A light scalar from deformations of Klebanov-Strassler  
(DE, arXiv:1401.3412 [hep-th])
On the glueball spectrum of walking backgrounds from wrapped-D5 gravity duals  
(DE, Maurizio Piai, arXiv:1212.2600 [hep-th])
Towards multi-scale dynamics on the baryonic branch of Klebanov-Strassler  
(DE, Jerome Gaillard, Carlos Nunez, Maurizio Piai, arXiv:1104.3963 [hep-th])
Light scalars from a compact fifth dimension  
(DE, Maurizio Piai, arXiv:1010.1964 [hep-th])
A light scalar from walking solutions in gauge-string duality  
(DE, Maurizio Piai, Carlos Nunez, arXiv:0908.2808 [hep-th])
Can we find walking dynamics using gauge/gravity duality?

- IR dynamics governed by an approximate fixed point:

![Graph showing coupling vs. energy with two energy levels labeled \( \Lambda \) and \( \Lambda' \)]

- Contrast with QCD:

![Graph showing coupling vs. energy with two energy levels labeled \( \Lambda \) and \( \Lambda' \)]
Motivation

Theoretical:
- Natural to consider strongly coupled field theories with more than one scale
- Potential for richer dynamics

Electro-Weak Symmetry Breaking:
- Strongly coupled theories could solve hierarchy problem
- Simple Technicolor models are ruled out experimentally
- Walking offers a way out: \( (\Lambda/\Lambda_*)^\gamma \), large anomalous dimension \( \gamma \)
Techni-dilaton

- Spontaneously broken approximate scale invariance
- Could this lead to a light scalar, the dilaton (pseudo-Goldstone of dilatations), in the spectrum?
- Such a light scalar would couple to the Standard Model fields in a way that mimicks the Higgs
How to compute at strong coupling?

- AdS/CFT is a duality between $\mathcal{N} = 4$ SYM and Type IIB String Theory on $\text{AdS}_5 \times S^5$:

\[
(A_\mu, 6 \times \Phi, 4 \times \lambda) \quad \text{Full String Theory} \quad \text{Type IIB Supergravity on } \text{AdS}_5 \times S^5
\]

\[\lambda = g_{YM}^2 N_c\]

- Allows to study strongly coupled dynamics in field theory
- The extra bulk dimension (the radial coordinate $r$) is related to energy scale in the field theory, and thus the bulk is in a sense a geometrical representation of the RG flow of the dual theory
What does this have to do with walking dynamics?
- The walking region can be thought of as the theory flowing near an IR fixed point.
- This near conformality means that we can apply ideas from AdS/CFT.

These fall into two classes:
- Phenomenological bottom-up models where the matter content in the bulk is put in by hand.
  (Example: \[ \int d^4xdr \sqrt{-g} \left[ R - \frac{1}{2} (\partial \Phi)^2 - V(\Phi) \right] \])
- Top-down models which have their origin in string theory constructions, and therefore are on firmer ground.
We will focus on top-down approaches

Questions:
- Can we build top-down models from string theory with walking dynamics?
- Do they contain a light scalar in the spectrum?
Outline of talk

- Walking dynamics from wrapped D5-branes
- Spectrum
- Deformations of Klebanov-Strassler
- Conclusions and open questions
Let us consider a top-down model obtained from string theory

- D5 system:
  - D5-branes wrapped on $S^2$
  - This gives us an $\mathcal{N} = 1$ SUSY field theory
Type IIB supergravity ansatz \((ds^2, F_3, \phi)\):

\[
ds^2 = e^{2p-x} ds_5^2 + (e^{x+g} + a^2 e^{x-g})(e_1^2 + e_2^2) + e^{x-g} (e_3^2 + e_4^2 + 2a(e_1 e_3 + e_2 e_4)) + e^{-6p-x} e_5^2,
\]

\[
ds_5^2 = dr^2 + e^{2A} dx_{1,3}^2,
\]

\[
F_3 = N \left[ -e_5 \wedge (e_4 \wedge e_3 + e_2 \wedge e_1 + b(e_4 \wedge e_1 - e_3 \wedge e_2)) + dr \wedge (\partial_r b (e_4 \wedge e_2 + e_3 \wedge e_1)) \right],
\]

