The KMI lattice project
— exploring for technicolor from QCD —

Yasumichi Aoki
Kobayashi-Maskawa Institute (KMI), Nagoya University

for the KMI lattice collaboration

@ KMI Inauguration Conference
KMI lattice collaboration members

- YA, T.Aoyama, M.Kurachi, T.Maskawa, K.Nagai, H.Ohki,
- K.Yamawaki, T.Yamazaki
- K.Hasebe
- A.Shibata
Origin of the mass of fundamental particles

— Standard Model —

• Higgs mechanism:
  • VEV of scalar field breaks global gauge symmetry $\rightarrow$ NG boson (massless)
  • NG boson absorbed as longitudinal component of $W, Z \rightarrow$ massive $W, Z$
  • Yukawa interaction gives mass to fermions
  • fundamental scalar: UV power divergence
    • gauge hierarchy problem (fine tuning)
Origin of the mass of fundamental particles — Technicolor (alternative to Higgs mechanism) —

- Techni-fermion condensate $<\overline{T_R T_L}>$ at low energy (like $<\overline{q_R q_L}>$ in QCD)
  - breaks chiral symmetry
  - produces techni-pion $\pi_{TC}$ (composite, like pion in QCD)
  - longitudinal component of $W, Z$
  - $M_W = M_Z \cos \theta_W = g F_\pi / 2$ ($F_\pi = \nu_{\text{weak}} = 246$ GeV)
- no power divergence $\Rightarrow$ no fine tuning necessary
- fermion masses $\Rightarrow$ extended technicolor (ETC)
- for suppressed FCNC with appropriate size of fermion masses $\Rightarrow$ walking TC
Walking Technicolor

- key: to realize suppressed FCNC and appropriate size of fermion masses

- renormalized gauge coupling
  - to run very slowly (walking)
  - logarithmically divergent at low energies → to produce techni pions

- mass anomalous dimension
  - large: $\gamma_m \sim 1$

[Yamawaki-Bando-Matsumoto]
conformal window and walking coupling
- non-Abelian gauge theory with $N_f$ massless fermions -
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conformal window and walking coupling
- non-Abelian gauge theory with $N_f$ massless fermions -

- Walking Technicolor could be realized just below the conformal window

$N_f$

$N_f^{AF}$

$N_f^{crit}$

Asymptotic non-free

Conformal window

Walking Technicolor

QCD-like

$\alpha(\mu)$
conformal window and walking coupling
- non-Abelian gauge theory with $N_f$ massless fermions -

- Walking Technicolor could be realized just below the conformal window

- crucial information: $N_f^{\text{crit}}$ & mass anomalous dimension around $N_f^{\text{crit}}$
SU(3) gauge theory with fundamental fermions

- perturbation theory
  - 2 loop universal running coupling at fixed point & 1 loop anomalous dim
    - $N_f^{\text{crit}} \approx 8.05$
    - $\alpha^* \approx 0.04, \gamma^* \approx 0.03$ for $N_f=16 \rightarrow$ likely in conformal phase
    - $\alpha^* \approx 0.8, \gamma^* \approx 0.5$ for $N_f=12$
SU(3) gauge theory with fundamental fermions

- perturbation theory
  - 2 loop universal running coupling at fixed point & 1 loop anomalous dim
    - $N_f^{\text{crit}} \approx 8.05$
    - $\alpha^* \approx 0.04$, $\gamma^* \approx 0.03$ for $N_f=16$ → likely in conformal phase
    - $\alpha^* \approx 0.8$, $\gamma^* \approx 0.5$ for $N_f=12$
      - requires non-perturbative method
most reliable method is lattice gauge theory

- success in QCD in SM: first principles calculation became possible
  - hadron spectrum
  - weak matrix elements: decay constants, bag parameters, form factors
  - running gauge coupling
- same quantity is indispensable and quite informative for technicolor
  - mass of the composite states
  - techni-pion decay constant
  - running technicolor coupling
KMI computer
KMI computer

• non GPU nodes
  • 148 nodes
  • 2x Xenon 3.3 GHz
  • 24 TFlops (peak)

• GPU nodes
  • 23 nodes
  • 3x Tesla M2050
  • 39 TFlops (peak)
Inauguration Ceremony of March 2nd, 2011
1st flagship project on \( \phi \)

- SU(3) + large \( N_f \) fundamental fermions

- utilize knowledge and tools developed in past ~30 years of Lattice QCD
  - reinforced by the knowledge from the real world

- investigates spectrum: techni pion mass, decay constant
SU(3) gauge theory with large $N_f$ [fundamental rep.]

