

SU(2) gauge theory with many flavors of domain-wall fermions

- Introduction
- Setup
- Result of simulation: $N_f=2, 4, 6, 8$
Static potential/meson masses/decay constant
- Summary/outlook

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Introduction

- **SU(2) gauge theory**

- Many works on confinement mechanism, finite temperature/density
- Beyond standard model: technicolor, conformal window, dark matter
 - F. Bursa et al. (2011), H. Ohki et al. (2010), Lewis, Pica, Sannino (2012)
T. Karavirta et al. (2012), M. Hayakawa et al. (2013)

- **Chiral dynamics depending on gauge group and fermion repr.**

- Different symmetry breaking pattern

Fundamental

$$\text{SU}(2): \text{SU}(2N_f) \rightarrow \text{Sp}(2N_f)$$

$$\text{SU}(N) \ N > 2: \text{SU}(N_f) \times \text{SU}(N_f) \rightarrow \text{SU}(N_f)$$

Adjoint

$$\text{SU}(N): \text{SU}(2N_f) \rightarrow \text{SO}(2N_f)$$

- Dependence on number of flavors
- Finite temperature/density
- Eigenvalue distribution – comparison to random matrix theory

Strategy

- Chiral symmetric fermion is better device

- Overlap: best symmetry, high numerical cost, involved setup (Aoki phase, etc.)
H.M., Kikukawa, Yamada, Nagai, Lattice 2010, 2009
- Domain-wall: good properties, numerically feasible
 - Approaches to overlap with large N_s
 - Residual mass probes explicit chiral symmetry violation

Domain-wall



Improved domain-wall



Overlap

- Topology changes
- Large lattice is possible

Measurement:

- Spectroscopy
- Eigenvalue/vectors, etc.

- Fixed topology
- p -regime/ ϵ -regime

Present work

- Lattice actions:
 - Iwasaki gauge action
 - Standard domain-wall fermions: $N_f=2, 4, 6, 8$
- Survey of N_f -dependence with fixed setup
 - Applicability of domain-wall (and overlap) fermions
 - Confining/conformal feature ?
 - Static potential
 - Meson masses and decay consts/residual mass
 - Eigenmodes of domainwall/overlap fermion operators (underway)
- Fundamental setup: making basis for further studies
 - Finite T/μ , adjoint fermions
 - Comparison with improved domain-wall, dynamical overlap
 - Condition to access the ε -regime

Setup

- Lattice size: $16^3 \times 32$, $N_s=16$
- Configuration generation: Hybrid Monte Carlo
 - Domain-wall/Pauli-Villars
 - Omelyan integrator + multi-time step (2-level)
 - About 1000 trajectories at each parameter set

Nf	beta	m
2	0.85	0.20, 0.10, 0.05
	0.90	0.20, 0.10, 0.05
4	0.85	0.20, 0.10, 0.05, 0.03
	0.90	0.20, 0.10, 0.05
6	0.80	0.20, 0.10, 0.05
	0.85	0.20, 0.10, 0.05
	0.90	0.20, 0.10, 0.05
8	0.80	0.20, 0.10, 0.05
	0.85	0.20, 0.10, 0.05

Resources/environment

- **Machines**

- Hitachi SR16000, IBM Blue Gene/Q at KEK
- ϕ at KMI, Nagoya Univ.



- **Code:**

- Bridge++ (C++)
 - Cf. S.Ueda's poster
- Fortran code



- **JLDG (Japan Lattice Data Grid)**

- for fast data transfer



Domain-wall fermion

- Standard domain-wall fermion action

$$\begin{aligned}
 S_{DW} = & \sum_{x,s} \bar{\psi}(x,s) D_W(x,y; -M_0) \psi(y,s) \\
 & - \frac{1}{2} \sum_{x,s} \bar{\psi}(x,s) [(1 - \gamma_5) \psi(x,s+1) + (1 + \gamma_5) \psi(x,s-1) - 2\psi(x,s)] \\
 & + m [\bar{\psi}(x,1) P_R \psi(x,L_s) + \bar{\psi}(x,L_s) P_L \psi(x,1)]
 \end{aligned}$$

$$D_W(x,y; M) = M \delta_{x,y} - \frac{1}{2} \sum_{\mu=1}^4 \left\{ (1 - \gamma_\mu) U_\mu(x) \delta_{x+\hat{\mu},y} + (1 + \gamma_\mu) U_\mu^\dagger(x - \hat{\mu}) \delta_{x-\hat{\mu},y} - 4\delta_{x,y} \right\}$$

- M_0 : domain-wall height, m : fermion mass
- L_s : extent of 5-th direction
- Boundary conditions: $P_R \psi(s=0) = P_L \psi(s=L_s+1) = 0$
- 4D fermion field:

$$\begin{aligned}
 q(x) &= P_L \psi(x,s=1) + P_R \psi(x,s=L_s) \\
 \bar{q}(x) &= \bar{\psi}(x,s=1) P_R + \bar{\psi}(x,s=L_s) P_L
 \end{aligned}$$

Residual mass

- Residual mass

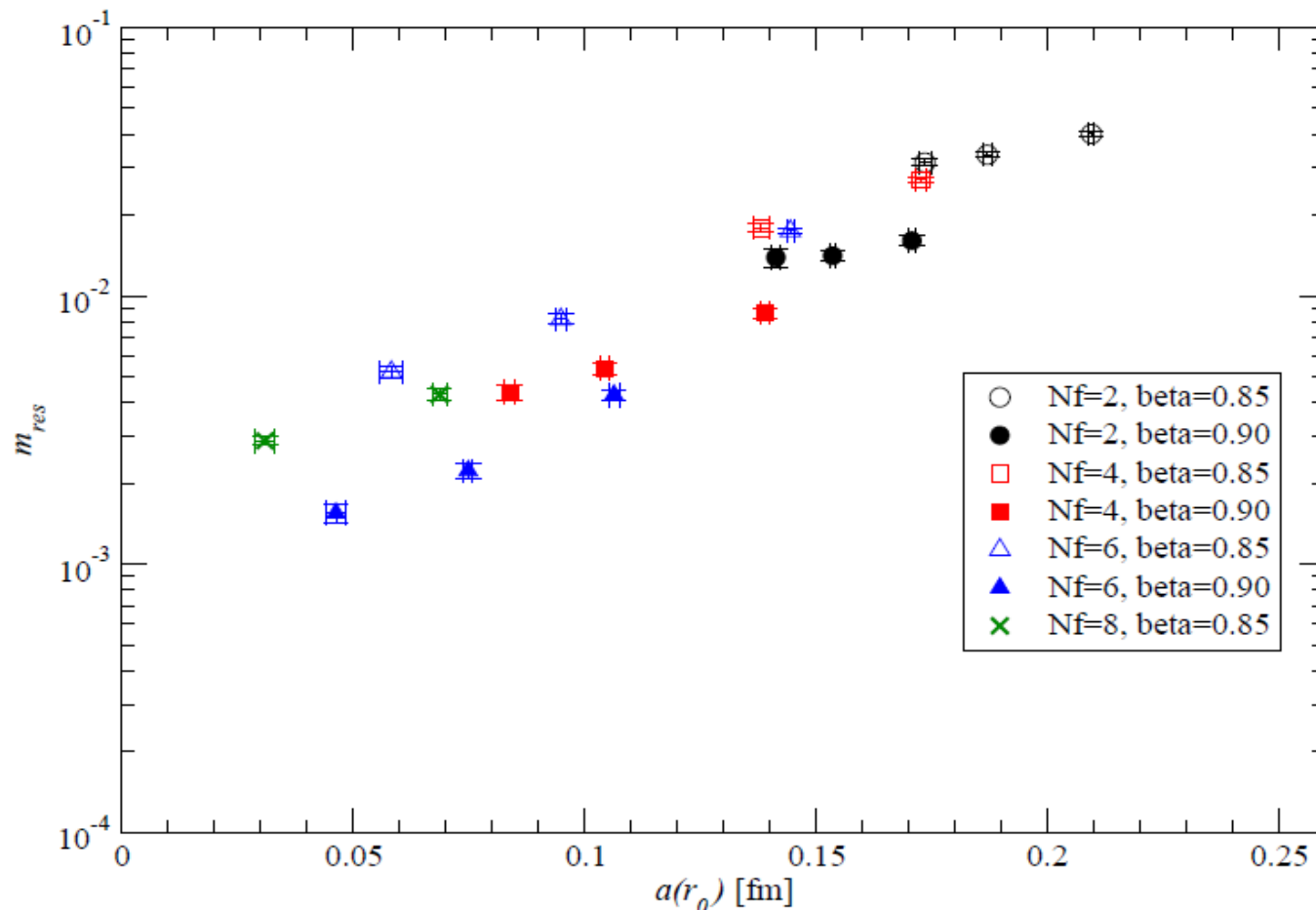
$$R(t) = \frac{\sum_{\vec{x}} \langle J_{5q}(\vec{x}, t) P(0) \rangle}{\sum_{\vec{x}} \langle P(\vec{x}, t) P(0) \rangle},$$

