Topological insights in many flavor QCD on the lattice

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Plan of Talk:

1. Introduction

2. Topological charge and susceptibility
   Mainly, in Nf=8
   ♠ Nf=8 is a candidate for the walking.

3. Eigenvalues and Anomalous dimension

4. Summary, Discussion
1. Introduction
Walking technicolor

\( N_f \) massless fermions + SU(\( N_{TC} \)) gauge at \( O(1) \) TeV

Model requirement:

- **Spontaneous chiral symmetry breaking**
- **Slow running (walking) coupling in wide scale range**
- **Large anomalous mass dimension \( \gamma^* \sim 1 \) in walking region**

- **Higgs \( \approx \) Light composite scalar**
  - pNGB (technidilaton)
  - of scale symmetry breaking

\[
m_{\text{Higgs}} / v_{\text{EW}} \sim 0.5 = m_\sigma / (\sqrt{N_d} F)
\]

\( F \) : decay constant, \( N_d \) : number of weak doublets

usual QCD \( m_\sigma / F \sim 4–5 \)
In the topological nature,

What happens in the walking/conformal phase?
In hadron phase:

★ Index theorem

\[ \text{fermionic chiral zero mode} \leftrightarrow \text{gluonic } F_{\mu\nu} \tilde{F}_{\mu\nu} \]

★ Banks-Casher relation

\[ \langle \bar{\psi}\psi \rangle = \lim_{m \to 0} \lim_{V \to \infty} \int d\lambda \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi \rho(\lambda = 0) \]

★ Leutwyler-Smilga relation

\[ \langle Q^2_{\text{top}} \rangle / V = \Sigma m_f / N_f \]

★ Eigenvalue distribution

\[ \rho(\lambda) \text{ in } p\text{- and } \varepsilon\text{-regime,} \]

\[ \nu(\lambda) = \int_{-\lambda}^{\lambda} \rho(\lambda) d\lambda \]

★ Flavor singlet Pseudo-scalar meson \(\leftrightarrow\) (famous) \(U(1)\) problem

Witten-Veneziano formula

\[ m_{\eta'}^2 = \lim_{N_c \to \infty} \frac{2N_f}{F_\pi^2} \chi|\text{quenched}, \]

\[ m_{\eta'}^2 + m_{\eta}^2 - 2m_K^2 = \mu_0^2 = \frac{4N_f \chi T}{f_\pi^2} \]

In Veneziano limit: \(O(1/N_c)\) expansion, because of \(f_\pi \sim \sqrt{N_c}\)

\[ \frac{\tilde{N}_f}{N_c} \ll 1 \]
2. Topological charge and susceptibility

Mainly, in $N_f=8$

♣ $N_f=8$ is a candidate for the walking.
Gradient flow (Wilson flow, Symanzik flow)

\[
\partial_t V(x, \mu) = -g_0^2 \{ \partial_{x,\mu} S_g(V_t) \} V_t(x, \mu), \quad V_t(x, \mu)|_{t=0} = U(x, \mu),
\]

Example) In the continuum QED,

\[
B_\mu = D_\nu G_{\nu \mu}, \quad B_\mu|_{t=0} = A_\mu,
\]

\[
B_\mu(t,x) = \int d^3y K_t(x-y) A_\mu(y) + \text{gauge terms}, \quad K_t(z) = \frac{e^{-\frac{|z|^2}{4t}}}{(4\pi t)^{3/2}}.
\]

In QCD and BSM,

Key technology in the lattice studies, nowadays.
Numerical integration of the Wilson flow

\[ \dot{V}_t = Z(V_t)V_t, \]

4th order Runge-Kutta method

\[ W_0 = V_t, \]
\[ W_1 = \exp\left\{ \frac{1}{4} Z_0 \right\} W_0, \]
\[ W_2 = \exp\left\{ \frac{8}{9} Z_1 - \frac{17}{36} Z_0 \right\} W_1, \]
\[ V_{t+\epsilon} = \exp\left\{ \frac{3}{4} Z_2 - \frac{8}{9} Z_1 + \frac{17}{36} Z_0 \right\} W_2, \]

where

\[ Z_i = \epsilon Z(W_i), \quad i = 0, 1, 2. \]

Our case:

Wilson flow \((S_{\text{Wilson}}) \rightarrow \text{Symanzik flow} \ (S_{\text{Symanzik}})\)

\[ \epsilon = 0.03 \]

(If \( \epsilon = 0.1 \) for instance, RK doesn't solve correctly or 5th order RK is needed.)
We re-use LatKMI configurations generated for the flavor-singlet scalar meson.

