

# The KMI lattice project

## — exploring for technicolor from QCD —

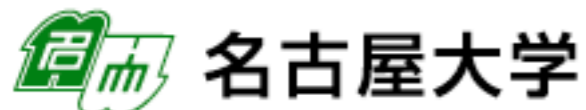
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Yasumichi Aoki

Kobayashi-Maskawa Institute (KMI), Nagoya University

for the KMI lattice collaboration

@ KMI Inauguration Conference



# KMI lattice collaboration members

YA, T.Aoyama, M.Kurachi, T.Maskawa, K.Nagai, H.Ohki,



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名古屋大学



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愛知大学  
AICHI UNIVERSITY



A.Shibata



# Origin of the mass of fundamental particles

## — Standard Model —

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- Higgs mechanism:
  - VEV of scalar field breaks global gauge symmetry  $\rightarrow$  NG boson (massless)
  - NG boson absorbed as longitudinal component of  $W, Z \rightarrow$  massive  $W, Z$
  - Yukawa interaction gives mass to fermions
  - fundamental scalar: UV power divergence
    - gauge hierarchy problem (fine tuning)

# Origin of the mass of fundamental particles

## — Technicolor (alternative to Higgs mechanism) —

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- Techni-fermion condensate  $\langle T_R T_L \rangle$  at low energy (like  $\langle q_R q_L \rangle$  in QCD)
  - breaks chiral symmetry
  - produces techni-pion  $\pi_{TC}$  (composite, like pion in QCD)
  - longitudinal component of  $W, Z$
  - $M_W = M_Z \cos \theta_W = g F_\pi / 2$  ( $F_\pi = v_{\text{weak}} = 246 \text{ GeV}$ )
- no power divergence  $\rightarrow$  no fine tuning necessary
- fermion masses  $\rightarrow$  extended technicolor (ETC)
- for suppressed FCNC with appropriate size of fermion masses  $\rightarrow$  walking TC

# Walking Technicolor

- key: to realize suppressed FCNC and appropriate size of fermion masses

[Yamawaki-Bando-Matsumoto]

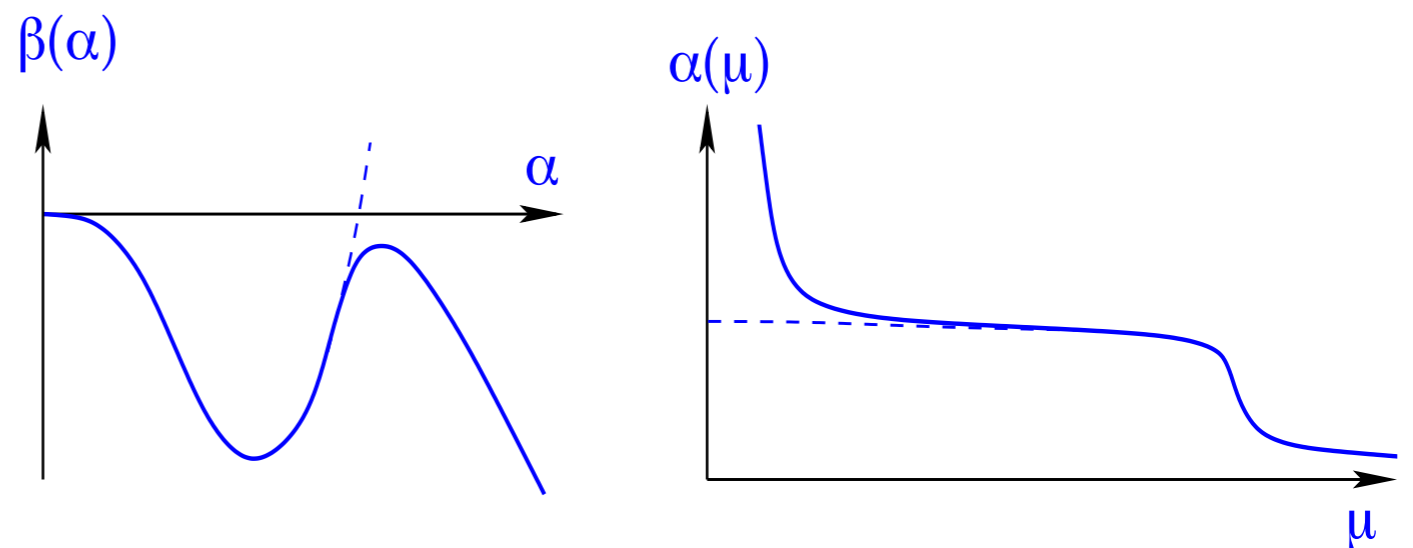
- renormalized gauge coupling

- to run very slowly (walking)

- logarithmically divergent at low energies → to produce techni pions

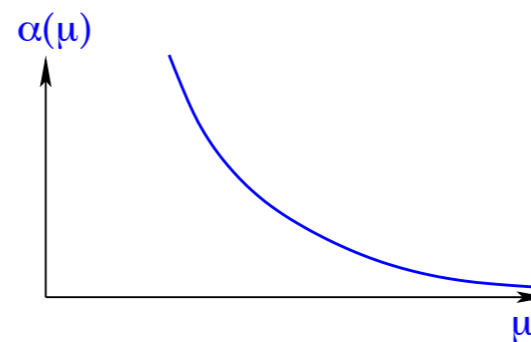
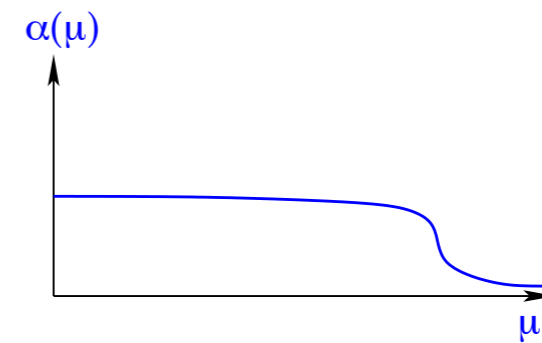
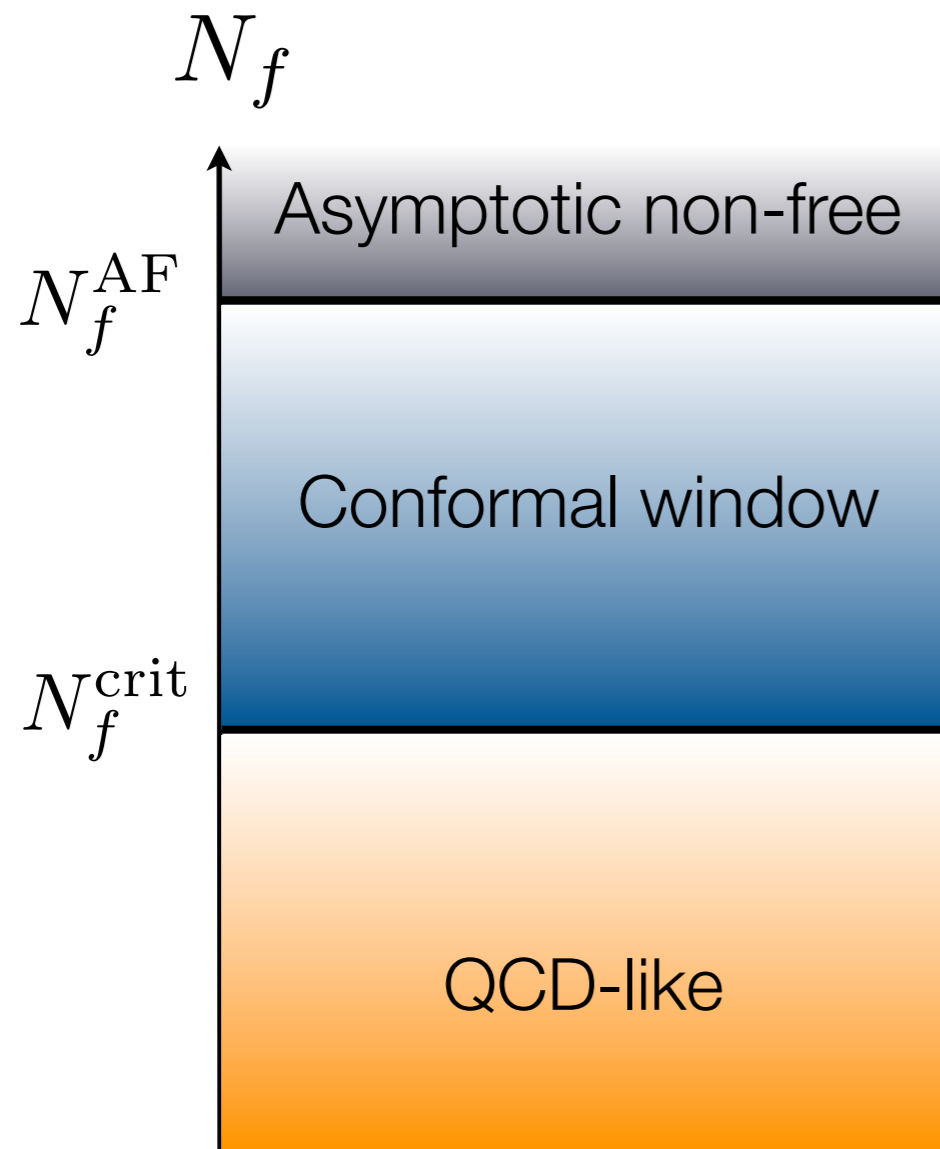
- mass anomalous dimension

- large:  $\gamma_m \sim 1$



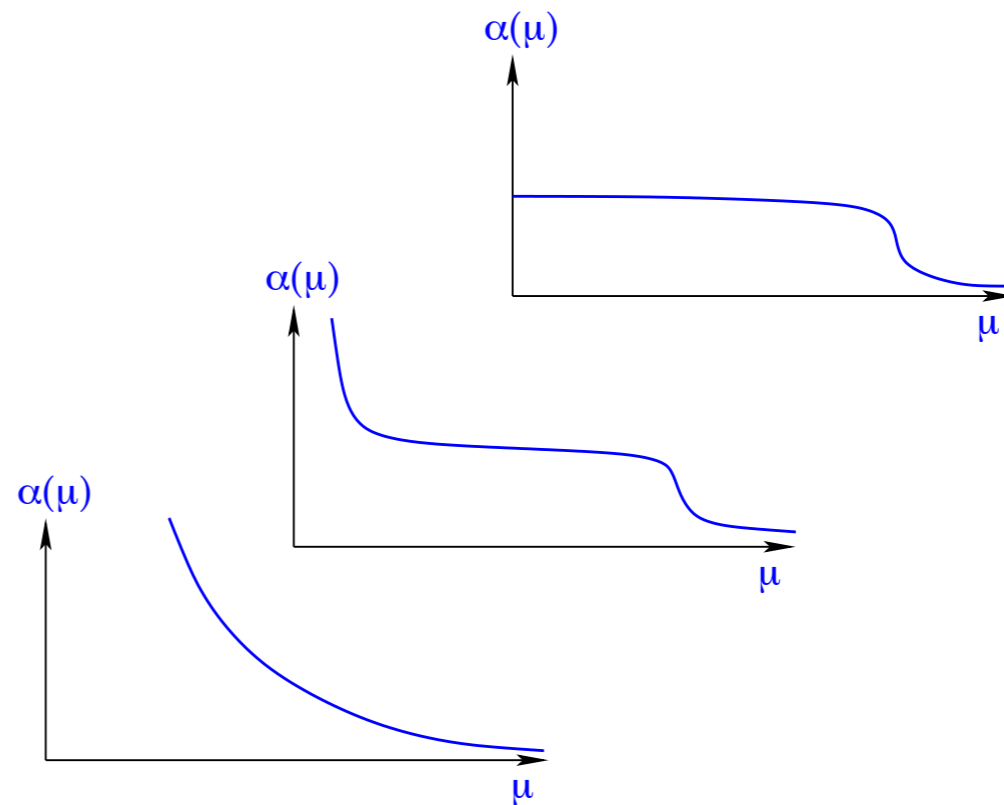
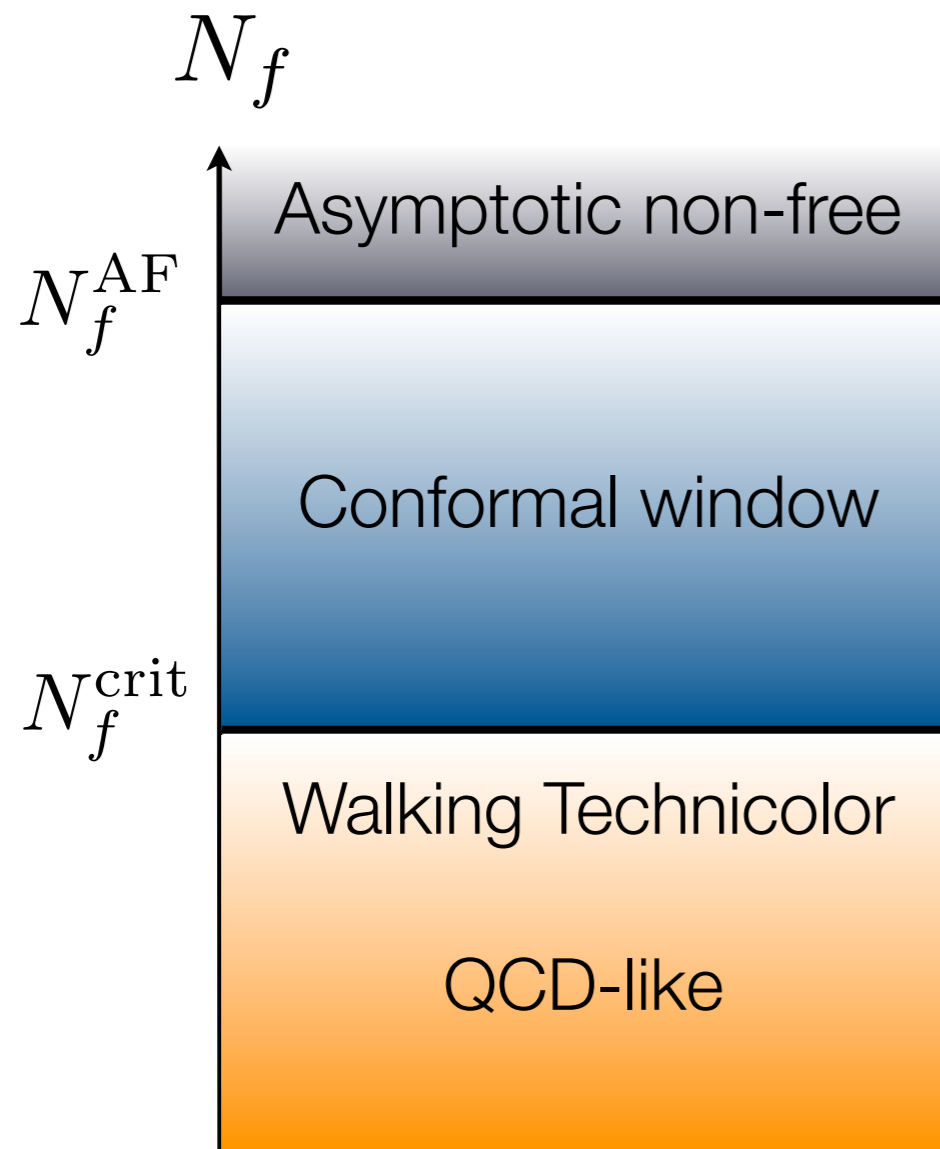
# conformal window and walking coupling