$$J_{5q}(x) = -\bar{\psi}(x, L_s/2) P_L \psi(x, L_s/2 + 1) + \bar{\psi}(x, L_s/2 + 1) P_R \psi(x, L_s/2).$$

- Averaged over large t
- Quantifies explicit chiral symmetry breaking of domain-wall formulation

Nf-dependence of residual mass

- General tendency: m_{res} decreases as lattice spacing increases
- To go below present residual mass, larger Ns or improved domain-wall are necessary



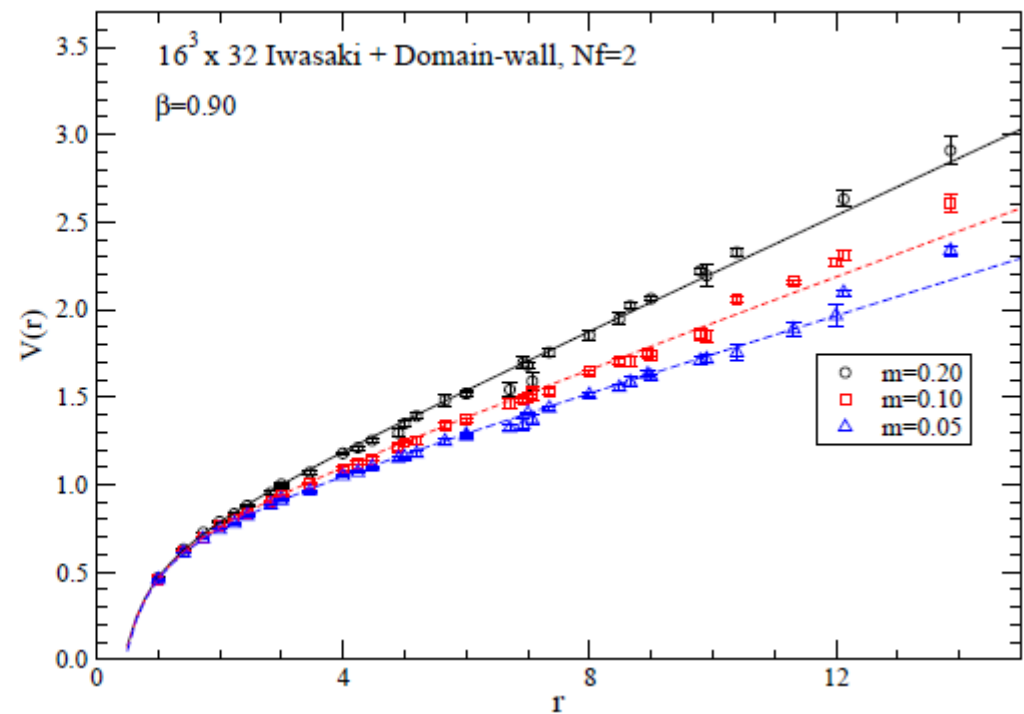
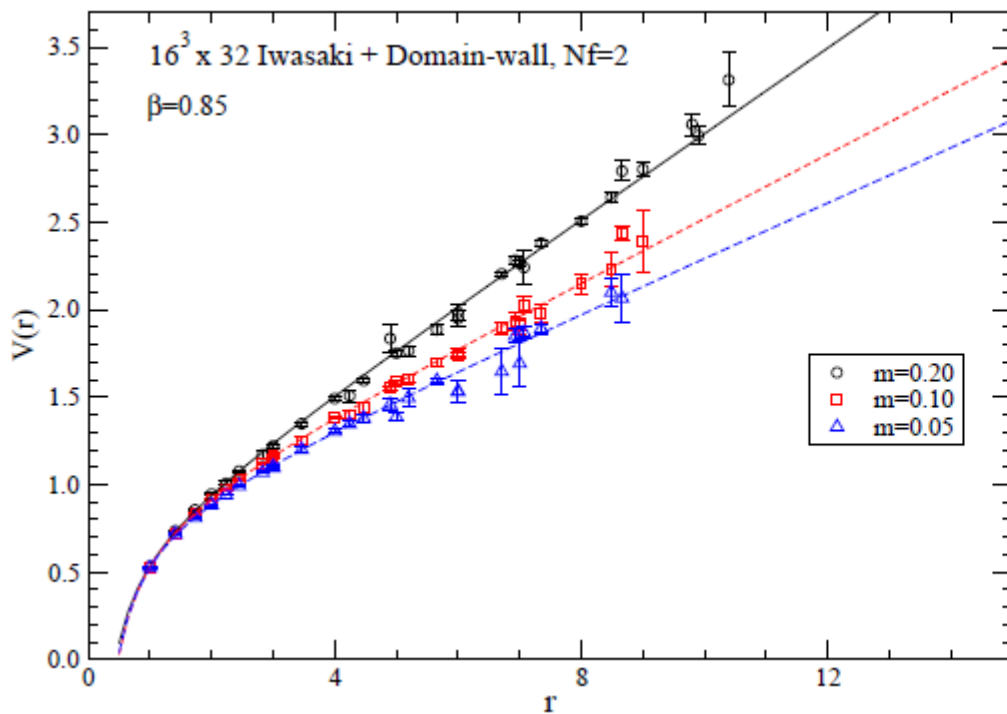
Static potential: Nf=2

Nf	beta	m
2	0.85	0.20, 0.10, 0.05
	0.90	0.20, 0.10, 0.05

- Static potential

- Fitted to $V(r) = \text{const} - A/r + \sigma r$

- Sommer scale r_0 (or string tension) \rightarrow "lattice spacing"

