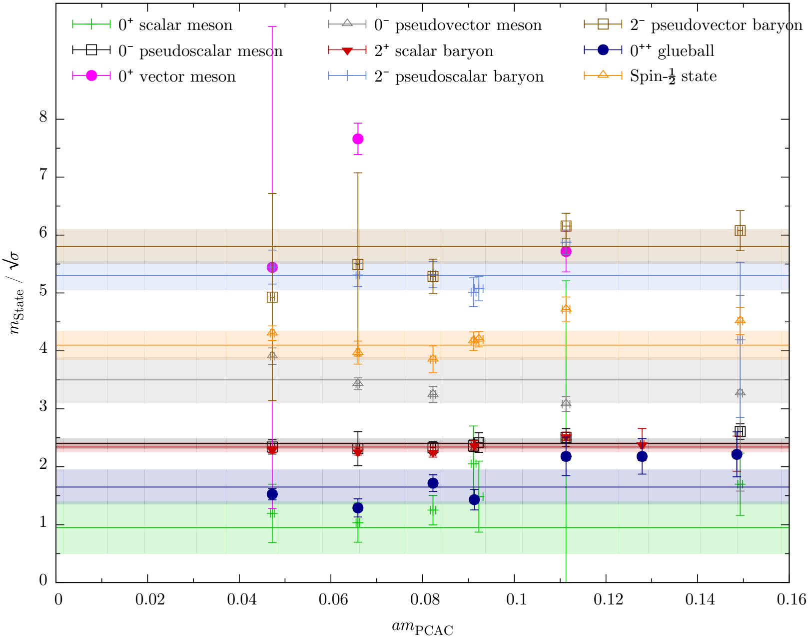
Spectrum



0⁺ scalar meson
0⁻ pseudoscalar meson
Spin-½ state
2⁺ scalar baryon
0⁺⁺ glueball
√σ
am_{State}
am_{PCAC}

Spectral ratios



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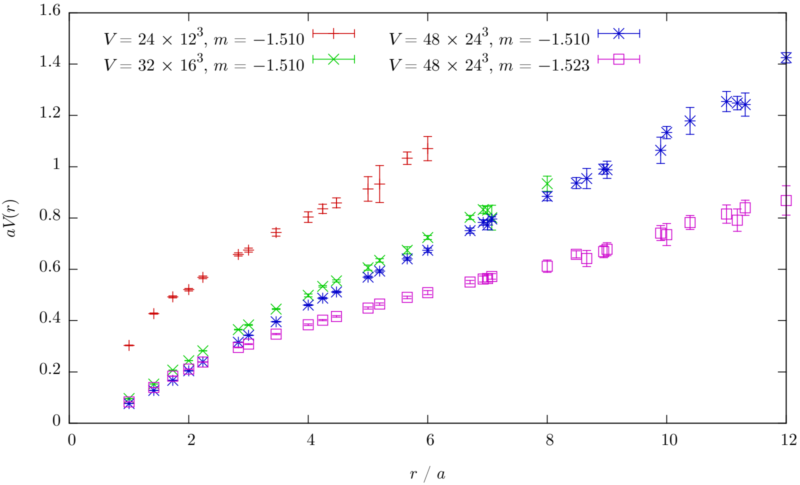

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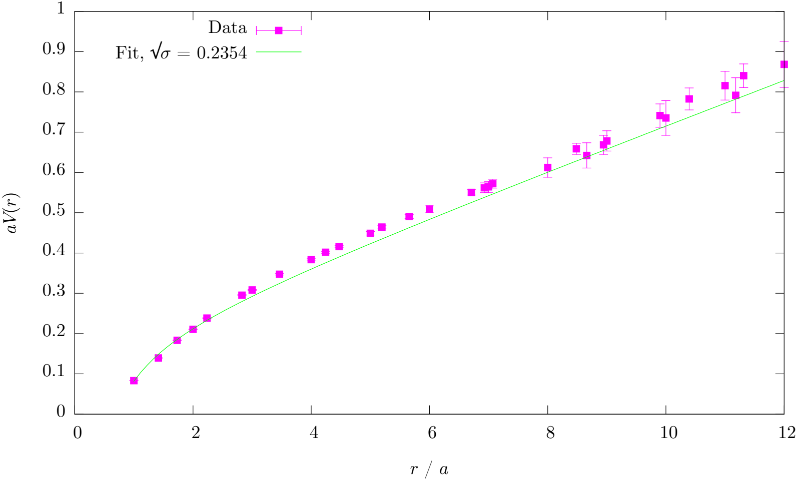

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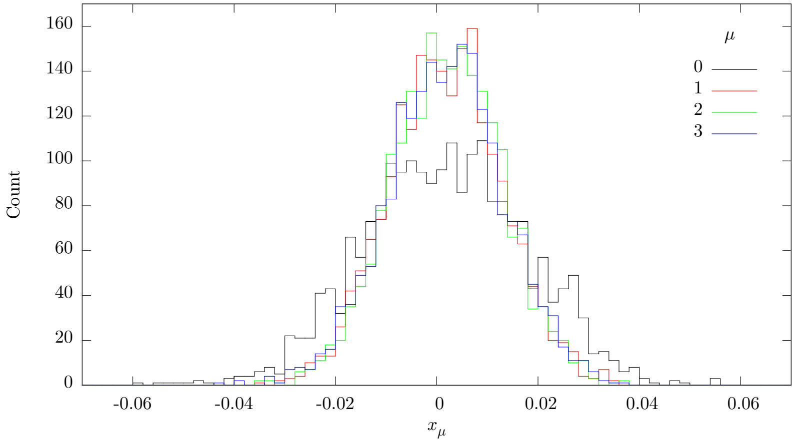
Center symmetry