- Our goals:
  - Understand the $n_f$ dependence of the theory
  - Find the conformal window
  - Find the walking regime and investigate mass anomalous dimension

- Status:
  - $N_f=16$ likely conformal
  - $N_f=12$: controversial
  - $N_f=10$: one study showing evidence of IR fixed point. Some more...
  - $N_f=8$: studies suggesting no IR fixed point $\leftrightarrow$ one for conformal
  - $N_f=6$: confining: enhancement of condensation
our approach

- study $N_f$ dependence systematically using single set up of the lattice simulation
  - target: $N_f=(0), 4, 8, 12, 16$
  - this talk mainly focuses on $N_f=12$ (most controversial in the community)
    - $N_f=12$ poster [Ohki]
    - $N_f=16$ poster [Yamazaki] (deep in conformal window ?)
      - results with 2 lattice spacings and a trial lattice spacing determination
    - $N_f=8$ poster [Nagai] (candidate for WTC?)
    - Swinger-Dyson approach and comparison with lattice $N_f=4, 12$ [Kurachi]
simulation strategy

• use of improved staggered action
  • to get nearly continuum results from non-zero lattice spacing
  • to reduce flavor violation for good SU(N) chiral symmetry
  • bound to \( N_f = 4 \) n

• we use MILC version of HISQ (Highly Improved Staggered Quark) action
  • Asqtad + \( g^2 a^2 \) taste exchange interaction & up to \((ma)^4\) removed, but
  • use tree level Symanzik gauge action
  • no \((ma)^2\) improvement (no interest to heavy quarks)
  • = HISQ/tree (HotQCD collaboration)
HISQ action

• proposed by HPQCD collaboration for
  • smaller taste violation than other approaches
  • better handling of heavy quarks

• being used in simulations (slightly changed versions)
  • MILC: Nf=2+1+1 QCD
  • HOTQCD: QCD thermodynamics: Bazavov-Petreczky (Lat’10 proceedings)
    • HISQ/tree is best of [HISQ/tree, Asqtad, stout]
      for flavor (taste) symmetry, dispersion relation
HISQ action

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      for flavor (taste) symmetry, dispersion relation

Figure 2: RMS pion mass when $m_{\pi} = 140$ MeV. See details in the text.
simulation procedure

• using MILC code v7
  • changed to do simple HMC (remove R) with 3g1f Omelyan integrator
  • note: our $\beta = 6/g^2$

• global search for $\beta$ & m with small volume

• measure meson spectrum
  • in particular Goldstone pion mass and decay constants

• varying volume
### $N_f=12$ SU(3): current situation

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now our results come.
now our results come.

all the following results are preliminary...
$n_f=12$: pion mass and decay constant, $\beta=3.5$
$n_f=12$: pion mass: fit for $\chi$ broken scenario?

$\beta=3.5$

$\chi^2$: $8^3 \times 12$, $12^3 \times 24$, $24^3 \times 32$
$n_f=12$: pion mass: fit for $\chi$ broken scenario?

$\beta=3.5$

\[ m_{\pi}^2 = a m^2 + b m + c \]
$n_f=12$: pion mass: fit for $\chi$ broken scenario?

$$a m_q^2 + b m_q + c$$

$\beta=3.5$
\( n_f=12 : \) pion mass : fit for \( \chi \) broken scenario?

- \( a m_q^2 + b m_q + c \)
- \( c=-0.090(5), \chi^2/dof=1.1 \)
\( n_f=12 : \) pion mass : fit for \( \chi \) broken scenario?