where

\[
e_1 = - \sin \theta \, d\phi,
\]
\[
e_2 = d\theta,
\]
\[
e_3 = \cos \psi \sin \tilde{\theta} \, d\tilde{\phi} - \sin \psi \, d\tilde{\theta},
\]
\[
e_4 = \sin \psi \sin \tilde{\theta} \, d\tilde{\phi} + \cos \psi \, d\tilde{\theta},
\]
\[
e_5 = d\psi + \cos \tilde{\theta} \, d\tilde{\phi} + \cos \theta \, d\phi
\]
Finding solutions:

- Write down BPS equations for the background fields
- These can be repackaged into a single second order differential equation (Hoyos, Nunez, Papadimitriou 2008):

\[ P'' + P' \left[ \frac{P' + Q'}{P - Q} + \frac{P' - Q'}{P + Q} - 4 \coth(2\rho) \right] = 0, \]

\[ Q(\rho) = N_c (2\rho \coth(2\rho) - 1) \]

- Map from \( P \) to solutions in Type IIB supergravity:

\( P \rightarrow \{p, x, g, \phi, A, a, b\} \)
Example: Maldacena-Nunez \( (P = 2N_c \rho) \)

- Non-singular: in the IR, \( S^2 \) shrinks to zero size, while the size of \( S^3 \) stays finite (deformed conifold)
- 4d gauge coupling constant \( \lambda = \frac{g^2 N_c}{8\pi^2} = \frac{N_c \coth \rho}{P} \)

- One scale: confinement scale
Walking dynamics from wrapped D5-branes

Walking backgrounds (UV behaviour: $P \sim e^{4\rho/3}$):
(Nunez, Papadimitriou, Piai 2008)

Two scales: confinement scale and end of walking region $\rho_*$
Walking dynamics from wrapped D5-branes

Walking backgrounds (UV behaviour: Maldacena-Nunez): (DE, Nunez, Piai 2009)

Two scales: confinement scale and end of walking region $\rho_*$
Is there a light scalar in the spectrum of glueballs? Compute spectrum holographically:

- Expand EOMs to linear order in fluctuations around the background
- Impose appropriate BCs on fluctuations in the IR and UV
- The values of momenta $K^2 (= -M^2)$ for which solutions exist give us the spectrum
In practice, it is easier to work in five dimensions:

- Consistent truncation to a 5d non-linear sigma model with fields \( \Phi = (g, p, x, \phi, a, b) \):
  
  (Berg, Haack, Mück, 2005)

\[
S_{5d} = \int d^4x dr \sqrt{g} \left[ \frac{R}{4} - \frac{1}{2} G_{ab}(\Phi) \partial_M \Phi^a \partial^M \Phi^b - V(\Phi) \right]
\]

- Metric with warp factor \( A \):

\[
ds^2 = dr^2 + e^{2A} dx_{1,3}^2
\]

(1)

- Any solution of 5d system solves 10d EOMs
\[ V = \frac{e^{-2(g+2(p+x))}}{128} \left[ 16 \left( a^4 + 2 \left( e^8 - e^{6p+2x} \right)^2 - 1 \right) a^2 + e^{4g} - 4e^g + 6p + 2x \left( 1 + e^{2g} \right) + 1 \right] + \\
\quad e^{12p+2x+\phi} \left( 2e^{2g} (a - b)^2 + e^{4g} + \left( a^2 - 2ba + 1 \right)^2 \right) N_c^2 \right], \\
\]
\[ G_{ab} = \text{diag} \left( \frac{1}{2}, 6, 1, \frac{1}{4}, \frac{e^{-2g}}{2}, \frac{e^{-2x+\phi}N_c^2}{32} \right) \]
ADM-formalism: write the metric as (lapse function $n$ and shift vector $n^{\mu}$)

$$ds^2 = (n^{\mu} n_{\mu} + n^2)dr^2 + 2n_{\mu}dx^{\mu}dr + g_{\mu\nu}dx^{\mu}dx^{\nu}$$

(2)