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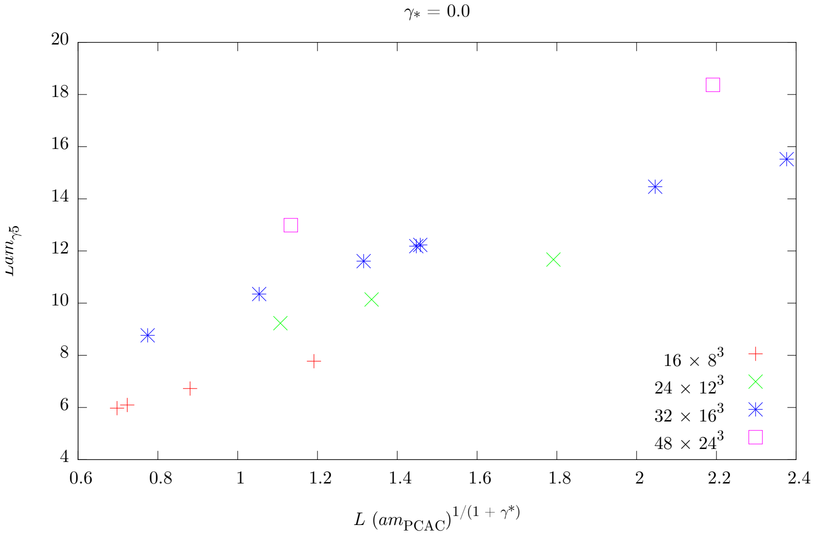

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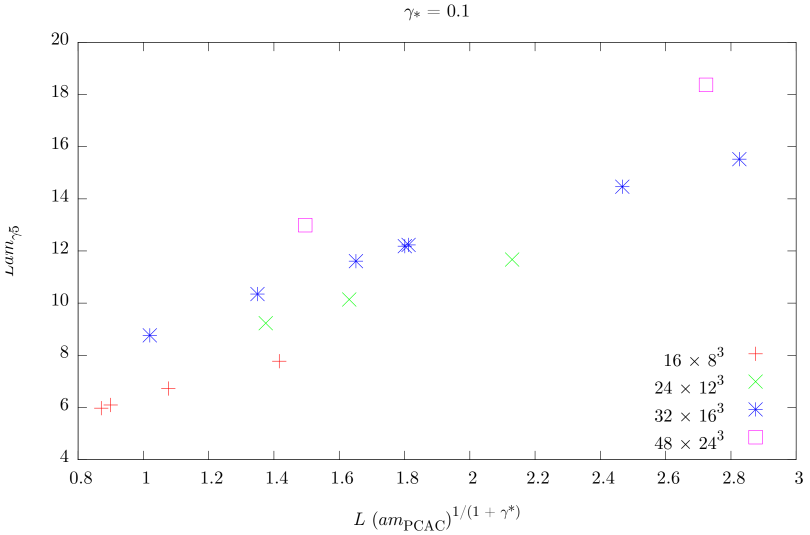

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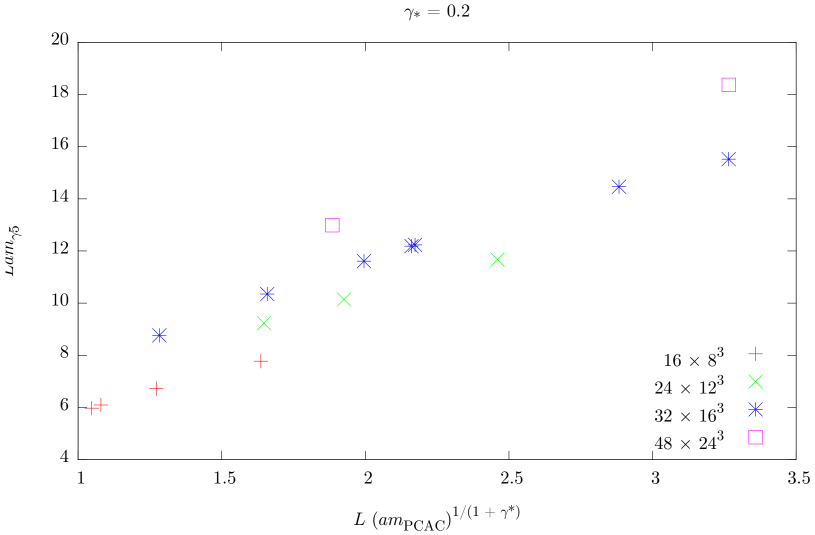

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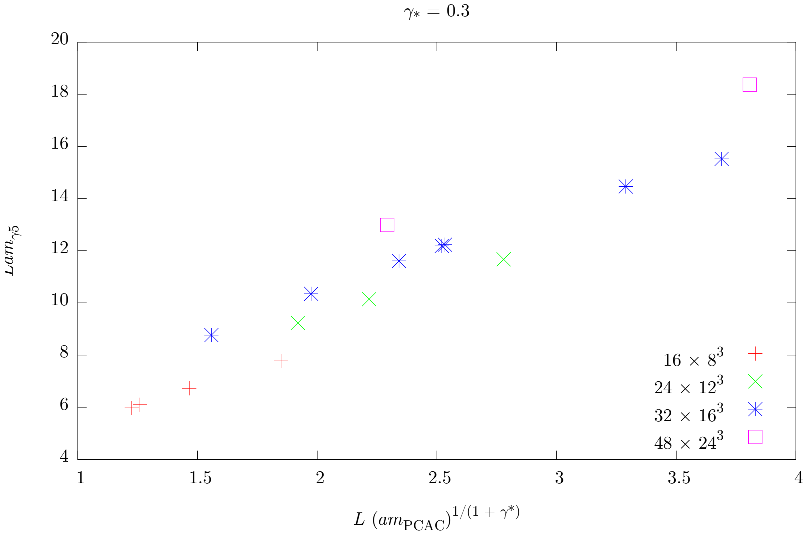