- \( a m_q^2 + b m_q + c \)

- \( c=-0.090(5), \chi^2/dof=1.1 \)

- \( c=0 \rightarrow \chi^2/dof=104 \)
\( n_f=12 \): pion mass: fit for \( \chi \) broken scenario?

\[ \beta = 3.5 \]

- \( a m_q^2 + b m_q + c \)
  - \( c = -0.090(5), \chi^2/dof = 1.1 \)
  - \( c = 0 \rightarrow \chi^2/dof = 104 \)

- \( a m_q^\delta \)
$n_f=12$: pion mass: fit for $\chi$ broken scenario?

$\beta=3.5$

- $a m_q^2 + b m_q + c$
  - $c=-0.090(5)$, $\chi^2/dof=1.1$
  - $c=0 \rightarrow \chi^2/dof=104$

- $a m_q^\delta$
  - $\delta=1.45(7)$, $\chi^2/dof=32$
$n_f=12$ : pion mass : fit for $\chi$ broken scenario?

$\beta=3.5$

- $a m_q^2 + b m_q + c$
  - $c=-0.090(5), \chi^2/dof=1.1$
  - $c=0 \rightarrow \chi^2/dof=104$

- $a m_q^\delta$
  - $\delta=1.45(7), \chi^2/dof=32$
  - $\Rightarrow \gamma^*=0.38(7)$
nf=12 : pion decay constant

\[ \beta = 3.5 \]

- \( a m_q^2 + b m_q + c \)
  - \( c = 0.021(3), \chi^2/\text{dof}=1.7 \)
  - \( c = 0 \rightarrow \chi^2/\text{dof}=17 \)

- \( b m_q^\delta \)
  - \( \delta = 0.681(9), \chi^2/\text{dof}=2.3 \)
  - \( \gamma^* = 0.47(2) \)
hyper scaling

- mass deformation in a massless conformal theory: Miransky 1999.
- mass dependence is described by anomalous dimensions at IRFP
  - quark mass anomalous dimension $\gamma^*$
  - operator anomalous dimension
- meson mass and pion decay constant obey same scaling
  \[ m_\pi = c_m m_f^{1+\gamma^*} \quad f_\pi = c_f m_f^{1+\gamma^*} \]
- finite size scaling formula (Del Debbio et al)
  - scaling variable: \( x = Lm_f^{1+\gamma^*} \)
  \[ Lf_\pi = F(x) \quad Lm_\pi = G(x) \]
$m_{\pi}$: finite size hyper scaling $N_f=12$, $\beta=3.5$
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- optimal: $\gamma^*=0.4--0.6$
$f_\pi$: finite size hyper scaling $N_f=12$, $\beta=3.5$
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\begin{center}
\begin{tabular}{c|c|c|c}
$L_f \pi$ & $8^3 \times 12$ & $12^3 \times 24$ & $24^3 \times 32$ \\
\hline
$\gamma=0.6$
\end{tabular}
\end{center}
$f_\pi$: finite size hyper scaling $N_f=12, \beta=3.5$

- optimal: $\gamma^*=0.4--0.6$
\( \gamma^* \) from a fit: \( \beta=3.5 \)

- \( y = b L m_q^{1/(1+\gamma^*)} + c \) for large \( x \) where linearity is observed

\[ \chi^2/\text{dof}=7.2 \]

\[ \chi^2/\text{dof}=16.3 \]

• errors are statistical only
\( \gamma^* \): extending calculation towards continuum limit

- from poster by Ohki: \( N_f=12 \)
  - \( \beta=3.5 \) not included due to non-uniform aspect ratio etc...
- consistent with conformal hypothesis
- errors are statistical only
- consistency between: \( m_\pi \) and \( f_\pi \)
- tends to decreases towards the continuum limit, BUT, it could be
  - due to lattice artifact (UV), reduced physical volume (IR) or other sys err.?
N_f=4 from poster by Kurachi

**Lattice results**

\[ \frac{L_m}{1 + \gamma_m} \]

\[ L_f \]

\[ \beta = 3.5 \]

\[ N_f = 12 \]

\[ N_f = 4 \]

no scaling observed
$N_f=8$ from poster by Nagai

**ChPT analysis in $N_f=8$**

> $\chi$ SB phase, analyzed by ChPT ??

