Rethinking naturalness

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CP3 Origins
Cosmology & Particle Physics

Can the Higgs be elementary ?

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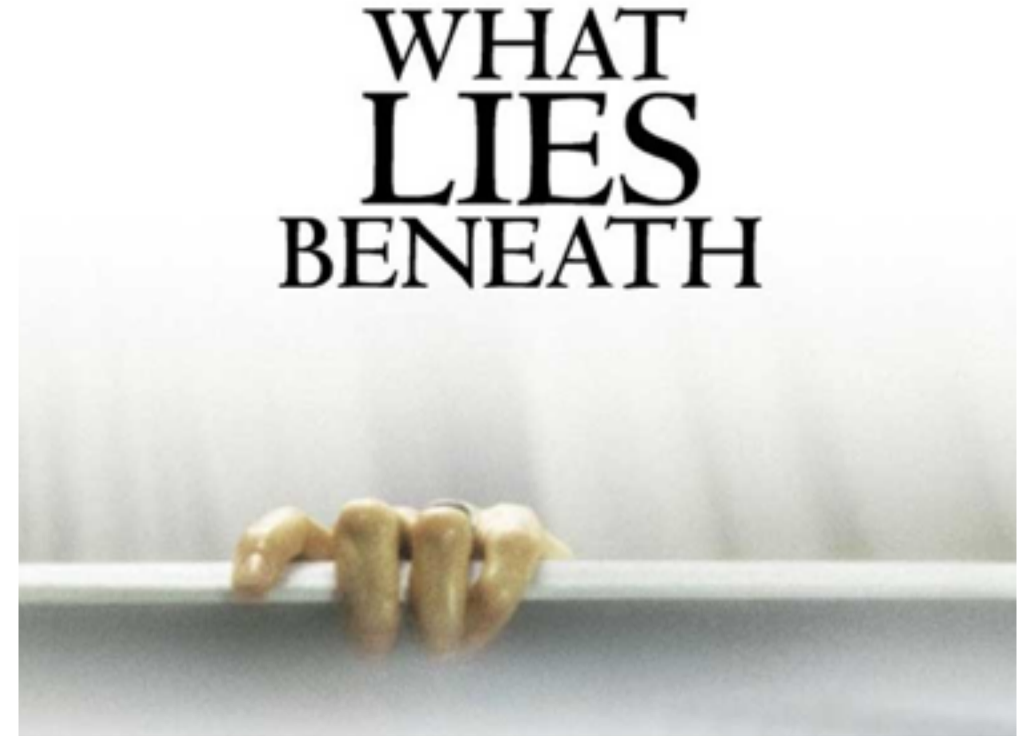
Plan



- Degrees of (un)naturality
- SM ado & Triviality
- Interacting UV-FP for Gauge-Yukawa theories
- Beyond asymptotic freedom

Back to the drawing board

RG (un)-naturality

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_r)^2 - \frac{1}{2}m^2\phi_r^2 - \frac{\lambda}{4!}\phi_r^4 + \frac{\delta_Z}{2}(\partial_\mu \phi_r)^2 - \frac{\delta_m}{2}\phi_r^2 - \frac{\delta_\lambda}{4!}\phi_r^4$$

$$\phi_B \equiv \sqrt{Z}\phi_r \quad \delta_Z \equiv Z - 1 \quad m^2 \equiv m_0^2 Z - \delta_m \quad \delta_\lambda \equiv \lambda_0 Z^2 - \lambda$$

$$Z = 1 + f_1(\lambda, g_i)\log\frac{\Lambda^2}{m_0^2} + \ldots \qquad \delta_m = f_2(\lambda, g_i)\Lambda^2 + \ldots$$

$$m^2 = m_0^2\left(1 + f_1(\lambda, g_i)\log\frac{\Lambda^2}{m_0^2}\right) - f_2(\lambda, g_i)\Lambda^2$$

Shades of (un)naturality

$$m^2 = m_0^2\left(1 + f_1(\lambda, g_i)\log\frac{\Lambda^2}{m_0^2}\right) - f_2(\lambda, g_i)\Lambda^2$$

- Standard model: cancel m_0 against cutoff
- Coleman-Weinberg (CW): idem with radiative EWSB
- Classical conformality* $\Lambda = 0, \quad m_0 = 0$
- Delayed naturality = Veltman Cond. $f_2 = 0$ Pert.
- CW + Delayed naturality $f_2 = 0, \quad m_0 = 0$

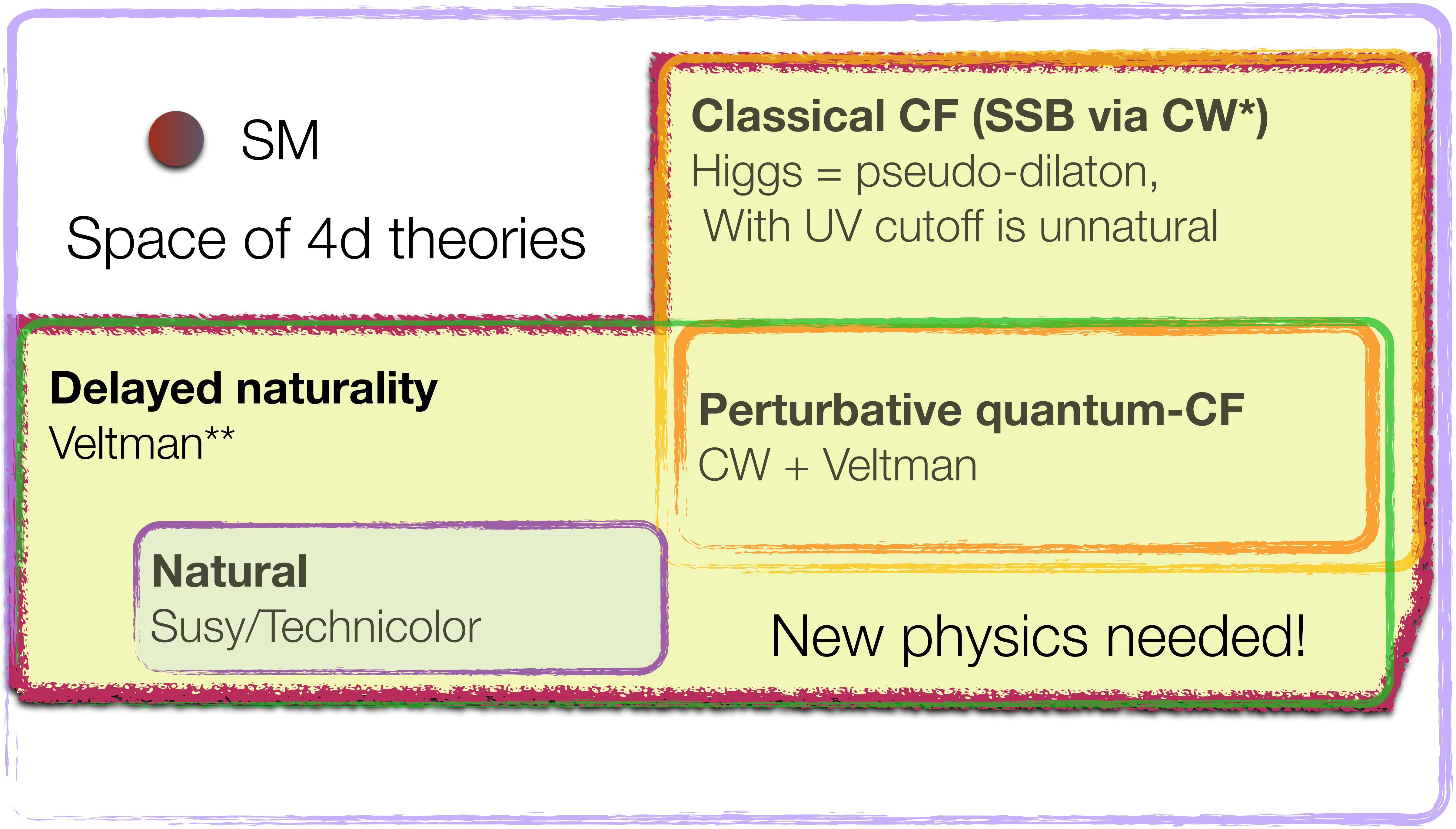
*Without a UV completion is indistinguishable from cancelling against cutoff

Natural theories

$$m^2 = m_0^2(1 + f_1(\lambda, g_i)\log\frac{\Lambda^2}{m_0^2}) - f_2(\lambda, g_i)\Lambda^2$$

- A symmetry exists protecting $f_2 = 0$
- Cutoff is physical as in composite models

