

Large-scale magnetic fields, non-Gaussianity, and gravitational waves from inflation

Reference: **Physical Review D 91, 043509 (2015)**
[arXiv:1411.4335 [astro-ph.CO]]

Sakata Memorial KMI Workshop on “Origin of Mass and Strong Coupling Gauge Theories” (SCGT15)

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I. Introduction

- **Galactic magnetic fields**

$$\sim \mu\text{G}$$

[Sofue *et al.*, *Annu. Rev. Astron. Astrophys.* 24, 459 (1986)]

- **Magnetic fields in clusters of galaxies**

$$0.1 - 10\mu\text{G}$$

$$10\text{kpc} - \underline{1\text{Mpc}}$$

[Clarke *et al.*, *Astrophys. J.* 547, L111 (2001)]

* **Recent reviews (examples)**

[Kandus *et al.*, *Phys. Rep.* 505, 1 (2011)] [Yamazaki *et al.*, *Phys. Rep.* 517, 141 (2012)]

[Maleknejad *et al.*, *Phys. Rep.* (2013)]

[Durrer and Neronov, *Astron. Astrophys. Rev.* 21, 62 (2013)]

Origin of cosmic magnetic fields

1. Astrophysical process: **Plasma instability**

[Biermann and Schlüter, *Phys. Rev.* **82**, 863(1951)]

[Weibel, *Phys. Rev. Lett.* **2**, 83 (1959)]

[Hanayama *et al.*, *Astrophys. J.* **633**, 941 (2005)]

[Fujita and Kato, *Mon. Not. Roy. Astron. Soc.* **364**, 247 (2005)]

2. Cosmological processes:

▪ **Electroweak/Quark-hadron phase transitions**

[Baym, Bödeker and McLerran, *Phys. Rev. D* **53**, 662 (1996)]

[Quashnock, Loeb and Spergel, *Astrophys. J.* **344**, L49 (1989)]

▪ **Density perturbations** [Ichiki *et al.*, *Science* **311**, 827 (2006)]

[Kobayashi, *et al.*, *Phys. Rev. D* **75**, 103501 (2007)]

Origin of cosmic magnetic fields (2)

- **Coherence scale**
- **Strength**

→ **It is difficult to obtain the results to explain the observations.**

⇒ **The most natural origin of large-scale magnetic fields:**

Electromagnetic quantum fluctuations generated at the inflationary stage

Inflationary cosmology

[Sato, Mon. Not. Roy. Astron. Soc. 195, 467 (1981)]

[Guth, Phys. Rev. D 23, 347 (1981)]

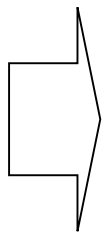
[Starobinsky, Phys. Lett. B 91, 99 (1980)]

[Linde, Phys. Lett. B 108, 389 (1982)]

[Albrecht and Steinhardt, Phys. Rev. Lett. 48, 1220 (1982)]

Inflation:

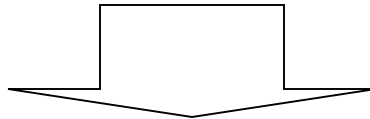
Exponential cosmic acceleration in the early universe



- 1. Homogeneity, Isotropy, and Flatness**
- 2. Primordial density perturbations**

Obstacle

- **Friedmann-Lemaître-Robertson-Walker (FLRW) metric is conformally flat.**
- **The Maxwell theory is conformally invariant.**



The conformal invariance has to be broken at the inflationary stage.

Breaking mechanisms

1. Coupling of a scalar field to electromagnetic fields

[Ratra, *Astrophys. J.* 391, L1 (1992)]

[KB and Yokoyama, *Phys. Rev. D* 69, 043507 (2004); 70, 083508 (2004)]

2. Non-minimal coupling of electromagnetic fields to gravity

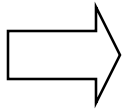
[Turner and Widrow, *Phys. Rev. D* 37, 2743 (1988)]

[KB and Sasaki, *JCAP* 0702, 030 (2007)]

3. Trace anomaly

[Dolgov, *Phys. Rev. D* 48, 2499 (1993)]

Motivation and Purpose (1)



By comparing the theoretical predictions of a toy model with observations, we obtain phenomenological implications on moduli inflation.

