What is Technicolor?

- Gedanken world: If EW symmetry $SU(2)_L \times U(1)_Y$ were unbroken at GeV energies, QCD would break it via strongly-coupled Higgs mechanism.

- Pions eaten to give mass to $W$ and $Z$ bosons of $O(30 \, \text{MeV})$.

- No Yukawa mechanism, so no fermion masses, plus much stronger EW couplings: many new phenomena. [Quigg-Shrock 2009]

- Basic Idea: Break EW symmetry at TeV scales by adding new fermions $(\bar{Q}, Q)$ with new strong interactions. [Weinberg, Susskind 1979]

- SM fermion mass: New gauge interactions broken at high scale $\Lambda_{ETC}$ couple SM fermions to techniquarks. [Dimopoulos-Susskind, Eichten-Lane 1979]

$$\text{Masses: } \frac{(\bar{Q}Q)(\bar{qq})}{\Lambda_{ETC}^2} \quad \text{FCNC's: } \frac{(\bar{qq})(\bar{qq})}{\Lambda_{ETC}^2} \quad \Lambda_{ETC} \gtrsim 1000 \, \text{TeV}$$

Why did Technicolor fall out of favor?

- QCD-like strong interactions at the TeV scale can drive the Higgs mechanism, but face phenomenological challenges:
  - Either flavor changing neutral currents (FCNC) are too large or generated SM fermion masses are too small.
  - Precision EW oblique corrections ($S$ parameter) in tension with experiment.

- A resolution: TeV strong interactions are not like QCD.
- A problem: How well do we really understand generic strongly interacting theories other than QCD?
- A solution: Lattice field theory is only now powerful enough to begin the study of strongly-coupled theories beyond QCD.
Where to look for non-QCD theories?

- For $N_f = 0–1$, confinement but no NG bosons.
- For $N_c = 2$, enhanced chiral symmetry means special case: Pattern of symmetry breaking yet to be determined.
- Pert. theory indicates IRFP for $N_f \lesssim 5.5 \cdot N_c$.
- Phenomenological success of large $N_c$ calculations suggest QCD-like theories for $N_f = 2–3$ and $N_c \geq 3$.
- Simplest search strategy: start from QCD and increase $N_f$. 
Can the running coupling be our guide?

- In QCD, \( g(L) \) is asymptotically free and runs rapidly until SSB and confinement: \( g(L_c) = g_c \).
- As \( N_f \) increases, the running slows down.
- For large \( N_f \), \( g(L) \) flows to \( g_* \) at IR fixed point (IRFP). No SSB, no Technicolor.
- Walking theories may exist nearby theories with strongly-coupled IRFP: \( g_* \approx g_c \).
- Unlike QCD, walking theories would have two dynamically generated scales: \( L_I \) and \( L_c \), and in rare cases \( L_I \ll L_c \).
- In Walking Technicolor, \( L_I^{-1} = \Lambda_{ETC} \sim 1000 \text{ TeV} \) and \( L_c^{-1} = \Lambda_{TC} \sim 1 \text{ TeV} \).
- How does walking help Technicolor’s FCNC problem?
Non-perturbative SF running coupling

Walking Dynamics

- The relevant scale for mass generation is $\Lambda_{ETC}$, so the relevant condensate is renormalized at that scale: $\langle \overline{Q}Q \rangle$ at $\Lambda_{ETC}$.

  $$\text{Masses: } \frac{\langle \overline{Q}Q \rangle}{\Lambda_{ETC}^2} \text{ FCNC's: } \frac{\langle \overline{q}q \rangle}{\Lambda_{ETC}^2} \quad \Lambda_{ETC} \gtrsim 1000 \text{ TeV}$$

- The condensate is renormalized using the anomalous dimension $\gamma(\mu)$. In QCD-like theories, $\gamma(\mu) \ll 1$ for $\mu \gg \Lambda_{TC}$. Leads to $\log(\Lambda_{ETC} / \Lambda_{TC})$ enhancement.

  $$\langle \overline{Q}Q \rangle_{\Lambda_{ETC}} = \langle \overline{Q}Q \rangle_{\Lambda_{TC}} \exp \left[ \int_{\Lambda_{TC}}^{\Lambda_{ETC}} \frac{\gamma(\mu)}{\mu} d\mu \right]$$

- Walking dynamics ($\gamma \sim 1$) leads to power-enhanced condensates.

  $$\frac{\langle \overline{Q}Q \rangle}{F_{\pi T}^3} \sim \frac{\langle \overline{q}q \rangle}{f_{\pi}^3} \left( \frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^\gamma$$

- Now, a hierarchy of SM fermion masses can be generated while suppressing FCNC.
LSD Program Overview

- SU(2) and SU(3) gauge theories with $N_f$ domain wall fundamental fermions.

