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SCGT15

IR fixed points in SU(3) Gauge Theories

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Plan of Talk

Introduction

Phase structure (brief review of our previous works)

Scaling relations based on RG

Set up

Results

Interpretation

Conclusions

Objectives

Identify IR fixed points in SU(3) Gauge Theories

with Nf fundamental fermions

within the conformal window $N_f^c \leq N_f \leq 16$

N_f^c ?

anomalous mass dimension γ^* ?

meson propagator on the fixed point in the continuum limit ?

Strategy

Propose a novel RG method

based on the scaling behavior of the propagator
through the RG analysis with a finite IR cut-off

Constructive approach

Define gauge theories as the continuum limit of lattice gauge theories

$N_x = N_y = N_z = N \quad N_t = rN$ (r aspect ratio) r=4 in this work

take the limit a->0 and N -> infinity

with $L = aN \text{ and } L_t = aN_t$ fixed

when L and/or Lt finite => IR cutoff

Conformal theories:
IR cutoff: an indispensable ingredient
in contrast with QCD

Constructive approach (2)

Important steps

1. Clarify the phase structure
2. Clarify what kind of phase exists
3. Clarify the boundary of the phases
4. Clarify the location of UV or IR fixed points

our earlier works: step 1. ~ 3.

The phase diagram for various number of flavors 7 \le Nf \le 300

Phys. Rev. Lett. 69(1992), 21
Phys. Rev. D69(2004), 014507

The phase diagram for Nf \le 6

Phys. Rev. D54(1996), 7010

A new phase "conformal region" in addition to the confining region and deconfining region

Phys.Rev. D87 (2013) 7, 071503
Phys.Rev. D89 (2014) 114503

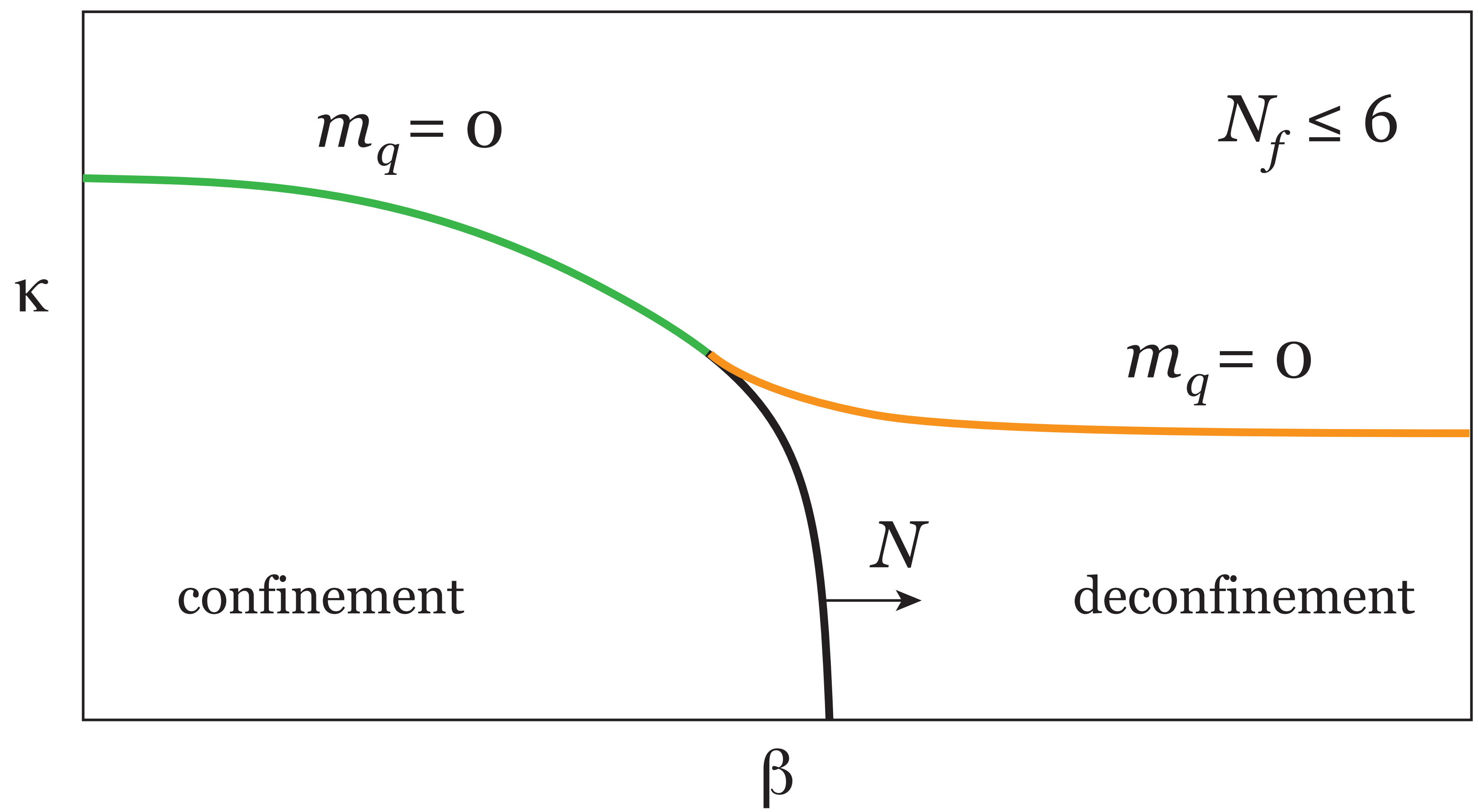
we intend to perform step 4 in this work

Phase Diagram: $N_f \leq 6$

as in 2004

Chiral transition on the massless line starting from the UVFP

The finite temperature phase transition in the quenched QCD transition and the chiral transition move toward larger beta, as N increases.



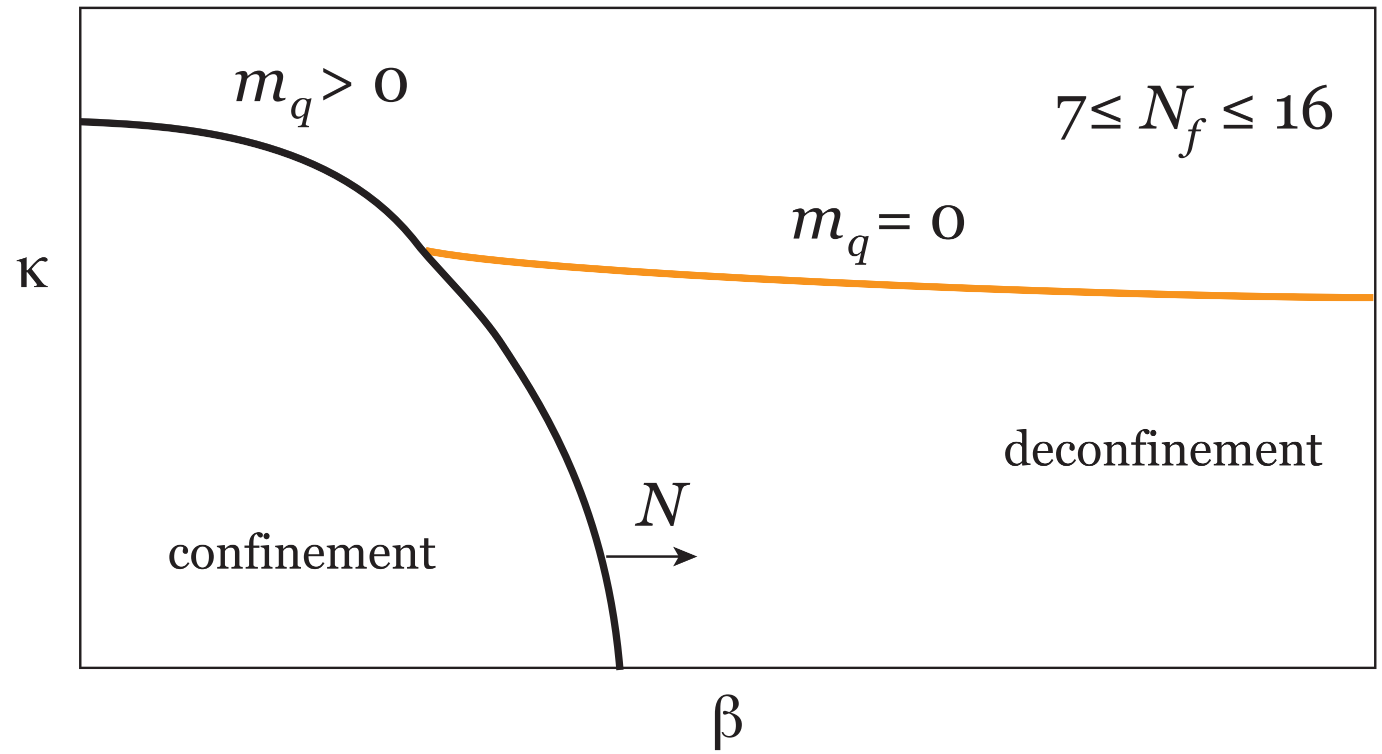
Phase Diagram: $7 \leq N_f \leq 16$

as in 2004

Complicated due to lack of chiral symmetry

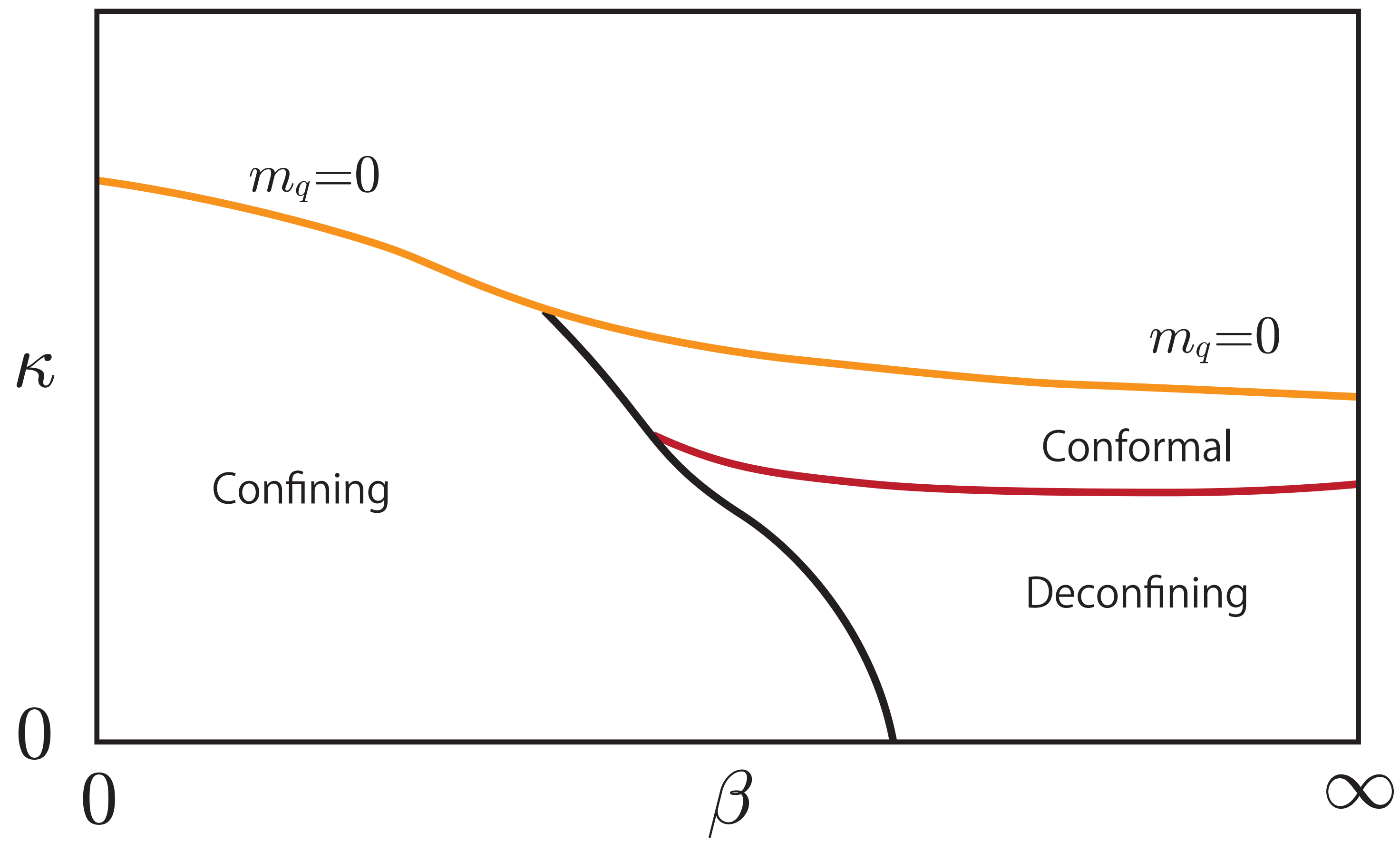
1. the massless line from the UVFP hits the bulk transition
2. no massless line in the confining phase at strong coupling region

