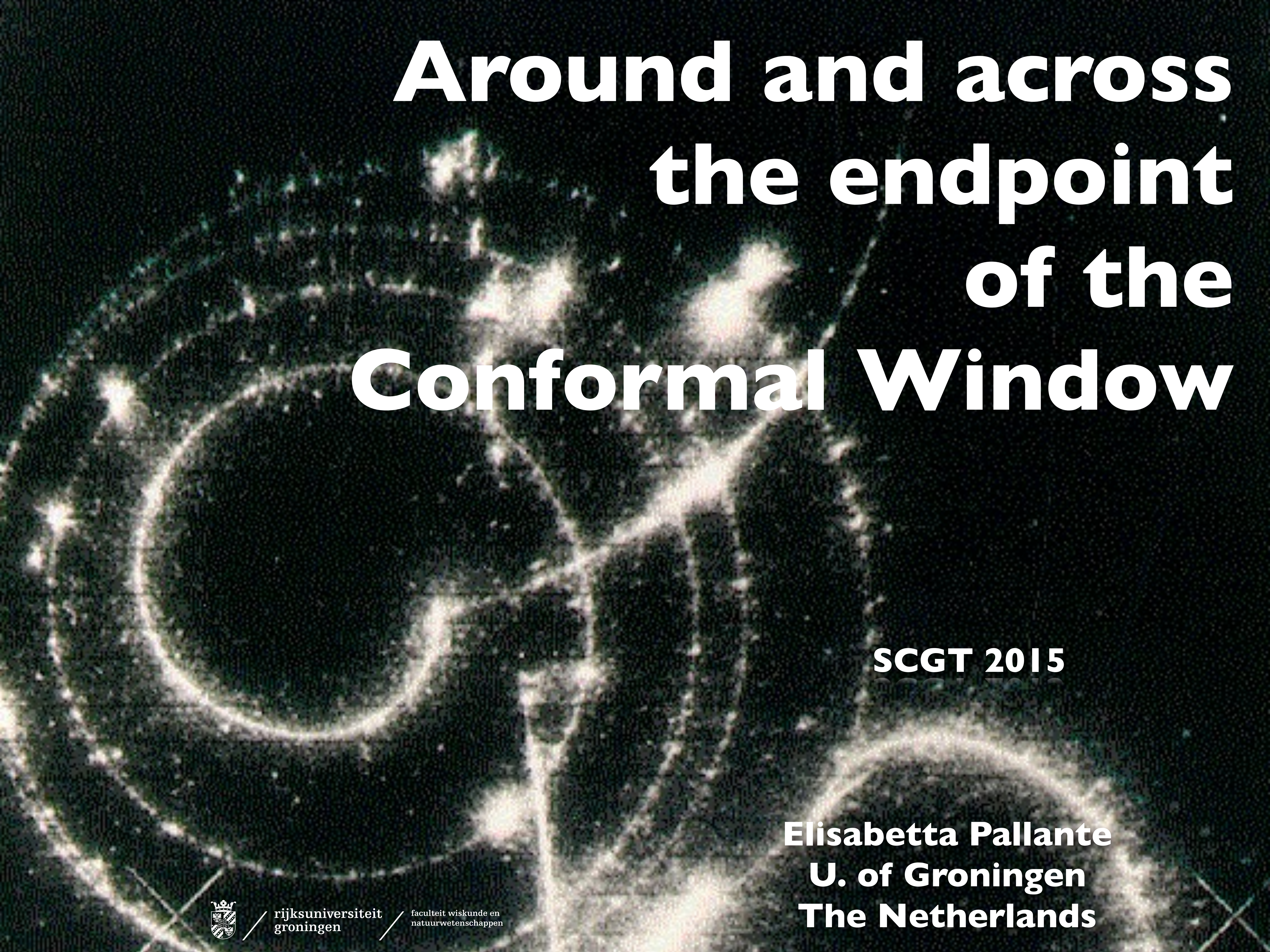

Around and across the endpoint of the Conformal Window

SCGT 2015

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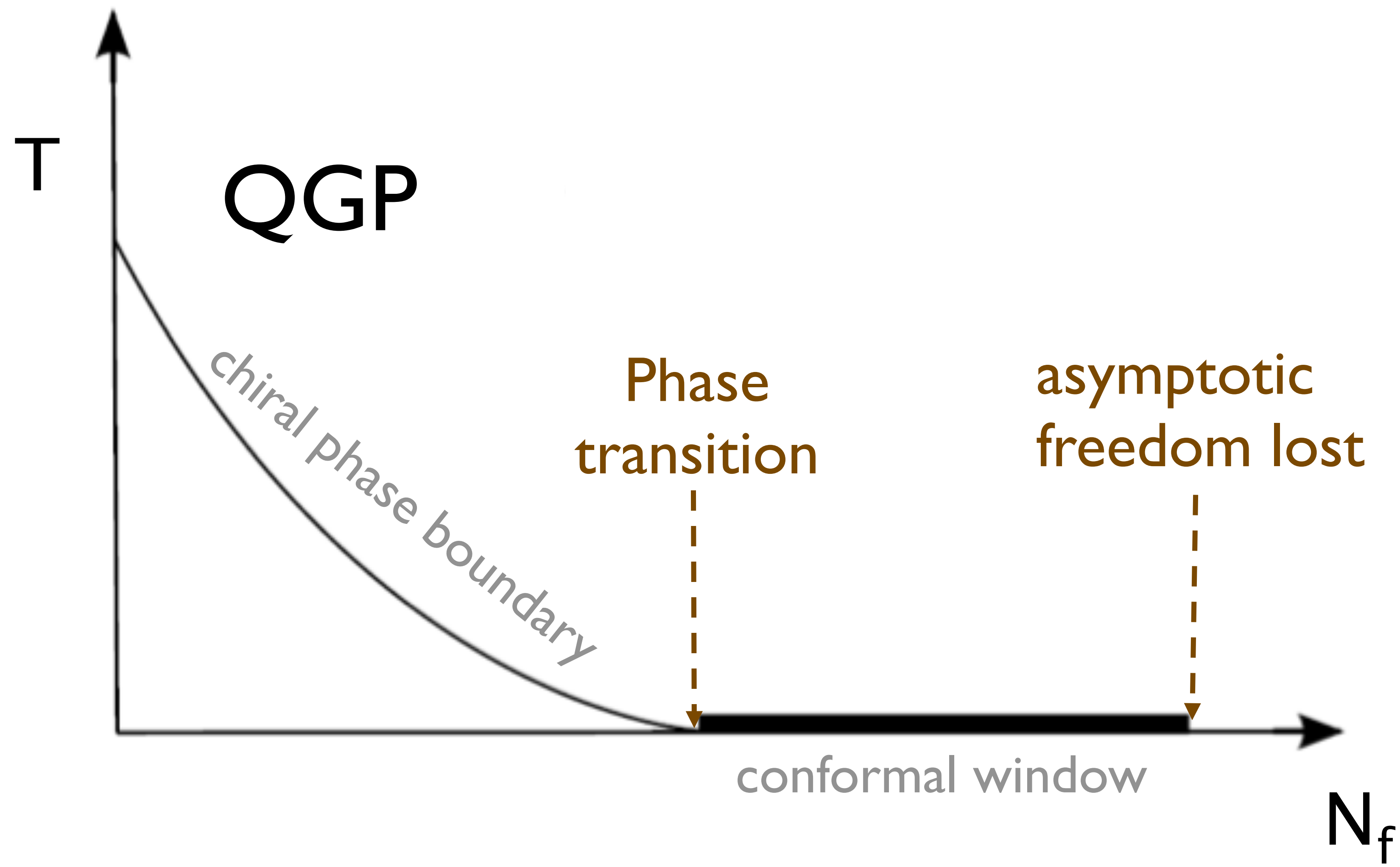
Outline