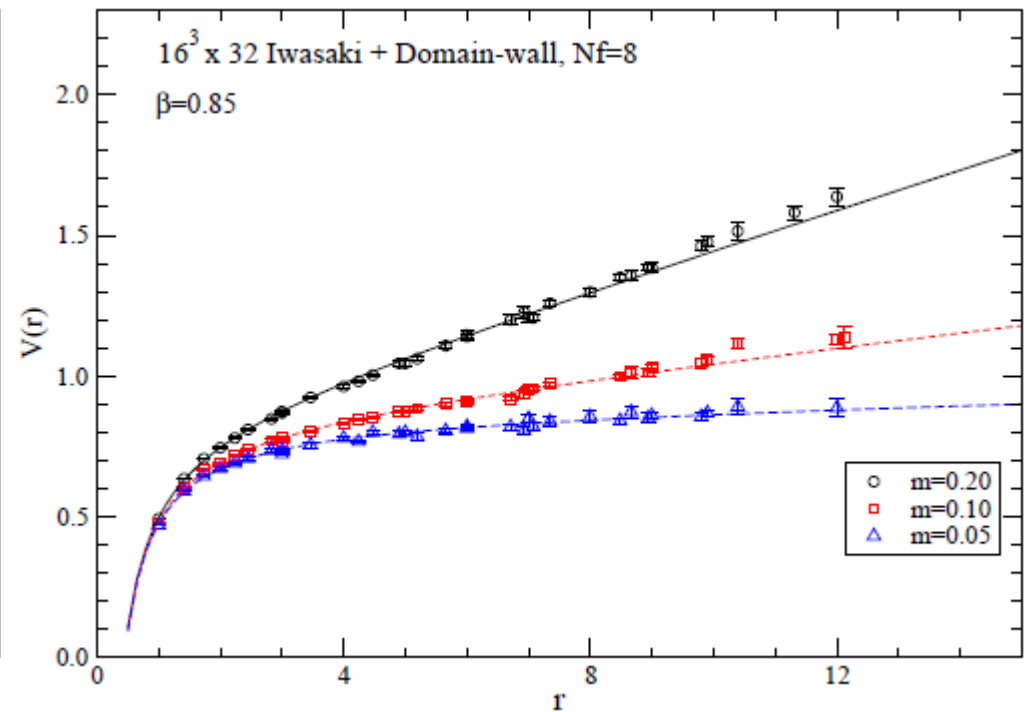
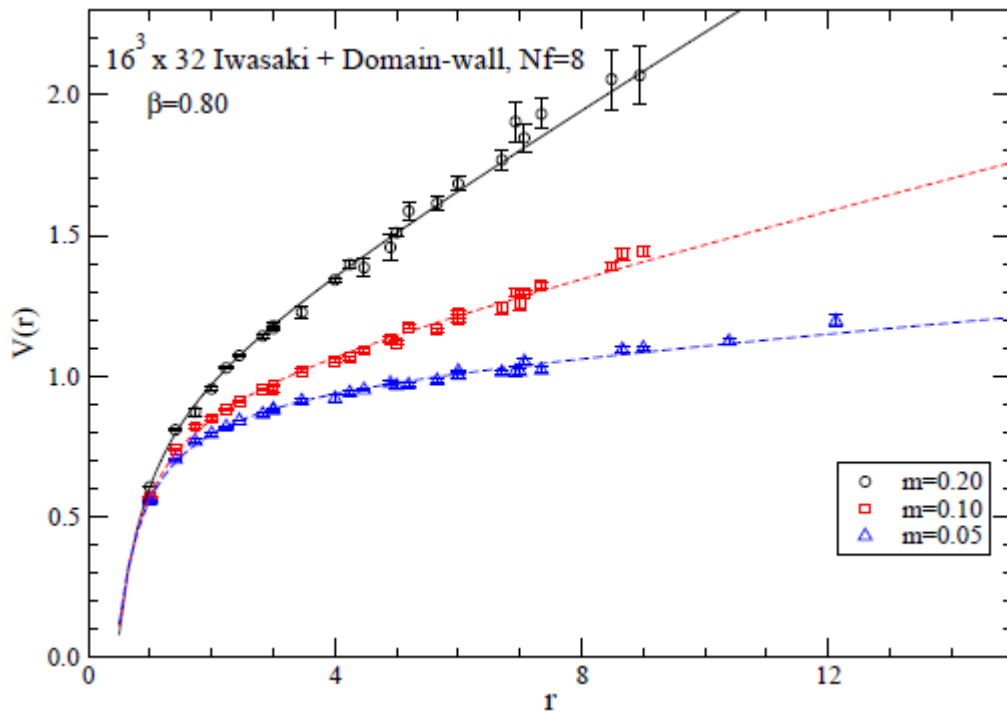


Static potential: $N_f=8$

N_f	beta	m
8	0.80	0.20, 0.10, 0.05
	0.85	0.20, 0.10, 0.05

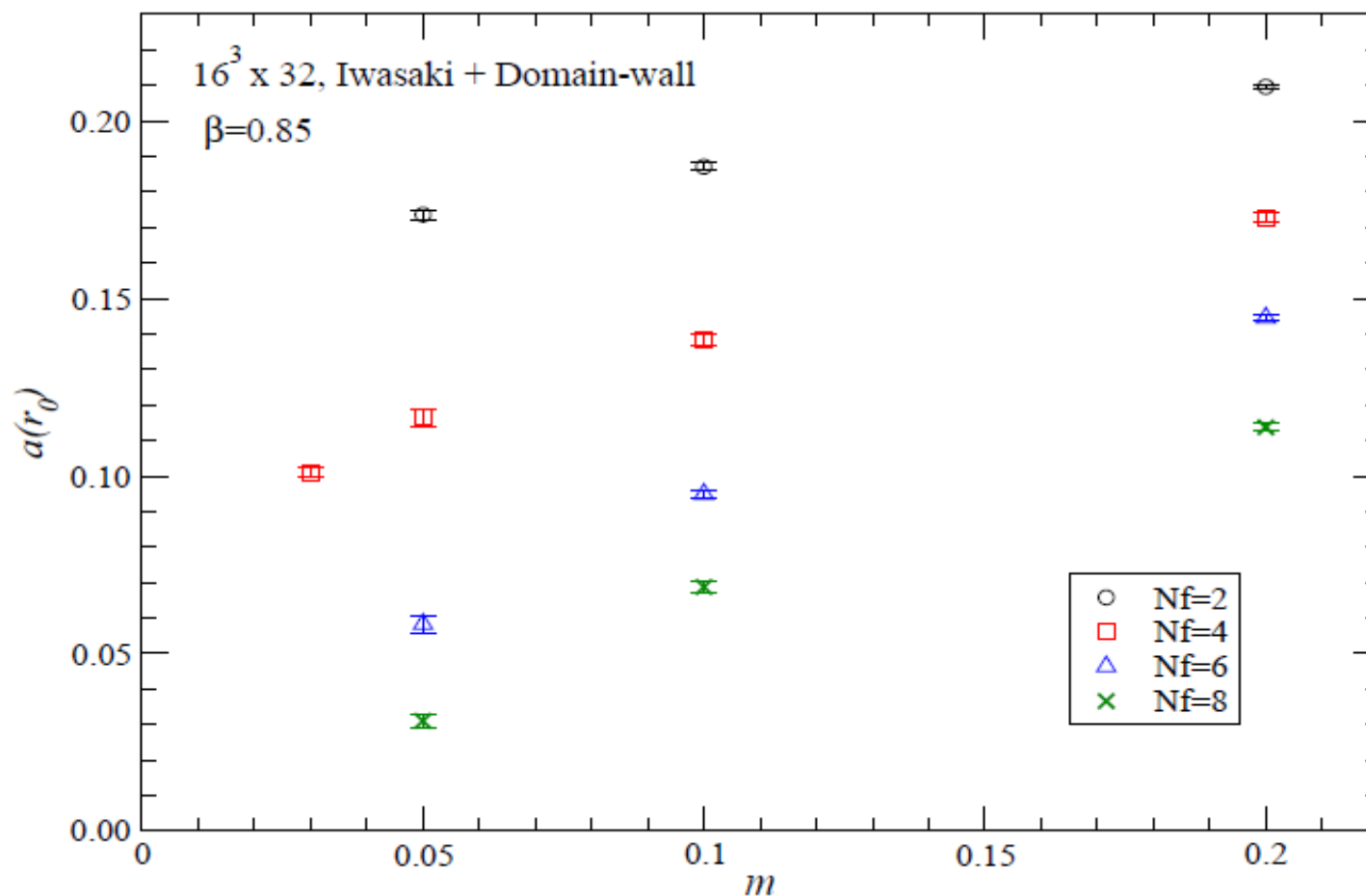
- Static potential

- Bare mass dependence is large
- String tension seems to vanish as m goes to zero