massless quark line only in the deconfining phase

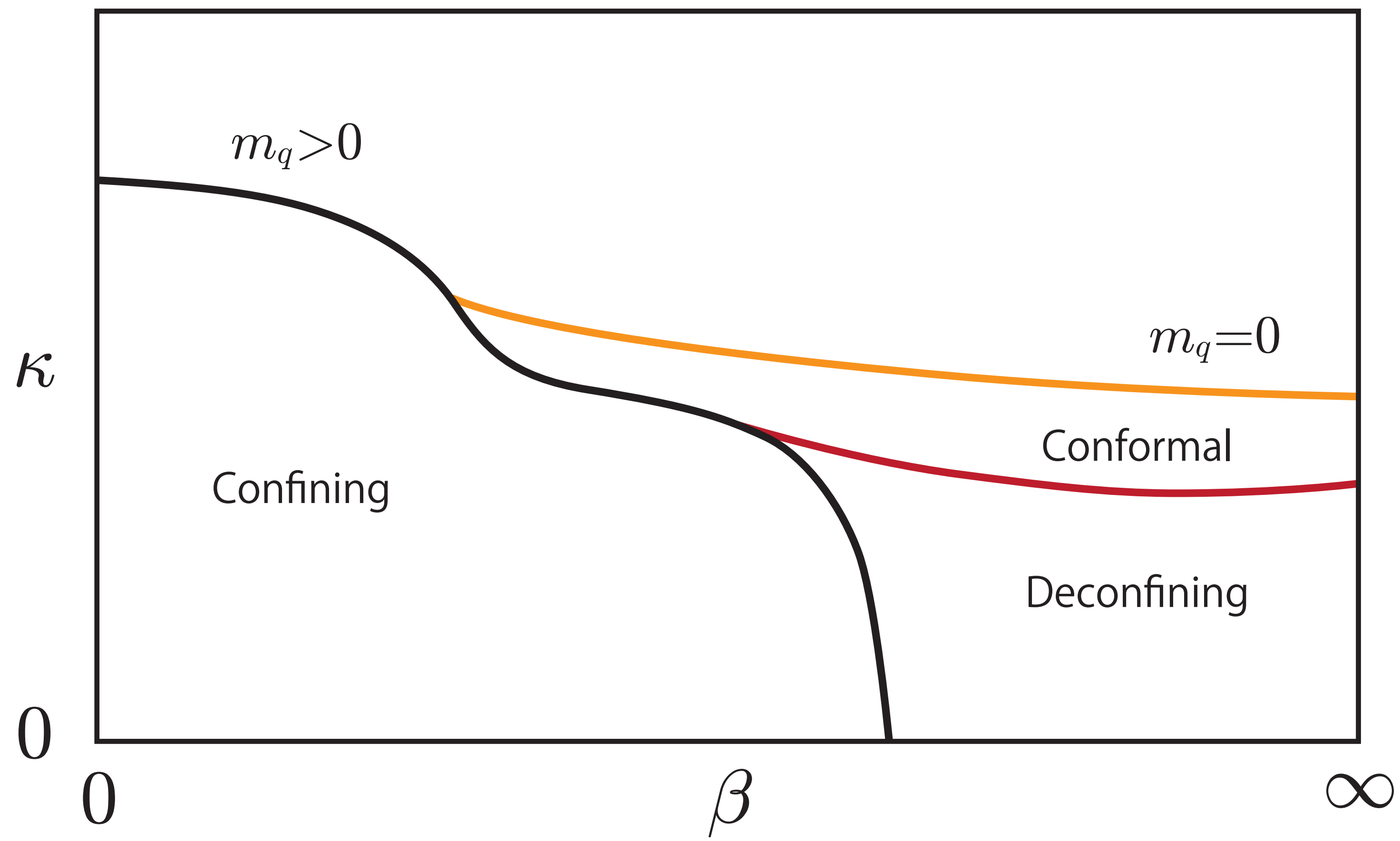


as in 2014

$N_f \leq 6$



$7 \leq N_f \leq 16$



Conformal region

A new concept “conformal theories with an IR cutoff”

