# QCD and gauge/string duality

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## §1. Introduction

**How to understand strong coupling dynamics of QCD?**

low energy hadron physics, sQGP, $\cdots$

- numerical analysis
- effective theory
  - ◇ chiral perturbation
  - ◇ hidden local symmetry
- **gauge/string duality** — Maldacena, ...

  AdS/CFT and its extension. As an application,

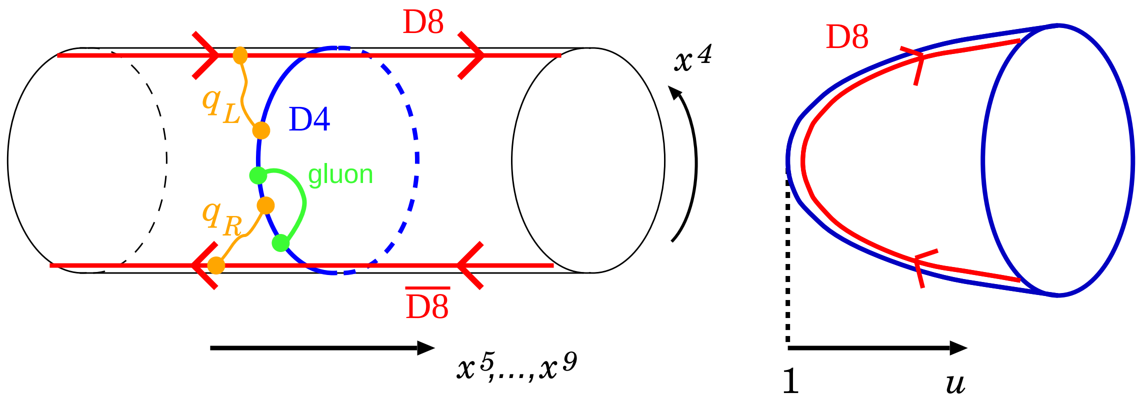
$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Kovtun-Son-Starinets

# A string dual of large $N_c$ QCD

Sakai-Sugimoto '04

Intersecting D4/D8/$\overline{\text{D8}}$-brane configuration in type IIA superstring




**Claim:**

**5d $U(N_f)$ YM-CS theory on D8 governs hadron dynamics**

$$S = S_{\rm YM} + S_{\rm CS}$$

$$S_{\rm YM} = -\kappa \int d^4x dz \ \mathrm{tr} \left[ \frac{1}{2} h(z) \mathcal{F}_{\mu\nu}^2 + k(z) \mathcal{F}_{\mu z}^2 \right]$$

$$S_{\rm CS} = \frac{N_c}{24\pi^2} \int_{M^4\times\mathbb{R}} \omega_5^{U(N_f)}(\mathcal{A}) = \frac{N_c}{24\pi^2} \int_{M^4\times\mathbb{R}} \mathrm{tr} \left( \mathcal{A}\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5 \right)$$

$$\kappa = \frac{\lambda N_c}{216\pi^3} \ , \quad \lambda = g_{\rm YM}^2 N_c \ , \quad h(z) = (1+z^2)^{-1/3} \ , \quad k(z) = 1 + z^2$$

## Results from the model

- chiral symmetry $U(N_f)_L \times U(N_f)_R$ manifest
- simple geometric explanation of $\chi$SB
- incorporating an infinite tower of mesons including $\rho, a_1, \cdots$ — Bando-Kugo-Yamawaki, ... Son-Stephanov, ...
- revival of an old idea of baryons as solitons — Skyrme, Witten, ... Atiyah-Manton, ...

# §2. Meson spectrum

**Mesons as KK states** of $\mathcal{A}$ along the warped 5th direction

$$\mathcal{A}_\mu(x,z) = \sum_{n\geq 1} B_\mu^{(n)}(x)\,\psi_n(z)\ ,$$

$$\mathcal{A}_z(x,z) = \sum_{n\geq 1} \varphi^{(n)}(x)\phi_n(z) + \varphi^{(0)}(x)\phi_0(z)\ ,$$

$$-k^{1/3}\partial_z(k\,\partial_z\psi_n) = \lambda_n\psi_n\ ,\quad \kappa\int dz k^{-1/3}\,\psi_n\psi_m = \delta_{nm}\ ,\quad \phi_n \propto \partial_z\psi_n\ .$$

It is found that

| | $\varphi^{(0)}$ | $v_\mu^n = B_\mu^{(2n-1)}$ | $a_\mu^n = B_\mu^{(2n)}$ |
|---|---|---|---|
| $J^{PC}$ | $0^{-+}$ | $1^{--}$ | $1^{++}$ |
| mass$^2$ | $0$ | $\lambda_{2n-1}M_{\rm KK}^2$ | $\lambda_{2n}M_{\rm KK}^2$ |

**Charge conjugation** and **parity transformation** comes from discrete symmetries of 5d YM-CS:

$$C: \ \mathcal{A}_\mu \to -\mathcal{A}_\mu^T \ , \ \mathcal{A}_z \to -\mathcal{A}_z^T \ , \ z \to -z$$
$$P: \ (x_1, x_2, x_3, z) \to -(x_1, x_2, x_3, z)$$

