Testing a model for the puzzling spin 0 mesons

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Work with A. H. Fariborz, R. Jora and M.N. Shahid :

Phys. Rev. D **79**, 074014 (2009); arXiv: hep-ph: 0902.2825

Phys. Rev. D **83**, 034018 (2011); arXiv: hep-ph: 1012.4868

arXiv: hep-ph: 1106.4538

arXiv: hep-ph: 1108.3581

Evolution of the Sakata Model

1) S. Sakata, Prog. Theor. Phys. **16**, 686 (1956).
Fundamental spin ½ hadronic fields are postulated to be:

$$p, n, \lambda$$

The mesons are objects like $p\bar{n}$. This enables one to construct schematically all the observed hadrons. It also suggests a natural SU(3) "flavor" symmetry which, I believe, was first studied at Nagoya.
(Note also that a leptonic analog was suggested to be:

$$\nu, \; e, \; \mu$$

by Gamba, Marshak and Okubo.)

2) A few years later the three fundamental hadronic fields were replaced by the fractionally charged quarks:

$$u, d, s \, .$$

Mesons are now objects like $u\bar{d}$.

3) The age of discovery. Between 1974 and 1995, three more quarks were discovered. This brings the total picture to:

$$u, d, s, c, b, t \ .$$

It is now easier to describe the quarks as:

$$q_a \ , \quad a = 1 \,..\, 6 \ .$$

(Will any more be discovered at LHC?)

Actually, during this period, it was discovered that the strong dynamics is described by an "SU(3) color" gauge theory so we must add a color index:

$$q_{aA} \ , \quad a = 1..6, \ A = 1..3$$

But we are still not done. If we regard this symbol as representing a Fermi-Dirac field, we know that the left and right handed projections enter differently into the unified weak interaction theory. Thus we must distinguish the two. Let us agree to leave the left handed index alone and put a dot on the index for the right handed projection.

In this language, a spin zero meson made of a quark and an antiquark can be schematically described as:

$$M_a^{\dot{b}} = (q_{\dot{b}A})^\dagger \gamma_4 \frac{1+\gamma_5}{2} q_{aA}$$

Using a matrix notation, $M_a^{\dot{b}} \rightarrow M_{a\dot{b}} \rightarrow M$, these spin zero mesons can be decomposed into scalar and pseudoscalar pieces:

$$M = S + i\phi, \quad M^\dagger = S - i\phi$$

If all six quarks were massless, the symmetry of the color gauge theory would be

$$SU(6)_L \times SU(6)_R \times U(1)_{VECTOR}$$

Actually, the first three quarks are relatively light so the reduced symmetry, $SU(3)_L \times SU(3)_R \times U(1)_V$ (while spontaneously broken) forms the basis of the successful chiral perturbation theory scheme.

Considering the spin 0 mesons

By counting, there are nine light (i.e. made as quark antiquark composites from the lightest three quarks) pseudoscalar mesons and nine light scalar mesons.

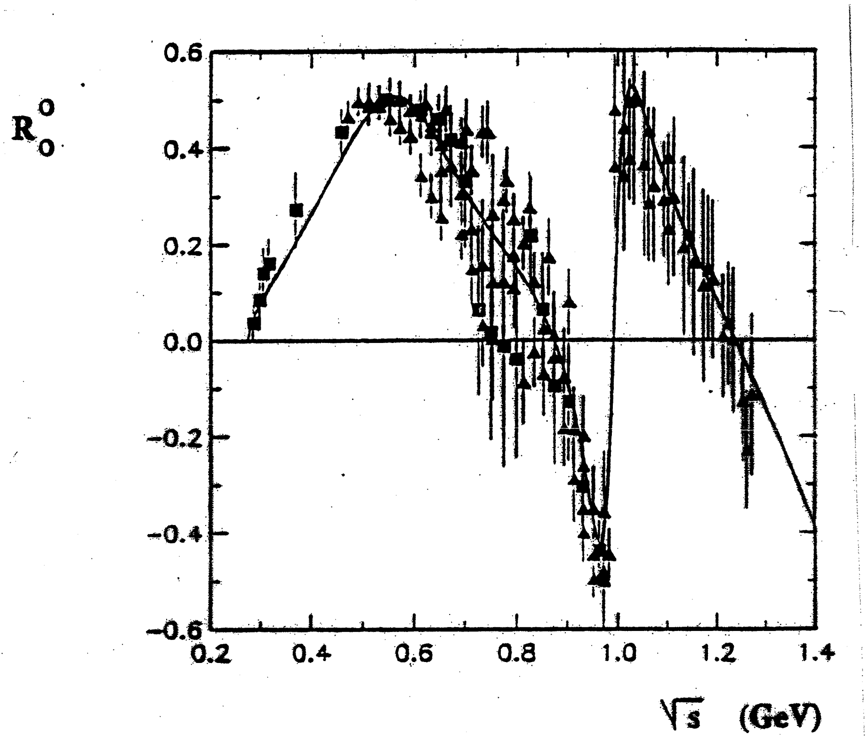
Theorist's perspective:

There are eight zero mass pseudoscalar Nambu-Goldstone bosons and one heavier pseudoscalar (since the axial U(1) symmetry is intrinsically broken by the axial anomaly and can't be spontaneously broken).

