Confinement/deconfinement phase transition in SU(3) Yang-Mills theory in view of dual superconductivity

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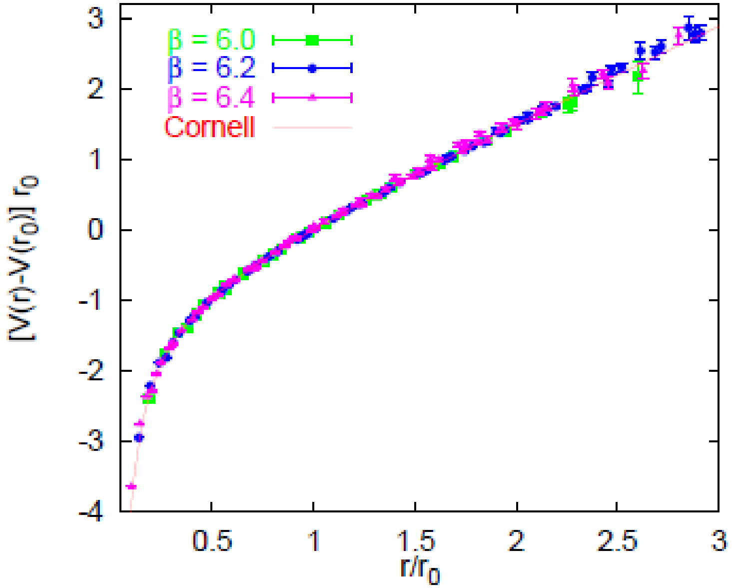
Introduction

- Quark Confinement follows from the area law of the Wilson loop average. [Wilson,1974]

Non-Abelian Wilson loop $\left\langle \mathrm{tr}\left[\mathscr{P}\exp\left\{ig\oint_C dx^\mu \mathscr{A}_\mu(x)\right\}\right]\right\rangle_{\mathrm{YM}}^{\mathrm{no\ GF}} \sim e^{-\sigma_{NA}|S|}$,

Mechanism of confinement

- Dual superconductivity is a promising mechanism for quark confinement. [Y.Nambu (1974). G.'t Hooft, (1975). S.Mandelstam, (1976) A.M. Polyakov (1975)]



G.S. Bali, [hep-ph/0001312], Phys. Rept. **343, 1–136 (2001)**

dual superconductivity

superconductor

- Condensation of electric charges (Cooper pairs)
- Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole
- Linear potential between monopoles

dual superconductor

- **Condensation of magnetic monopoles**
- **Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark**
- **Linear potential between quarks**

Evidences for the dual superconductivity (I)

By using Abelian projection

String tension (Linear potential)

- ☐ Abelian dominance in the string tension [Suzuki & Yotsuyanagi, 1990]
- ☐ Abelian magnetic monopole dominance in the string tension [Stack, Neiman and Wensley,1994][Shiba & Suzuki, 1994]

Chromo-flux tube (dual Meissner effect)

- ☐ Measurement of (Abelian) dual Meissner effect
- ◆ Observation of chromo-electric flux tubes and Magnetic current due to chromo-electric flux
- ◆ Type the super conductor is of order between Type I and Type II [Y.Matsubara, et.al. 1994]

- ✓ **only obtained in the case of special gauge such as MA gauge**
- ✓ **gauge fixing breaks the gauge symmetry as well as color symmetry**

The evidence for dual superconductivity (II)

Gauge decomposition method (a new lattice formulation)

- **Extracting the relevant mode *V* for quark confinement** by solving the defining equation in the gauge independent way (gauge-invariant way)

➢ For SU(2) case, the decomposition is a lattice compact representation of the Cho-Duan-Ge-Faddeev-Niemi-Shabanov (CDGFNS) decomposition.

➢ For SU(N) case, the formulation is the extension of the SU(2) case.

➔ **we have showed in the series of works that**

➢ V-field dominance, magnetic monopole dominance in string tension,

➢ chromo-flux tube and dual Meissner effect.

➢ The first observation on quark confinement/deconfinement phase transition in terms of dual Meissner effect

Plan of talk

- Introduction
- dual superconductivity at zero temperature (brief review)
  - Linear potential and string tension
  - Dual Meissner effects
  - Monopole condensation as induced magnetic currents by quark-antiquark pair
- Confinement/deconfinement phase transition at finite temperature
  - Appearance and disappearance of flux tubes
- Summary and outlook

EVIDENCE OF DUAL SUPERCONDUCTIVITY AT ZERO TEMPERATURE

A new formulation of Yang-Mills theory (on a lattice)

Decomposition of SU(N) gauge links

- For SU(N) YM gauge link, there are several possible options of decomposition *discriminated by its stability groups*:

☐ SU(2) Yang-Mills link variables: unique $U(1) \subset SU(2)$

☐ SU(3) Yang-Mills link variables: Two options

maximal option : $U(1) \times U(1) \subset SU(3)$

✓ Maximal case is a gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group)

minimal option : $U(2) \cong SU(2) \times U(1) \subset SU(3)$

✓ Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the non-Abelian Stokes theorem

The decomposition of SU(3) link variable: minimal option

$$W_C[U] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu\rangle \in C} U_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

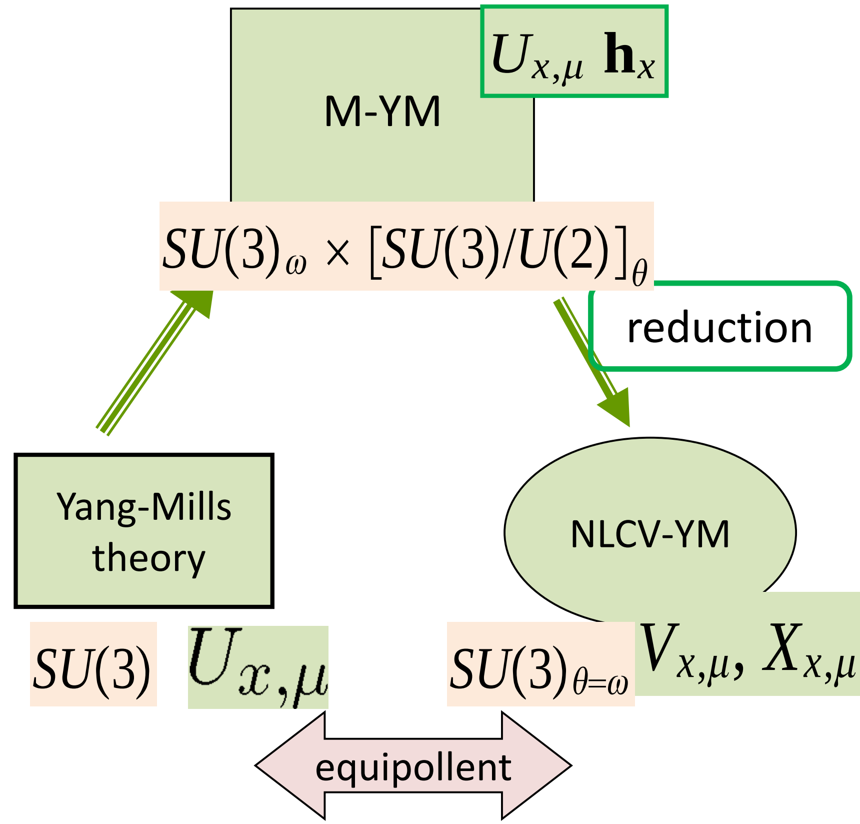
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \boxed{\Omega_x X_{x,\mu} \Omega^\dagger_x}$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \mathrm{Tr}\left[P \prod_{\langle x, x+\mu\rangle \in C} V_{x,\mu} \right] / \mathrm{Tr}(\mathbf{1})$$



$$W_C[U] = \mathrm{const.}\, W_C[V] \quad !!$$

Defining equation for the decomposition

Phys.Lett.B691:91-98,2010 ; arXiv:0911.5294（hep-lat）

Introducing a color field $\mathbf{h}_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the defining equation of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu^\epsilon[V]\mathbf{h}_x = \frac{1}{\epsilon}(V_{x,\mu}\mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N}\exp(-a_x^{(0)}\mathbf{h}_x - i\sum_{i=1}^{3} a_x^{(l)}u_x^{(i)}) = 1,$$

which correspond to the continuum version of the decomposition, $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$,

$$D_\mu[\mathcal{V}_\mu(x)]\mathbf{h}(x) = 0, \qquad \mathrm{tr}(\mathcal{X}_\mu(x)\mathbf{h}(x)) = 0.$$

Exact solution (N=3)

$$X_{x,\mu} = \hat{L}_{x,\mu}^\dagger(\det\hat{L}_{x,\mu})^{1/N}g_x^{-1} \quad V_{x,\mu} = X_{x,\mu}^\dagger U_{x,} = g_x\hat{L}_{x,\mu}U_{x,}(\det\hat{L}_{x,\mu})^{-1/N}$$

$$\hat{L}_{x,\mu} = \left(\sqrt{L_{x,\mu}L_{x,\mu}^\dagger}\right)^{-1}L_{x,\mu}$$

$$L_{x,\mu} = \frac{N^2-2N+2}{N}\mathbf{1} + (N-2)\sqrt{\frac{2(N-2)}{N}}(\mathbf{h}_x + U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1}) + 4(N-1)\mathbf{h}_x U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1}$$

continuum version by continuum limit

$$\mathbf{V}_\mu(x) = \mathbf{A}_\mu(x) - \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] - ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)],$$

$$\mathbf{X}_\mu(x) = \frac{2(N-1)}{N}[\mathbf{h}(x),[\mathbf{h}(x),\mathbf{A}_\mu(x)]] + ig^{-1}\frac{2(N-1)}{N}[\partial_\mu\mathbf{h}(x),\mathbf{h}(x)].$$

Non-Abelian Stokes theorem and decomposition

From the non-Abelian Stokes theorem, we can show Wilson loop operator can rewritten by the decomposed variable V with minimal option.