Motivation and Purpose (2)

Model (a) Coupling of the electromagnetic field to a scalar field

(b) That to a pseudo scalar field (inflaton)

Both couplings: Novel point

* **In the past works: Only (b)**

E.g.: [Barnaby and Peloso, Phys. Rev. Lett. 106, 181301 (2011)]

[Barnaby, Namba and Peloso, JCAP 1104, 009 (2011)]

Observables

(i) Large-scale magnetic fields

(ii) Non-Gaussianity

(iii) Tensor-to-scalar ratio

II. Model

$$\mathcal{L} = \underbrace{\left(\frac{M_P^2}{2} R \right)}_{\text{Gravity term}} - \frac{1}{4} X F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} g_{\text{ps}} \frac{Y}{M} F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - U(\Phi) - \frac{1}{2} g^{\mu\nu} \partial_\mu Y \partial_\nu Y - V(Y)$$

Breaking of the conformal invariance

$M_P \simeq 2.4 \times 10^{18}$ GeV : Reduced Planck mass

R : Scalar curvature

Model (2)

$$\mathcal{L} = \frac{M_P^2}{2} R \left(\underbrace{-\frac{1}{4} X F_{\mu\nu} F^{\mu\nu}}_{\text{red line}} \right) - \frac{1}{4} g_{\text{ps}} \frac{Y}{M} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$-\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - U(\Phi) - \frac{1}{2} g^{\mu\nu} \partial_\mu Y \partial_\nu Y - V(Y)$$

Coupling of a scalar field Φ to the electromagnetic field

$$X \equiv \exp(-\lambda\Phi/M_P), \quad \lambda : \text{Normalization constant}$$

$U(\Phi)$: Potential of Φ

$$F_{\mu\nu} = \nabla_\mu F_\nu - \nabla_\nu F_\mu, \quad F_\mu : \text{U}(1)_Y \text{ gauge field}$$

Model (3)

$$\mathcal{L} = \frac{M_P^2}{2} R - \frac{1}{4} X F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} g_{\text{ps}} \frac{Y}{M} F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - U(\Phi) - \frac{1}{2} g^{\mu\nu} \partial_\mu Y \partial_\nu Y - \underline{V(Y)}$$

Coupling of a pseudo scalar field Y (inflaton) to the electromagnetic field

g_{ps} : Dimensionless constant

$\tilde{F}_{\mu\nu}$: Dual tensor of $F_{\mu\nu}$

M : Mass scale

$V(Y) \approx \bar{V} - \frac{1}{2} m^2 Y^2$: Potential of Y \bar{V} : Normalization constant

(Feature of moduli inflation)

m : Mass of Y

Equations of motion (EoM) (1)

Flat FLRW space-time

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

$a(t)$: Scale factor

EoM for Φ

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{dU(\Phi)}{d\Phi} = 0$$

$H = \dot{a}/a$: Hubble parameter

$$\dot{} = \partial/\partial t$$

EoM for Y (inflaton)

$$\ddot{Y} + 3H\dot{Y} + \frac{dV(Y)}{dY} = 0$$

Equations of motion (EoM) (1)

EoM for the $U(1)_Y$ gauge field

* $F_0 = \partial_j F^j(t, \mathbf{x}) = 0$: Coulomb gauge

$$\ddot{F}_i(t, \mathbf{x}) + \underbrace{\left(H + \frac{\dot{X}}{X} \right)}_{\uparrow} \dot{F}_i(t, \mathbf{x}) - \frac{1}{a^2} \partial_j \partial_j F_i(t, \mathbf{x}) - \underbrace{\frac{g_{\text{ps}}}{M} \frac{1}{aX} \dot{Y} \epsilon^{ijk} \partial_j F_k(t, \mathbf{x})}_{\uparrow} = 0$$

Breaking of the conformal invariance

ϵ_{ijk} : Totally antisymmetric tensor
 ($\epsilon_{123} = +1$)

Slow-roll inflation

$$a(t) = a_k \exp [H_{\text{inf}} (t - t_k)] \longleftarrow \underline{\dot{Y}^2/2 \ll V(Y)}$$

$$Y = Y_k \exp \left\{ \frac{3}{2} \left[-1 \pm \sqrt{1 + \left(\frac{2m}{3H_{\text{inf}}} \right)^2} \right] H_{\text{inf}} (t - t_k) \right\}$$

$$k/(a_k H_{\text{inf}}) = 1$$

H_{inf} : Hubble parameter at the inflationary stage

a_k : Value of the scale factor at $t = t_k$

t_k : Time when a comoving wavelength $2\pi/k$ of the $U(1)_Y$ gauge field first crosses the horizon during inflation