$$m_q \leq \Lambda_{IR}$$

meson propagators show a power-modified Yukawa-type decay

Two sets of Conformal window

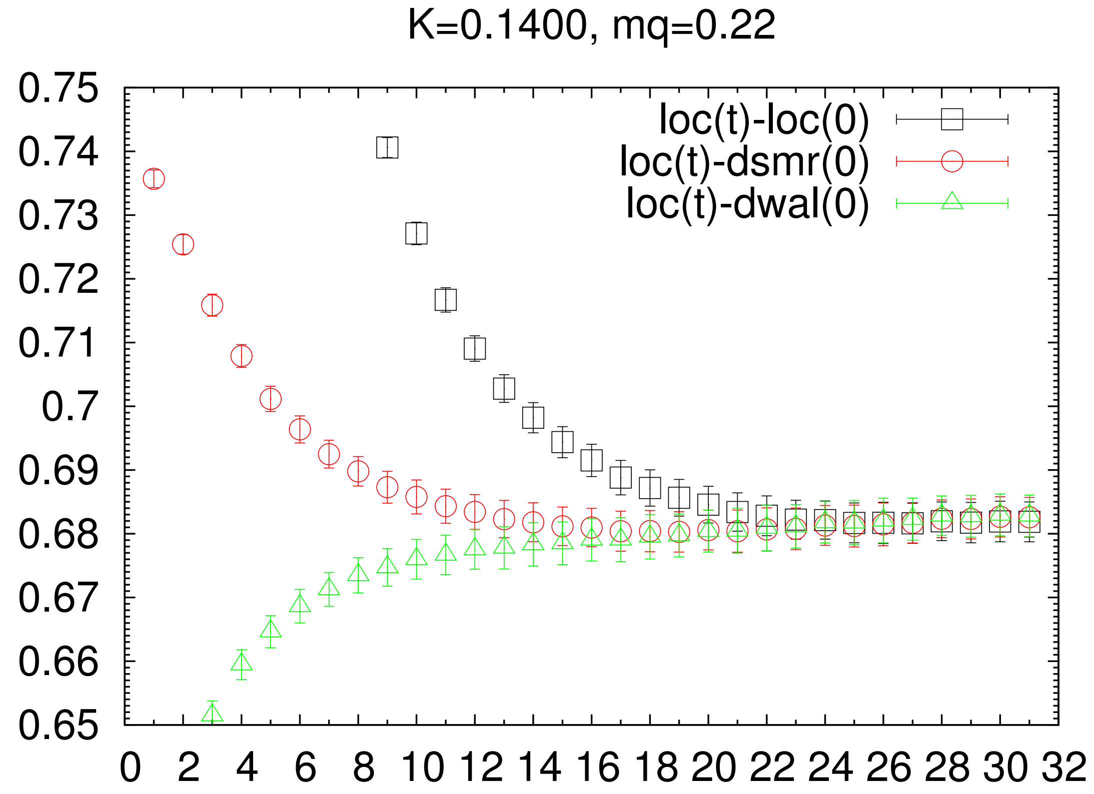
Large Nf and QCD in high temperature

Nf=7 ~ T/Tc =1.0 ~2.0: unparticle meson model

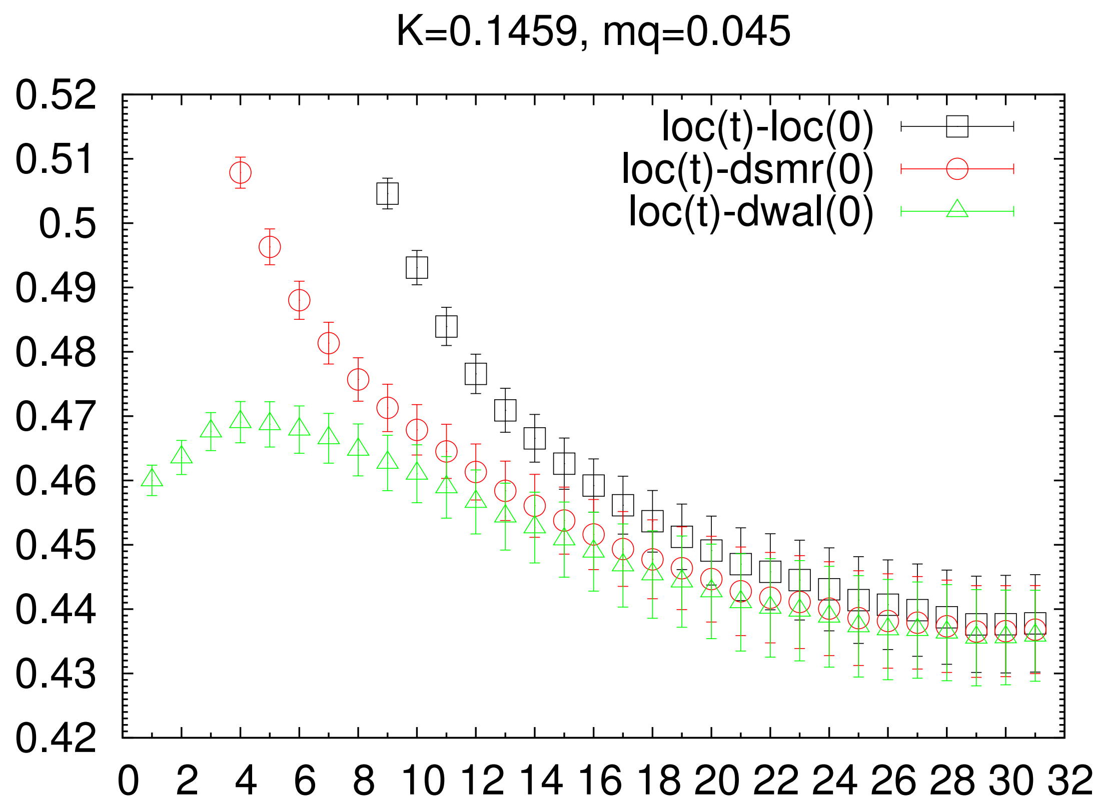
strongly support the conjecture that

the conformal window: $7 \leq N_f \leq 16$

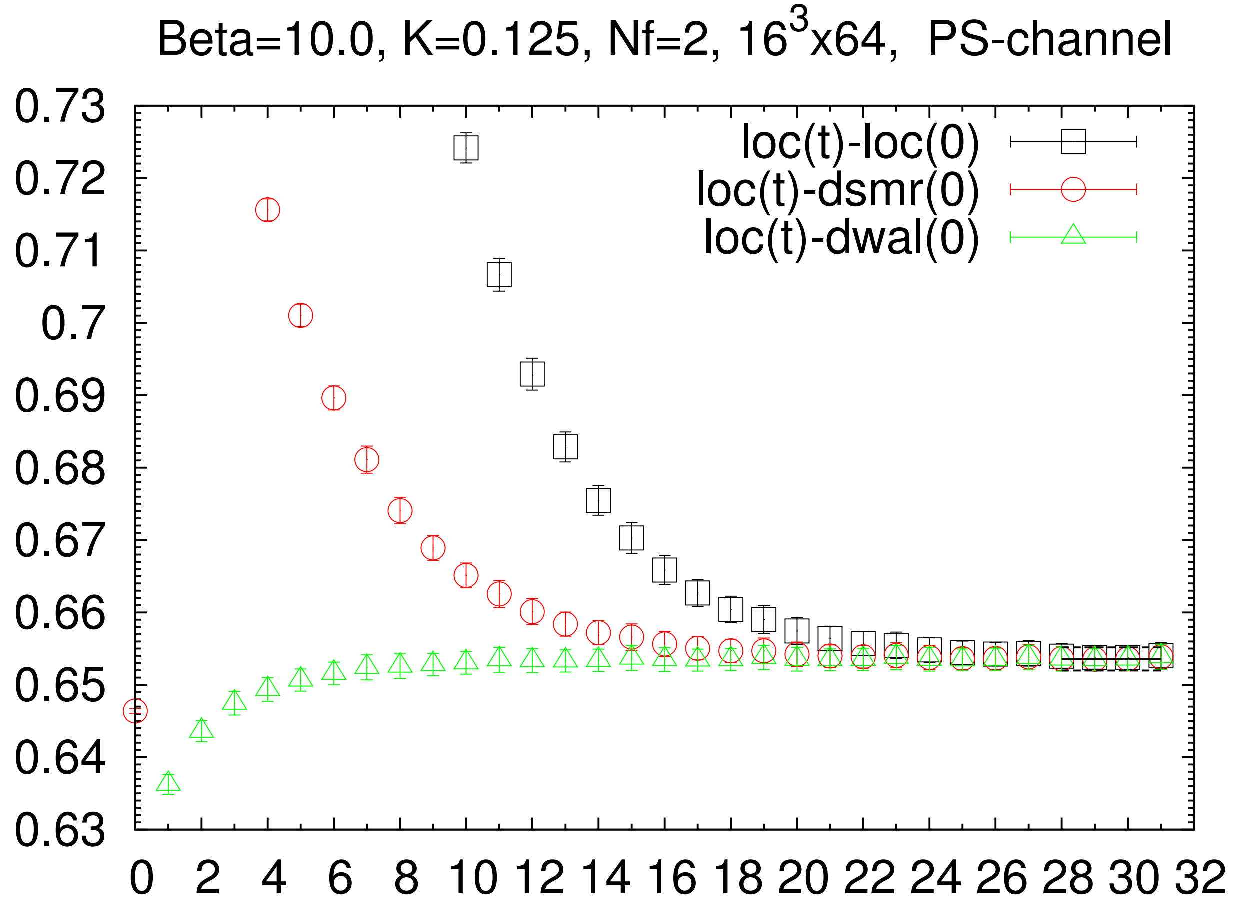
Nf=7 confining



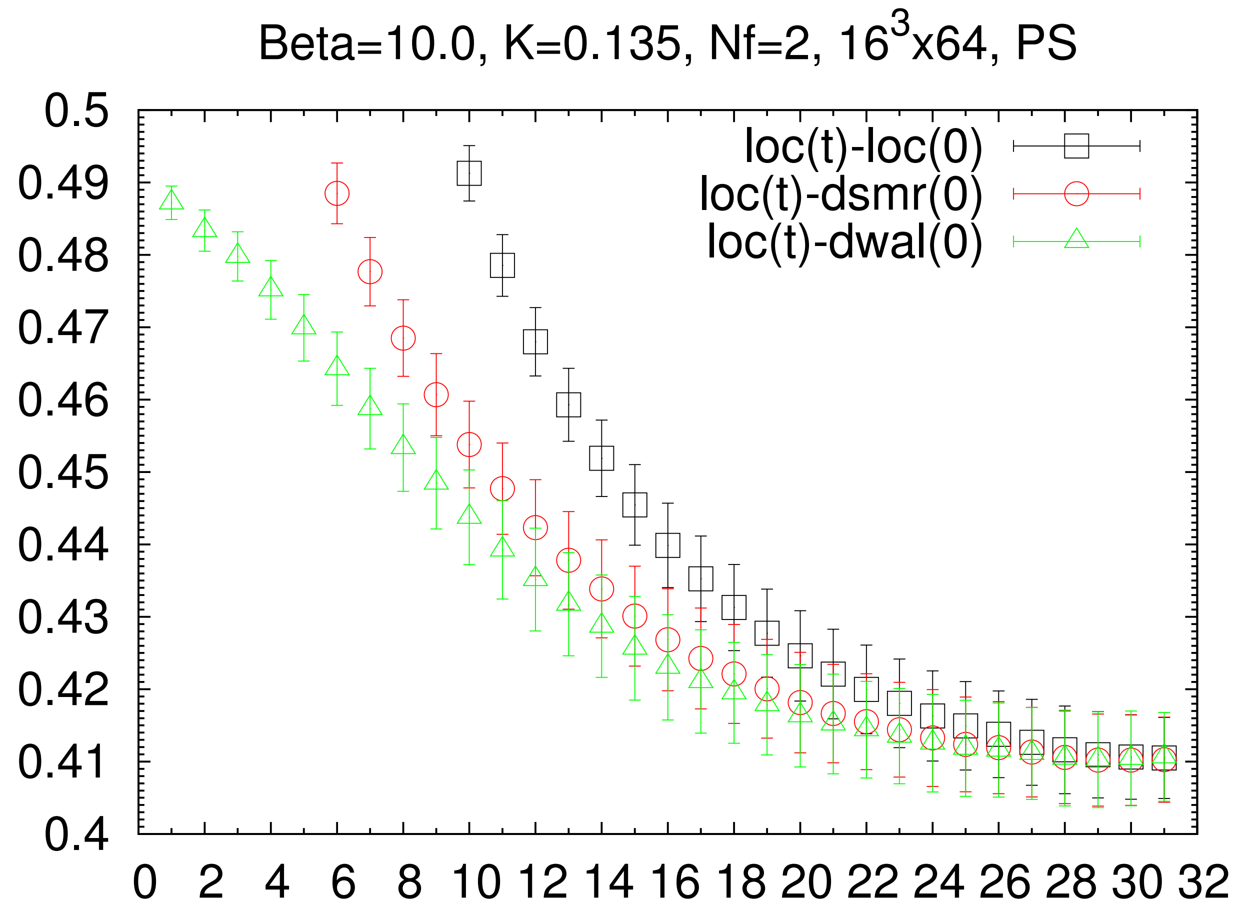
Nf=7 conformal



Nf=2 deconfining



Nf=2 conformal



Scaling relations from RG

RG equation for the propagator at vicinity of IRFP

e.g. Del Debbio, Zwicky 2010

RG
scale change

$$G(t; g, m_q, N, \mu) = \left(\frac{N^{'}}{N}\right)^{3-2\gamma} G(t'; g', m_q{}', N', \mu).$$

$$N^{'} = N/s$$
$$t^{'} = t/s$$

at the IRFP

$$g' = g = g^*$$
$$m_q' = m_q = 0$$
$$\gamma = \gamma^*.$$

Simplified expression

$$\tilde{G}(\tau, N) = G(t, N)$$
$$\text{with } \tau = t/N_t.$$

Scaling relation 1

$$\tilde{G}(\tau; N) = \left(\frac{N^{'}}{N}\right)^{3-2\gamma^*} \tilde{G}(\tau; N^{'}) \ .$$

Scaled effective mass

$$\mathfrak{m}(t,N) = \frac{N}{N_0} \ln \frac{G(t,N)}{G(t+1,N)}.$$

$N \to \infty$

$$\mathfrak{m}(\tau,N) = -\frac{1}{N_0}\partial_\tau \ln G(\tau,N)$$

Scaling relation 2

$$\mathfrak{m}(\tau,N) = \mathfrak{m}(\tau,N^{'})$$

Stringent condition for the IR fixed point

Strategy

With given Nf, choose \beta, and tune mq ~ 0.0

Calculate the meson propagator on the lattices

with size 8^3x32, 12^3x48 and 16^3x64

Plot the scaled effective mass

In general, three points and lines do not coincide

repeat this process

narrow the region of \beta in such a way that the three approach together

finally find the \beta at which three pots and lines coincide

within the standard error

identify the \beta IR fixed point

Stage and Tools

SU(3) gauge theories with Nf quarks in the fundamental representation

Action: the RG gauge action (called the Iwasaki gauge action)

Wilson fermion action

Nf = 7, 8, 12, 16

Lattice size: 8^3x32, 12^3x48, 16^3 x 64

Boundary conditions: periodic boundary conditions

an anti-periodic boundary conditions (t direction) for fermions

Algorithm: Blocked HMC for 2N and RHMC for 1 : Nf=2N + 1

Statistics: 1,000 +1,000 ~ 4000 trajectories

Computers: U. Tsukuba: CCS HAPACS; KEK: HITAC 16000

Measurement

Plaquette

Polyakov loop

quark mass

$$m_q = \frac{\langle 0|\nabla_4 A_4|\mathrm{PS}\rangle}{2\langle 0|P|\mathrm{PS}\rangle}$$

meson propagator

$$G_H(t) = \sum_x \langle \bar{\psi}\gamma_H \psi(x,t) \bar{\psi}\gamma_H \psi(0)\rangle$$

effective mass

$$\frac{\cosh(m_H(t)(t - N_t/2))}{\cosh(m_H(t)(t+1-N_t/2))} = \frac{G_H(t)}{G_H(t+1)}$$

$$m_H(t) = \ln \frac{G_H(t)}{G_H(t+1)}$$

N	N_{traj}	acc	plaq	m_q
$N_f = 16,\ \beta = 10.5,\ K = 0.1292$				
16	2000	0.59(1)	0.922 55(1)	−0.0063(1)
12	4000	0.77(1)	0.922 55(1)	−0.0053(1)
08	4000	0.89(1)	0.922 57(1)	0.0003(5)
$N_f = 12,\ \beta = 3.0,\ K = 0.1405$				
16	3000	0.68(1)	0.744 16(2)	−0.002(1)
12	3000	0.84(1)	0.744 15(1)	−0.002(1)
08	4000	0.94(1)	0.744 19(2)	0.004(1)
$N_f = 8,\ \beta = 2.4,\ K = 0.147$				
16	4000	0.72(1)	0.676 20(1)	−0.007(1)
12	4000	0.84(1)	0.676 20(1)	−0.006(3)
08	3000	0.93(1)	0.676 22(2)	−0.0005(5)
$N_f = 7,\ \beta = 2.3,\ K = 0.14877$				
16	4000	0.72(1)	0.659 31(1)	−0.0017(2)
12	4000	0.85(1)	0.659 31(1)	−0.0005(3)
08	5000	0.94(1)	0.659 41(3)	0.0047(6)

Results

Nf=16

Perturbation:
beta function up to two loops
RG scheme independent

β^* =11.5

On the other hand,

$$\beta_1 = \beta_2 + c_{12}$$

$$\beta_{\mathrm{RG}} = \beta_{\overline{\mathrm{MS}}} - 0.3$$

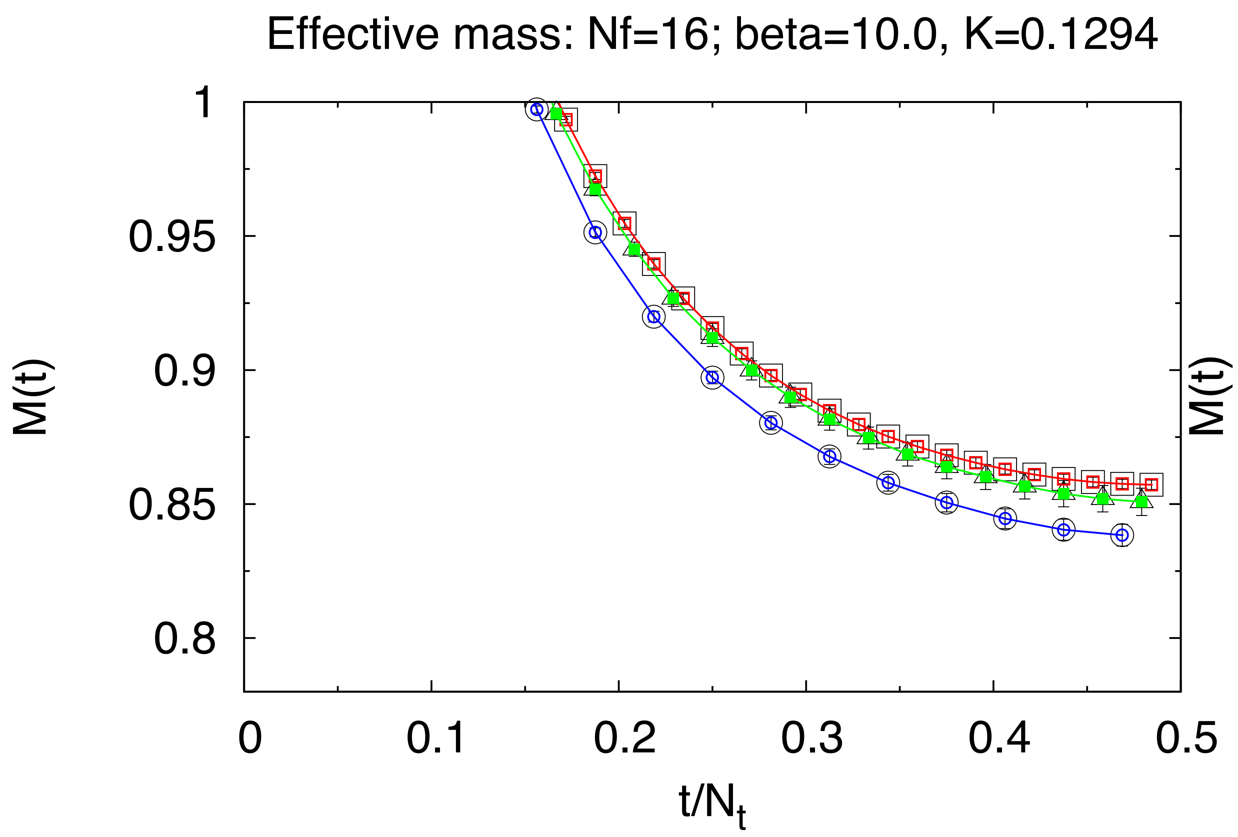
and

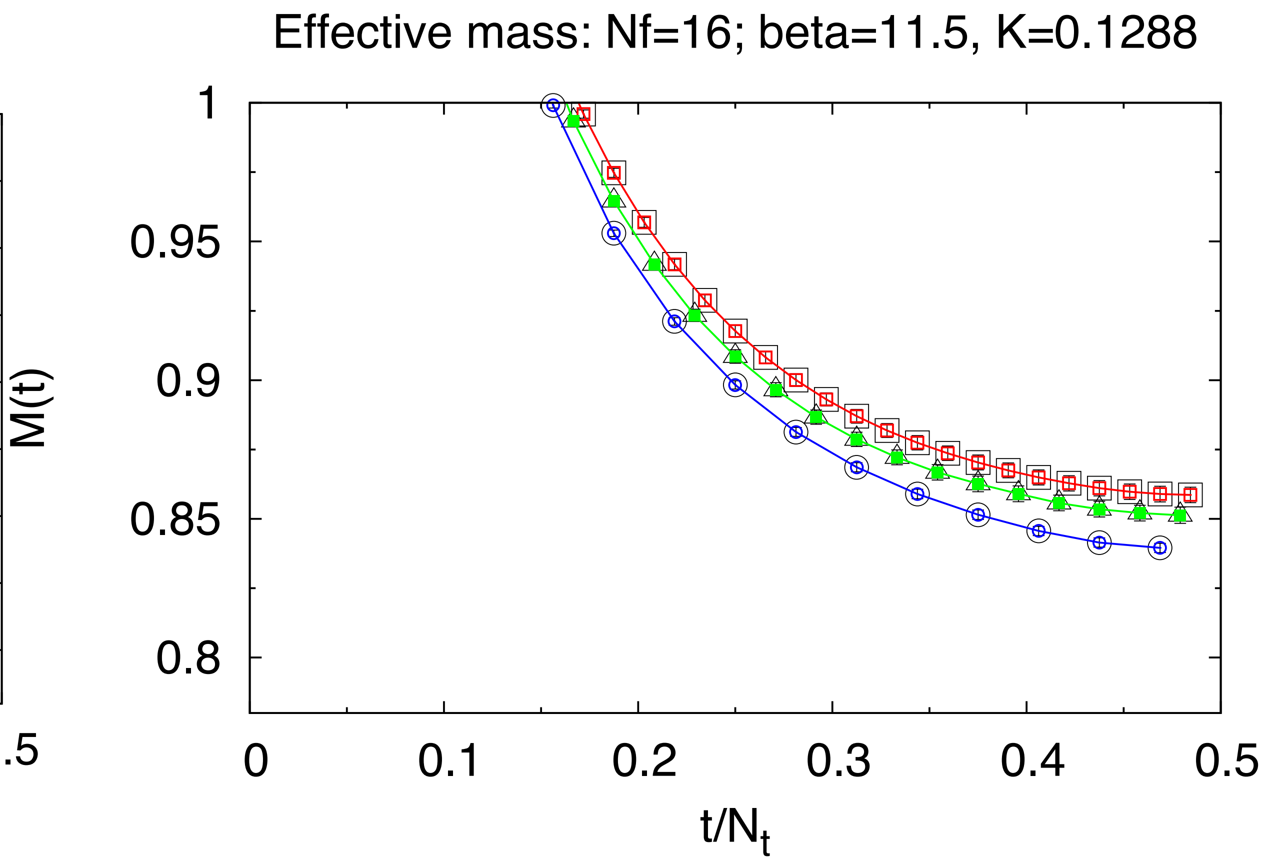
$$\beta_{\mathrm{one-plaquette}} = \beta_{\overline{\mathrm{MS}}} + 3.1.$$