In order to do chiral perturbation theory calculations which automatically recover the "current- algebra" theorems as a starting point it is convenient to neglect the scalar mesons. This is elegantly done by "integrating them out". Hence the nine scalar mesons should have infinite mass, or at least be very heavy.

Experimentalist's perspective:

Look at the data. This is not so easy. The starting point is a partial wave analysis of pi-pi scattering data in the I=J=0 channel:



The experimental data for the real part of the J = I = 0 amplitude are illustrated by the points with error bars. It looks complicated; the fit shown by the solid line (M. Harada, F. Sannino, JS) corresponds to putting in

a) chiral Lagrangian background
b) rho meson background
c) broad "sigma" meson around 550 MeV
d) "f_0 (980)" candidate

A number of other groups have found similar results.

Similar treatments of the pi-K scattering and the pi-eta scattering have produced evidence for a spin 0 scalar strange meson (kappa) and agreement with the experimental indication of another J = 0, I = 1 resonance, a_0 (980). That yields a putative J = 0 scalar nonet:

I = 0 : m(σ) = 550 MeV

I = ½ : m(κ) = 800 M eV

I = 1 : m(a_0) = 980 MeV

I = 0 : m(f_0) = 980 MeV

This may be compared with the (most standard) vector nonet:

I = 0 : m(ω) = 783 MeV $n\bar{n}$

I = 1: m(ρ) = 776 Mev $n\bar{n}$

I = ½ : m (K^*) = 892 MeV $n\bar{s}$

I = 0 : m(ϕ) = 1020 MeV $s\bar{s}$

In the standard vector nonet, the masses increase from the almost degenerate I = 0 and I =1 particles to the lone I = 0 particle ϕ. Roughly this is due to the strange quark, s being heavier than than the non strange ones, n. However, for the J =0 scalar nonet candidates, the mass dependence is exactly reversed! A long time ago, Jaffe argued that a nonet made of two quarks and two anti-quarks would have this reversed behavior. Let us accept this hypothesis for the time being.

Now, it was found (D. Black, A. Fariborz, S. Moussa, S. Nasri, JS) that the same form of the complicated partial wave amplitude, R^0_0 could be obtained by employing a simple linear sigma model using the field M = S + i ϕ mentioned above. Similarly reasonable descriptions of the scattering partial waves involving the κ and a_0 scalar states could also be obtained. But , by construction, *both* the pseudoscalar and scalar nonets start out as quark- anti-quark objects in the model.

Furthermore, there are some extra scalar and pseudoscalar states floating around. Also, the lighter scalar masses are much lower than where they are expected to be according to the reasonable non-relativistic quark model. In that model, the lowest mass nonets (below 1 GeV) are the pseudoscalars and the vectors. The next highest nonets (somewhat above 1 GeV) are the scalars, tensors and two axials (with different C properties).

So, the situation concerning the spin 0 chiral partners seems to call for clarification. For this purpose, D. Black et al proposed that there might be two chiral spin 0 nonets - one of quark antiquark type (M) and the other of two quark – two antiquark type ($M' = S' + i\phi'$) and that they be allowed to mix with each other. This mixing is expected to lead to level repulsion, which could make the lighter scalars even lighter and the heavier ones even heavier.

What would the schematic quark structure of the " four quark" chiral nonet, M' look like ?

There are three possibilities:

"Molecular" type :

$$M^{(2)\dot{b}}_{a} = \epsilon_{acd}\, \epsilon^{\dot{b}\dot{e}\dot{f}} (M^{\dagger})^{c}_{\dot{e}} (M^{\dagger})^{d}_{\dot{f}}$$

Color triplet diquark- anti diquark type:

$$M^{(3)\dot{f}}_{g} = (L^{gA})^{\dagger} R^{\dot{f}A} ,$$

$$L^{gE} = \epsilon^{gab} \epsilon^{EAB} q^{T}_{aA} C^{-1} \frac{1+\gamma_5}{2} q_{bB} ,$$

$$R^{gE} = \quad '' \quad \frac{1-\gamma_5}{2}$$

Color sextet diquark- anti diquark type:

$$M^{(4)\dot{f}}_{g} = (L^{g}_{\mu\nu,AB})^{\dagger} R^{\dot{f}}_{\mu\nu,AB} ,$$

$$L^{g}_{\mu\nu,AB} = \epsilon^{gab} q^{T}_{aA} C^{-1} \sigma_{\mu\nu} \frac{1+\gamma_5}{2} q_{bB}$$

etc.