K.-I. Kondo PRD77 085929(2008)

$$
\begin{aligned}
W_c[\mathbf{A}] &= \mathrm{tr}\left[P \exp ig \oint_C \mathbf{A}_\mu(x) dx^\mu \right] / \mathrm{tr}(\mathbf{1}) \\
&= \int d\mu[\xi]_\Sigma \exp\left\{ -ig \int_{\Sigma:\partial\Sigma = C} dS^{\mu\nu} \frac{2}{\sqrt{3}} tr(\mathbf{n}\mathcal{F}_{\mu\nu}[\mathbf{V}]) \right\} \\
&= \int d\mu[\xi]_\Sigma \exp\{ -ig(K, \Xi_\Sigma) - ig(J, N_\Sigma) \},
\end{aligned}
$$

$$
K := \delta^* F, \quad \Xi_\Sigma := \delta^* \Theta_\Sigma \Delta^{-1},
$$

$$
J := \delta F, \; N_\Sigma := \delta \Theta_\Sigma \Delta^{-1}
$$

Further applying the Hodge decomposition, the magnetic monopole *k* is derived without using the Abelian projection

The lattice version is defined by using plaquette:

$$
\Theta^8_{\mu\nu} := -\arg \mathrm{Tr}\left[\left(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}} \mathbf{h}_x \right) V_{x,\mu} V_{x+\mu,\mu} V^\dagger_{x+\nu,\mu} V^\dagger_{x,\nu} \right],
$$

$$
k_\mu = 2\pi n_\mu := \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \partial_\nu \Theta^8_{\alpha\beta},
$$

■ SU(3) Yang-Mills theory

- In confinement of fundamental quarks, a restricted non-Abelian variable ***V***, and the extracted non-Abelian magnetic monopoles play the dominant role (dominance in the string tension), in marked contrast to the Abelian projection.

gauge independent "Abelian" dominance

$$\frac{\sigma_V}{\sigma_U} = 0.92$$

$$\frac{\sigma_V}{\sigma_{U^*}} = 0.78 - 0.82$$

Gauge independent non-Abalian monople dominance

$$\frac{\sigma_M}{\sigma_U} = 0.85$$

$$\frac{\sigma_M}{\sigma_{U^*}} = 0.72 - 0.76$$

U^* is from the table in R. G. Edwards, U. M. Heller, and T. R. Klassen, Nucl. Phys. B517, 377 (1998).

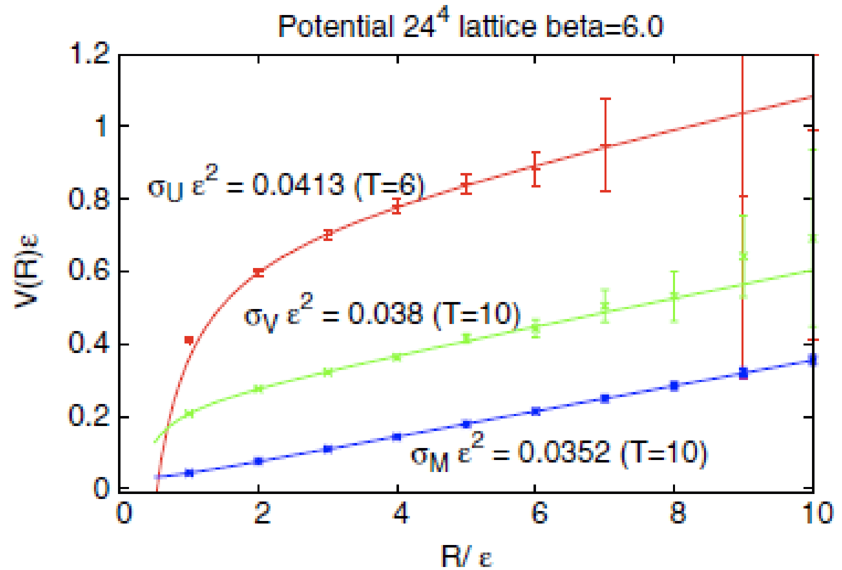


FIG. 1 (color online). $SU(3)$ quark-antiquark potentials as functions of the quark-antiquark distance R: (from tob to bottom) (i) full potential $V_f(R)$ (red curve), (ii) restricted part $V_r(R)$ (green curve), and (iii) ma;gnetic-monopole part $V_m(R)$ (blue curve), measured at $\beta = 6.0$ on 24^4 using 500 configurations where ϵ is the lattice spacing.

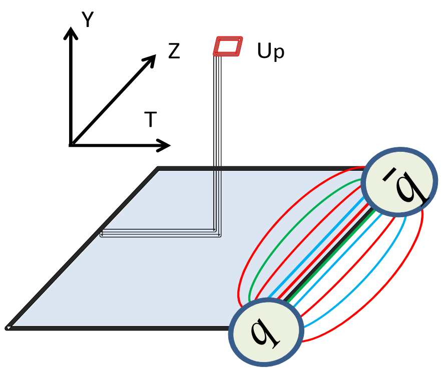
PRD 83, 114016 (2011)

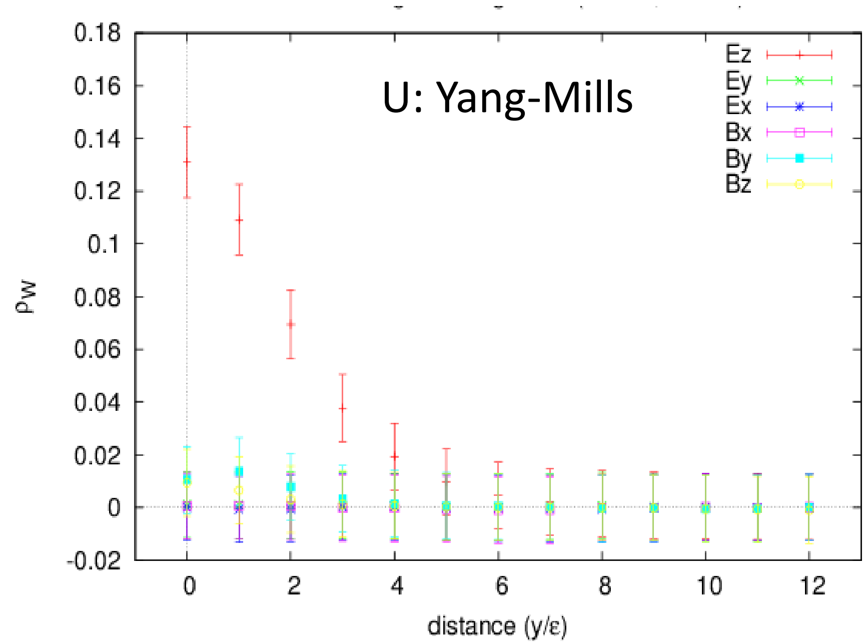
Chromo flux

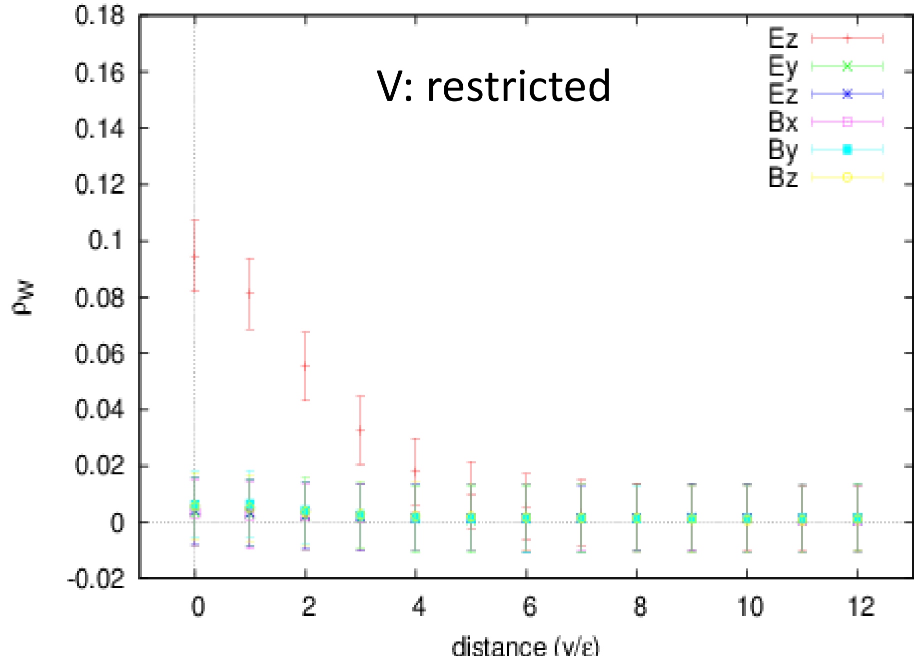
$$\rho_W = \frac{\langle \mathrm{tr}(W L U_p L^\dagger) \rangle}{\langle \mathrm{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \mathrm{tr}(W) \mathrm{tr}(U_p) \rangle}{\langle \mathrm{tr}(W) \rangle}$$

Gauge invariant correlation function: This is settled by Wilson loop (W) as quark and antiquark source and plaquette (Up) connected by Wilson lines (L). N is the number of color (N=3) [Adriano Di Giacomo et.al. PLB236:199,1990 NPBB347:441-460,1990]

$\mathrm{tr}(U_p L W L^\dagger)$

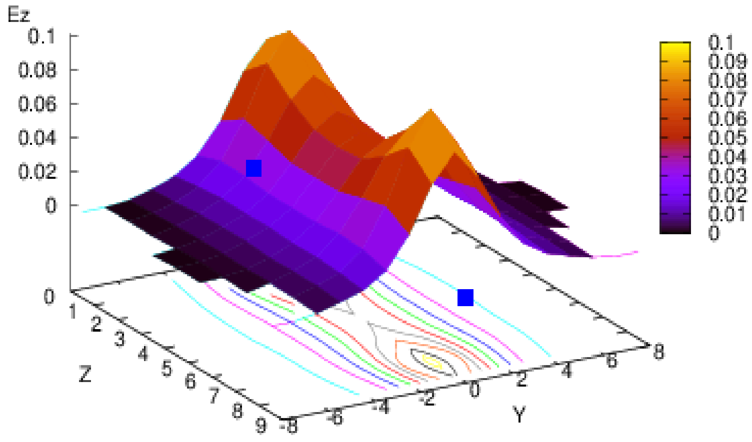




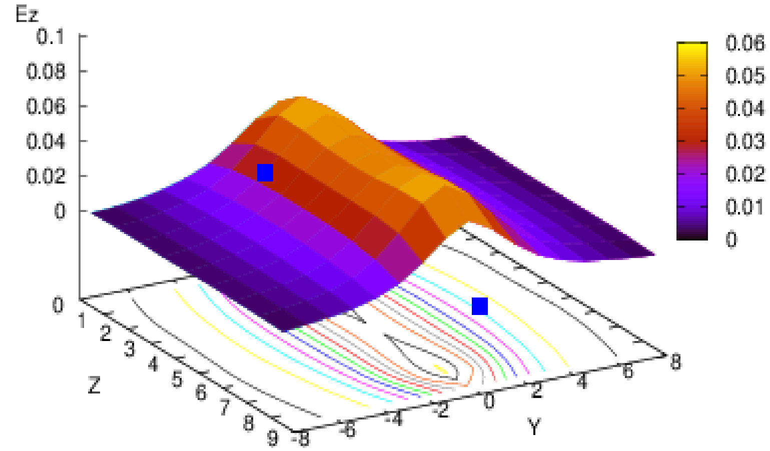


Chromo-electric (color flux) Flux Tube

Original YM filed



Restricted field



A pair of quark-antiquark is placed on z axis as the 9x9 Wilson loop in Z-T plane. Distribution of the chromo-electronic flux field created by a pair of quark-antiquark is measured in the Y-Z plane, and the magnitude is plotted both 3-dimensional and the contour in the Y-Z plane.