Tree-level Symanzik gauge + HISQ fermions

$L^3 \times (4L/3)$

Periodic boundary condition in spatial-dir.

Anti-PBC in time-dir.

$L=12, 18, 24, 30, 36, 42, (48)$
In $N_f=8$ at $m_f=0.06$ on $L=24$

**Qtop vs Symanzik flow time**

![Graph showing Qtop vs Symanzik flow time for different configurations.]

**Qtop vs #trajectory**

![Graph showing Qtop vs #trajectory for different configurations.]

**t^2E_{plaq} vs flow time**

![Graph showing $t^2E_{plaq}$ vs flow time for different configurations.]

Scale determination: $t^2\langle E \rangle_{t=t_0} = 0.3$
$N_f = 8, m = 0.08, L = 24$

$N_f = 8, m = 0.04, L = 30$

$N_f = 8, m = 0.012, L = 42$
$N_f = 12, \beta = 3.7, m = 0.16, L = 18$

$N_f = 12, \beta = 4.0, m = 0.04 L = 36$

$N_f = 4, m = 0.01, L = 20$
\[ N_f = 4, m = 0.01, L = 20 \]

\[ N_f = 12, \beta = 3.7, m = 0.16, L = 18 \]
$t_0$ vs $m_\pi$ : Scale determination:

\[ t^2 \langle E \rangle |_{t=t_0} = 0.3 \]

- Nf=4; flat in $m_\pi \to 0$
- Nf=12; blowup in $m_\pi \to 0$
- Nf=8; ? in $m_\pi \to 0$

$t^2E$ vs $t$ (flow time) in Nf=16

We can't extract $t_0$ for the current $N_f = 16$ data, as $t^2E$ flattens off before it reaches 0.3.
Nf=8 Finite volume study of $\chi_{top}$

At small mf on small Vol., Qtop is almost frozen.
Analysis of $\chi_{\text{top}} - (1)$

In hadron phase: (ChPT)

$$\chi = C m_f + f(a)$$

$$\chi^{1/4} = (C m_f + f(a))^{1/4}$$

(ChPT) and Hyperscaling

→ It seems to be good.

$\gamma = 1.04(4)$ in $N_f=8$

$\gamma = 0.43(5)$ in $N_f=12$
Analysis of $\chi_{\text{top}} - (2)$

$\chi^{1/4}/m_\pi$ vs $m_\pi$

$\chi^{1/4}/\sqrt{8t_0}$ vs $\sqrt{8t_0}$

- $N_f = 4$: $\chi^{1/4}/m_\pi$ & $\chi^{1/4}/\sqrt{8t_0}$ steep slope for $m_\pi$ & $\sqrt{8t_0} \to 0$
- $N_f = 12$: $\chi^{1/4}/m_\pi$ & $\chi^{1/4}/\sqrt{8t_0}$ flat for $m_\pi$ & $\sqrt{8t_0} \to 0$
- $N_f = 8$: between $N_f = 4$ and 12
Summary-(1): $Q_{\text{top}}$ and $\chi_{\text{top}}$

From $Q_{\text{top}}$ and $\chi_{\text{top}}$, it is difficult with current data to determine whether $N_f=8$ is confining/walking/conformal.

$$\gamma = 1.04(4)$$

However, $\chi_{\text{top}}$ in $N_f=8$ is just between $N_f=4$ and 12.

$$\begin{cases} 
\text{If } \chi_{\text{top}} \text{ in } N_f=4 \text{ is regarded as in the hadron phase} \\
\text{If } \chi_{\text{top}} \text{ in } N_f=12 \text{ is regarded as in the conformal phase}
\end{cases}$$

$$\gamma (N_f=12) = 0.43(5)$$

$\Rightarrow$ $N_f=8$ is in the near-conformal/walking phase. (near the conformal edge)