The distinction between molecular and diquark- antidiquark pieces is not fundamental because of the Fierz identity:

$$8\, M_a^{(2)\dot{b}} = 2\, M_a^{(3)\dot{b}} - M_a^{(4)\dot{b}}$$

We focus on chiral invariance as the main goal and assume that dynamics selects some linear combination of the above,

$$M' = S' + i\phi'$$

to be bound. Then we construct a "toy" linear sigma model to study the consequences. We do not make any a priori assumptions about what are the "two quark" and "four quark " contents of the 18 scalar and 18 pseudoscalar states which emerge but let the model, with some experimental inputs tell us the answer.

Terms in the model Lagrangian density

Kinetic terms:

$- 1/2\ \mathrm{Tr}(\partial_\mu M\ \partial_\mu M^\dagger)\ -1/2\ \mathrm{Tr}(\partial_\mu M' \partial_\mu M'^\dagger)$

Symmetry breaking quark mass terms:

$-2\ \mathrm{Tr}\ (AS)$,

where $A = \mathrm{diag}(A_1, A_2, A_3)$ are proportional to the three light quark masses.

Chiral invariant interaction terms:

$-c_2\ \mathrm{Tr}\ (MM^\dagger) + c_4^a \mathrm{Tr}(MM^\dagger MM^\dagger) + d_2\ \mathrm{Tr}\ (M'M'^\dagger)$

$+c_3^a\ (\varepsilon_{\dot{a}\dot{b}\dot{c}}\varepsilon^{def} M_d^{\dot{a}} M_e^{\dot{b}} M'^{\dot{c}}_f + \mathrm{h.c.})$

(There are about 20 renormalizable invariant terms; we just kept those with 8 or less underlying quarks)

Terms to mock up the U(1) axial anomaly:

$C_3 [\, \gamma_1 \ln (\det M/\det M^\dagger) + (1-\gamma_1) \ln((\mathrm{Tr}(MM'^\dagger)/\mathrm{Tr}(M'M^\dagger))$

(Both types appear in the 3-flavor 't Hooft type instanton calculation)

Needed scalar field vacuum values:

$\langle S_1^1 \rangle = \langle S_2^2 \rangle, \langle S_3^3 \rangle, \langle S'^1_1 \rangle, \langle S'^3_3 \rangle$

Input values: (8)

m_π = 137 MeV, F_π = 131 MeV, m[a_0(980] = 984.7 ± 1.2 MeV,

m[a_0(1450)] = 1474±19 MeV, m[π(1300)] = 1300±100 MeV,

A_3/A_1 = 20 – 30, two mass parameters for the four I = 0 η' s .

Predictions for the pseudoscalars:

State	$\bar{q}q$%	$\bar{q}\bar{q}qq$%	m (GeV)
π	85	15	0.137
π'	15	85	1.215
K	86	14	0.515
K'	14	86	1.195
η_1	89	11	0.553
η_2	78	22	0.982
η_3	32	68	1.225
η_4	1	99	1.794

Not surprisingly, the lower mass particles of each isospin turn out to be dominantly of quark antiquark type. The heavier particles of each type turn out to be dominantly of two quark – two antiquark type. Note that all four I=0 particles mix with each other to some extent. Also note that there are enough candidates to fill two nonets.

Predictions for the scalars:

State	$\bar{q}q\%$	$\bar{q}\bar{q}qq\%$	m (GeV)
a	24	76	0.984
a'	76	24	1.474
κ	8	92	1.067
κ'	92	8	1.624
f_1	40	60	0.742
f_2	5	95	1.085
f_3	63	37	1.493
f_4	93	7	1.783

Here the situation is opposite. The lower lying states are predominantly of two quark- two anti quark type. For example the lighter isovector a is 76 % "four quark" while the heavier isovector a is 24 % "four quark".
The famous $\sigma = f_1$ is 40 % quark antiquark and 60 % "four quark".
The f_0 (980) = f_2 is 95% of "four quark" type.