- non-Abelian gauge theory with  $N_f$  massless fermions -



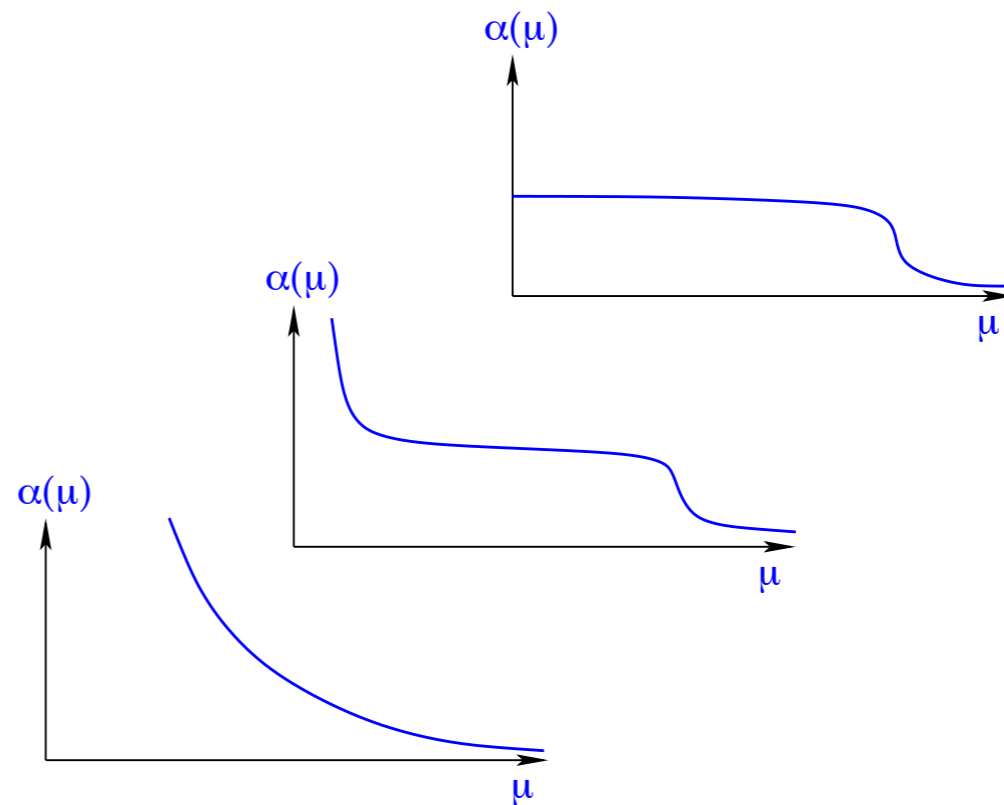
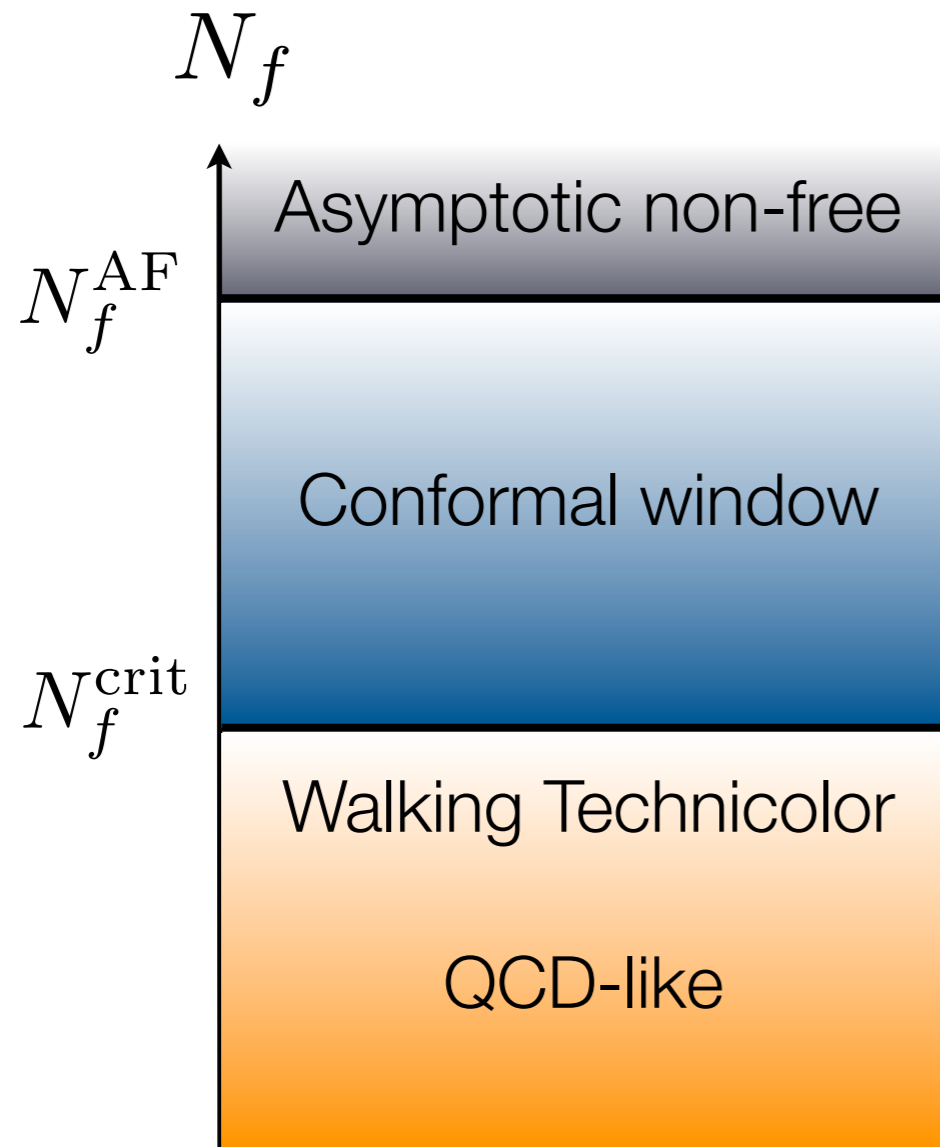
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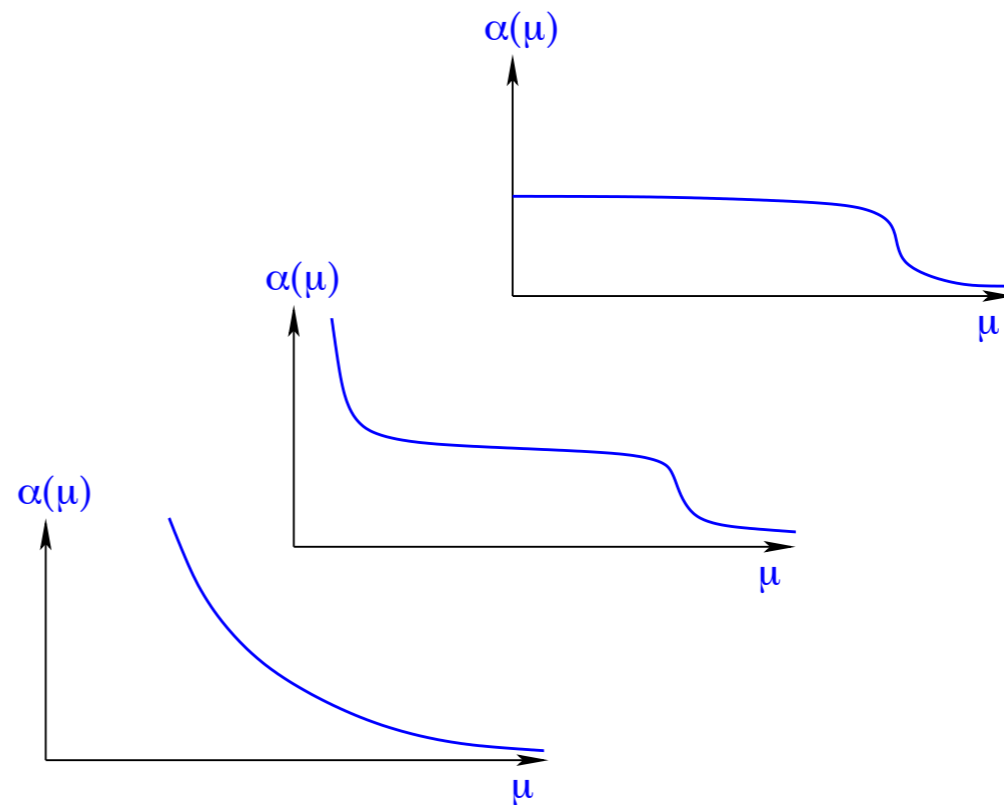
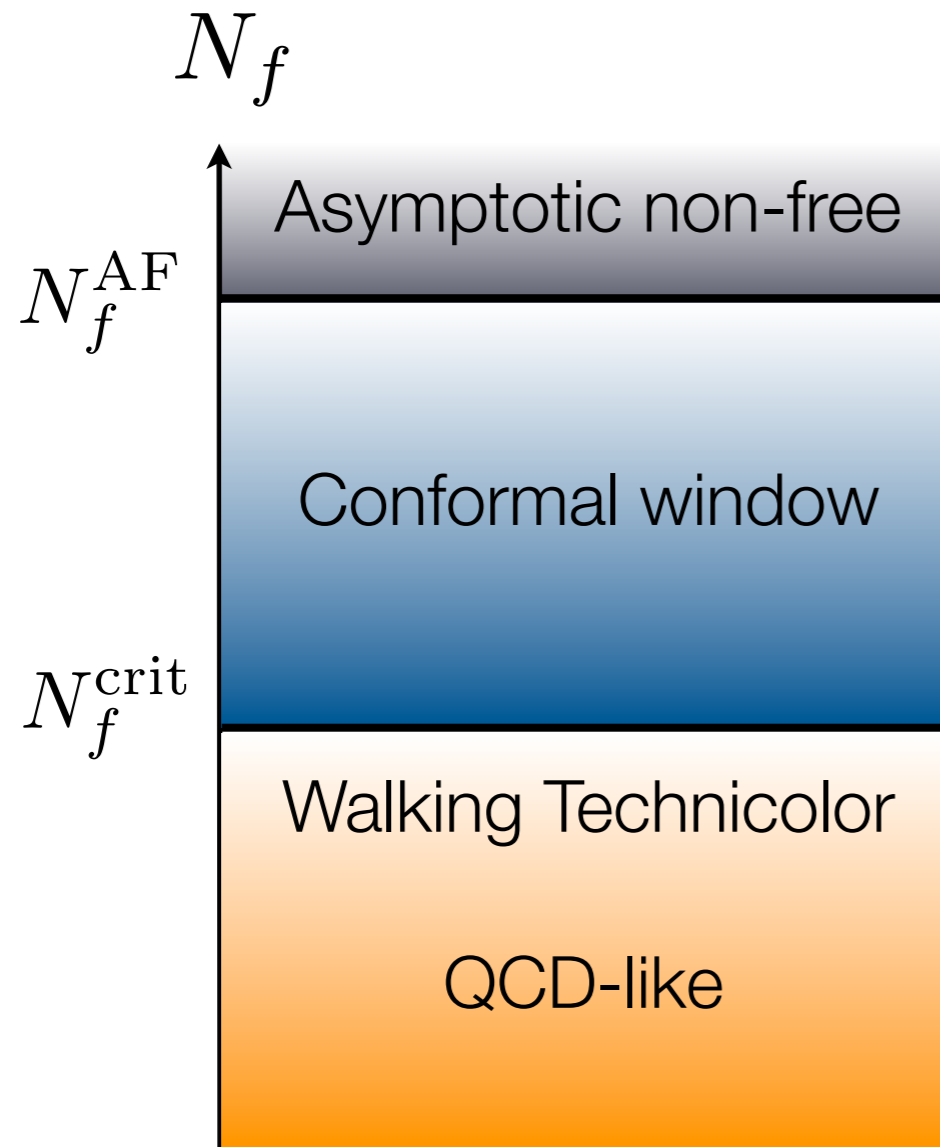


- Walking Technicolor could be realized just below the conformal window



# conformal window and walking coupling

- non-Abelian gauge theory with  $N_f$  massless fermions -



- Walking Technicolor could be realized just below the conformal window
- crucial information:  $N_f^{crit}$  & mass anomalous dimension around  $N_f^{crit}$

# SU(3) gauge theory with fundamental fermions

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- perturbation theory
  - 2 loop universal running coupling at fixed point & 1 loop anomalous dim
    - $N_f^{\text{crit}} \sim 8.05$
    - $\alpha^* \sim 0.04, \gamma^* \sim 0.03$  for  $N_f=16$   $\rightarrow$  likely in conformal phase
    - $\alpha^* \sim 0.8, \gamma^* \sim 0.5$  for  $N_f=12$

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    - $\alpha^* \sim 0.8, \gamma^* \sim 0.5$  for  $N_f=12$ 
      - $\rightarrow$  requires non-perturbative method

# most reliable method is lattice gauge theory

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- success in QCD in SM: first principles calculation became possible
  - hadron spectrum
  - weak matrix elements: decay constants, bag parameters, form factors
  - running gauge coupling
- same quantity is indispensable and quite informative for technicolor
  - mass of the composite states
  - techni-pion decay constant
  - running technicolor coupling

# KMI computer

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# KMI computer

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# KMI computer



- non GPU nodes
  - 148 nodes
  - 2x Xenon 3.3 GHz
  - 24 TFlops (peak)
- GPU nodes
  - 23 nodes
  - 3x Tesla M2050
  - 39 TFlops (peak)



# Inauguration Ceremony of $\varphi$

March 2nd, 2011

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# 1st flagship project on $\varphi$

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- SU(3) + large  $N_f$  fundamental fermions
- utilize knowledge and tools developed in past ~30 years of Lattice QCD
  - reinforced by the knowledge from the real world
- investigates spectrum: techni pion mass, decay constant

# SU(3) gauge theory with large $N_f$ [fundamental rep.]

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- our goals:
  - understand the  $n_f$  dependence of the theory
  - find the conformal window
  - find the walking regime and investigate mass anomalous dimension
- status:
  - $N_f=16$  likely conformal
  - $N_f=12$ : controversial
  - $N_f=10$ : one study showing evidence of IR fixed point. Some more...
  - $N_f=8$ : studies suggesting no IR fixed point  $\leftrightarrow$  one for conformal
  - $N_f=6$ : confining: enhancement of condensation

# our approach

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- study  $N_f$  dependence systematically using single set up of the lattice simulation
  - target:  $N_f=(0), 4, 8, 12, 16$
  - this talk mainly focuses on  $N_f=12$  (most controversial in the community)
    - $N_f=12$  poster [Ohki]
    - $N_f=16$  poster [Yamazaki] (deep in conformal window ?)
      - results with 2 lattice spacings and a trial lattice spacing determination
    - $N_f=8$  poster [Nagai] (candidate for WTC?)
    - Swinger-Dyson approach and comparison with lattice  $N_f=4, 12$  [Kurachi]

# simulation strategy

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- use of improved staggered action
  - to get nearly continuum results from non-zero lattice spacing
  - to reduce flavor violation for good SU(N) chiral symmetry
  - bound to  $N_f=4$  n
- we use MILC version of HISQ (Highly Improved Staggered Quark) action
  - Asqtad +  $g^2 a^2$  taste exchange interaction & up to  $(ma)^4$  removed, but
  - use tree level Symanzik gauge action
  - no  $(ma)^2$  improvement (no interest to heavy quarks)
  - = HISQ/tree (HotQCD collaboration)

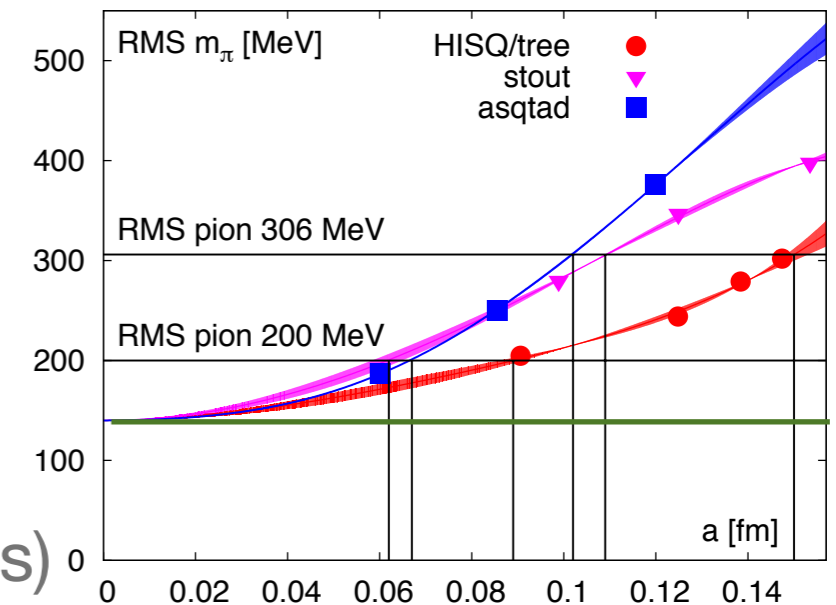
# HISQ action

---

- proposed by HPQCD collaboration for
  - smaller taste violation than other approaches
  - better handling of heavy quarks
- being used in simulations (slightly changed versions)
  - MILC:  $N_f=2+1+1$  QCD
  - HOTQCD: QCD thermodynamics: Bazavov-Petreczky (Lat'10 proceedings)
    - HISQ/tree is **best** of [HISQ/tree, Asqtad, stout]  
for flavor (taste) symmetry, dispersion relation

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**Figure 2:** RMS pion mass when  $m_{\gamma_5} = 140$  MeV. See details in the text.

# simulation procedure

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- using MILC code v7
  - changed to do simple HMC (remove R) with 3g1f Omelyan integrator
  - note: our  $\beta=6/g^2$
- global search for  $\beta$  &  $m$  with small volume
- measure meson spectrum
  - in particular Goldstone pion mass and decay constants
- varying volume

# $N_f=12$ SU(3): current situation

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collaboration	conclusion	method	remarks
Fodor et al	$\chi$ Broken	spectrum	big $V$ , single lat.spgs.
Columbia	$\chi$ Broken	spectrum, $T_c$	naive KS
Deutchman et al	Conformal	spectrum, $T_c$	KS+Naik
Itou et al	Conformal	coupling	naive KS + cont.lim.
Appelquist et al	Conformal	coupling	non-exact algorithm
Appelquist et al	Conformal	spectrum	using Fodor's data
DeGrand	consistent with Conformal	spectrum	using Fodor's data



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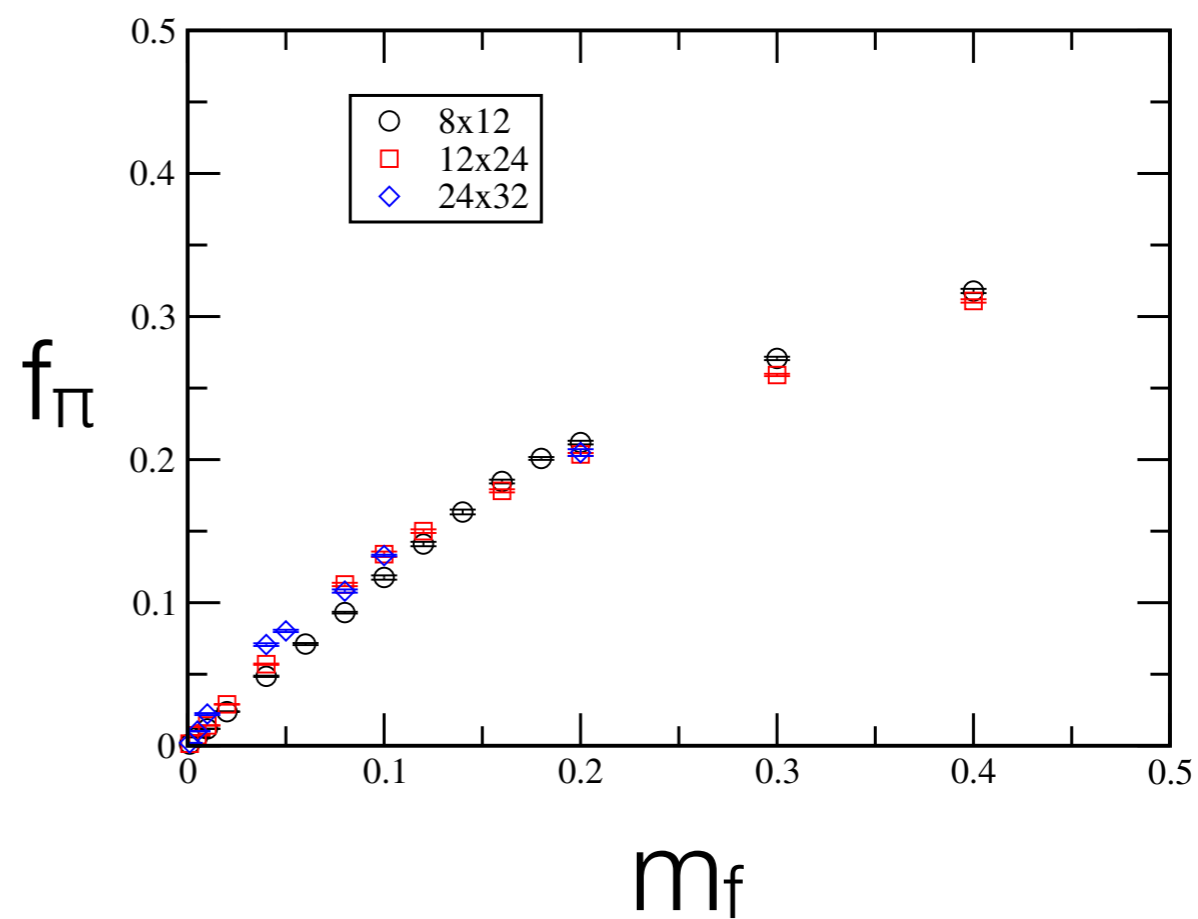
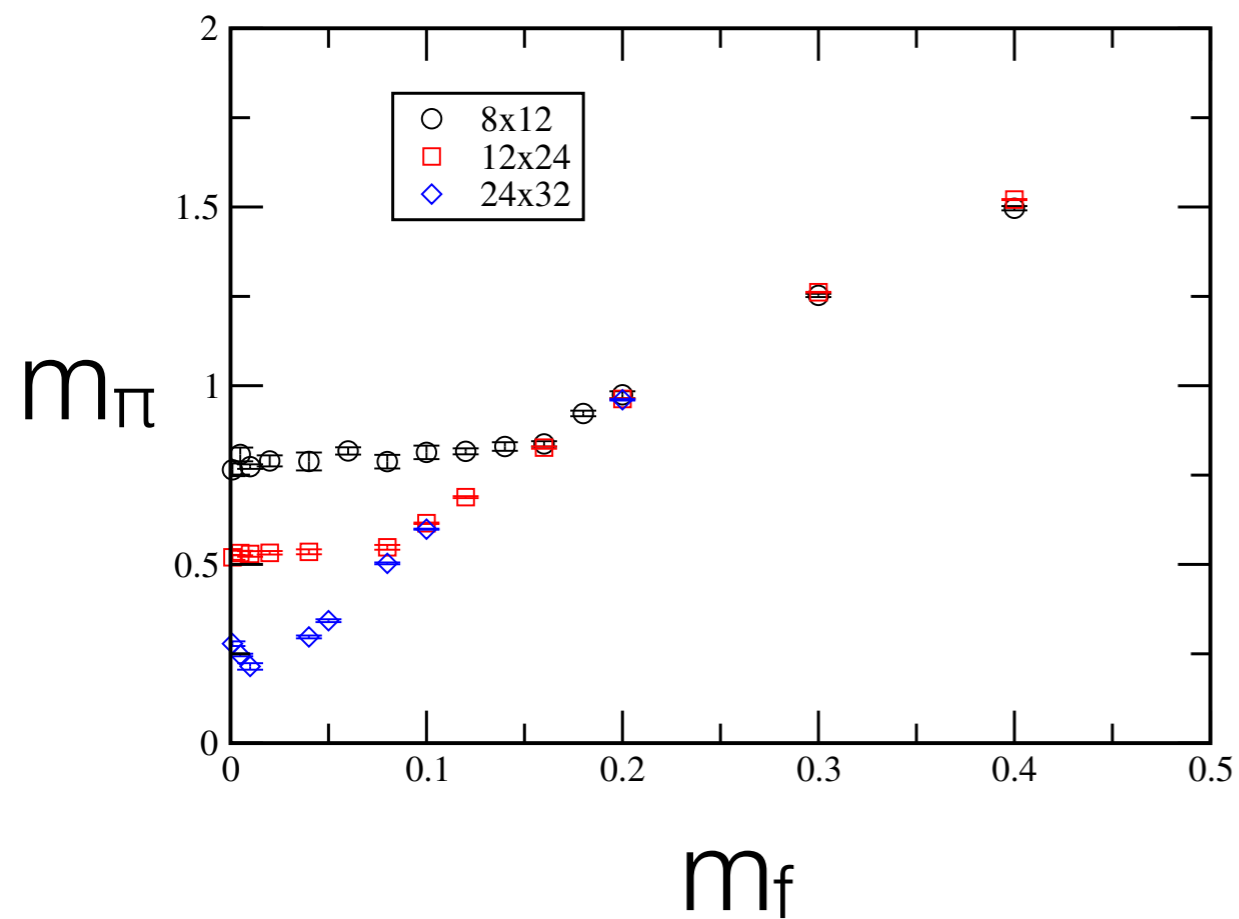
now our results come.

