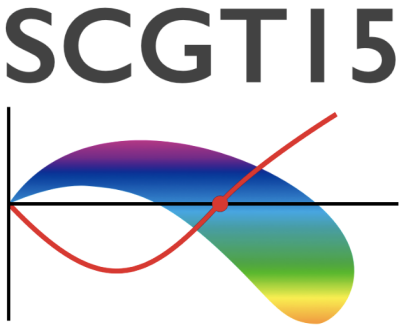
Scalar Mesons in Lattice QCD



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Kobayashi-Maskawa Institute
for the Origin of Particles and the Universe

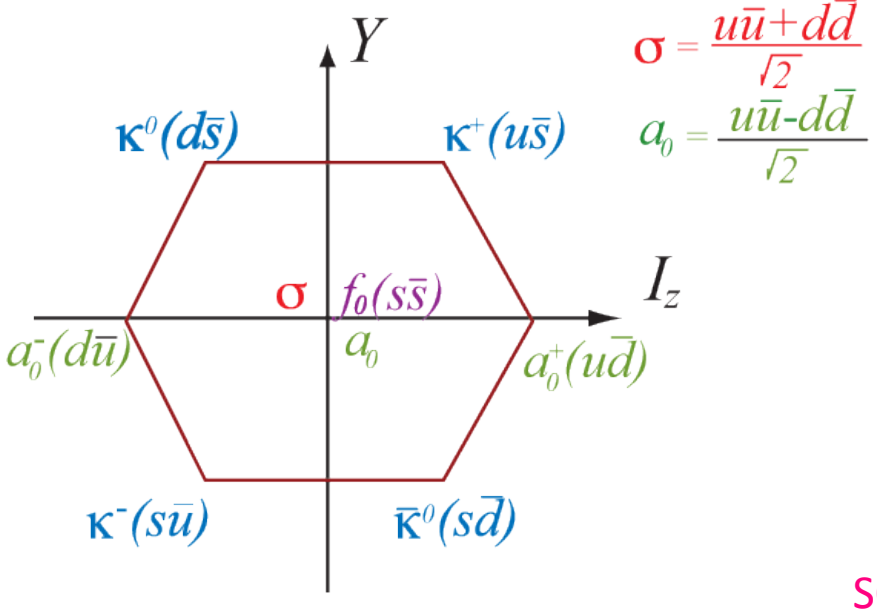
March 5, 2015@SCGT2015

Low Lying Scalar Mesons

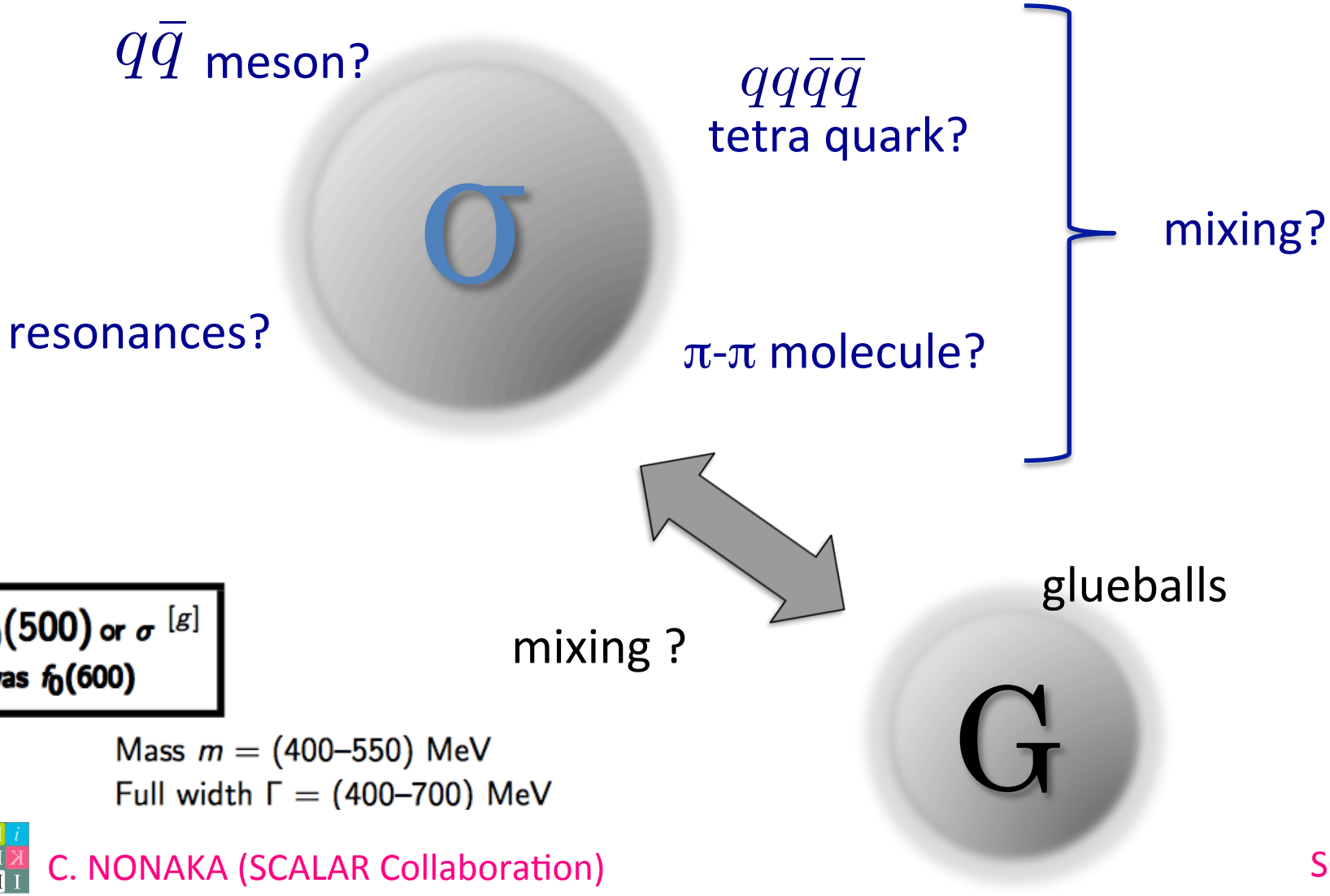
- Light scalar Mesons
  - σ meson, $I=0, J^{PC}=0^{++}$: light σ, $m_\sigma \sim m_\rho$ ~~Quark model~~
  - Nuclear force, important for low energy hadron physics
  - re-identification of the σ: "$f_0(600)$ of σ" in PDG2002

    existence of σ pole: reanalysis of π-π scattering phase shift

    Igi and Hikasa, PRD59(1999)034005
  - κ meson, $I=1/2, J^{PC}=0^{++}$: m_κ ~800 MeV
- Nonet scalar states



What is the σ ?



$f_0(500)$ or σ [g]
was $f_0(600)$

Mass m = (400–550) MeV
Full width Γ = (400–700) MeV

Scalar Mesons in Lattice QCD

σ

as $q\bar{q}$ meson

quench

1987 screening mass — *DeTar and Kogut, PRD36(1987)2828*

$\approx$

2000 $q^2\bar{q}^2$ — *Alford and Jaffe, NPB578(2000)367*

mixing with glueball — *Lee and Weingarten, PRD61(2000)014015*

dynamical

2001 +glueball — *McNeile and Michael, PRD63(2001)114503*

$m_\sigma < m_\pi$??

2002 disconnected diagram — *SCALAR ,NPProc.Suppl.106(2002)272*

2003 domain wall fermion, propagators in quench — *Prelovsek and Orginos, NPProc.Suppl.119(2003)822*

2004 disconnected diagram — *SCALAR,PRD70 (2004)034504*

Sigma Meson as Two Quark State

SCALAR, Phys. Rev. D70 (2004)034504

- Operator (two flavor)

$$\hat{\sigma}(x) \equiv \sum_{\substack{c=1 \\ \text{color}}}^{3} \sum_{\substack{\alpha=1 \\ \text{Dirac}}}^{4} \frac{\bar{u}^c_\alpha(x) u^c_\alpha(x) + \bar{d}^c_\alpha(x) d^c_\alpha(x)}{\sqrt{2}}$$

Quark model

- Propagator

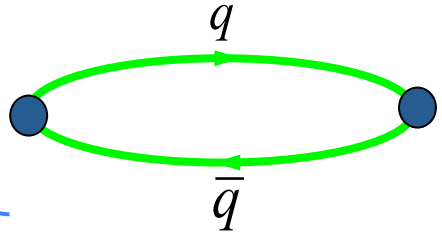
$$G(y,x) = -\langle \mathrm{Tr} W^{-1}(x,y) W^{-1}(y,x) \rangle$$

connected

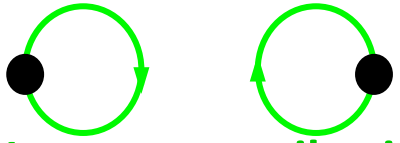
$$+ 2\langle (\sigma(y) - \langle \sigma(y) \rangle)(\sigma(x) - \langle \sigma(x) \rangle) \rangle$$

disconnected

Connected diagram



Disconnected diagram



- Vacuum contribution

Simulation Setup

SCGT15

SCALAR, Phys. Rev. D70 (2004)034504

- Full QCD, Hybrid Monte Carlo

  Plaquette gauge action, Wilson Fermion

- $\beta = 4.8 \quad \kappa = 0.1846, 0.1874, 0.1891$

CP-PACS, Phys. Rev. D60 (1999)114508

- Lattice size $8^3 \times 16$
- Disconnected diagrams

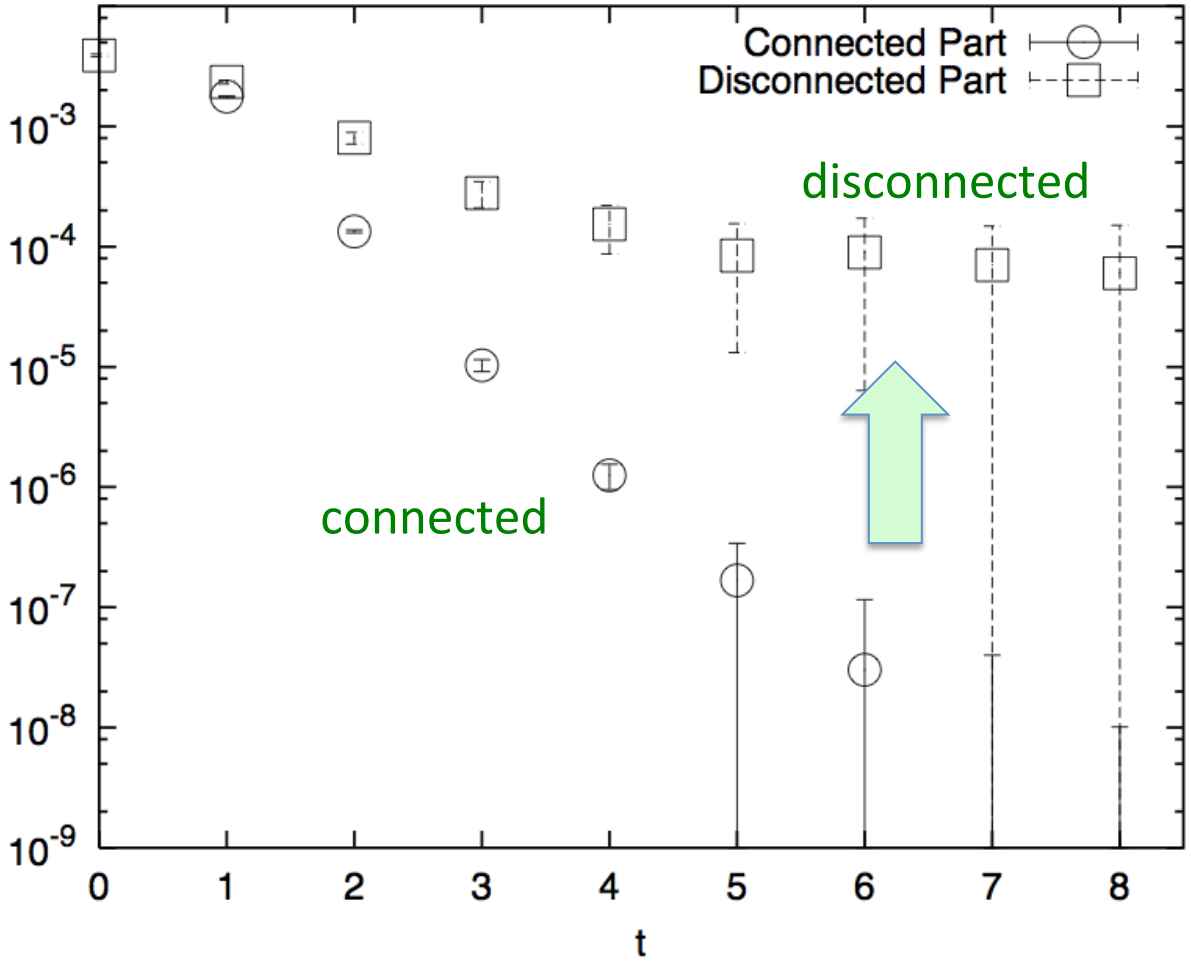
  Z_2 noise method (number of noise: 1000)