As an example,

$$\frac{m^2_{a_1(1260)}}{m^2_\rho} = \begin{cases} \frac{\lambda_2}{\lambda_1} \simeq \frac{1.57}{0.669} \simeq 2.35 & \text{(our model)} \\ \frac{(1230 \text{ MeV})^2}{(776 \text{ MeV})^2} \simeq 2.51 & \text{(experiment)} \end{cases}$$

$$\frac{m^2_{\rho(1450)}}{m^2_\rho} = \begin{cases} \frac{\lambda_3}{\lambda_1} \simeq \frac{2.87}{0.669} \simeq 4.29 & \text{(our model)} \\ \frac{(1465 \text{ MeV})^2}{(776 \text{ MeV})^2} \simeq 3.56 & \text{(experiment)} \end{cases}$$

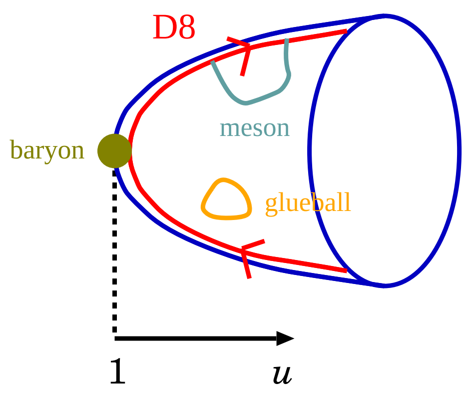
**Problems**: How to obtain mesons that are not included in 5d YM-CS?

- **mesons with $J > 1$**
- **those of $J^{PC} = 1^{+-}$ and $1^{-+}$**
- **massive scalar mesons**

This is done by **quantizing an open string ending ending on D8**

**dictionary**

- mesons as **open strings** ending on D8
- baryons as a **D4-brane** wrapped around $S^4$ — Hata-Sakai-Sugimoto-Yamato, ...
- glueballs as **closed strings** — Csaki et.al., de Mello Koch et.al., Brower et.al.,...



This is difficult to perform because

- curved background with RR-flux
- D8 curved as well

These problems are addressed using $1/\lambda$ **expansions**:

Imoto-Sakai-Sugimoto '09

step 1

In the large $\lambda = g_{\rm YM}^2 N_c$ limit, the curved background can be approximated with flat $\mathbb{R}^{9,1}$. The mass spectrum on D8 is given by

$$-p^2 = \frac{N}{\alpha'_{\rm eff}} = \frac{4}{27}\lambda M_{\rm KK}^2 N \equiv m_0^2 \ , \quad N \in \mathbb{Z}_{\geq 1} \ .$$

step 2

Remove the exotic states that are **not composed of the quark and the gluon**.

D4/D8 system has exotic symmetries that have no counterpart in realistic QCD. Since the quarks and gluon are neutral under them,

- the genuine QCD mesons are neutral
- exotic states carry a nontrivial charge of the unphysical symmetries

We are led to focus on only the **string modes propagating in** $\mathbb{R}^{4,1} \subset \mathbb{R}^{8,1}$.

step 3

Incorporate the 1st-order $1/\lambda$ correction into the mass shell condition, which amounts to turning on a potential term along the extra 5th direction. We obtain

$$M_n^2 = \frac{N}{\alpha'_{\rm eff}} + \sqrt{\frac{2N}{\alpha'_{\rm eff}}} \left( n + \frac{1}{2} \right) M_{\rm KK} \ .$$

Here $n = 1, 2, \cdots$.

step 4 Assign $P$ **and** $C$ **parity**

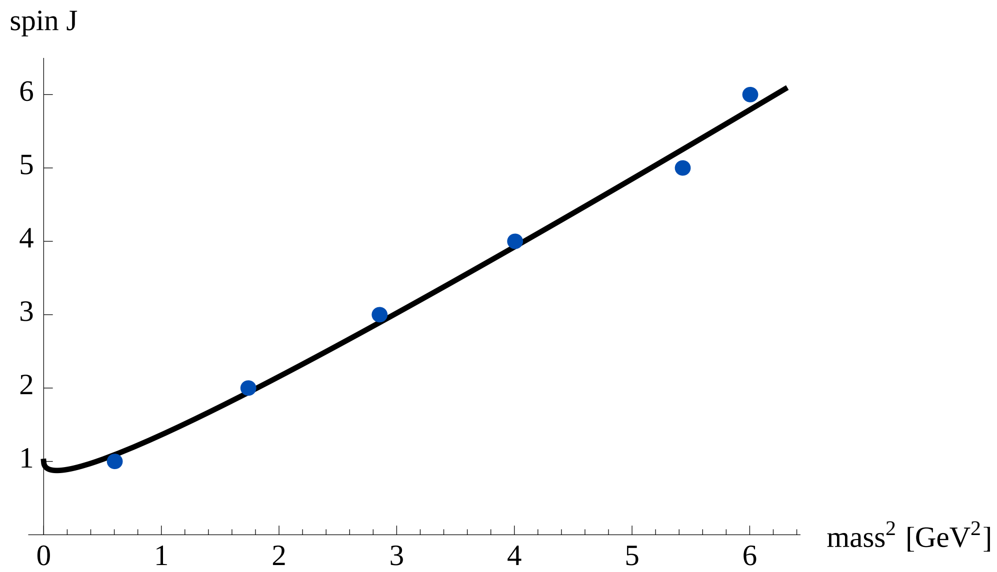
$$P = I_{123z} \ , \quad C = I_{489}\, \Omega\, (-)^{F_L}$$