Quantization of $U(1)_Y$ gauge field

Canonical momenta $\pi_0 = 0, \quad \pi_i = X_a \dot{F}_i(t, \mathbf{x})$

Canonical commutation relation \mathbf{k} : Comoving wavenumber

$$[F_i(t, \mathbf{x}), \pi_j(t, \mathbf{y})] = i \int \frac{d^3 k}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} [\delta_{ij} - (k_i k_j / k^2)] \quad k = |\mathbf{k}|$$

$$F_i(t, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} \left[\hat{b}(\mathbf{k}) \underline{F_i(t, \mathbf{k})} e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{b}^\dagger(\mathbf{k}) \underline{F_i^*(t, \mathbf{k})} e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

$$[\hat{b}(\mathbf{k}), \hat{b}^\dagger(\mathbf{k}')] = \delta^3(\mathbf{k} - \mathbf{k}'), \quad [\hat{b}(\mathbf{k}), \hat{b}(\mathbf{k}')] = [\hat{b}^\dagger(\mathbf{k}), \hat{b}^\dagger(\mathbf{k}')] = 0$$

$\hat{b}(\mathbf{k}), \hat{b}^\dagger(\mathbf{k})$: Annihilation and creation operators

Normalization condition

$$F_i(k, t) \dot{F}_j^*(k, t) - \dot{F}_j(k, t) F_i^*(k, t) = \frac{i}{X_a} [\delta_{ij} - (k_i k_j / k^2)]$$

Circular polarization of $U(1)_Y$ gauge field

- x^3 axis to lie along the spatial momentum k direction

→ Transverse directions x^I ($I = 1, 2$)

- **Circular polarization:** $F_{\pm}(k, t) \equiv F_1(k, t) \pm iF_2(k, t)$

$$\ddot{F}_{\pm}(k, t) + \left(H_{\text{inf}} + \frac{\dot{X}}{X} \right) \dot{F}_{\pm}(k, t) + \left[1 \pm \frac{g_{\text{ps}}}{M} \frac{\dot{Y}}{X} \left(\frac{k}{a} \right)^{-1} \right] \left(\frac{k}{a} \right)^2 F_{\pm}(k, t) = 0$$

$$C_+(k, t) \equiv \frac{F_+(k, t)}{F_+(k, t_k)} : \text{Normalized amplitude}$$

Evolution of $C_+(k, t)$ (1)