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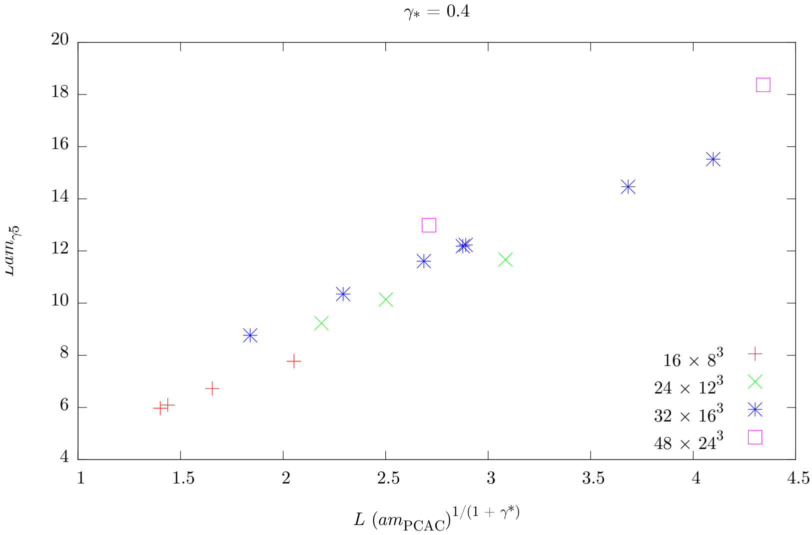

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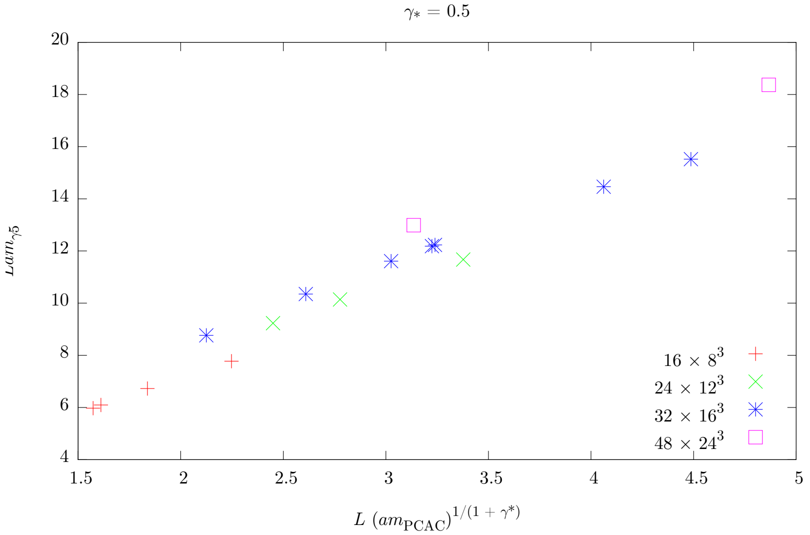

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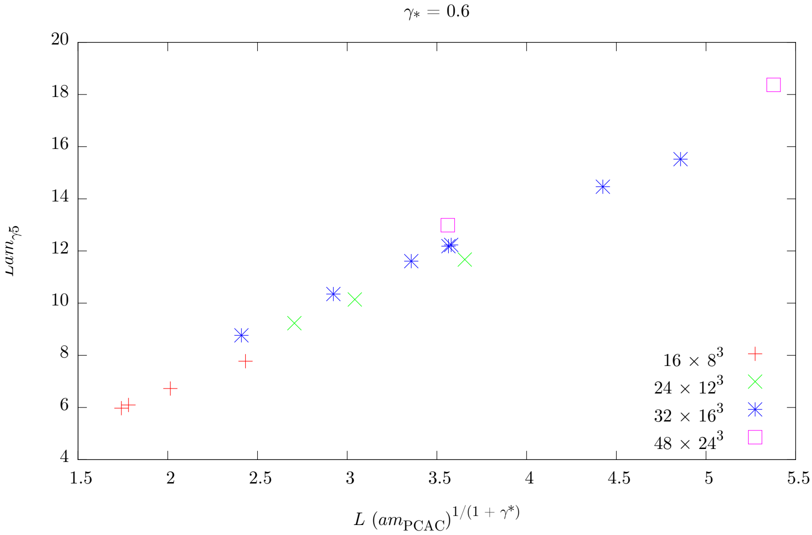

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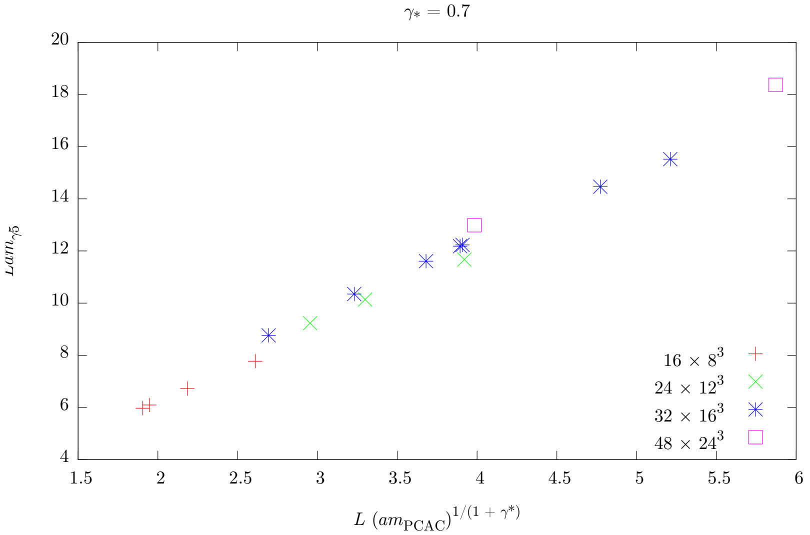

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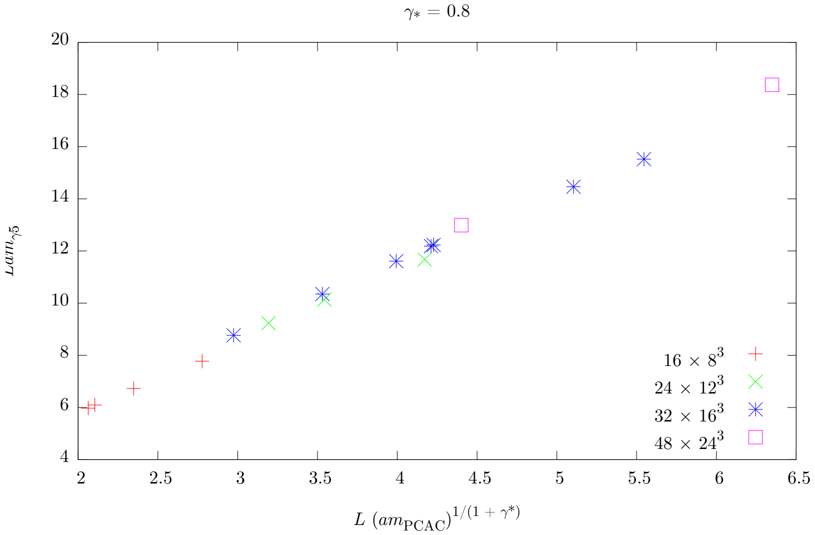