- Initial focus on SU(3): code readiness and QCD experience.

- Preparing SU(2) code for production.

- Majority of flops so far spent on SU(3) with $N_f=2,6,10$.

- Exploration of IR: QCD-like, conformal or “walking”.

- Phenomenology: $S$ parameter, condensate enhancement/mass anomalous dimension, $WW$ scattering, dark matter form factors.

- Published results: PRL 104, 071601 (2010); PRL 106, 231601 (2011).

- Four new publications in draft.
**LSD**: Comparing $N_f = 2$ and $N_f = 6$

- Why $N_f = 6$? It’s very unlikely to walk...
- On largest computers, calculations still limited to lattices where $L/a \leq 64$.
- A walking theory should be studied on lattices where $L/a \sim 256–1024$.
- Can precursors to walking be seen in slowly running theories?
- Lattice scales chosen to match confinement scale physics to $\sim 10\%$.
- Usual caveats (finite $L, m, a$) expected to get worse with increasing $N_f$. 
LSD: Condensate Enhancement

- Tricky to compare scale dependent quantities in two different theories.

- Definition of Enhancement:

  \[ \frac{\langle \bar{\psi} \psi \rangle^{(N_f)}}{\langle \bar{\psi} \psi \rangle^{(2)}} \mid_{5M_\rho} \equiv R(5M_\rho) \approx \frac{\exp \left( \int_{(5M_\rho)}^{(5M_\rho)} \frac{\gamma(\alpha)}{\pi \beta(\alpha)} \mid_{N_f} \ d\alpha \right)}{\exp \left( \int_{(5M_\rho)}^{(5M_\rho)} \frac{\gamma(\alpha)}{\pi \beta(\alpha)} \mid_{N_f=2} \ d\alpha \right)} \]

- GMOR Ratios

  \[ R = \frac{\langle \bar{\psi} \psi \rangle^{\text{CF}}}{F^3_\pi} = \frac{\langle \bar{\psi} \psi \rangle^{\text{CM}}}{M^3_\pi} = \frac{\langle \bar{\psi} \psi \rangle^{\text{FM}}}{2mF_\pi} \quad \text{as} \ m \to 0 \]

- Chiral extrapolation

  \[ R_{XY,\tilde{m}} = \frac{R^{(N_f)}_{(2)}}{R^{(2)}} [1 + \tilde{m} (\alpha_{XY10} + \alpha_{11} \log \tilde{m})] , \quad \tilde{m} = \sqrt{m^{(N_f)} m^{(2)}} \]

- Perturbative estimates of enhancement: \( R(5M_\rho) \sim 1.2–1.3 \) (lat scheme)

- Enhancement bigger than expected. **Is this a precursor to walking?**
• \( T=0 \) in lattice calculations due to isospin symmetry.

• Vertical bands based on [JLQCD PRL 101, 242001; RBC-UKQCD PRD 81, 014504; LSD PRL 106, 231601].
**LSD**: Polarization Tensor for $S$ Parameter

- $S$ for $N_f/2$ EW doublets

$$S = 4\pi \frac{N_f}{2} \left[ \Pi'_{VV}(0) - \Pi'_{AA}(0) \right] + \Delta S_{SM}$$

$$= \frac{1}{3\pi} \int_0^\infty ds \left\{ \frac{N_f}{2} \left[ R_V(s) - R_A(s) \right] - \frac{1}{4} \left[ 1 - \left( 1 - \frac{m_h^2}{s} \right)^3 \Theta(s - m_h^2) \right] \right\}$$

- Pade(1,2) fit of $\Pi_{V-A}(Q^2)$ assumes $Q^{-2}$ scaling as $Q^2 \to \infty$ [1st WSR].

- Slope shows decreasing trend with decreasing mass for $N_f = 6$.

- *n.b.* smaller $S$ for fewer EW doublets.
**LSD**: Flavor dependence of $\Pi'_{V-A}(0)$

- Polarization tensor computed for one EW doublet.
- Filled symbols $M_P \cdot L \geq 4$.
- Plot vs. $M_P^2$ instead of $m$, in units of $M_{V0}$.
- $\Pi' \sim \log M_P^2$ as $M_P^2 \to 0$.
- Free field value for $\Pi' = 1/2\pi = 0.159...$
Flavor dependence of $S$ Parameter

• Very naïve scaling for $S$

$$S \propto \frac{N_f N_c}{2 \cdot 3}$$

• Walking conjectured to reduce $S$ by parity doubling, e.g. single-pole dominance:

$$S \sim 4\pi \left( \frac{N_f}{2} \right) \left[ \frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2} \right]$$

• After $\Delta S_{SM}$ subtraction, $S$ reduced relative to naïve scaling for $N_f=6$. **Is it a precursor of walking behavior?**

• *n.b.* $S$ for $N_f=6$ still log divergent until spectrum of PNGB’s is fixed.
Flavor dependence of parity partners

- Note slope of $M_V$ vs. $M_P^2$ roughly independent of $N_f$, not true for $M_V$ vs. $m$.