- The energy flow
- Topology
- The spectrum

Phase diagram: T-N_f plane



The energy flow

$$E = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

$$\dot{B}_\mu = D_\nu G_{\nu\mu} + \lambda(=1)\, D_\mu \partial_\nu B_\nu$$

$$B_\mu|_{t=0} = A_\mu$$

Wilson flow (modified)

Perturbation theory close to trivial UVFP [LUSCHER 2010]

$$t^2 \langle E \rangle = \frac{3}{4\pi} \alpha(q) \left\{ 1 + k_1 \alpha(q) + O(\alpha^2) \right\}$$

$$q \equiv \mu = 1/\sqrt{8t}$$

Wilson flow "time" t

$$k_1 = \frac{1}{4\pi} \left\{ 11\gamma_E + \frac{52}{3} - 9 \ln 3 - N_f \left(\frac{2}{3}\gamma_E + \frac{4}{9} - \frac{4}{3} \ln 2 \right) \right\} = 1.0978 + 0.0075\, N_f$$

- It does not need renormalisation (NLO in perturbation theory at least)
- Residual "t" dependence due to breaking of conformal symmetry

Larger Wilson flow time/beyond perturbation theory

Confining $t^2\langle E\rangle \sim t$

IRFP $\langle E\rangle = \frac{1}{4}\langle \mathrm{Tr(GG)}\rangle = C\frac{\alpha^*}{t^{2+\gamma_G/2}}$

Trace anomaly constrains this anomalous dimension

Trace anomaly of QCD

$$T^\mu_\mu = \frac{\beta(\alpha)}{16\pi\alpha^2}\,\mathrm{Tr}G^2 + \text{fermionic terms}$$

$$\beta(\alpha) \equiv \frac{d\alpha(\mu)}{d\ln\mu} \qquad \alpha \equiv \frac{g^2}{4\pi}$$

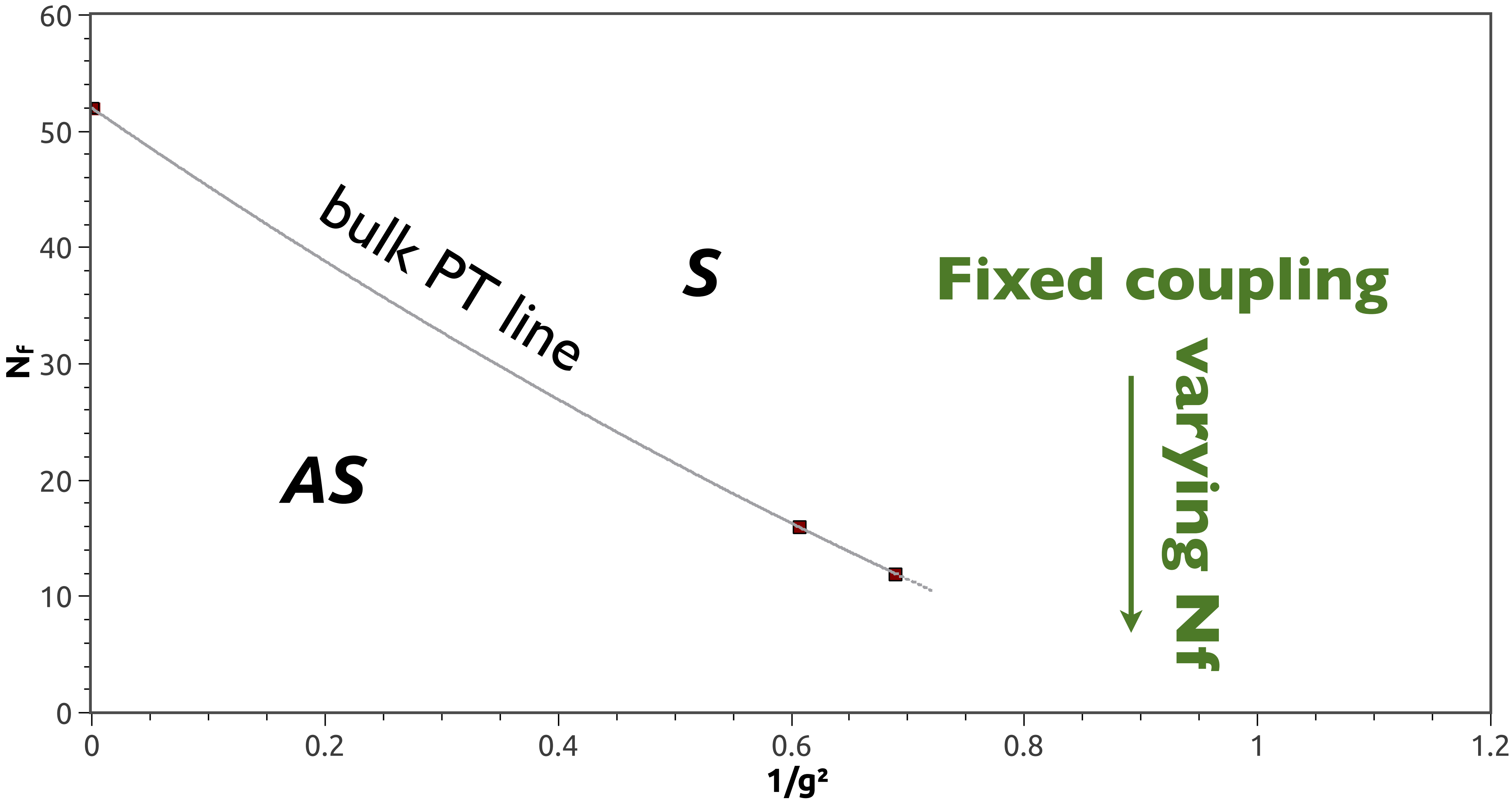
Scaling of a quantum operator

$$\Delta_O O = \frac{dO}{d\ln\mu} \quad O(\mu) \sim \mu^{\Delta_O} \qquad \Delta_O = \Delta_c + \gamma_O$$