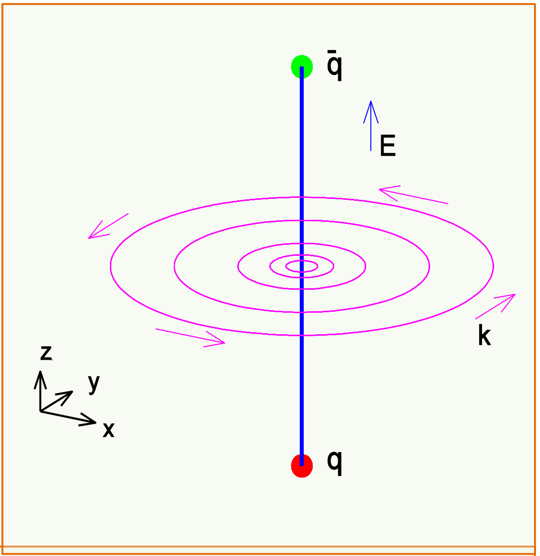
Flux tube is observed for V-field case. :: dual Meissner effect

Magnetic current induced by quark and antiquark pair

Yang–Mills equation (Maxell equation) fo rrestricted field V_μ, the magnetic current (monopole) can be calculated as

$$k = \delta^* F[V] = {}^* dF[V],$$

where $F[V]$ is the field strength of V, d exterior derivative, $*$ the Hodge dual and δ the coderivative $\delta := {}^* d^*$, respectively.



$\mathbf{k} \neq 0 \Rightarrow$ signal of monopole condensation.
Since field strengthe is given by $F[\mathbf{V}] = d\mathbf{V}$, and $\mathbf{k} = {}^* dF[\mathbf{V}] = {}^* ddF[\mathbf{V}] = 0$
(Bianchi identity)

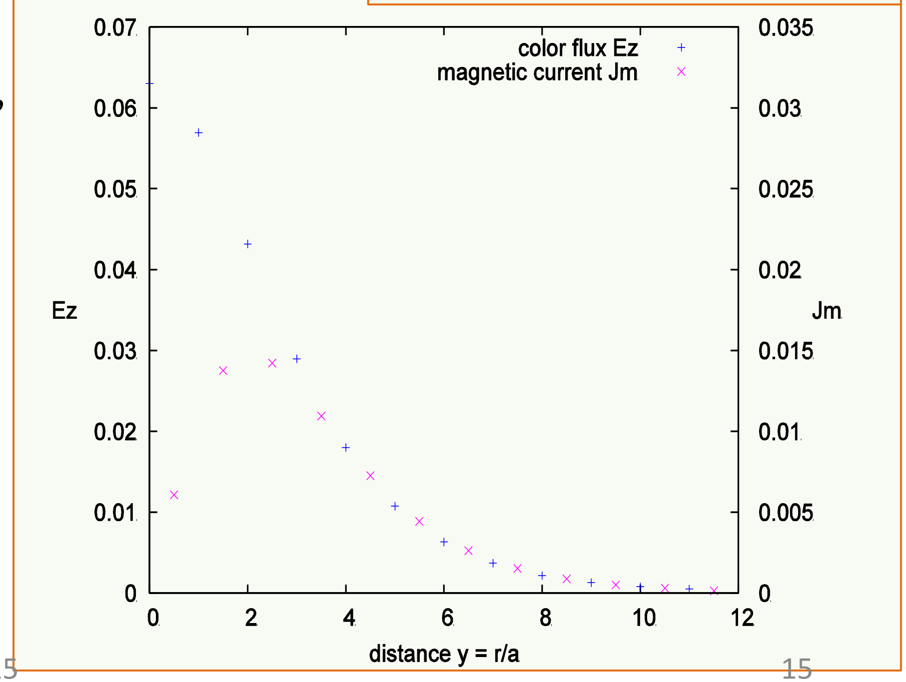


Figure: (upper) positional relationship of chromo-electric flux and magnetic current. (lower) combination plot of chromo-electric flux (left scale) and magnetic current(right scale).

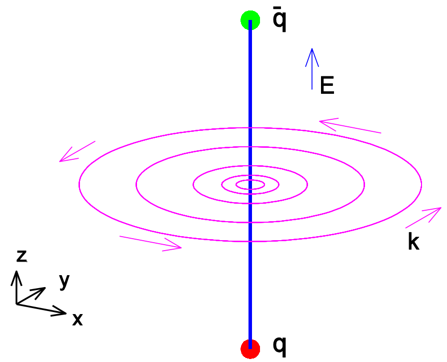
Type of dual superconductivity (Ginzburg-Landau theory)

Ginzburg–Landau equation

$$D_\mu D^\mu \phi - \lambda(\phi^* \phi - \mu^2/\lambda^2)\phi = 0$$

Ampere equation

$$\partial^\nu F_{\mu\nu} + iq[\phi^*(D_\mu \phi) - (D_\mu \phi)^* \phi] = 0$$



J.R.Clem J. low Temp. Phys. 18 427 (1975)

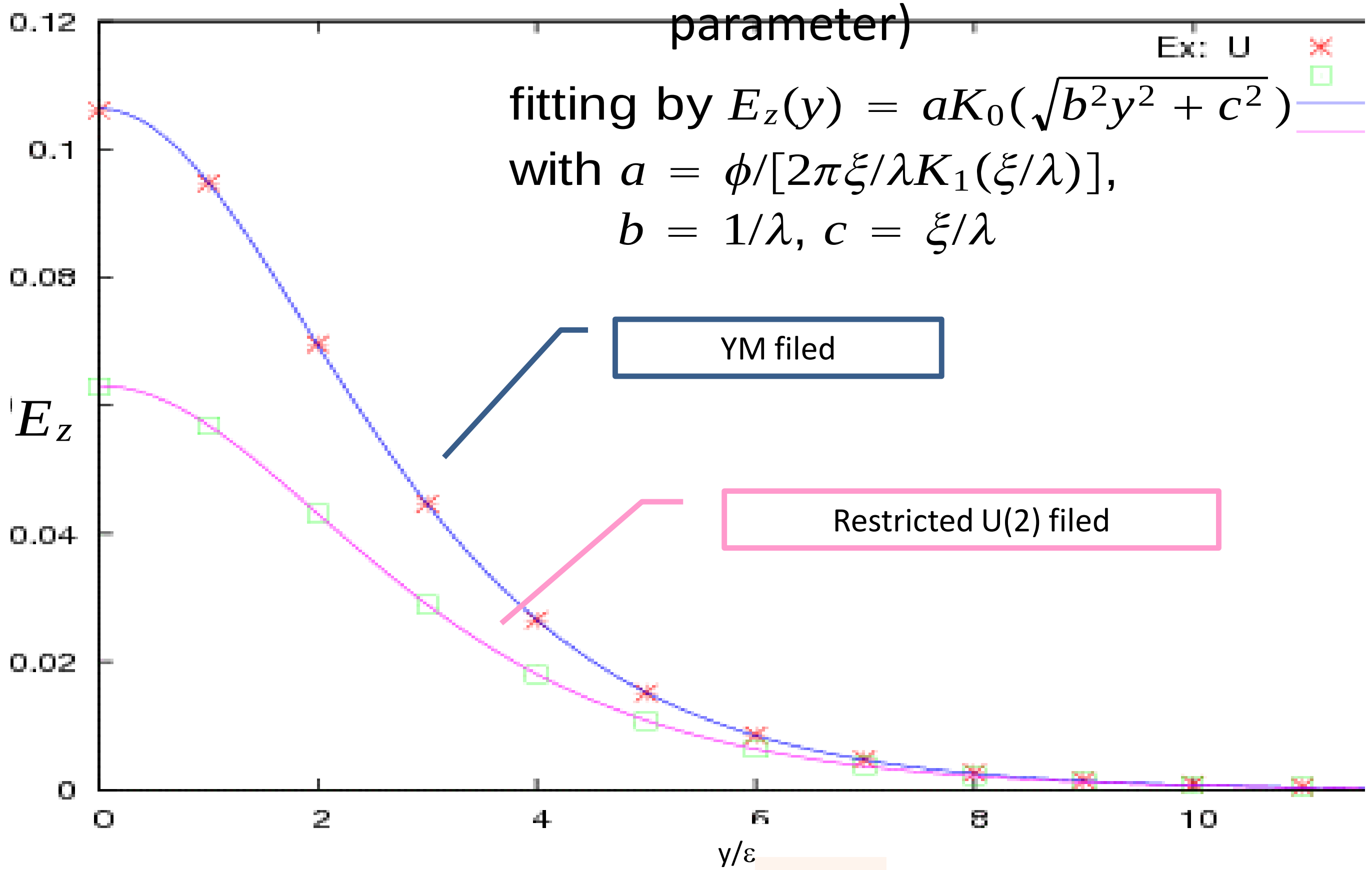
The profile of chromo-electric flux in the super conductor is given by

$$E_z[y] = \frac{\Phi_0}{2\pi} \frac{1}{\xi\lambda} \frac{K_0(R/\lambda)}{K_1(\xi/\lambda)}, \quad R = \sqrt{y^2 + \xi^2}$$

K_ν : the modified Bessel function of the ν-th order, λ the parameter corresponding to the London penetration length, ξ a variational core radius parameter, and Φ_0 external flux.

❖ this formula is for the super conductor of U(1) gauge field.

Type of dual superconductivity (Ginzburg-Landau) parameter)



fitting by $E_z(y) = aK_0(\sqrt{b^2y^2+c^2})$

with $a = \phi/[2\pi\xi/\lambda K_1(\xi/\lambda)]$,

$b = 1/\lambda,\ c = \xi/\lambda$

	λ/ϵ	ξ/ϵ	$a\epsilon^2$	Φ_0	κ
Yang-Mills	1.65	3.24	1.09	2.00	0.43
restricted U(2)	1.81	3.36	0.567	1.33	0.45

Ginzburg-Landau (GL) parameter

$\kappa = \sqrt{2}/(\xi/\lambda)\sqrt{1-K_0^2(\xi/\lambda)/K_1^2(\xi/\lambda)}$.