Expand to linear order in fluctuations $\{ \varphi^a, \nu, \nu^{\mu}, h^{TT}_{\mu\nu}, h, H, \epsilon^{\mu} \}$ around the background:

$$\Phi^a = \Phi^a + \varphi^a,$$

$$n = 1 + \nu,$$

$$n^{\mu} = \nu^{\mu},$$

$$g_{\mu\nu} = e^{2A}(\eta_{\mu\nu} + h_{\mu\nu}),$$

with

$$h^{\mu}_{\nu} = h^{TT\mu}_{\nu} + \partial^{\mu}\epsilon_{\nu} + \partial_{\nu}\epsilon^{\mu} + \frac{\partial^{\mu}\partial_{\nu}}{\Box}H + \frac{1}{3}\delta^{\mu}_{\nu}h$$
Linearized equations of motion for spin-0 sector:

\[
\begin{align*}
&\left[ D_r^2 + 4A' D_r - e^{-2A} K^2 \right] \alpha^a - \\
&\left[ V^a_c - \mathcal{R}^a_{bcd} \bar{\Phi}^b \bar{\Phi}^d + \frac{4(\bar{\Phi}'^a V_c + V^a \bar{\Phi}'_c)}{3A'} + \frac{16V \bar{\Phi}'^a \bar{\Phi}'_c}{9A'^2} \right] \alpha^c = 0
\end{align*}
\]

with

\[
\alpha^a = \phi^a - \frac{\bar{\Phi}'^a}{6A'} h
\]
What boundary conditions should we impose?

- Usually, one picks the normalizable modes in the UV, and imposes regularity in the IR.
- We put $\varphi^a = 0$ in the IR and UV, which corresponds to:

$$- \frac{2\Phi'^a\Phi'^b}{3A'} D_r a^b \bigg|_{IR,UV} = \left[ e^{-2A} q^2 + \frac{A'}{2} \partial_r \left( \frac{A''}{A'^2} \right) \right] a^a \bigg|_{IR,UV}$$

- This automatically picks the normalizable mode in simple examples (e.g. Goldberger-Wise) and reproduces the same spectrum in all examples we have tried (e.g. Maldacena-Nunez, Klebanov-Strassler).
Spectrum of scalar glueballs for different values of $\rho_*$:

![Graph showing the spectrum of scalar glueballs with different $\rho_*$ values.](image)

Light scalar whose mass is suppressed by the length of the walking region.
UV behaviour of the walking backgrounds discussed so far:
- Not asymptotically AdS (either MN or dim-8 operator)
- The dictionary is less well-defined
- It is not easy to identify the QFT that is dual to a particular geometry
The baryonic branch of Klebanov-Strassler is parameterized by a dim-2 VEV $\langle \text{Tr}(\bar{A}A - \bar{B}B) \rangle$

Maldacena-Nunez can be thought of as taking the limit $\langle \text{Tr}(\bar{A}A - \bar{B}B) \rangle \rightarrow \infty$

Can we find walking dynamics on the baryonic branch of Klebanov-Strassler?

This would lead to better UV asymptotics (well-known duality cascade)
Solution generating technique (Maldacena, Martelli 2009):

- Start with a solution to the D5 system \((ds^2, F_3, \phi)\)
- Generate a new (rotated) solution \((ds^{(r)}^2, F_3^{(r)}, H_3^{(r)}, F_5^{(r)}, \phi^{(r)})\):

\[
ds^{(r)}^2 = e^{\phi/2} \left[ (1 - \kappa^2 e^{2\phi})^{-1/2} ds_{1,3}^2 + (1 - \kappa^2 e^{2\phi})^{1/2} ds_{6}^2 \right],
\]

\[
\phi^{(r)} = \phi,
\]

\[
F_3^{(r)} = F_3,
\]

\[
H_3^{(r)} = -\kappa e^{2\phi} \ast_6 F_3,
\]

\[
F_5^{(r)} = -\kappa (1 + \ast_{10}) \text{vol}_{(4)} \wedge d \left( e^{-2\phi} - \kappa^2 \right)^{-1}
\]

- D5 system \(\rightarrow\) D3/D5 system
- Preserves SUSY
Applying to the walking backgrounds with $P \sim e^{4\rho/3}$ in the UV (dim-8 operator)