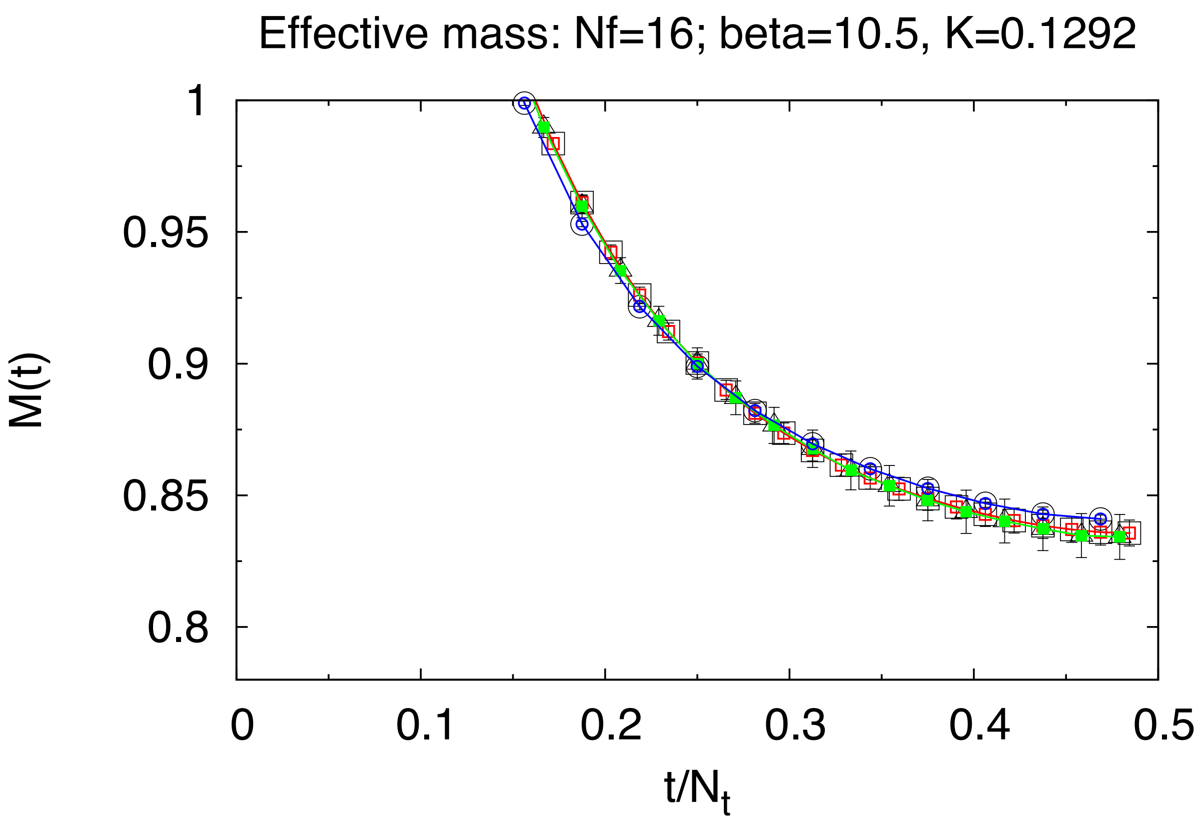
higher order contribution will be large for one-plaquette action