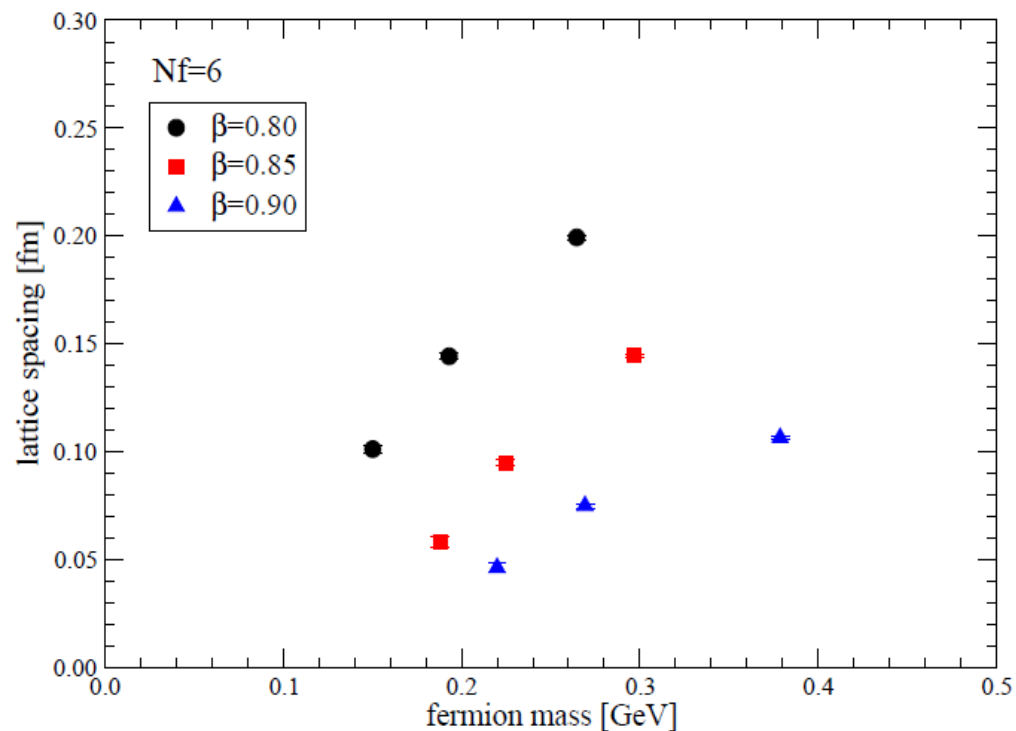
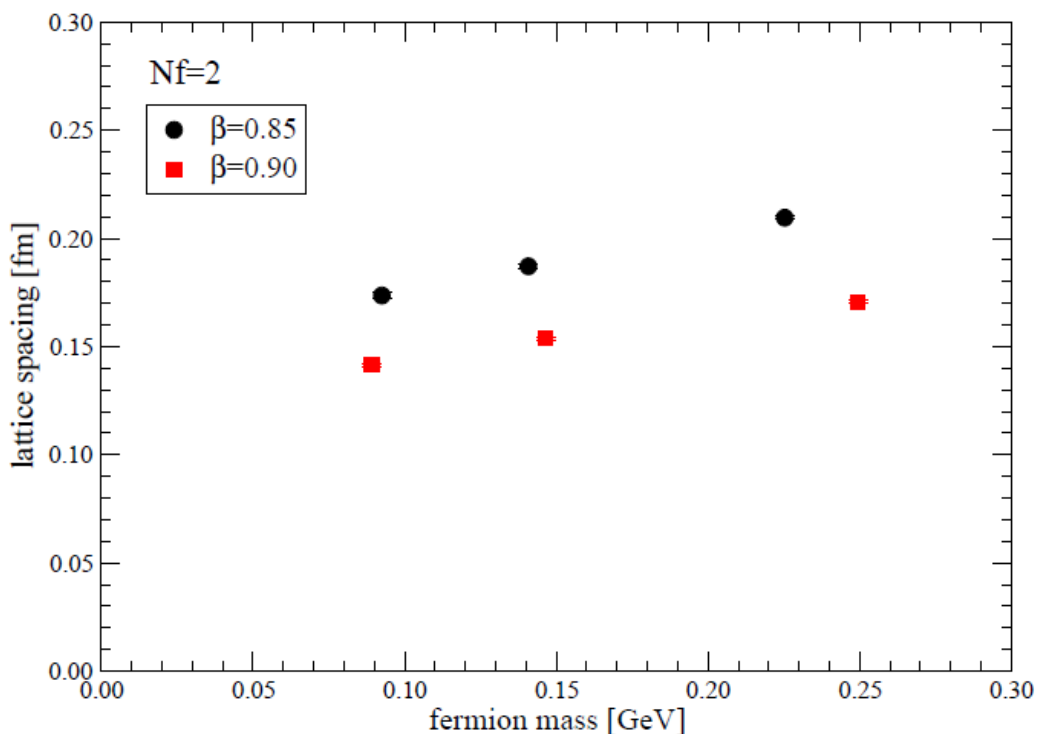
Nf dependence of static potential

- "lattice spacing" (lattice scale) set by $r_0 = 0.49$ fm
- Nf-dependence grows as Nf increases.
- For Nf=8 (and 6), confining feature seem to disappear at $m=0$.