Non renormalization of T^μ_μ implies $\Delta_{T^\mu_\mu} = 4 \text{ in d=4}$

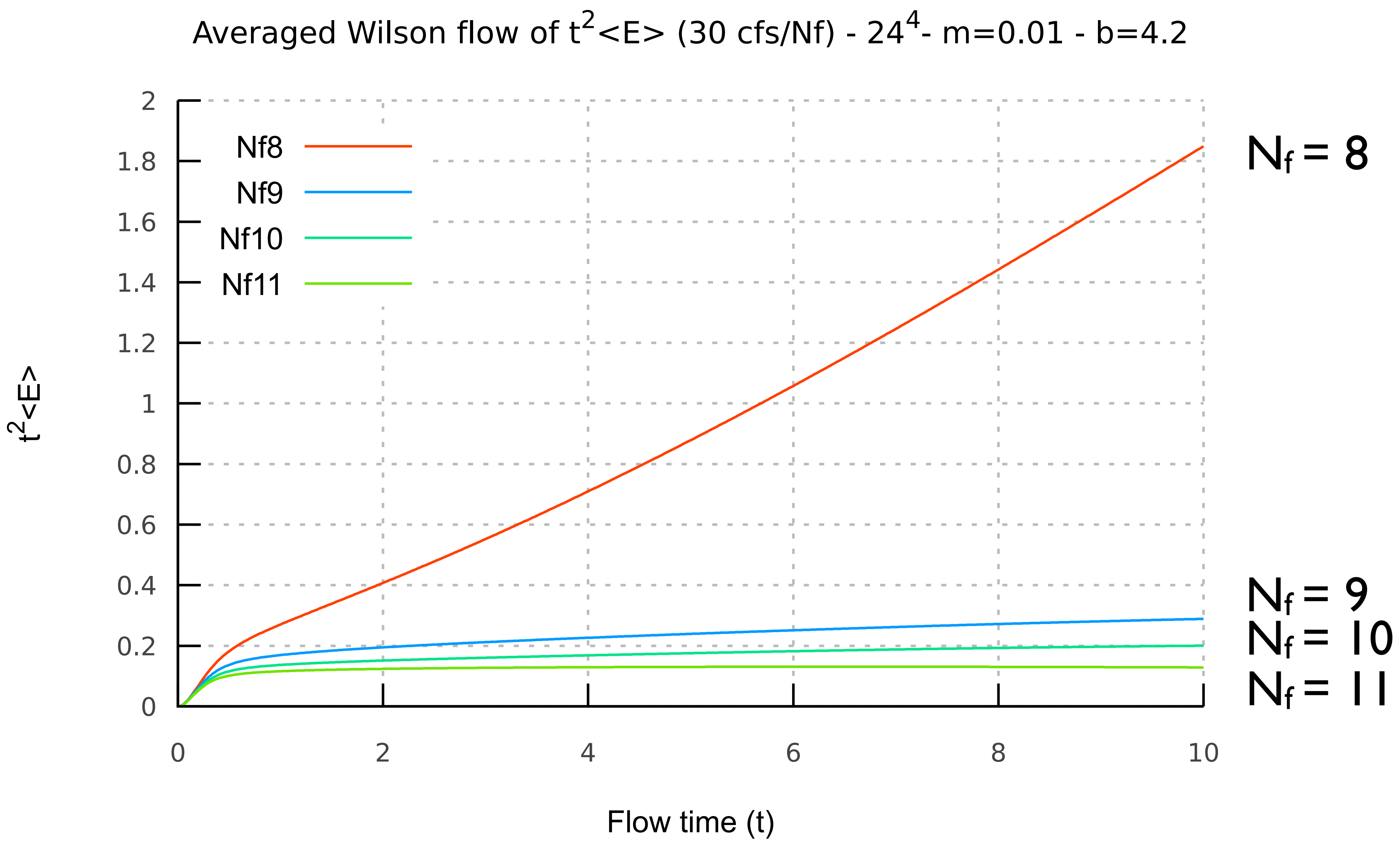
$$\Delta_G = 4 - \beta'(\alpha) + \frac{2}{\alpha}\beta(\alpha) \qquad \boxed{\gamma_G = -\beta'(\alpha^*) \ \text{IRFP}}$$