Type I $\kappa < \kappa_c = 1/\sqrt{2} \simeq 0.707$

Type II $\kappa > \kappa_c$

String tension and dual Meissner effect: SU(2)

Phys.Rev. D91 (2015) 3, 034506

YM field

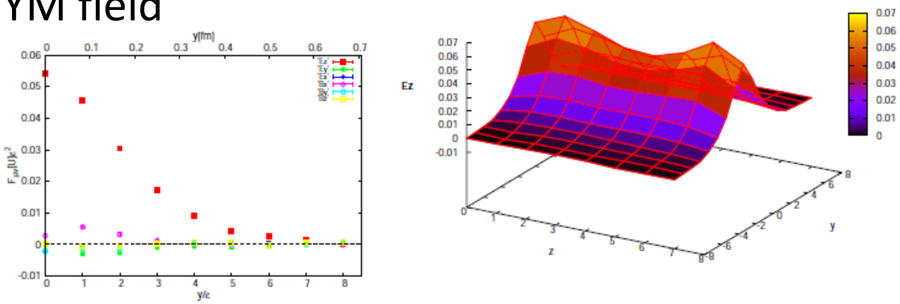
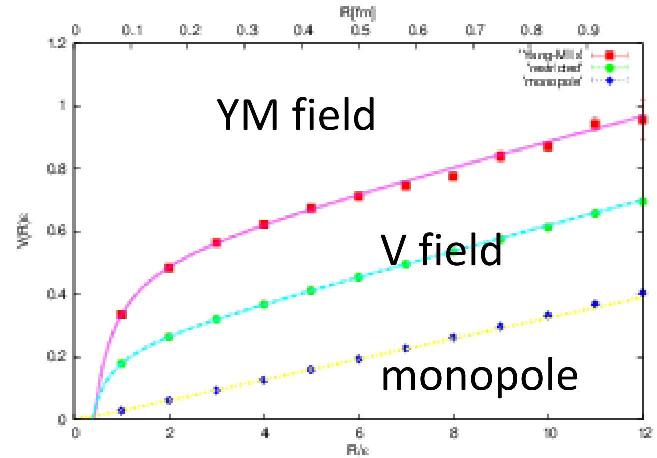


FIG. 3: The chromoelectric and chromomagnetic fields obtained from the full field U on 24^4 lattice at $\beta = 2.5$. (Left panel) y dependence of the chromoelectric field $E_i(y) = F_{4i}(y)$ $(i = x, y, z)$ at fixed $z = 4$ (mid-point of $q\bar{q}$). (Right panel) The distribution of $E_z(y, z)$ obtained for the 8×8 Wilson loop with $\bar{q}$ at $(y, z) = (0, 0)$ and q at $(y, z) = (0, 8)$.



V field

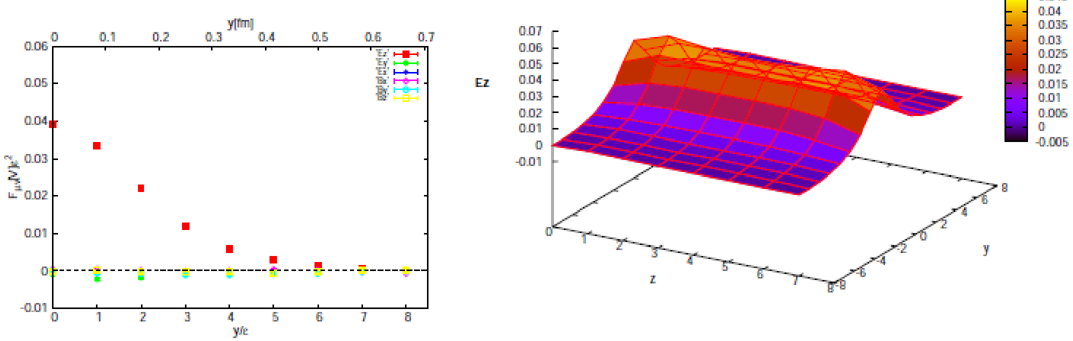


FIG. 4: The chromoelectric field obtained from the restricted field V on 24^4 lattice at $\beta = 2.5$.

$$\frac{\sigma_{\text{rest}}}{\sigma_{\text{full}}} = (102 \pm 2)\% \quad (\text{on } 24^4 \text{ lattice at } \beta = 2.5),$$

$$\frac{\sigma_{\text{mono}}}{\sigma_{\text{rest}}} = (83 \pm 1)\% \Longrightarrow \frac{\sigma_{\text{mono}}}{\sigma_{\text{full}}} = (85 \pm 2)\%$$

Magnetic current and GL parameter: SU(2)

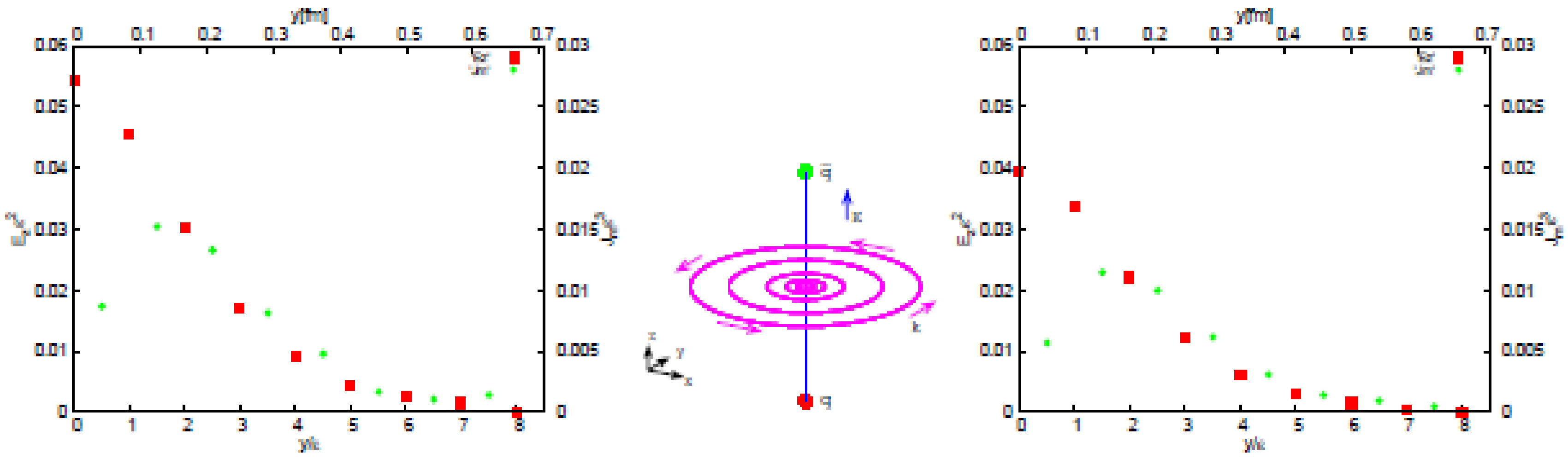


FIG. 6: The magnetic-monopole current $\mathbf{k}$ induced around the chromoelectric flux along the z axis connecting a pair of quark and antiquark. (Center panel) The positional relationship between the chromoelectric field E_z and the magnetic current $\mathbf{k}$. (Left panel) The magnitude of the chromoelectric field E_z and the magnetic current $J_m = |\mathbf{k}|$ as functions of the distance y from the z axis calculated from the original full variables. (Right panel) The counterparts of the left graph calculated from the restricted variables.

Thus we have obtained the GL parameter for the full field κ_U and the restricted field κ_V:

$$\kappa_U = 0.484 \pm 0.070 \pm 0.026, \quad \kappa_V = 0.377 \pm 0.079 \pm 0.018.$$

Type I $\quad \kappa < \kappa_c = 1/\sqrt{2} \simeq 0.707$

Type II $\quad \kappa > \kappa_c$

Confinement / deconfinement phase transition

- We measure the chromo-flux generated by a pair of quark and antiquark at finite temperature applying our new formulation of Yang-Mills theory on the lattice.
- The quark-antiquark source can be given **by a pair of Polyakov loops** in stead of the Wilson loop.
- Convensionally, average of Ployakov loops <P> is used as order parameter of the phase transition.
- In the view of dual superconductivity
  - ☐ Confinement phase :: dual Meissner effect
  - ➔ generation of the chromo-flux tube and induced magnetic current (monopole)
  - ☐ Deconfinement phase :: disappearance of dual Meissner effect.
  - ➔ Disappearance of the magnetic currents!?

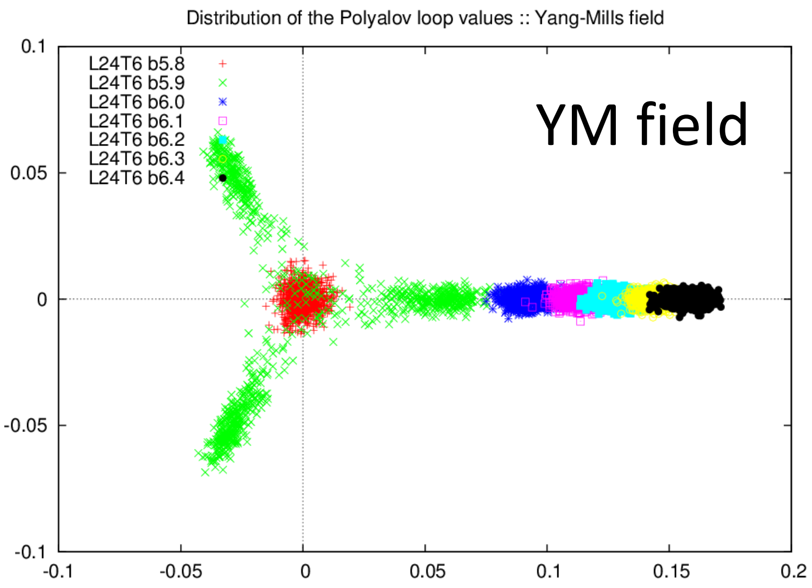
Lattice set up

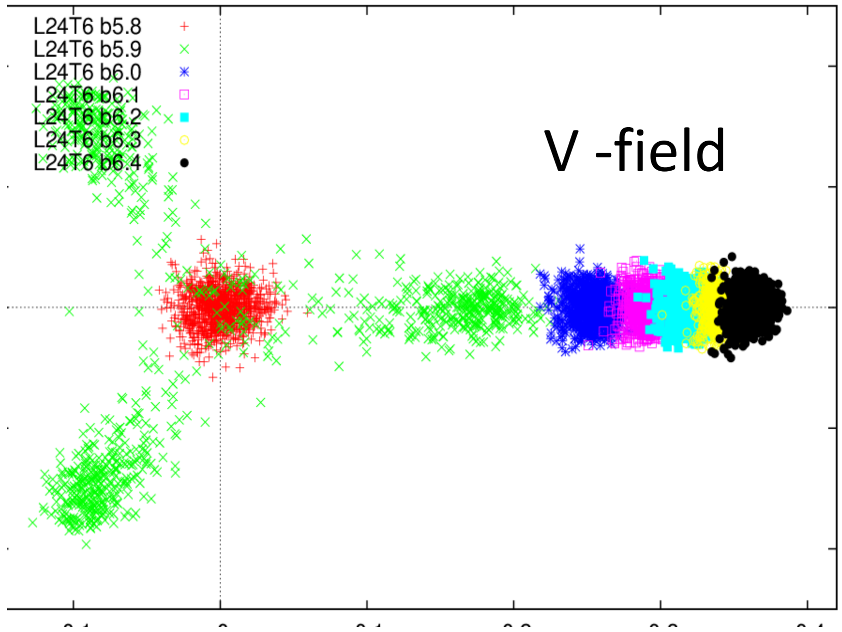
- Standard Wilson action
- 24^3 x 6 lattice
- Temperature is controlled by using β (=6/g^2);