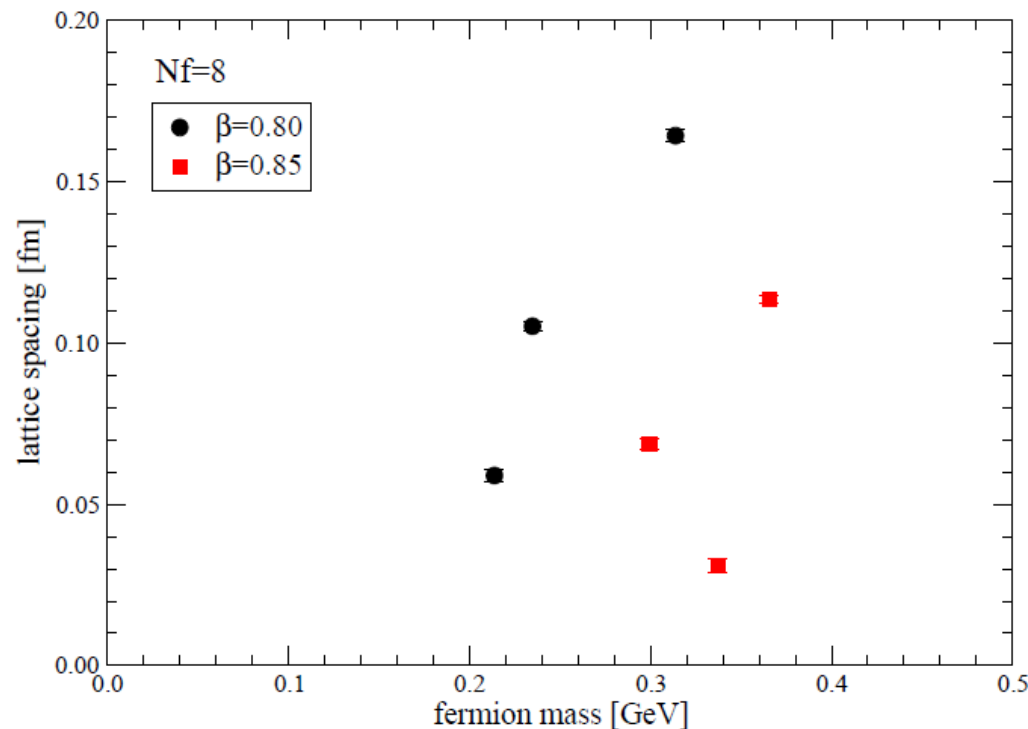
Nf dependence of static potential

- Lattice spacing vs fermion mass (residual + bare)
 - No renormalization for fermion mass
- Extrapolation to massless limit seems successful for Nf=2 while not for Nf=6, and 8 (next page)



Nf dependence of static potential

- Nf=8: massless limit is hardly taken
- Confining feature seems to disappear at massless limit
- **Caution: at beta=0.85 and m=0.05, volume might be too small**



Meson spectroscopy

- **Partial quenched analysis**

- ← Large sea quark mass dependence for large N_f
- Valence fermion mass dependence is observed on given config.

- **Measurement**

- Meson correlators are measured in standard way → mass, decay const.
- Solving linear equations only for one of color components is needed (thanks to pseudoreality of SU(2))

$$\begin{aligned} D_{DW}(x, s; y, s') &= C^{-1}(-i\sigma_2)^{-1} R D_{DW}(y, s'; x, s)^T R C(-i\sigma_2), \\ &= C^{-1}(-i\sigma_2)^{-1} \gamma_5 D_{DW}(x, s; y, s')^* \gamma_5 C(-i\sigma_2) \end{aligned}$$

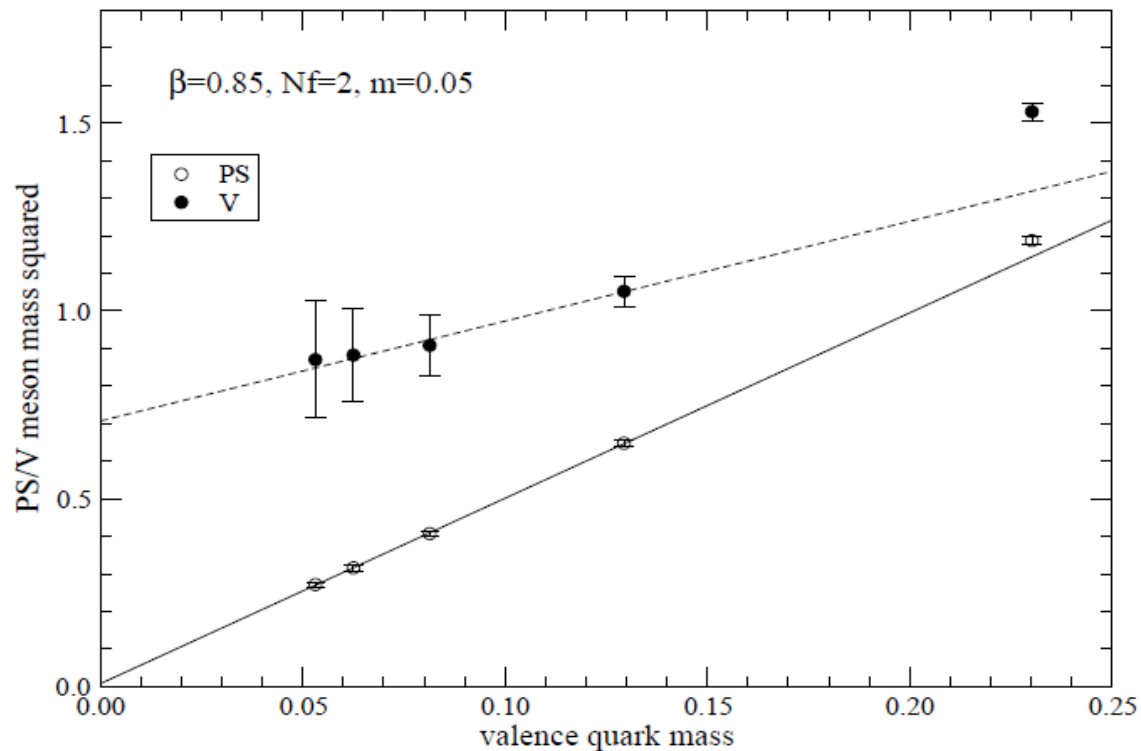
→ baryon and meson correlators are identical

Lewis, Pica and Sannino (2012)

- Currently only with local-local correlators

PS and V meson spectra: $N_f=2$

- Valence fermion mass dependence: similar behavior as SU(3) case
- Fermion mass: bare + residual masses



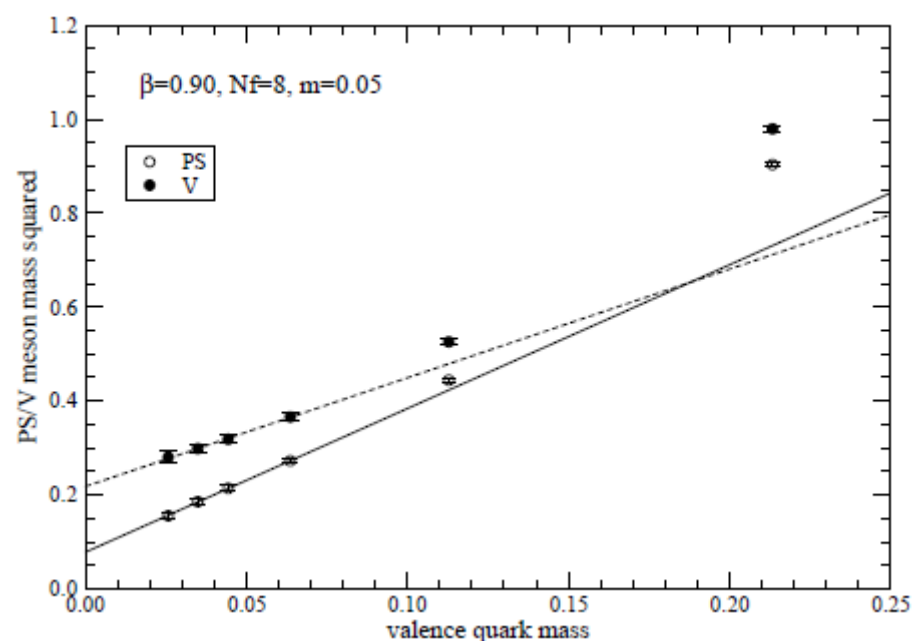
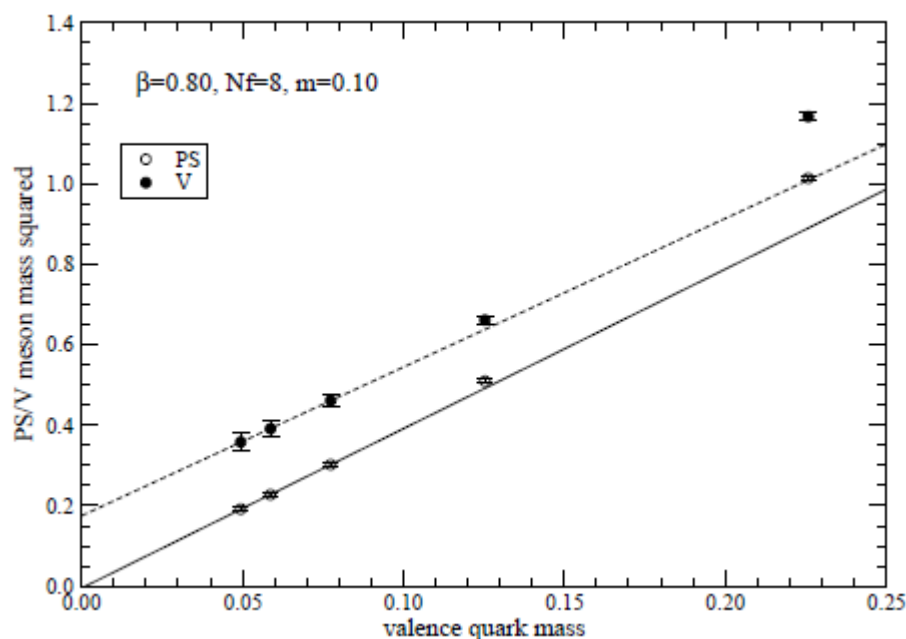
PS and V meson spectra: $N_f=8$