κ	0.1846	0.1874	0.1891	
statistics [1)]	1110	860	730	
m_π/m_ρ [2)]	0.8291(12)	0.7715(17)	0.7026(32)	*CP-PACS*
m_π/m_ρ [3)]	0.825(2)	0.757(2)	0.693(3)	*our results*

Disconnected Diagrams

- Propagators

SCALAR, Phys. Rev. D70 (2004)034504

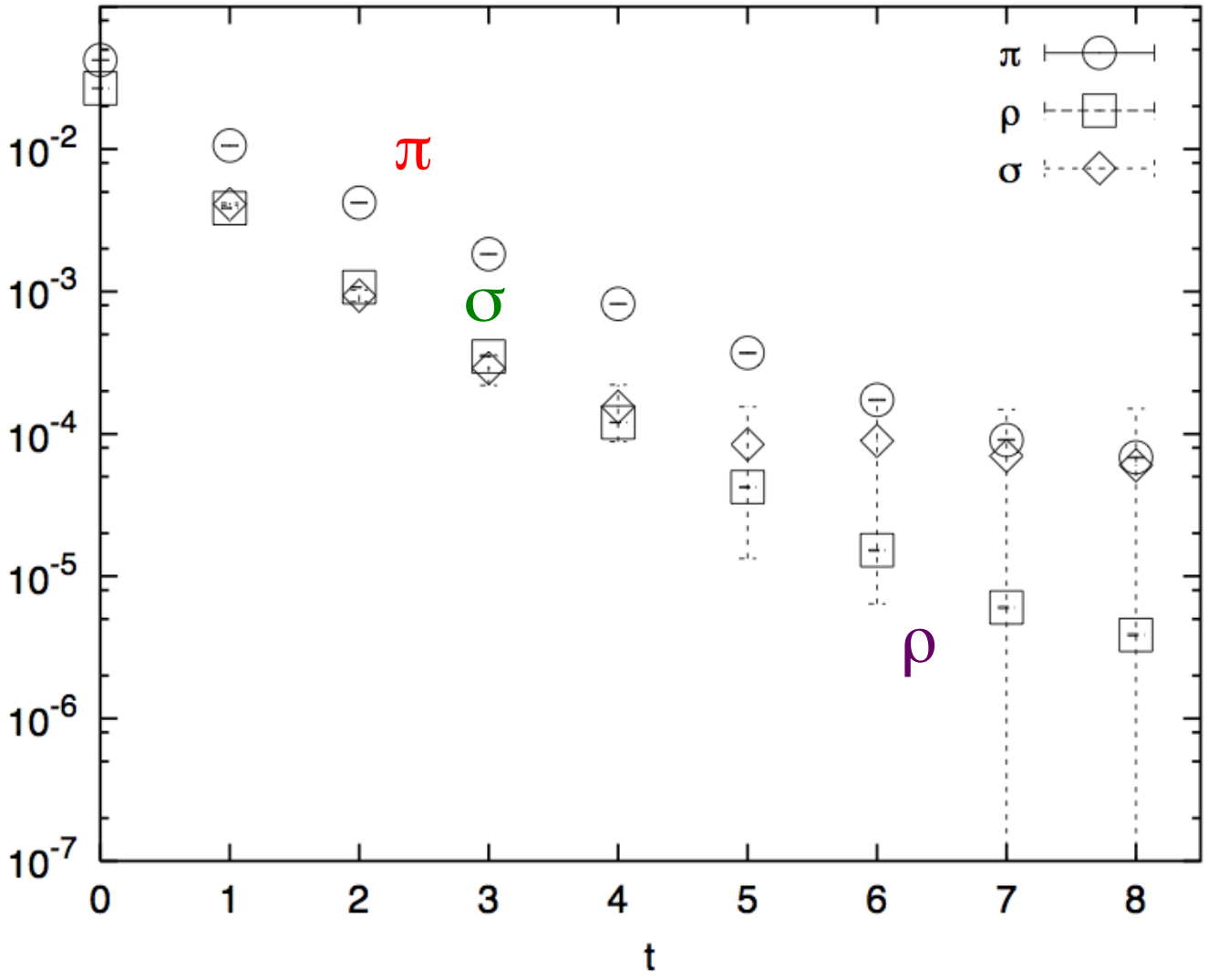


- Due to the existence of disconnected diagram, m_σ becomes smaller.

Light Scalar Meson

SCALAR, Phys. Rev. D70 (2004)034504

- Propagators

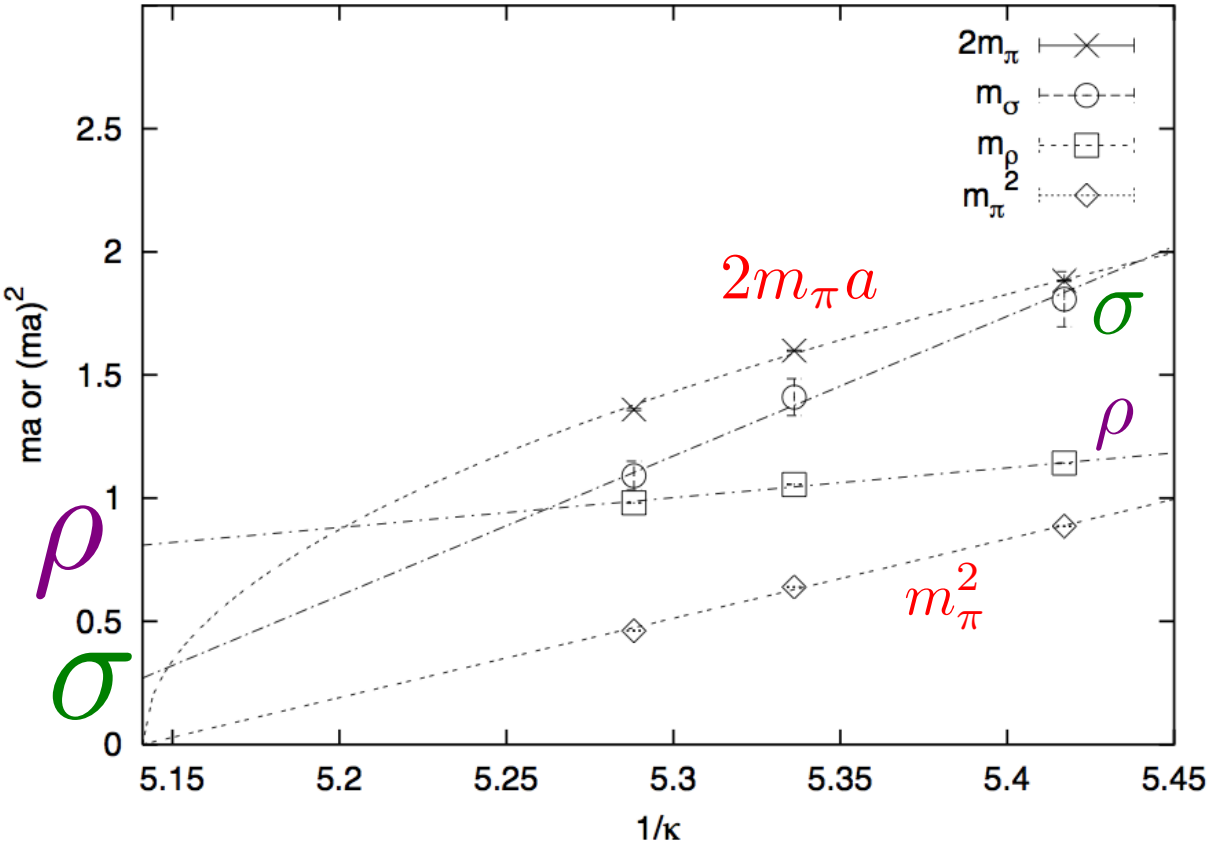


$m_\sigma \sim m_\rho$

Light Scalar Meson

SCGT15

SCALAR, Phys. Rev. D70 (2004)034504



- Only connected diagrams

$$m_\sigma > 2m_\rho$$

- Disconnected diagrams

$$m_\sigma \sim m_\rho$$

- At chiral limit

$$m_\sigma < m_\rho$$

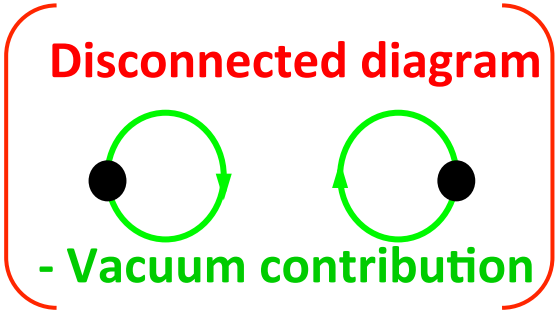
κ	0.1846	0.1874	0.1891
m_σ/m_ρ [3)]	1.6(1)	1.34(8)	1.11(6)
$m_{\mathrm{connect}}/m_\rho$ [3)]	2.40(2)	2.44(3)	2.48(4)

Sigma Meson

Sigma meson as two quark state

SCALAR, Phys. Rev. D70 (2004)034504

For light sigma meson,
the disconnected diagram
is important.



If the glueball states
were not heavy...

mixing with glueballs and tetra quarks....

$qq - \bar{q}\bar{q}$

Sigma Meson

Sigma meson as two quark state

SCALAR, Phys. Rev. D70 (2004)034504

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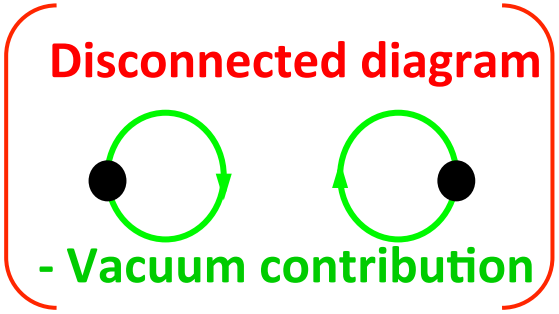


If the glueball states
were not heavy...

mixing with glueballs and tetra quarks....

