Hidden Local Symmetry
as
magnetic gauge theory

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celebration of 30th anniversary of Hidden Local Symmetry in QCD

SCGT15 workshop, Nagoya, March 6, 2015
Vector mesons as gauge bosons

It is probably necessary that the vector mesons are described as the gauge bosons if there is an effective field theory description for them.

Sakurai 60's
Bando, Kugo, Uehara, Yamawaki, Yanagida ’85
Son, Stephanov ’03
Sakai, Sugimoto ’04
Hidden Local Symmetry

[Bando, Kugo, Uehara, Yamawaki, Yanagida ’85]

Two-parameter model for $\pi$-$\rho$-$\gamma$ system.

\[
\mathcal{L} = -\frac{1}{4g_H^2} F^a_{\mu\nu} F^{a\mu\nu} + \frac{a f^2}{2} \text{tr} \left[ |D_\mu U_L|^2 + |D_\mu U_R|^2 \right] + \frac{(1 - a) f^2}{4} \text{tr} \left[ |\partial_\mu (U_L U_R)|^2 \right].
\]
It is quite successful.

Maybe there is an effective description.
In this talk,

We discuss the possibility that the relation between the QCD and the Hidden Local Symmetry is the electric-magnetic duality.

$\rho$ meson = magnetic gauge boson?

[Komargodski ’10][RK ’11]
[Harada, Yamawaki ’99]
Crazy?

This isn’t necessarily a crazy idea.

- **Hidden Local Symmetry** ($\rho$ meson as a gauge boson)
- **Seiberg duality** (Low energy description of UV free gauge theory as IR free magnetic gauge theory)
Unification

$\rho$ meson = magnetic gauge boson?

This provides us with a unified picture of hadron world.

$$\langle \bar{q}q \rangle \neq 0 \leftrightarrow \langle \bar{m}m \rangle \neq 0$$

chiral symmetry breaking

(non-abelian) monopole condensation = confinement

electric-magnetic duality
If the $\rho/\omega$ meson are the magnetic gauge boson, the string made of the mesons should be the confining string.

Linear potential

[Nambu’74, Mandelstam ’75, ’t Hooft ’75]
linearized HLS

$$\mathcal{L} = -\frac{1}{4} F^{(\omega)}_{\mu\nu} F^{(\omega)}_{\mu\nu} - \frac{1}{4} F^{(\rho) a}_{\mu\nu} F^{(\rho) a}_{\mu\nu}$$

$$+ \frac{f^2}{2} \text{Tr} \left[ |D_\mu H_L|^2 + |D_\mu H_R|^2 \right]$$

$$- V(H_L, H_R).$$

Hidden Local Symmetry with the Higgs bosons.
String made of mesons

construct a string configuration made of $\rho$, $\omega$, and $f_0$ and calculate an energy.

\begin{align*}
g_\rho &= (340 \text{ MeV})^2, \\
m_\rho &= 770 \text{ MeV}, \\
\sim &\quad m_\omega \\
m_S &= m_A = 980 \text{ MeV}, \\
(\text{scalar meson masses})
\end{align*}

\[
V(R) = -\frac{A}{R} + \sigma R, \quad A = 0.25 \quad \sqrt{\sigma} = 400 \text{ MeV}
\]
Not bad.

Lattice QCD

\[ V(R) = -\frac{A}{R} + \sigma R. \]

\[ A \sim 0.25 - 0.5, \quad \sqrt{\sigma} \sim 430 \text{ MeV}. \]
trial (theoretical)

QCD $\quad$ add SUSY breaking $\quad$ SUSY QCD

Hidden Local Symmetry? $\quad$ dual magnetic theory

add SUSY breaking

Seiberg duality
Model

<table>
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<tr>
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<th>$SU(N_e)$</th>
<th>$SU(N_f)_L$</th>
<th>$SU(N_f)_R$</th>
<th>$U(1)_B$</th>
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auxiliary flavors $\rightarrow$ massive

$W = mQ'\bar{Q}'$

enhanced symmetry $\rightarrow$ gauging

SUSY $\rightarrow$ break by hand

$\mathcal{L}_{\text{soft}} = -\tilde{m}^2(|Q|^2 + |\bar{Q}|^2 + |Q'|^2 + |\bar{Q}'|^2) - \left(\frac{m\lambda}{2}\lambda\lambda + \text{h.c.}\right) - (BmQ'\bar{Q}' + \text{h.c.})$
continuous path?

approximately supersymmetric

Seiberg duality can be used

dynamical scale

small mass parameters

large mass parameters

QCD
dual picture

electric

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<tr>
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<th>SU($N_c$)</th>
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magnetic
gauged

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<td>$2(N_f - N_c)/N_f$</td>
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<tr>
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<tr>
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<td>0</td>
<td>1 + Adj.</td>
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<td>$\bar{Z}$</td>
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<td>$(2N_f - N_c)/N_f$</td>
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Higgsing this direction gives massive $\rho/\omega$ mesons and massless pions.