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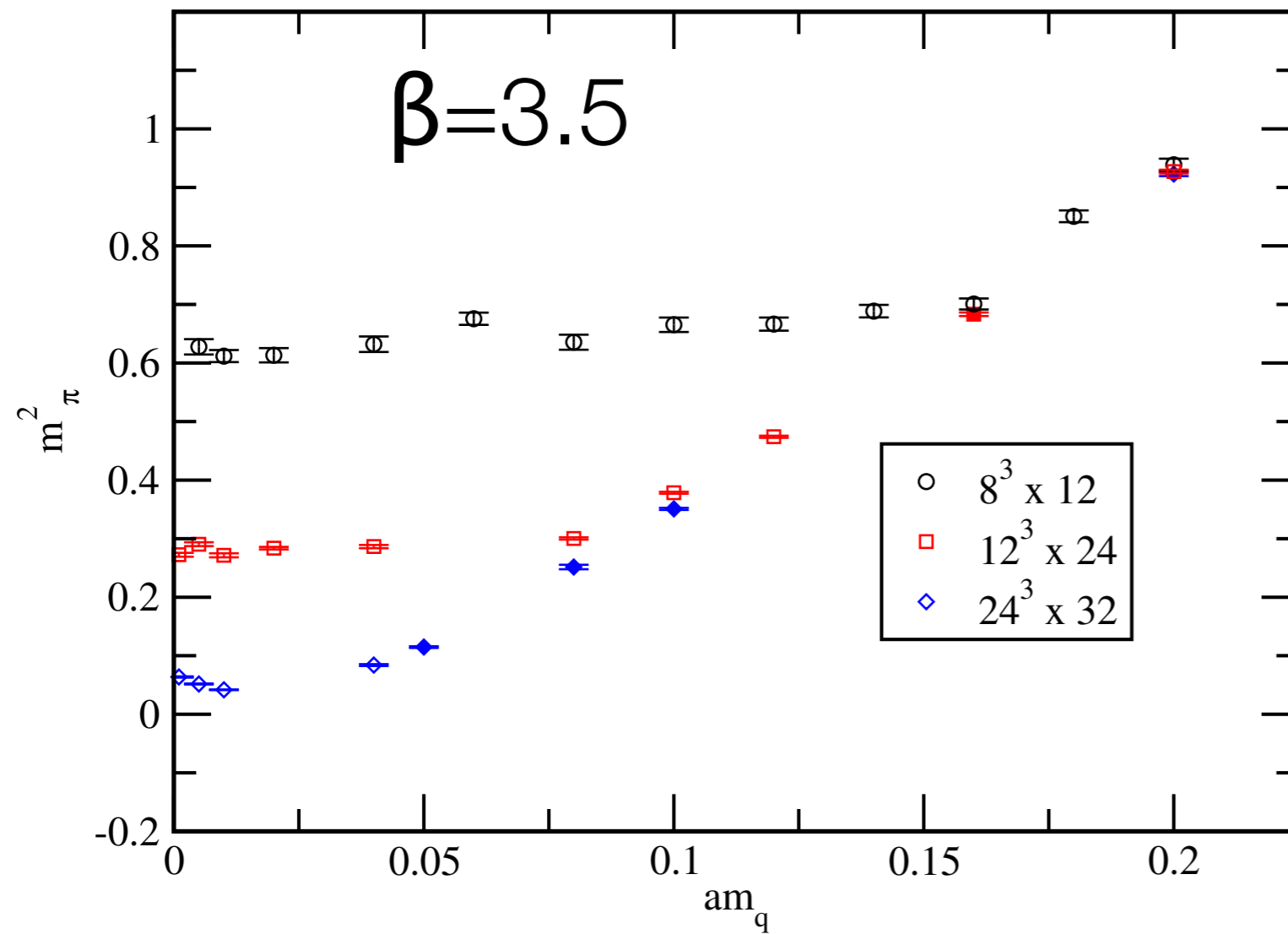
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all the following results are preliminary...

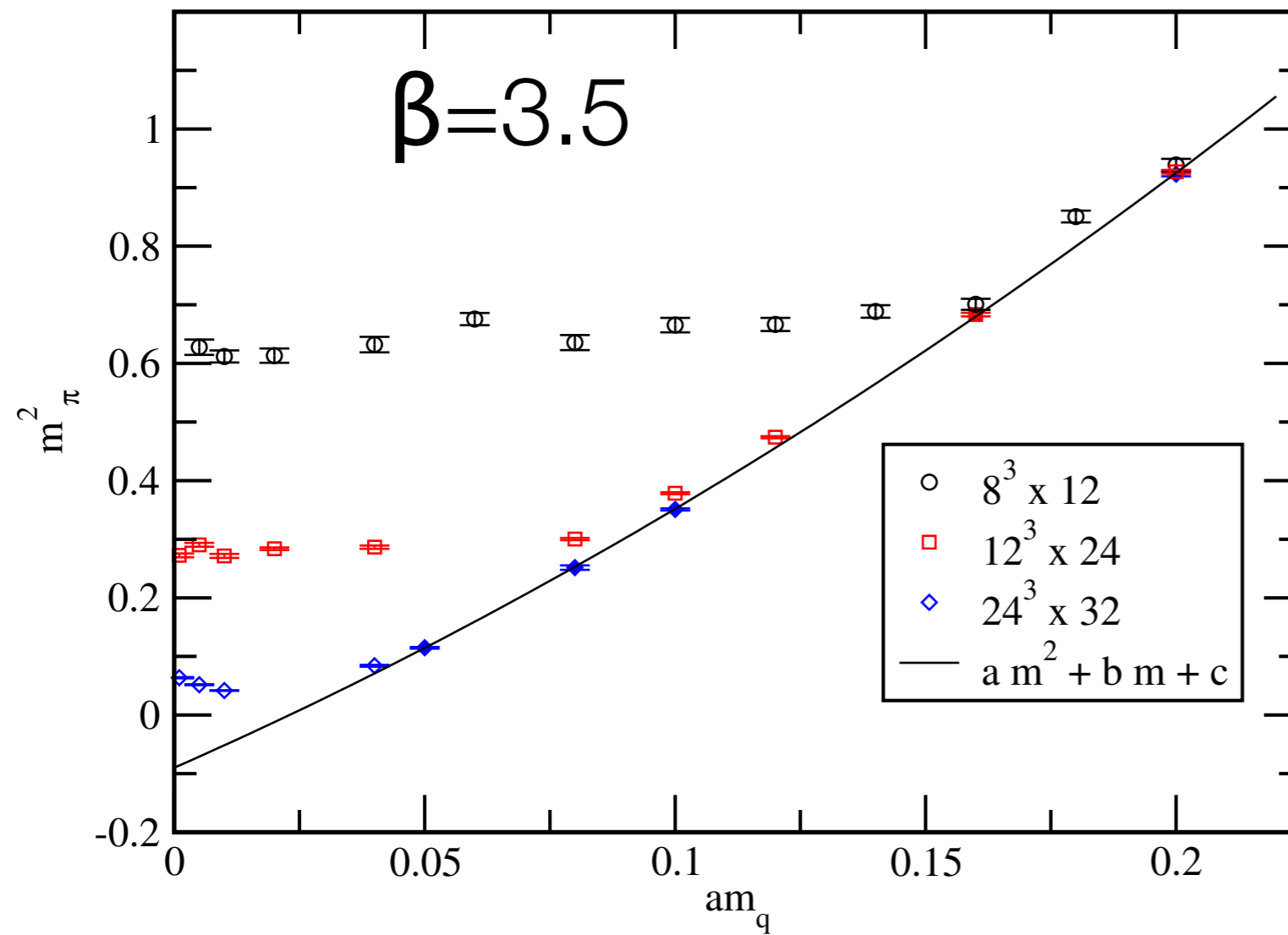
# $n_f=12$ : pion mass and decay constant, $\beta=3.5$



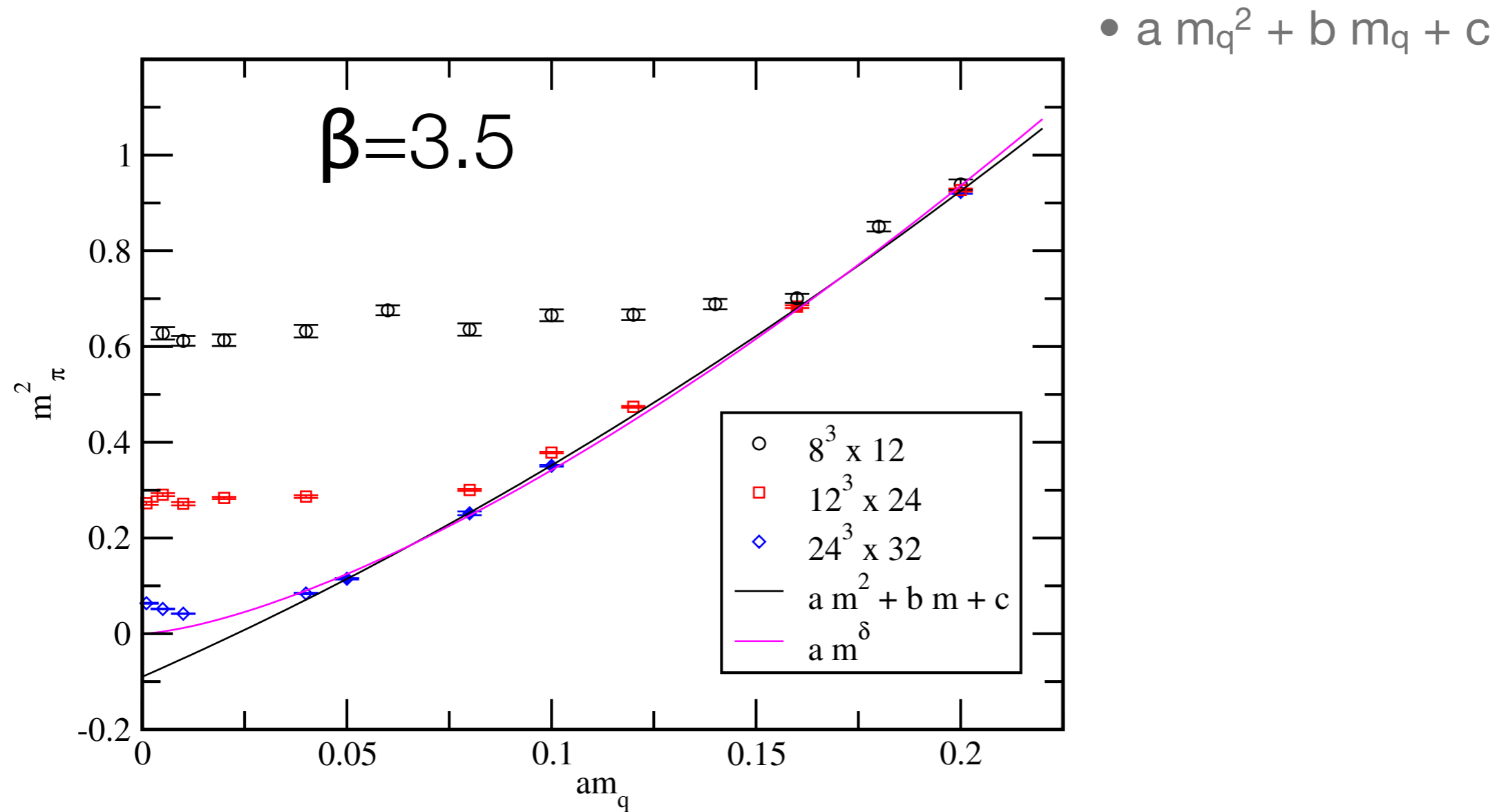
# $n_f=12$ : pion mass : fit for $\chi$ broken scenario ?



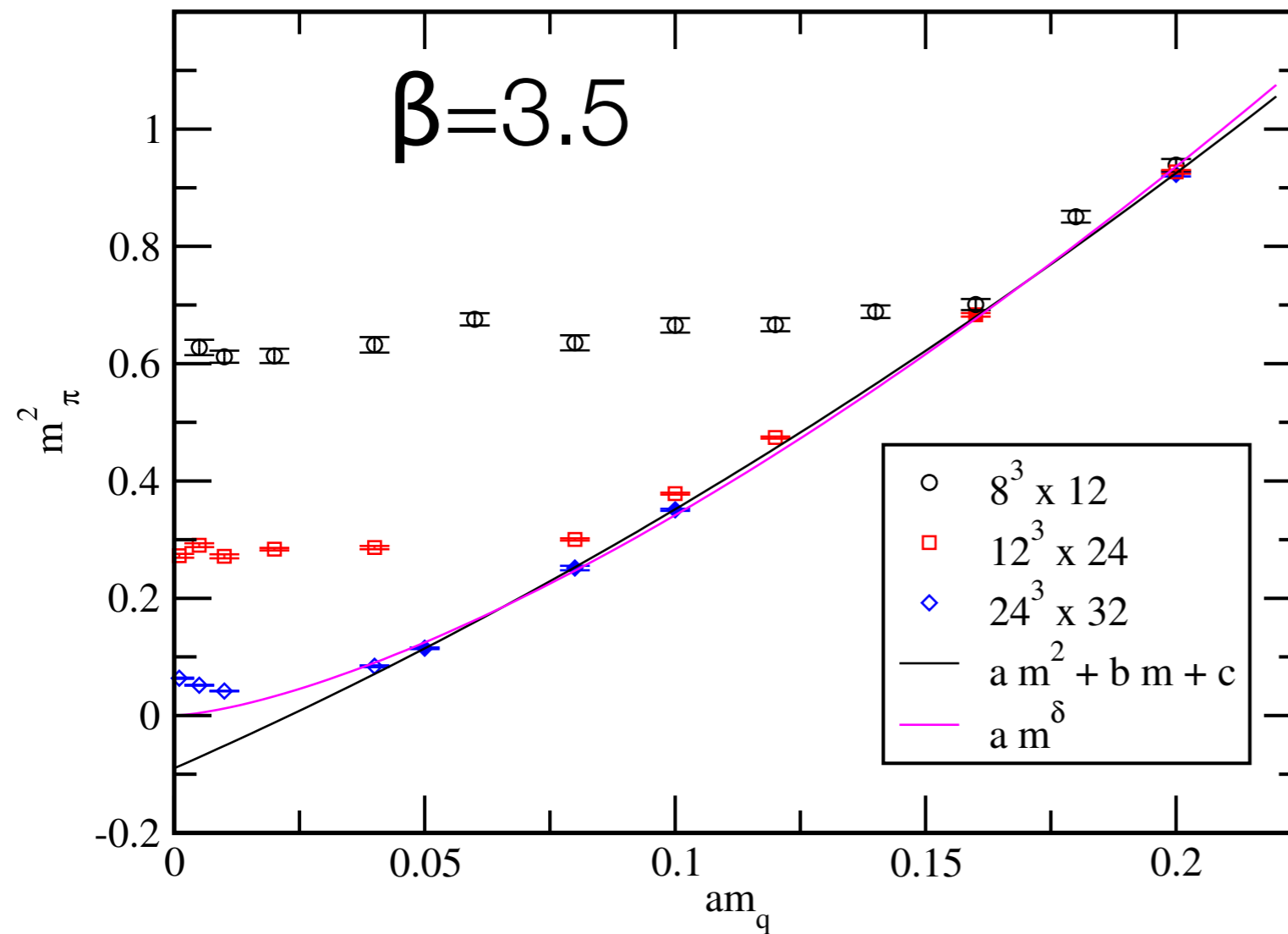
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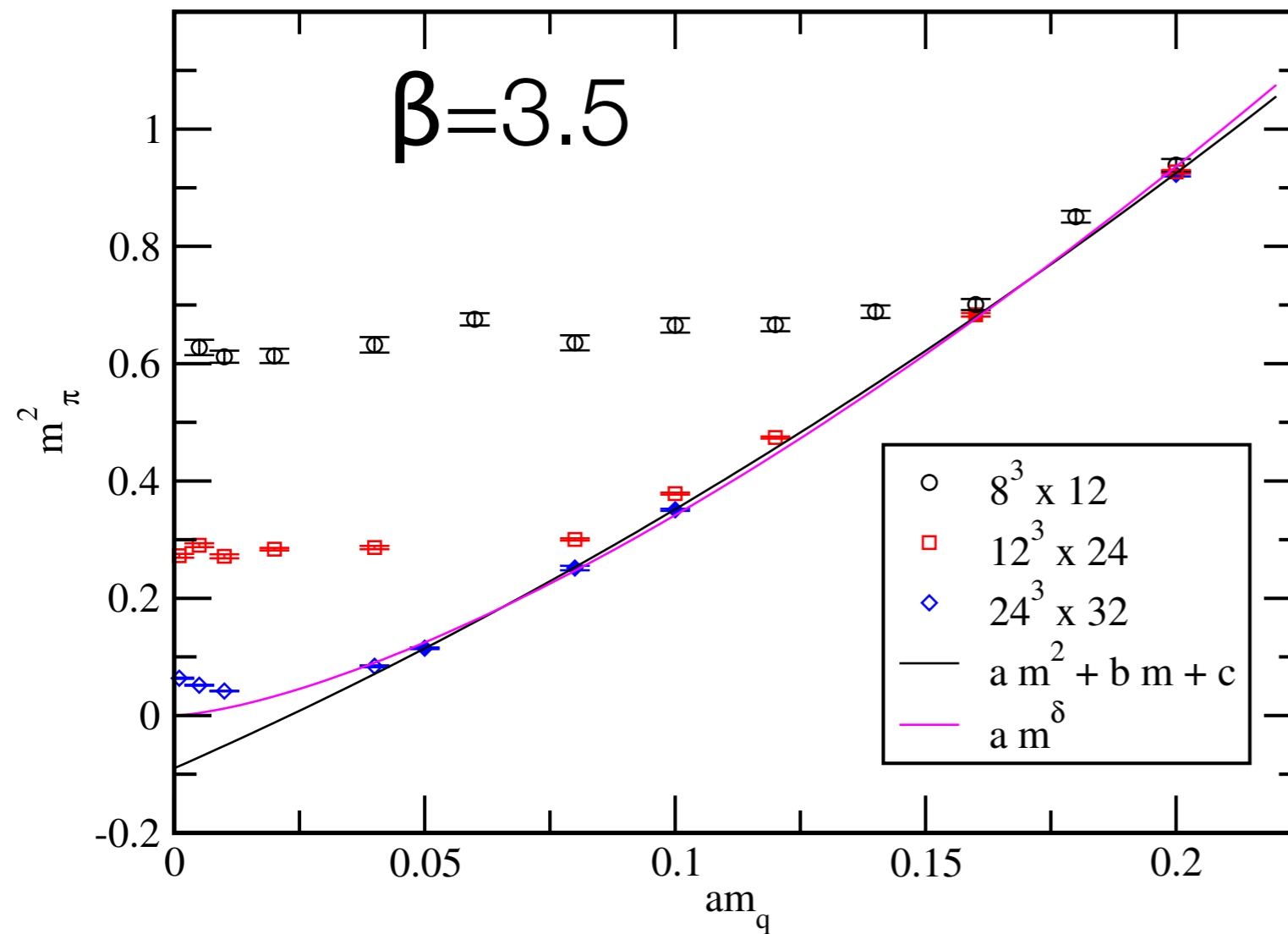
# $n_f=12$ : pion mass : fit for $\chi$ broken scenario ?