$qq-\bar{q}\bar{q}$

➡ Sigma meson as four quark state

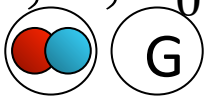
SCALAR(+Wakayama), arXiv1412.3909[hep-lat]

Scalar Mesons in Lattice QCD

SCGT15

Full QCD

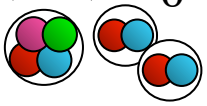
2004 σ:  *SCALAR, PRD70(2004)034504*

2006 σ, κ, a_0:  *UKQCD,PRD74(2006)114505*

UKQCD,PRD74(2006)014508

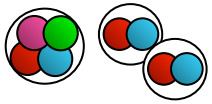
2007 κ:  *SCALAR, PLB652(2007)250*

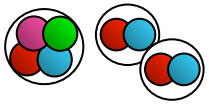
2009 σ, κ, a_0: *S.Prelovsek et al, PRD79(2009)014503*

2010  *S.Prelovsek et al, PRD82(2010)094507*

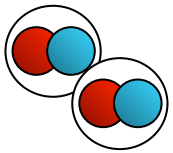
} connected diagrams

2012 κ, a_0:  *BGR,PRD85(2012)034508*

2013 κ, a_0:  *ETM,JHEP1304(2013)137*

2014 σ:  *SCALAR(+Wakayama), arXiv1412.3909[hep-lat]*

connected + singly disconnected diagrams



Molecule

- Operator (two flavor) The lightest pseudoscalar mesons

Jaffe, Phys.Rept.409(2005) 1

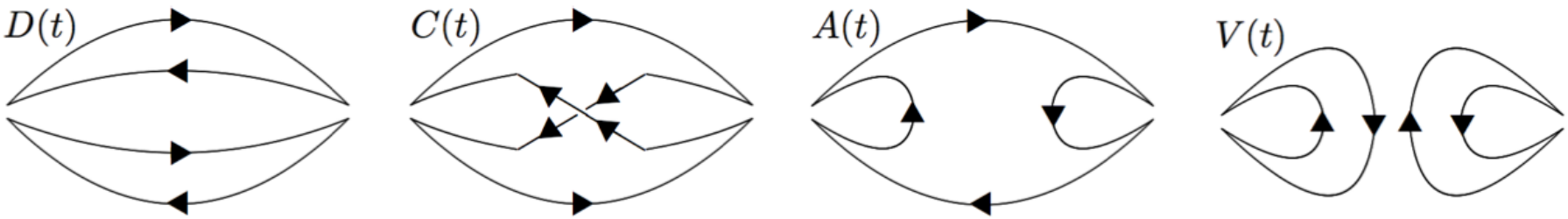
$$\mathcal{O}^{\rm molec}(t) = \frac{1}{\sqrt{3}}\left[\mathcal{O}^{\pi^+}(t)\mathcal{O}^{\pi^-}(t) - \mathcal{O}^{\pi^0}(t)\mathcal{O}^{\pi^0}(t) + \mathcal{O}^{\pi^-}(t)\mathcal{O}^{\pi^+}(t)\right]$$

$$\mathcal{O}^{\pi^+}(t) = -\sum_{\mathbf{x}\,a} \bar{d}^a(t,\mathbf{x})\gamma_5 u^a(t,\mathbf{x}) \quad \mathcal{O}^{\pi^-}(t) = \sum_{\mathbf{x}\,a} \bar{u}^a(t,\mathbf{x})\gamma_5 d^a(t,\mathbf{x})$$

$$\mathcal{O}^{\pi^0}(t) = \frac{1}{\sqrt{2}}\sum_{\mathbf{x}\,a}\left[\bar{u}^a(t,\mathbf{x})\gamma_5 u^a(t,\mathbf{x}) - \bar{d}^a(t,\mathbf{x})\gamma_5 d^a(t,\mathbf{x})\right]$$

- Propagators

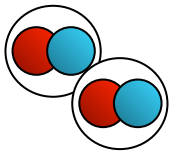
$$G^{\rm molec}(t) = 2\left[D(t) + \frac{1}{2}C(t) - 3A(t) + \frac{3}{2}V(t)\right]$$



connected diagrams

Singly disconnected diagram

doubly disconnected diagram



Molecule

SCGT15

- Operator (two flavor) The lightest pseudoscalar mesons

Jaffe, Phys.Rept.409(2005) 1

$$\mathcal{O}^{\rm molec}(t) = \frac{1}{\sqrt{3}}\left[\mathcal{O}^{\pi^+}(t)\mathcal{O}^{\pi^-}(t) - \mathcal{O}^{\pi^0}(t)\mathcal{O}^{\pi^0}(t) + \mathcal{O}^{\pi^-}(t)\mathcal{O}^{\pi^+}(t)\right]$$

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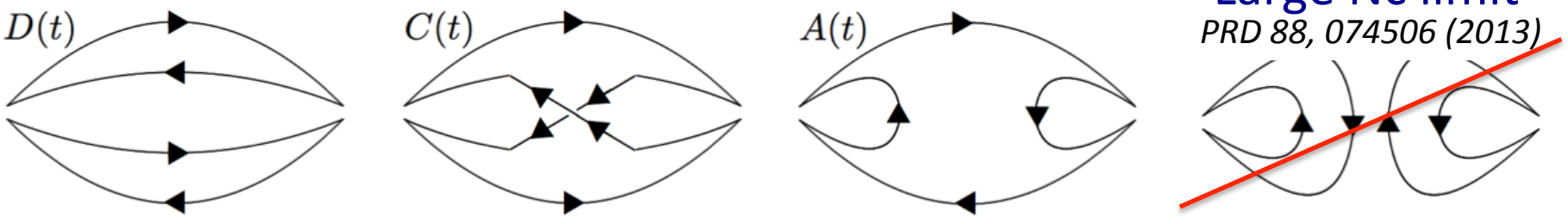
$$G^{\rm molec}(t) = 2\left[D(t) + \frac{1}{2}C(t) - 3A(t) + \frac{3}{2}V(t)\right]$$

Large Nc limit

PRD 88, 074506 (2013)



connected diagrams

Singly disconnected diagram

doubly disconnected diagram

Tetra

- Operator (two flavor) The lightest diquarks

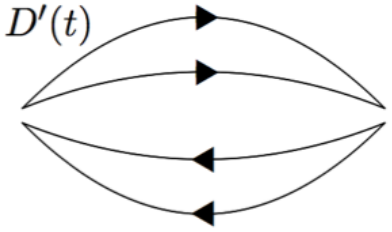
Jaffe, Phys.Rept.409(2005) 1

$$\mathcal{O}^{\rm tetra}(t) = \sum [ud]^a(t)[\bar{u}\bar{d}]^a(t)$$

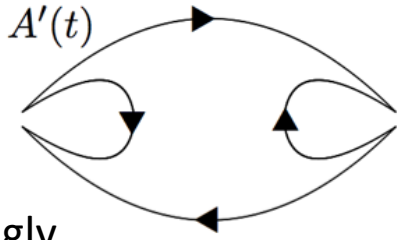
$$[ud]^a(t) = \frac{1}{2}\sum_{\mathbf{x}\,b,c} \epsilon^{abc}\left[u^{Tb}(t,\mathbf{x})\ C\gamma_5\ d^c(t,\mathbf{x}) - d^{Tb}(t,\mathbf{x})\ C\gamma_5\ u^c(t,\mathbf{x})\right]$$

$$[\bar{u}\bar{d}]^a(t) = \frac{1}{2}\sum_{\mathbf{x}\,b,c} \epsilon^{abc}\left[\bar{u}^{b}(t,\mathbf{x})\ C\gamma_5\ \bar{d}^{Tc}(t,\mathbf{x}) - \bar{d}^{b}(t,\mathbf{x})\ C\gamma_5\ \bar{u}^{Tc}(t,\mathbf{x})\right]$$

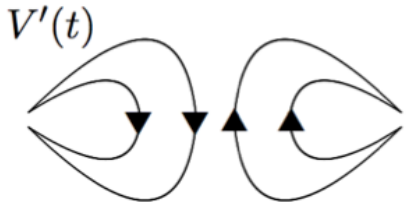
- Propagators $G^{\rm tetra}(t) = 2\Big[2\left(D_1'(t)+D_2'(t)\right) - 2\left(A_1'(t)+A_2'(t)+A_3'(t)+A_4'(t)\right) + \left(V_1'(t)+V_2'(t)+V_3'(t)+V_4'(t)\right)\Big]$



connected diagrams



Singly disconnected diagram



doubly disconnected diagram

Tetra

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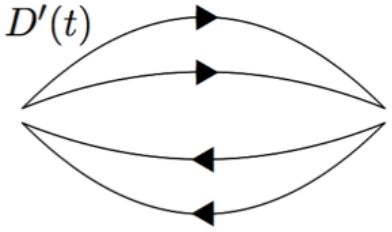
$$[\bar{u}\bar{d}]^a(t) = \frac{1}{2}\sum_{\mathbf{x}\,b,c} \epsilon^{abc}\left[\bar{u}^{b}(t,\mathbf{x})\ C\gamma_5\ \bar{d}^{Tc}(t,\mathbf{x}) - \bar{d}^{b}(t,\mathbf{x})\ C\gamma_5\ \bar{u}^{Tc}(t,\mathbf{x})\right]$$

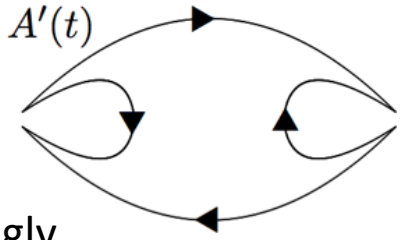
- Propagators $G^{\rm tetra}(t) = 2\Big[2\left(D_1'(t)+D_2'(t)\right) - 2\left(A_1'(t)+A_2'(t)+A_3'(t)+A_4'(t)\right) + \left(V_1'(t)+V_2'(t)+V_3'(t)+V_4'(t)\right)\Big]$

Large Nc limit

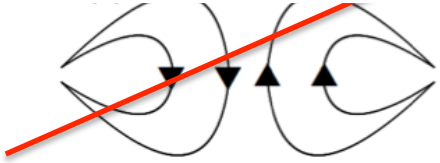
PRD 88, 074506 (2013)



connected diagrams



Singly disconnected diagram



doubly disconnected diagram

Simulation Setup

SCGT15

arXiv1412.3909[hep-lat]

- 2 flavor full QCD: *CP-PACS Phys. Rev. D 63, 034502 (2001)*

  Hybrid Monte Carlo (HMC) with the clover Wilson action

  $C_{SW} = 1.68 \quad \beta = 1.7$ Lattice size: $8^3 \times 16$

- Heavy quark masses, large statistics
- Disconnected diagrams: Z_2 noise method with truncated eigenmode approach noise: 120 X 16 eigenvector : 12

TABLE I: Masses of π and ρ and number of configurations.