  β=5.8, 5.9, 6.0, 6.1, 6.2, 6.3
- Measurement by 1000 configurations

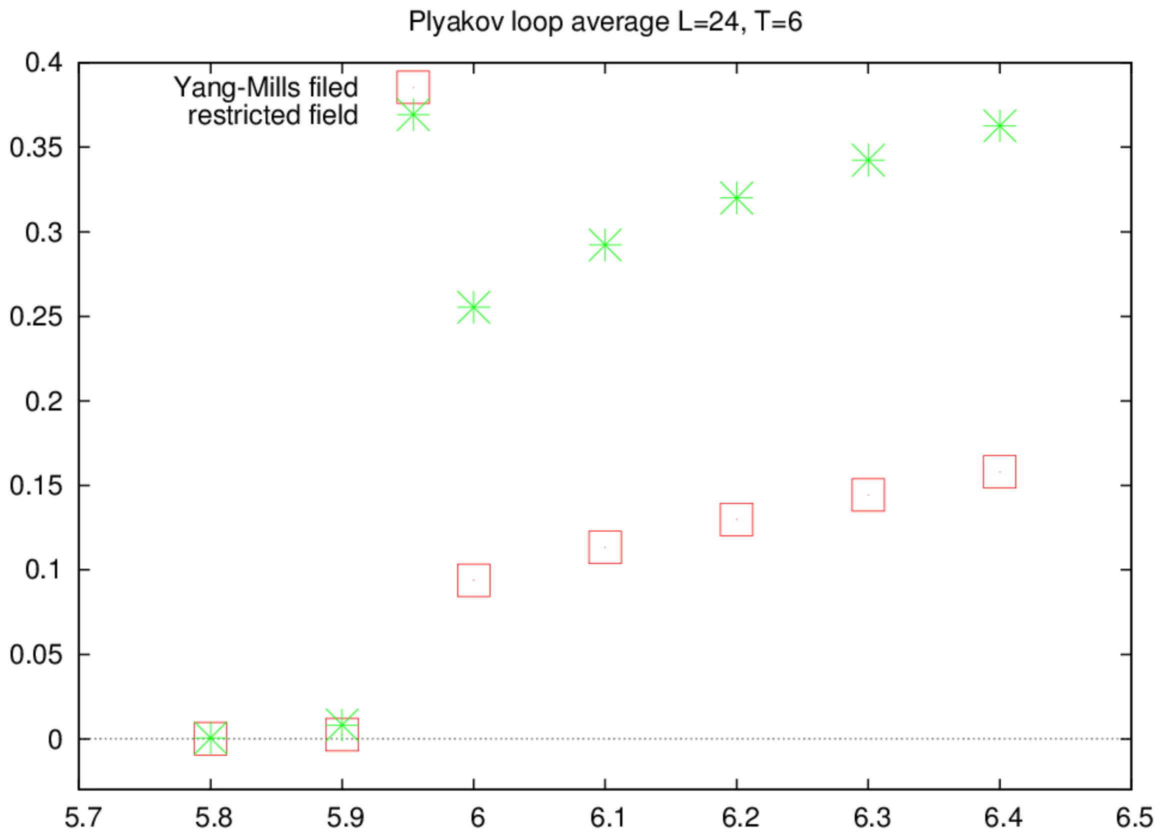
Distribution of Polyakov loop

$$P_U(x) = \mathrm{tr}\left(\prod_{t=1}^{Nt} U_{(x,t),4}\right) \text{ for original Yang–Mills filed}$$
$$P_V(x) = \mathrm{tr}\left(\prod_{t=1}^{Nt} V_{(x,t),4}\right) \text{ for restricted field}$$

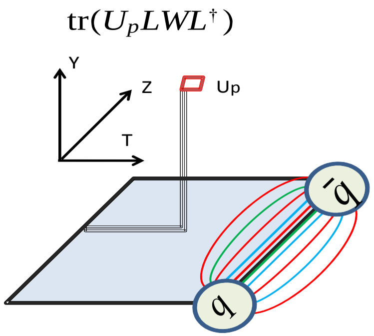


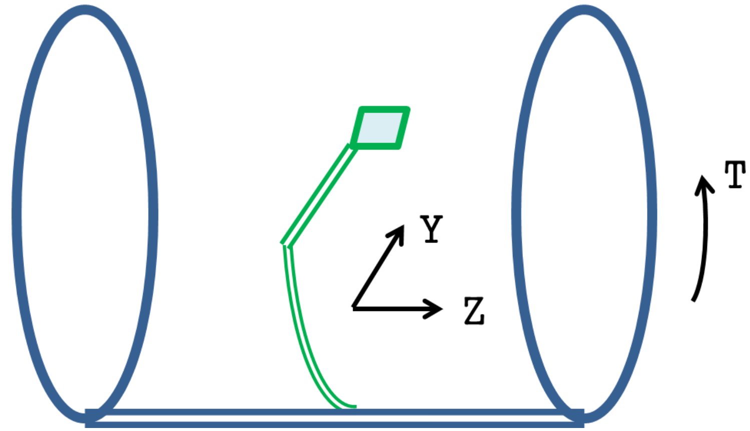


Polyakov loop average



Chromo-electric flux at finite temperature

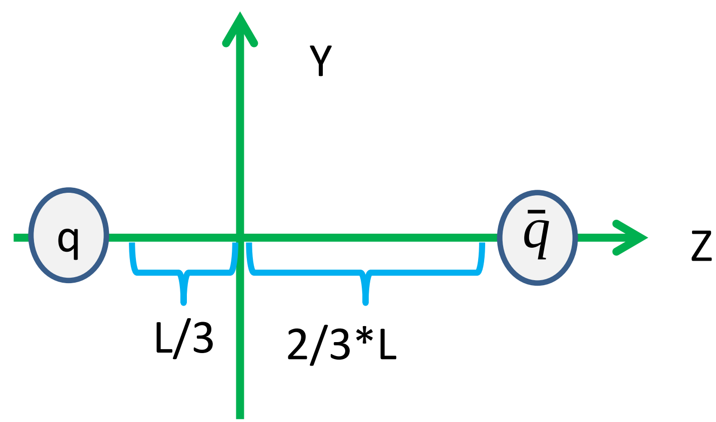




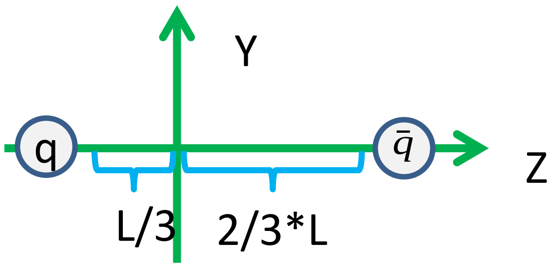
Size of Wilson loop T-direction = Nt
➔The quark and antiquark sources are given by **Plyakov loops**.

$$\rho_W = \frac{\langle \mathrm{tr}(W L U_p L^\dagger)\rangle}{\langle \mathrm{tr}(W)\rangle} - \frac{1}{N}\frac{\langle \mathrm{tr}(W)\mathrm{tr}(U_p)\rangle}{\langle \mathrm{tr}(W)\rangle}$$

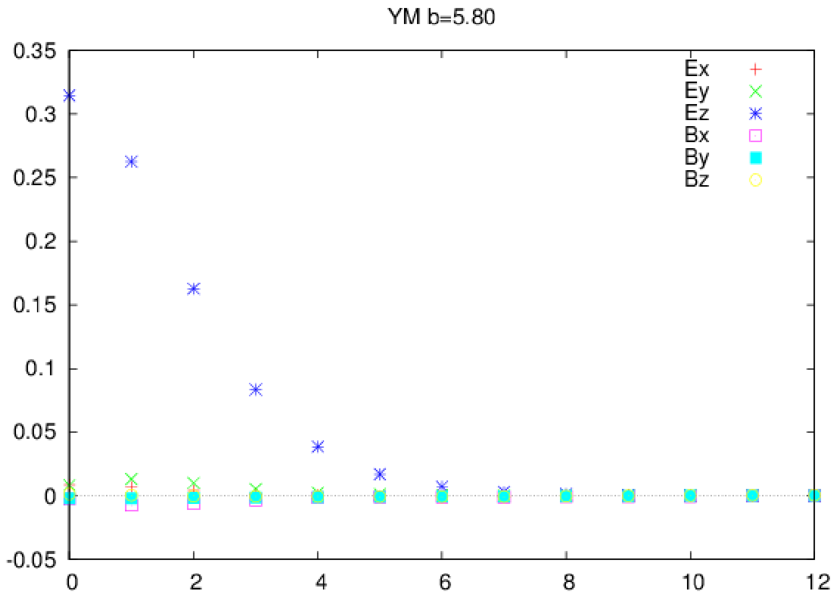
$$F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}}\,\rho_W(x)$$