- $a m_q^2 + b m_q + c$

- $c = -0.090(5)$ ,  $\chi^2/\text{dof} = 1.1$

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  - $\delta=1.45(7)$ ,  $\chi^2/\text{dof}=32$

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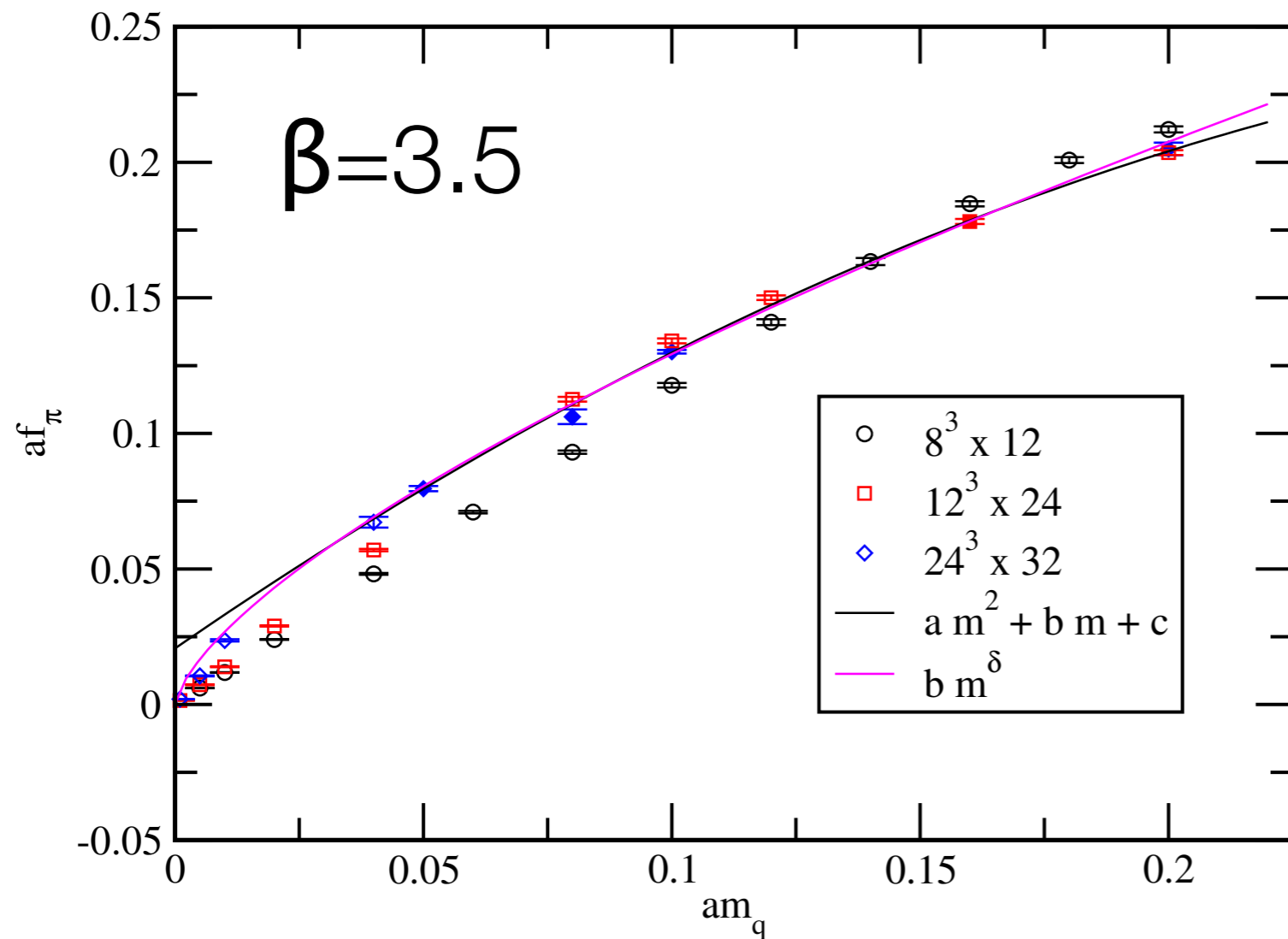
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- $a m_q^\delta$ 
  - $\delta=1.45(7)$ ,  $\chi^2/\text{dof}=32$
  - $\Rightarrow \gamma^*=0.38(7)$

# $n_f=12$ : pion decay constant



- $a m_q^2 + b m_q + c$

- $c=0.021(3), \chi^2/\text{dof}=1.7$

- $c=0 \rightarrow \chi^2/\text{dof}=17$

- $b m_q^\delta$

- $\delta=0.681(9), \chi^2/\text{dof}=2.3$

$\Rightarrow \gamma^*=0.47(2)$

# hyper scaling

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- mass deformation in a massless conformal theory: Miransky 1999.
- mass dependence is described by anomalous dimensions at IRFP
  - quark mass anomalous dimension  $\gamma^*$
  - operator anomalous dimension
- meson mass and pion decay constant obey same scaling

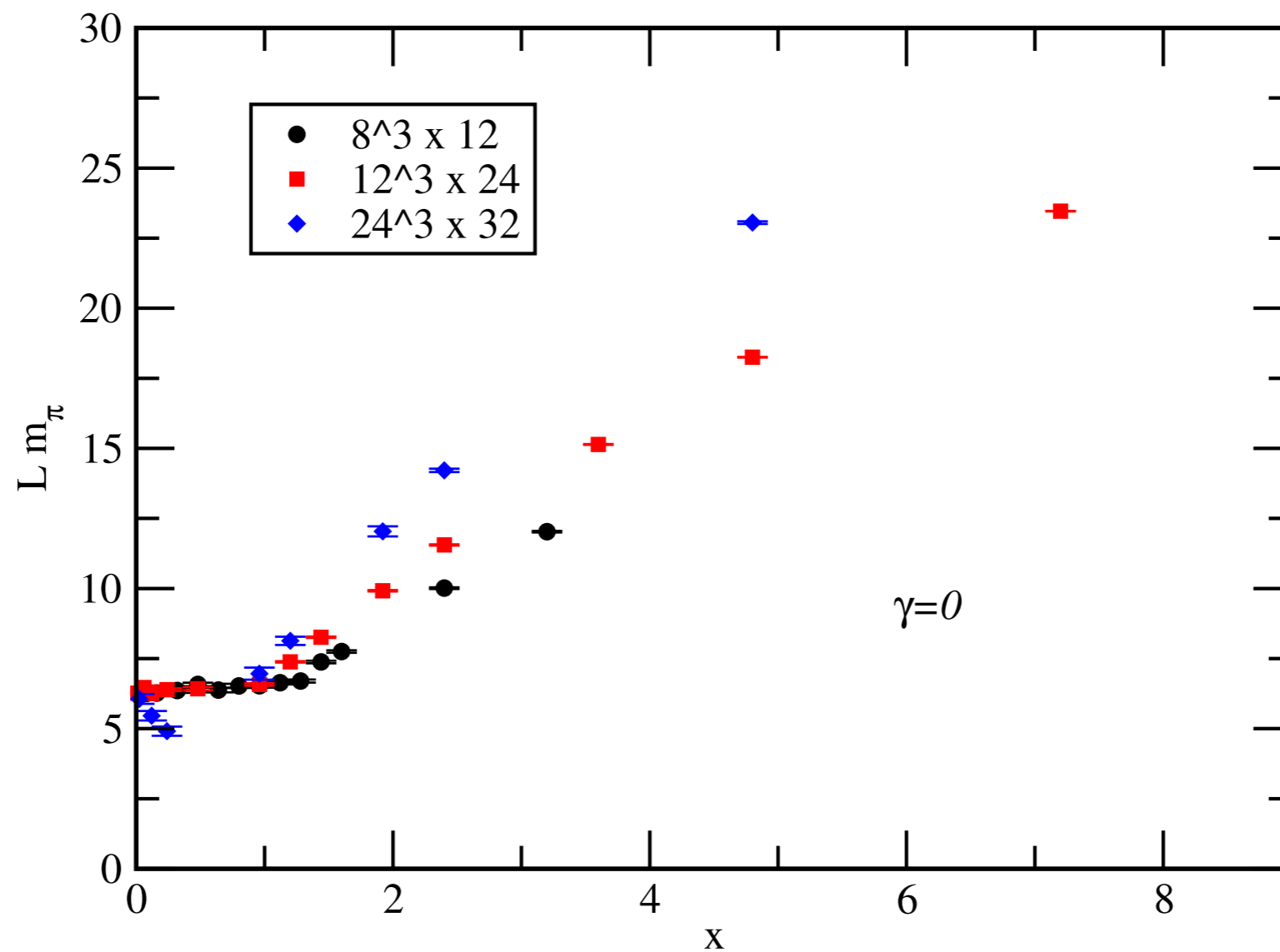
$$m_\pi = c_m m_f^{\frac{1}{1+\gamma^*}} \quad f_\pi = c_f m_f^{\frac{1}{1+\gamma^*}}$$

- finite size scaling formula (Del Debbio et al)

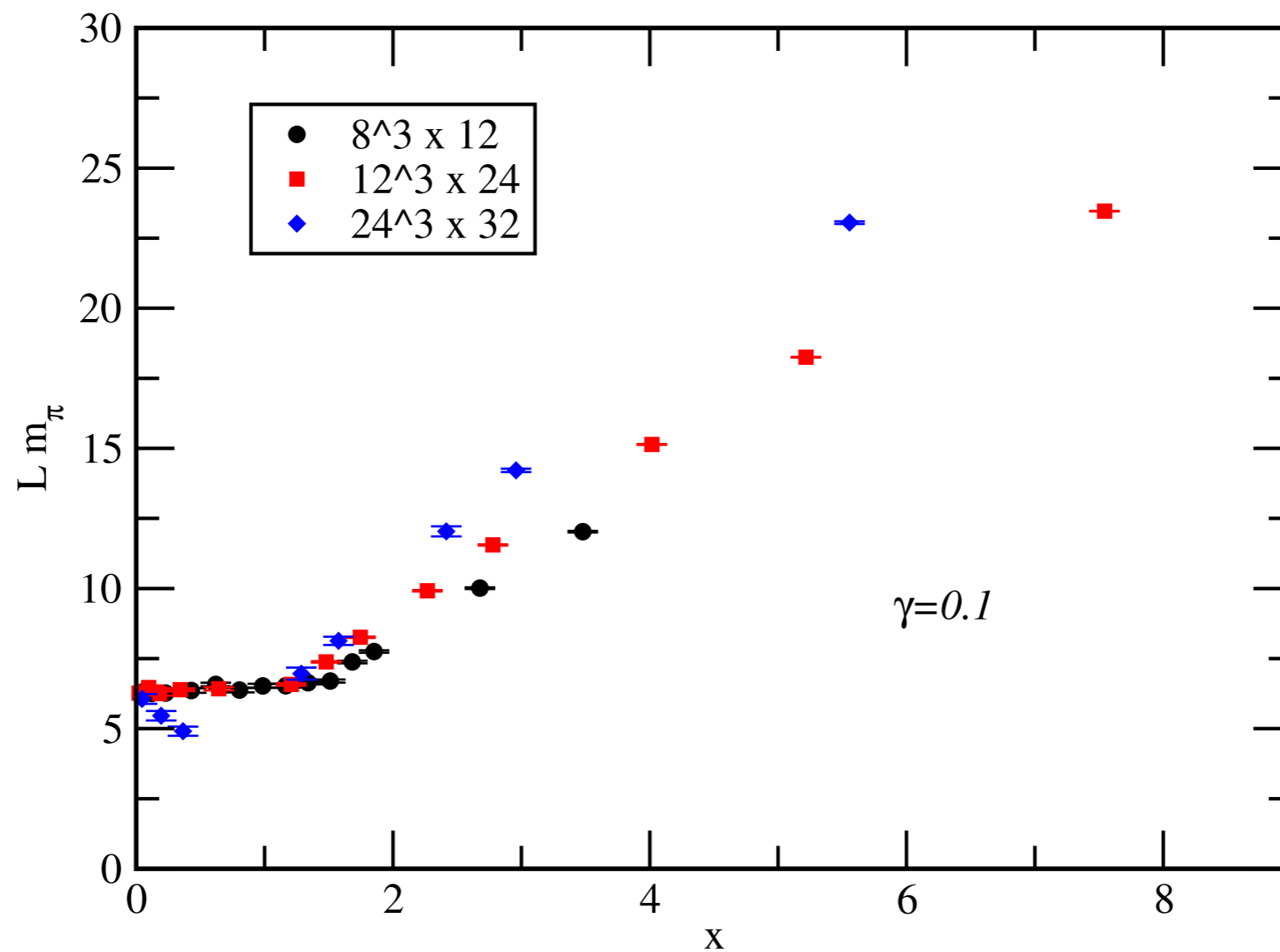
- scaling variable:  $x = L m_f^{\frac{1}{1+\gamma^*}}$

$$L f_\pi = F(x) \quad L m_\pi = G(x)$$

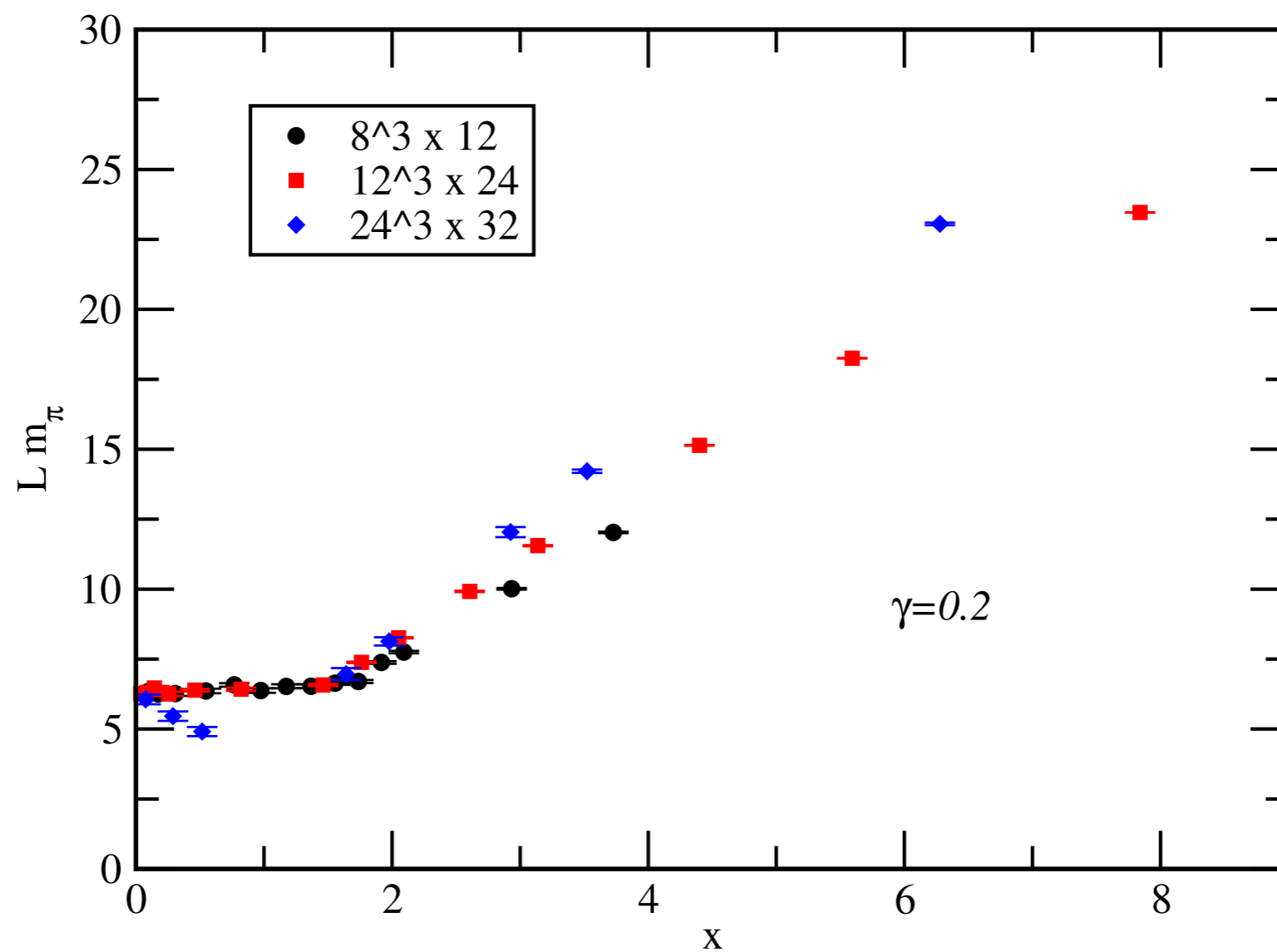
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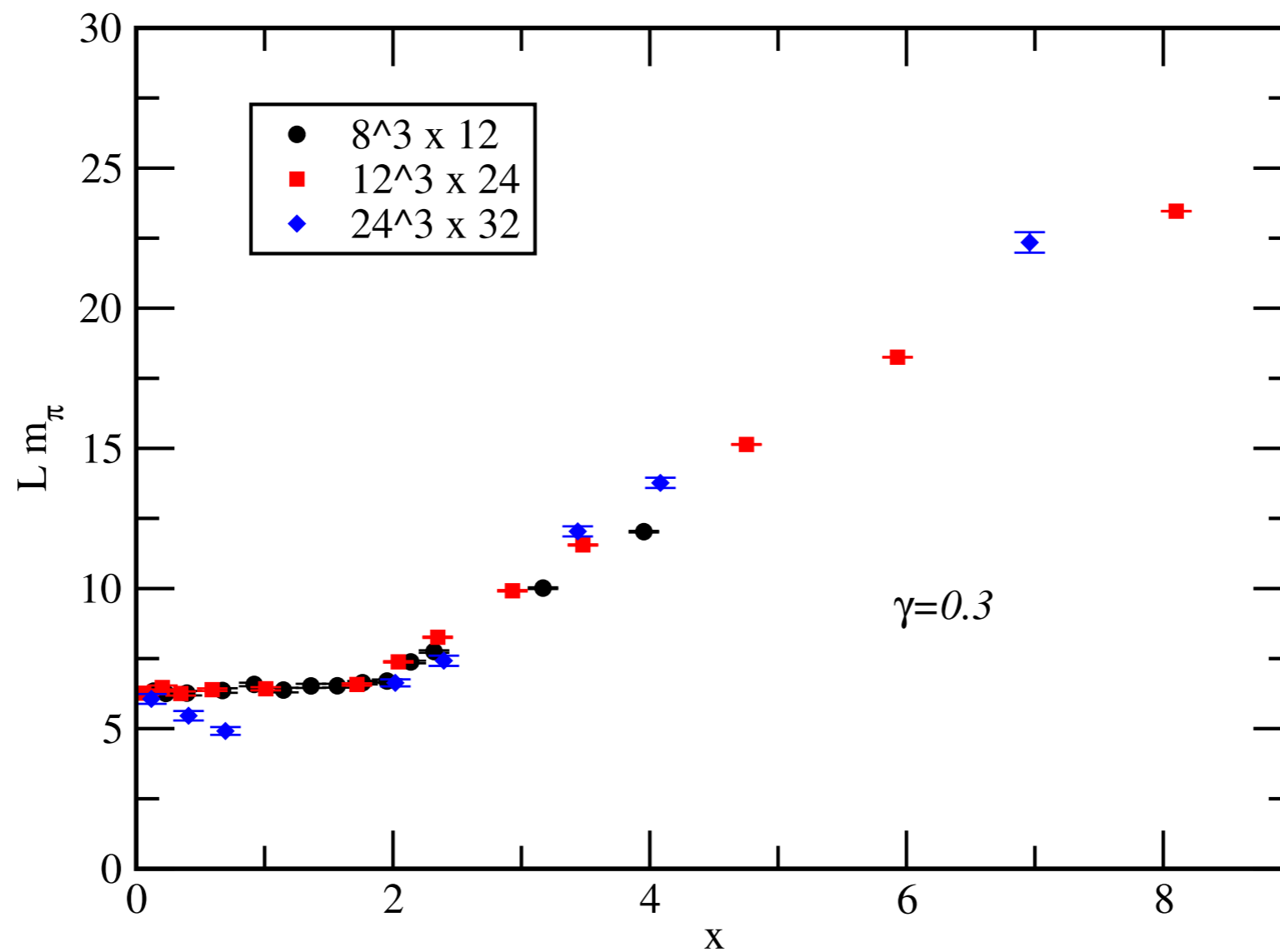