κ	$m_\pi a$	m_π MeV	$m_\rho a$	m_ρ MeV	configurations [a]
0.146	1.018(2)	747(27)	1.431(4)	1050(39)	16496
0.147	0.930(2)	682(25)	1.358(6)	996(38)	14344
0.148	0.827(4)	607(23)	1.304(10)	956(39)	11720

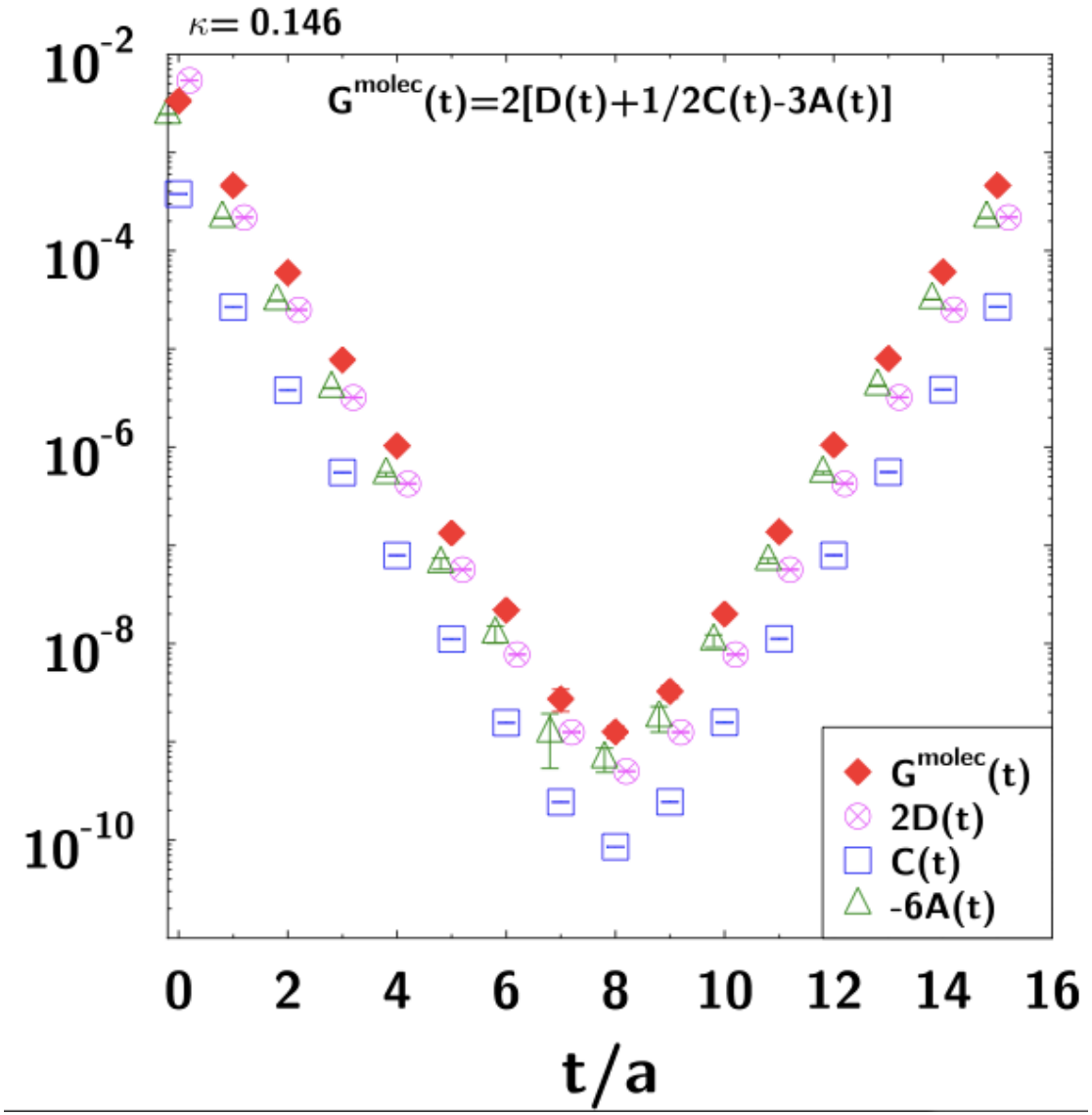
 $\kappa_c = 0.152(6), a = 0.269(9)$ fm

Caveat

- “Molecule” contains mixing with tetra and two quark state
- “Tetra” contains mixing with molecule and two quark state
- Application of the variational method for the possible interpolators is needed.

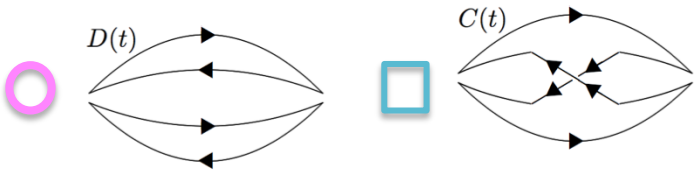
Propagators of Molecule

arXiv1412.3909[hep-lat]

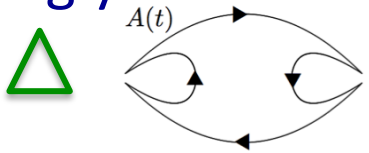


◆ total

Connected diagrams



Singly disconnected diagram

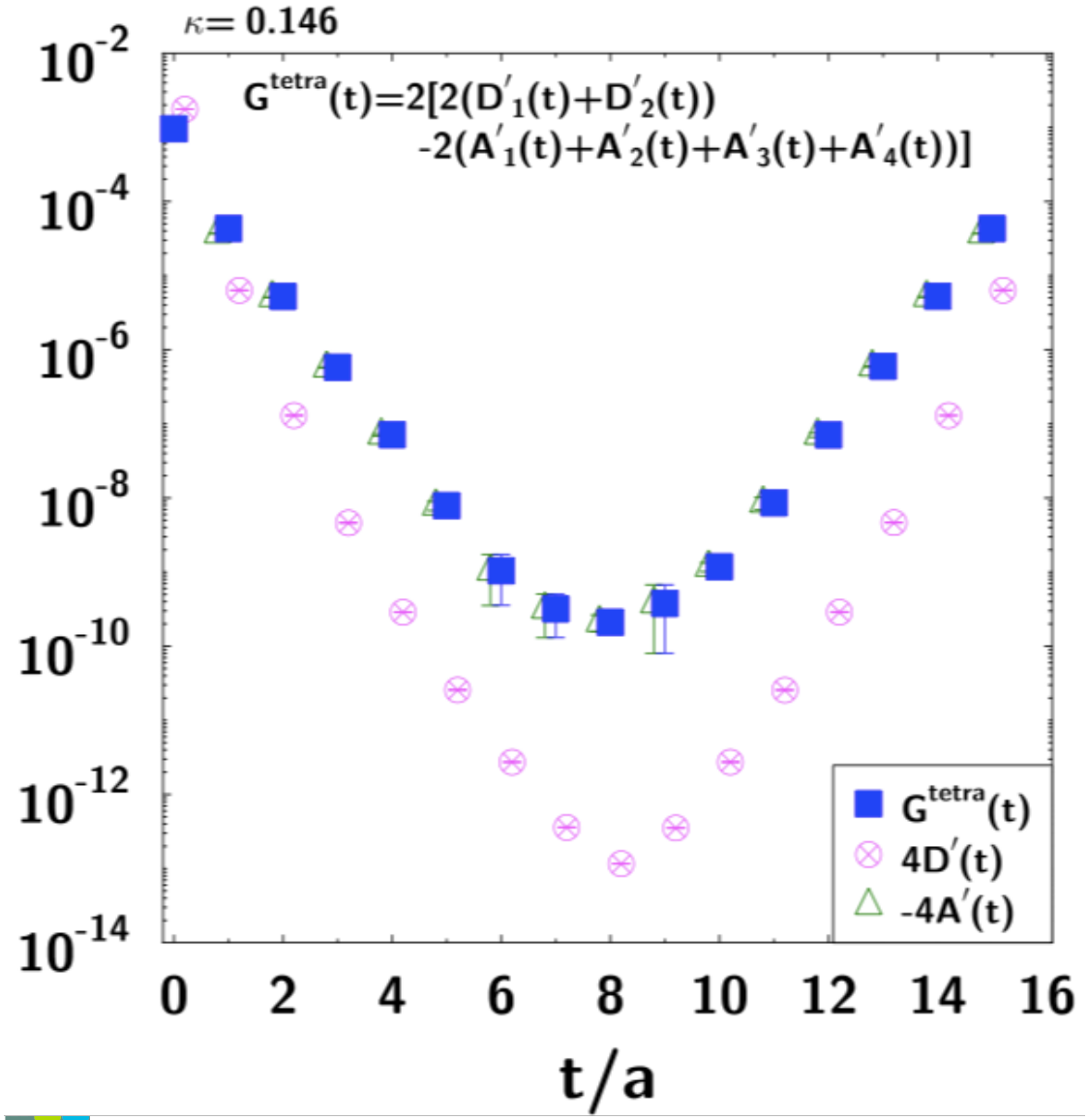


- Singly disconnected diagram is dominant.
- Slopes (~masses) of them are almost the same.

Propagators of Tetra

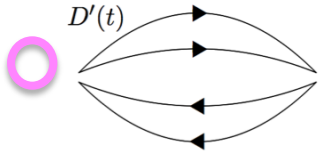
SCGT15

arXiv1412.3909[hep-lat]