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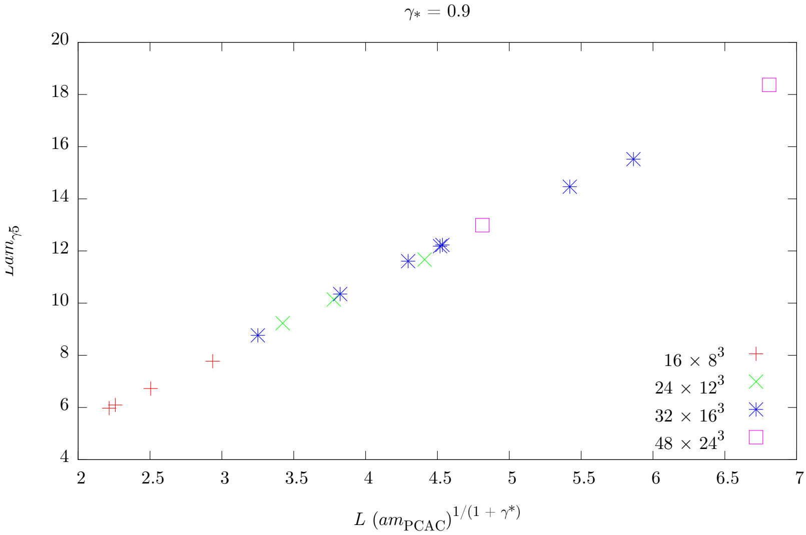

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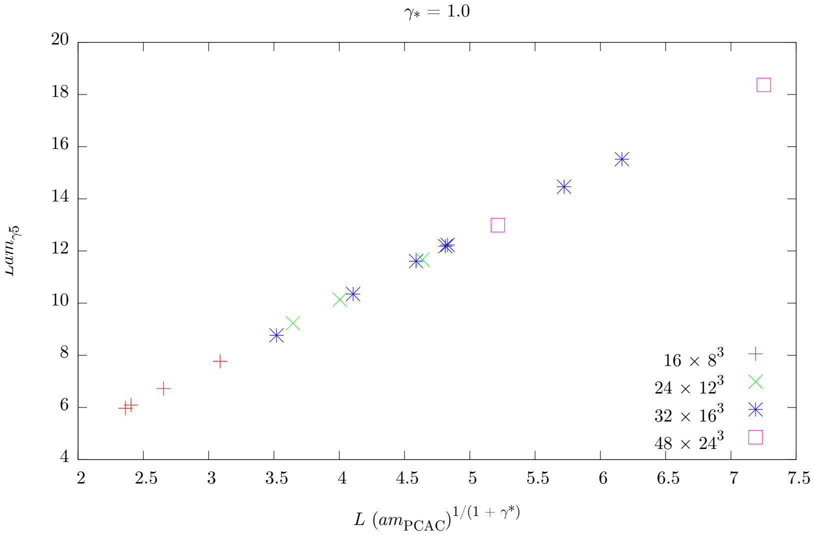

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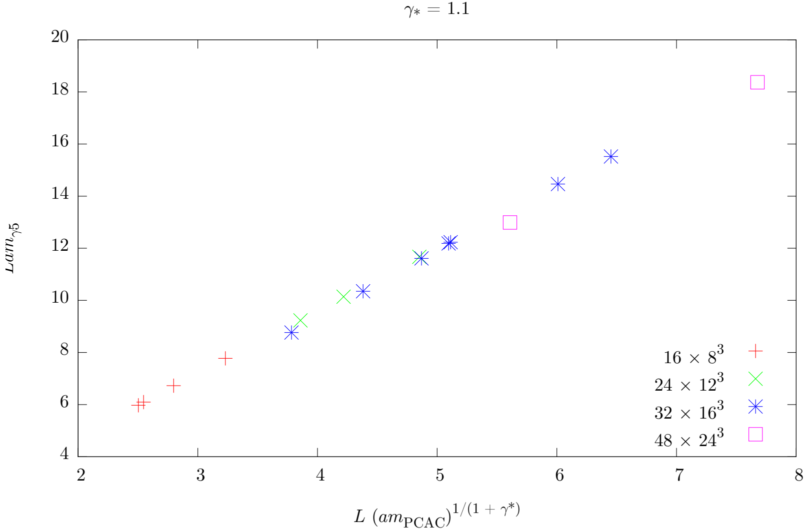

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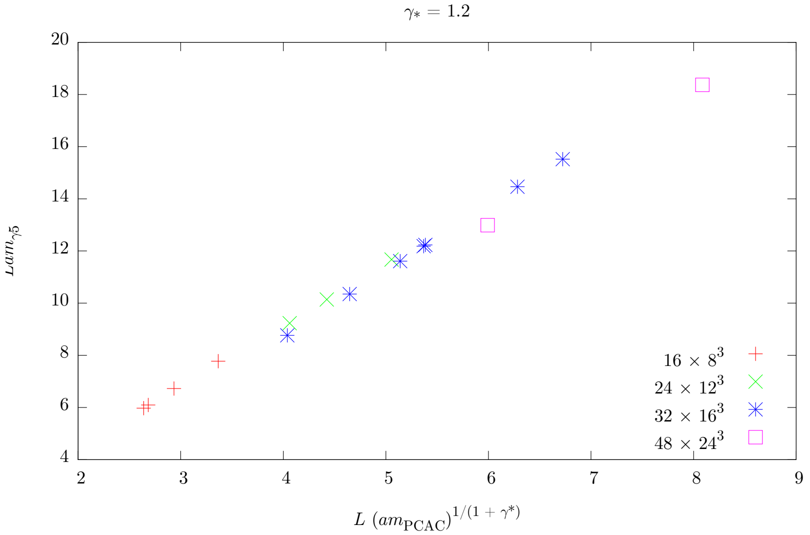