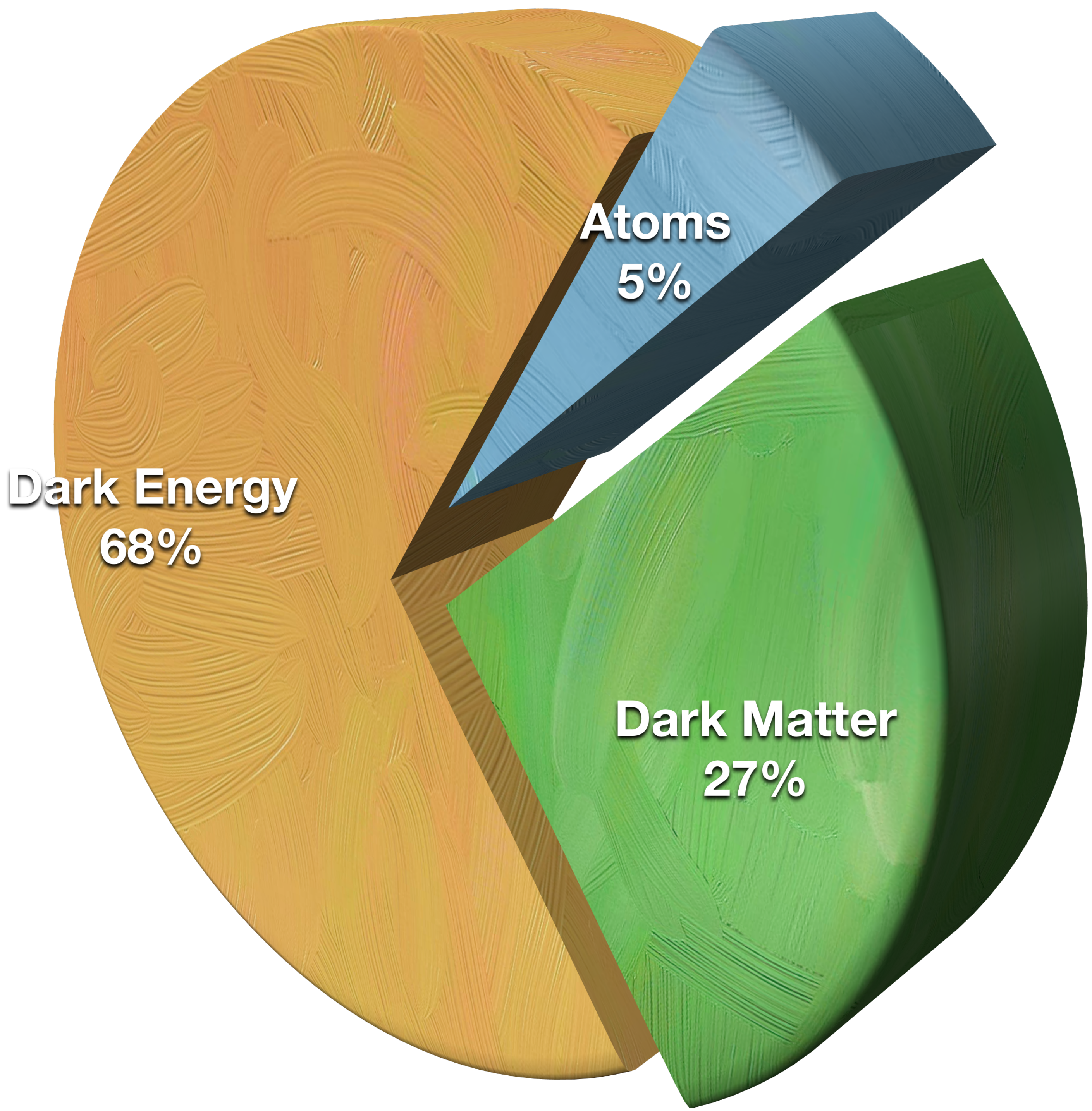
Degrees of naturality



* CW = Coleman-Weinberg

**Perturbative cancellation of quadratic divergences

Much ado for 5%



95% is unknown!

Richer than 5%? Most likely!

The Standard Model ado

Fields:

Gauge fields + fermions + scalars

Interactions:

Gauge: SU(3) x SU(2) x U(1) at EW scale

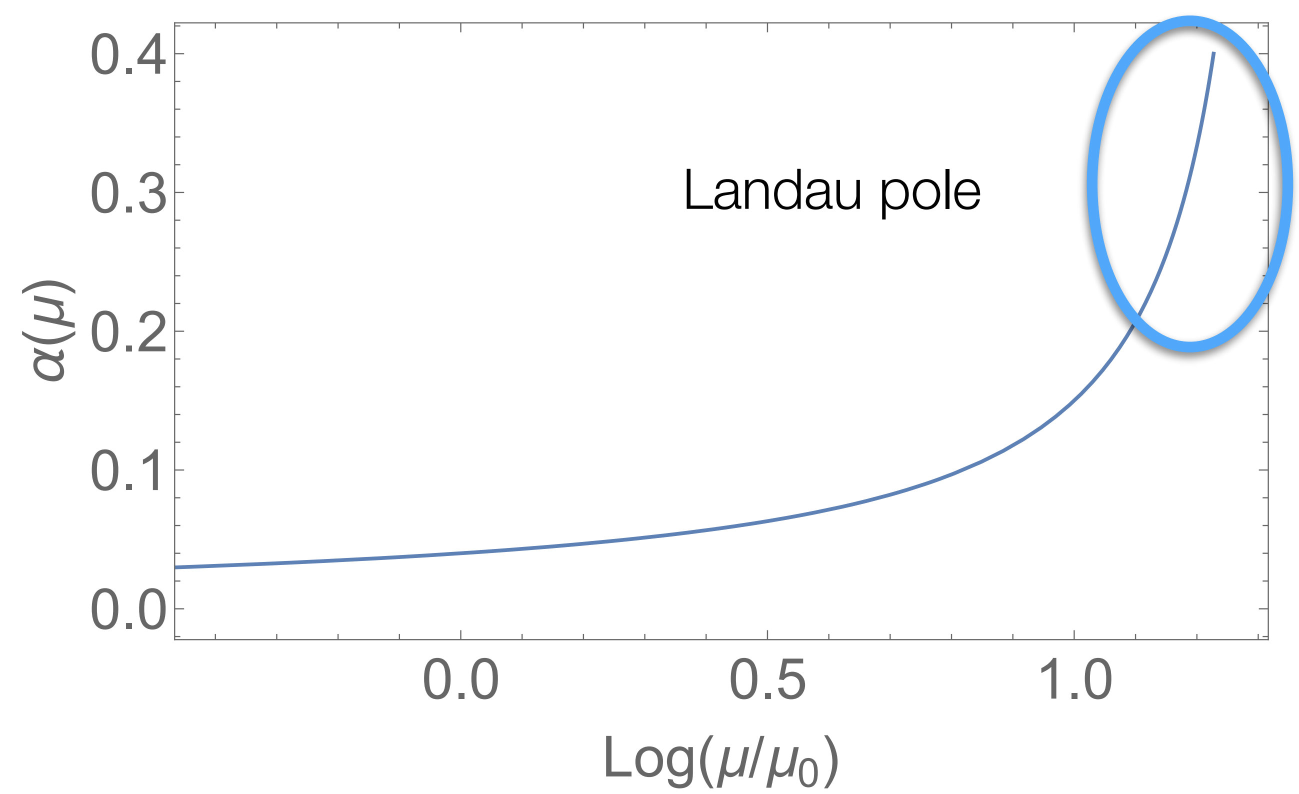
Yukawa: Fermion masses/Flavour

Culprit: Higgs

Scalar self-interaction

Two main issues

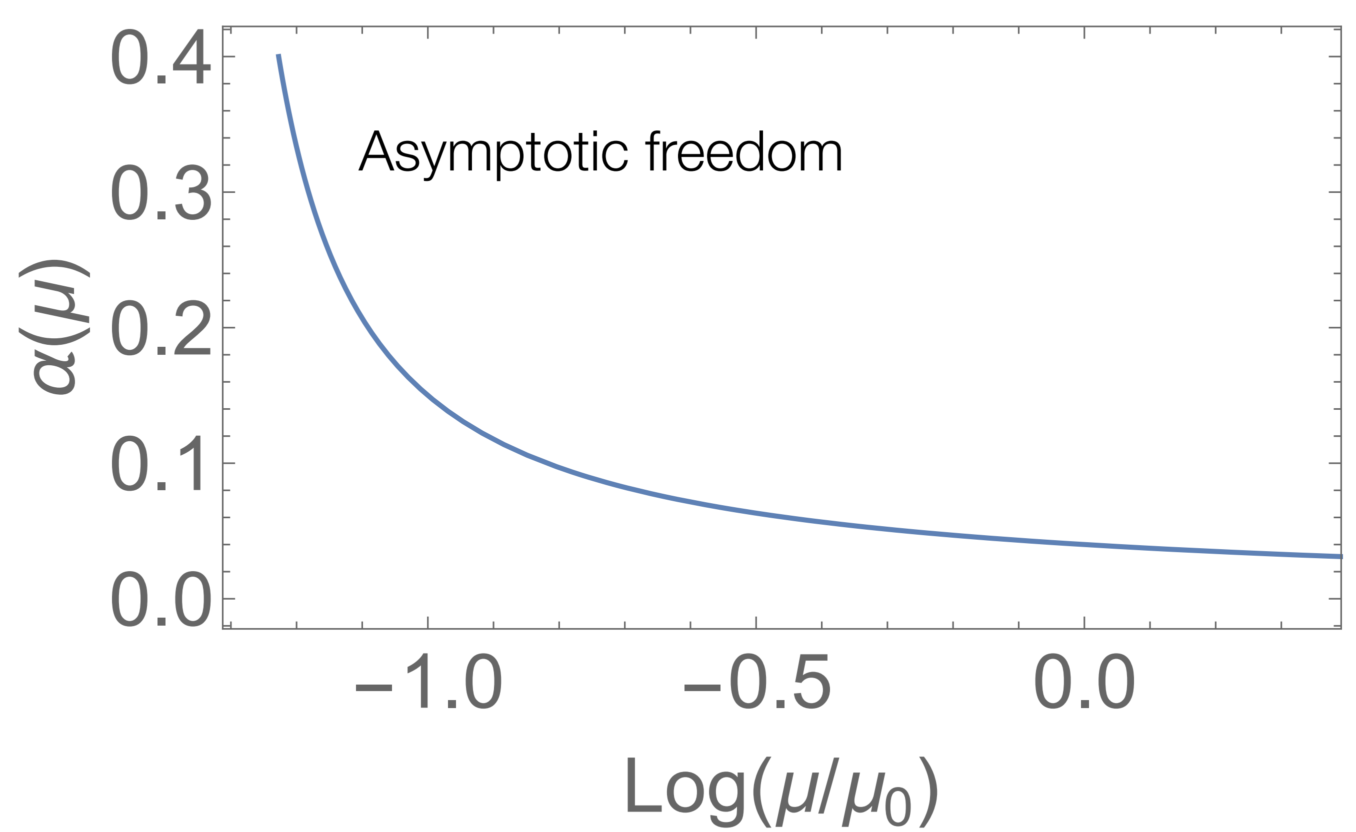
- EW scale stability
- UV triviality (Landau Pole)



The Compositeness Solution

- EW scale = Composite scale
- UV non-interacting
- Not ruled out

Arbey et al. 1502.04718



Elementary solution ?