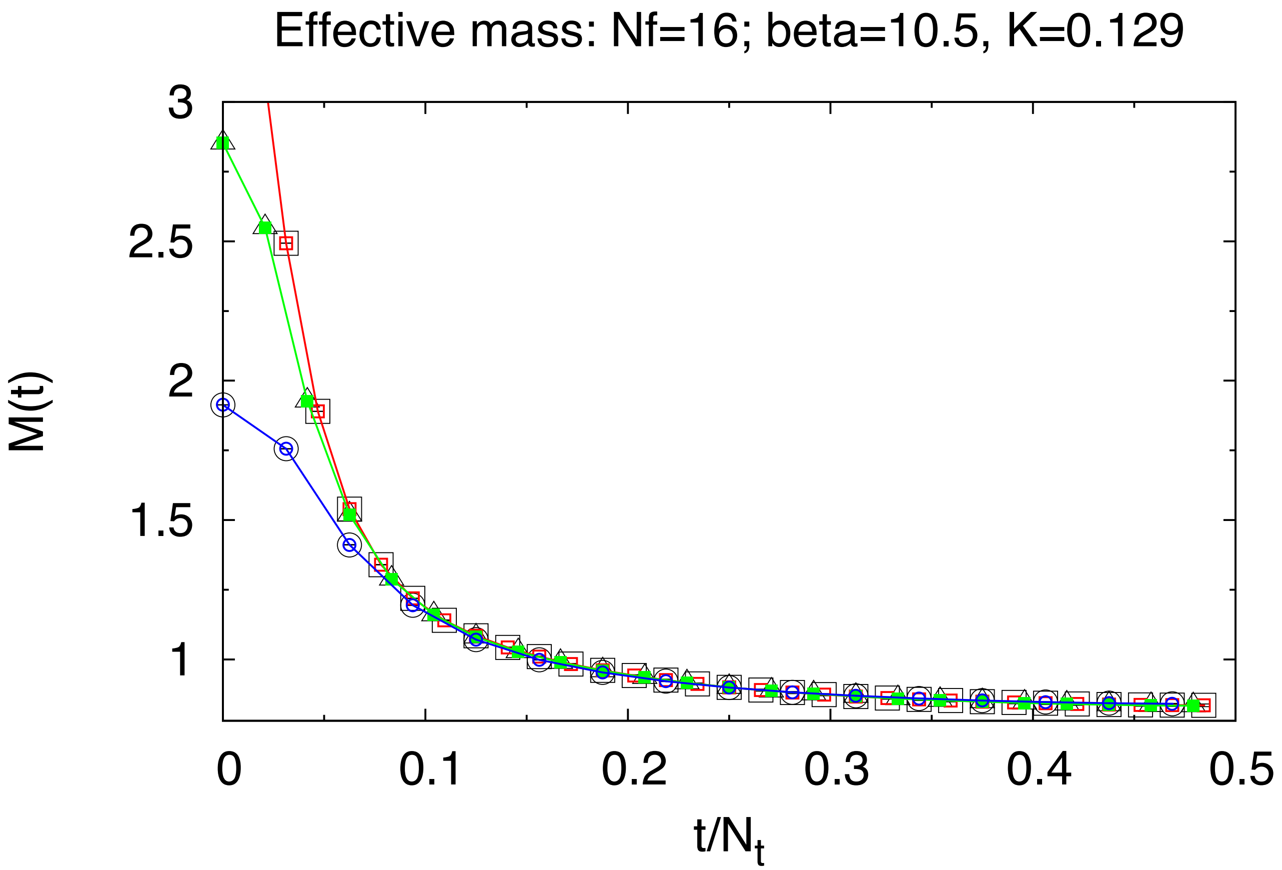
may expect \beta_RG ~11.2

Nf=16

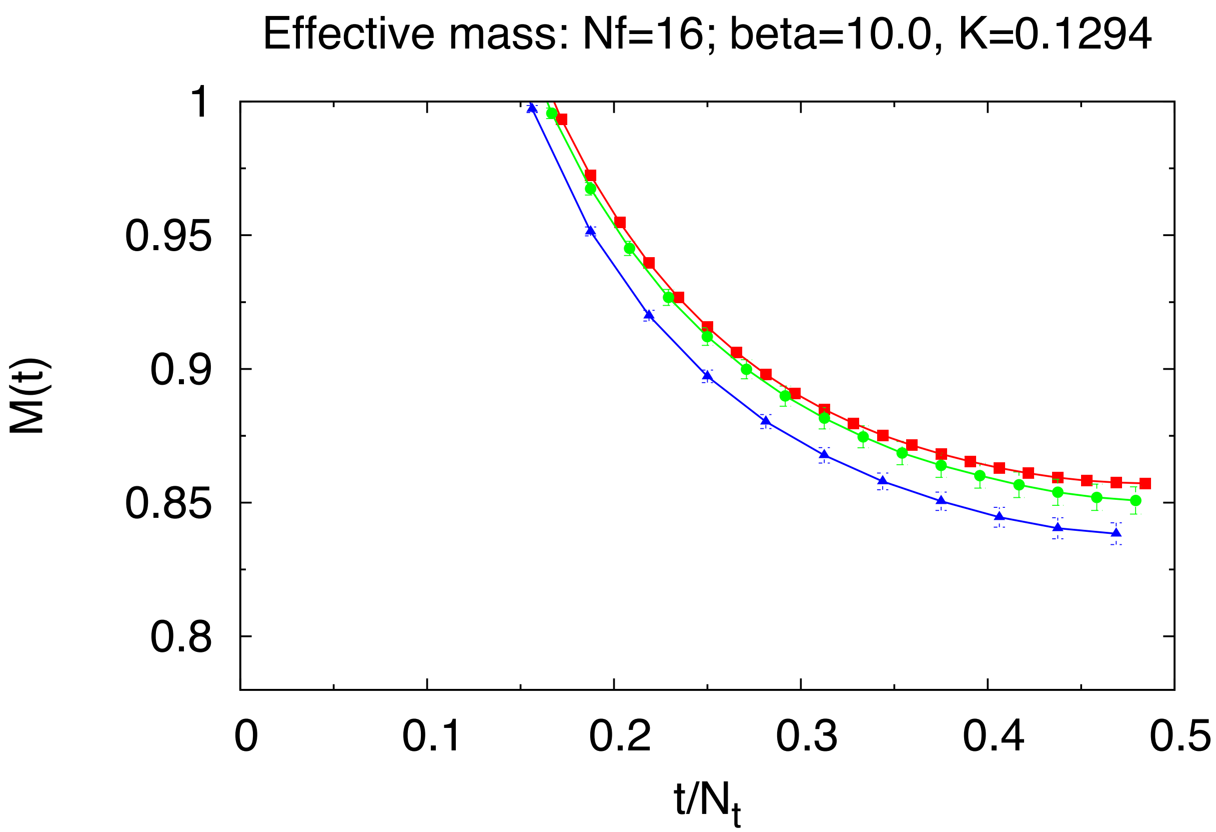

Effective mass: Nf=16; beta=10.0, K=0.1294
M(t)
t/N_t

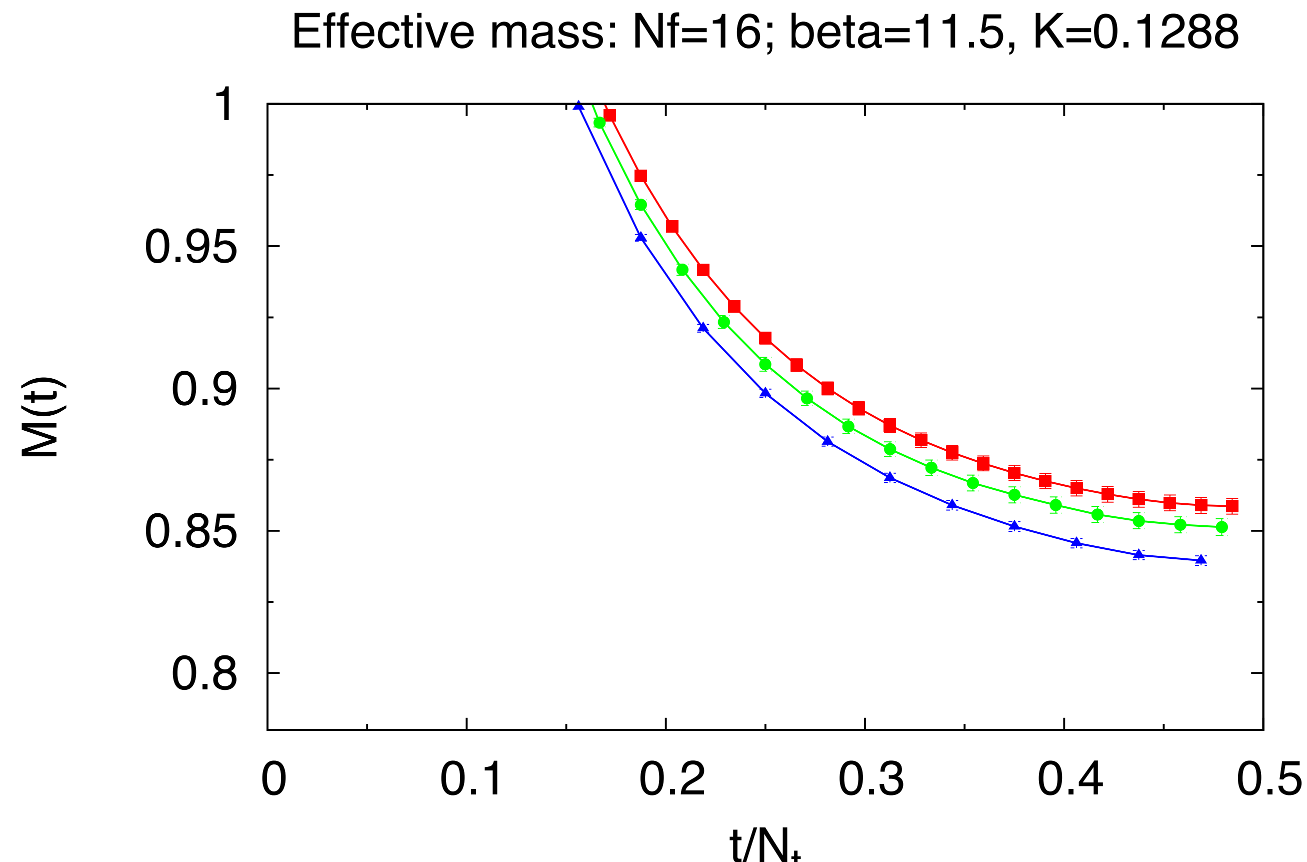

Effective mass: Nf=16; beta=11.5, K=0.1288
M(t)
t/N_t

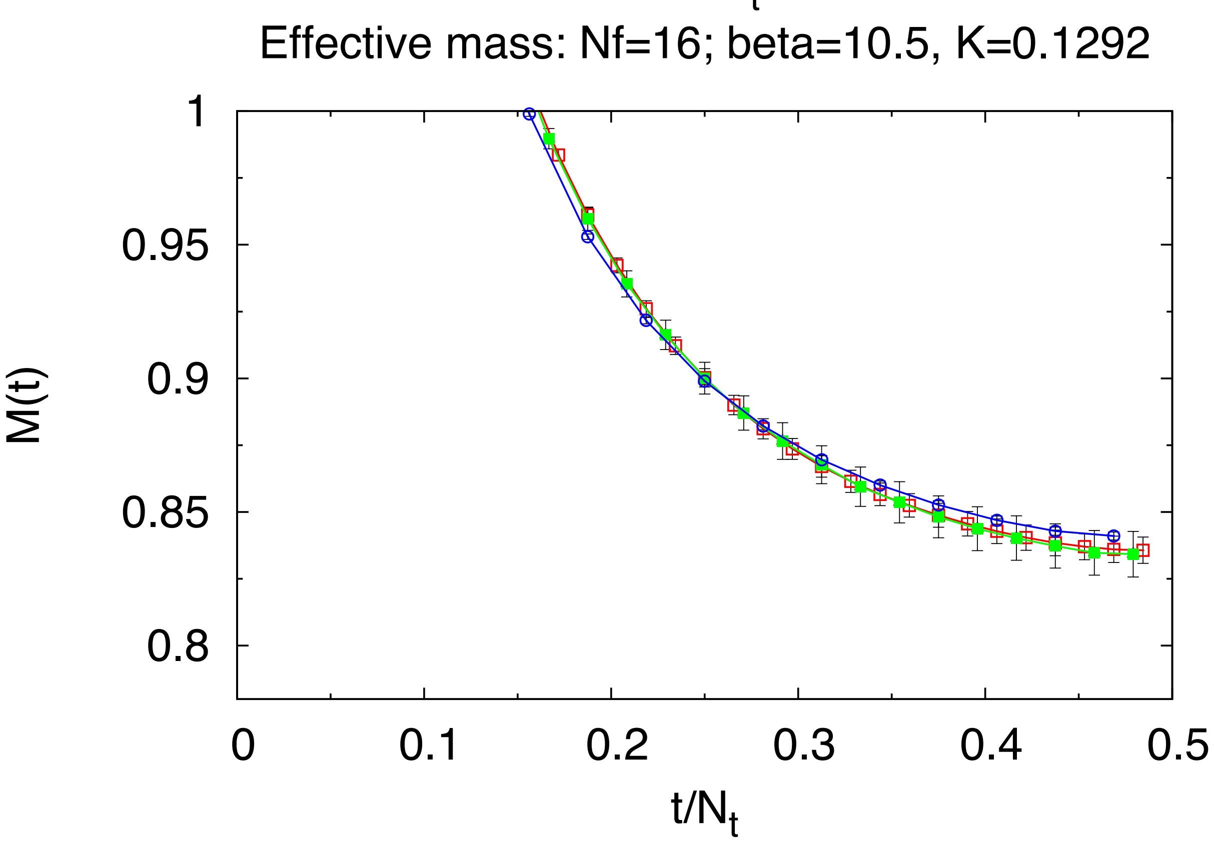

Effective mass: Nf=16; beta=10.5, K=0.1292
M(t)
t/N_t

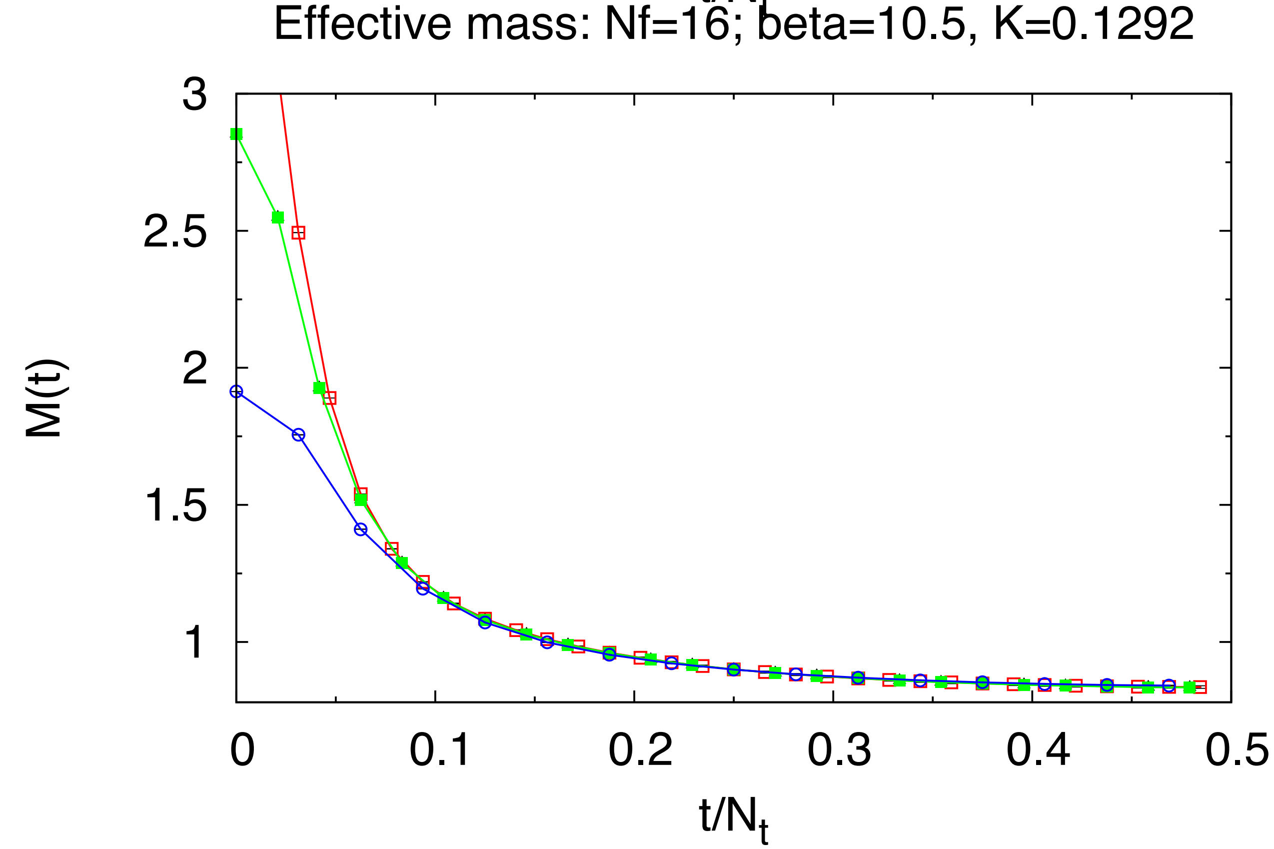

Effective mass: Nf=16; beta=10.5, K=0.129
M(t)
t/N_t

Nf=16

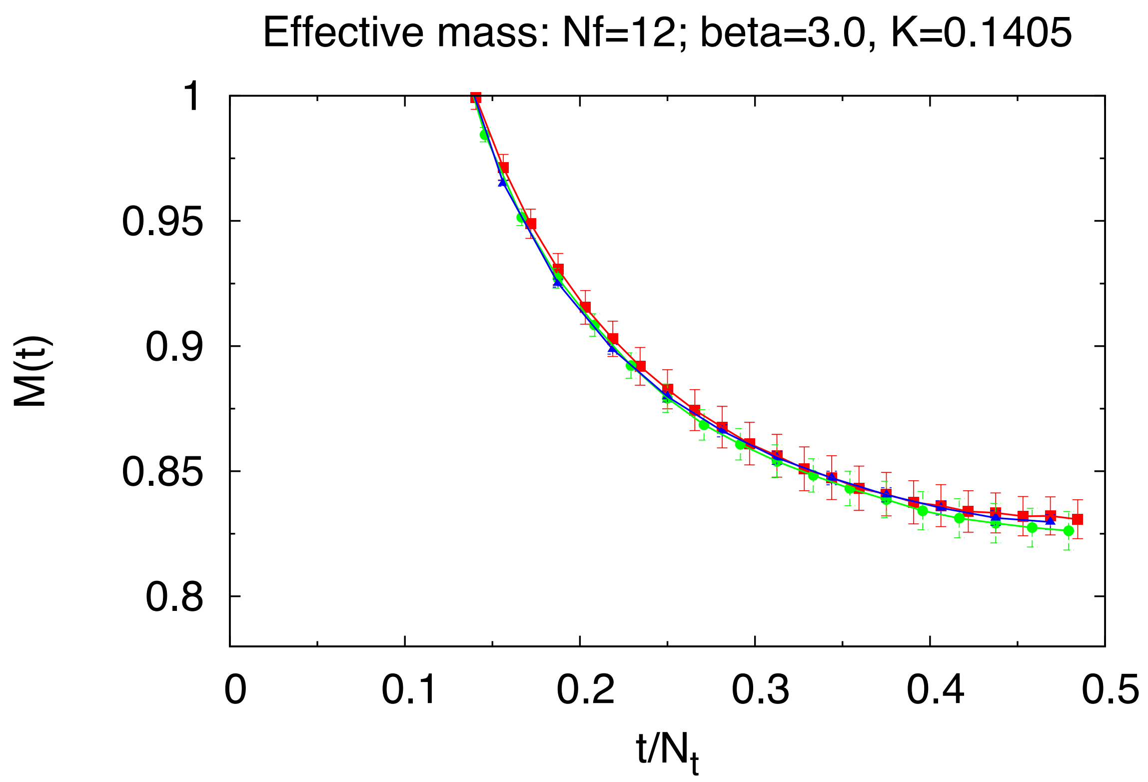

Effective mass: Nf=16; beta=10.0, K=0.1294
M(t)
t/N_t
1
0.95
0.9
0.85
0.8
0
0.1
0.2
0.3
0.4
0.5


Effective mass: Nf=16; beta=11.5, K=0.1288
M(t)
t/N_t
1
0.95
0.9
0.85
0.8
0
0.1
0.2
0.3
0.4
0.5

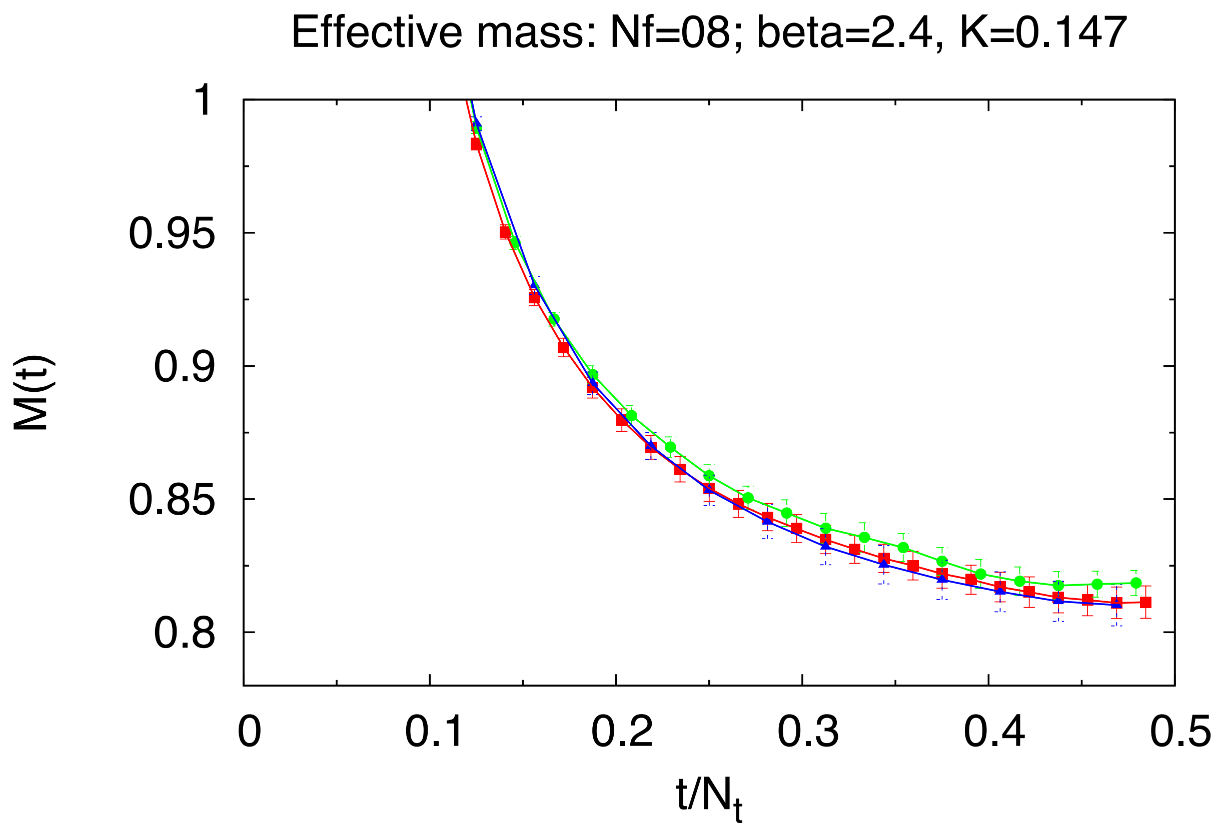

Effective mass: Nf=16; beta=10.5, K=0.1292
M(t)
t/N_t
1
0.95
0.9
0.85
0.8
0
0.1
0.2
0.3
0.4
0.5


Effective mass: Nf=16; beta=10.5, K=0.1292
M(t)
t/N_t
3
2.5
2
1.5
1
0
0.1
0.2
0.3
0.4
0.5

