# Holographic Estimate of Isospin splitting in hadron mass

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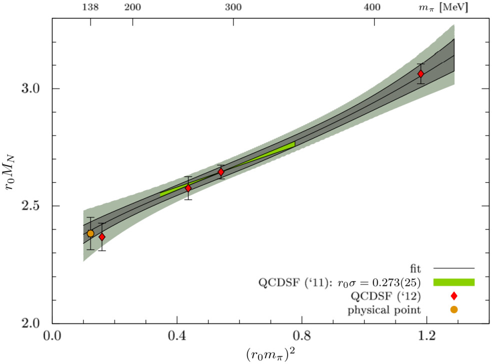
SCGT15, KMI, Nagoya University

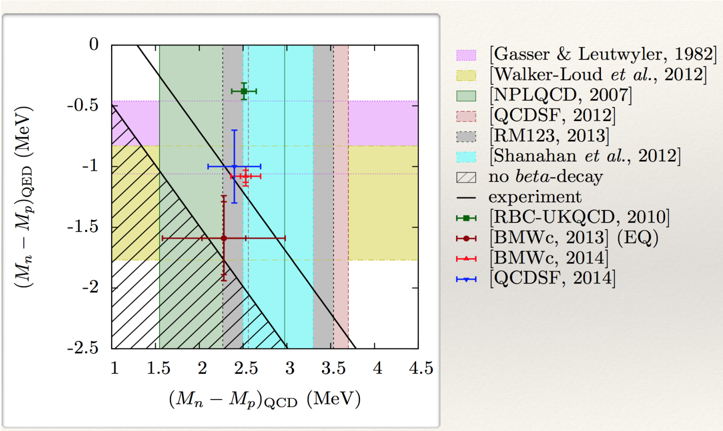
Based on arXiv:1409.8139

# Introduction

- Solving QCD is hard, because quarks and gluons are not right degrees of freedom at low energy, though fundamental.
- Lattice (QCDSF12)



## Lattice (Portelli at ICHEP 2014)



# EM mass of Pions - Vacuum Alignment

- In QCD the chiral symmetry is spontaneously broken,

$$G \longrightarrow H$$

- Pions and Kaons are massless in the chiral limit, because they are fluctuations on the vacuum manifold, $G/H$, along the flat directions of broken generators. (Goldstone 1961)
- When the chiral symmetry is approximate, however, the vacuum degeneracy is lifted and they become massive, the pseudo-Nambu-Golstone bosons. (Weinberg 1972)

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# Mass of Pions - Vacuum Alignment

- Current quark mass lifts vacuum degeneracy and gives pion mass. In the isospin limit by PCAC

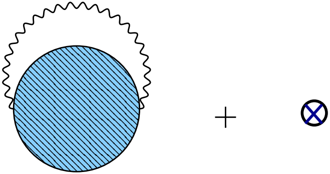
$$F_\pi^2 m_\pi^2 = 2m_q \langle q\bar{q} \rangle \ .$$

- The pion mass in the isospin limit in hQCD (Aharony+Kutasov; Hashimoto et al '08)

$$m_\pi^2 = \frac{1}{TV} \left. \frac{\delta^2}{\delta\pi^2} S_{\rm hQCD} \right|_{\pi=0}$$

# EM mass of Pions - Vacuum Alignment

- EM interaction breaks the isospin symmetry and contributes to the vacuum energy, lifting its degeneracy.
- The corrections to the vacuum energy is



$$\Delta E_{\text{vac}} = -\frac{e^2}{2} \int \mathrm{d}^4x \, \Delta^{\mu\nu}(x) \langle 0 | U^\dagger \, \mathrm{T} J_\mu^{Q_{\text{em}}}(x) J_\nu^{Q_{\text{em}}}(0) U | 0 \rangle \, .$$

with $U = \exp(2i\pi/F_\pi)$ .

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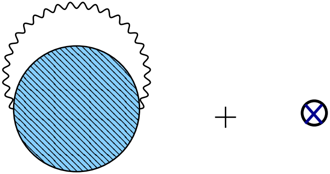

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# EM mass of Pions - Vacuum Alignment

- The EM mass is now

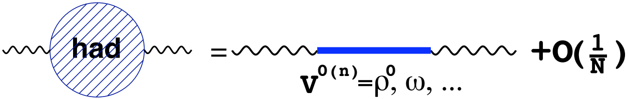
$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{\partial^2}{\partial\pi_+\partial\pi_-}\Delta E[U]\Big|_{U=1} = e^2 M^2\,,$$

where

$$M^2 = \frac{1}{F_\pi^2}\int \mathrm{d}^4x\Delta^{\mu\nu}(x)\,\langle 0|T\left[V_\mu^3(x)V_\nu^3(0) - A_\mu^3(x)A_\nu^3(0)\right]|0\rangle\ .$$