Does an UV interacting safe 4D gauge theory exist?

Can we lose asymptotic freedom?

Exact Interacting UV Fixed Point
in
4D Quantum Gauge Theories

With D. Litim, 1406.2337, JHEP

$SU(N_C)$ Gauge-Yukawa Template

$$L_{\rm YM} = -\frac{1}{2}\,{\rm Tr}\left[F_{\mu\nu}F^{\mu\nu}\right]$$

$$L_F = i\,{\rm Tr}\left[Q\gamma^{\mu}D_{\mu}Q\right]$$

$$L_{\rm H} = {\rm Tr}\left[\partial_{\mu}H^{\dagger}\partial^{\mu}H\right]$$

$$L_{\rm Self} = -u\,{\rm Tr}\left[(H^{\dagger}H)^2\right] - v\,{\rm Tr}\left[(H^{\dagger}H)\right]^2$$

$$L_{\rm Y} = y\,\left({\rm Tr}\left[\overline{Q}_L H Q_R\right] + {\rm h.c.}\right)$$

Global symmetry $SU(N_F) \times SU(N_F) \times U_V(1)$

Veneziano Limit

- Normalised couplings

$$\alpha_g = \frac{g^2\,N_C}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2\,N_C}{(4\pi)^2}\,, \quad \alpha_h = \frac{u\,N_F}{(4\pi)^2}\,, \quad \alpha_v = \frac{v\,N_F^2}{(4\pi)^2}$$

$$\frac{v}{u} = \frac{\alpha_v}{\alpha_h N_F}$$

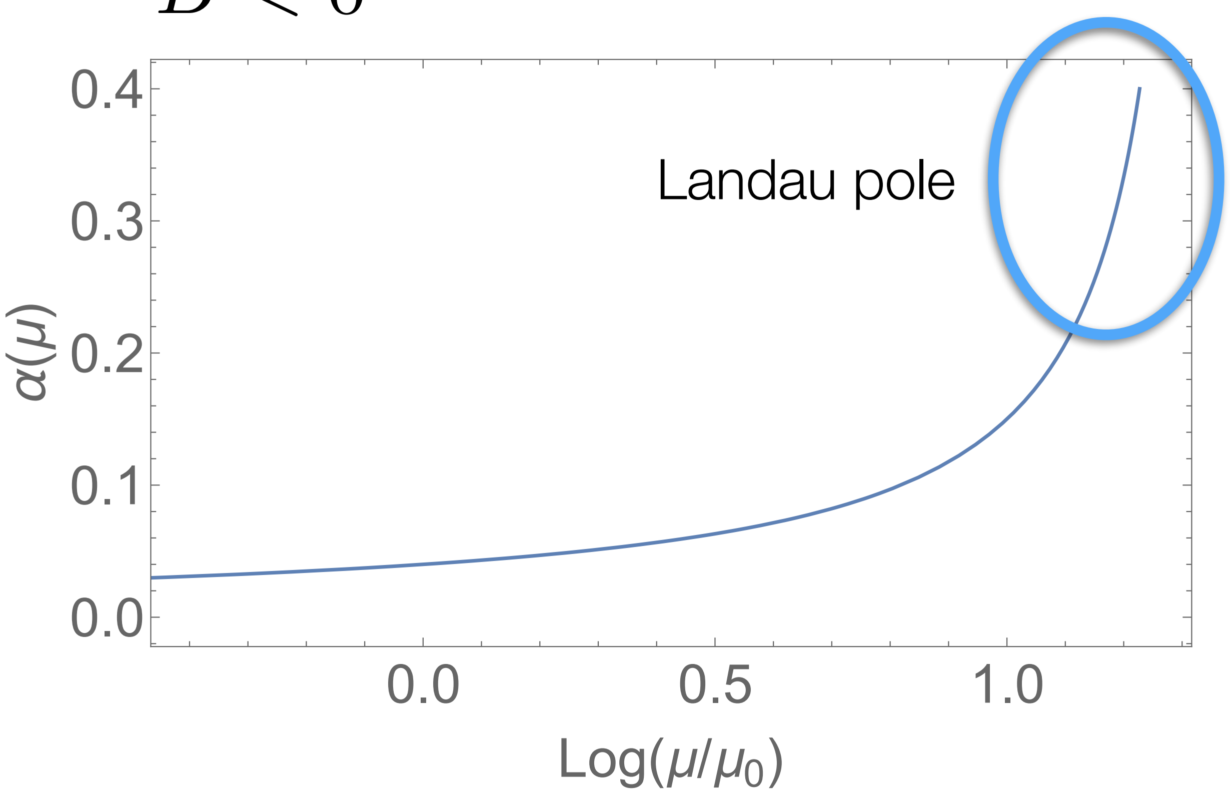
At large N $\quad \dfrac{N_F}{N_C} \in \Re^+$

Non-Asymptotically Free

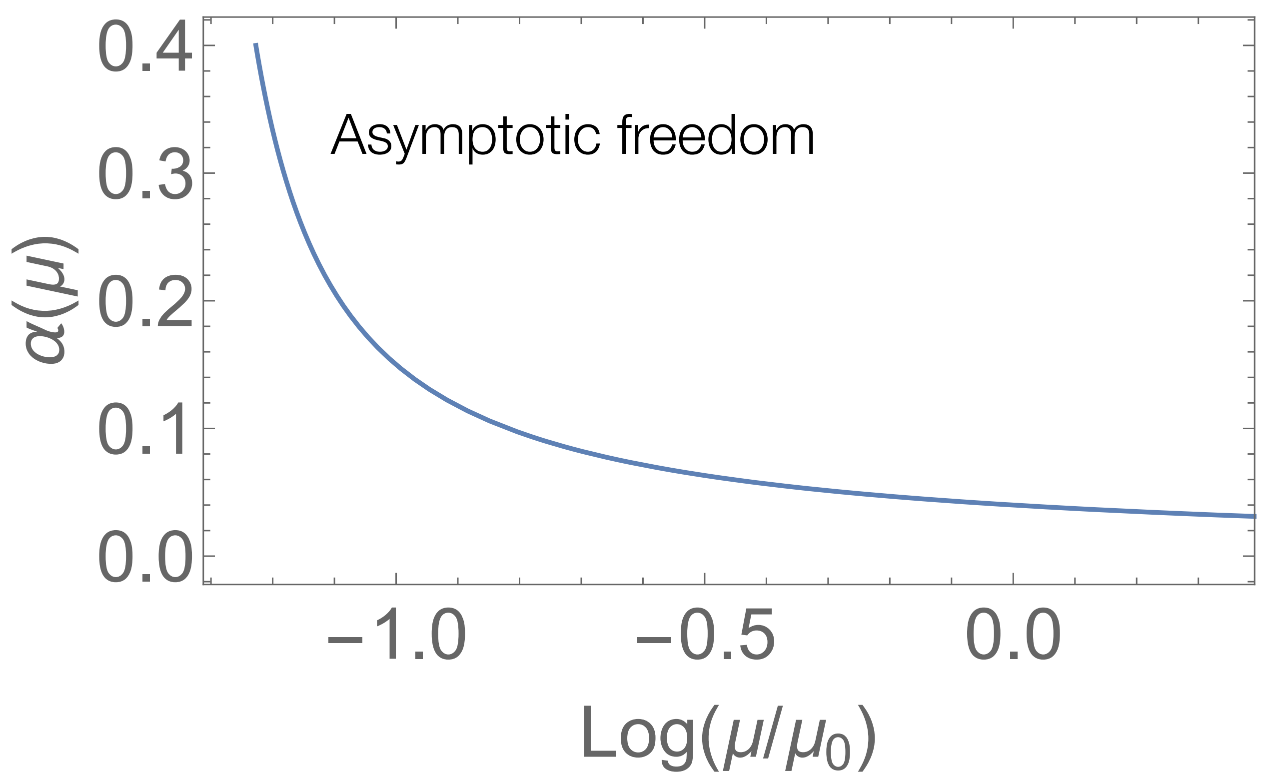
$$\beta_g = \partial_t \alpha_g = -B\alpha_g^2$$