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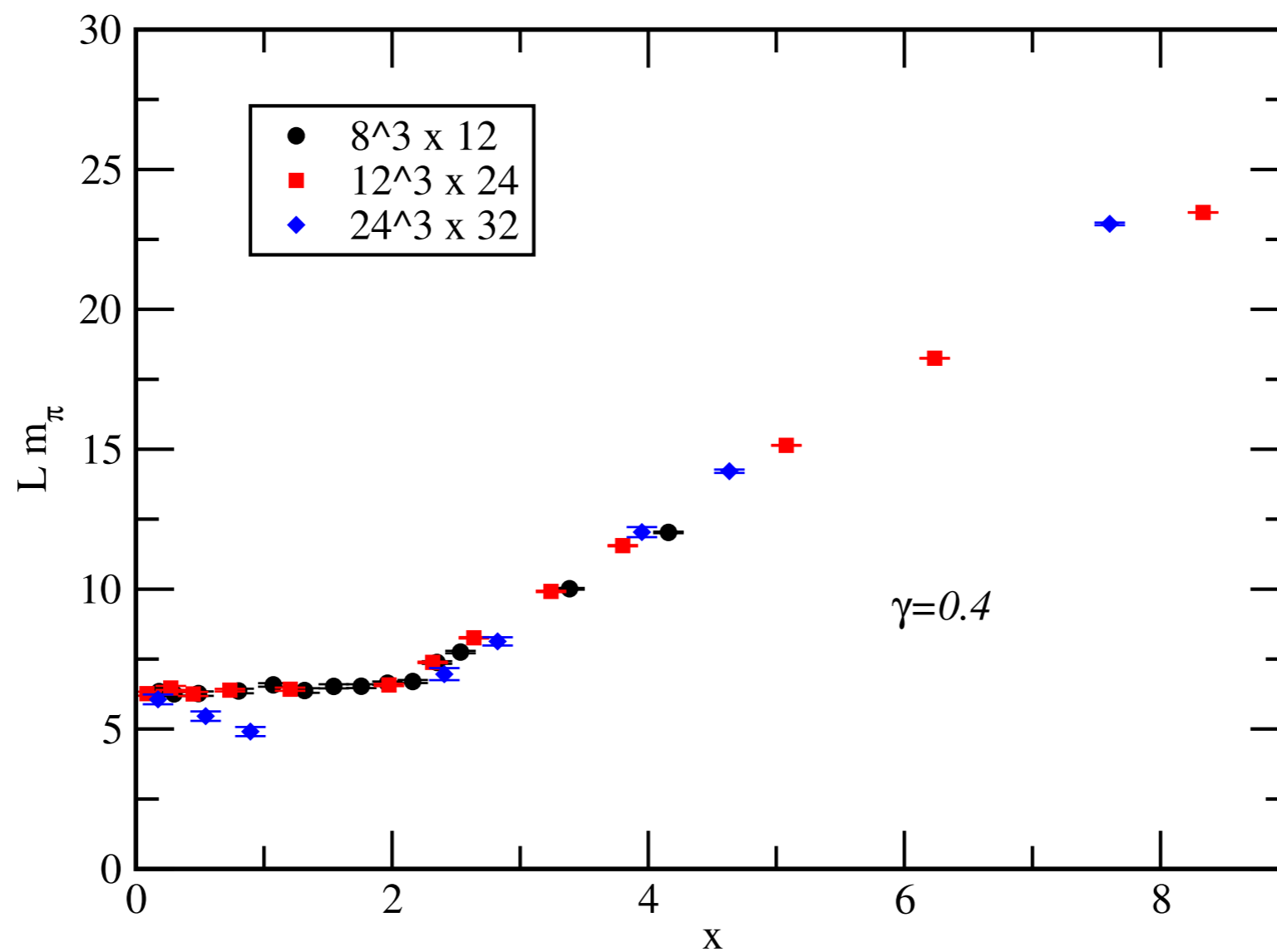




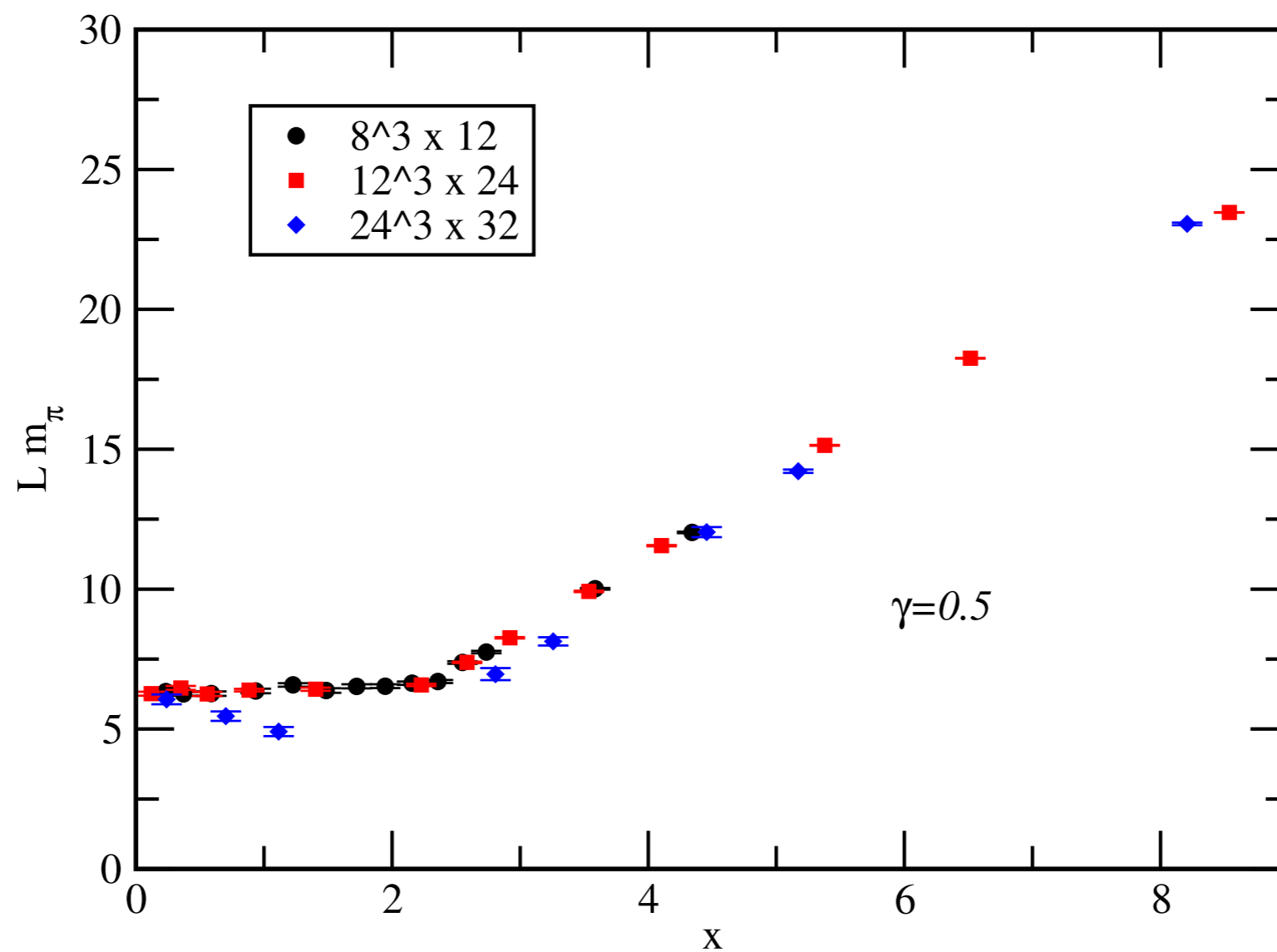
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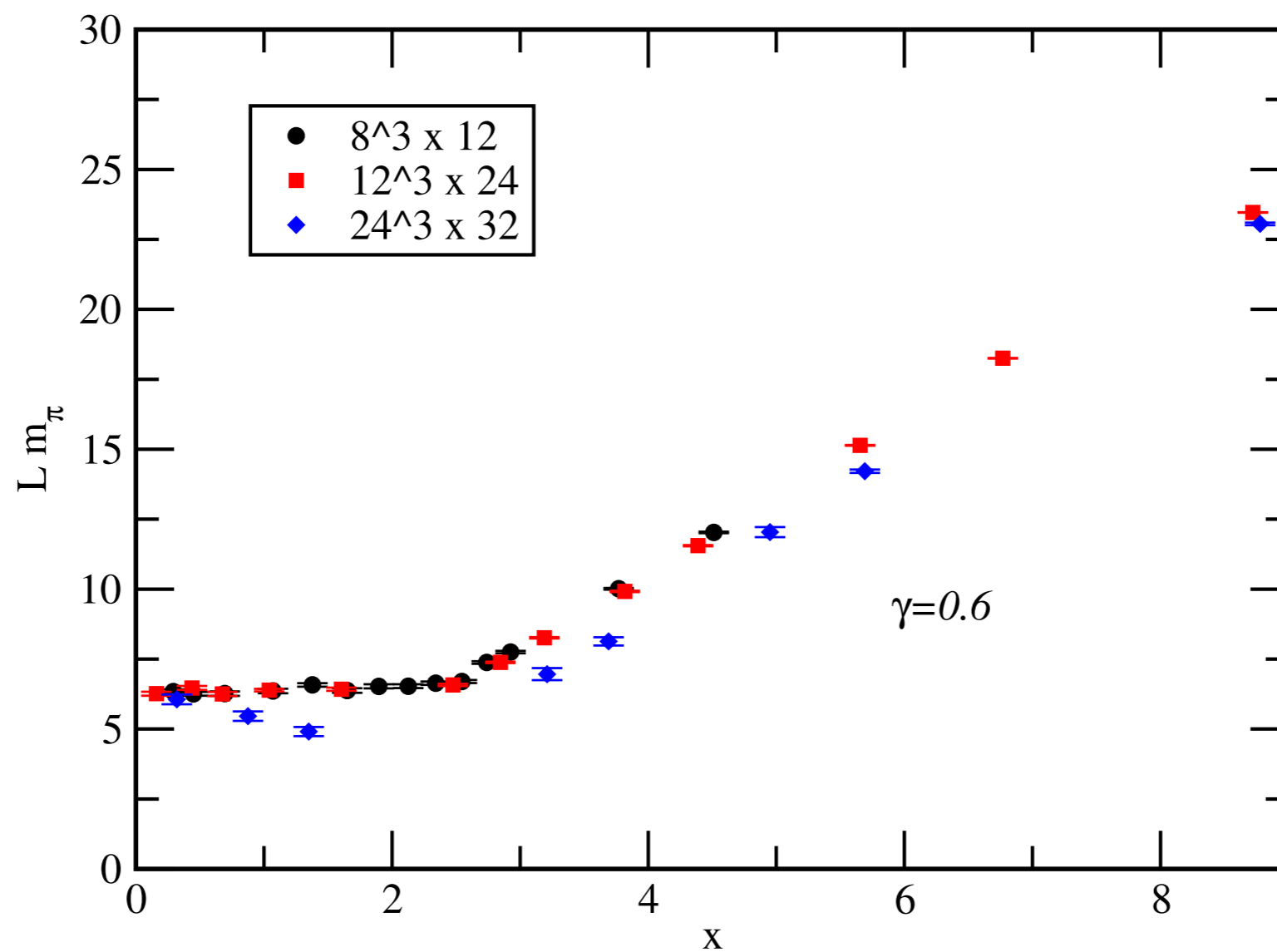
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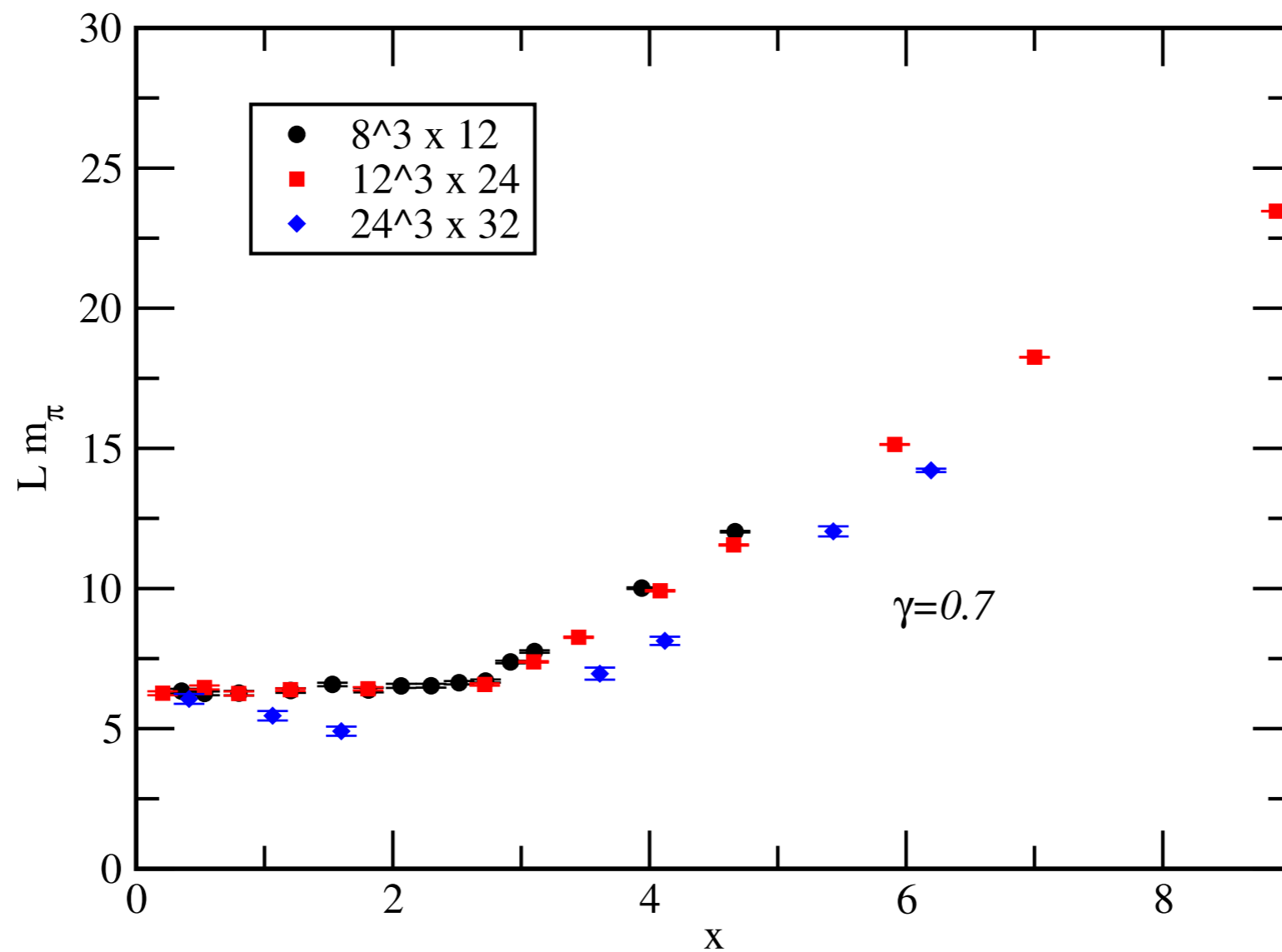
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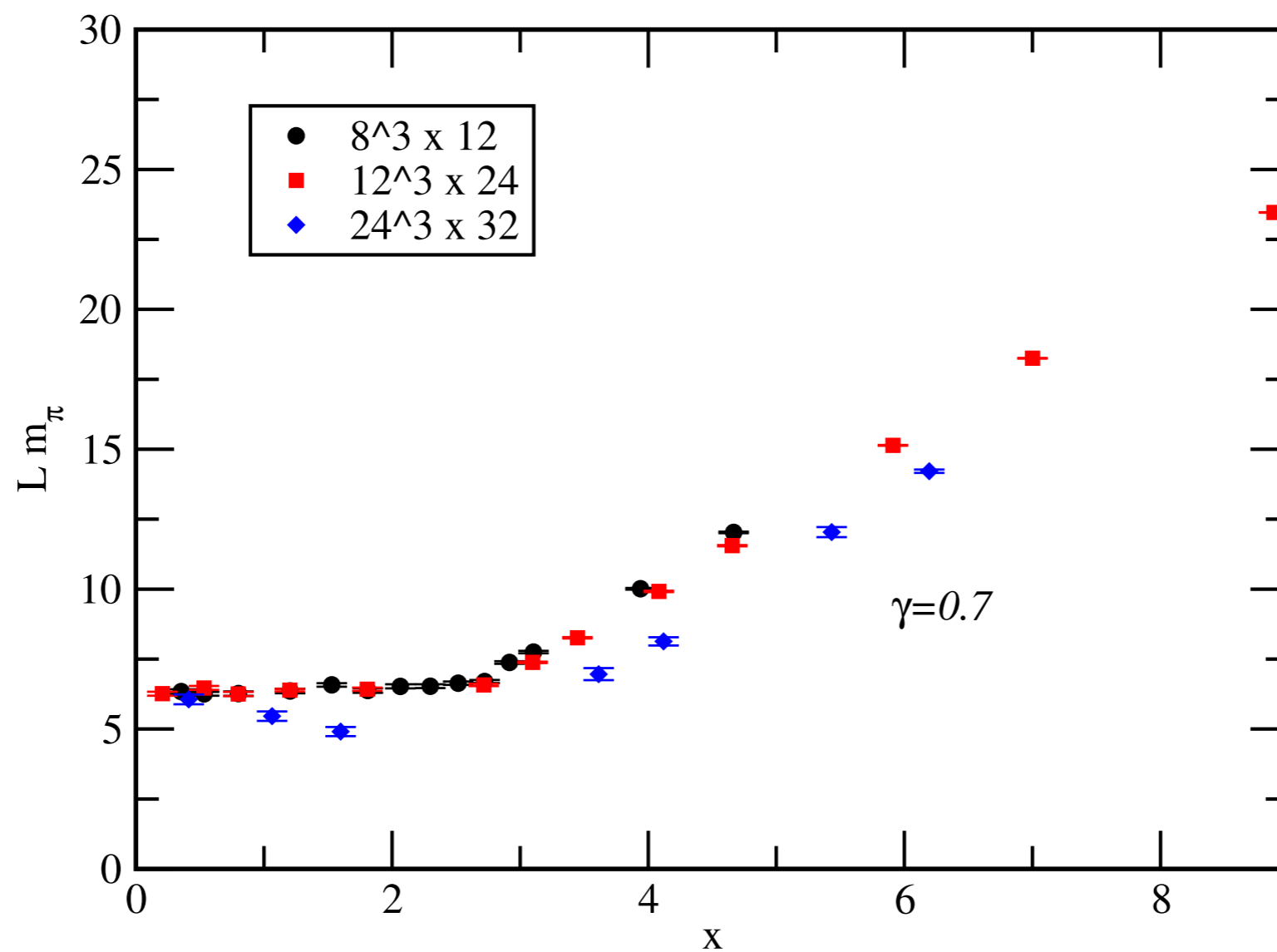
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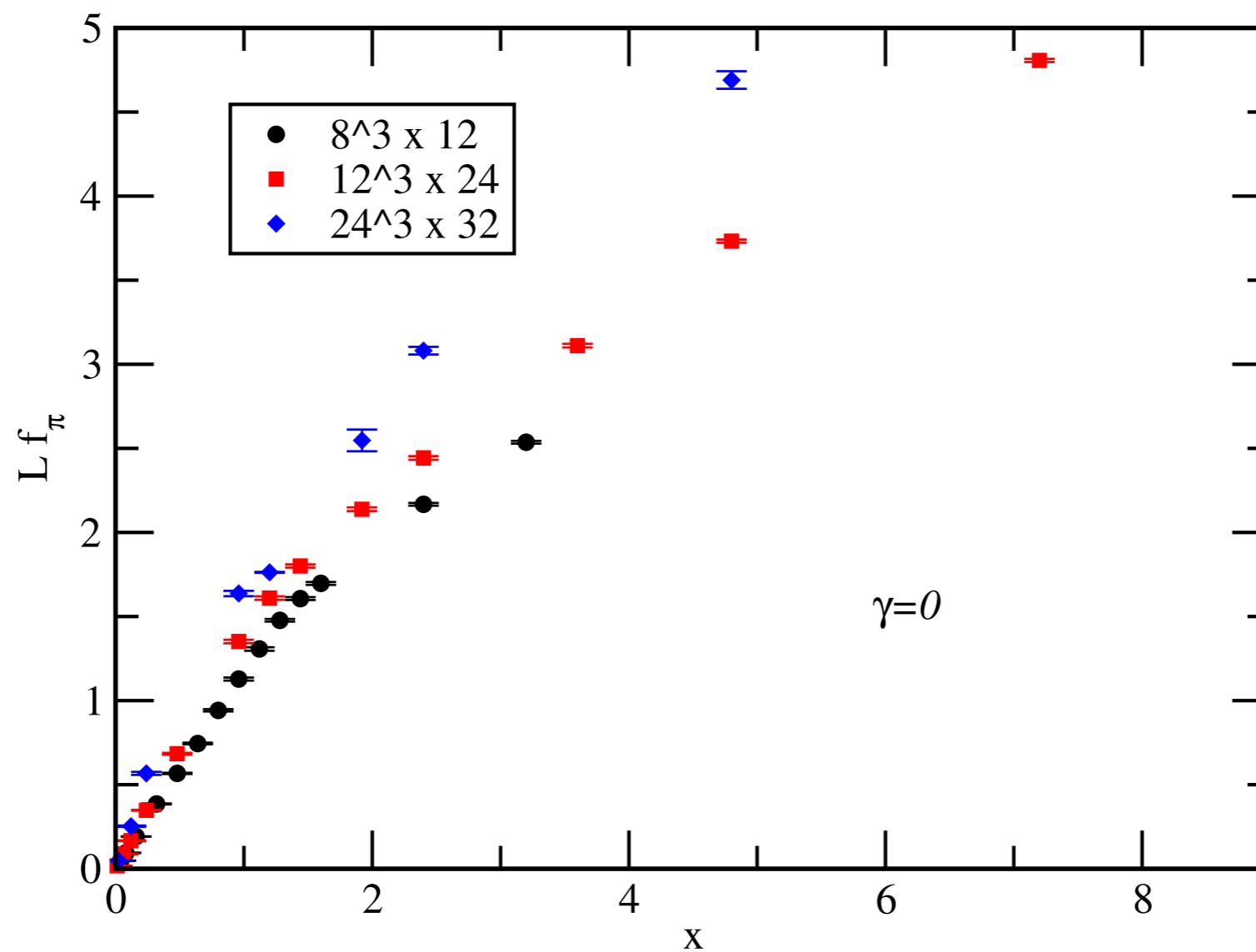