Illustration of method on the lattice



w L. Robroek

PRELIMINARY



...and do not use this figure to decide where the endpoint lies

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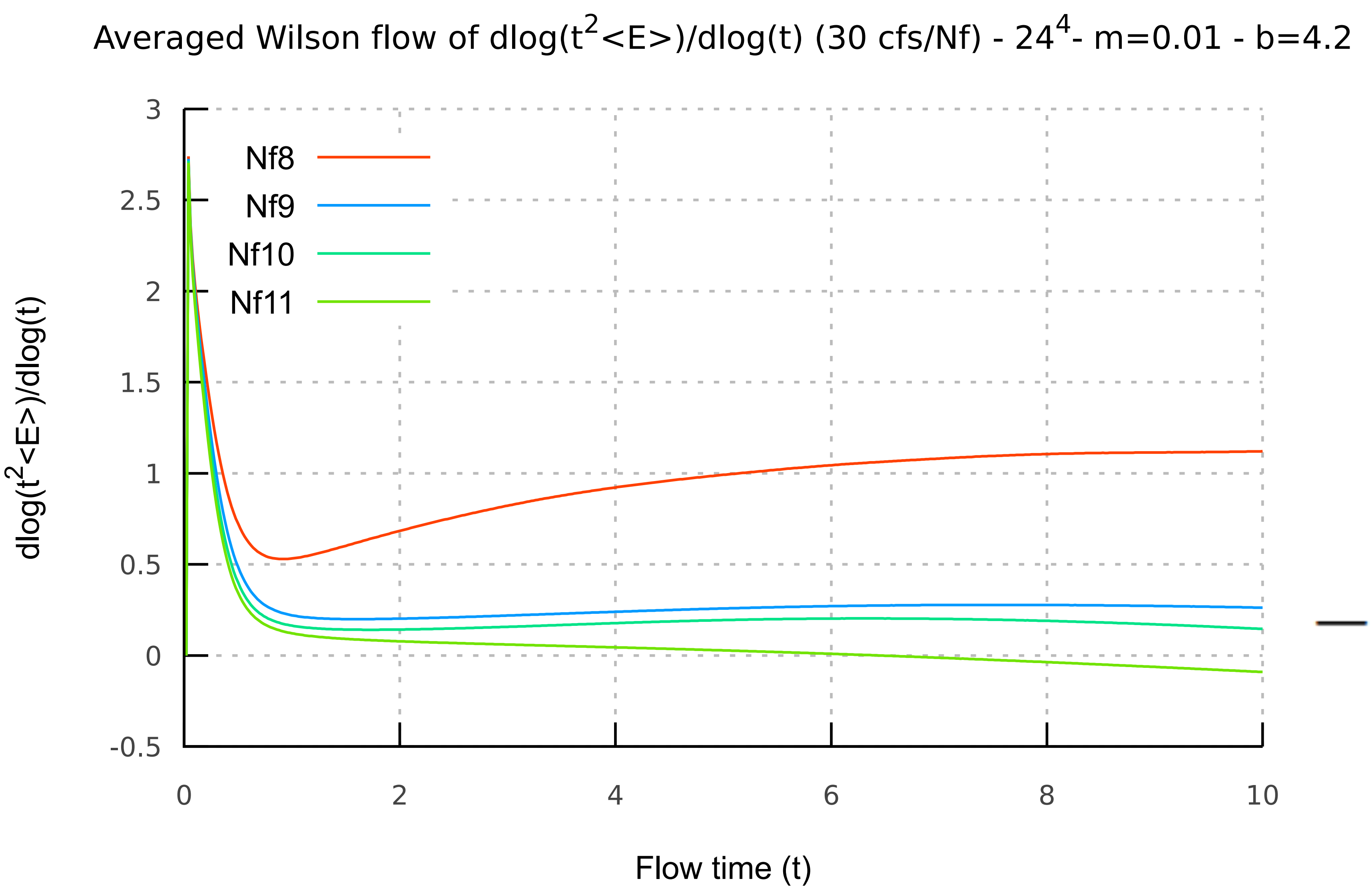
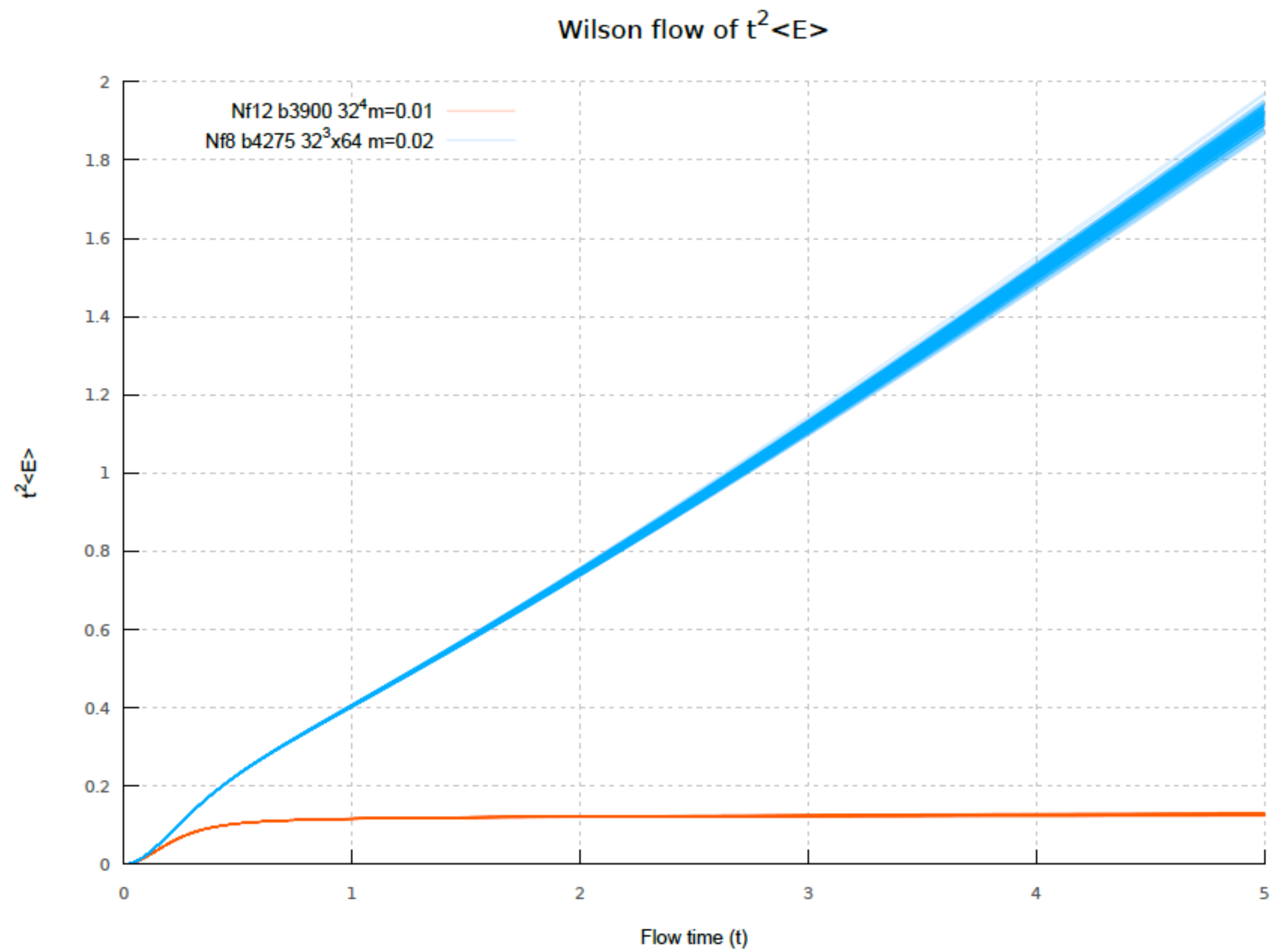
PRELIMINARY



$-\gamma_G/2$

Nf=8 and 12 configurations by Lombardo, Miura, Nunes, EP 2013, 2014



not averaged

w L. Robroek

Anomalous dimension in perturbation theory

$$\alpha_* = -\frac{\beta_0}{\beta_1}$$

$$\beta'(\alpha_*) = -\frac{\beta_0^2}{\beta_1}$$

[Ritbergen Vermaseren Larin 1997]

2 loop		
N_f	$a_* = \alpha_*/\pi$	$\beta'(\alpha_*)$
8	No zero	
9	~ 1.6	~ 4
10	~ 0.7	~ 1.7
12	~ 0.23	~ 0.3
4 loop		
8	~ 0.5	~ 2.6
9	~ 0.35	~ 1.5
10	~ 0.25	~ 0.8
12	~ 0.15	~ 0.25

Topology
(and the U(1) axial anomaly)

3 symmetries play a role at the endpoint

- Conformal symmetry
- Chiral (flavour) symmetry
- U(1) axial

How many order parameters ?