$$t = \ln \frac{\mu}{\mu_0}$$

$B < 0$



$B > 0$

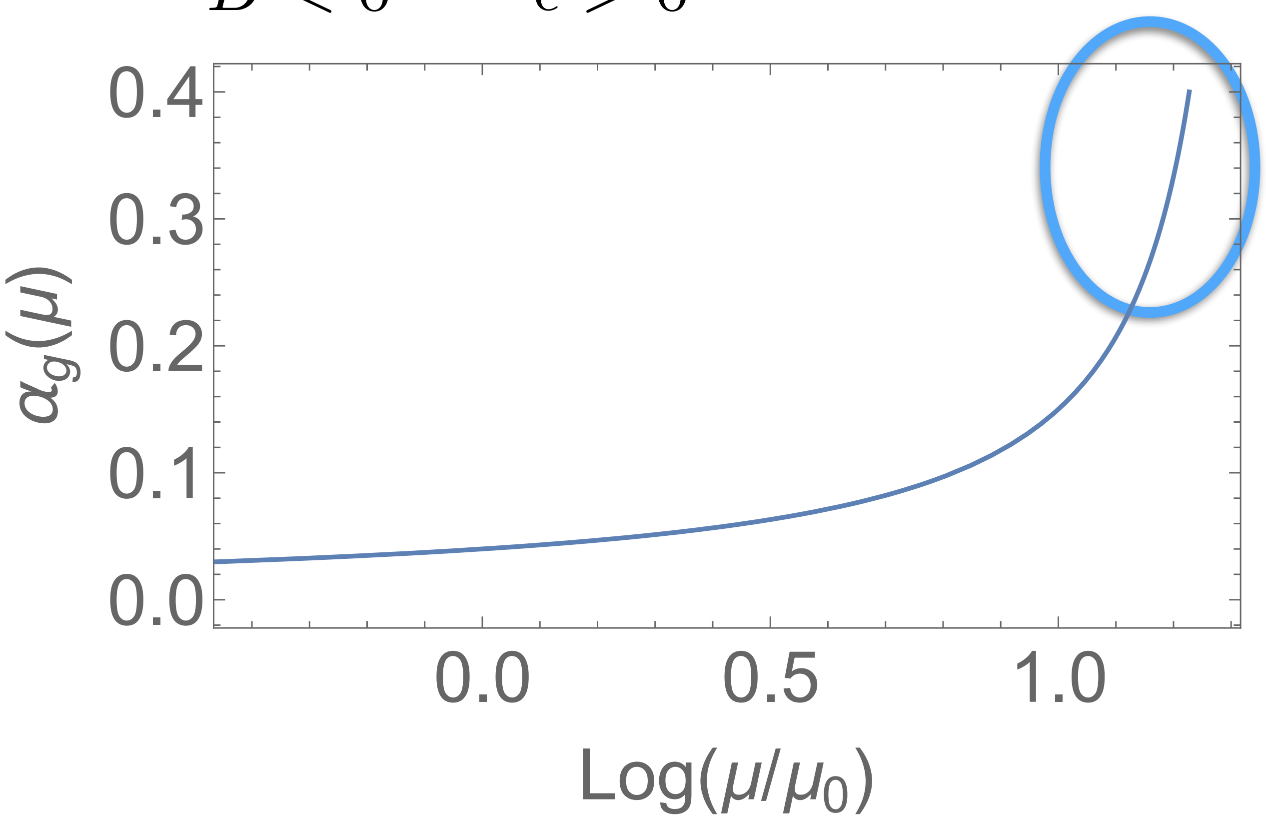


Small parameters

$$B = -\frac{4}{3}\epsilon$$

$$\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$$

$B < 0 \qquad \epsilon > 0$



$$0 \leq \epsilon \ll 1$$

Landau Pole ?

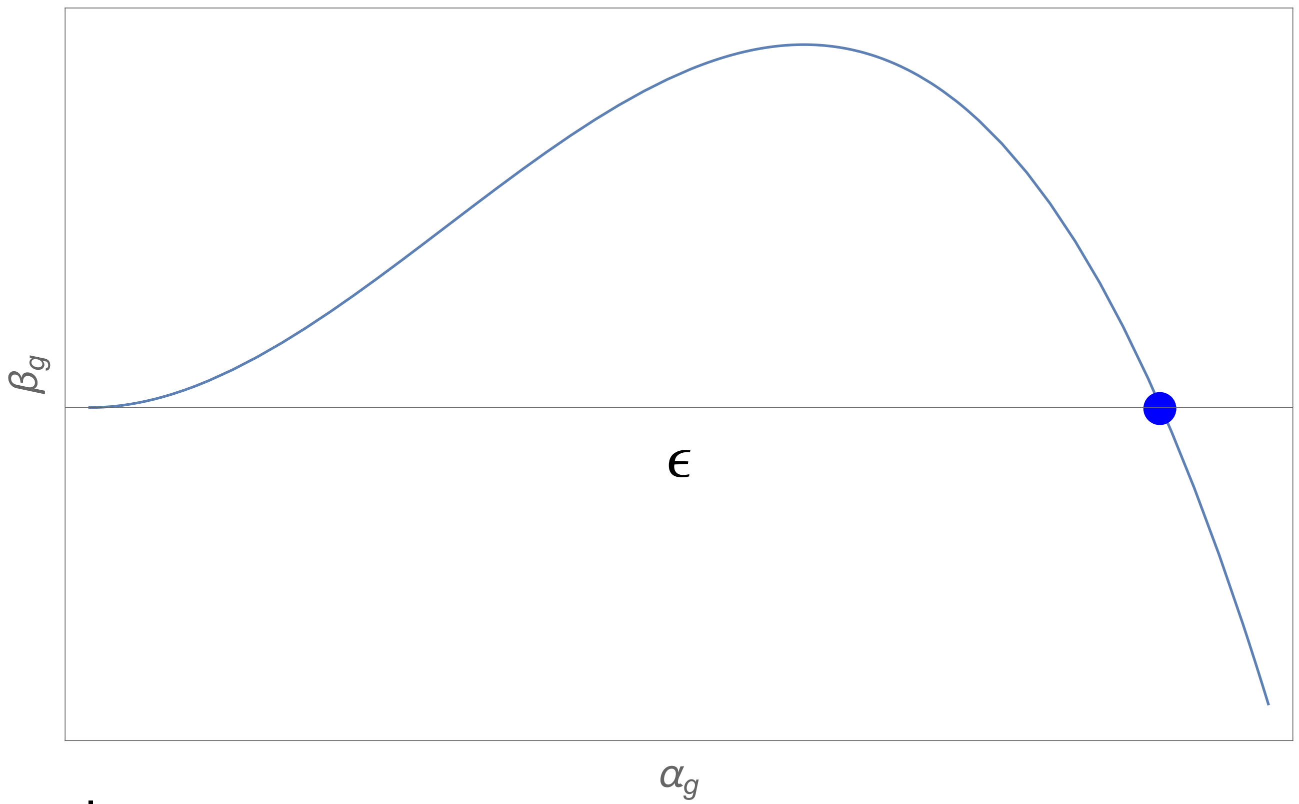
Can NL help?

$$\beta_g = -B\alpha_g^2 + C\alpha_g^3$$

$$B = -\frac{4}{3}\epsilon$$

$$0 \leq \alpha_g^* \ll 1 \quad \text{iff} \quad C < 0$$

$$\alpha_g^* = \frac{B}{C} \propto \epsilon$$



Impossible in Gauge Theories with Fermions alone

Caswell, PRL 1974

Add Yukawa

$$\beta_g = \alpha_g^2 \left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right]$$

$$\beta_y = \alpha_y \left[(13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g \right]$$

Computation abides Weyl consistency conditions

Osborn 89 & 91, Jack & Osborn 90

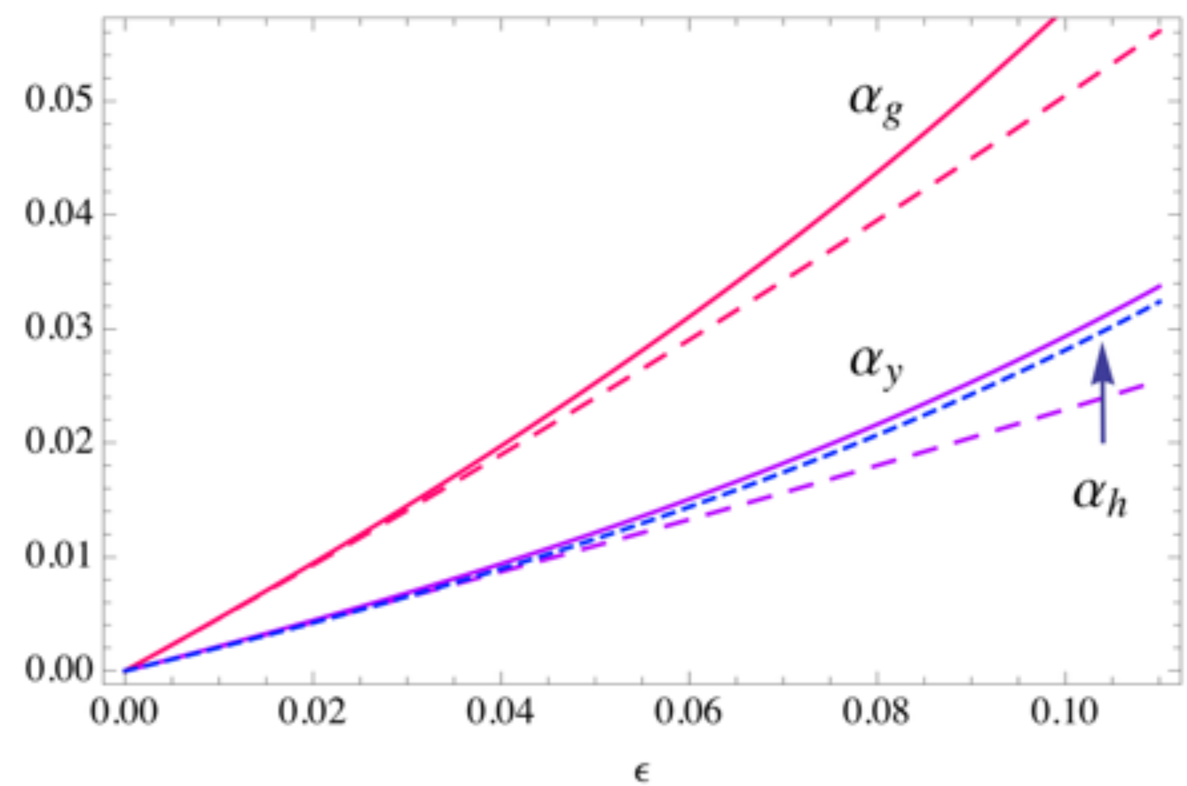
Antipin, Gillioz, Mølgaard, Sannino [a-theorem] 1303.1525

Antipin, Gillioz, Krog. Mølgaard, Sannino [SM vacuum stability] 1306.3234

NLO - Fixed Points

- Gaussian fixed point

$$(\alpha_g^*, \alpha_y^*) = (0, 0)$$



- Interacting fixed point

$$\alpha_g^* = \frac{26\epsilon + 4\epsilon^2}{57 - 46\epsilon - 8\epsilon^2} = \frac{26}{57}\epsilon + \frac{1424}{3249}\epsilon^2 + \frac{77360}{185193}\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$\alpha_y^* = \frac{12\epsilon}{57 - 46\epsilon - 8\epsilon^2} = \frac{4}{19}\epsilon + \frac{184}{1083}\epsilon^2 + \frac{10288}{61731}\epsilon^3 + \mathcal{O}(\epsilon^4)\,.$$