We have to confirm this conjecture.
3. Eigenvalues and Anomalous dimension
★ Contribution of $\lambda \sim 0$ region (IR)

$$\langle \bar{\psi} \psi \rangle = \lim_{m \to 0} \lim_{V \to \infty} \int d\lambda \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi \rho(\lambda = 0)$$

Banks-Casher relation

$\rho(\lambda)$ is measured in the dynamical gauge background.

$$\Rightarrow \rho_{m_f}(\lambda) = \langle \rho(\lambda) \rangle_{m_f}$$

$$\langle \mathcal{O} \rangle_{m_f} = \int dU \mathcal{O}(U) \det(i\lambda(U) + m_f) \exp(-S_g(U))$$

$$\nu(\lambda, m_f) = \int_0^\lambda \rho_{m_f}(\omega) d\omega$$

$$\nu(\lambda, m_f) = d_1 \lambda^{\alpha+1}$$

If the system is in the conformal,

$$\nu(\lambda, m_f) = d_1 \lambda^{\alpha+1}$$

$$\alpha + 1 = \frac{4}{1 + \gamma}$$

$\gamma$ is a function of $\lambda$ and $m_f$; $\gamma = \gamma(\lambda, m_f)$

$$\alpha + 1 = \frac{\ln \nu_2 - \ln \nu_1}{\ln \lambda_2 - \ln \lambda_1}$$

where $\lambda_2 = \lambda_1 + \Delta$

$\Rightarrow$ local $\gamma$ ($\gamma_{\text{eff}}$)
Nf=8, mf=0.04, mode number counting, \( \nu(\lambda) \)

Nf=8, mf=0.04, spectral density, \( \rho_{mf}(\lambda) \)
Effective $\gamma_{\text{eff}}$ from $\rho(\lambda)$ and $\nu(\lambda)$

from $\rho_{\text{mf}}(\lambda)$

$\gamma_{\text{eff}} = 0.5 \sim 0.7$ for $\lambda > 0.1 \sim 0.2$ ($\gg m_f$)

This value is smaller than $\gamma$ from the spectroscopy.
Behavior and Care in $\lambda \sim 0$ region:

At $\lambda \sim 0$, $\rho(\lambda)$ has the strong dependence of $\Delta \lambda$ (=bin size of histogram). For finer $\Delta \lambda$, $\rho(\lambda \sim 0)$ has a peak. → mass deformed effect?

$$\rho_{m_f}(\lambda) = c_0 + c_1 \lambda^\alpha$$ is not valid.

$$\nu(\lambda, m_f) = \int_0^\lambda \rho_{m_f}(\omega) d\omega$$

$$\nu(\lambda, m_f) = d_0 \lambda + d_1 \lambda^{\alpha+1}$$ doesn’t describe exactly this behavior.
Behavior and Care in $\lambda \sim 0$ region:

At $\lambda \sim 0$, $\rho(\lambda)$ has the strong dependence of $\Delta \lambda$ (=bin size of histogram). For finer $\Delta \lambda$, $\rho(\lambda \sim 0)$ has a peak. \textcolor{red}{$\rightarrow$ mass deformed effect?}

Is it similar to the $\varepsilon$-regime ChPT?

c.f. H.Fukaya et al., PRL104(2010)122002
Furthermore, to consider $\lambda \sim 0$ (IR) region

Size independent for $\lambda > 0.2 \gg mf$

It seems that $\nu(\lambda)$ is independent of $mf$ for $\lambda > 0.2 \gg mf$

For $\lambda < 0.1$, $\nu(\lambda)$ is not simple power behavior.

$\Rightarrow \gamma$ obtained from $\lambda > 0.1 \sim 0.2 \equiv$ the anomalous dimension?
Summary-(2): EV and $\gamma$

From EV distribution and the mode number counting,

$$\gamma = 0.5 \sim 0.7 \ (N_f=8) \ \text{for} \ \lambda > 0.1 \sim 0.2 \ (\gg m_f)$$

smaller than that from spectroscopy ($\gamma = 0.7 \sim 1.0$)

Why? $\Rightarrow$ We should make clear.

We estimate $\gamma$ from large $\lambda$ region. ($\lambda \gg m_f$)

$\Rightarrow$ near UV $\lambda$? (very far from IR?)