PRL 106, 231601 (2011)
Conclusions

• For SU(3) running coupling studies for various $N_f$ suggest a walking theory may exist for $10 \leq N_f \leq 12$ flavors.

• Direct study of walking theories beyond the current capabilities of largest computers, best algorithms, ...

• Searches for precursors of walking behavior as the running slows with increasing $N_f$ support the vision that a walking theory can solve Technicolor’s phenomenological problems.

• For $N_f = 6$, non-perturbative condensates are enhanced and $S$ parameter reduced relative to perturbative expectations.

• Technicolor remains a viable option for physics at the TeV scale.

• Last week: “Lattice Meets Experiment for BSM”, www.usqcd.org
A Dozen Lattice BSM Efforts Worldwide

- Aoyama et al.
- Del Debbio et al.
- Catteral et al.
- Hietanen et al.
- Yamada et al.
- DeGrand et al.
- Deuzeman et al.
- LSD
  - A. Hasenfratz
  - Jin-Mawhinney
  - Kogut-Sinclair

LHC
How can the lattice address Technicolor?

• Technicolor scenario has Higgs mechanism driven by TeV-scale strong interactions with spontaneous symmetry breaking (SSB) and Nambu-Goldstone (NG) bosons.

• QCD has these features and been studied on the lattice for decades, recently with much success.

• Other strongly-coupled gauge theories likely have these features, i.e. other flavors ($N_f$), colors ($N_c$), etc.

• Lattice studies can search for the right combination that enables Technicolor to satisfy phenomenological constraints.

• Unfortunately, other theories are usually computationally more expensive than QCD for calculation: $\propto N_f^{3/2}, N_c^3, d(R)^3$
Technicolor on the Lattice (II)

- Tools developed for study of Lattice QCD:
  - Non-perturbative Running Coupling
  - Non-perturbative Renormalization of Operators
  - Light Hadron and Glueball Spectrum
  - Chiral Observables (condensate, Dirac eigenvalues)
  - Thermodynamic Observables ($T_c$, EoS)

- Are tools optimized for QCD useful for non-QCD studies?
  - Exception: Monte Carlo methods using Wilsonian RG?
  - Can finite-size scaling methods be adapted from stat. mech.?
Flavor dependence of NLO ChiPT

\[ M_\pi^2 = 2mB \left\{ 1 + \frac{2mB}{(4\pi F)^2} \left[ 2\alpha_8 - \alpha_5 + N_f (2\alpha_6 - \alpha_4) + \frac{1}{N_f} \log \frac{2mB}{(4\pi F)^2} \right] \right\} \]

\[ F_\pi = F \left\{ 1 + \frac{2mB}{(4\pi F)^2} \left[ \frac{1}{2} (\alpha_5 + N_f \alpha_4) - \frac{N_f}{2} \log \frac{2mB}{(4\pi F)^2} \right] \right\} \]

\[ \langle \bar{q}q \rangle = F^2 B \left\{ 1 + \frac{2mB}{(4\pi F)^2} \left[ \frac{1}{2} (2\alpha_8 + \eta_2) + 2N_f \alpha_6 - \frac{N_f^2 - 1}{N_f} \log \frac{2mB}{(4\pi F)^2} \right] \right\} \]

- The leading non-analytic terms are enhanced in the condensate and \( f_\pi \) but suppressed in \( (M_\pi)^2 \).
- The \( \alpha_i \sim O(1) \) low energy constants.
- \( \eta_2 \sim O(a^{-2}) \) contact term: UV-sensitive slope for condensate.
Non-analytic flavor factors in NNLO ChiPT

- Small NLO coeff for $M_{\pi^2}$ is not generic and doesn’t persist to higher orders.
- Can NNLO formulae help us extrapolate $N_f \gg 2$ results?
• NNLO ChiPT fits work fine for $N_f=2$.
• NNLO expression for general $N_f$ recently derived by Bijnens and Lu [JHEP11(2009)116].
Preliminary: $\chi$PT Radius of Convergence

- Smaller quark masses needed for reliable NNLO extrapolation for $N_f > 2$ [E.T. Neil et al., PoS(CD09)088].
- On $32^3 \times 64$, $m \approx 0.01$: $M_\pi \cdot L \sim 4$ and $F_\pi \cdot L \sim 1$. 48$^3 \times 64$ lattices needed to reach smaller quark masses.