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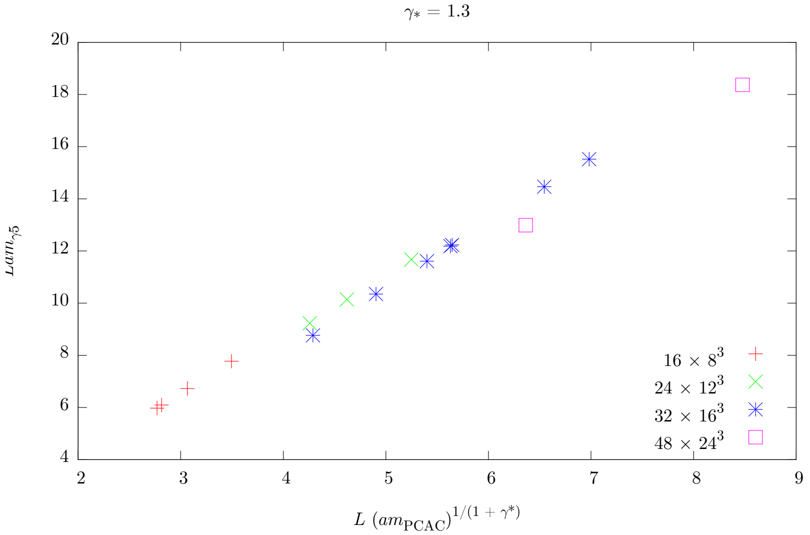

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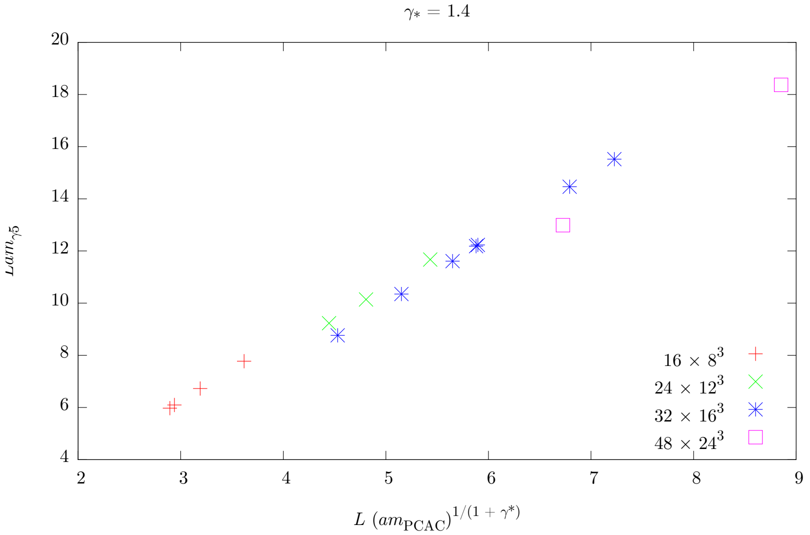

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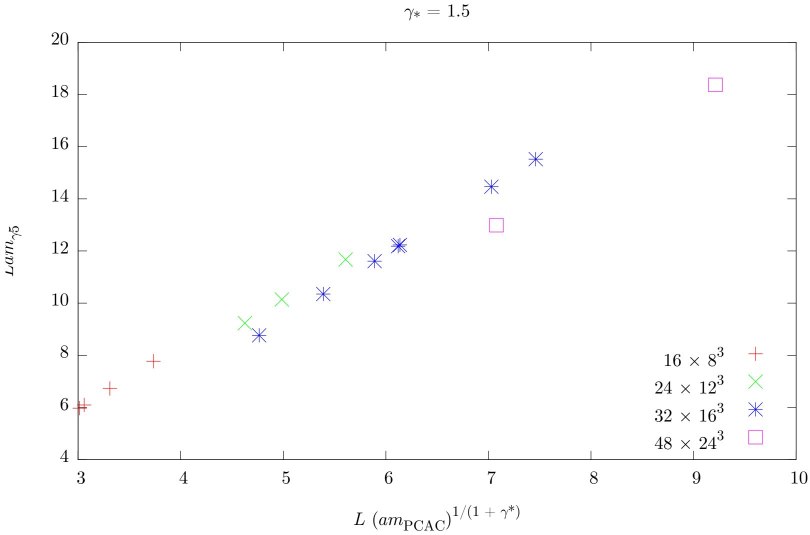

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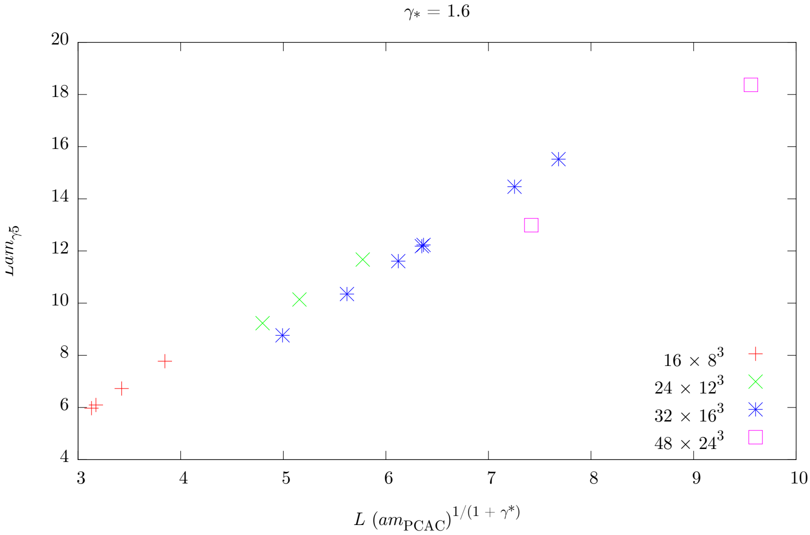