- $N_f=8$

- As sea fermion mass decreases, deviation from GMOR relation
- Finite size effect ?

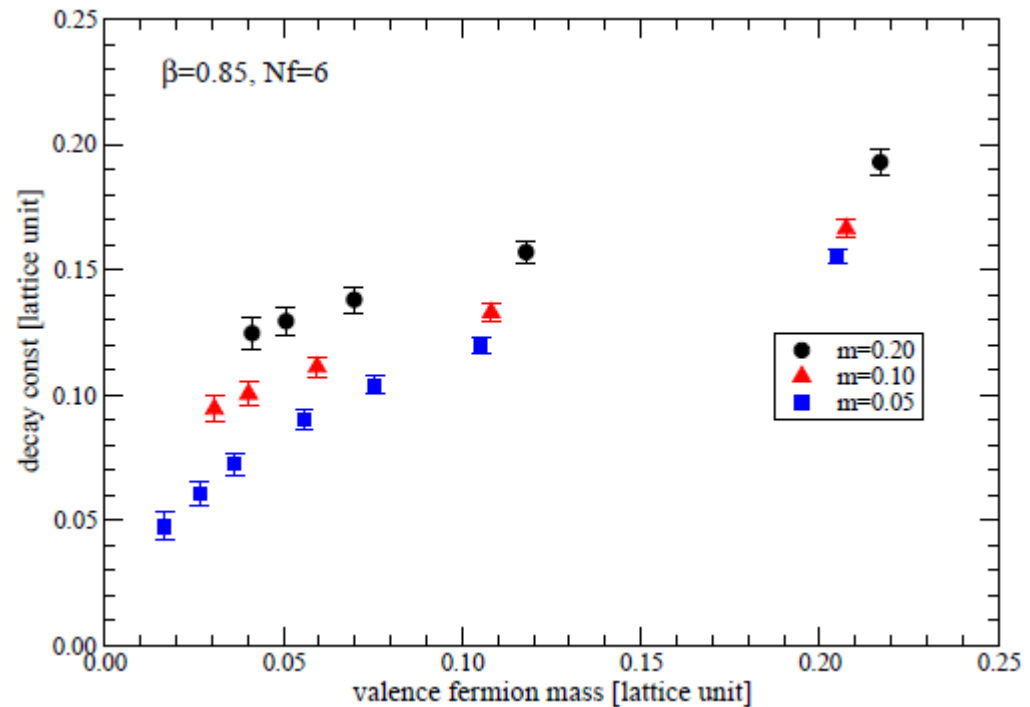
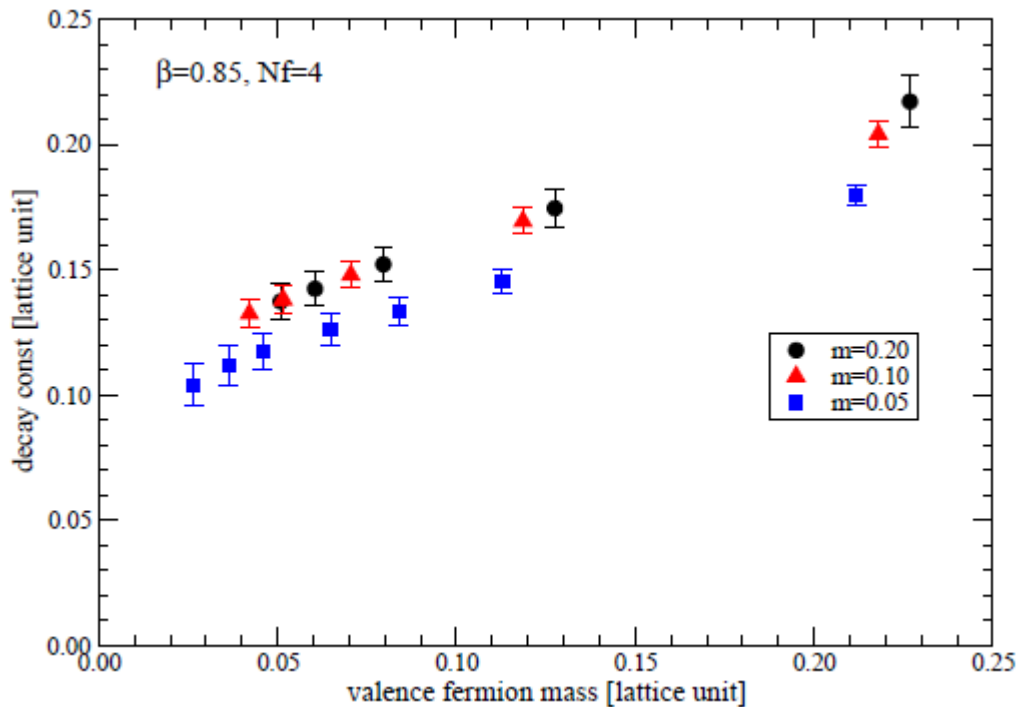
- **Caution:**

- Ground state plateau behavior is transient
→ to be compared with smeared correlation functions