Note that these masses are "tree level" ones. For the σ the unitarity corrections (which involve computing the $\pi\pi$ scattering amplitude in the model) reduce the predicted mass to 477MeV!

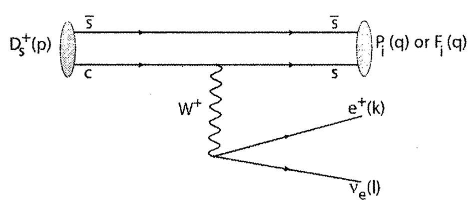
Experimental information on scalar mesons:

Typically it comes from partial wave analyses of scattering processes. Another source of information arises from Dalitz analyses of multiparticle final states in non-leptonic decays.

Recently, the CLEO collaboration got a simple neat determination of the mass and width of the f_0 (980) [f_2 in the notation above] from the semileptonic decay of a charmed meson:

$$D_s^+ (1968) \rightarrow f_0 (980) + e^+ + \nu_e$$

(arXiv:0907.3201[hep-exp]. This process corresponds to the picture,



Predicting the semi-leptonic decay widths:

It is interesting to us that in addition to the decay into what we called f_2, there also exist decays into f_1, f_3 and f_4 which should be easily predictable in the model. All four of these particles arise as predicted mixtures of the standard basis states as:

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \left(L_0^{-1}\right) \begin{pmatrix} (S_1^1 + S_2^2)/\sqrt{2} \\ S_3^3 \\ (S_1'^1 + S_2'^2)/\sqrt{2} \\ S_3'^3 \end{pmatrix} \quad \begin{matrix} n\bar{n} \\ s\bar{s} \\ ns\bar{n}\bar{s} \\ nn\bar{n}\bar{n} \end{matrix}$$

where L_0^{-1} is predicted for a typical parameter choice to be:

$$L_0^{-1} = \begin{pmatrix} 0.60 & 0.20 & 0.60 & 0.49 \\ -0.11 & 0.19 & 0.64 & -0.74 \\ 0.79 & -0.50 & -0.39 & -0.47 \\ 0.06 & -0.96 & 0.27 & -0.02 \end{pmatrix}$$

Similarly, there are four predictions of the model for decays of the D_s^+ into the pseudoscalar singlets η_i + leptons.

The required hadronic information consists of the vector (for decays into the η_i) and the axial vector (for decays into the f_i) Noether currents of the model. For the three flavor model these currents take the well known forms:

$$
\begin{aligned}
V^b_{\mu a}(total) &= V^b_{\mu a} + V'^b_{\mu a}, \\
A^b_{\mu a}(total) &= A^b_{\mu a} + A'^b_{\mu a}.
\end{aligned}
$$

$$
\begin{aligned}
V^b_{\mu a} &= i\phi^c_a \overset{\leftrightarrow}{\partial_\mu} \phi^b_c + i\tilde{S}^c_a \overset{\leftrightarrow}{\partial_\mu} \tilde{S}^b_c + i(\alpha_a - \alpha_b)\partial_\mu \tilde{S}^b_a, \\
A^b_{\mu a} &= \tilde{S}^c_a \overset{\leftrightarrow}{\partial_\mu} \phi^b_c - \phi^c_a \overset{\leftrightarrow}{\partial_\mu} \tilde{S}^b_c + (\alpha_a + \alpha_b)\partial_\mu \phi^b_a,
\end{aligned}
$$

$$
\begin{aligned}
V'^b_{\mu a} &= i\phi'^c_a \overset{\leftrightarrow}{\partial_\mu} \phi'^b_c + i\tilde{S}'^c_a \overset{\leftrightarrow}{\partial_\mu} \tilde{S}'^b_c + i(\beta_a - \beta_b)\partial_\mu \tilde{S}'^b_a, \\
A'^b_{\mu a} &= \tilde{S}'^c_a \overset{\leftrightarrow}{\partial_\mu} \phi'^b_c - \phi'^c_a \overset{\leftrightarrow}{\partial_\mu} \tilde{S}'^b_c + (\beta_a + \beta_b)\partial_\mu \phi'^b_a,
\end{aligned}
$$

Noether currents in the four flavor case:

Clearly, an extension is needed to accommodate the D_s^+ particle. One's first thought is to just sum 1 – 4 instead of 1 - 3 in the currents above.