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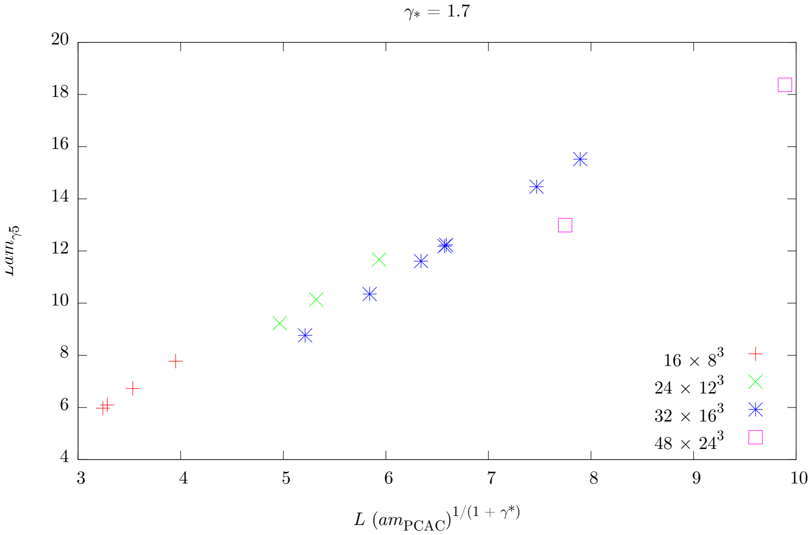

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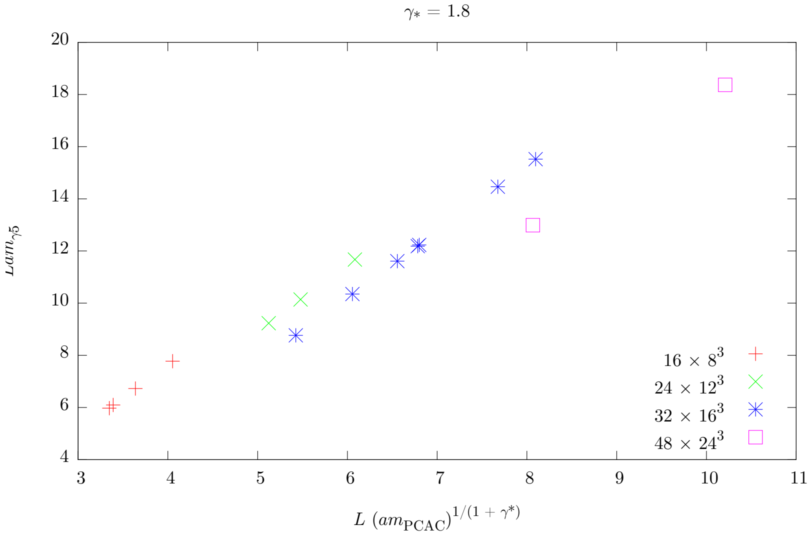

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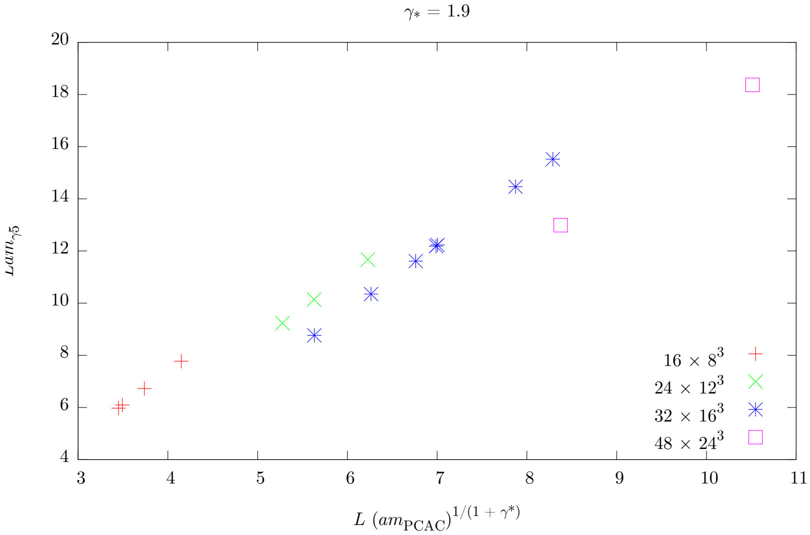

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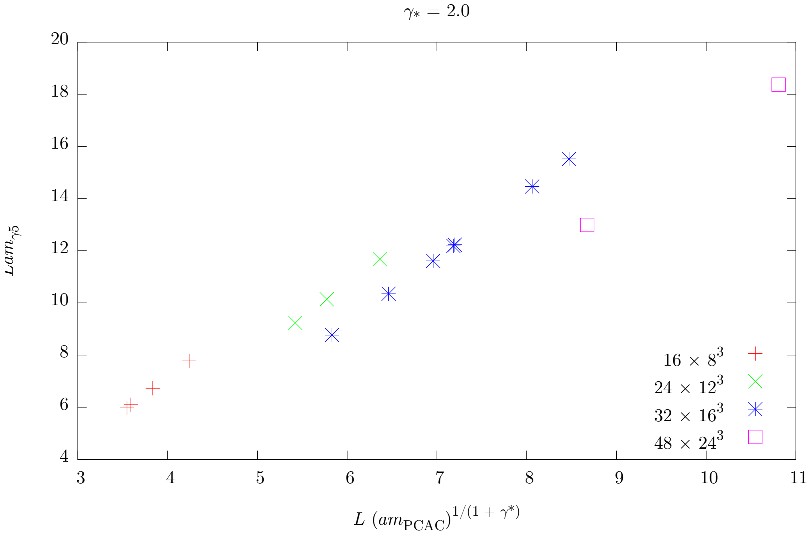