Consistent with Vafa-Witten theorem.
SUSY breaking terms in the dual picture

[Cheng, Shadmi '98][Arkani-Hamed, Rattazzi '98]
[Karch, Kobayashi, Kubo, Zoupanos '98]
[Luty, Rattazzi '99][Abel, Buican, Komargodski '11]

\[
\mathcal{L}_{\text{soft}} = -\tilde{m}_q^2 (|q|^2 + |\bar{q}|^2 + |q'|^2 + |\bar{q}'|^2) - \tilde{m}_M^2 (|Y|^2 + |Z|^2 + |\bar{Z}|^2 + |\Phi|^2) \\
- \left( \frac{m_\lambda}{2} \tilde{\lambda} \tilde{\lambda} + \tilde{B} m_\lambda \Lambda Y + A h (q' Y \bar{q}' + q' Z \bar{q} + q \bar{Z} q' + q \Phi \bar{q}) + \text{h.c.} \right).
\]

\[
\tilde{m}_q^2 = -\frac{N_c - 2N_f}{3(N_f + N_c)} D_R,
\]

\[
\tilde{m}_M^2 = \frac{2(N_c - 2N_f)}{3(N_f + N_c)} D_R,
\]

\[
\frac{m_\lambda}{g^2} = -\frac{2N_c - N_f}{16\pi^2} F_\phi,
\]

\[
\frac{m_\lambda}{g_H^2} = -\frac{2N_f - N_c}{16\pi^2} F_\phi,
\]

\[
A = 2(\gamma_M + \gamma_q + \gamma_{\bar{q}}) F_\phi,
\]
Indeed

In the free-magnetic range ($N_f < N_c/2$),

$$\frac{\tilde{m}^2}{\tilde{m}_q^2} = \frac{2N_c - N_f}{-N_c + 2N_f} < 0$$

[Luty, Rattazzi ’99]

positive mass$^2$ for squarks in the electric picture

\[\rightarrow\]

negative mass$^2$ for squarks in the magnetic picture!

\[\rightarrow\] triggers squark condensation!
Indeed, one can find a vacuum where SU($N_f$) magnetic gauge group is Higgsed and the chiral symmetry is spontaneously broken as well.

Realization of HLS as the dual gauge theory.
In Seiberg dualities, relation between confinement and Higgsing (namely electric and magnetic) is not completely clear. This is because the dualities are between massless degrees of freedom.

Motivate us to consider N=2 SUSY model.
**N=2 SUSY model**

Non zero $\langle Q \rangle = \mu$ reduces to N=2 QCD.

Adding a superpotential term reduce to N=1 SUSY

$$W \ni \frac{m}{2} \text{Tr} \Phi_1^2.$$
D-brane construction

Following Witten, one can construct 4D gauge theory by branes and solve this theory.

\[ N_f D4 \]

\[ N_c D4 \]

\[ \text{NS}5_1 \]

\[ \text{NS}5_2 \]

\[ \text{NS}5_3 \]

One can get full quantum information from the SW curve.
Exact results (SW curve)

For $N_c=3$, $N_f=2$ ($\mu=0$, $m=0$), we get

$$\Lambda_2^3 t^3 - (v - \phi_1)'(v - \phi_2)'(v - \phi_3)' t^2 + (v - \hat{\phi}_1)(v - \hat{\phi}_2)(v - \hat{\phi}_3)' t + \Lambda_1 v^2 = 0,$$

$$\hat{\phi}_1 + \hat{\phi}_2 + \hat{\phi}_3 = -\Lambda_1, \quad \phi_1' + \phi_2' + \phi_3' = 0.$$ 