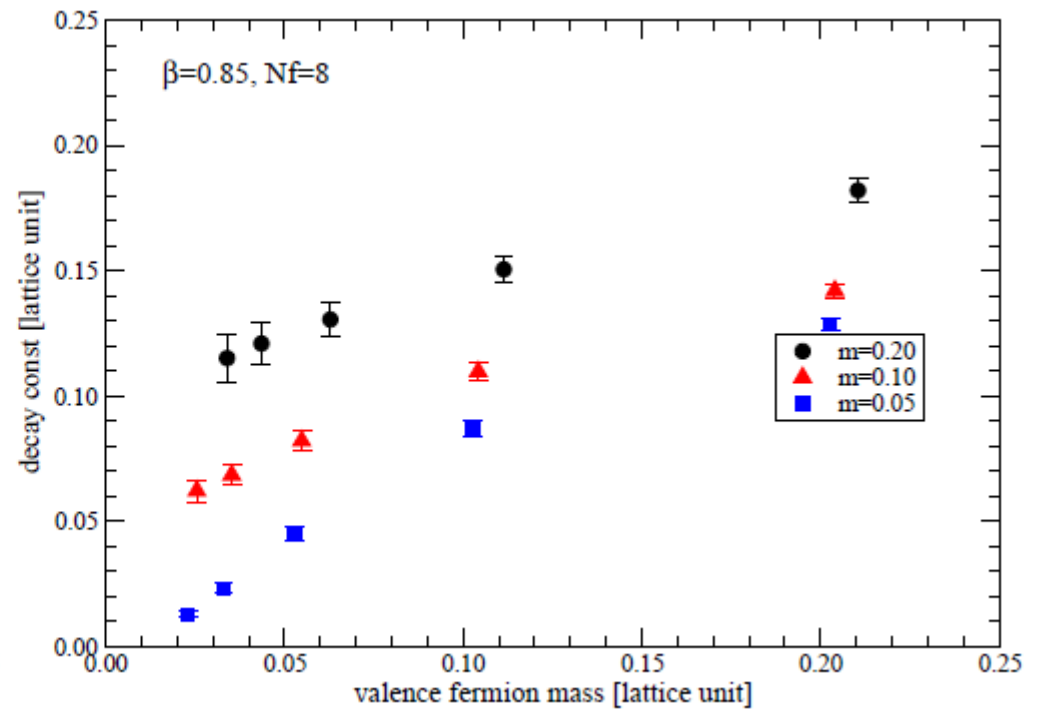
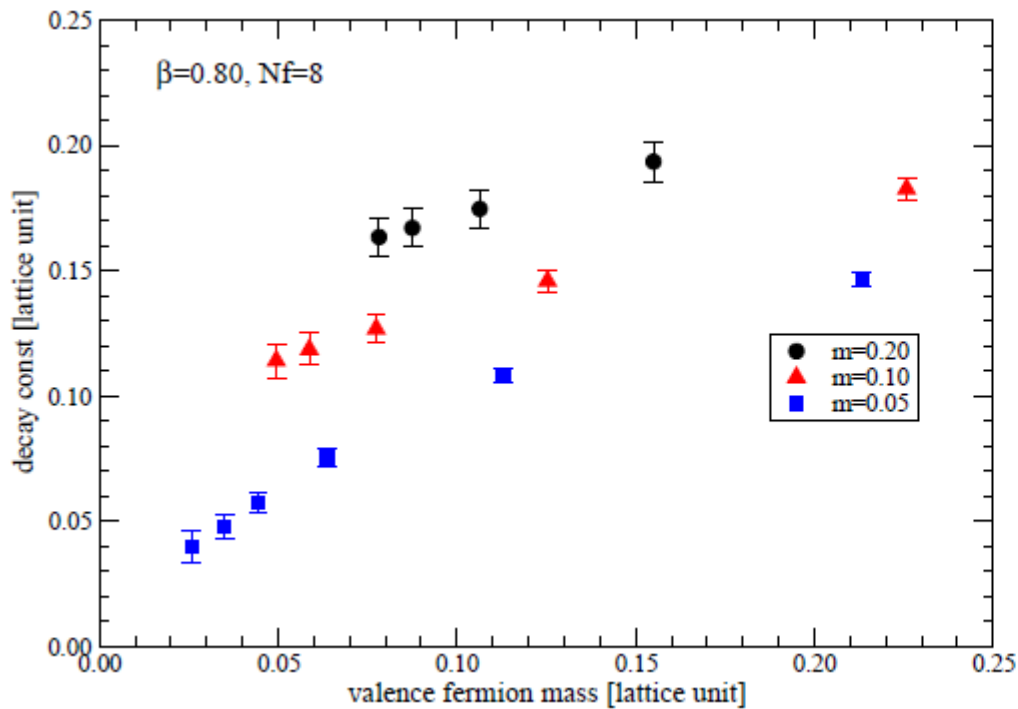
Decay constant

- $N_f=4$
 - Approaches to finite value as valence mass goes to zero
- $N_f=6$
 - Sea fermion mass dependence becomes larger



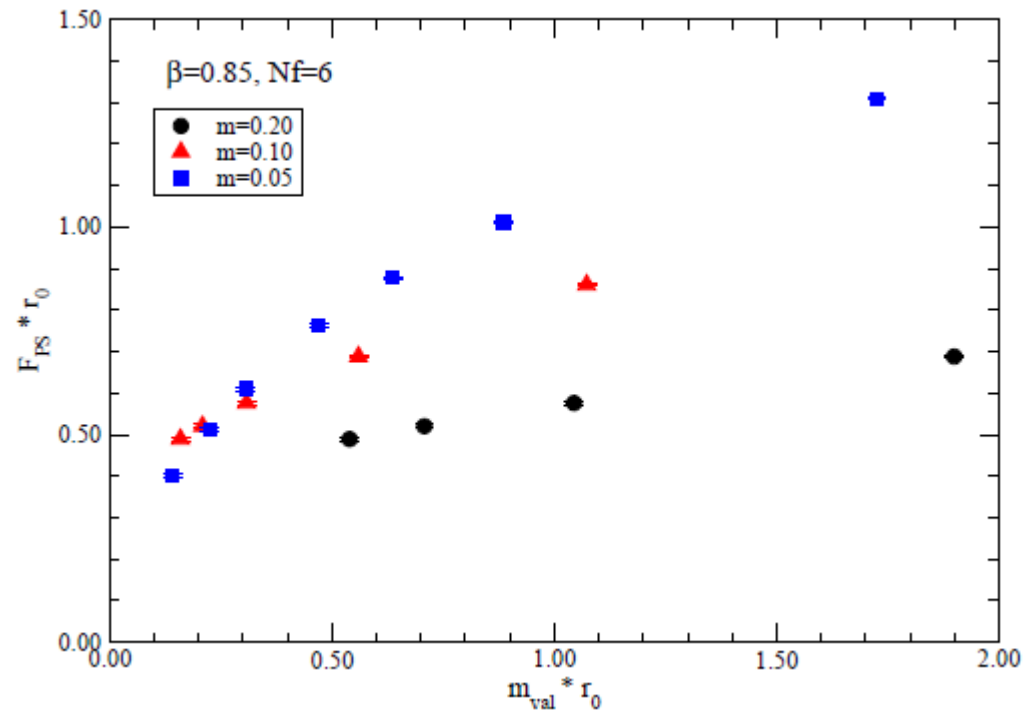
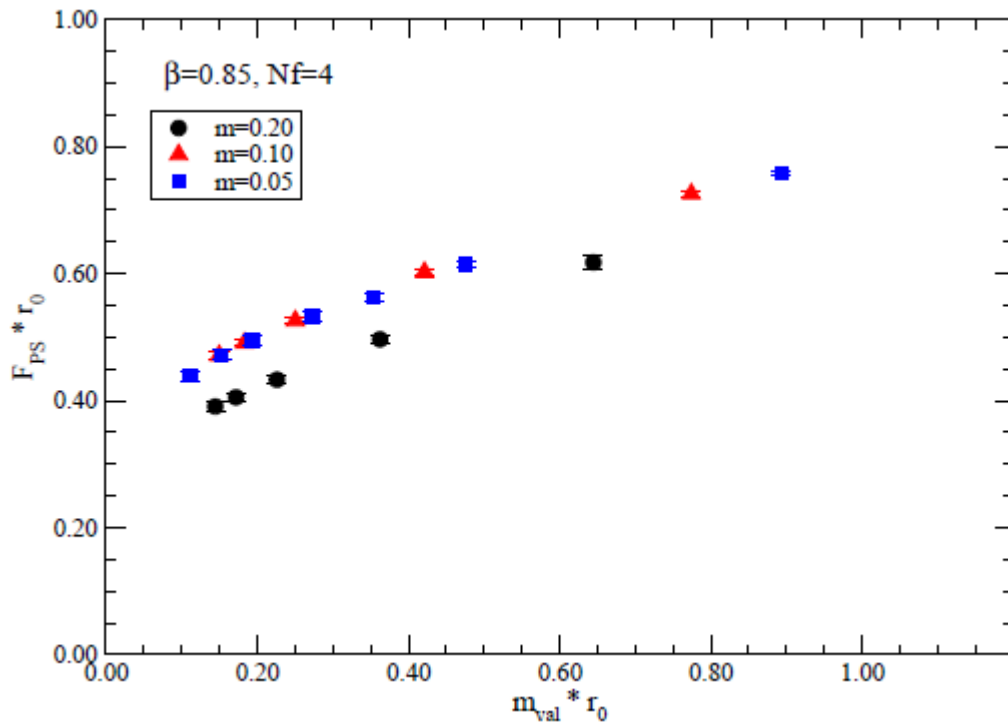
Decay constant