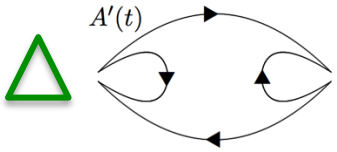


■ total

Connected diagram



Singly disconnected diagram

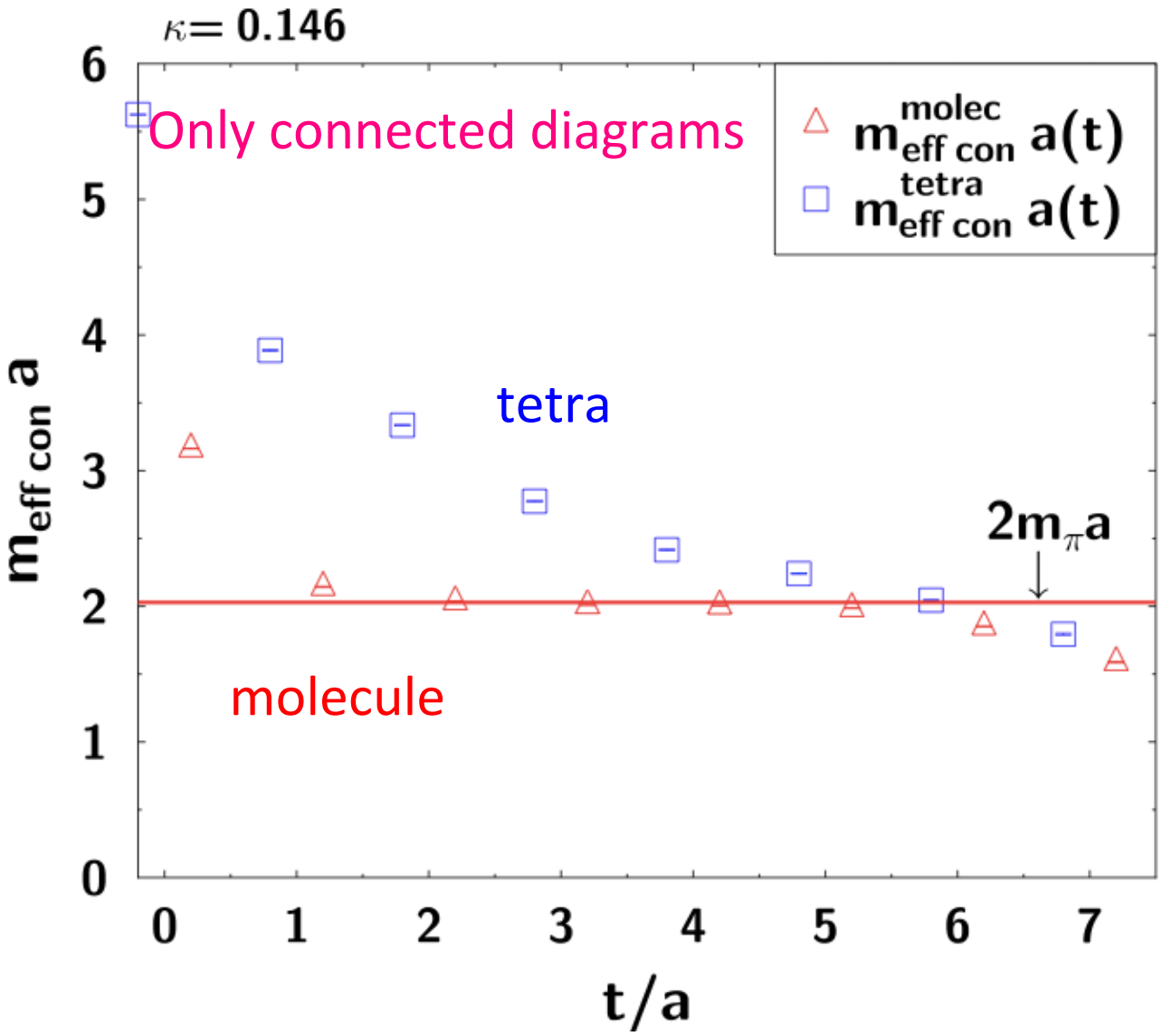


- Singly disconnected diagram is dominant.
- Due to the singly disconnected diagram, the mass of tetra becomes smaller.

Effective Masses

SCGT15

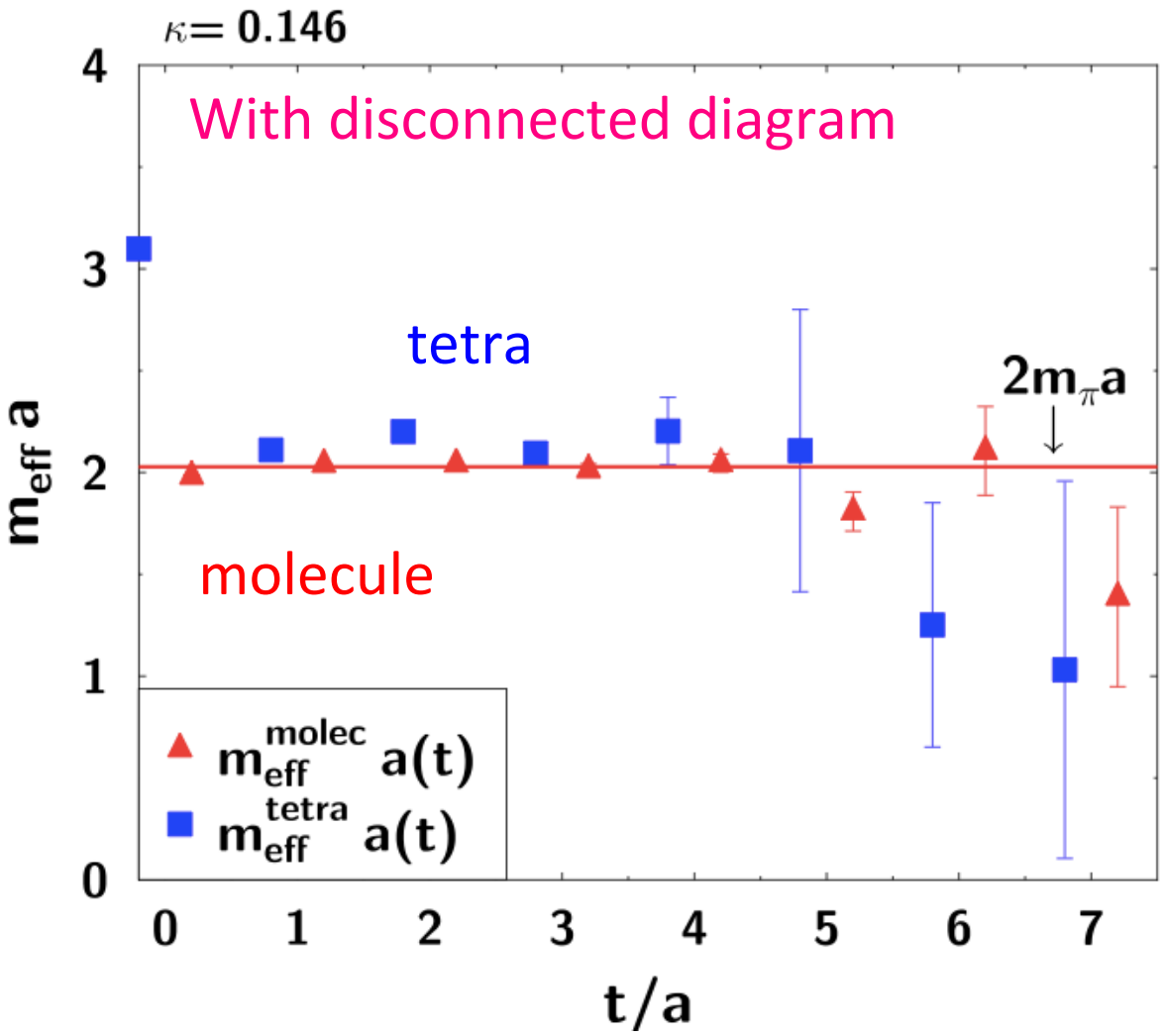
arXiv1412.3909[hep-lat]

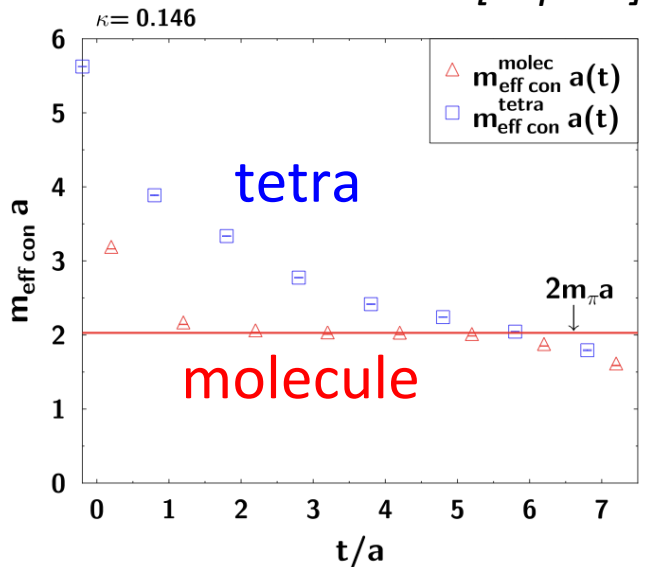


- Molecule
  -Clear plateau
  ~ $2m_\pi$
  π-π scattering state?
- Tetra
  -No clear plateau

Importance of Disconnected Diagrams

arXiv1412.3909[hep-lat]



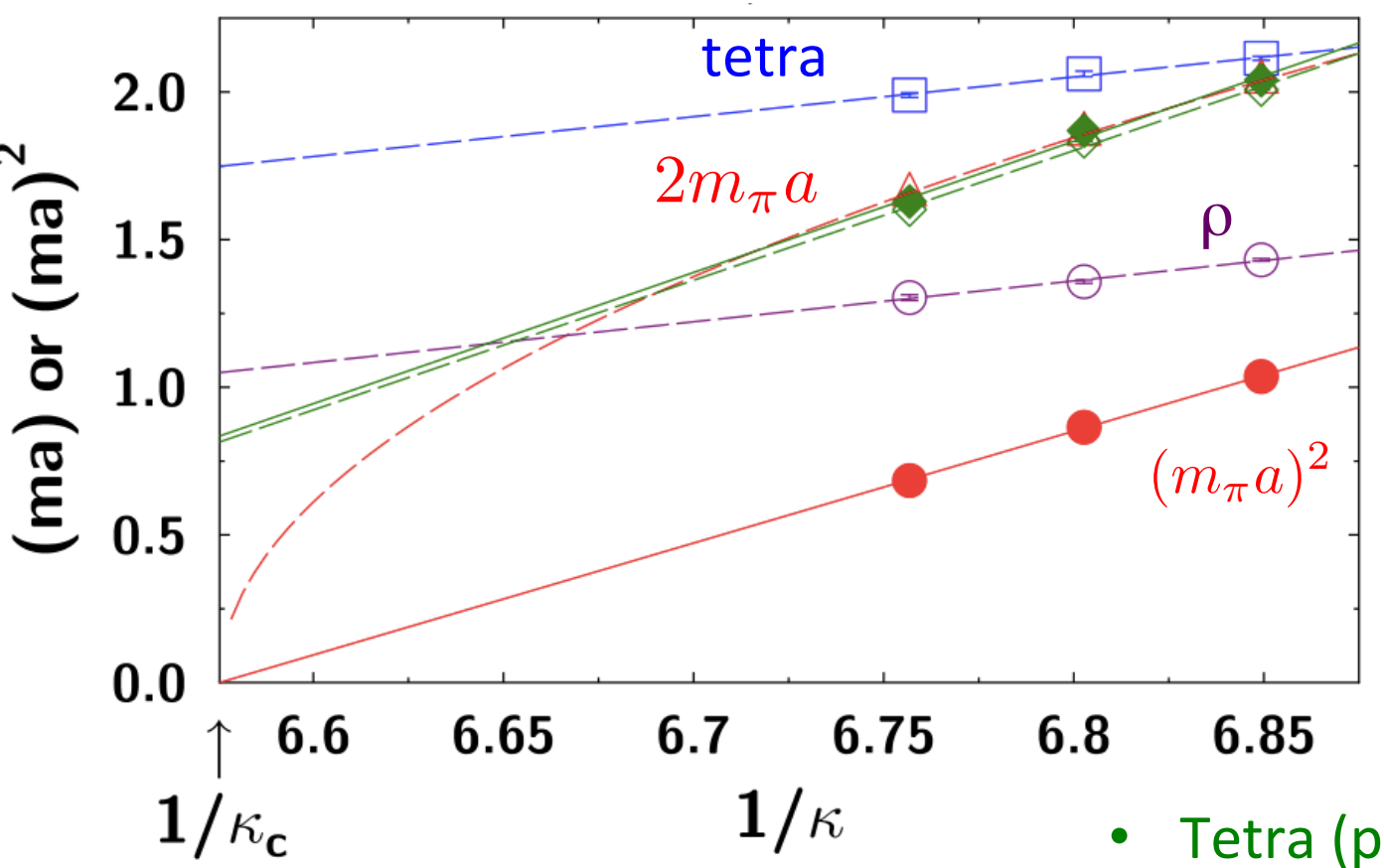


- Molecule
  -same as that of connected diagrams
  - same as $2m_\pi$
  π-π scattering state?
- Tetra
  - plateau ?

κ Dependence of Effective Masses

SCGT15

arXiv1412.3909[hep-lat]



with disconnected diagram

◆ $m^{molec}\ a$

◇ $m^{molec}_{con}\ a$

connected diagram

- Molecule
  - Small effect of disconnected diagram
  - π-π scattering state?