Topological charge and susceptibility

$$\boxed{Q = \frac{1}{32\pi^2}\int_V d^4x\, G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}}$$

/winding number

$$\boxed{\chi_t = \frac{\langle Q^2\rangle}{V} = \frac{\partial^2 Z}{\partial\theta^2}\bigg|_{\theta=0}}$$

second moment of Q distribution

The U(1) axial anomaly

$$\partial_\mu J^5_\mu = \frac{g^2 N_f}{16\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} + \text{explicit fermion mass contribution}$$

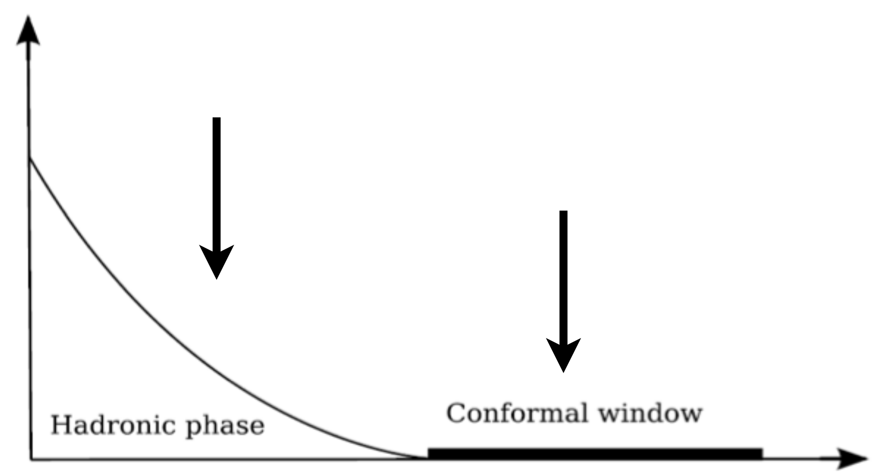
Axial current conserved for

- $N_f = 0$
- $N_c = \infty$ relevant for AdS/CFT arguments

The effective restoration of the U(1) axial symmetry

[SEE ALSO:
DEUZEMAN, LOMBARDO, EP 2010
LOMBARDO, MIURA, NUNES, EP 2012, 13]

$N_f > 2, T > T_c$ QCD shares a lot with Conformal Window
One difference: ∃ IRFP



For $N_f > 2$ there is one relevant order parameter

$\langle \bar{\psi}_L \psi_R \rangle$	Chiral condensate
$\langle \det \bar{\psi}^f_L \psi^f_R \rangle \sim \Lambda^{3N_f}$	U(1) axial order parameter (irrelevant for IRFP physics)

It suggests one single phase transition for chiral and U(1) axial (effective) restoration

$\Rightarrow$ sharp change in topological susceptibility

$\sim$ sharp change in instanton distribution

[e.g.
Del Debbio, Panagopoulos, Vicari 2004 T>Tc
Parnachev&Zhitnitsky, Zhitnitsky 2013 CW

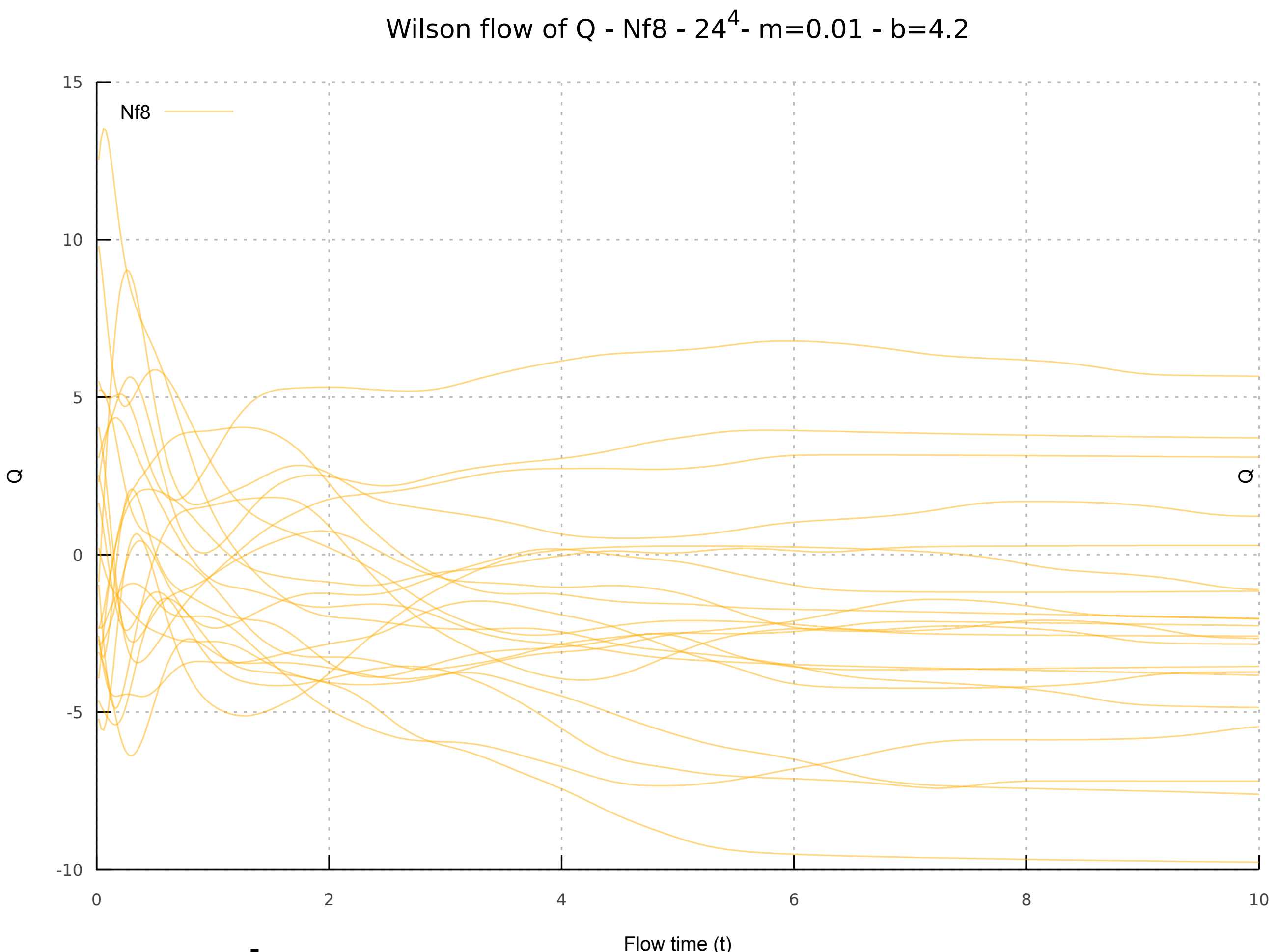
Reverse ‘t Hooft argument:

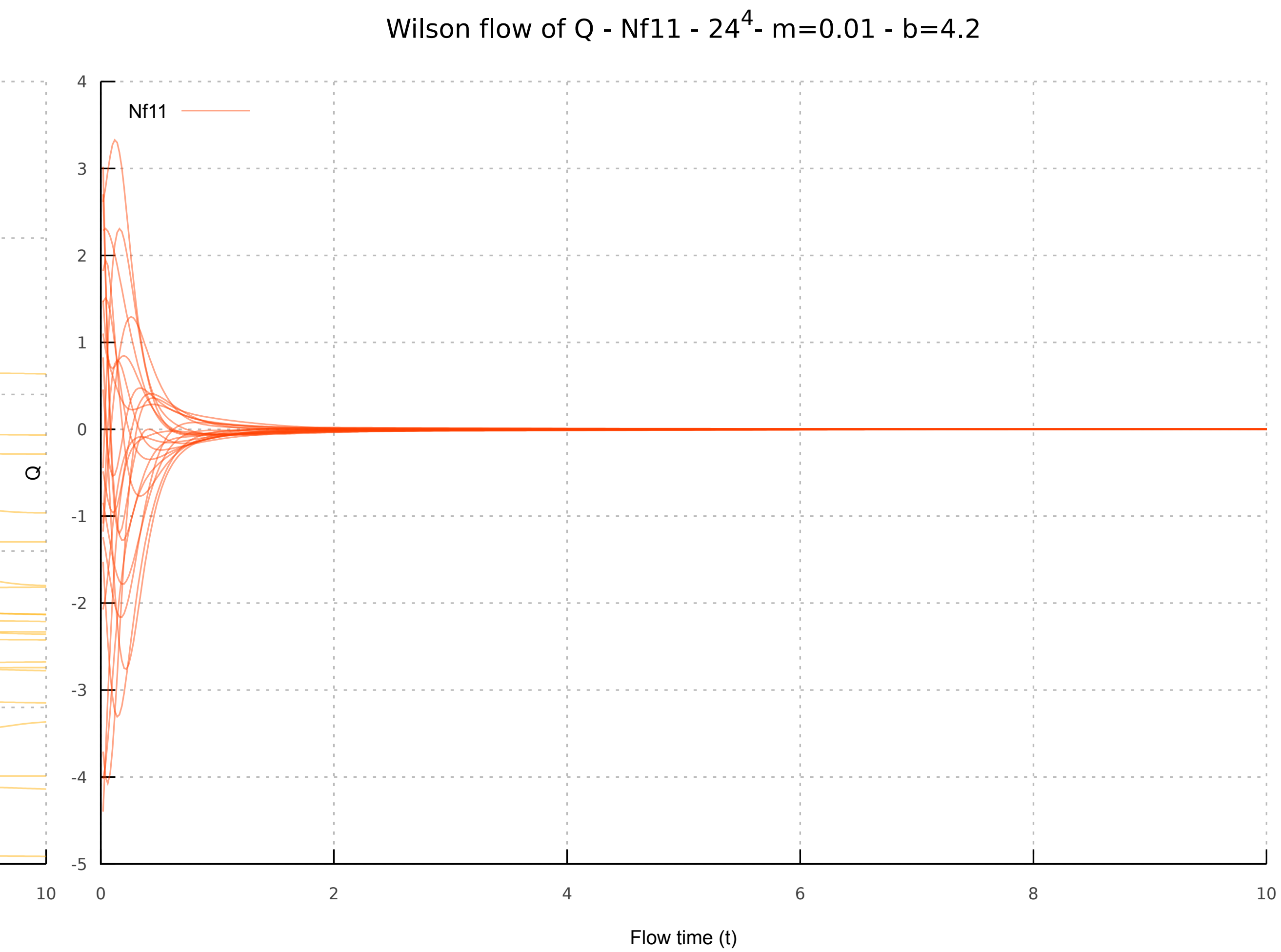
Restored U(1) axial means the absence/suppression of instantons, which carry topological charge. If we suppress instantons, $G\tilde{G}$ essentially vanishes.

$$\chi_t \sim e^{-\gamma N} \cos\theta$$

This effect may be sharper than at finite T (due to existence of IRFP)

Illustration: Wilson flow of topological charge





N_f=8 N_f=11

Rough: plausibly large lattice spacing effects in addition to broken chiral symmetry

Dirac operator zero modes

Atiyah-Singer Index Theorem

$$\boxed{Q = \sum_s \chi_s = n_+ - n_-}$$

$Q \neq 0$ from zero modes ($\chi_s = \pm 1$)

Banks-Casher

$$\boxed{-\langle \bar{\psi}\psi(m=0)\rangle = \lim_{m \to 0} m \int_{-\infty}^{\infty} d\lambda \, \frac{\rho(\lambda)}{\lambda^2 + m^2} = \pi \, \rho(0)}$$

Vanishing chiral condensate implies $\rho(0)=0$, and does not contradict $Q=0$.
It still agrees with exponential suppression of topological susceptibility.
However, no evident distinction between $N_f=2$ and $N_f>2$.

2-point functions

[inspired by old work by Lagae, Kogut, Sinclair 1998 see also Deuzeman, Lombardo, Nunes da Silva, EP 2011, and later]

$$\begin{tikzcd} C_{\vec{\pi}} \arrow[r, leftrightarrow] \arrow[d, leftrightarrow, "U_A(1)"'] & C_{\sigma} \arrow[d, leftrightarrow] \\ C_{\vec{\delta}} \arrow[r, leftrightarrow, "SU(N_f)"'] & C_{\eta'} \end{tikzcd}$$

Assume exact chiral symmetry: do spectral decomposition

$$\begin{aligned} C_{\vec{\pi}} &= -(\nu=0) + (\nu=1) \\ C_{\vec{\delta}} &= -(\nu=0) - (\nu=1) \\ C_{\sigma} &= -(\nu=0) - (\nu=1) + N_f(\nu=1) \\ C_{\eta'} &= -(\nu=0) + (\nu=1) - N_f(\nu=1) \end{aligned}$$

disconnected

$N_f > 2$: degeneracy under $SU(N_f)$ and $U_A(1)$ implies the absence of contributions from nonzero topological charge sectors.