[Giveon, Pelc '97][Erlich, Naqvi, Randall '98][Carlino, Konishi, Murayama '00]

from this, one can identify the vacua which remain after turning on $\mu$ and $m$:

$$\rho = \frac{\Lambda_1^2}{2} + c_1 \Lambda^2, \quad \sigma = c_2 \Lambda^3, \quad \rho' = c_1 \Lambda^2, \quad \sigma' = -\Lambda_2^3 + c_2 \Lambda^3,$$

$$\rho = \frac{1}{2}(\hat{\phi}_1^2 + \hat{\phi}_2^2 + \hat{\phi}_3^2), \quad \rho' = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2),$$

$$\sigma = \hat{\phi}_1 \hat{\phi}_2 \hat{\phi}_3, \quad \sigma' = \phi_1' \phi_2' \phi_3',$$

$$\Lambda^{14} = \Lambda_1 \Lambda_2^3.$$

$$(c_1, c_2) = \left( \frac{2i}{\sqrt{3}}, \pm \frac{8\sqrt{2}}{9 \cdot 3^{1/4}}(1 + i) \right), \left( \frac{-2i}{\sqrt{3}}, \pm \frac{8\sqrt{2}}{9 \cdot 3^{1/4}}(1 - i) \right)$$

[Carlino, Konishi, Murayama '00]
For magnetic IR free effective theory below \( \Lambda_1 \) 

Consider the parameter region with \( N_c = 3, N_f = 2 \)

For \( \Lambda_1 \gg \Lambda_2, \mu \ll \Lambda_1 \) VEV of \( Q \)

Effective theory below \( \Lambda_1 \)

[Argyres, Plesser, Seiberg '96]
SU(3)$_2$ factor gets strong at a scale

\[ \Lambda' = (\Lambda_1 \Lambda_2^3)^{1/4} \]

There is a point at which massless monopoles appear.

[Carlino, Konishi, Murayama '00]

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<th>SU(2)$_1$</th>
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<td>0</td>
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effective theory below $\Lambda'$
$W \ni \frac{m}{2} \text{Tr}\Phi_1^2.$

$W \ni e\Phi_{D1}\bar{e} - e_1\Phi_{D2}\bar{e}_1 - e_2\Phi_{D2'}\bar{e}_2 + m\Lambda_1x_1\Phi_{D1} + m\Lambda'x_2\Phi_{D2} + m\Lambda'x_2'\Phi_{D2'}$,

condensations of $e, e_1, e_2$

$magnetic\ gauge\ group$

$<e>$ would not cause confinement since $U(1)_{B'}$ is not dynamical

$SU(2)_1 \quad U(1)_{1'} \quad U(1)_2 \quad U(1)_{2'}$

\begin{tabular}{c|c|c|c|c|c|c}
 q & 2 & 1/2 & 0 & 0 & 2 & 0 \\
 e & 1 & -1 & 0 & 0 & 1 & 0 \\
 $e_1$ & 1 & 0 & 1 & 0 & 1 & 0 \\
 $e_2$ & 1 & 0 & 0 & 1 & 1 & 0 \\
\end{tabular}

$magnetic\ gauge\ group$

$SU(2)_{f} \quad U(1)_{B}$

\begin{tabular}{c|c|c}
 q & 2 & 3/2 \\
\end{tabular}

$SU(2)_{f} \quad U(1)_{B}$

\begin{tabular}{c|c|c}
 q & 2 & 0 \\
\end{tabular}

effective theory below $(m\Lambda')^{1/2}$

[Argyres, Plesser, Seiberg '96]
turn on $\mu$

$$W \ni -\frac{3}{2} \text{Tr}(q \Phi_X \bar{q}) + \mu^2 \Phi_X,$$

$$\rightarrow \quad q = \bar{q} = \frac{\mu}{\sqrt{3}} \cdot 1,$$

$\text{SU}(2)_1 \times \text{SU}(2)_f \rightarrow \text{SU}(2)_{1+f}$

color-flavor locking

magnetic gauge boson $\rightarrow$ vector meson ($\rho$)

$\text{U}(1)_X$ breaking $\rightarrow$ string formation

$\rightarrow$ quark confinement

This is what we wanted.
We have seen that Quiver deformation provides us with an understanding of HLS as the magnetic gauge theory.
Summary

• The success of the Hidden Local Symmetry in QCD may have a deep reason.

• We studied the possibility that the HLS is actually the magnetic picture of QCD.

• Electric-magnetic duality in SUSY QCD looks like a promising approach.

• It is exciting that the confining string maybe made of mesons.