- $N_f=8$
 - Sea fermion mass dependence is amplified
 - At $\beta=0.85$, no finite massless limit



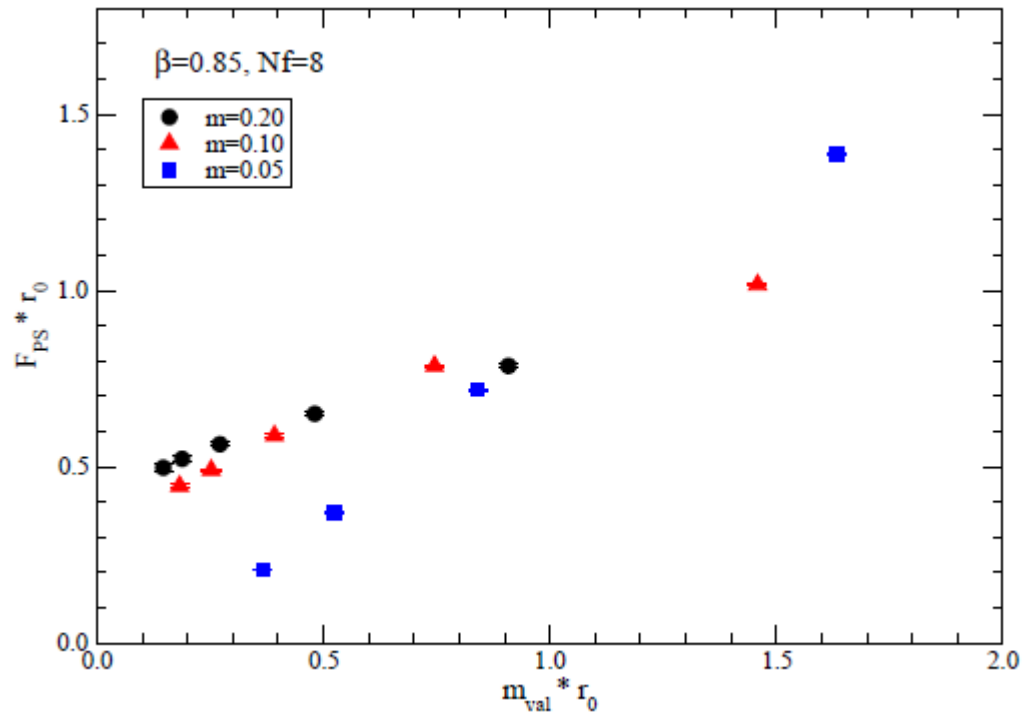
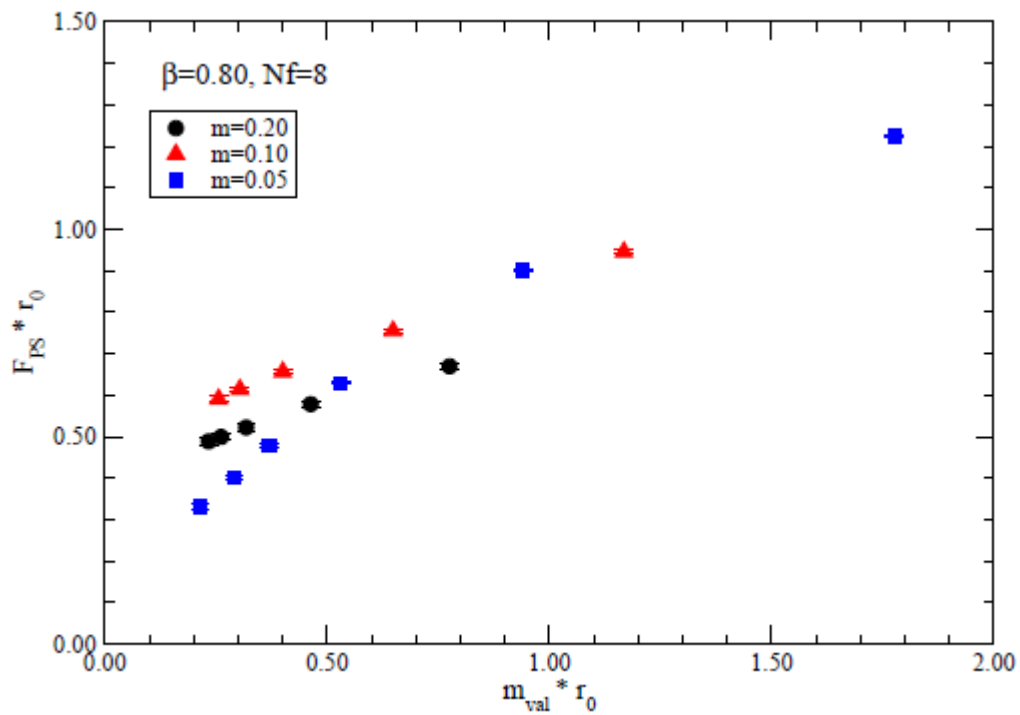
Decay constant

- Scaled by Sommer's scale r_0 from static potential
 - Both F_{PS} and r_0 give lattice scales in QCD
 - Dimensionless combination
 - $N_f=4, 6$: Similar tendency as results in lattice units



Decay constant

- $N_f=8$ (scaled)
 - Consistent with no finite massless limit



Summary/outlook

- Summary

- SU(2) gauge theory with domainwall fermions of $N_f=2,4,6,8$
- $N_f=8$: confining feature tends to disappear at small m
- $N_f=6$ is similar, but no unusual behavior in PS and V meson spectra
- More detailed analyses are underway

- To do

- So far analysis based on confining/chiral symmetry broken phase were applied
- Analysis to test conformality such as hyperscaling is planned
- Locality of domainwall/overlap fermion to be confirmed
- Method to improve signals needed for other channels
- Eigenmodes of domainwall/overlap fermion operators

Summary/outlook

- Outlook

- Improvement of domain-wall fermion (to reduce residual mass)
- finite temperature/density
- Other gauge group, fermion repr. : adjoint fermions
- Dynamical overlap (fixed topology) in epsilon-regime