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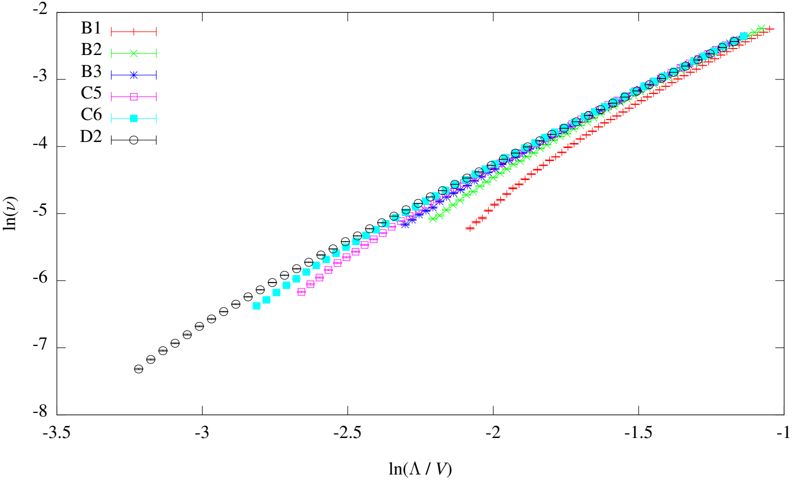
from Patella [arxiv:1204.4432]

Mode number results



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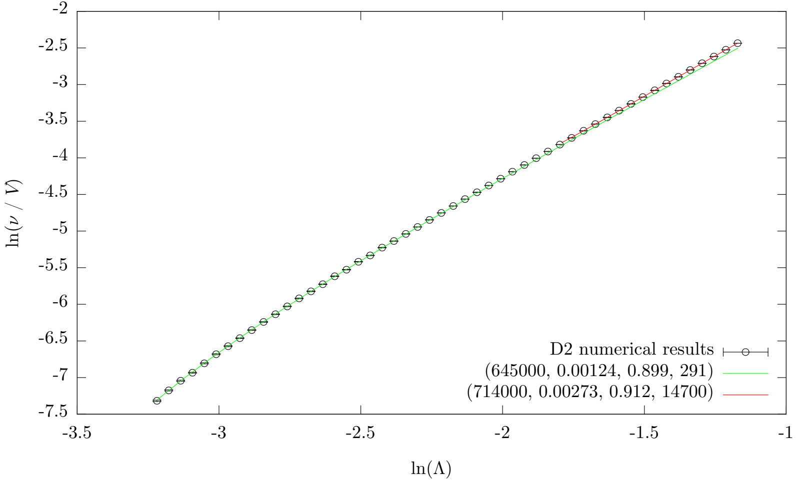

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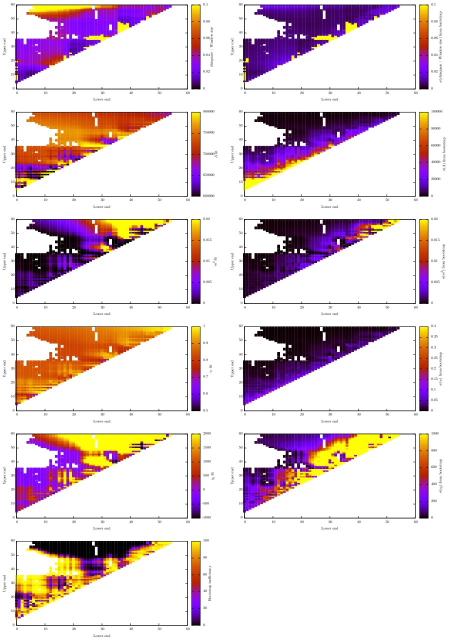

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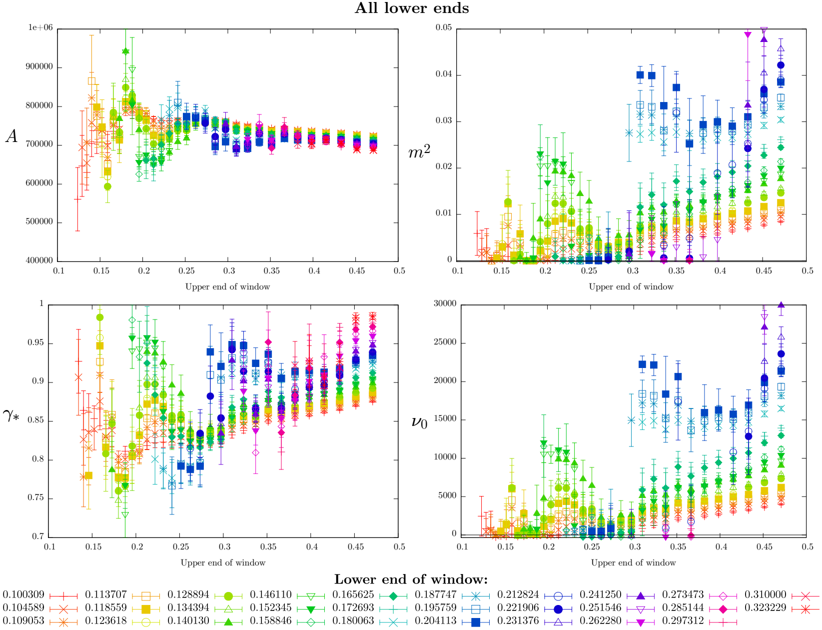