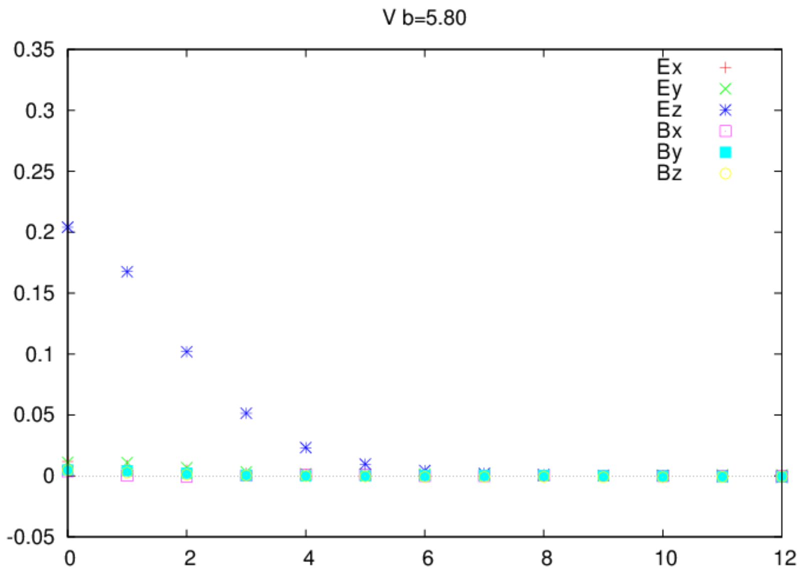
Chromo-flux β=5.8



YM field



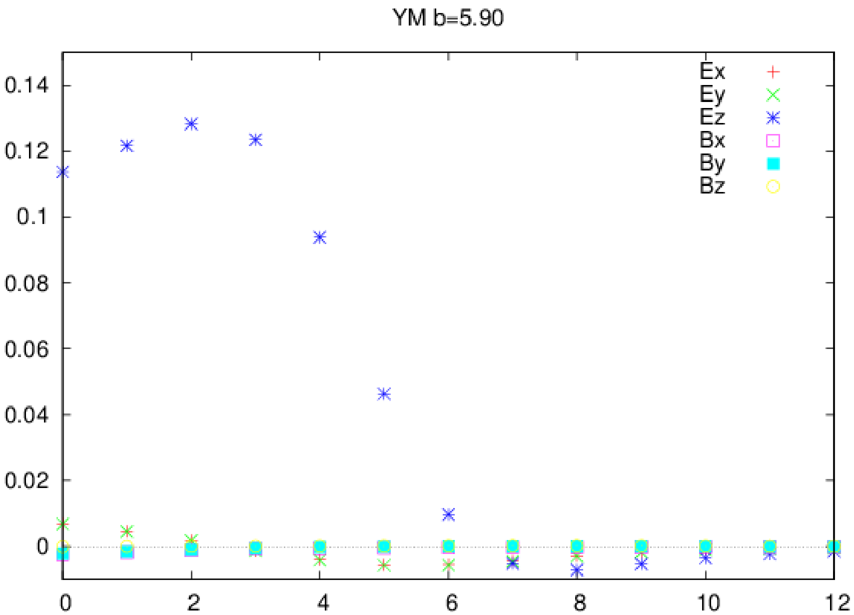
V field

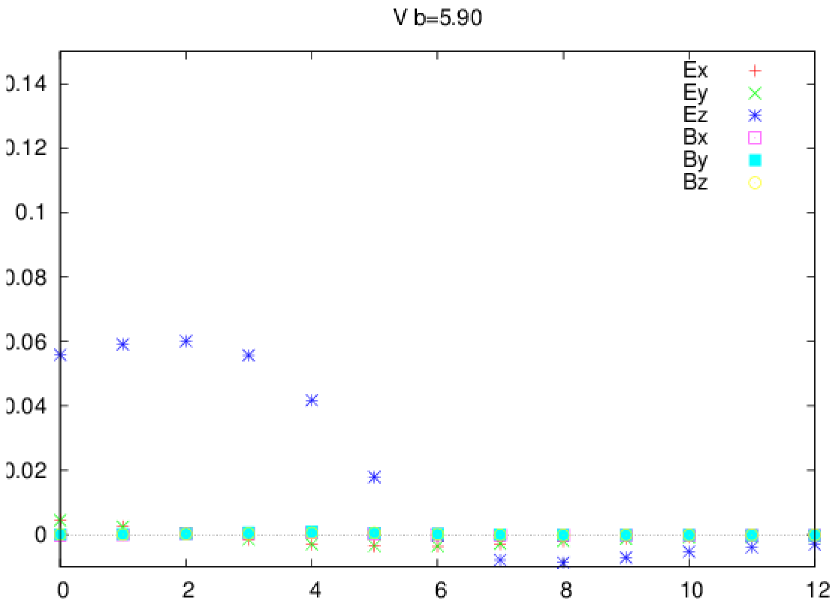


Chromo-flux β=5.9

YM field

V field

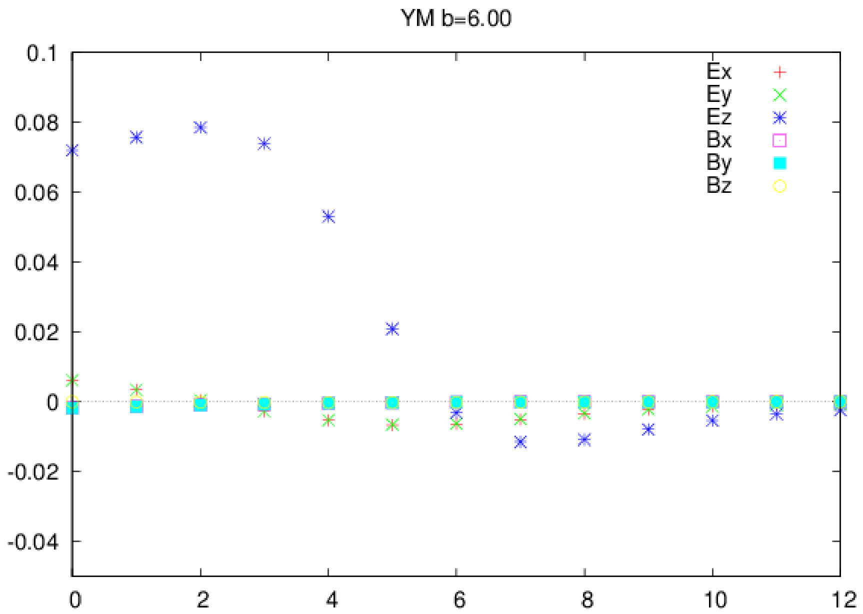


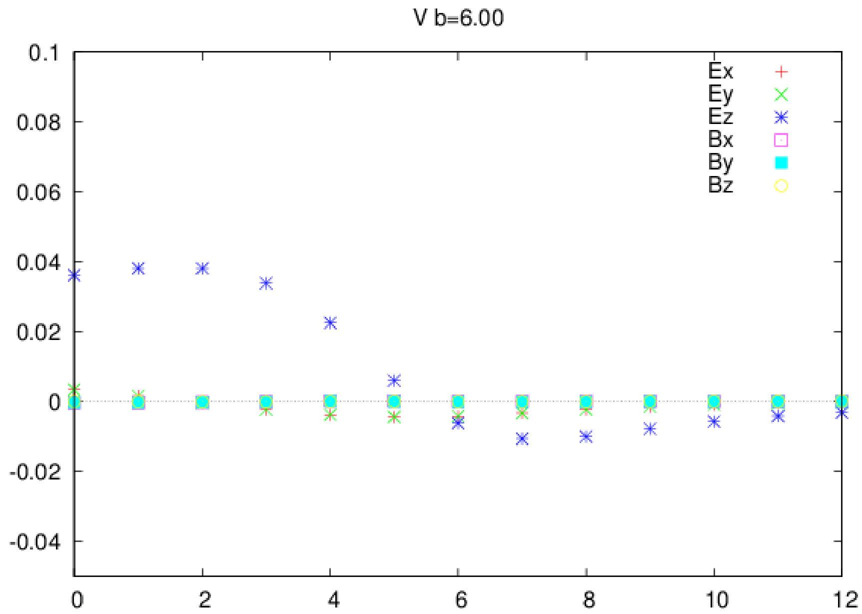


Chromo-flux β=6.0

YM field

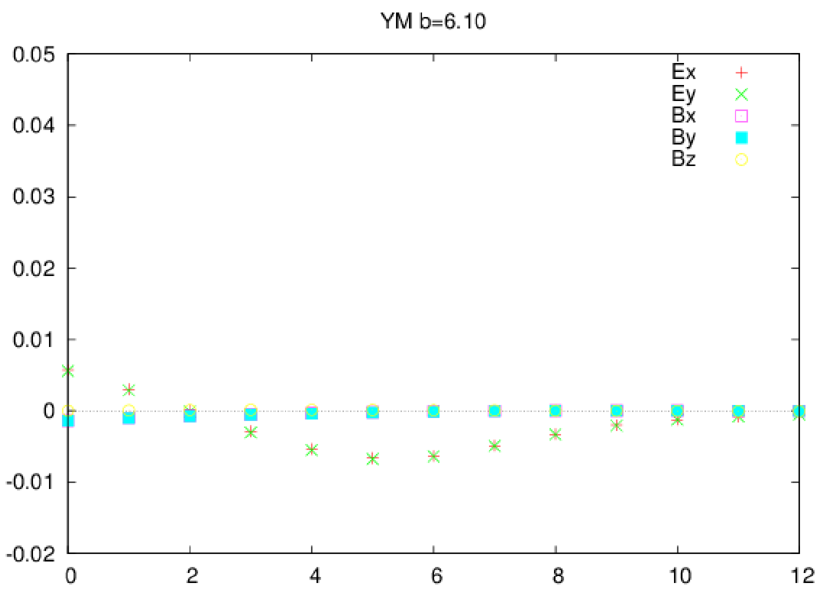
V field



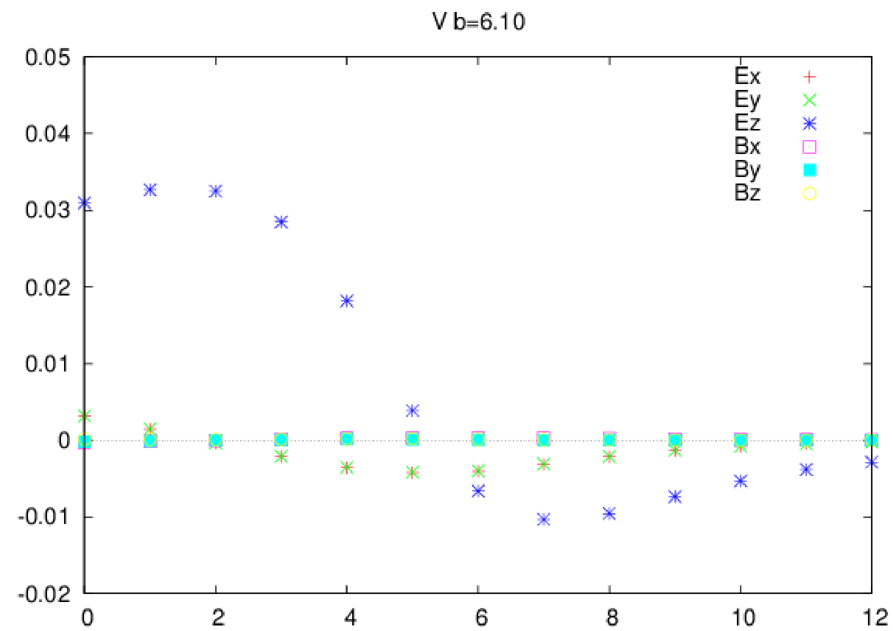


Chromo-flux β=6.1

YM field

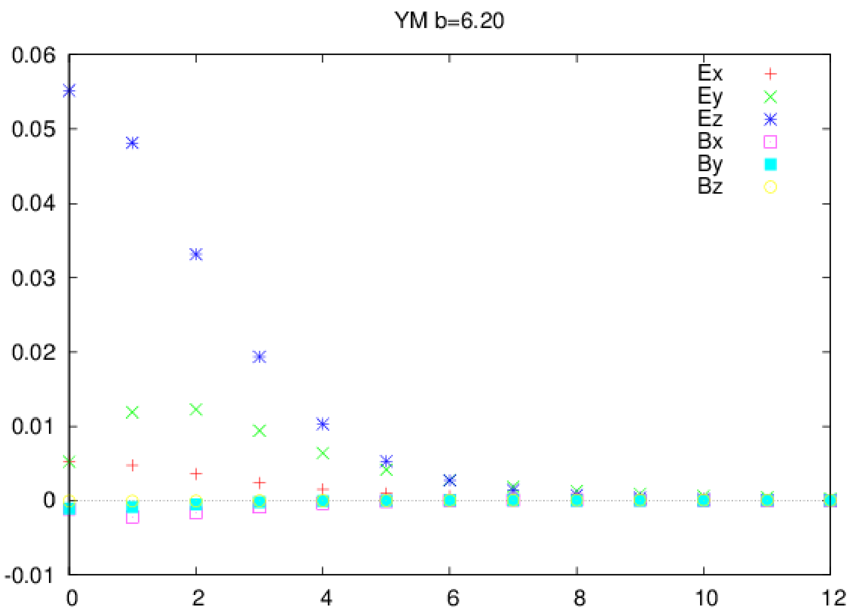


V field

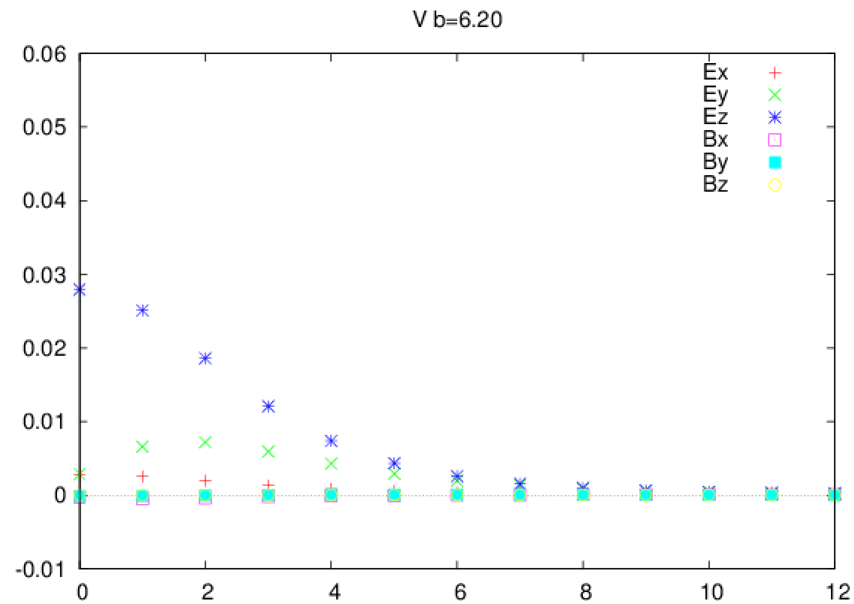


Chromo-flux β=6.2

YM field

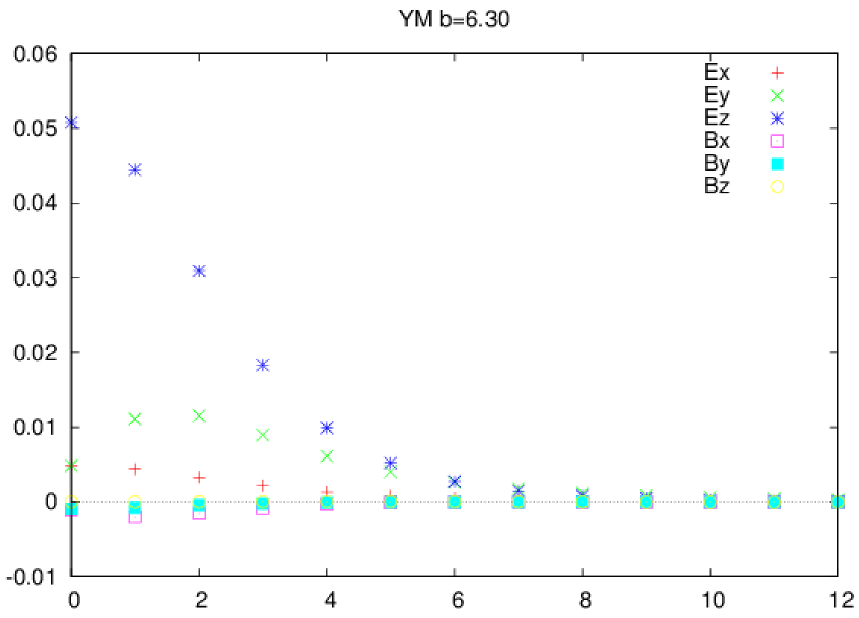


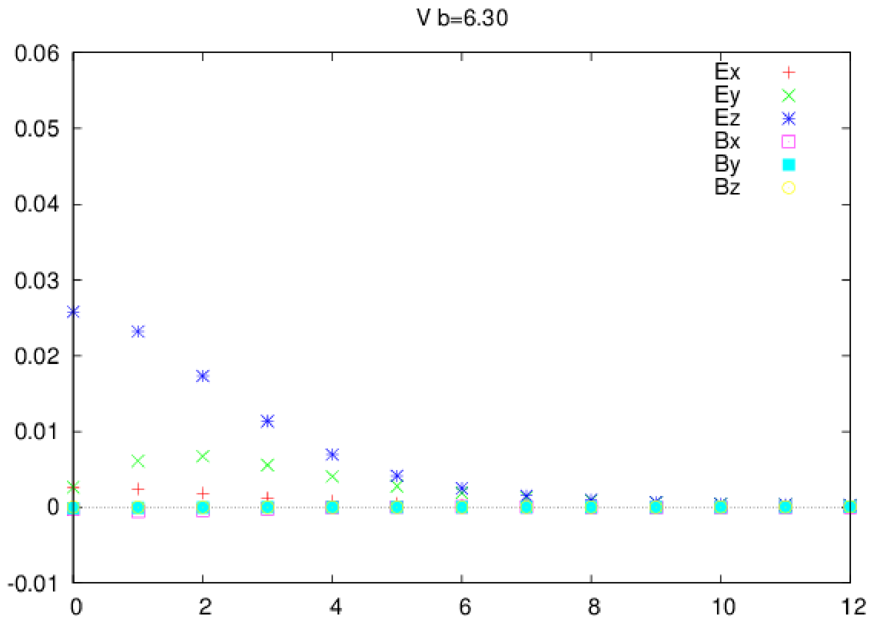