# EM mass of Pions - Vacuum Alignment

- The current-current correlators are given in hQCD as




$$\Pi_V(q^2) = \sum_{n=1}^{\infty} \frac{g_{v^n}^2}{(q^2 - m_{v^n}^2) m_{v^n}^2}$$

$$\Pi_A(q^2) = \sum_{n=1}^{\infty} \frac{g_{a^n}^2}{(q^2 - m_{a^n}^2) m_{a^n}^2}$$

# EM mass of Pions - Vacuum Alignment

- The EM mass becomes (Das et al 1967)

$$e^2 M^2 = \frac{3e^2}{f_\pi^2} \int \frac{\mathrm{d}^4 Q}{(2\pi)^4} \left[\Pi_V(-Q^2) - \Pi_A(-Q^2)\right] + \delta m_\pi^2(\Lambda)\,,$$
$$= \frac{3e^2}{8\pi^2 f_\pi^2} \sum_n \left[ g_{v^n}^2 \ln\left(\frac{\Lambda}{m_{v^n}}\right) - g_{a^n}^2 \ln\left(\frac{\Lambda}{m_{a^n}}\right)\right] - \frac{3e^2}{16\pi^2}\Lambda^2 + \delta m_\pi^2$$

- $m_{\pi^\pm} - m_{\pi^0} \simeq 3.5 \sim 7.2\ \mathrm{MeV}$ for $\Lambda^2 = 14 \sim 15\ M_{\mathrm{KK}}^2$ in SS model. (Exp) $4.6\ \mathrm{MeV}$.

# Baryons in hQCD

- Baryons in large N QCD are realized as solitons (Witten '79).
- hQCD, being a 5d gauge theory, admits a topologically conserved current:

$$J^M = \frac{1}{32\pi^2}\epsilon^{MNLPQ}\,\mathrm{tr}\left(F_{NL}F_{PQ}\right)$$

- This topological current couples to U(1) bulk (quark number) gauge fields through Chern-Simons term

$$B^\mu = \frac{1}{8\pi^2}\int \mathrm{d}z\,\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left(F_{\nu\rho}F_{\sigma z}\right)$$

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# Baryons in hQCD

- Baryon number in hQCD is nothing but the instanton number:

$$\int \mathrm{d}^3x\,\mathrm{d}z\,J^0(x,z) = \frac{1}{32\pi^2}\int \mathrm{d}^3x\,\mathrm{d}z\,F^a_{IJ}\tilde{F}^{aIJ} = \int \mathrm{d}^3x B^0(x) = N_B$$

- The energy of soliton from the DBI action at the leading order

$$\begin{aligned}\mathcal{E}_0 &= \kappa\int \mathrm{d}^3x\mathrm{d}z\left[\frac{1}{4}\left(1+z^2\right)^{-1/3}\left(F^a_{ij}\right)^2+\frac{1}{2}\left(1+z^2\right)\left(F^a_{iz}\right)^2\right]\\ &= \kappa\int \mathrm{d}^3x\,\mathrm{d}z\,\left(1+z^2\right)^{1/3}\vec{E}^a\cdot\vec{B}^a\,,\end{aligned}$$

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# Baryons in hQCD

- For a small size baryon at $z = 0$

$$\frac{1}{8\pi^2}\,\vec{E}^a \cdot \vec{B}^a = \frac{6}{\pi^2} \cdot \frac{\rho^4}{\left(\rho^2 + \vec{x}^2 + z^2\right)^4}$$

- The energy from DBI becomes

$$\mathcal{E}_0 = m_B^{(0)} \left(1 + \frac{1}{6}\,\rho^2 + \cdots\right)$$

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# Baryons in hQCD

- Since the baryons carry U(1) charge, it has a Coulomb energy:

$$
\begin{aligned}
\mathcal{E}_{\mathrm{C}} &= \kappa \int \mathrm{d}^3x\mathrm{d}z \left[-\frac{1}{2}\left(1+z^2\right)^{-1/2} E_i^2 - \frac{1}{2}\left(1+z^2\right) E_z^2\right] \\
&\quad + \frac{1}{2} N_c \int \mathrm{d}^4x\mathrm{d}z\, A_0 J^0 \\
&= \frac{1}{4} N_c \int \mathrm{d}^3x\mathrm{d}z\, A_0 J^0 \\
&\simeq \frac{\kappa}{2} \int \mathrm{d}^3x\, \mathrm{d}z\, E_r^2 = \frac{N_c^2}{40\pi^2\kappa} \cdot \left(\frac{1}{\rho^2} + \mathcal{O}(1)\right).
\end{aligned}
$$

# Baryons in hQCD

- By minimizing the soliton energy, we find the size of baryons:

$$\rho_B \simeq \frac{1}{\pi}\left(\frac{3}{40}\right)^{1/4}\sqrt{\frac{N_c}{2\kappa}} = \frac{9.6}{\sqrt{\lambda}}$$

- For a small size soliton we may write an effective action (HRYY '08):

$$S_{\rm hQCD} = \int_{x,w}\left[-i\bar{\mathcal{B}}\gamma^m D_m\mathcal{B} - im_b(w)\bar{\mathcal{B}}\mathcal{B} + \kappa(w)\bar{\mathcal{B}}\gamma^{mn}F_{mn}^{SU(2)_I}\mathcal{B} + \cdots\right]$$