$C_+(k, t)$

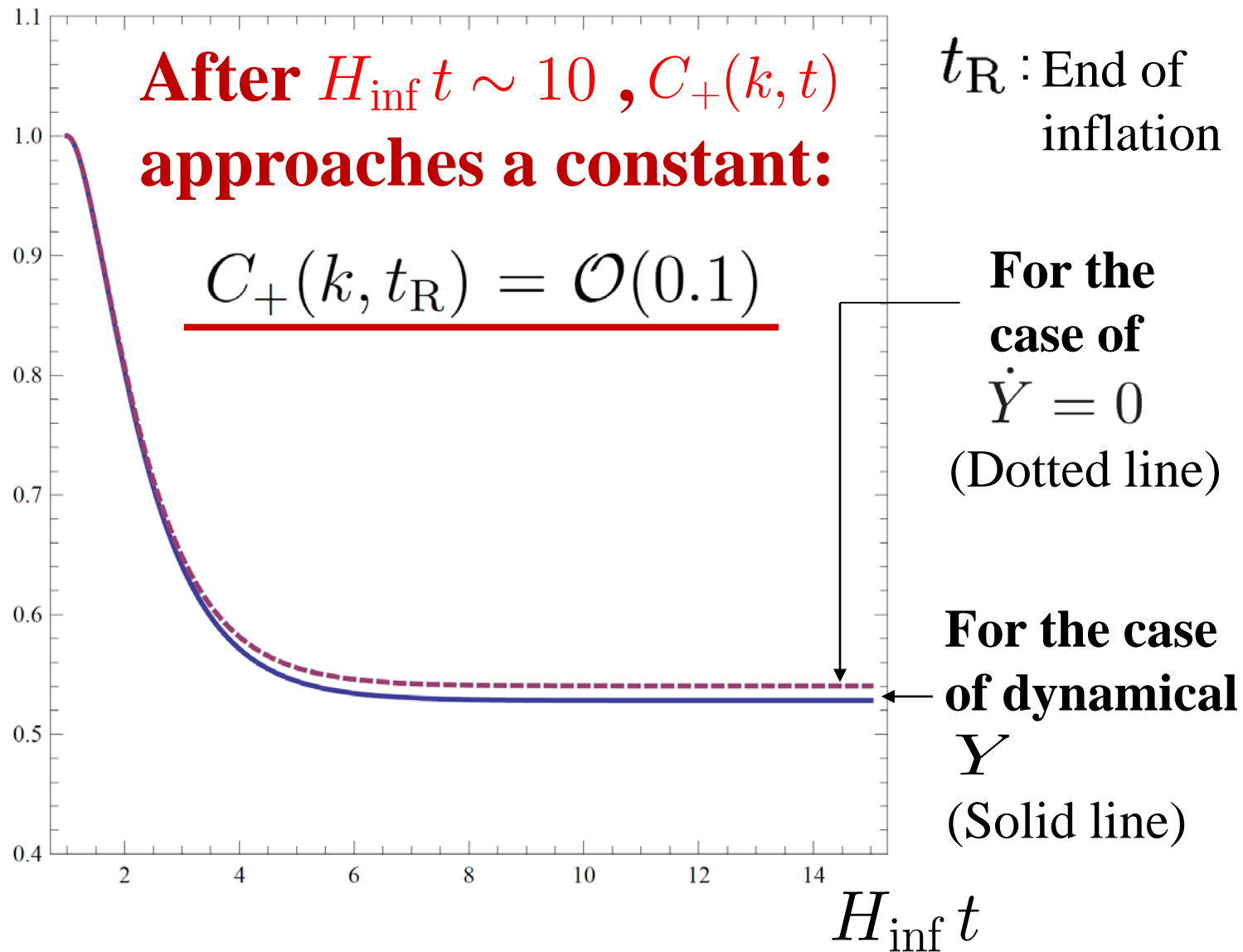


Fig. 1

Evolution of $C_+(k, t)$ (2)

Values of parameters for Fig.1

$$X(t_k) \equiv \exp(\chi_k), \quad \chi_k = -0.940$$

$$H_{\text{inf}} = 1.0 \times 10^{10} \text{GeV}, \quad m = 2.44 \times 10^9 \text{GeV}$$

$$Y_k = 7.70 \times 10^{-2} M_{\text{P}}, \quad M = 1.0 \times 10^{-1} M_{\text{P}}$$

$$\bar{V} = 5.07 \times 10^{-17} M_{\text{P}}^4, \quad g_{\text{ps}} = 1.0$$

Evolution of $C_+(k, t)$ (3)

- **COBE normalization**

(Amplitude of power spectrum of the curvature perturbations)

$$\Delta_{\mathcal{R}}^2(k) = 2.4 \times 10^{-9} \quad \text{at} \quad k = 0.002 \text{Mpc}^{-1}$$

- **Planck result**

(Spectral index of power spectrum of the curvature perturbations)

$$n_s = 0.9603 \pm 0.0073 \text{ (68\% CL)} \quad \text{at} \quad k = 0.05 \text{Mpc}^{-1}$$

[Ade *et al.* [Planck Collaboration], *Astron. Astrophys.* **571**, A22 (2014)]

$$n_s \simeq 1 - 6\epsilon_V + 2\eta_V \quad \text{Slow-roll parameters:}$$

$$\epsilon_V \equiv \frac{M_{\text{P}}^2}{2} \left(\frac{V'(Y)}{V(Y)} \right)^2, \quad \eta_V \equiv M_{\text{P}}^2 \frac{V''(Y)}{V(Y)}$$

- $\bar{V} = 3H_{\text{inf}}^2 M_{\text{P}}^2$: **Friedmann equation**

III. Current strength of the magnetic fields

Proper magnetic field

$$B_{Y_i}^{\text{proper}}(t, \mathbf{x}) = \frac{1}{a^2} B_{Y_i}(t, \mathbf{x}) = \frac{1}{a^2} \epsilon_{ijk} \partial_j F_k(t, \mathbf{x})$$

- **Instantaneous reheating at $t = t_R$**
- **After reheating, $\sigma_c \gg H$.**