However there is a problem with this simple extension. A fourth quark flavor will not allow the construction of a two quark- two antiquark state which has the same chiral SU(4) transformation property as the one quark – one antiquark state. For example, trying a molecule form would result in:

$$M_{ag}^{(2)\dot{b}\dot{h}} = \epsilon_{agcd}\epsilon^{\dot{b}\dot{h}\dot{e}\dot{f}} \left(M^\dagger\right)_{\dot{e}}^{c} \left(M^\dagger\right)_{\dot{f}}^{d}.$$

But, instead of transforming under $SU(4)_L \times SU(4)_R$ as $(4, \bar{4})$ as desired it transforms as $(6, \bar{6})$ owing to the two sets of antisymmetric indices which appear!

Thus, we assume that there are no two quark- two antiquark components for the mesons containing a charm quark. The kinetic terms for the model may thus be written as:

$$\mathcal{L} = -\frac{1}{2}Tr^4(\partial_\mu M \partial_\mu M^\dagger) - \frac{1}{2}Tr^3(\partial_\mu M' \partial_\mu M'^\dagger)$$

where the superscript means that the first term should be summed over the heavy quark index also but not the second term.

In the presently needed currents, only the unprimed fields appear:

$$\begin{aligned} V^a_{\mu 4}(total) &= V^a_{\mu 4} = i\phi^c_4 \overleftrightarrow{\partial_\mu} \phi^a_c + iS^c_4 \overleftrightarrow{\partial_\mu} S^a_c, \\ A^a_{\mu 4}(total) &= A^a_{\mu 4} = S^c_4 \overleftrightarrow{\partial_\mu} \phi^a_c - \phi^c_4 \overleftrightarrow{\partial_\mu} S^a_c. \end{aligned}$$

where the unspecified indices can run from 1 to 4.

Note that the currents do not contain any unknowns; their normalization is given by the component which is the electric current. Then the unintegrated decay widths into any of the four isoscalar 0^+ mesons or four isoscalar 0^- mesons is given by:

$$\frac{d\Gamma}{d|\mathbf{q}|} = \frac{G_F^2 |V_{cs}|^2}{12\pi^3} \left\{ \begin{array}{c} ((R_0)_{2i})^2 \\ ((L_0)_{2i})^2 \end{array} \right\} m(D_s) \frac{|\mathbf{q}|^4}{q_0}.$$

where q_μ is the final meson four-momentum and V_{cs} is the Kobayashi Maskawa matrix element.

Table I summarizes the calculations for the pseudoscalar mesons and Table II, the calculations for the scalar mesons.

m_i (MeV)	$(R_0)_{2i}$	$(q_{max})_i$ (MeV)	Γ_i (MeV)
553	0.661	906.20	4.14×10^{-11}
982	0.512	739.00	7.16×10^{-12}
1225	-0.546	602.74	2.57×10^{-12}
1794	0.051	166.31	2.65×10^{-17}

TABLE I: pseudoscalars.

m_i (MeV)	$(L_0)_{2i}$	$(q_{max})_i$ (MeV)	Γ_i (MeV)
477	0.199	933.23	4.56×10^{-12}
1037	0.189	710.79	7.80×10^{-13}
1127	-0.050	661.30	3.62×10^{-14}
1735	-0.960	219.21	3.85×10^{-14}

TABLE II: scalars.

Experimental data, at the moment exist only for three of these eight decay modes:

$$\begin{aligned}
\Gamma(D_s^+ \to \eta e^+ \nu_e) &= (3.5 \pm 0.6) \times 10^{-11} \quad \text{MeV} \\
\Gamma(D_s^+ \to \eta' e^+ \nu_e) &= (1.29 \pm 0.30) \times 10^{-11} \quad \text{MeV} \\
\Gamma(D_s^+ \to f_0 e^+ \nu_e) &= (2.6 \pm 0.4) \times 10^{-12} \quad \text{MeV}
\end{aligned}$$

The model agrees with experiment for the decay of η_1, is about 30 % less than experiment for η_2 and is about 1/3 the experimental value for f_2 . Allowing variations in the input quantities (like the exact mass chosen for m[π(1300)] can improve the latter result. But it is clear that this simple model is certainly in the "right ball park".

Summary:

The light spin 0 pseudoscalar mesons appear to be of $q\bar{q}$ type.

The light spin 0 scalar mesons appear to be of $qq\bar{q}\bar{q}$ type.

Chiral symmetry is a symmetry of massless QCD so they should be chiral partners. So, how can we reconcile these two statements.?

Proposed solution here: Introduce a chiral $q\bar{q}$ multiplet *and* a chiral $qq\bar{q}\bar{q}$ multiplet. They mix and the lightest pseudoscalars are mainly $q\bar{q}$ while the lightest scalars are mainly $qq\bar{q}\bar{q}$.

Many of the 8 isoscalars in this scheme might be studied experimentally via the semi-leptonic decays of the $D_s^+(1968)$.