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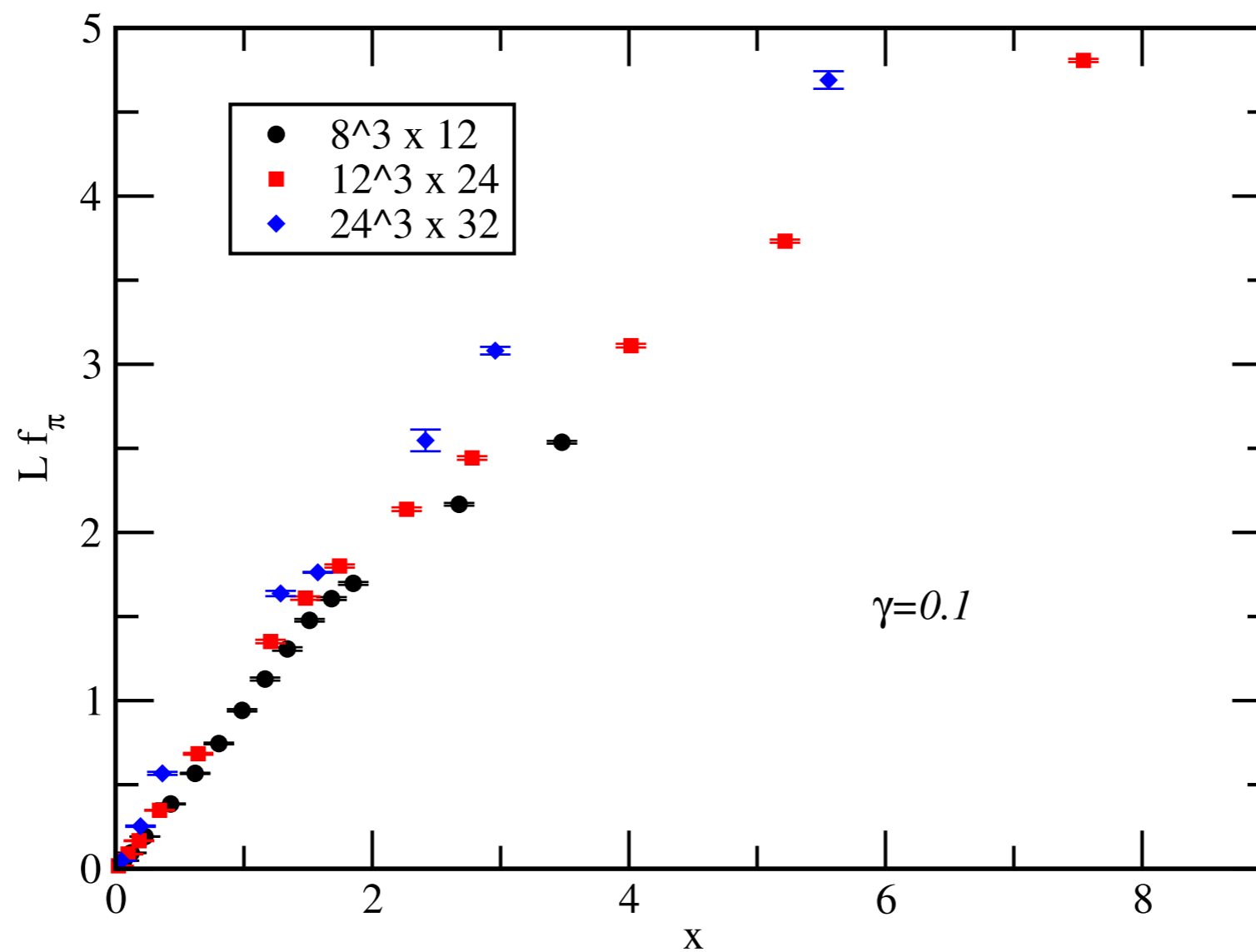


- optimal:  $\gamma^*=0.4--0.6$

# $f_\pi$ : finite size hyper scaling $N_f=12$ , $\beta=3.5$

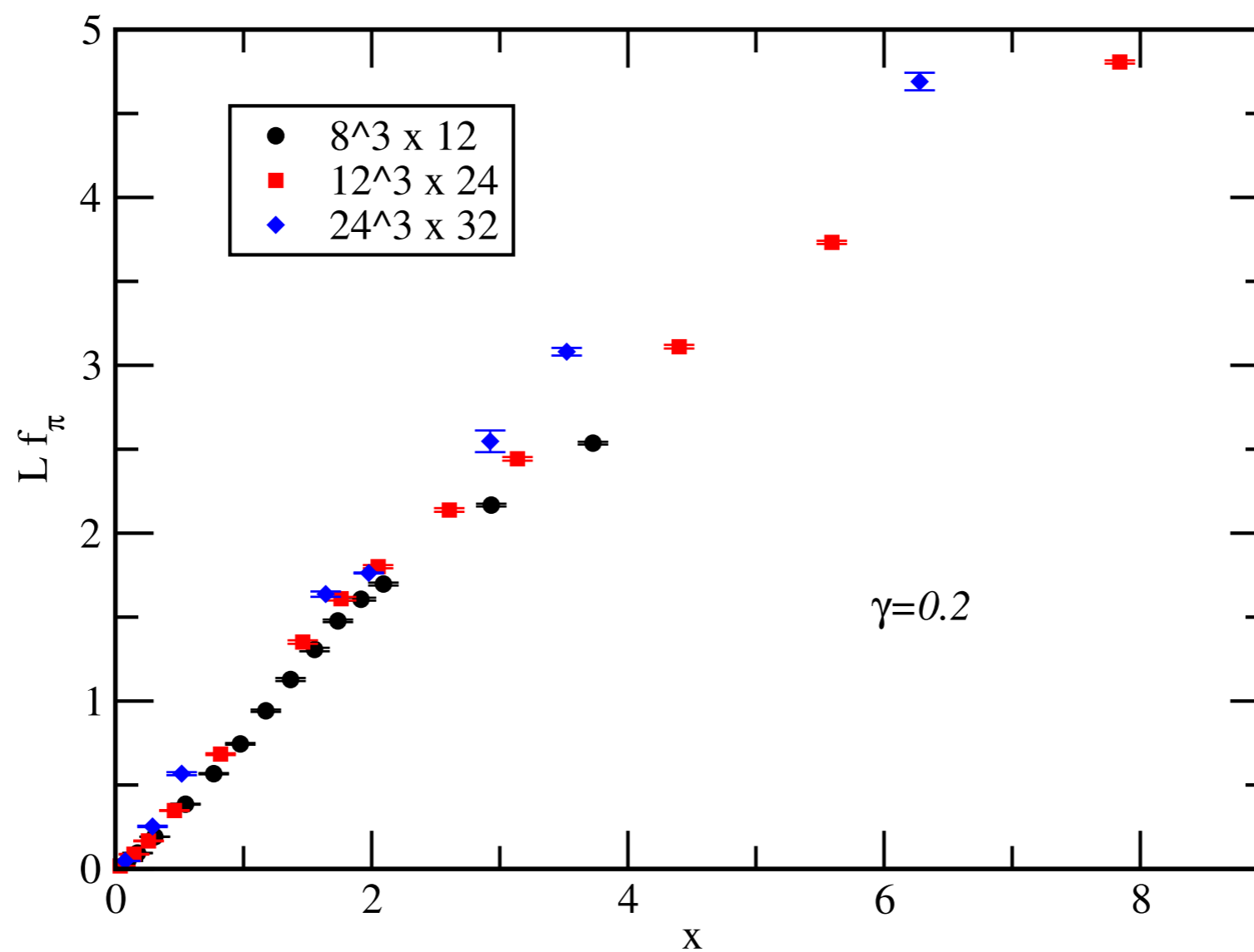


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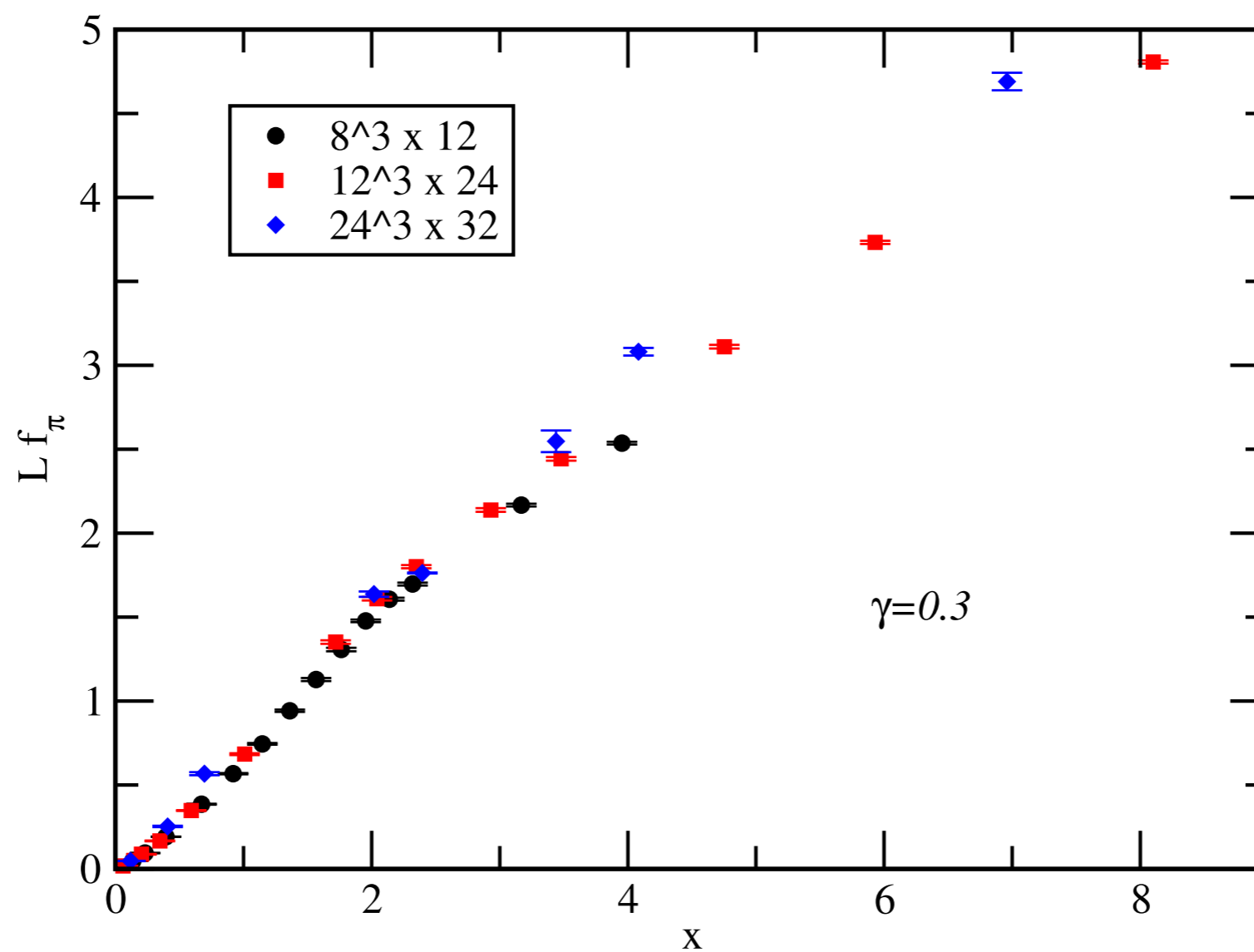