- Tetra (plateau ?)
  - Disconnected diagram is important.
  - small overlap to ground state of molecule

Summary

- Scalar meson as two quark

Phys. Rev. D70 (2004)034504

  - For the light sigma meson, the disconnected diagram is important.
  - Mixing with glueballs and four quark state

- Scalar meson as four quark

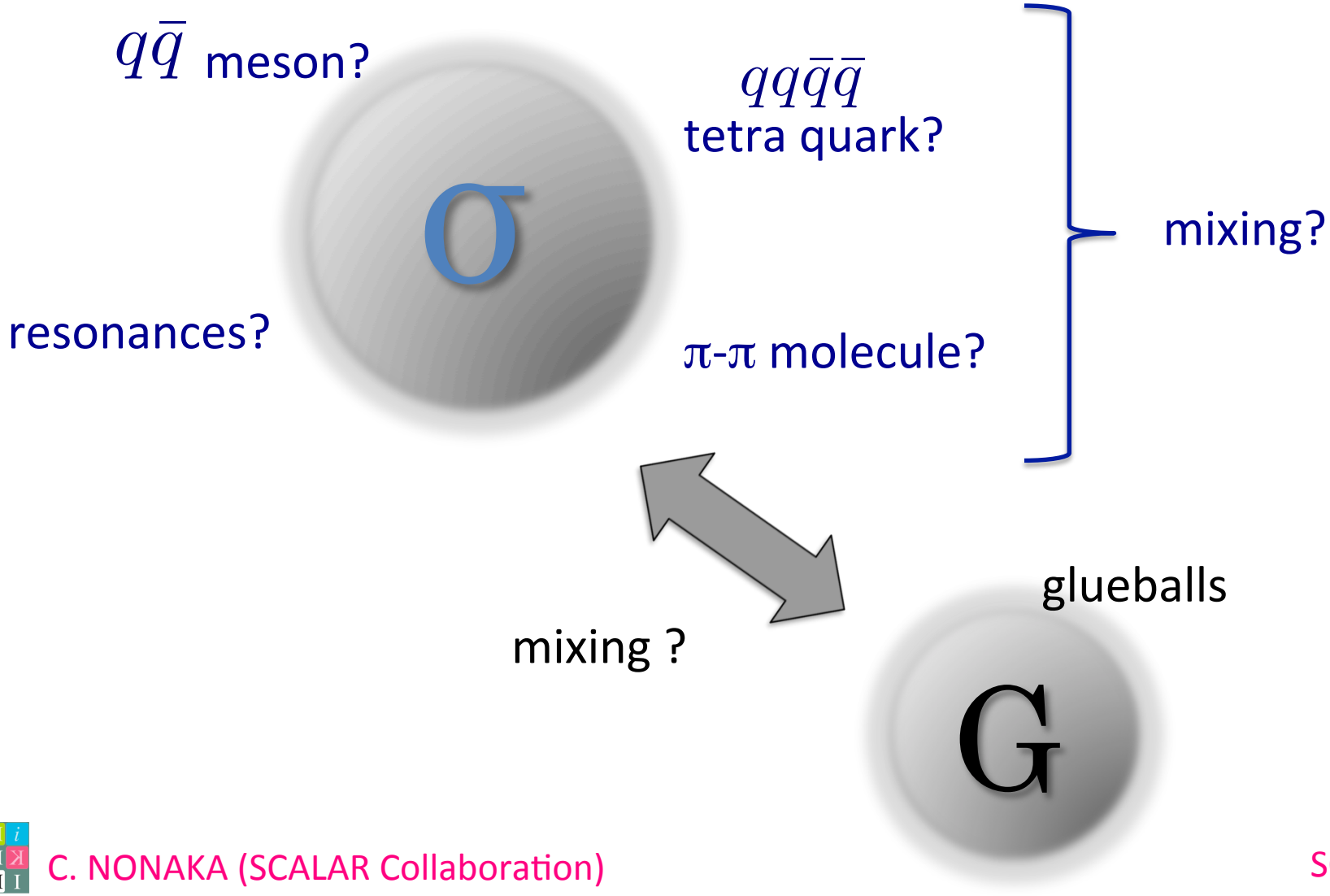
arXiv1412.3909[hep-lat]

  - The disconnected diagrams are important.
  - "Molecule": π-π scattering state?
  - "Tetra": small overlap to the ground state

Future work

  - Doubly disconnected diagram, lighter quark mass, larger lattice
  - Variational method for possible interpolators
  - Finite temperature
  - κ,a_0 mesons

σ Meson


$q\bar{q}$ meson?
$qq\bar{q}\bar{q}$
tetra quark?
σ
mixing?
resonances?
π-π molecule?
mixing ?
glueballs
G

Backup

Order estimation in large N_c limit

SCGT15

One loop of the color line contributes by the power of N_c.

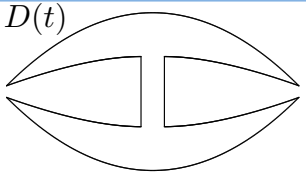
One gluon coupling contributes by the power of $1/\sqrt{N_c}$.

Replacement with color lines

One quark line
↓
One color line

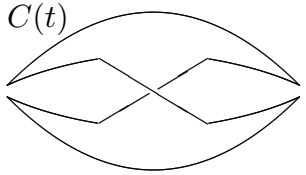
One gluon line
↓
Two color lines

$D(t)$



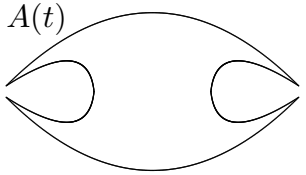
$$\sim \mathcal{O}\left((N_c)^{1} \times \left(\frac{1}{\sqrt{N_c}}\right)^{2}\right) \sim \mathcal{O}(1)$$

$C(t)$



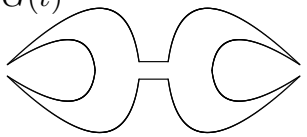
$$\sim \mathcal{O}\left((N_c)^{1} \times \left(\frac{1}{\sqrt{N_c}}\right)^{0}\right) \sim \mathcal{O}(N_c)$$

$A(t)$



$$\sim \mathcal{O}\left((N_c)^{1} \times \left(\frac{1}{\sqrt{N_c}}\right)^{0}\right) \sim \mathcal{O}(N_c)$$

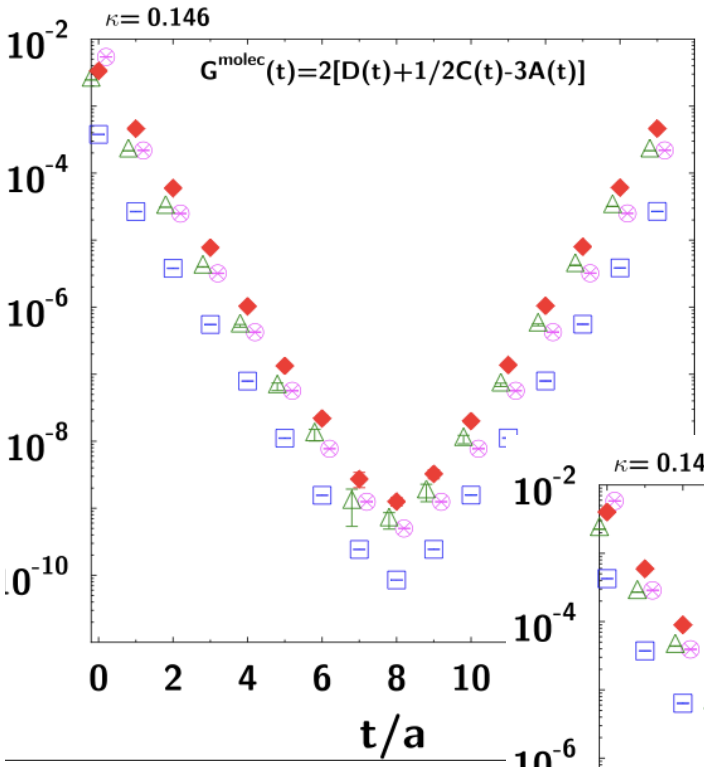
$G(t)$

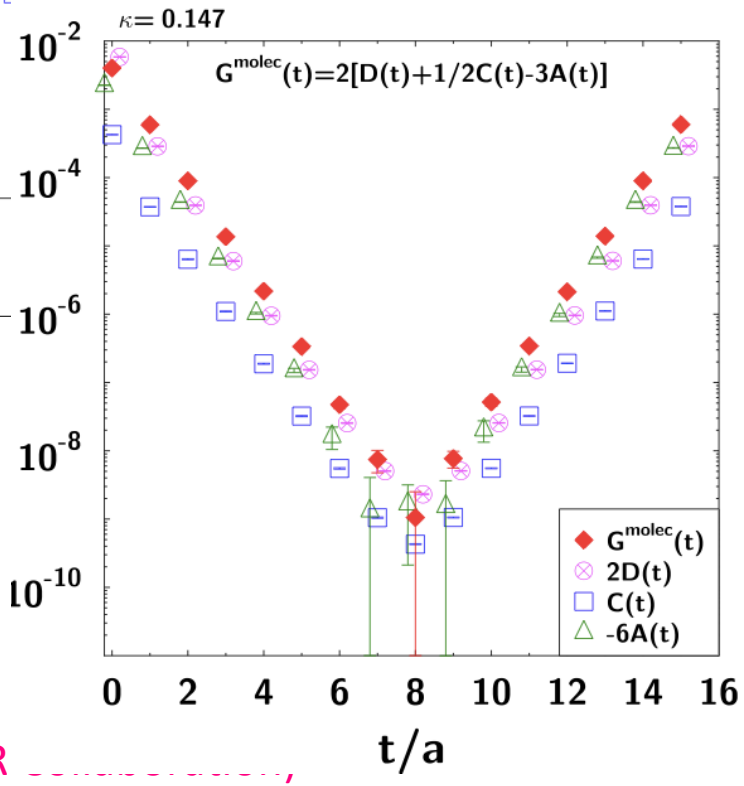


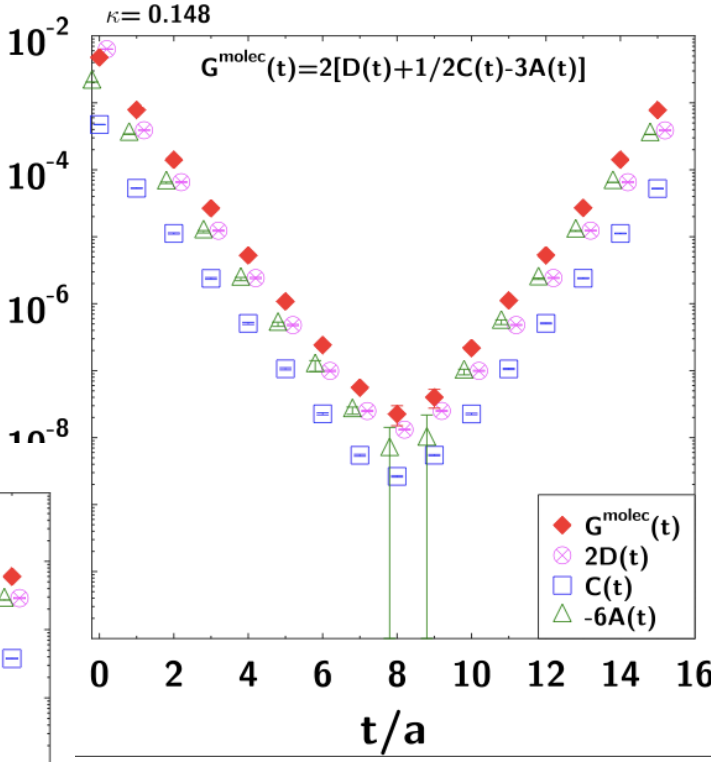
$$\sim \mathcal{O}\left((N_c)^{1} \times \left(\frac{1}{\sqrt{N_c}}\right)^{2}\right) \sim \mathcal{O}(1)$$