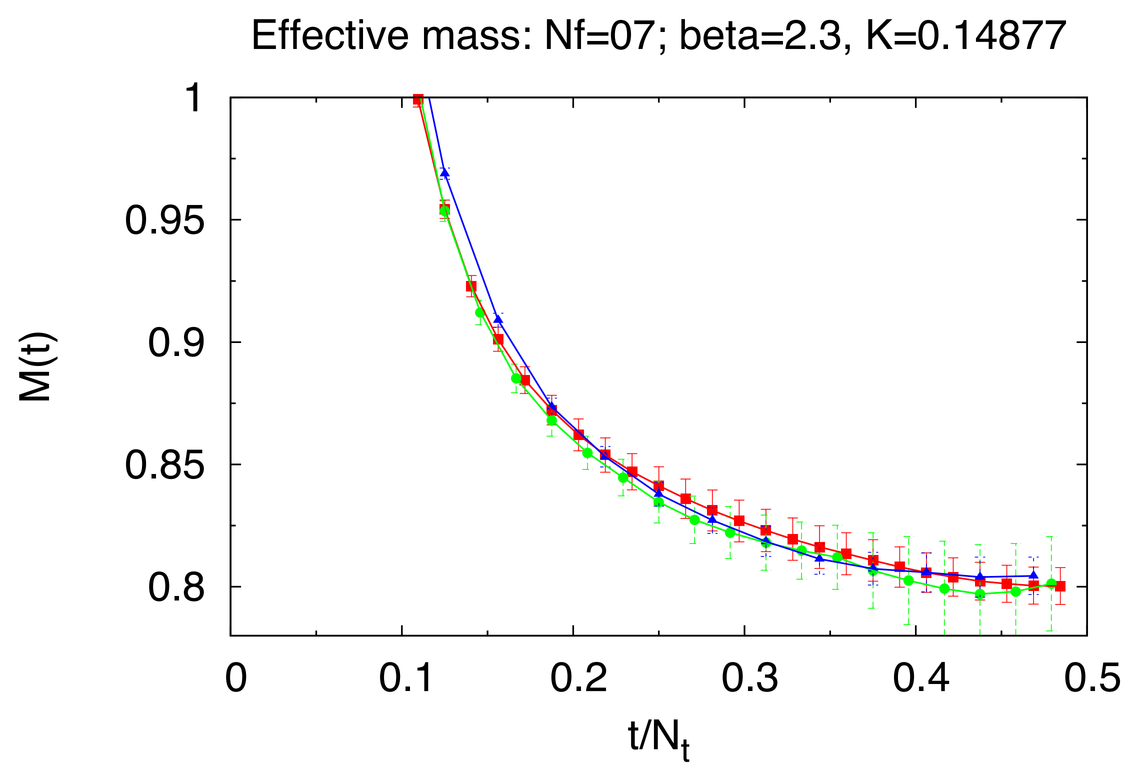
Nf=12, 8, 7



Effective mass: Nf=12; beta=3.0, K=0.1405
M(t)
1
0.95
0.9
0.85
0.8
0 0.1 0.2 0.3 0.4 0.5
t/N_t



Effective mass: Nf=08; beta=2.4, K=0.147
M(t)
1
0.95
0.9
0.85
0.8
0 0.1 0.2 0.3 0.4 0.5
t/N_t



Effective mass: Nf=07; beta=2.3, K=0.14877
M(t)
1
0.95
0.9
0.85
0.8
0 0.1 0.2 0.3 0.4 0.5
t/N_t

The location of IR fixed points

$N_f = 16$: $\beta^* = 10.5 \pm 0.5$
$N_f = 12$: $\beta^* = 3.0 \pm 0.1$
$N_f = 8$: $\beta^* = 2.4 \pm 0.1$
$N_f = 7$: $\beta^* = 2.3 \pm 0.05$

The conformal window

$$7 \leq N_f \leq 16$$

Continuum limit of propagators at IRFP

continuum limit of scaled effective mass is given
by the limit N --> infinity

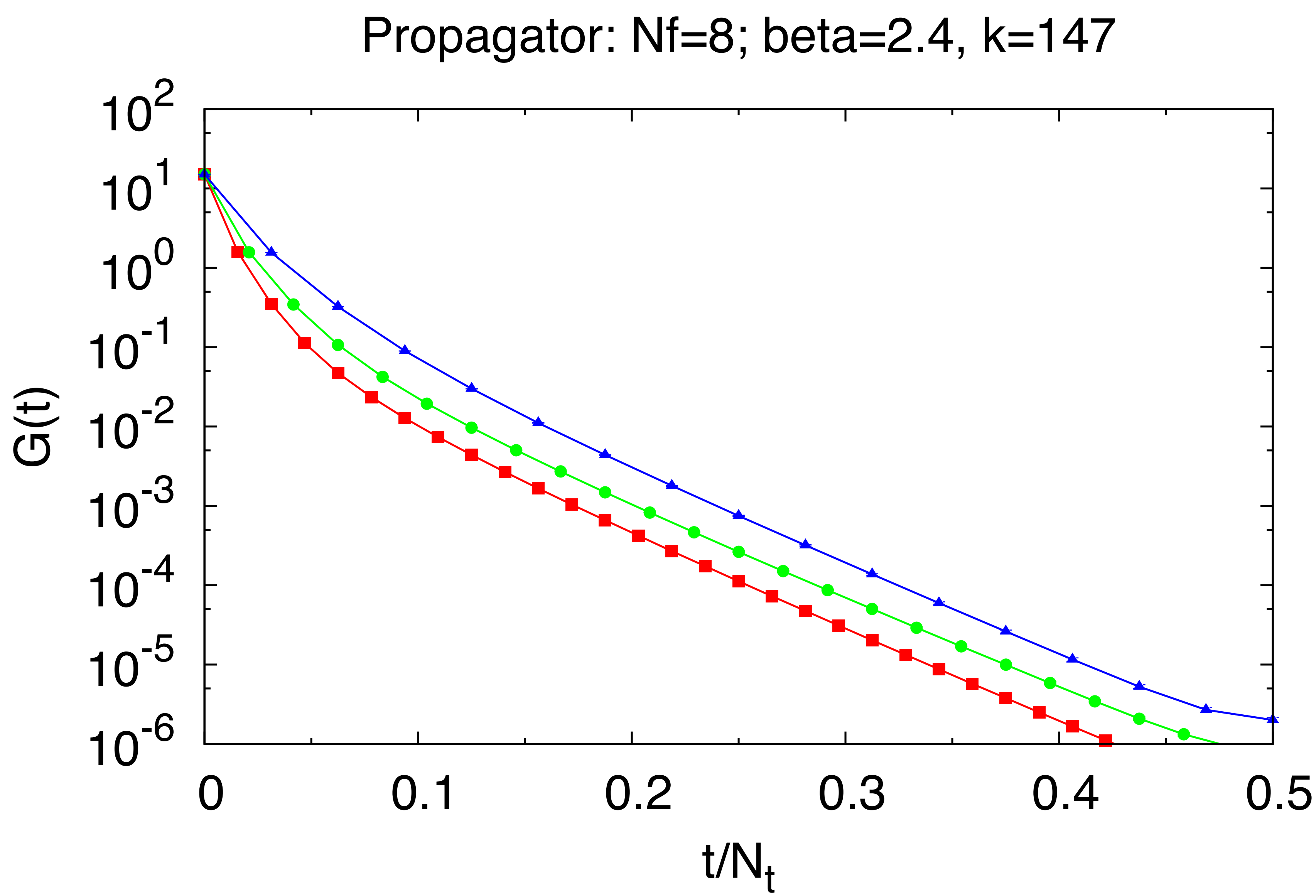
Even up to N=16, the limit is almost realized for $\tau \ge 0.1$.
As N becomes larger, it will be realized for $\tau \le 0.1$

Note the limit depends on the aspect ratio and boundary conditions,
but not on L= N a

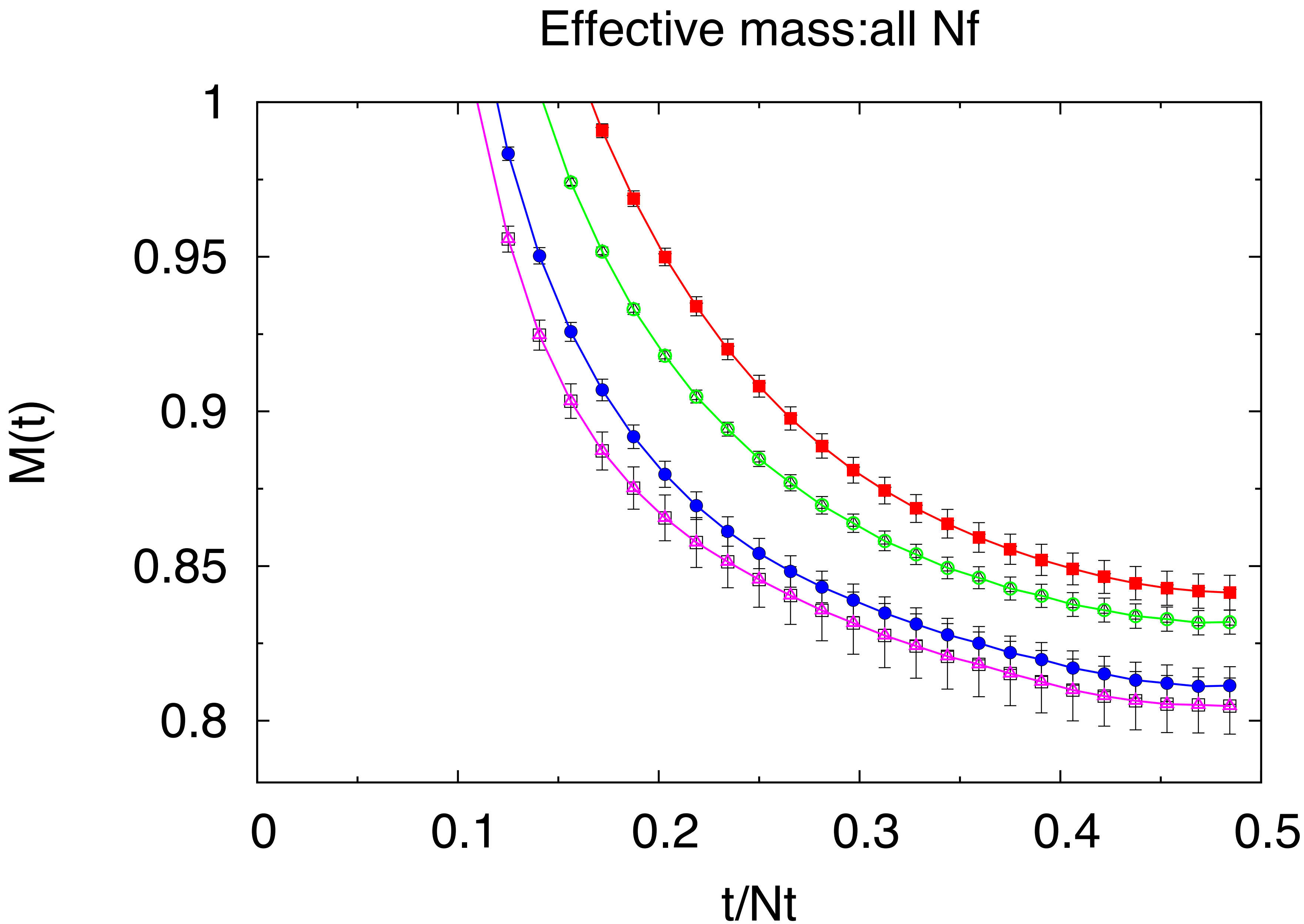
Note that local-local propagators are not local observables,
due to the summation over the space coordinates

Scaling relation for propagators

$$\tilde{G}(\tau; N) = \left(\frac{N'}{N}\right)^{3-2\gamma^*} \tilde{G}(\tau; N') \ .$$