V field



Chromo-flux β=6.3

YM field V field

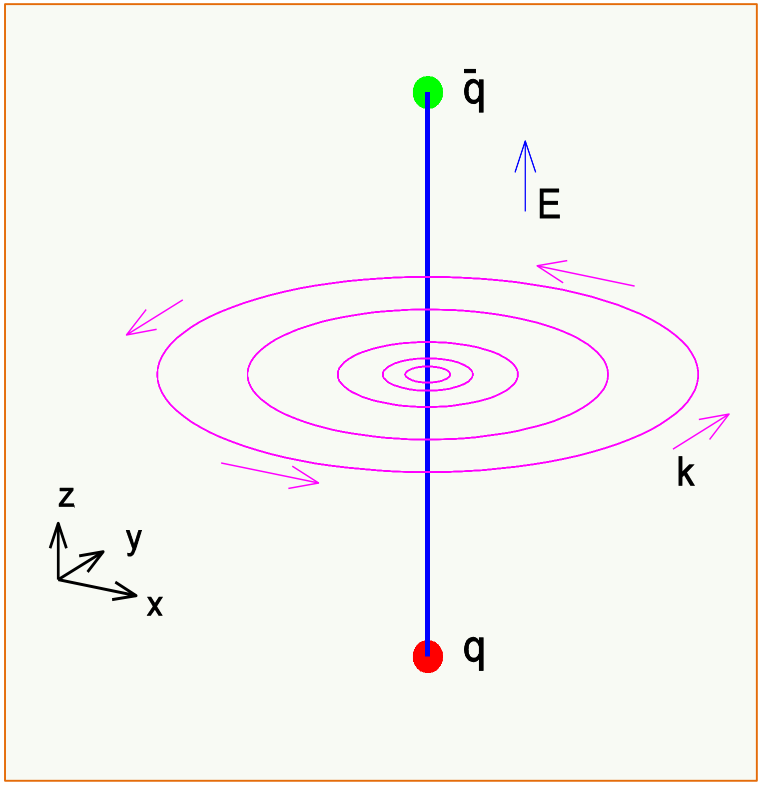




Chromo-magnetic current (monopole current)

- To know relation to the monopole condensation, we further need the measurement of magnetic current in Maxell equation for V field.

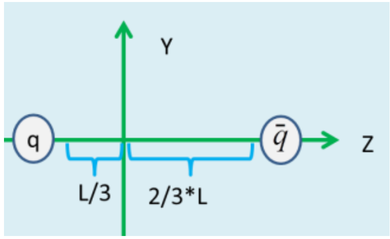
$$k = \delta^* F[V] = {}^* dF[V]$$



$\mathbf{k} \neq 0 \Rightarrow$ signal of monopole condensation.
Since field strengthe is given by $F[\mathbf{V}] = d\mathbf{V}$,
and $\mathbf{k} = {}^*dF[\mathbf{V}] = {}^*ddF[\mathbf{V}] = 0$
(Bianchi identity)

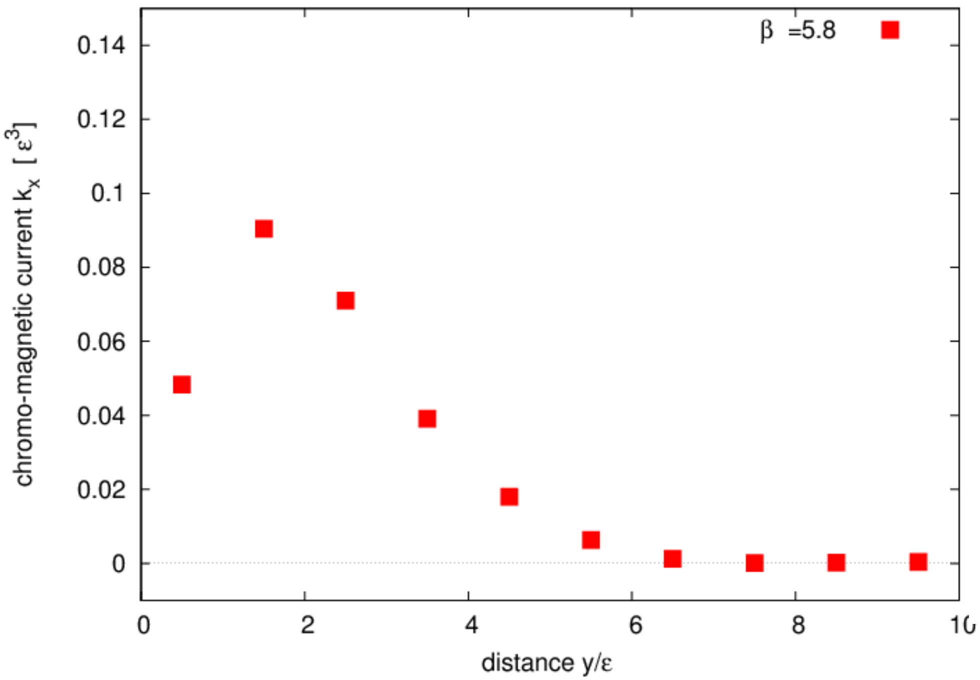
Chromo-magnetic (monopole) current β=5.8