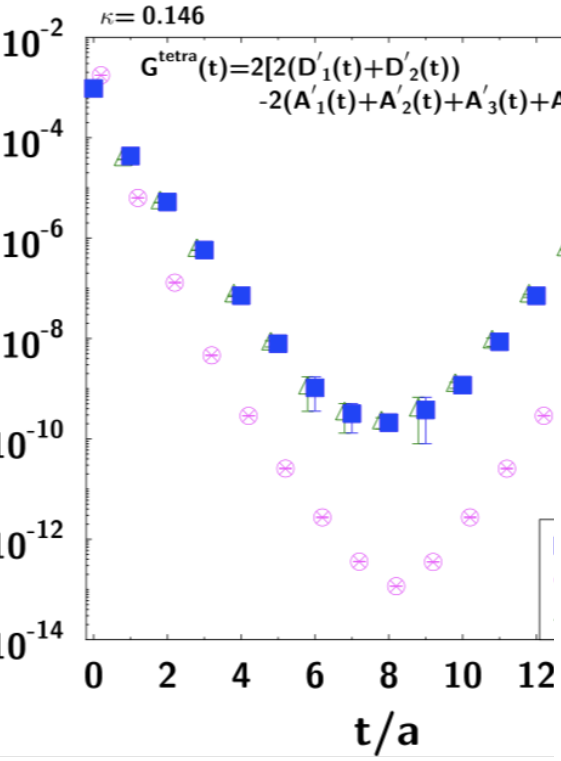
We neglect the doubly disconnected diagrams, because in PRD 88, 074506 (2013), they suggest that the contribution of the doubly disconnected diagrams are N_c order smaller than one of the singly disconnected diagrams.

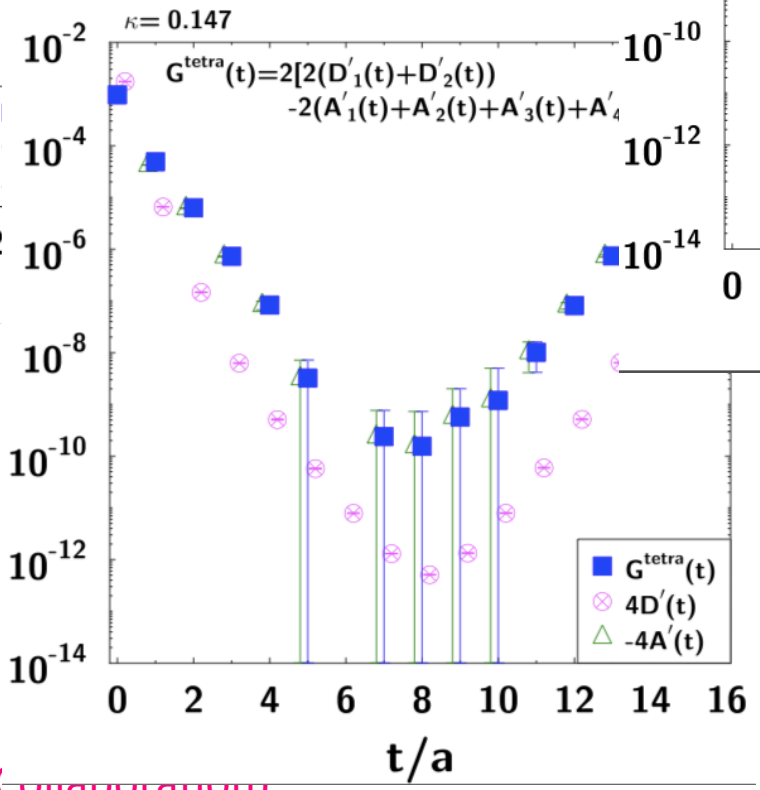
Quark Mass Dependence

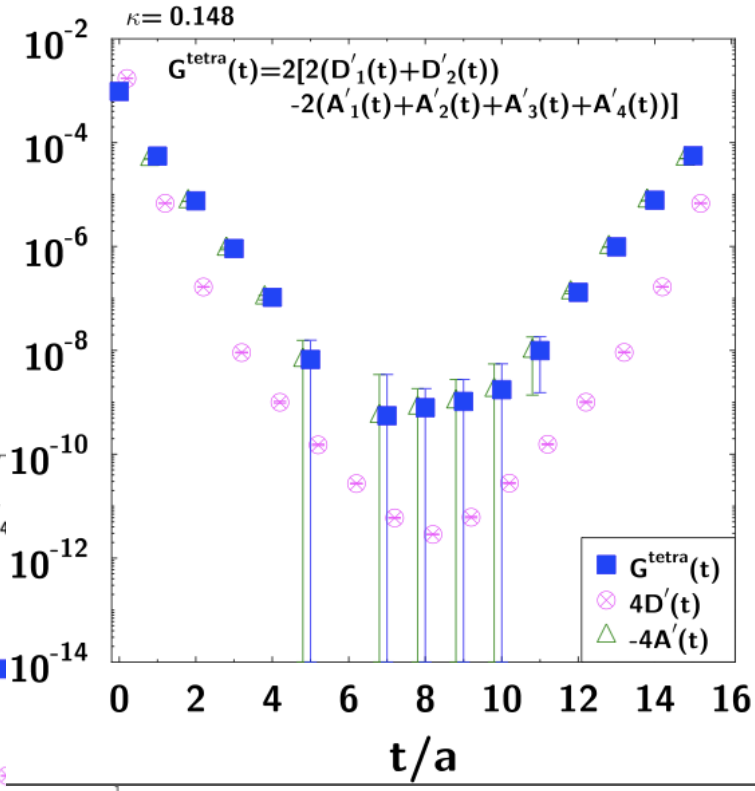




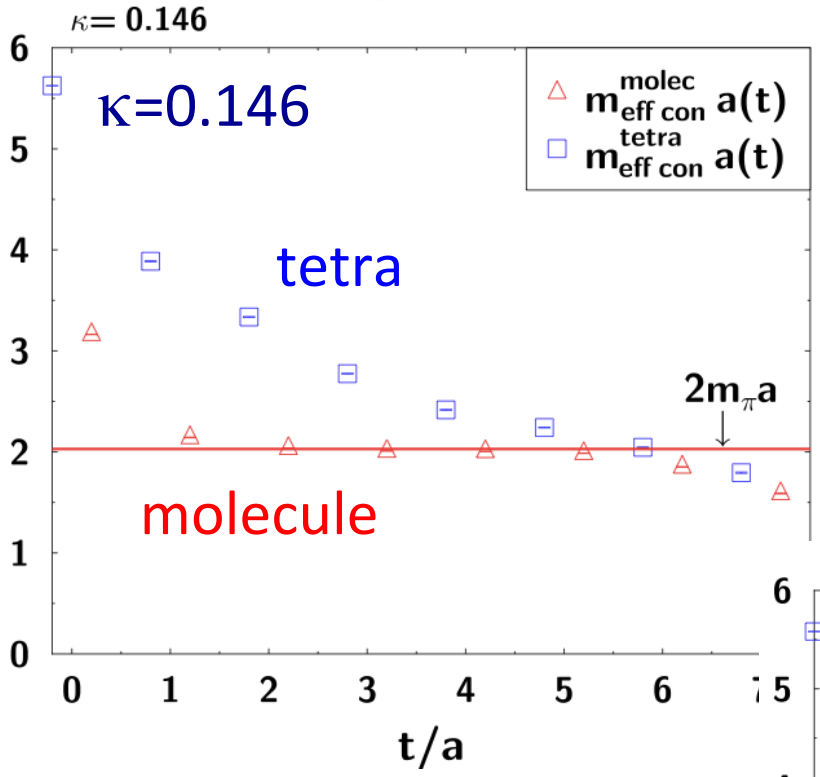


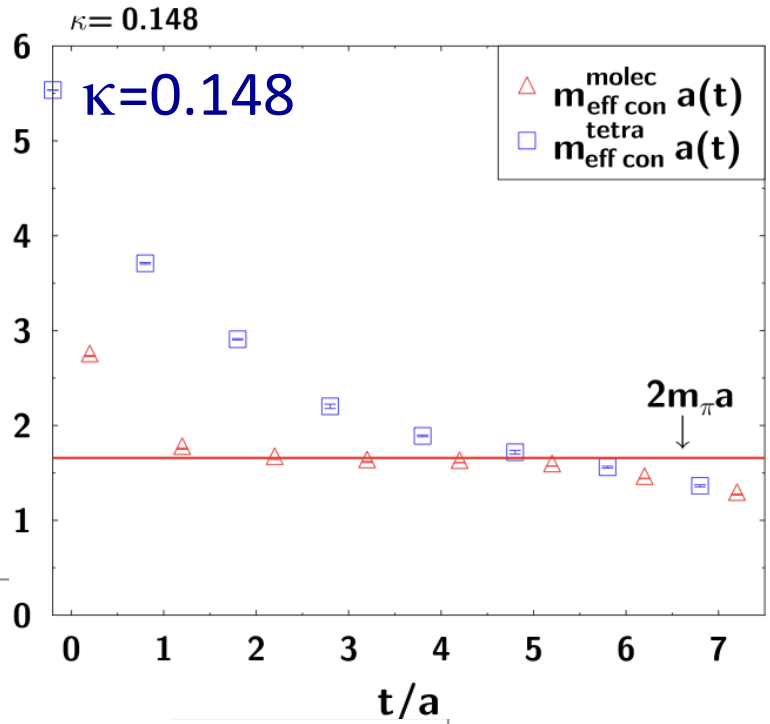

κ= 0.146
G^tetra(t)=2[2(D'1(t)+D'2(t))
-2(A'1(t)+A'2(t)+A'3(t)+A'4(t))]
t/a

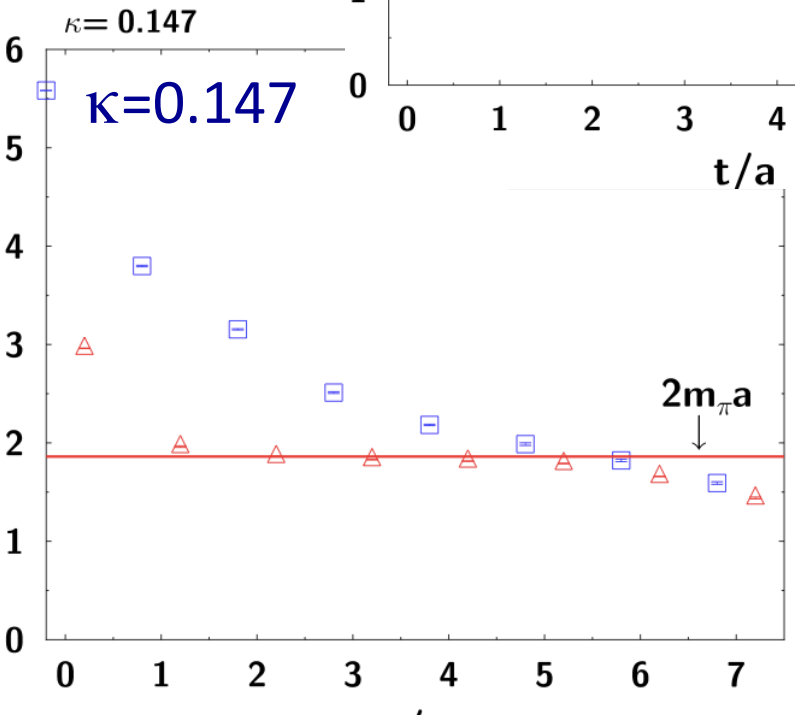

κ= 0.147
G^tetra(t)=2[2(D'1(t)+D'2(t))
-2(A'1(t)+A'2(t)+A'3(t)+A'4
G^tetra(t)
4D'(t)
-4A'(t)
t/a