Effective mass for Nf=16, 12, 8, 7

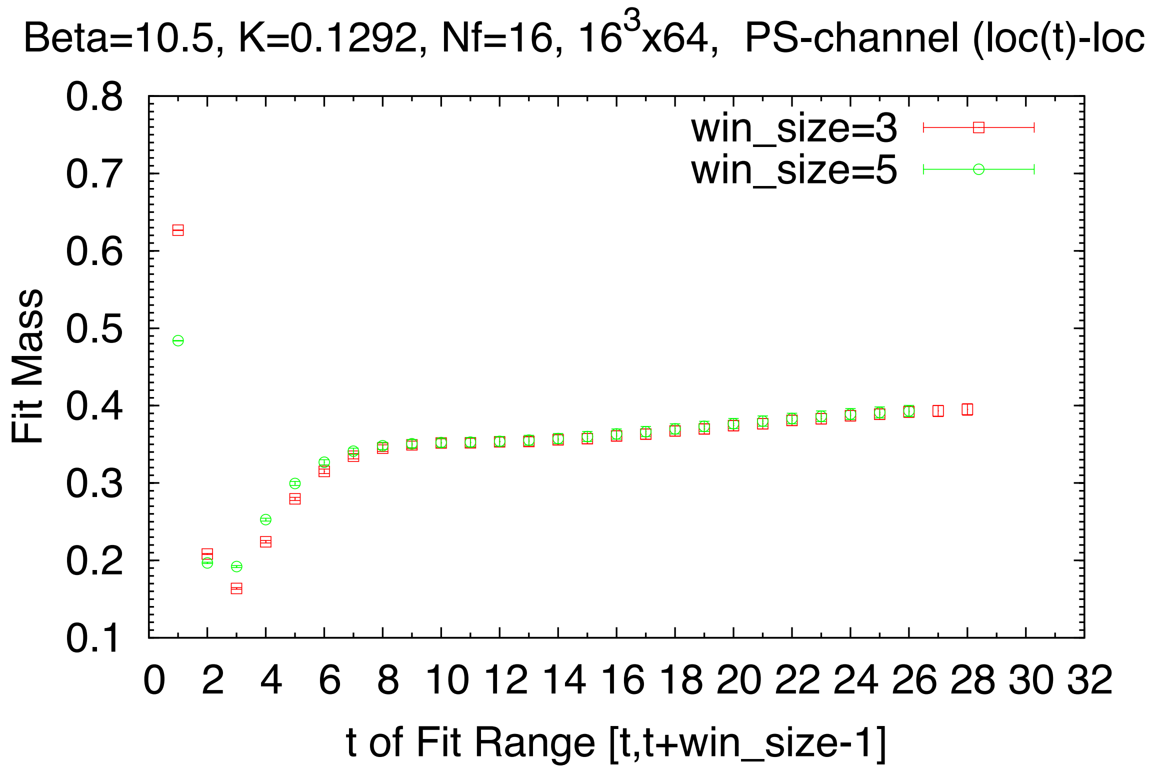


Local analysis of propagators

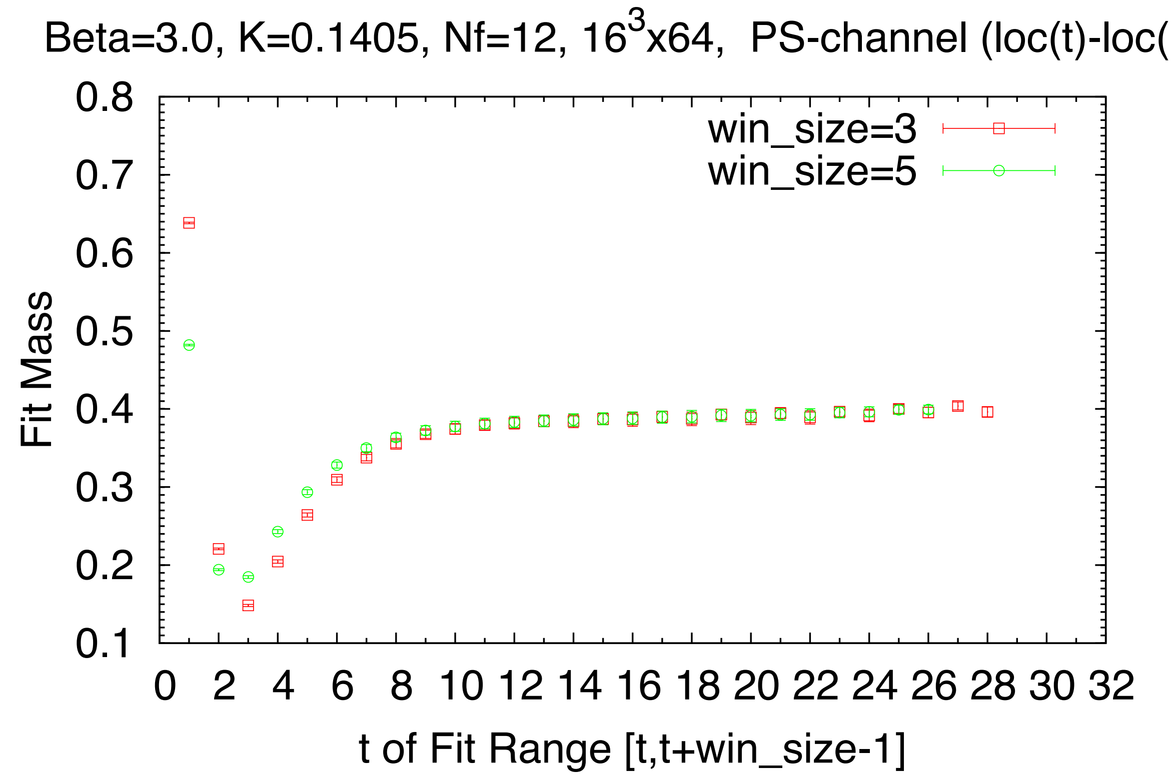
$$G(t) = c(t)\,\frac{\exp(-m(t)\,t)}{t^{\,\alpha(t)}}$$

parametrization using data at three points
useful for seeing the characteristics

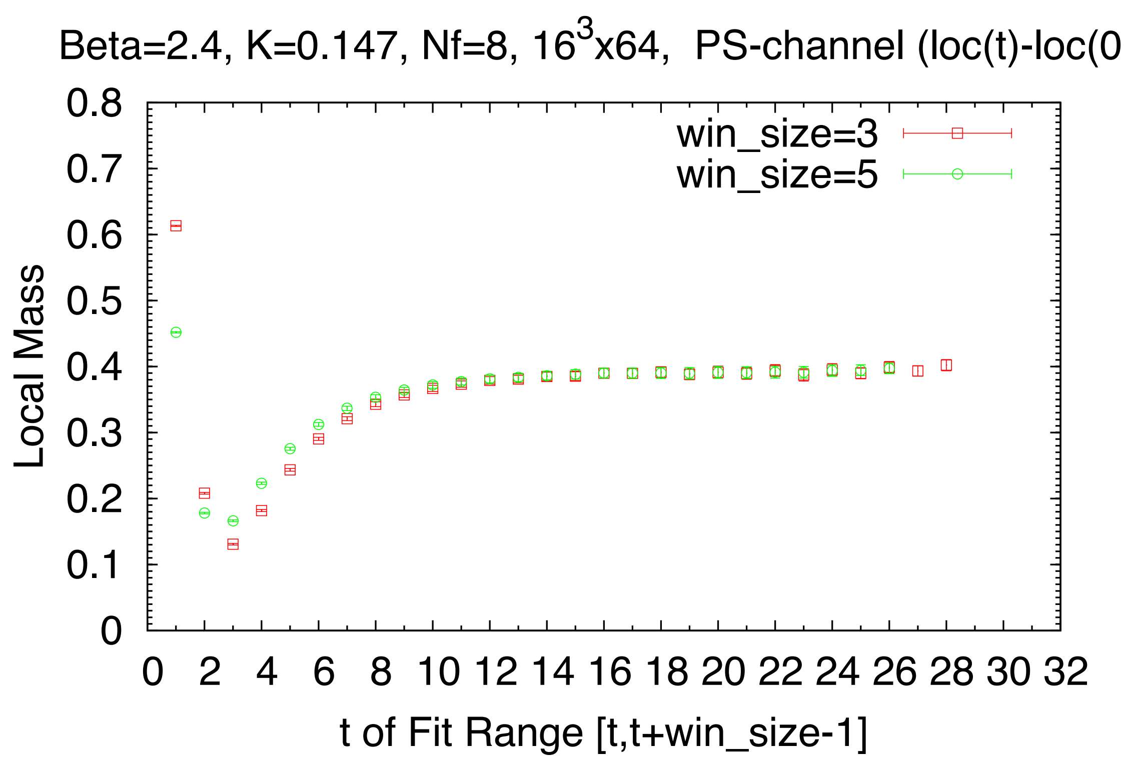
local mass



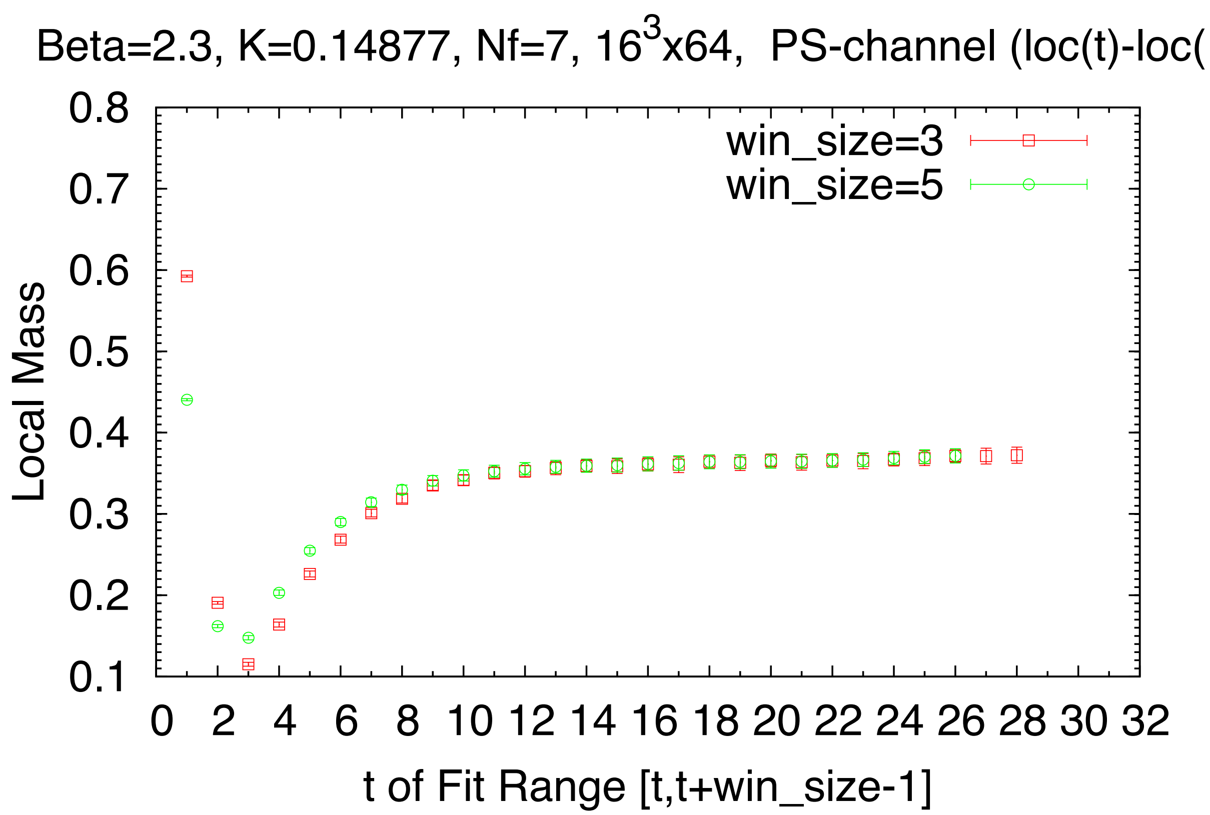
Beta=10.5, K=0.1292, Nf=16, 16^3x64, PS-channel (loc(t)-loc
win_size=3
win_size=5
Fit Mass
t of Fit Range [t,t+win_size-1]



Beta=3.0, K=0.1405, Nf=12, 16^3x64, PS-channel (loc(t)-loc(
win_size=3
win_size=5
Fit Mass
t of Fit Range [t,t+win_size-1]

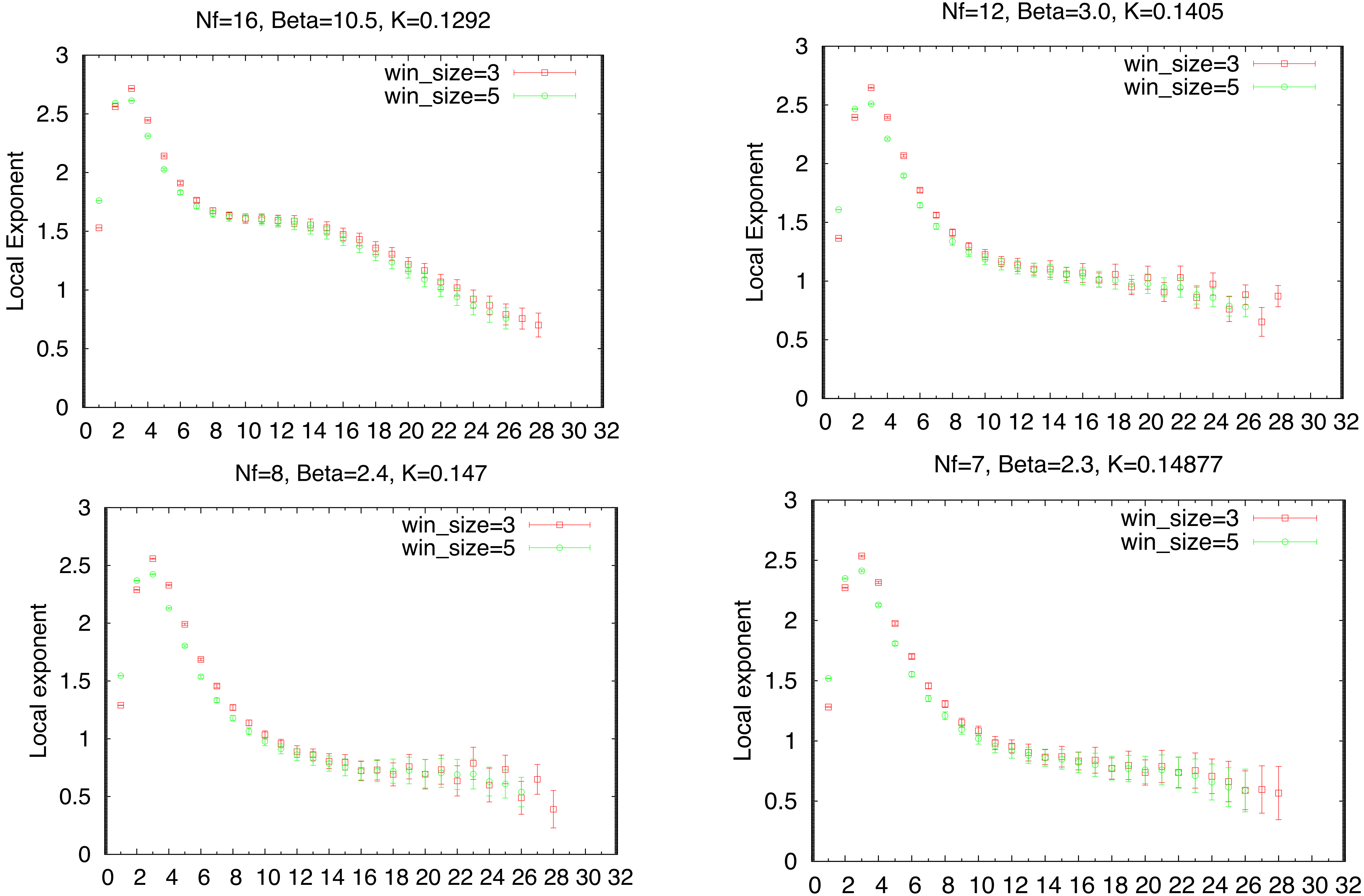


Beta=2.4, K=0.147, Nf=8, 16^3x64, PS-channel (loc(t)-loc(0
win_size=3
win_size=5
Local Mass
t of Fit Range [t,t+win_size-1]