## §3 Comparison with experiments

1. For a given $N \geq 1$, the maximal spin state is fixed uniquely with $J = N + 1$.

$$J = 1 + \alpha'_{\rm eff}\, M_n^2 + \sqrt{2}\,\alpha'_{\rm eff}\left(n + \frac{1}{2}\right) M_{\rm KK} M_n + \mathcal{O}(\lambda^{-1}) \ .$$

**Regge behavior with a nonlinear correction in $M_n^2$**




A plot of the modified Regge trajectory with $\alpha'_{\rm eff} = 1.1$ GeV$^{-2}$, $M_{\rm KK} = 949\,{\rm MeV}$. The dots represent the mesons in the **$\rho$-meson trajectory**.

2. The $(N,n)=(1,0)$ state with label $J^{PC}$ includes

$$2^{++}\ ,\quad 1^{--}\ ,\quad 1^{+-}\ ,\quad 0^{-+}\ ,\quad 0^{++}\times 3\ ,$$

which are predicted to have mass $1290\,\mathrm{MeV}$ by setting

$$\alpha'_{\mathrm{eff}}=1.1\,\mathrm{GeV}^{-2}\ ,\qquad M_{\mathrm{KK}}=949\,\mathrm{MeV}\ .$$

- $2^{++}$ is identified with $a_2(1320)$.
- The rest should be identified with mesons with mass $\sim 1290\,\mathrm{MeV}$.

3. The $(N,n)=(2,0)$ state includes

$$3^{--}\ ,\quad 2^{++}\ ,\quad 2^{-+}\times 2\ ,\quad 2^{--}\ ,\quad 1^{--}\times 7\ ,\quad 1^{+-}\times 4\ ,\quad 1^{++}\times 3\ ,$$
$$1^{-+}\ ,\quad 0^{++}\times 2\ ,\quad 0^{-+}\times 6\ ,$$

which are predicted to have mass $1680\,\mathrm{MeV}$.

- $3^{--}$ is identified with $\rho_3(1690)$.
- The rest should be identified with mesons with mass $\sim 1680\,\mathrm{MeV}$

| | | | | | | |
|---|---|---|---|---|---|---|
| $0^{-+}(\pi)$ | $135^{SS}$ | $1300^{(1,0)}$ | $1800^{(2,0)}$ | | | |
| $0^{++}(a_0)$ | $980^{*}$ | $1450^{(1,0)}$ | | | | |
| $1^{--}(\rho)$ | $770^{SS}$ | $1450^{(1,0)\mathrm{or}SS}$ | $1570^{\triangle(1,0)\mathrm{or}SS}$ | 1700 | $1900^{\triangle}$ | $2150^{\triangle}$ |
| $1^{++}(a_1)$ | $1260^{SS}$ | $1640^{\triangle(2,0)}$ | | | | |
| $1^{+-}(b_1)$ | $1235^{(1,0)}$ | | | | | |
| $1^{-+}(\pi_1)$ | $1400^{*}$ | $1600^{*(2,0)}$ | | | | |
| $2^{++}(a_2)$ | $1320^{(1,0)}$ | $1700^{\triangle(2,0)}$ | | | | |
| $2^{-+}(\pi_2)$ | $1670^{(2,0)}$ | $1880^{(2,0)}$ | $2100^{\triangle}$ | | | |
| $3^{--}(\rho_3)$ | $1690^{(2,0)}$ | $1990^{\triangle}$ | $2250^{\triangle}$ | | | |
| $4^{++}(a_4)$ | $2040^{(3,0)}$ | | | | | |
| $5^{--}(\rho_5)$ | $2350^{\triangle(4,0)}$ | | | | | |
| $6^{++}(a_6)$ | $2450^{\triangle(5,0)}$ | | | | | |

Isovector mesons from meson summary table in PDG. $\triangle$ indicates a poor evidence. $*$ indicates non-$\bar{q}q$ states.

## Comments

- We have obtained the meson states that were missed in 5d YM-CS.
- As $N, n$ grows, our model predicts more redundant states that are not found in PDG data.
  - ◇ In large $N_c$ QCD, there exists an infinite number of stable mesons. For finite $N_c$, it is possible that
    - ⋆ some of them form a bound state
    - ⋆ some correspond to wide resonances with width of $\mathcal{O}(1/N_c)$.
  - ◇ No one-to-one correspondence between our model and experiments expected
- Those states identified with the mesons match experiments quantitatively.