For $\kappa = e^{-\phi_{\text{UV}}}$, the rotated backgrounds behave asymptotically like Klebanov-Strassler in the UV

The field theory dual to KS is more well-understood: $SU(N+M) \times SU(N)$ gauge group, bifundamental matter $A_i$ and $B_i$ ($i = 1, 2$) in representations $(N+M, \bar{N})$ and $(\bar{N}+M, N)$, superpotential $\mathcal{W} = \lambda_1 \text{Tr}(A_iB_jA_kB_l)\epsilon^{ik}\epsilon^{jl}$
Properties of the rotated solutions ($\kappa = e^{-\phi_{UV}}$):

- The dim-8 operator is no longer present, making the UV well-defined.
- There is a dim-3 VEV, the gaugino condensate.
- There is a dim-2 VEV, $\langle \text{Tr}(A\bar{A} - B\bar{B}) \rangle \neq 0$, signalling that we are on the baryonic branch of Klebanov-Strassler.
- There is a dim-6 VEV, $\langle \text{Tr}W^2\bar{W}^2 \rangle \neq 0$.
- Compute Wilson loops $\Rightarrow$ confinement.
A particularly simple case, turn off dim-2 VEV:

\[ ds_{10}^2 = h(\rho)^{-1/2} ds_{1,3}^2 + h(\rho)^{1/2} ds_6^2, \quad \text{(string frame)} \]

\[ ds_6^2 = \frac{e^{4/3} K(\rho)}{4} \left[ \frac{2}{3 K(\rho)^3} (4d\rho^2 + e_5^2) + 2(e_1 e_3 + e_2 e_4) + \cosh(2\rho) \sum_{i=1}^4 e_i^2 \right], \]

\[ \phi = \phi_0, \]

\[ B_2 = f(\rho) \left[ e_1 \wedge e_2 + e_3 \wedge e_4 + \text{sech}(2\rho)(e_1 \wedge e_4 - e_2 \wedge e_3) \right], \]

\[ F_3 = N \left[ - e_5 \wedge (e_4 \wedge e_3 + e_2 \wedge e_1 + b(\rho)(e_4 \wedge e_1 - e_3 \wedge e_2)) + b'(\rho) d\rho \wedge (e_4 \wedge e_2 + e_3 \wedge e_1) \right], \]

\[ F_5 = (1 + *_{10}) f_5, \quad f_5 = dc_4, \quad c_4 = e^{-\phi_0} h(\rho)^{-1} \text{vol}_{1,3}, \]

where

\[ b(\rho) = \frac{2\rho}{\sinh(2\rho)}, \quad K(\rho) = \frac{(f_0 - 4\rho + \sinh(4\rho))^{1/3}}{2^{1/3} \sinh(2\rho)}, \]

\[ h(\rho) = \frac{64Ne^{\phi_0}}{e^{8/3}} \int_{\rho}^{\infty} d\tilde{\rho} \frac{f(\tilde{\rho})}{\sinh^2(2\tilde{\rho}) K(\tilde{\rho})^2} \left( \frac{4\tilde{\rho}}{\sinh(4\tilde{\rho})} - 1 \right), \]

\[ f(\rho) = Ne^{\phi_0} \coth(2\rho)(1 - 2\rho \coth(2\rho)) \]

The dim-6 VEV is parameterized by \( f_0 \equiv e^{4\rho*} \) (KS corresponds to \( f_0 = 0 \))
Deformations of Klebanov-Strassler

Spectrum of scalar glueballs for different values of $\rho^*$:

Light scalar whose mass is suppressed by the length of the walking region.
Conclusions and open questions

Conclusions:
- We constructed top-down models with walking dynamics
- We found a light state suggestive of being a techni-dilaton

Open Questions:
- The backgrounds have a mild singularity in the IR: Wilson loop makes sense, $R$ and $R_{\mu\nu}R^{\mu\nu}$ are finite, but $R_{\mu\nu\delta\sigma}R^{\mu\nu\delta\sigma}$ diverges. Can we construct non-singular walking backgrounds?
- Spectrum when dim-2 VEV is finite?
- How to incorporate EWSB? Can we find U-shaped D-brane embeddings?