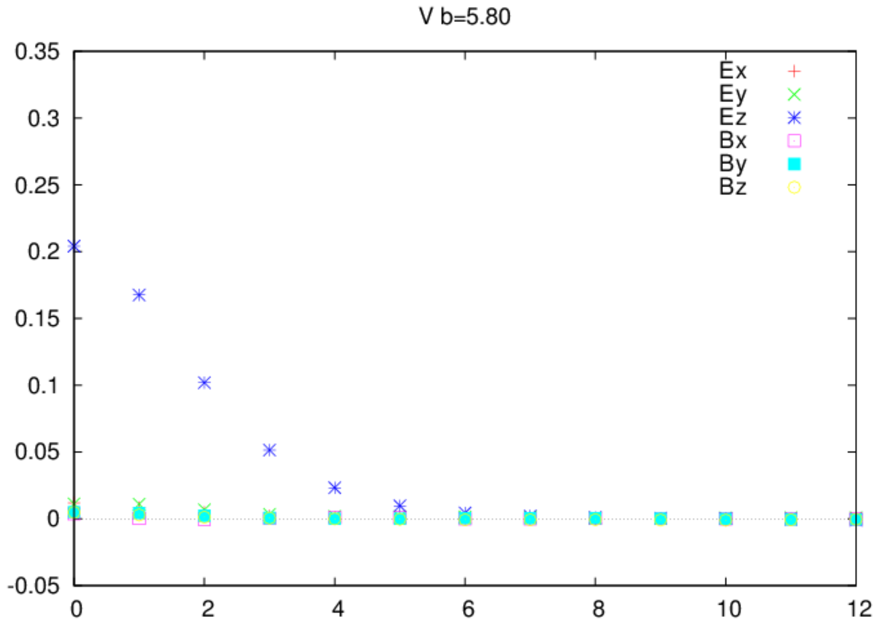
Confinement phase



chromo-magnetic current k_x

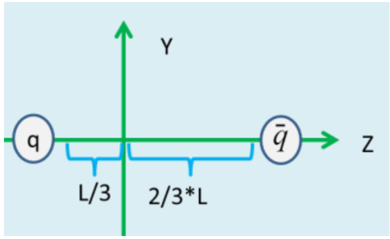
Chromo-flux



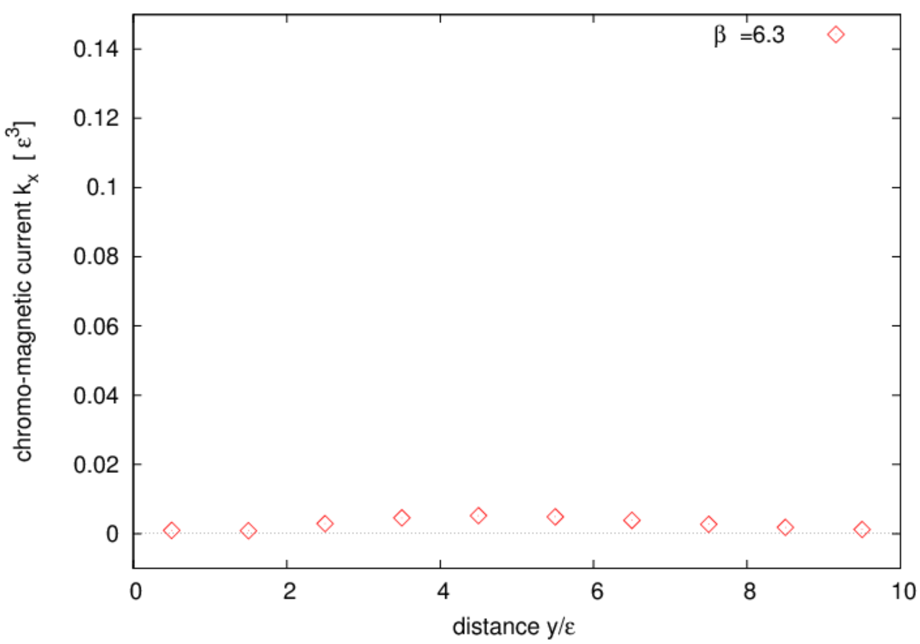


Chromo-magnetic (monopole) current β=6.3

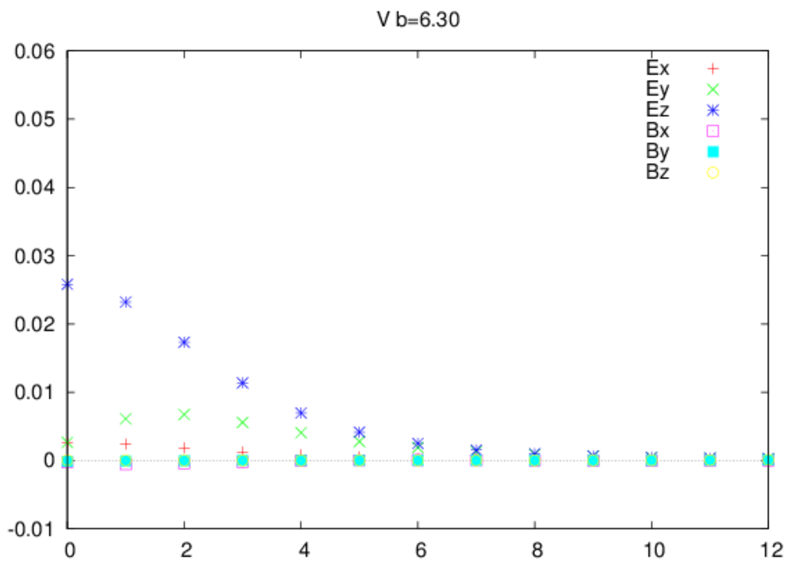
deconfinement phase



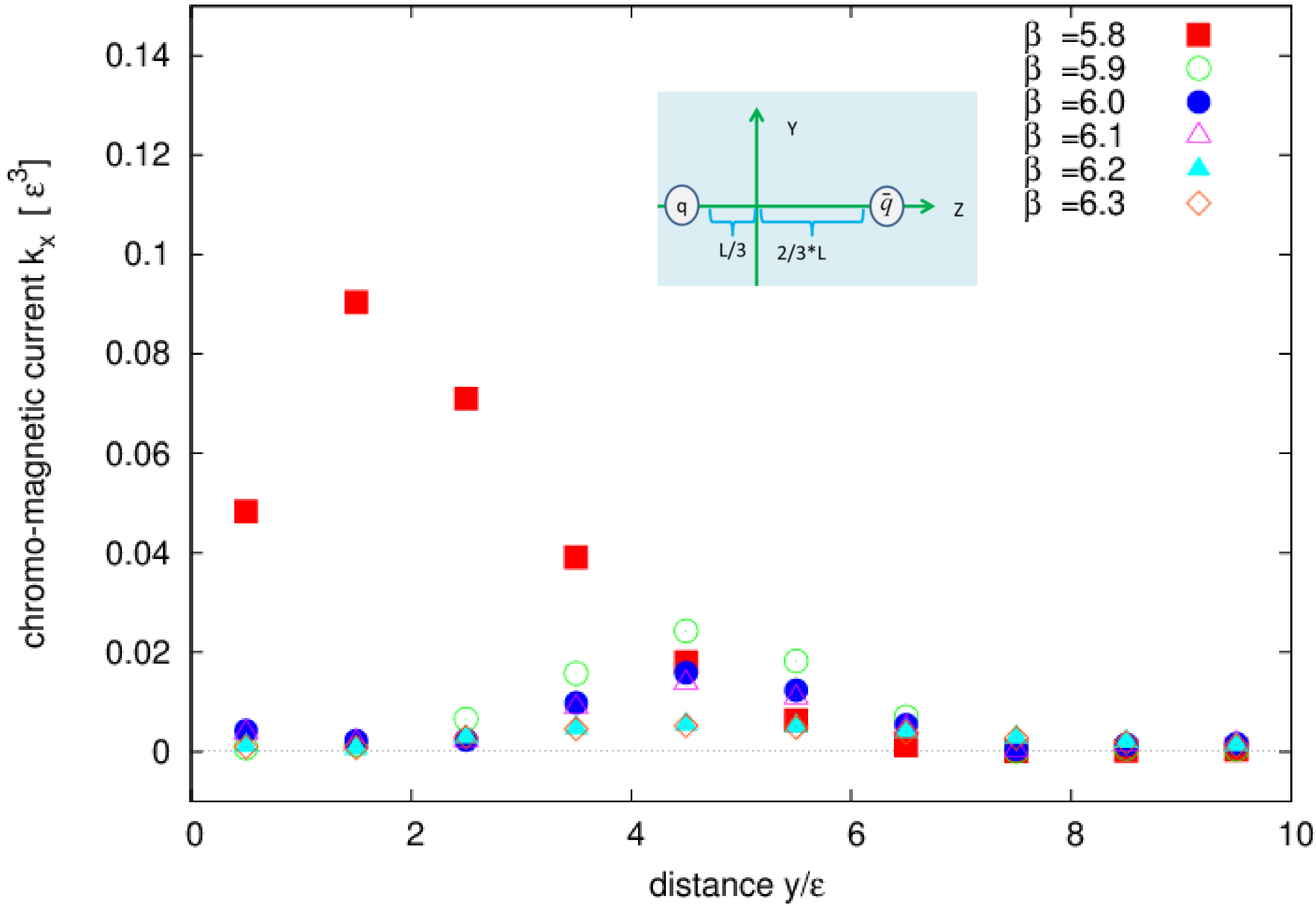
chromo-magnetic current k_x



Chromo-flux



Chromo-magnetic current k_x :: (combied plot)



Summary and out look

Summary

- **We investigate non-Abelian dual Meissner effects at finite temperature,** applying our new formulation of Yang-Mills theory on the lattice..

➢ In confinement phase, **observation of the chromo-electric flux tube and induced magnetic monopole**

➢ In deconfiment phase, **disappearance of the the chromo-electronic flux tube and vanishing the magnetic monopole**

➔The magnetic monopole plays the dominant role in confinement/ deconfinement phase transition.

➔ Confiment / deconfinement phasetransition can be described by the phase transition of the dual super conductivity.

Outlook

- Study of nature of the non-Abelian chromo-electric flux.

Thank you for your attention