σ_c : Cosmic conductivity

$$\Rightarrow B^{\text{proper}} \propto a^{-2}$$

Current strength of the magnetic fields (2)

$$\rho_B(L, t) \simeq \frac{1}{8\pi^2} \frac{1}{X(t_k)} \frac{1}{\sqrt{2\xi_k}} \exp \left[2 \left(\pi\xi_k - 2\sqrt{2\xi_k} \right) \right] \left(\frac{k}{a} \right)^4 |C_+(k, t_R)|^2$$

Tachyon instability

$$\propto (X(t_k))^{-1}$$

$$\xi \equiv \frac{1}{2} \frac{g_{\text{ps}}}{M} \frac{1}{X} \frac{\dot{Y}}{H_{\text{inf}}}, \quad \xi_k \equiv \xi(t = t_k)$$

$L = 2\pi/k$: Comoving scale

* $X(t_R) = 1$: **After inflation, the Maxwell theory is recovered.**

Current strength of the magnetic fields (3)

	$B(H_0^{-1}, t_0)$ [G]	H_{inf} [GeV]	m [GeV]	Y_k/M_{P}	$C_+(k, t_{\text{R}})$
(a)	7.15×10^{-64}	1.0×10^{11}	2.44×10^{10}	7.70×10^{-2}	0.528
(b)	7.15×10^{-64}	1.0×10^{10}	2.44×10^9	7.70×10^{-2}	0.528
(c)	2.33×10^{-64}	1.0×10^8	1.0×10^7	1.62×10^1	0.172
(d)	2.33×10^{-64}	1.0×10^6	1.0×10^5	1.62×10^1	0.172
(e)	2.85×10^{-64}	1.0×10^4	8.0×10^2	2.23×10^1	0.211
(f)	2.85×10^{-64}	1.0×10^2	8.0	2.23×10^1	0.211

Current strength of magnetic fields on the Hubble horizon scale H_0^{-1} :

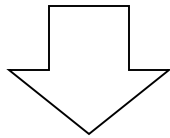
t_0 : Present time

For Fig. 1 $B(H_0^{-1}, t_0) \sim 10^{-64}$ G

IV. Non-Gaussianity of curvature perturbations

- **Perturbed Einstein equation**

Metric perturbation



= Matter density perturbation

① Scalar mode (Curvature perturbation)

→ **Temperature perturbation of the cosmic microwave background (CMB) radiation:**

$$\frac{\delta T}{T} \simeq 10^{-5}$$

(2) Vector mode

③ Tensor mode (Primordial gravitational wave)

Non-Gaussianity of curvature perturbations (2)

- **We suppose that the $U(1)_Y$ gauge field F_μ couples to another Higgs-like field φ , which develops a vacuum expectation value, through the kinetic term $|D\varphi|^2$.**

[Meerburg and Pajer, JCAP 1302, 017 (2013)]

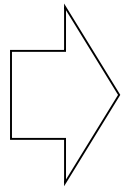
$$D_\mu \equiv \partial_\mu + ig'F_\mu : \text{Covariant derivative for } \varphi$$

g' : Gauge coupling

⇒ **The gauge symmetry can spontaneously be broken, so that the gauge field acquires its mass.**

Non-Gaussianity of curvature perturbations (3)

- **The number of e -folds of inflation could be changed by the perturbations of φ .**
- **The curvature perturbations can be generated through the perturbations of φ .**



The local-type non-Gaussianity in terms of the curvature perturbations is produced.

* δN formalism

[Sasaki and Stewart, Prog. Theor. Phys. 95, 71 (1996)]

[Starobinsky, JETP Lett. 42, 152(1985)]

[Lyth and Rodriguez, Phys. Rev. Lett. 95, 121302 (2005)]

Non-Gaussianity of curvature perturbations (4)

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + \underline{f_{\text{NL}}^{\text{local}}} (\Phi_L^2(\mathbf{x}) - \langle \Phi_L^2(\mathbf{x}) \rangle)$$



Quantity showing the non-Gaussianity

[Ade *et al.* [Planck Collaboration], *Astron. Astrophys.* **571**, A24 (2014)]

$\Phi_L(\mathbf{x})$: **Gravitational potential with the Gaussian statistical property**