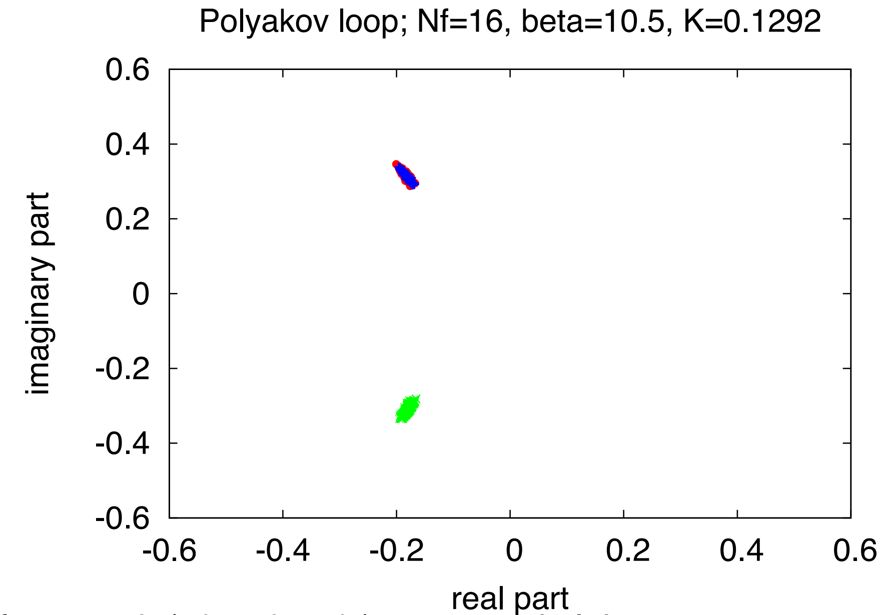
Beta=2.3, K=0.14877, Nf=7, 16^3x64, PS-channel (loc(t)-loc(
win_size=3
win_size=5
Local Mass
t of Fit Range [t,t+win_size-1]

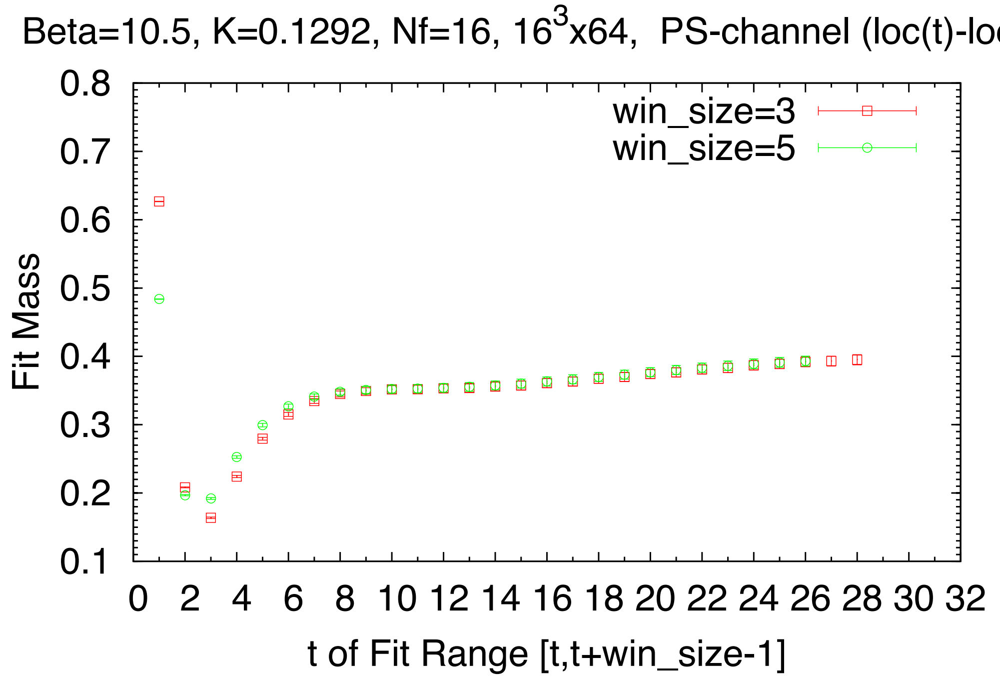
local exponent

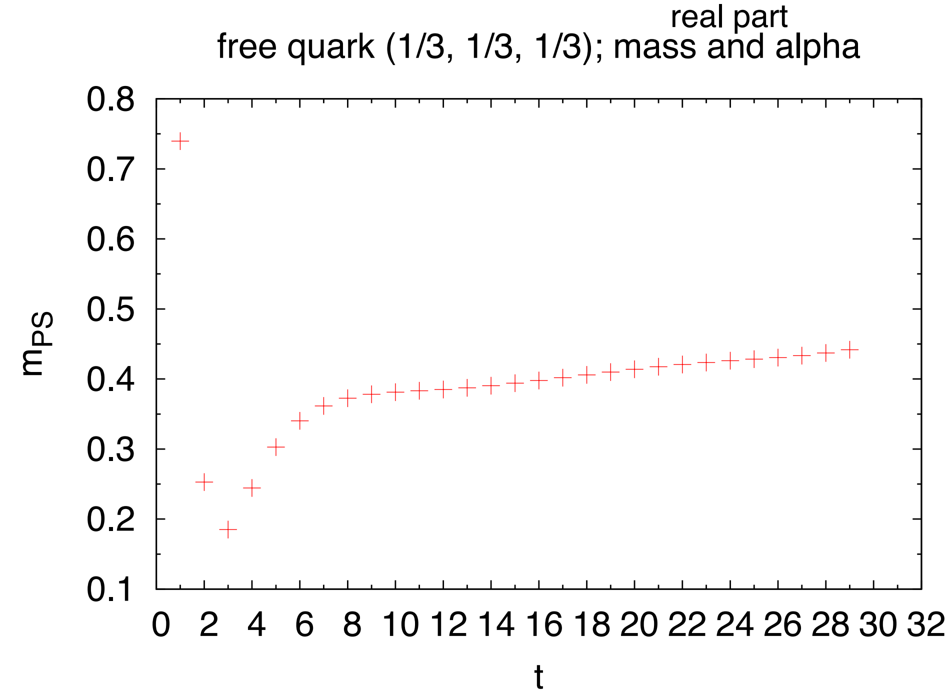


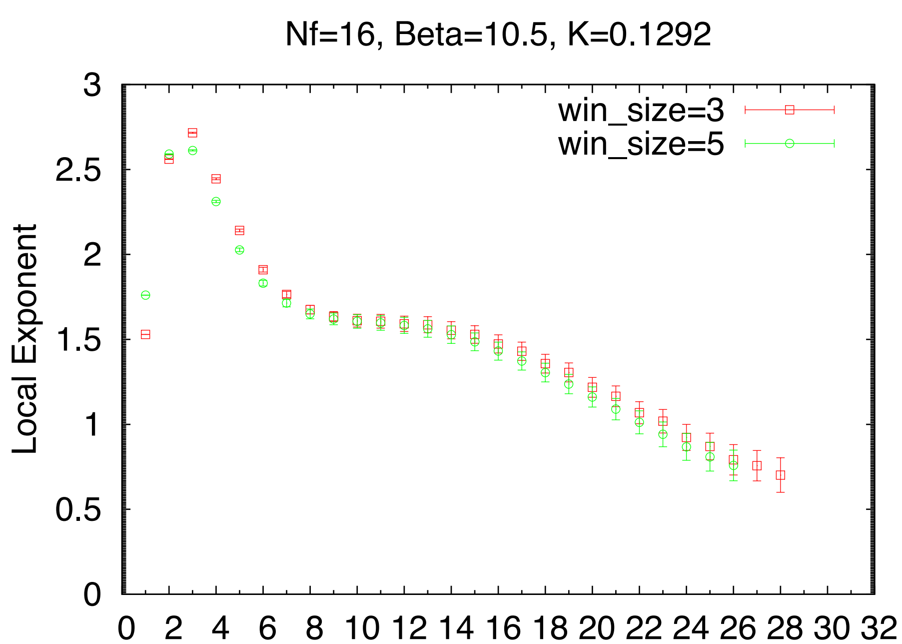
Nf=18

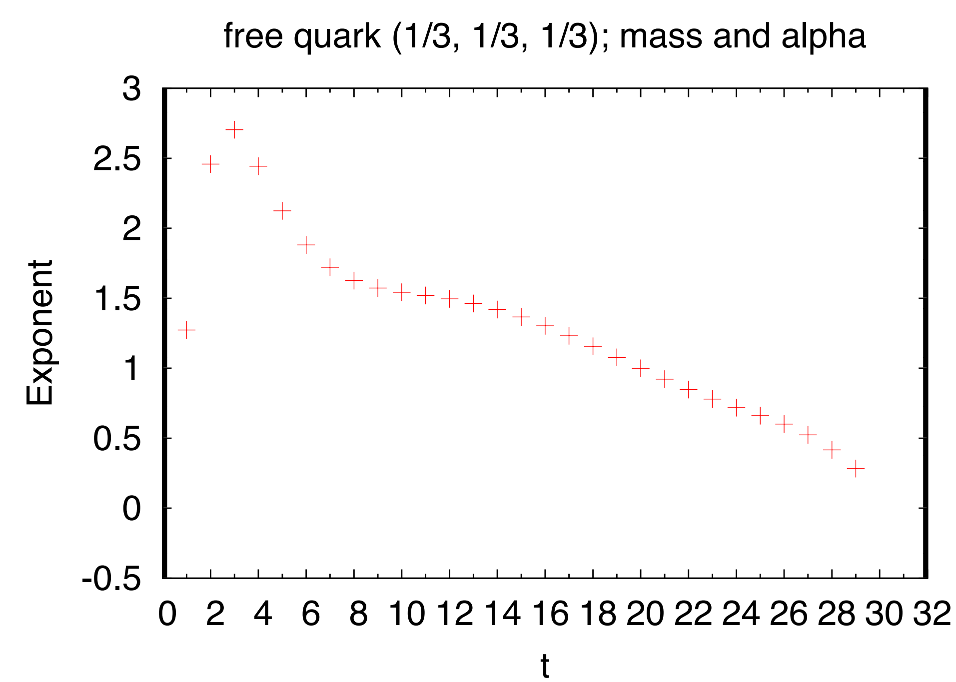
almost free particle in the Z(3) twisted vacuum











Nf=7, 8

- meson unparticle model*

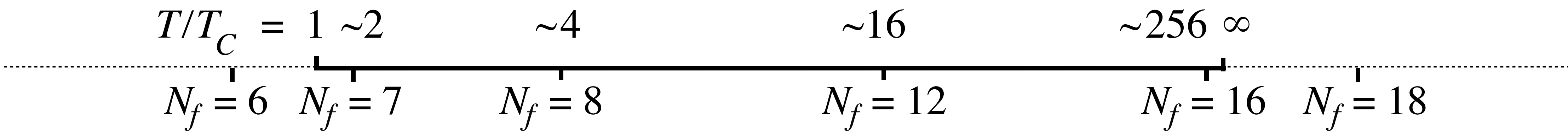
$$\langle O(p)O(-p)\rangle = \frac{1}{(p^2+m^2)^{2-\Delta}}$$

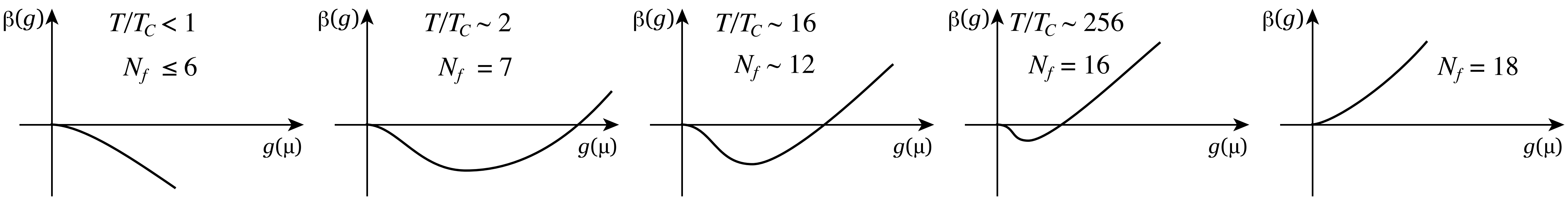
plateau at $t = 16 \sim 24$

$$2-\gamma^* \sim 0.8$$

$$\gamma^* \sim 1.2$$

Correspondence between two sets





γ^* =2.0

meson

γ^* ~1.3

unparticle

γ^* =0.0

free fermion
Z(3) twisted vacuum

Conclusions (cont.)

- two scaling relations are derived
- scaling of scaled effective masses provides a stringent test of IRFP
- able to identify the location of IRFP for Nf=7, 8, 12 and 16.
- established the conformal window
- continuum limit of propagators at IRFP is derived
- It depends on the aspect ratio and boundary conditions, not L=N a

Conclusions

- Nf=16 is similar to free fermions in the Z(3) twisted vacuum
- Nf=7 and 8 are consistent with meson unparticle model
- there is a nice correspondence between large Nf and high temperature.
- A lot of things should be done
  - Larger N and high statistics
  - estimate γ^* by several methods

Thank you !