Possible scenario

Endpoint conformal window

$\Updownarrow$

Restoration of SU(N_f) and U(1) axial

$\Updownarrow$

(at least) exponential suppression of topological susceptibility
(i.e. instanton distribution)

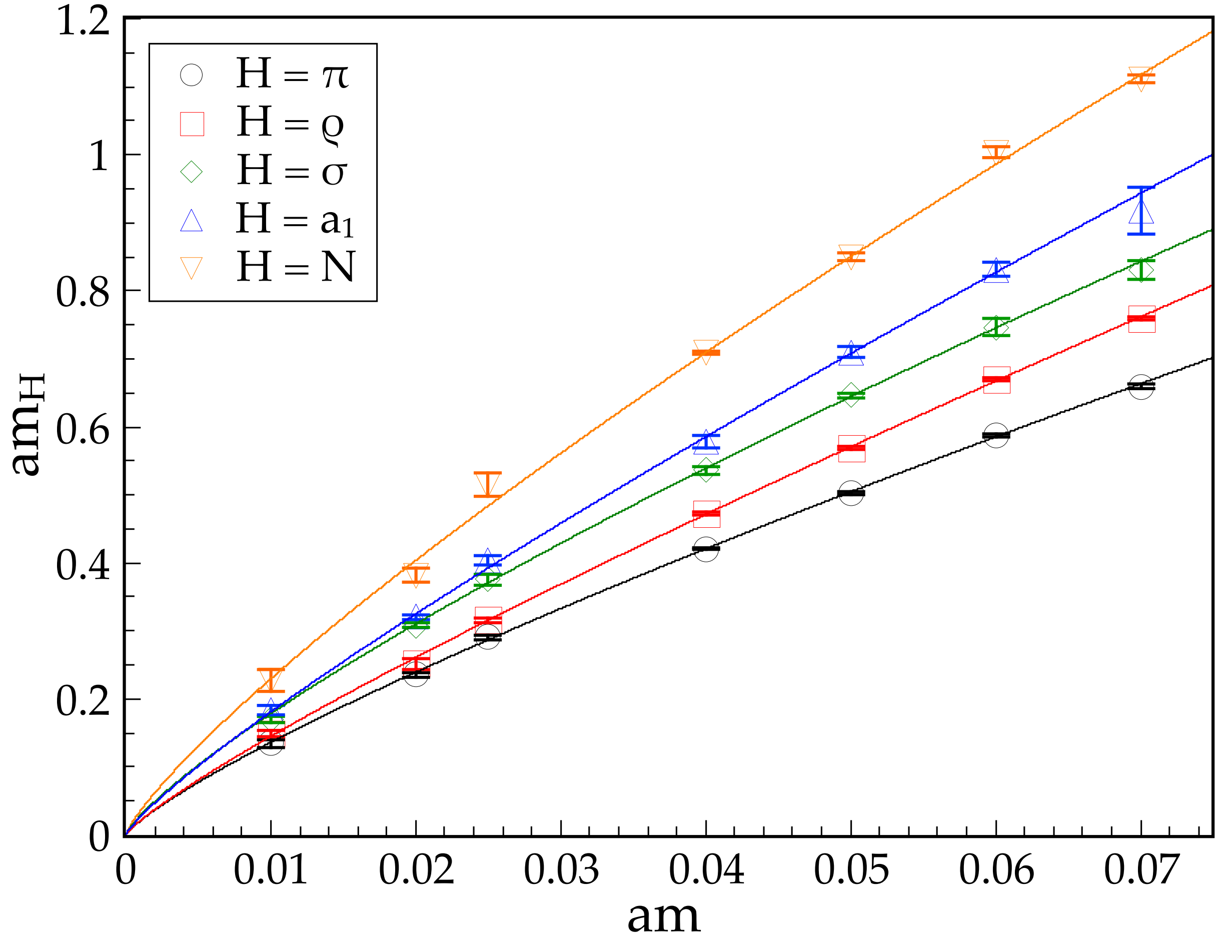
$\Downarrow$

Absence of nontrivial topological charge

The spectrum

IRFP with mass deformation

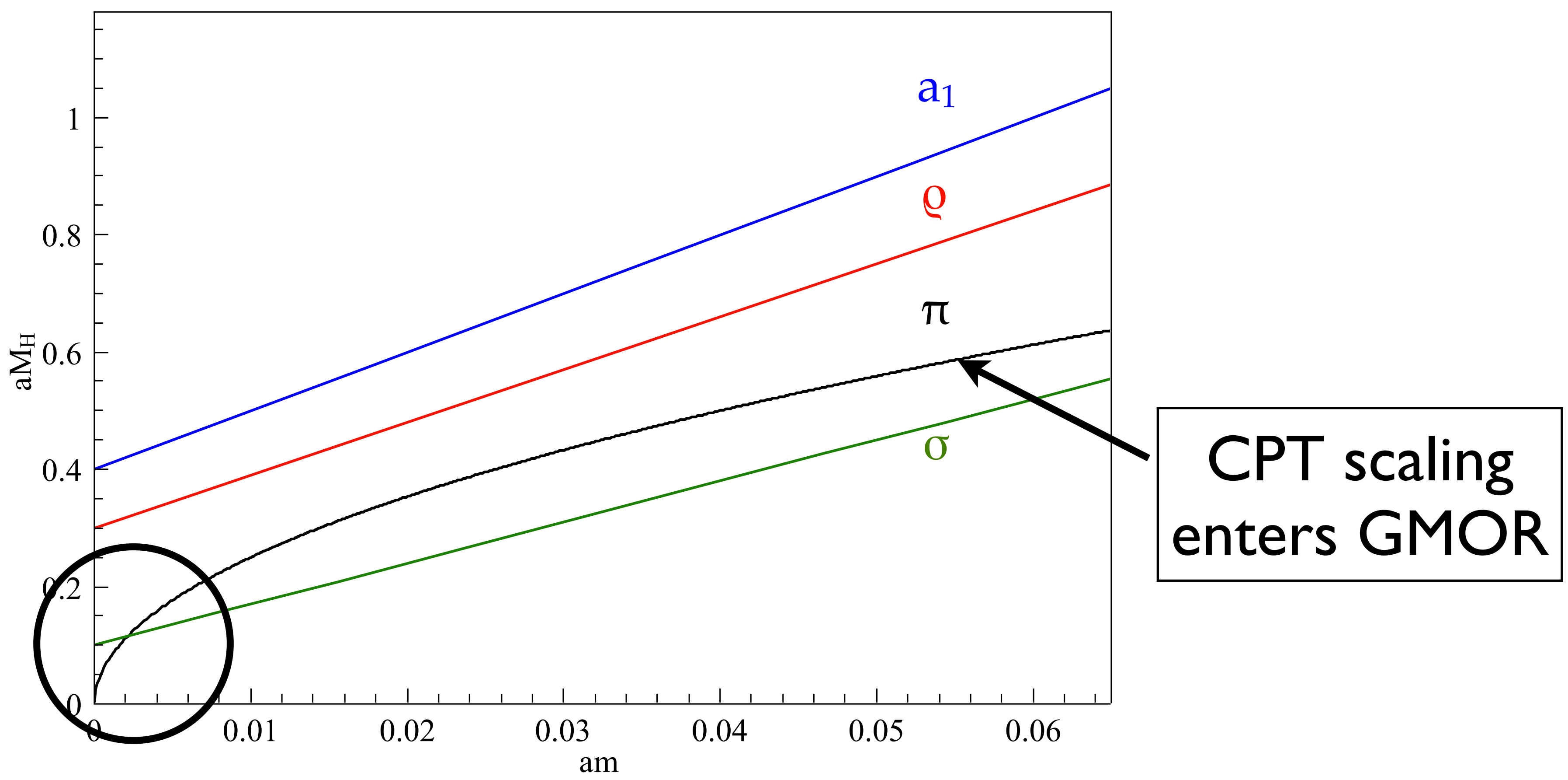
[Lombardo, Miura, Nunes da Silva, EP 2014]