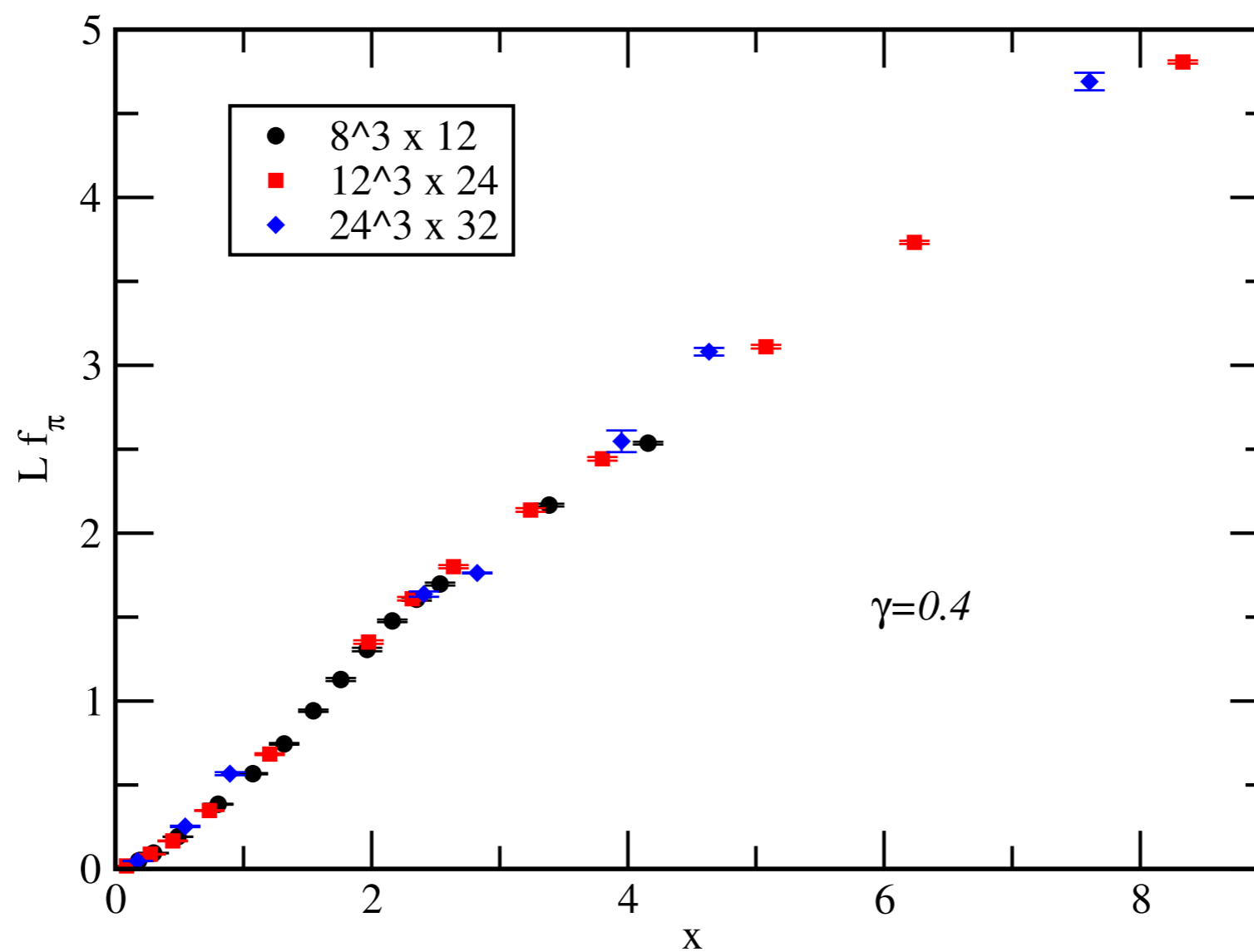
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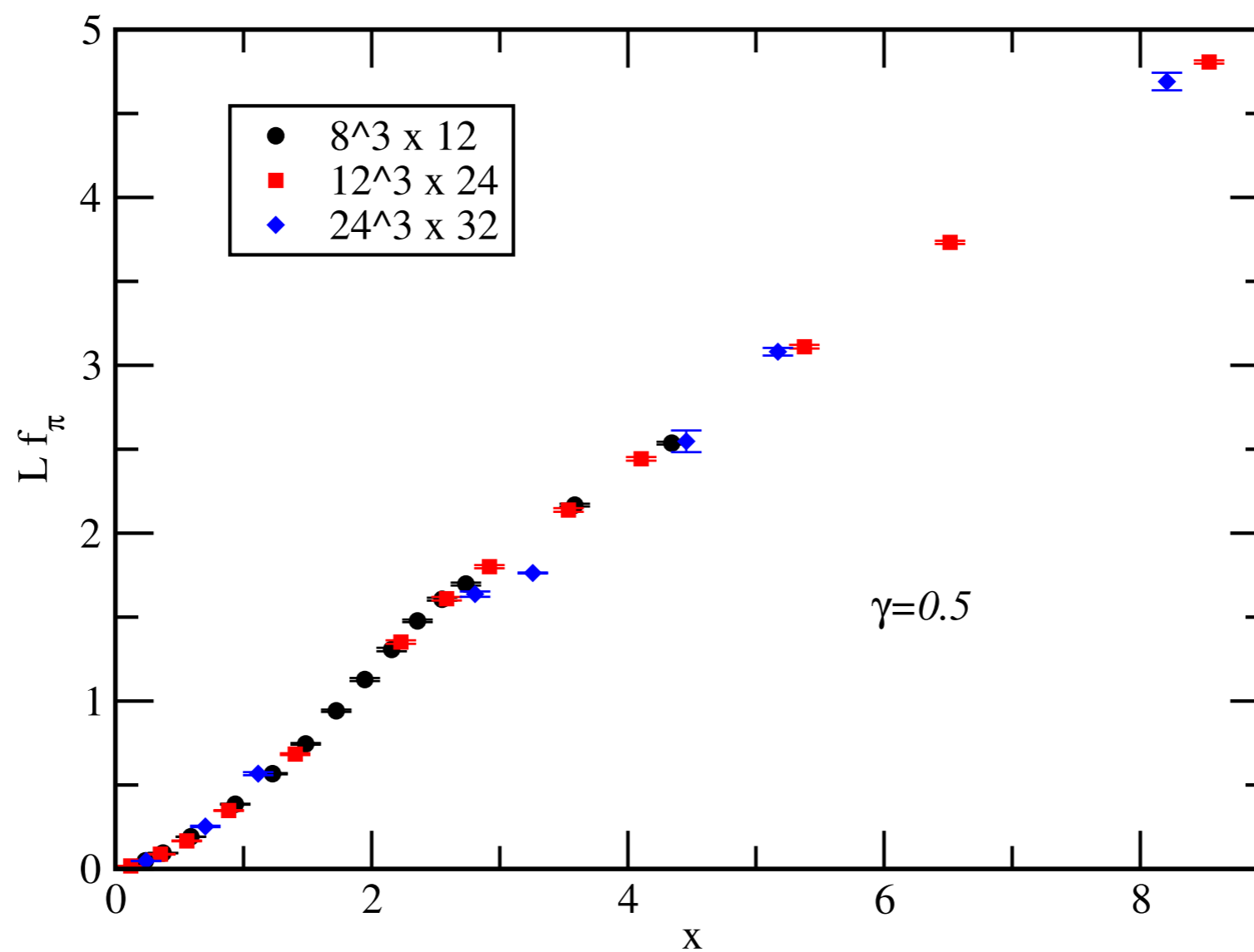
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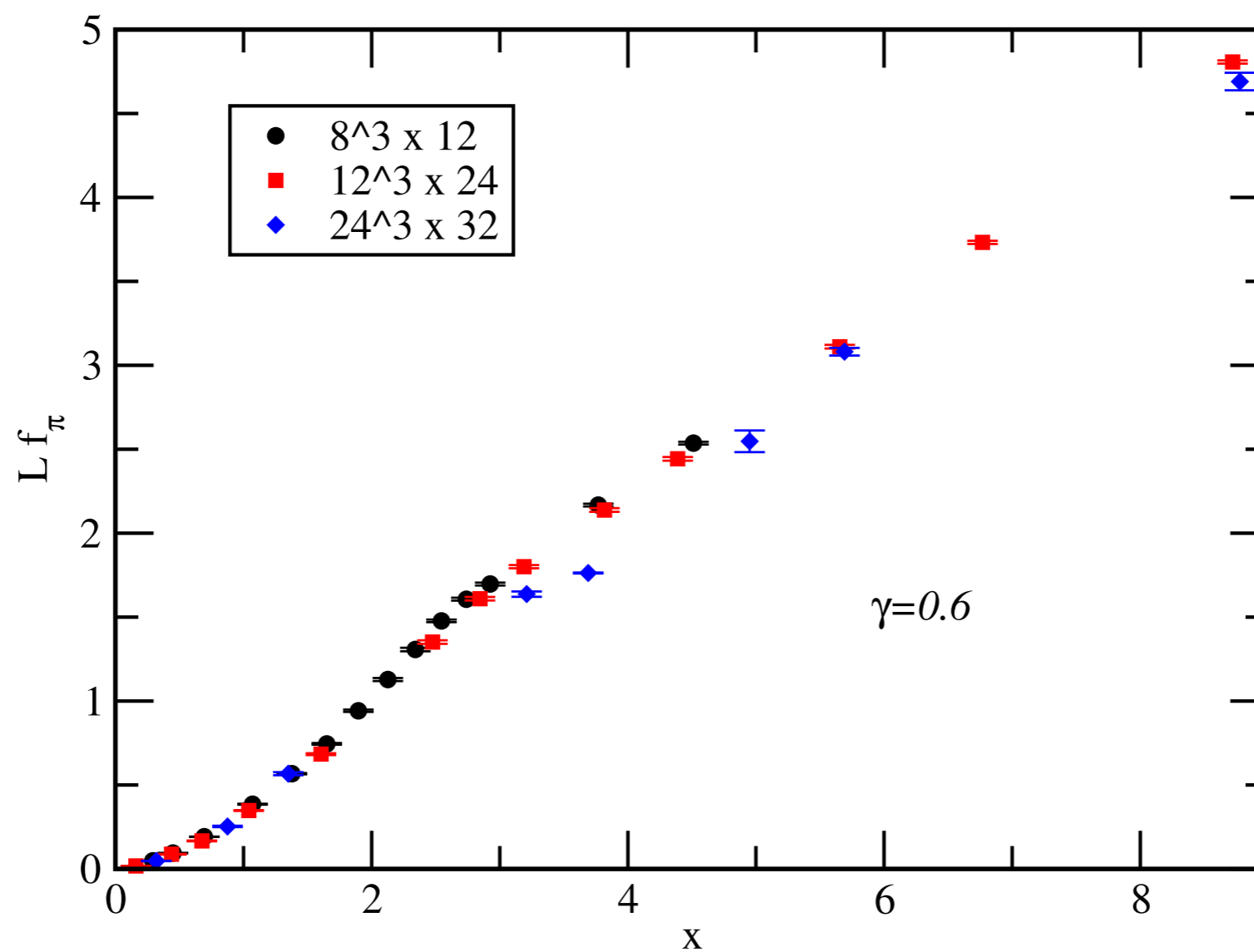
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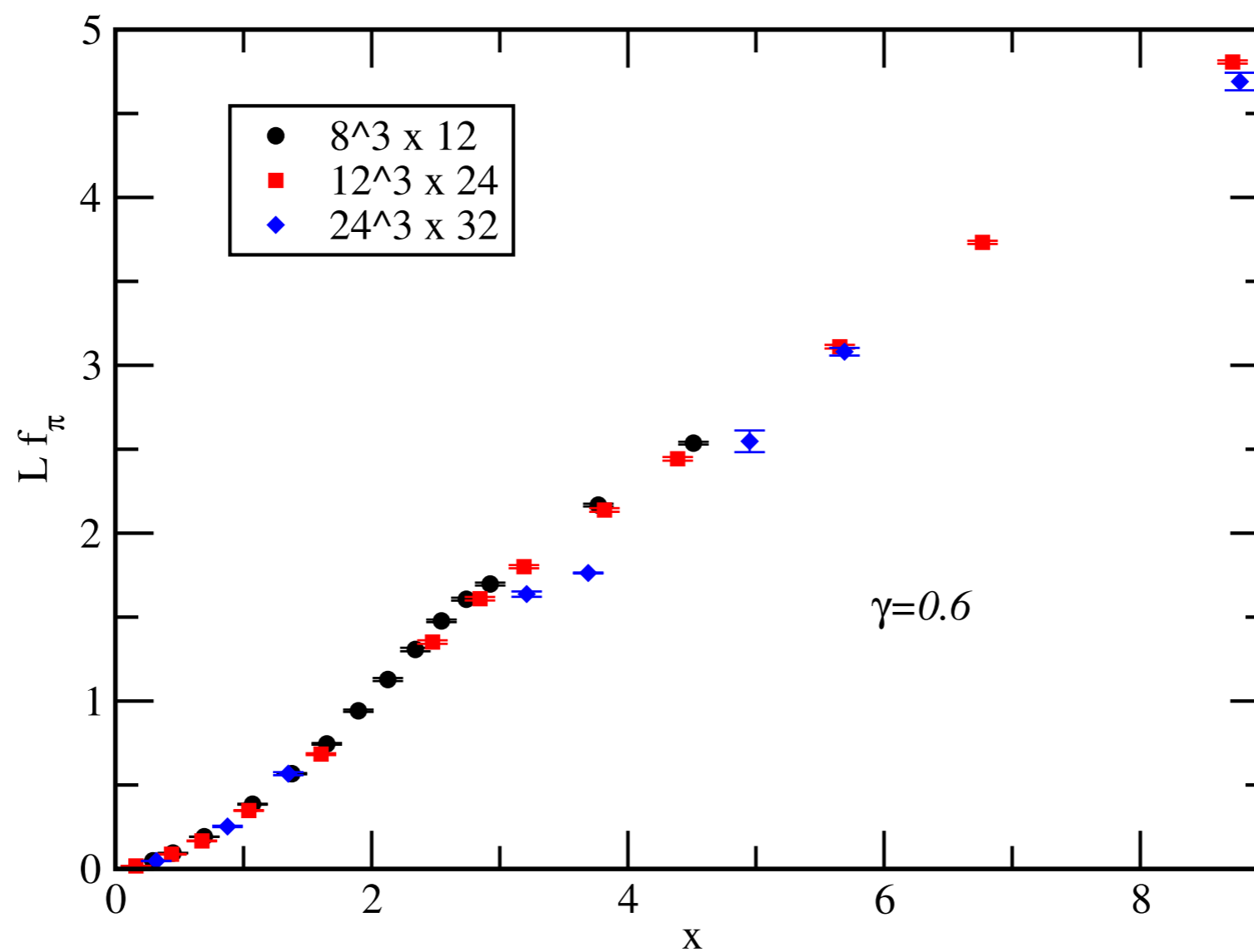
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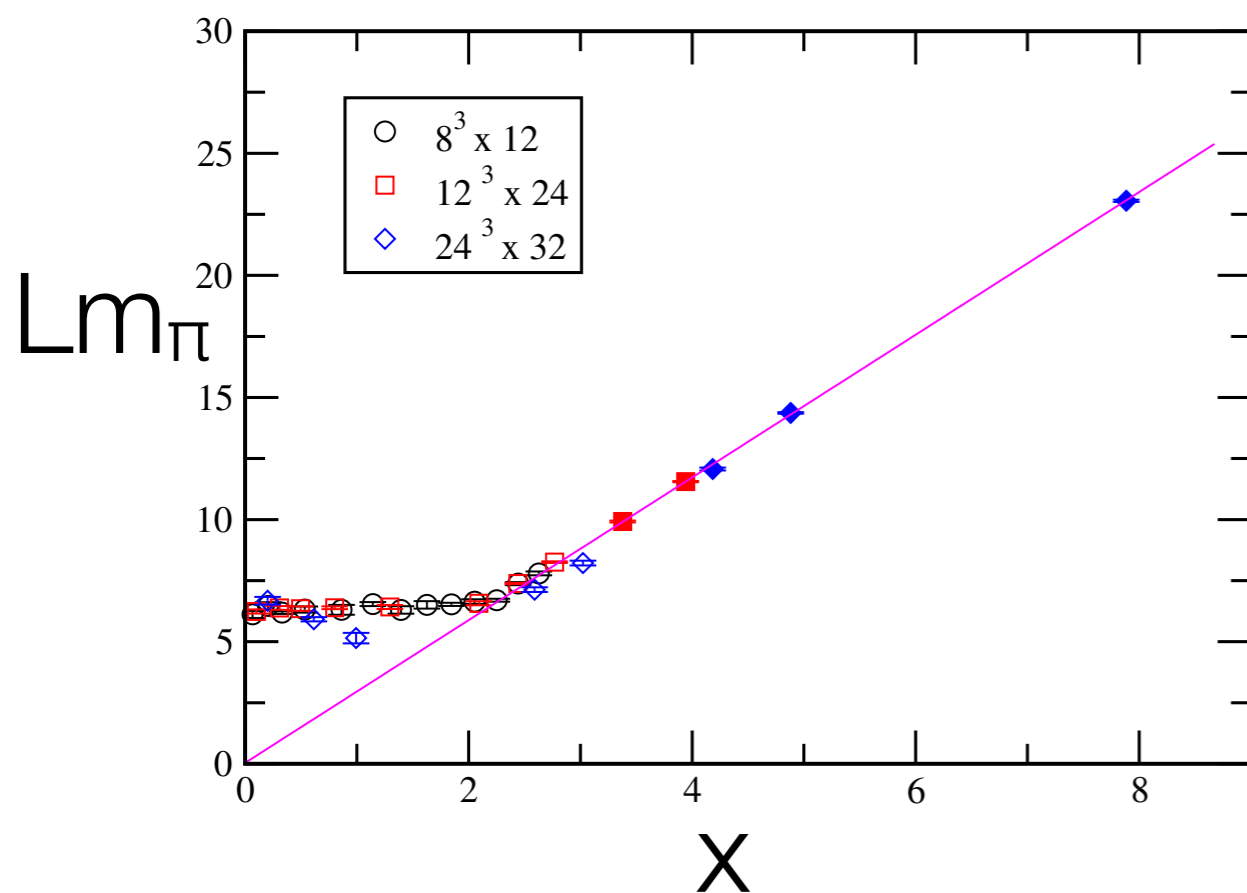
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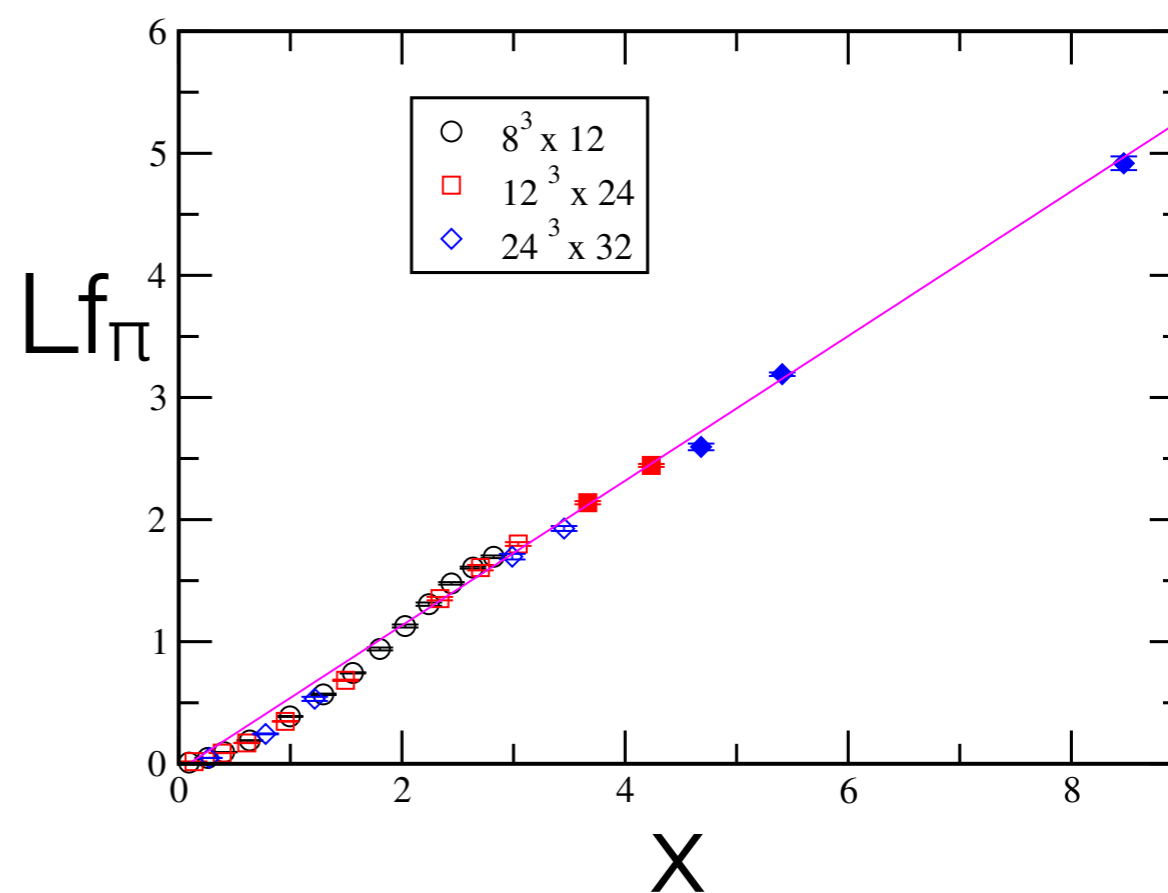
# $\gamma^*$ from a fit: $\beta=3.5$

- $y = b L m_q^{1/(1+\gamma^*)} + c$  for large  $x$  where linearity is observed



$$m_\pi: \gamma^* = 0.446(7)$$

$$\chi^2/\text{dof} = 7.2$$



$$f_\pi: \gamma^* = 0.545(2)$$

$$\chi^2/\text{dof} = 16.3$$

- errors are statistical only

# $\gamma^*$ : extending calculation towards continuum limit

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- from poster by Ohki:  $N_f=12$ 
  - $\beta=3.5$  not included due to non-uniform aspect ratio etc...
- consistent with conformal hypothesis

<b>beta</b>	<b>gamma* (<math>m_\pi</math>)</b>	<b>gamma* (<math>f_\pi</math>)</b>
3.7	0.44(1)	0.44(3)
4.0	0.39(1)	0.40(2)