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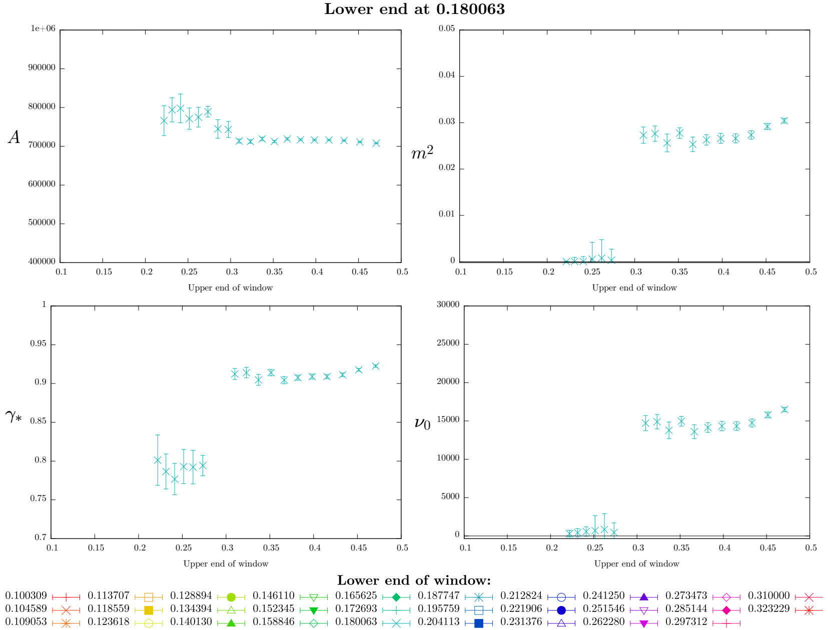

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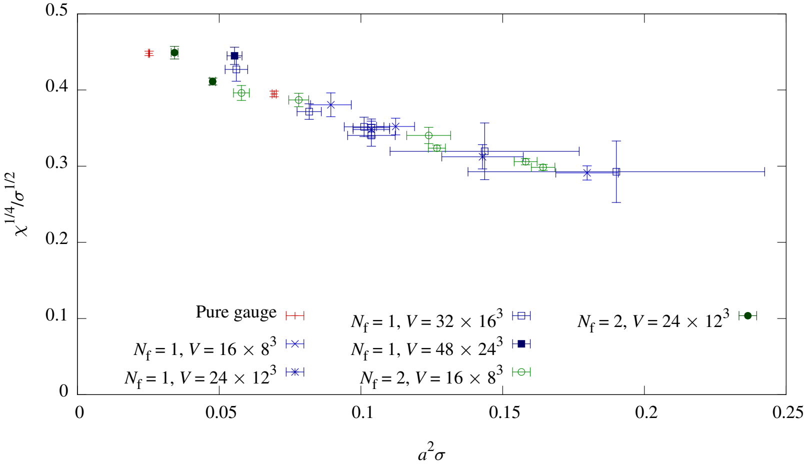

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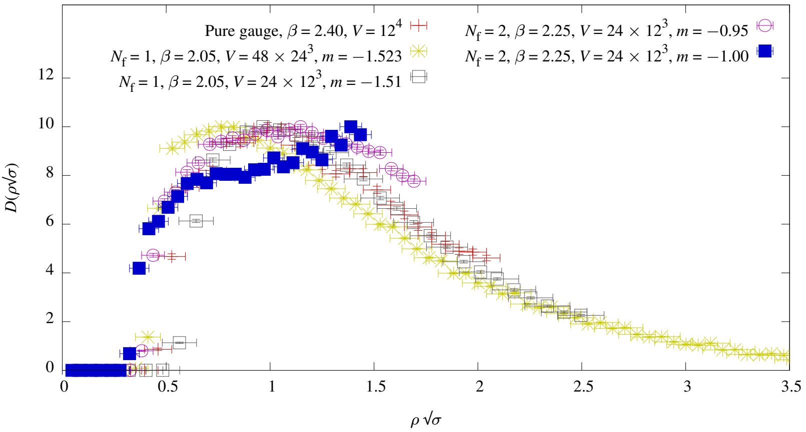
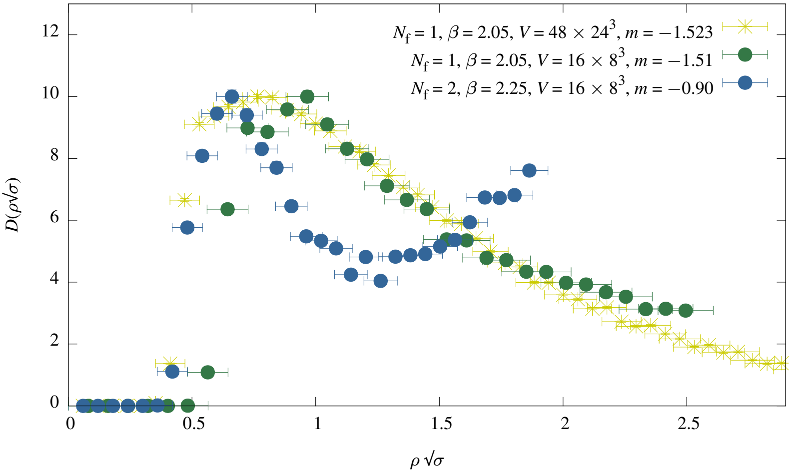
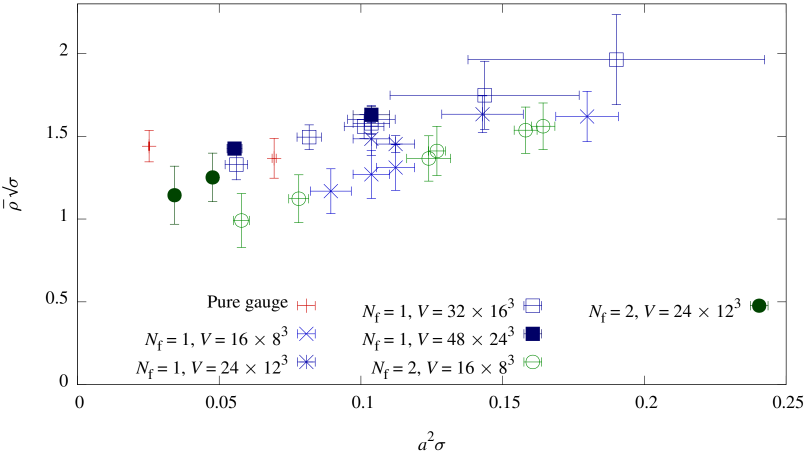


Instanton size distribution finite-volume effects



Average instanton size



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  - $\mathrm{SU}(3) + 8$, 12 fundamental flavours

ありがとうございました！

Back-up slides

Visualisation of topological charge distribution in 5D