Linearised RG Flow

$$\delta\alpha = (\alpha - \alpha_*) \propto \left(\frac{\mu}{\Lambda_c}\right)^{\vartheta}$$

$$\vartheta = \partial\beta/\partial\alpha|_*$$

- Stability Matrix

$$\beta_i = \sum_j M_{ij}\,(\alpha_j - \alpha_j^*) + \text{subleading}$$

$$M_{ij} = \partial\beta_i/\partial\alpha_j|_*$$

$$i = (g, y)$$

Scaling exponents: UV completion

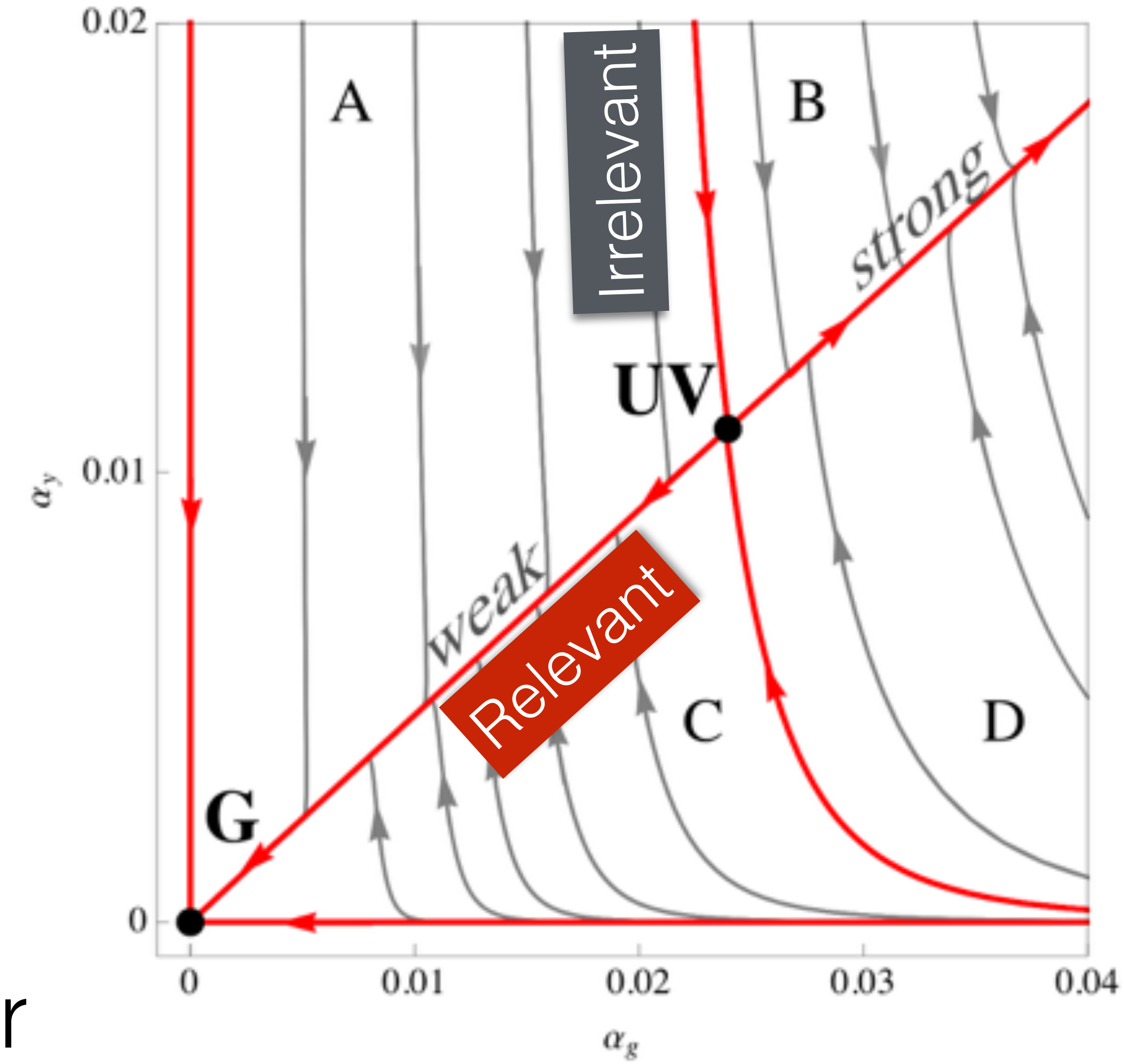
- Eigen values of M

$$\vartheta_1 = -\frac{104}{171}\epsilon^2 + \frac{2296}{3249}\epsilon^3 + \frac{1387768}{1666737}\epsilon^4 + \mathcal{O}(\epsilon^4)$$
$$\vartheta_2 = \frac{52}{19}\epsilon + \frac{9140}{1083}\epsilon^2 + \frac{2518432}{185193}\epsilon^3 + \mathcal{O}(\epsilon^4)\,.$$

$\vartheta_1 < 0$ Relevant direction

$\vartheta_2 > 0$ Irrelevant direction

A true UV fixed point to this order



NNLO - The scalars

- The scalar self-couplings

$$\beta_h = -(11 + 2\epsilon)\,\alpha_y^2 + 4\alpha_h(\alpha_y + 2\alpha_h)$$

Single trace

$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y)$$

Double trace

- Only single trace effect on Yukawa

$$\Delta\beta_y^{(2)} = \alpha_y \left\{ \frac{20\epsilon - 93}{6}\alpha_g^2 + (49 + 8\epsilon)\alpha_g\alpha_y - \left(\frac{385}{8} + \frac{23}{2}\epsilon + \frac{\epsilon^2}{2}\right)\alpha_y^2 - (44 + 8\epsilon)\alpha_y\alpha_h + 4\alpha_h^2 \right\}$$

$$\Delta\beta_g^{(3)} = \alpha_g^2 \left\{ \left(\frac{701}{6} + \frac{53}{3}\epsilon - \frac{112}{27}\epsilon^2\right)\alpha_g^2 - \frac{27}{8}(11 + 2\epsilon)^2\alpha_g\alpha_y + \frac{1}{4}(11 + 2\epsilon)^2(20 + 3\epsilon)\alpha_y^2 \right\}.$$

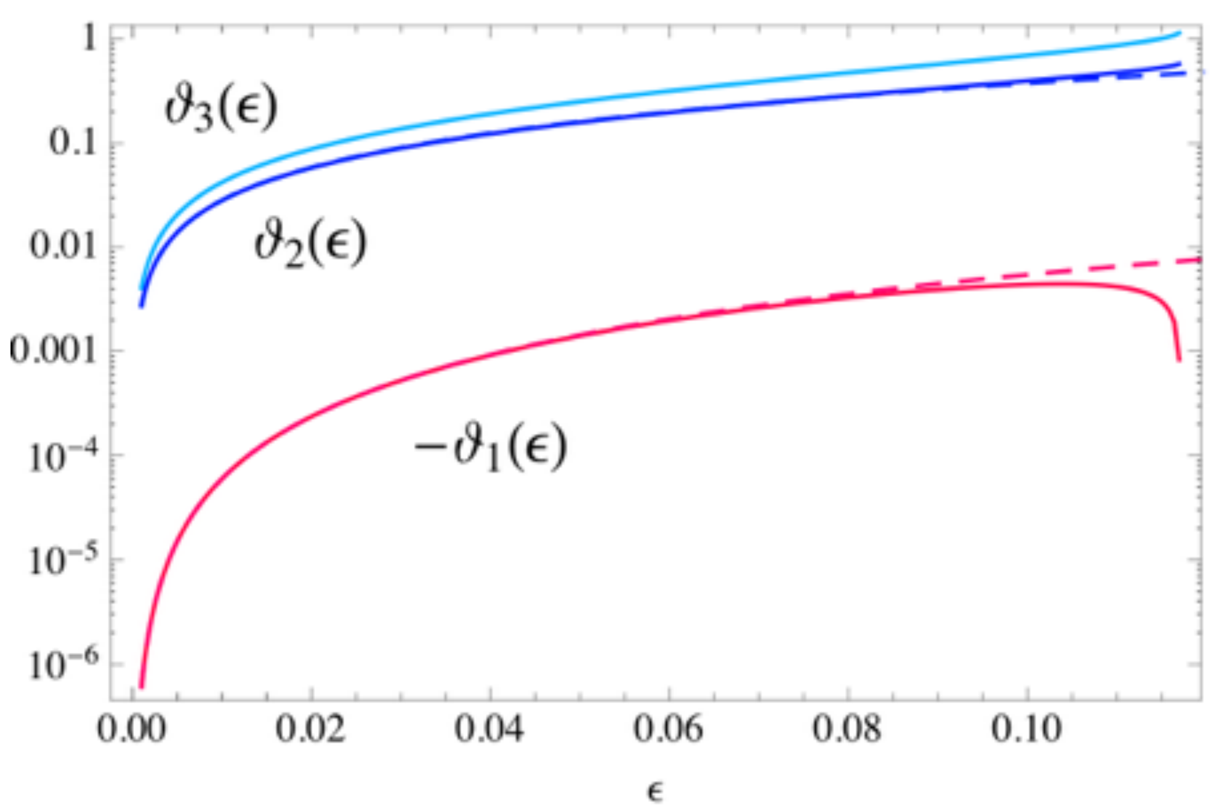
Double-trace coupling is a spectator

NNLO - All direction UV Stable FP

- Fixed point

$$\alpha_g^* = \quad 0.4561\,\epsilon + 0.7808\,\epsilon^2 + 3.112\,\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$\alpha_y^* = \quad 0.2105\,\epsilon + 0.5082\,\epsilon^2 + 2.100\,\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$\alpha_h^* = \quad 0.1998\,\epsilon + 0.5042\,\epsilon^2 + 2.045\,\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$\alpha_h^* \equiv \alpha_{h1}^* > 0$$



- Scaling exponents

$$\vartheta_1 = -0.608\,\epsilon^2 + 0.707\,\epsilon^3 + 2.283\,\epsilon^4 + \cdots$$
$$\vartheta_2 = \quad 2.737\,\epsilon \;\; + 6.676\,\epsilon^2 + \cdots$$
$$\vartheta_3 = \quad 4.039\,\epsilon \;\; + 14.851\,\epsilon^2 + \cdots\,.$$

$$i = (g, y, h)$$

Double - trace and stability

$$\alpha^*_{v1,v2} = -\frac{1}{19}\left(2\sqrt{23} \mp \sqrt{20 + 6\sqrt{23}}\right)\epsilon + \mathcal{O}(\epsilon^2)$$

- Is the potential stable at FP?
- Which FP survives?