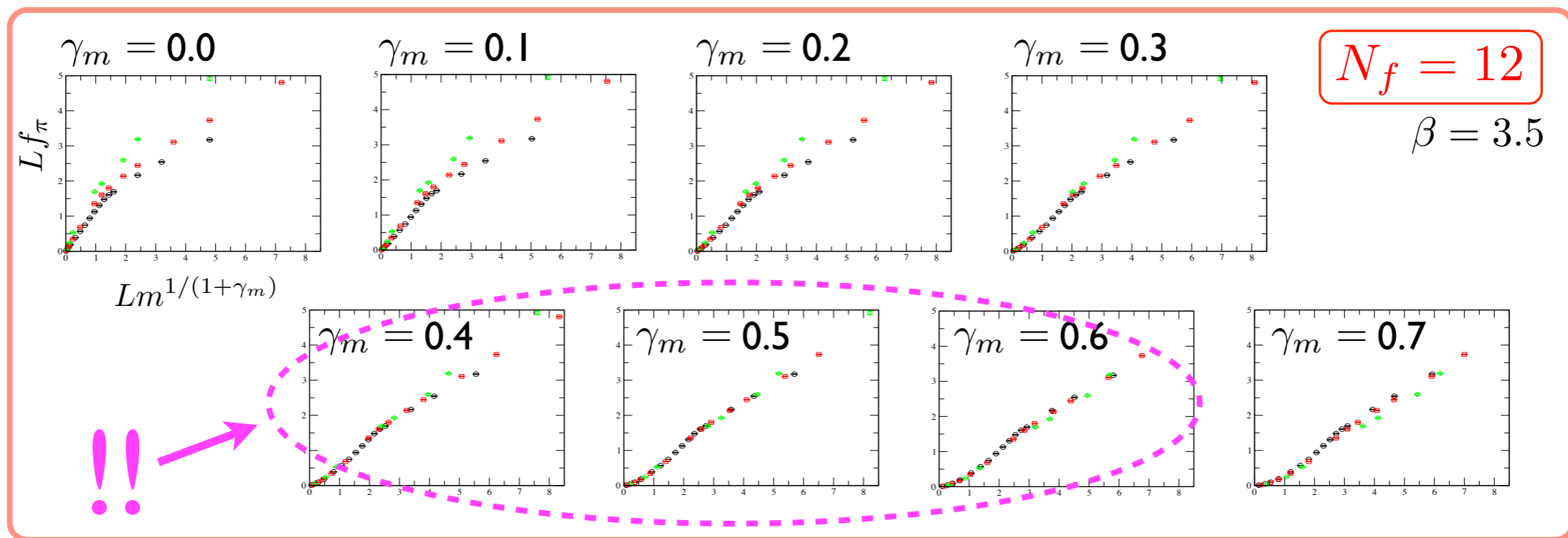
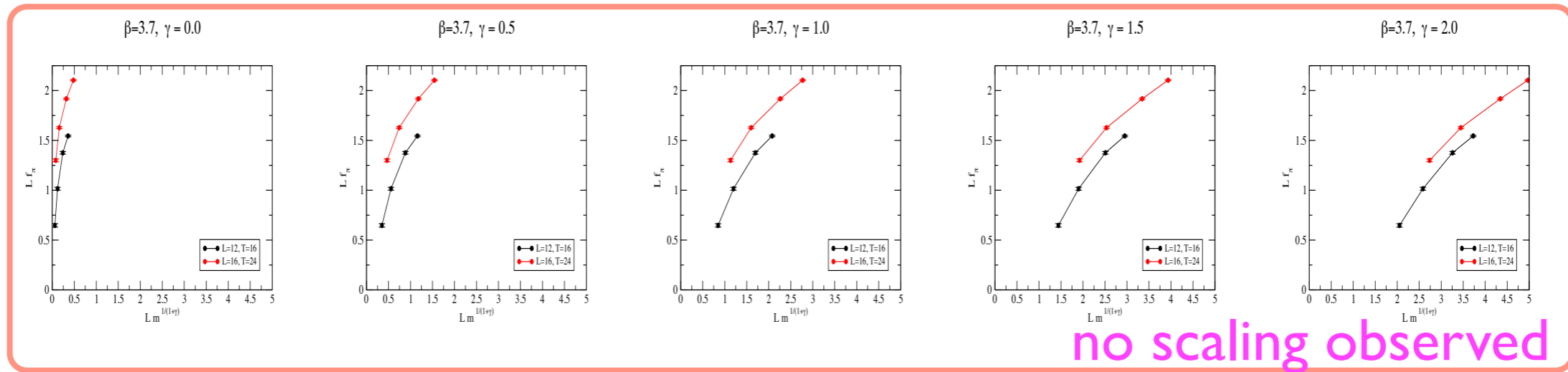
- errors are statistical only
- consistency between:  $m_\pi$  and  $f_\pi$
- tends to decrease towards the continuum limit, BUT, it could be
  - due to lattice artifact (UV), reduced physical volume (IR) or other sys err.?



# $N_f=4$ from poster by Kurachi

## Lattice results

$N_f = 4$

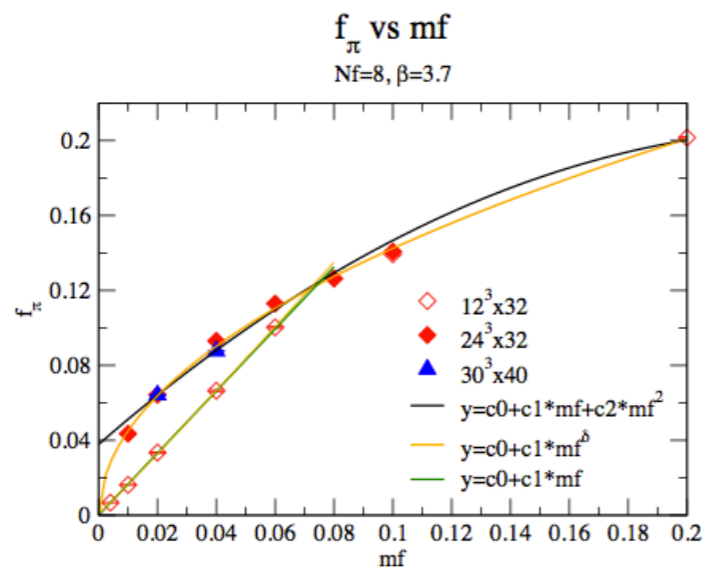
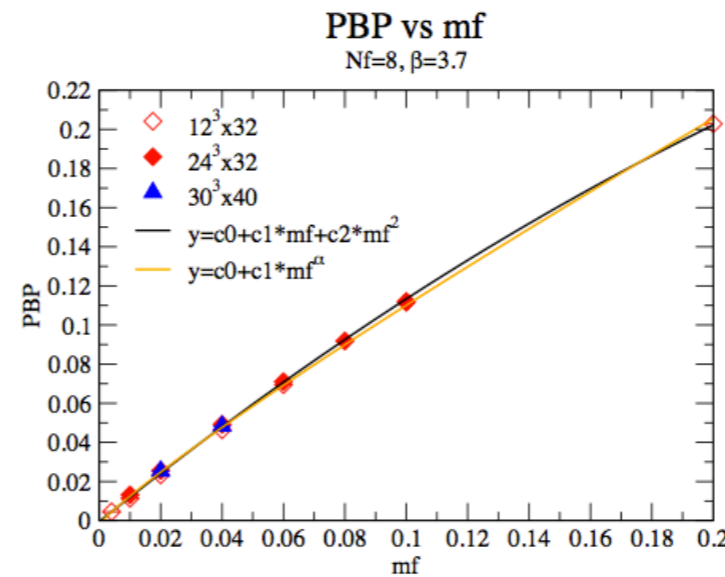
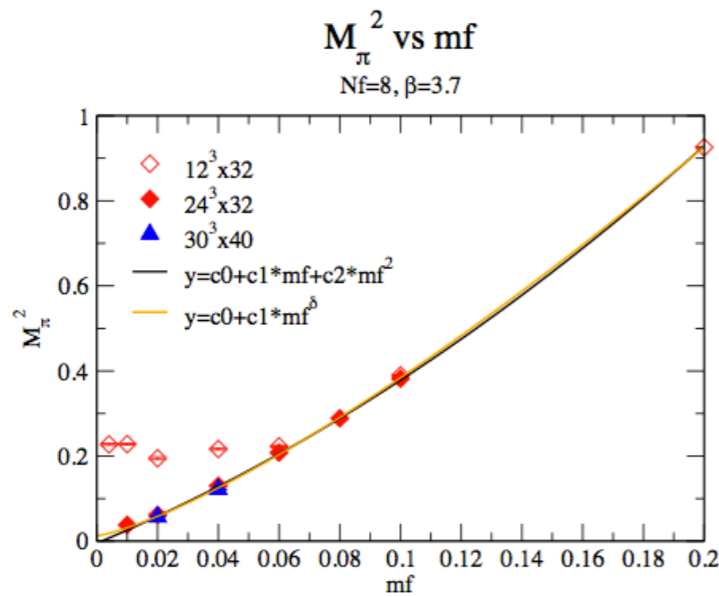


# $N_f=8$ from poster by Nagai

## ChPT analysis in $N_f=8$

➔  $\chi$  SB phase, analyzed by ChPT ??

5



• Quadratic fit:  
 $y=c_0+c_1*mf+c_2*mf^2$

• Power fit:  
(conformal-like)  
 $y=c_0+c_1*mf^\delta$

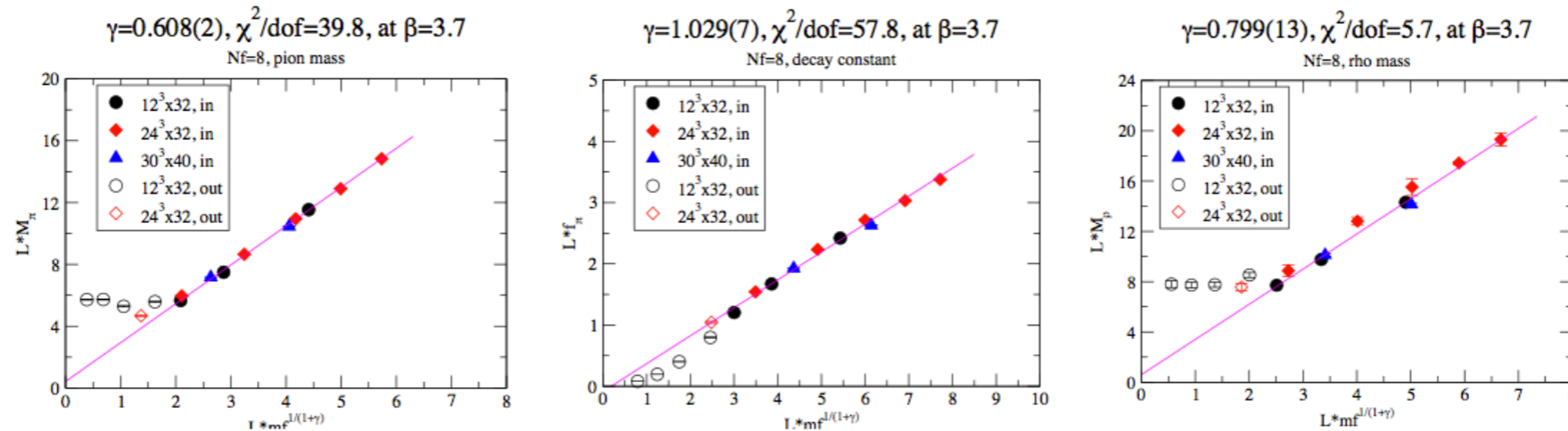
It's difficult to conclude  
that  $N_f=8$  is in the hadron phase.

➔ Conformal analysis

# $N_f=8$ from poster by Nagai

## Fit result of the hyperscaling in the conformal hypothesis for $N_f=8$

7



	$\beta = 3.6$	$\beta = 3.7$	$\beta = 3.8$	$\beta = 3.9$
$\gamma$ in $M_\pi$	0.608(2)	0.607(3)	0.563(3)	0.757(14)
$\gamma$ in $M_\rho$	0.766(40)	0.799(13)	0.862(59)	1.18(32)
$\gamma$ in $f_\pi$	1.02(1)	1.03(1)	0.98(1)	1.13(3)

Table:

It seems not to be simple hadronic phase:  $\gamma \neq 1$ . *c.f.*  $N_f=4$  case.

$\gamma \approx 1$ , walking ??

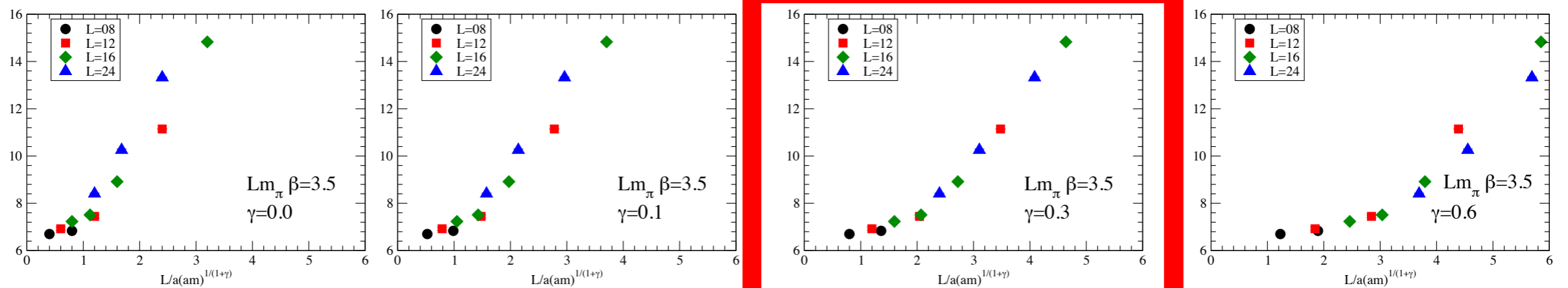
♠  $N_f=8$  shows the good behavior of the hyperscaling.

♠ Still,  $\gamma(M_\pi) < \gamma(M_\rho) < \gamma(f_\pi) \rightarrow$  not exact Conformal ??

# $N_f=16$ from poster by Yamazaki

## 5. Results of mass and finite size deformed case

Changein  $\gamma_*$  of  $Lm_\pi$  vs  $Lm_f^{\frac{1}{1+\gamma_*}}$  ( $\beta = 3.50$ )

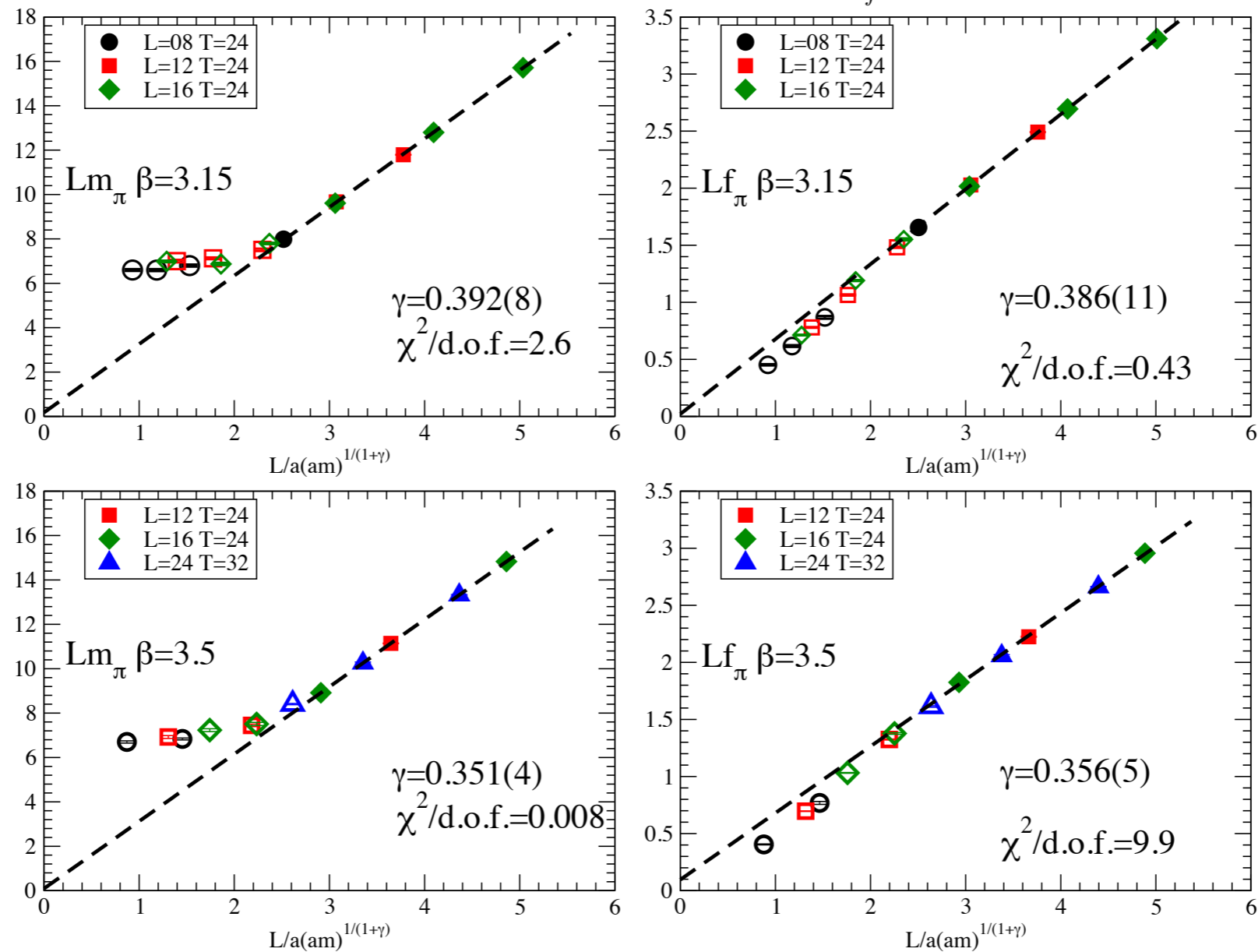


$\gamma_* \sim 0.3$  gives a nice scaling at larger value of  $x$ -axis.

$\frac{1}{1+\gamma_*}$

# $N_f=16$ from poster by Yamazaki

$Lm_\pi$  and  $Lf_\pi$  fit with asymptotic form :  $Lm_\pi = c_0 + c_1x$ ,  $x = Lm_f^{\frac{1}{1+\gamma_*}}$



$\gamma_*$  at  $\beta = 3.50$  is consistent with the one of mass deformed case.

Two  $\gamma_*$  from different observables reasonably agree with each other at both  $\beta$ .

However,  $\gamma_*$  at both  $\beta$  is much larger than the perturbative result,  $\gamma_*^{\text{pert}} \sim 0.015$ .

# summary

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- large  $N_f$  SU(3) gauge theory with fundamental rep. is being investigated
  - quest for the walking technicolor
- using a HISQ type fermion and the tree-level Symanzik gauge action
- aiming to explore a wide range of the  $N_f$  systematically
- This talk mainly described  $N_f=12$  study
  - three lattice spacings ( $\beta=6/g^2$ ) studied, with spatial size up to  $L_s=30$
  - pion mass and decay constant are studied
  - approximate finite size scaling for conformal scenario is observed
  - with the current lattice volume, results favor conformal theory
  - assuming an IR fixed point, mass anomalous dimension calculated
    - $\gamma^* \sim 0.4$

# comparison to other works on $N_f=12$ SU(3)

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collaboration	conclusion	method	remarks
Fodor et al	$\chi$ Broken	spectrum	big V, single lat.spgs.
Columbia	$\chi$ Broken	spectrum, $T_c$	naive KS
Deutchman et al	Conformal	spectrum, $T_c$	KS+Naik
Itou et al	Conformal	coupling	naive KS + cont.lim.
Appelquist et al	Conformal	coupling	non-exact algorithm
Appelquist et al	Conformal	spectrum	using Fodor's data
DeGrand	consistent with Conformal	spectrum	using Fodor's data
KMI	consistent with Conformal	spectrum	HSIQ, 3 lat.spgs.

# summary (continued)

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- $N_f=4$ 
  - clearly in  $\chi$  broken phase
  - finite size hyper scaling not observed
- $N_f=8$ 
  - more study needed for definite conclusion
- $N_f=12$ 
  - results are consistent with conformal hypothesis
- $N_f=16$ 
  - consistent with conformal, but with large anomalous dimension
  - study with weaker coupling necessary



# outlook

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- to meet our goals
- for  $N_f=8, 12$ 
  - larger size than  $L_s=30$  is needed to investigate further IR regime
    - make it possible to study lighter mass
  - glueball mass to check hyper scaling
  - masses for other mesons, baryons, flavor singlets: to check hyper scaling
- for  $N_f=16$ 
  - much weaker coupling



Thank you for your attention