What happens
just below the endpoint ?

$N_f = 12, \gamma_m \sim 0.25$

Even possible inversion of π - σ states at m > 0



If Conformal Phase Transition is realised, then a smooth variation of the dynamically generated mass scale leaves its footprint in the spectrum

Plausible is that the scalar state is <u>not</u> parametrically lighter. Why?

- Pions only Goldstone bosons

- Conformal symmetry explicitly broken at CPT

- Broken chiral symmetry implies $R_\pi \sim \dfrac{m_\pi^2}{m_\sigma^2} \underset{m\to 0}{\longrightarrow} 0$

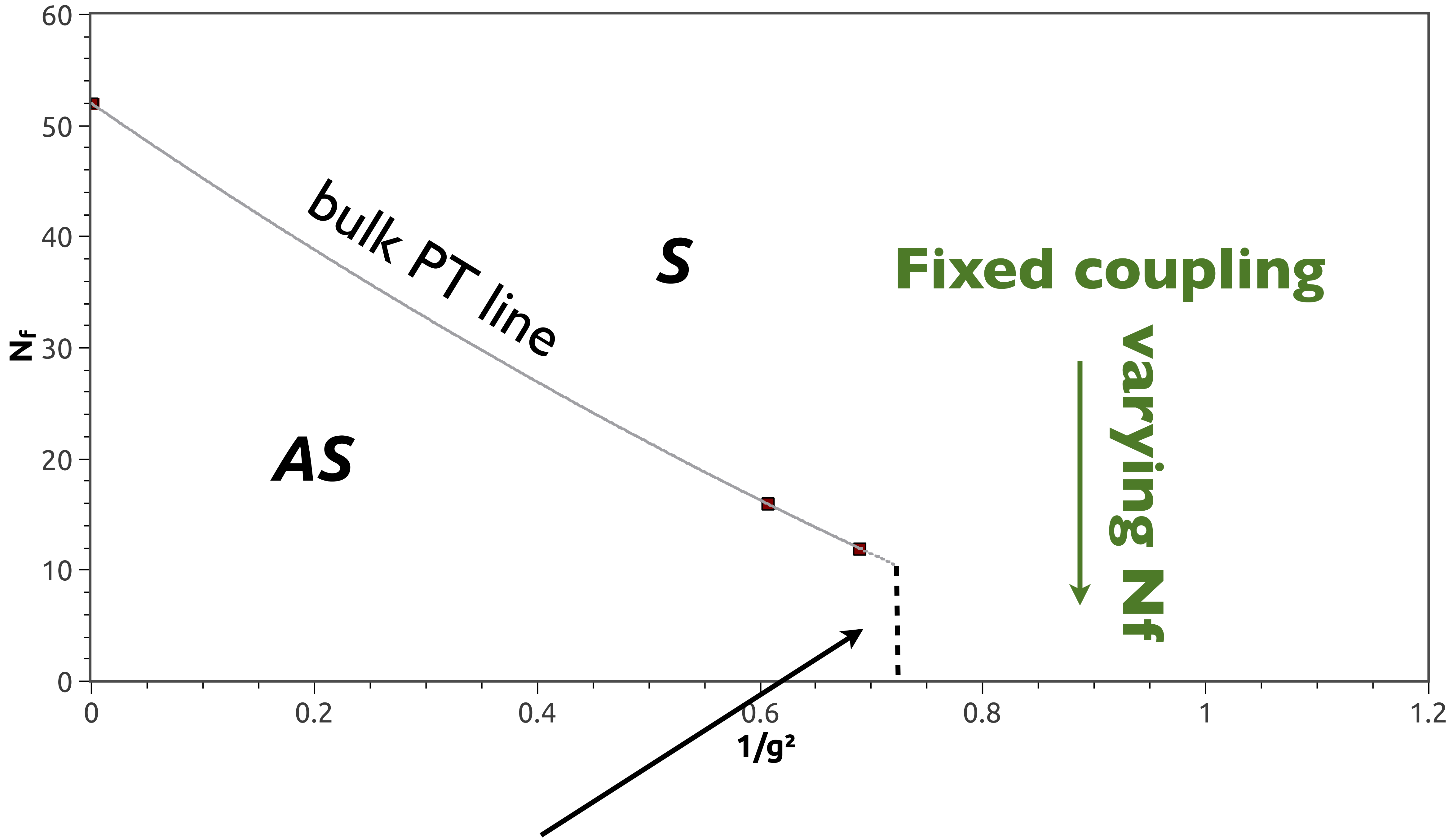
- This system smoothly flows into QCD

[SEE ALSO AdS/CFT:
KUTASOV, LIN, PARNACHEV 2012]

Summary

- Energy flow, topology and spectrum carry important footprints of the endpoint dynamics
- Interplay of 3 symmetries: conformal, chiral and U(1) axial
- One difference with $N_f > 2$, $T > T_c$: existence of IRFP. Analogy with quantum critical phenomena more natural than a first order phase transition driven by instabilities
- Chiral and U(1) axial effective restoration at the endpoint associated to (at least) exponential suppression of topological susceptibility (instantons) and absence of zero modes
- Possible π - σ inversion at finite fermion mass, with no dilaton

Backup slide



Possible separation between 2 chirally broken phases

Nf dependence effectively suppressed.