Moduli

Classical moduli space

$$V = u\,\mathrm{Tr}\,(H^\dagger H)^2 + v\,(\mathrm{Tr}\,H^\dagger H)^2$$

Use $U(N_f) x U(N_f)$ symmetry $\quad H_c = \mathrm{diag}(h_1, \ldots, h_{N_F})$

$$V = u \sum_{i=1}^{N_F} h_i^4 + v \left(\sum_{i=1}^{N_F} h_i^2 \right)^2 - 2\lambda (\textstyle\sum_i h_i^2 - 1)$$

If V vanishes on H_c it will vanish for any multiple of it

Litim, Mojaza, Sannino 1501.03061

Ground state conditions at any Nf

$$\alpha_h > 0 \quad \text{and} \quad \alpha_h + \alpha_v \geq 0 \qquad H_c \propto \delta_{ij}$$

$$\alpha_h < 0 \quad \text{and} \quad \alpha_h + \alpha_v/N_F \geq 0 \qquad H_c \propto \delta_{i1}$$

$$V_\phi = (4\pi)^2(\alpha_h + \alpha_v)\phi^4 \qquad \alpha_h^* + \alpha_{v_2}^* < 0 < \alpha_h^* + \alpha_{v_1}^*$$

Stability for $\alpha_{v_1}^*$

UV critical surface

(Ir)relevant directions implies UV lower dim. critical

$$\alpha_i = F_i(\alpha_g) \qquad \alpha_i(\mu) = \alpha_i^* + \sum_n c_n \, V_i^n \left(\frac{\mu}{\Lambda_c}\right)^{\vartheta_n} + \text{subleading}$$

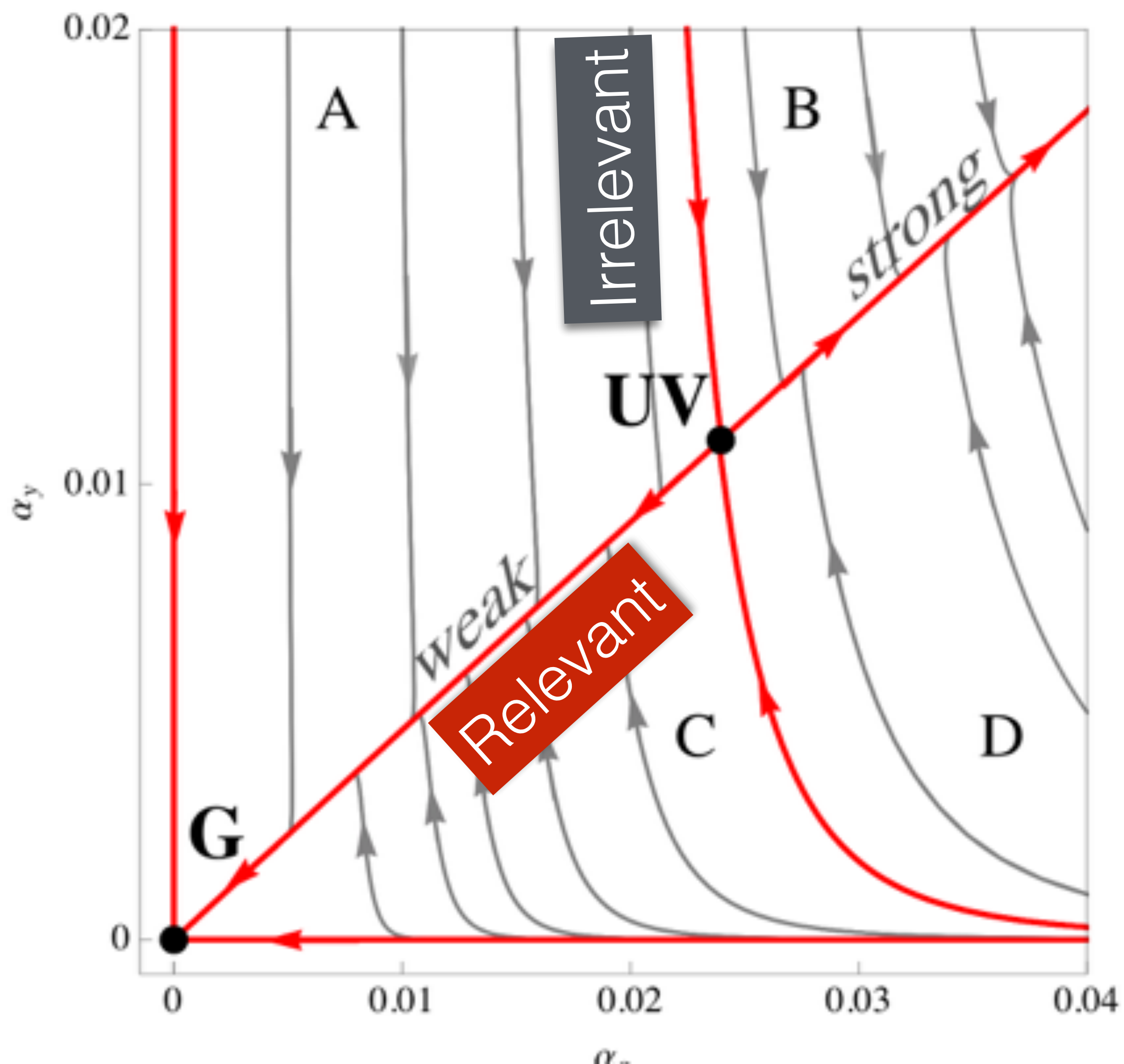
$$\begin{aligned}
F_y(\alpha_g) &= \quad (0.4615 + 0.6168\,\epsilon)\,\alpha_g - 0.1335\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\
F_h(\alpha_g) &= \quad (0.4380 + 0.5675\,\epsilon)\,\alpha_g - 0.09658\,\epsilon^2 + \mathcal{O}(\epsilon^3)\,, \\
F_v(\alpha_g) &= -(0.3009 + 0.3241\,\epsilon)\,\alpha_g + 0.1373\,\epsilon + 0.3828\,\epsilon^2 + \mathcal{O}(\epsilon^3)
\end{aligned}$$

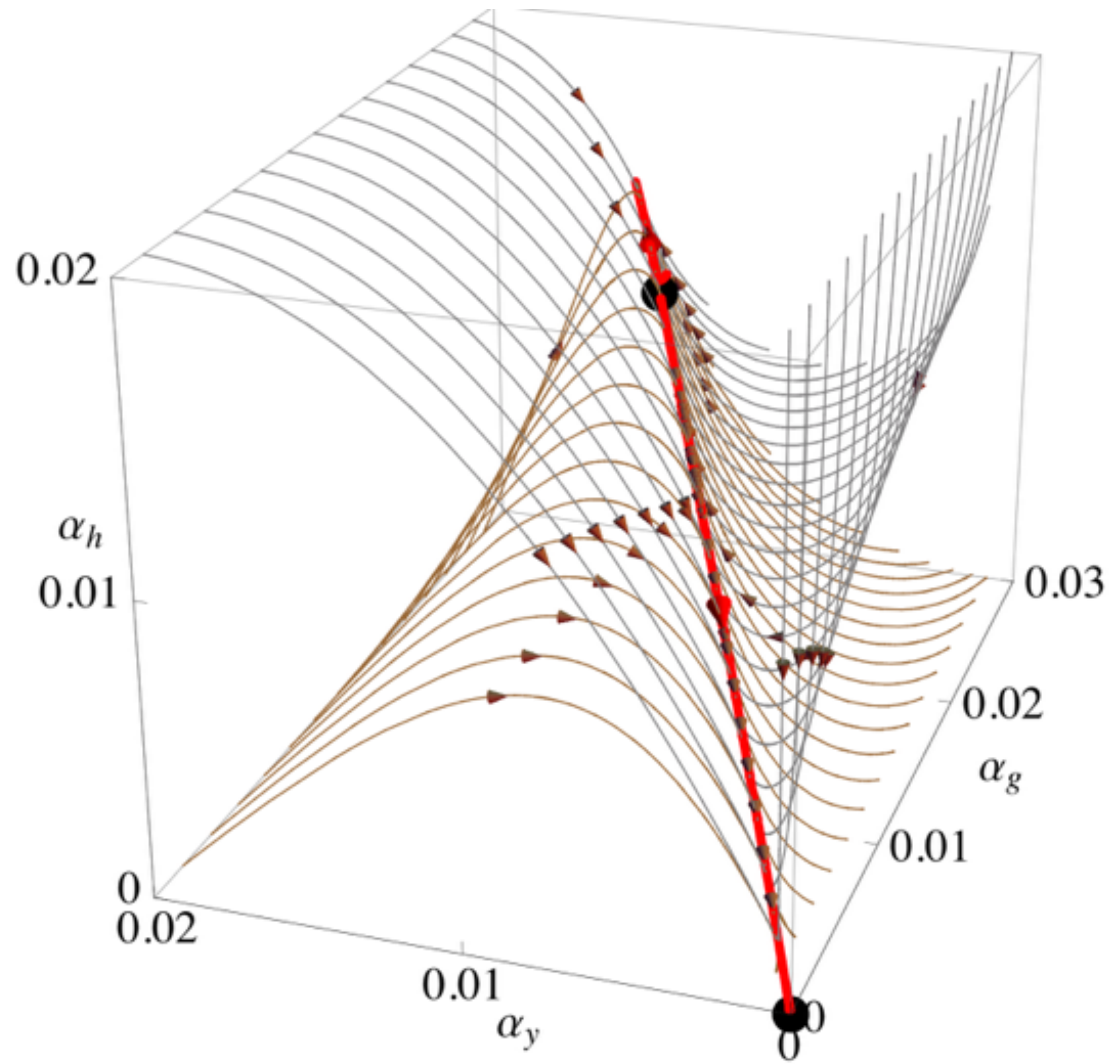
Near the fixed point

$$\alpha_g(\mu) = \alpha_g^* + \Big(\alpha_g(\Lambda_c) - \alpha_g^*\Big)\left(\frac{\mu}{\Lambda_c}\right)^{\vartheta_1(\epsilon)}$$