- **$M^2$ vs $mf$**
  - $N_f=8, \beta=3.7$
  - Quadratic fit: $y=c_0+c_1*mf+c_2*mf^2$
  - Conformal-like fit: $y=c_0+c_1*mf^a$

- **PBP vs $mf$**
  - $N_f=8, \beta=3.7$

- **$f_\pi$ vs $mf$**
  - $N_f=8, \beta=3.7$
  - Conformal analysis

It’s difficult to conclude that $N_f=8$ is in the hadron phase.
N_f=8 from poster by Nagai

Fit result of the hyperscaling in the conformal hypothesis for N_f=8

\[ \gamma = 0.608(2), \chi^2/\text{dof}=39.8, \text{ at } \beta=3.7 \]
N_f=8, pion mass

\[ \gamma = 1.029(7), \chi^2/\text{dof}=57.8, \text{ at } \beta=3.7 \]
N_f=8, decay constant

\[ \gamma = 0.799(13), \chi^2/\text{dof}=5.7, \text{ at } \beta=3.7 \]
N_f=8, rho mass

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<th>$\gamma$ in $M_{\pi}$</th>
<th>$\beta = 3.6$</th>
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<td>0.608(2)</td>
<td>0.607(3)</td>
<td>0.563(3)</td>
<td>0.757(14)</td>
<td></td>
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<tr>
<td>0.766(40)</td>
<td>0.799(13)</td>
<td>0.862(59)</td>
<td>1.18(32)</td>
<td></td>
</tr>
<tr>
<td>1.02(1)</td>
<td>1.03(1)</td>
<td>0.98(1)</td>
<td>1.13(3)</td>
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Table:

It seems not to be simple hadronic phase: $\gamma \neq 1$. c.f. N_f=4 case.

\[ \gamma \approx 1, \text{ walking ??} \]

\[ \spadesuit \text{ N_f}=8 \text{ shows the good behavior of the hyperscaling.} \]

\[ \spadesuit \text{ Still, } \gamma(M_{\pi}) < \gamma(M_{\rho}) < \gamma(f_{\pi}) \text{ not exact Conformal ??} \]
5. Results of mass and finite size deformed case

Changing $\gamma_*$ of $Lm_\pi$ vs $Lm_{\frac{1}{1+\gamma_*}} (\beta = 3.50)$

$\gamma_* \sim 0.3$ gives a nice scaling at larger value of $x$-axis.
$N_f=16$ from poster by Yamazaki

$Lm_\pi$ and $Lf_\pi$ fit with asymptotic form: $Lm_\pi = c_0 + c_1 x, \quad x = Lm_\pi^{\frac{1}{1+\gamma}}$

$\gamma_*$ at $\beta = 3.50$ is consistent with the one of mass deformed case. Two $\gamma_*$ from different observables reasonably agree with each other at both $\beta$.

However, $\gamma_*$ at both $\beta$ is much larger than the perturbative result, $\gamma^\text{pert}_* \sim 0.015$.  

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\[ \text{Poster: Ohki} \]
summary

- large $N_f$ SU(3) gauge theory with fundamental rep. is being investigated
  - quest for the walking technicolor
- using a HISQ type fermion and the tree-level Symanzik gauge action
- aiming to explore a wide range of the $N_f$ systematically
- This talk mainly described $N_f=12$ study
  - three lattice spacings ($\beta=6/g^2$) studied, with spatial size up to $L_s=30$
  - pion mass and decay constant are studied
  - approximate finite size scaling for conformal scenario is observed
  - with the current lattice volume, results favor conformal theory
  - assuming an IR fixed point, mass anomalous dimension calculated
    - $\gamma^* \sim 0.4$
comparison to other works on Nf=12 SU(3)

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summary (continued)

• $N_f=4$
  • clearly in $\chi$ broken phase
  • finite size hyper scaling not observed
• $N_f=8$
  • more study needed for definite conclusion
• $N_f=12$
  • results are consistent with conformal hypothesis
• $N_f=16$
  • consistent with conformal, but with large anomalous dimension
  • study with weaker coupling necessary
outlook

• to meet our goals

• for $N_f=8, 12$
  • larger size than $L_s=30$ is needed to investigate further IR regime
  • make it possible to study lighter mass
  • glueball mass to check hyper scaling
  • masses for other mesons, baryons, flavor singlets: to check hyper scaling

• for $N_f=16$
  • much weaker coupling
Thank you for your attention