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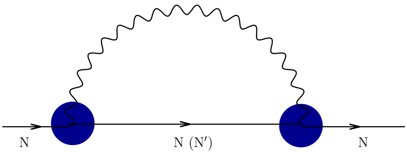
# EM mass of baryons - Radiative corrections

- The EM mass of nucleon at the leading order in $\alpha_{\rm em}$ is through vertex correction

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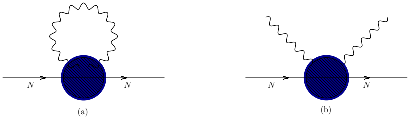

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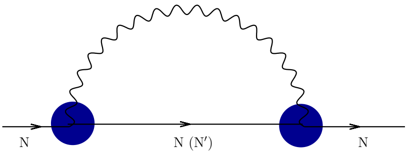


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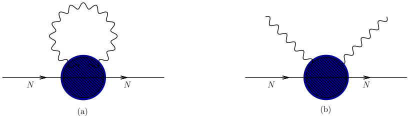


# EM mass of baryons - Radiative corrections

- The nucleon form factors are defined as

$$\langle p' | J^\mu(x) | p \rangle = e^{iqx} \, \bar{u}(p') \, \mathcal{O}^\mu(p, p') \, u(p) \, ,$$

By the Lorentz invariance and the current conservation we get

$$\mathcal{O}^\mu(p, p') = \gamma^\mu F_1(Q^2) + i \frac{\sigma^{\mu\nu}}{2m_N} q_\nu F_2(Q^2) \, .$$

# EM mass of baryons - Radiative corrections

- By AdS/CFT the EM form factors are given as

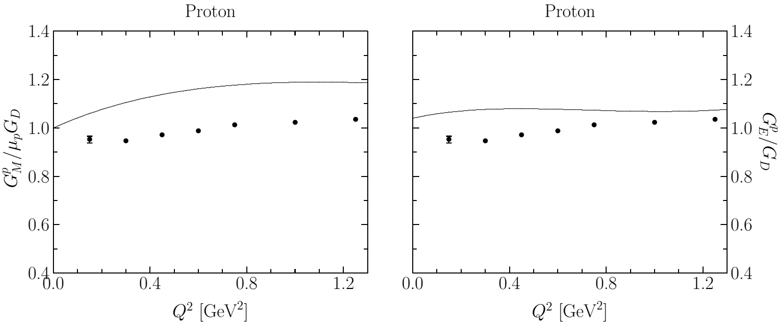
$$\begin{aligned} F_1(Q^2) &= \sum_{n=1}^{\infty} \left( g_{V,min}^{(n)} Q_{\rm em} + g_{V,mag}^{(n)} \, \tau^3 \right) \frac{\zeta_n m_n^2}{Q^2 + m_n^2} \,, \\ F_2(Q^2) &= F_2^3(Q^2)\, \tau^3 = \tau^3 \sum_{n=1}^{\infty} \frac{g_2^{(n)} \zeta_n m_n^2}{Q^2 + m_{2n}^2} \,, \end{aligned}$$

where

$$\begin{aligned} g_{V,min}^{(n)} &= \int_{-w_{max}}^{w_{max}} dw \, |f_L(w)|^2 \, \psi_{(n)}(w) \\ g_{V,mag}^{(n)} &= 2 \int_{-w_{max}}^{w_{max}} dw \, \kappa(w) \, |f_L(w)|^2 \, \partial_w \psi_{(n)}(w) \,, \\ g_2^{(n)} &= 4 m_N \int_{-w_{max}}^{w_{max}} dw \, \kappa(w) f_L^*(w) f_R(w) \psi_{(n)}(w) \,. \end{aligned}$$

# EM mass of baryons - Radiative corrections

- The EM form factors of nucleons for SS model (HRYY 2009):
  $G_D = 1/(1+Q^2/0.71)^2$, $G_M^p(Q^2) = F_1^p(Q^2) + F_2^p(Q^2)$,
  $G_E^p(Q^2) = F_1^p(Q^2) - \frac{Q^2}{4m_N^2} F_2^p(Q^2)$:



# EM mass of baryons - Radiative corrections

- The EM mass of nucleons becomes

$$\delta M_{p\,(n)} = e^2 \int \frac{\mathrm{d}^4 Q}{(2\pi)^4} \left[ F_1^{p\,(n)}(Q^2) \right]^2 \frac{3M_N}{Q^2 + M_N^2} \cdot \frac{1}{Q^2} \, .$$

- We find, keeping the first four vector mesons

$$\delta M_p = 0.494 \text{ MeV}, \quad \delta M_n = 0.018 \text{ MeV}.$$

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# Conclusion and Outlook

- We have calculated the EM mass of pions by the vacuum alignment.
- The EM masses of nucleons are calculated as radiative corrections in hQCD.

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