Phase Diagram

$$\vartheta_1 = -0.608\,\epsilon^2 + \mathcal{O}(\epsilon^3)$$
$$\vartheta_2 = \quad 2.737\,\epsilon + \mathcal{O}(\epsilon^2)$$
$$\vartheta_3 = \quad 4.039\,\epsilon + \mathcal{O}(\epsilon^2)$$
$$\vartheta_4 = \quad 2.941\,\epsilon + \mathcal{O}(\epsilon^2)\,.$$





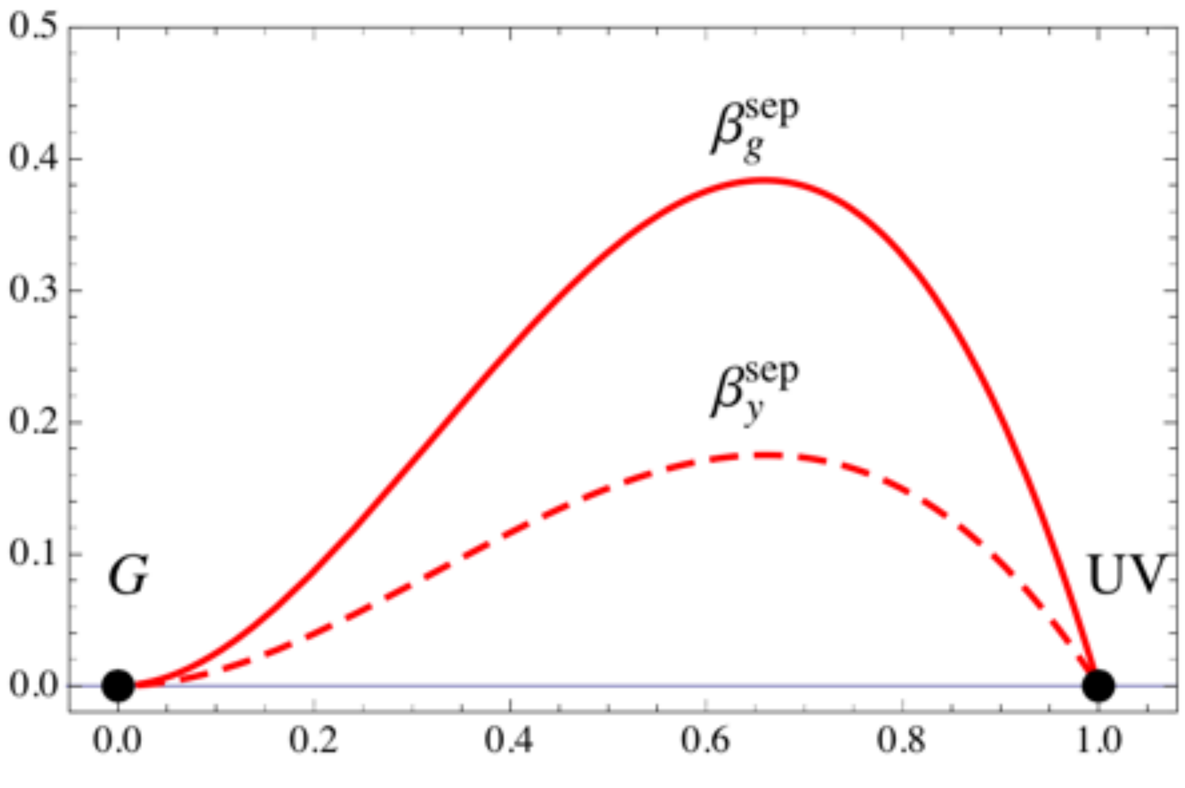
$$\vartheta_1 < 0 < \vartheta_2 < \vartheta_4 < \vartheta_3$$

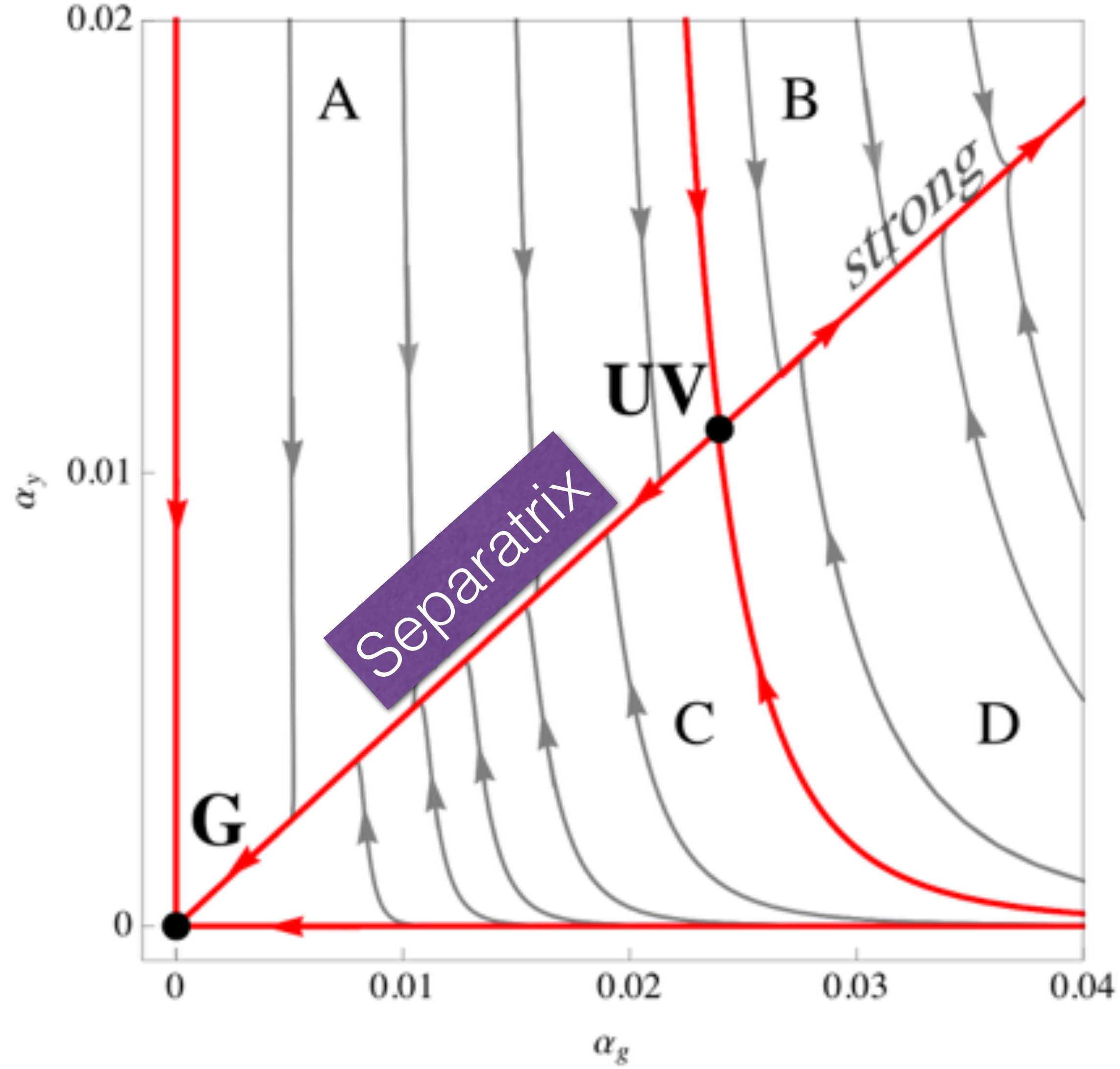
Separatrix = Line of Physics

Globally defined line connecting two FPs

$$\beta_g^{\text{sep}}(\alpha_g) \equiv \beta_g(\alpha_g, \alpha_y = F_y(\alpha_g))$$

$$\beta_y^{\text{sep}}(\alpha_g) \equiv \beta_y(\alpha_g, \alpha_y = F_y(\alpha_g))$$





Quantum Potential

The QP obeys an exact RG equation

$$\left(\mu_0 \frac{\partial}{\partial \mu_0} - \gamma(\alpha_j)\,\phi_c \frac{\partial}{\partial \phi_c} + \sum_i \beta_i(\alpha_j) \frac{\partial}{\partial \alpha_i}\right) V_{\text{eff}}(\phi_c, \mu_0, \alpha_j) = 0$$

$$H_c = \phi_c \delta_{ij} \qquad\qquad \gamma = -\frac{1}{2} d \ln Z / d \ln \mu$$