At $\lambda \sim 0$, the peak in $\rho (\lambda)$ appears. Is this similar to the $\epsilon$-regime ChPT?

(Due to mass deformed, $mf\neq 0$ effect ?)

Our simulation is done in $mf\neq 0$ dynamical gauge background.

$\Rightarrow$ How to take $mf \rightarrow 0$?

$\Rightarrow$ How to treat $\lambda \sim 0$ region?
Summary

- SU(3) gauge theories with 8 HISQ quarks.
  Preliminary (data updated: 2013→2014→2015)

- $F_{\mu\nu}\tilde{F}_{\mu\nu}$ + Gradient flow

- Topological charge & susceptibility $\Rightarrow \gamma = 1.04(4)$ in Nf=8
  it is difficult with current data to determine whether Nf=8 is confining/walking/conformal.

- Dirac Eigenvalue Distribution (Mode Number Counting)
  $\Rightarrow$ Anomalous dimension $\Rightarrow$ but, small value
  Care of $\lambda=0$ region and $mf\rightarrow 0$

We have a lot of issues to understand a large Nf QCD
Furthermore,
What we are doing, What we are planning.
Discussion:

(0) In each $Q_{top}$ sector
(1) Index theorem
(2) Banks-Casher relation
(3) Leutwyler-Smilga relation
(4) Flow $\Rightarrow$ smearing (smoothing)
(5) Glueball, String tension (Wilson loop), Polyakov loop
(6) Flavor singlet scalar meson (re-calc.), Cross Corr. with G-ball
(7) Flavor singlet Pseudo-scalar meson $\Rightarrow$ probe of $U(1)_{A}$

In conformal/walking phase
Eigenvalue in gradient flow configuration

\[ t: \text{flow time} \quad (t=0: \text{original configuration}) \]

After the flow,
it is possible to obtain the clear signal of EVs of staggered fermions.
EV behavior after the flow = spectral-“flow”

The cases that zero mode exists.

The case of no zero mode
Chirality in the spectral-"flow"

20 configs. (Nev=20)

$N_f=8$, $L=24$, $m_f=0.06$

$\Rightarrow$ After the flow, the (would-be) zero mode appears with definite chiralities$=\pm 1$.

$\Rightarrow$ good application to the topological insights
Discussion: In conformal/walking phase

(0) In each $Q_{\text{top}}$ sector
(1) Index theorem
(2) Banks-Casher relation
(3) Leutwyler-Smith relation
(4) Flow $\Rightarrow$ smearing (smoothing)
(5) Glueball, String tension (Wilson loop), Polyakov loop
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(7) Flavor singlet Pseudo-scalar meson $\Rightarrow$ probe of $U(1)_A$
Glueball mass \((0^{++})\)

effective mass, \(m_G = \log \frac{C_H(t)}{C_H(t+1)}\)

t=0: no plateau \(\rightarrow\) cannot extract \(m_G\)

t>0: plateau (?), better signal than \(t=0\) case \(\rightarrow m_G\) or \(m_G/\sqrt{t_0}\)
Discussion: In conformal/walking phase

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\( \eta' \) mass

\( m_{\eta'} \) in \( N_c < N_f < N_f^{cr} \) and \( N_f > N_f^{cr} \) ?

In the region of \( N_f/N_c << 1 \) or \( >> 1 \) ?

Witten-Veneziano formula, Veneziano limit ?

\( U(1)_A \) in conformal phase ?

\[
\langle \bar{\psi} \psi \rangle = \lim_{m \to 0} \lim_{V \to \infty} \int d\lambda \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi \rho(\lambda = 0) = 0
\]

\( \langle \partial_\mu j_\mu^5 \rangle, Q_{top}, \chi_{top}, \rho(\lambda) \) ?

\( m_{\eta'} \) ?
In Progress & In the near future

(0) In each Qtop sector
(1) Index theorem
(2) Banks-Casher relation
(3) Leutwyler-Smilga relation
(4) Flow $\Rightarrow$ smearing (smoothing)
(5) Glueball, String tension (Wilson loop), Polyakov loop
(6) Flavor singlet scalar meson (re-calc.), Cross Corr. with G-ball
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Thank you