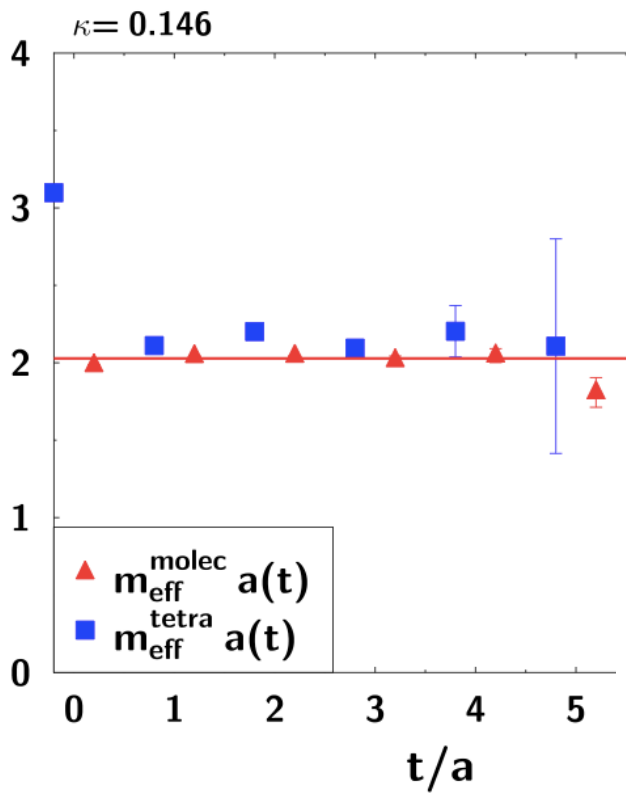

κ= 0.148
G^tetra(t)=2[2(D'1(t)+D'2(t))
-2(A'1(t)+A'2(t)+A'3(t)+A'4(t))]
G^tetra(t)
4D'(t)
-4A'(t)
t/a

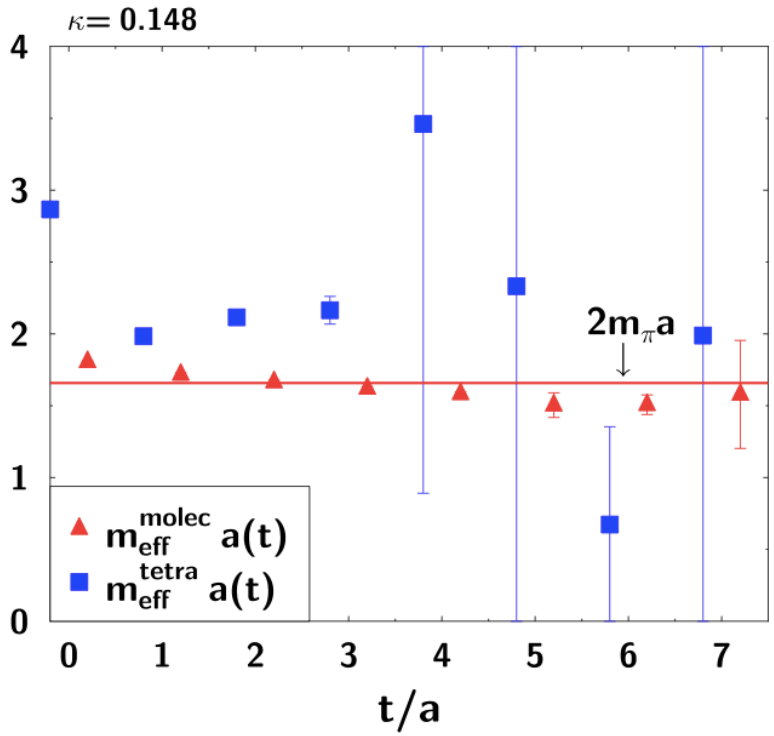
Quark Mass Dependence

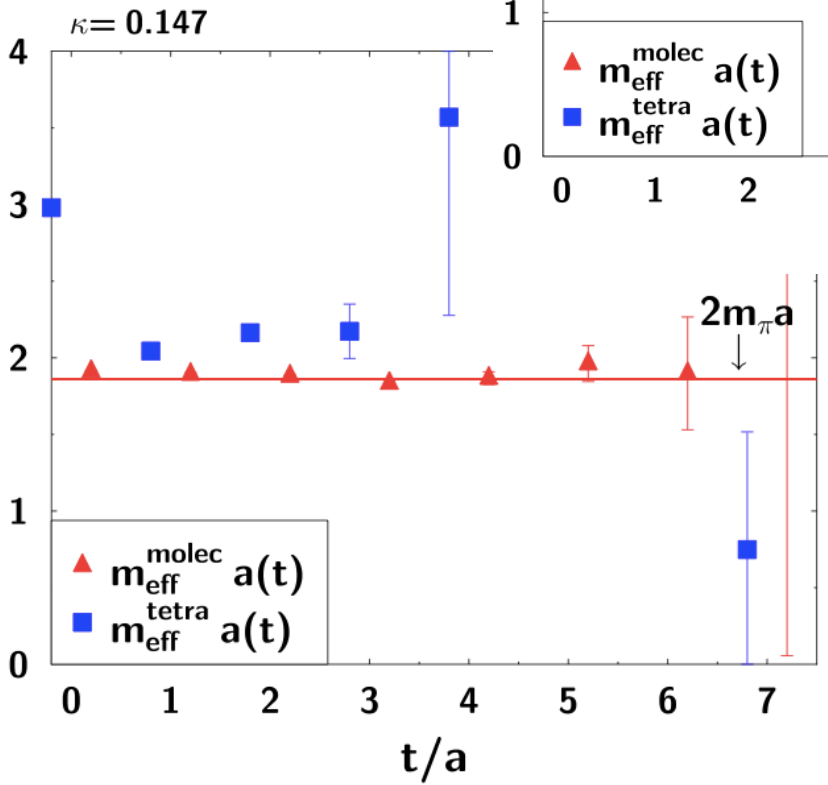


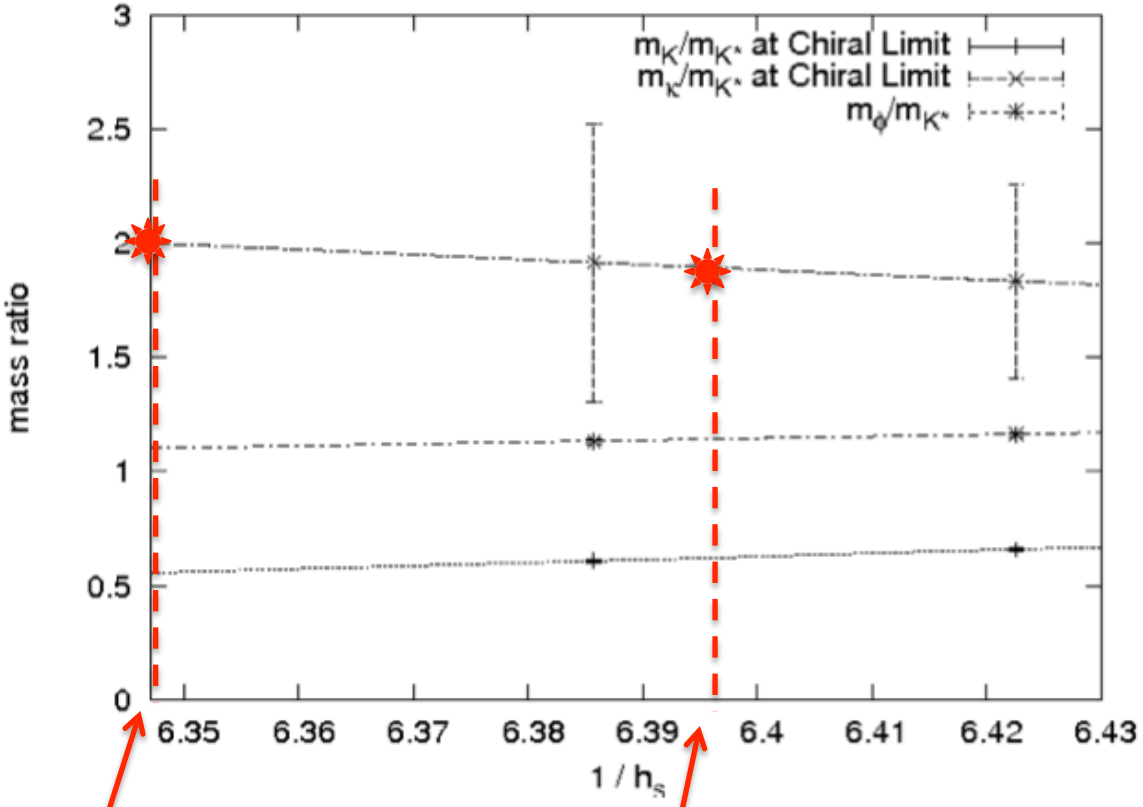





κ= 0.146
2m_πa
m_eff^molec a(t)
m_eff^tetra a(t)
t/a


κ= 0.148
2m_πa
m_eff^molec a(t)
m_eff^tetra a(t)
t/a


κ= 0.147
2m_πa
m_eff^molec a(t)
m_eff^tetra a(t)
t/a



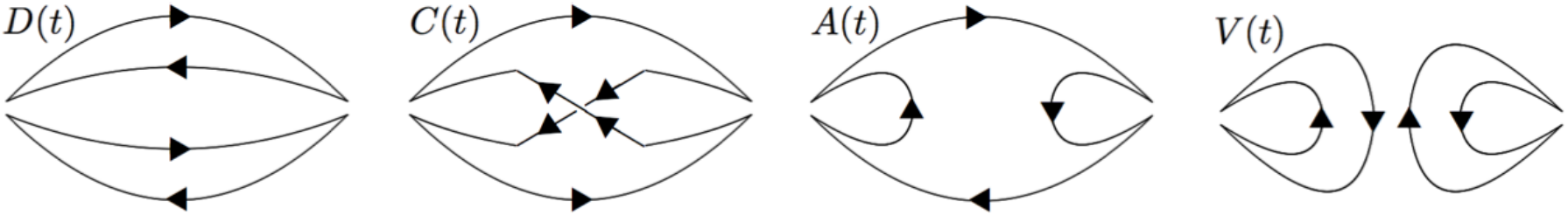
m_K/m_{K^*} input

m_ϕ/m_{K^*} input

h_s	0.1566	0.1557	0.1563(3)	0.1576(2)
$1/h_s$	6.3857	6.4226	6.396(13)	6.3452(80)
m_ϕ/m_{K^*}	1.135(10)	1.164(10)	1.143[1)]	–
m_K/m_{K^*}	0.6086(79)	0.6593(63)	0.623(11)	0.5556[1)]
m_κ/m_{K^*}	1.92(61)	1.84(43)	1.89(55)	2.00(80)

Molecule & Tetra

- Molecule



- Tetra