Resumming logs

Dimensional analysis $V_{\text{eff}}(\phi_c; \mu_0, \alpha_i) = \lambda_{\text{eff}}(\phi_c/\mu_0, \alpha_i) \cdot \phi_c^4$

$$\left(\phi_c \frac{\partial}{\partial \phi_c} + 4\,\bar{\gamma}(\alpha_j) - \sum_i \bar{\beta}_i(\alpha_j) \frac{\partial}{\partial \alpha_i} \right) \lambda_{\text{eff}}(\phi_c) = 0$$

$$\lambda_{\text{eff}}(\phi_c) = \lambda(\phi_c)\, \exp\left(-4 \int_{\mu_0}^{\phi_c} \frac{d\mu}{\mu}\, \bar{\gamma}(\mu) \right)$$

$$\bar{\gamma}(\alpha_i) = \frac{\gamma(\alpha_i)}{1 + \gamma(\alpha_i)}$$

The Potential

$$V_{\mathrm{cl}}(\phi_c) = \lambda_* \, \phi_c^4$$

$$\lambda_* = \epsilon \, \tfrac{16\pi^2}{19} (\sqrt{20 + 6\sqrt{23}} - \sqrt{23} - 1)$$

$$V_{\mathrm{eff}}(\phi_c) = \frac{V_{\mathrm{cl}}(\phi_c)}{1 + W(\phi_c)} \left(\frac{W(\phi_c)}{W(\mu_0)} \right)^{-4D/B}$$

$$-4D/B = 18/(13 \cdot \epsilon) > 0$$

Lambert Function

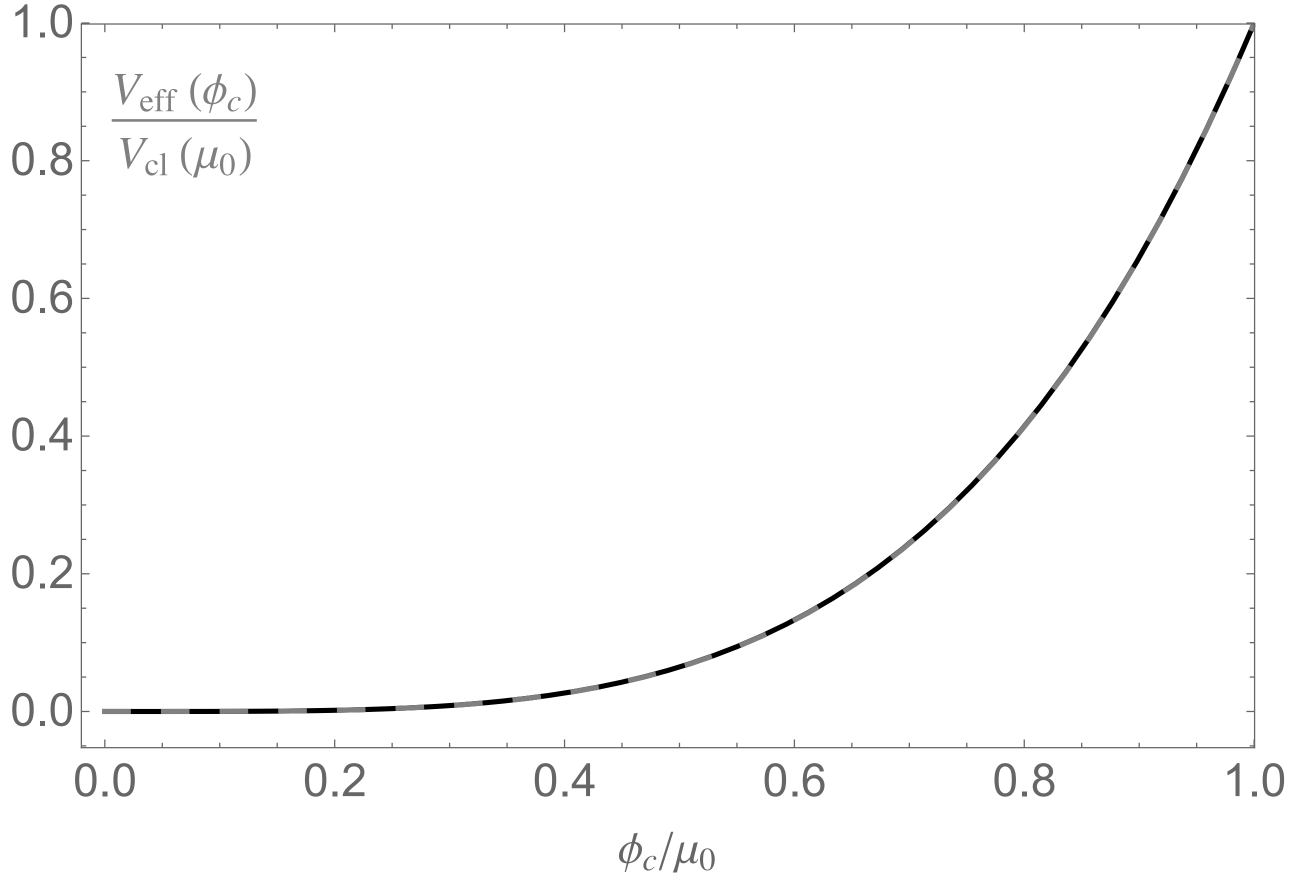
$$z = W \exp W$$

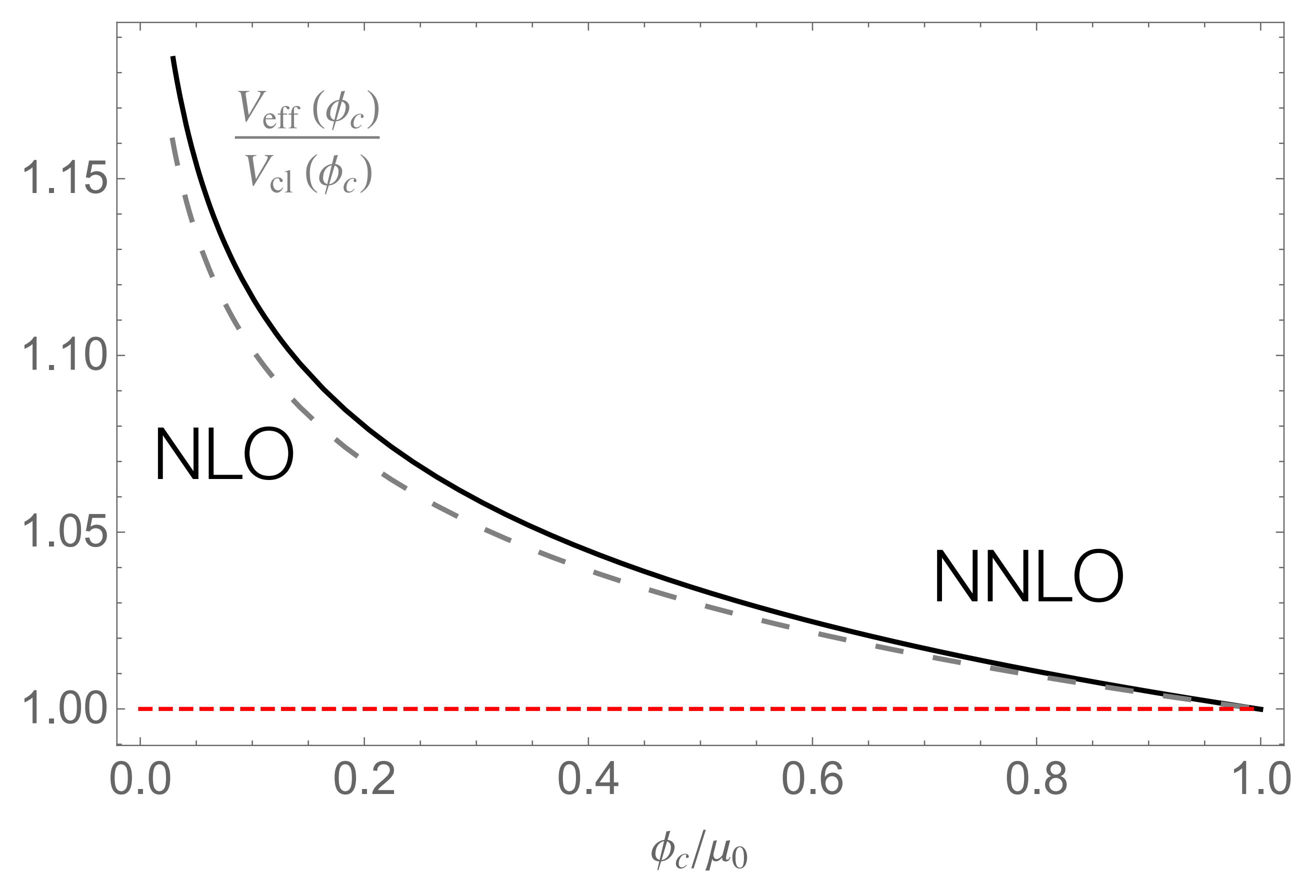
$$z = \left(\frac{\mu_0}{\mu} \right)^{-B \cdot \alpha_*} \left(\frac{\alpha_*}{\alpha_0} - 1 \right) \exp \left(\frac{\alpha_*}{\alpha_0} - 1 \right)$$

$$\alpha(\mu) = \frac{\alpha_*}{1 + W(\mu)}$$

Effective gauge coupling

Visualisation





QFT is controllably defined to arbitrary short scales

Summary

Gauge + fermion + scalars theories can be fund. at any energy scale

Exact results: independent on any scheme choice

Higgs mass squared operator is UV irrelevant

Existence of UV nontrivial Gauge-Yukawa theories

Discovered UV complete Non-Abelian QED-like theories

Outlook

Composite operators critical exponents, ...

Extend to other gauge theories

Extensions of the Standard Model

Models of DM and/or Inflation, 1412.8034 & 1503.00702

Hope for asymptotic safe quantum gravity?