Non-Gaussianity of curvature perturbations (5)

$$f_{\text{NL}}^{\text{local}} \approx 1.0 \times 10^{14} \frac{g'^4}{\xi^6} \frac{m^2}{H_{\text{inf}}^2}$$

g' : Gauge coupling [Meerburg and Pajer, JCAP 1302, 017 (2013)]

	$f_{\text{NL}}^{\text{local}}$	g'^2	H_{inf} [GeV]	m [GeV]	Y_k/M_{P}	$B(H_0^{-1}, t_0)$ [G]
(A)	2.70	1.13×10^{-5}	1.0×10^{13}	2.44×10^{12}	7.70×10^{-2}	7.15×10^{-64}
(B)	2.12×10^8	1.0×10^{-1}	1.0×10^{12}	2.44×10^{11}	7.70×10^{-2}	7.15×10^{-64}

$$\xi_k = 2.5590616$$

Planck result

$$f_{\text{NL}}^{\text{local}} = 2.5 \pm 5.7 \quad (68\% \text{ CL})$$

[Ade *et al.* [Planck Collaboration], arXiv:1502.01592]

V. Tensor-to-scalar ratio

Power spectrum of the curvature perturbation

$$\propto k^{n_s-1} \quad n_s : \text{Spectral index}$$

Tensor-to-scalar ratio

$$r_{\text{TS}} \equiv \frac{\text{Power spectrum of the tensor mode}}{\text{Power spectrum of the scalar mode}}$$

Tensor-to-scalar ratio (2)

$$r_{\text{TS}} = 16\epsilon_V(t_k), \quad \epsilon(t_k) = \frac{2M_{\text{P}}^2 m^4 Y_k^2}{(2\bar{V} - m^2 Y_k)^2}$$

[Barnaby and Peloso, Phys. Rev. Lett. **106**, 181301 (2011)]

[Barnaby, Namba and Peloso, JCAP **1104**, 009 (2011)]

	r_{TS}	H_{inf} [GeV]	m [GeV]	Y_k/M_{P}
(A)	1.87×10^{-5}	1.0×10^{13}	2.44×10^{12}	7.70×10^{-2}
(B)	1.87×10^{-5}	1.0×10^{12}	2.44×10^{11}	7.70×10^{-2}

Planck result

$$r_{\text{TS}} < 0.11 \text{ (95\% CL)} \quad \text{at} \quad k = 0.002 \text{Mpc}^{-1}$$

[Ade *et al.* [Planck Collaboration], arXiv:1502.02114]

Cf. [Ade *et al.* [Planck Collaboration], arXiv:1502.00612]

VI. Conclusions

- **We have studied the generation of the large-scale magnetic fields from a kind of moduli inflation.**
- **We have first estimated the explicit values of three cosmological observables such as the current magnetic fields on the Hubble horizon, local non-Gaussianity, and the tensor-to-scalar ratio.**
- **The local non-Gaussianity and tensor-to-scalar ratio obtained in this model are consistent with the Planck results.**

Acknowledgments

- **The presenter would like to sincerely appreciate the significant discussions with Professor Tatsuo Kobayashi and Professor Osamu Seto and their kind important suggestions and comments.**

Back up slides

Non-Gaussianity of curvature perturbations (4)

- If $\sigma_c \gg H$, the effect of the coupling between F_μ and φ on the resultant amplitude of the large-scale magnetic fields can be negligible.

[Bassett, Pollifrone, Tsujikawa and Viniegra, Phys. Rev. D 63, 103515 (2001)]

Axion monodromy inflation

$$V(Y) = AY^q, \quad q = 1$$

$$X = 1, \text{ (i.e., } \Phi = 0)$$

[Kobayashi, Seto and Yamaguchi, PTEP 2014, 103C01 (2014)]

[Higaki, Kobayashi, Seto and Yamaguchi, JCAP 1410, 025 (2014)]

$$Y = \bar{Y}t, \quad \bar{Y} = -\frac{A}{3H_{\text{inf}}}$$

$$\ddot{F}_i(t, \mathbf{x}) + H\dot{F}_i(t, \mathbf{x}) - \frac{1}{a^2}\partial_j\partial_j F_i(t, \mathbf{x}) + \frac{g_{\text{ps}}}{M} \frac{1}{a} \frac{A}{3H_{\text{inf}}} \epsilon^{ijk} \partial_j F_k(t, \mathbf{x}) = 0$$

$$\xi = -\frac{1}{6} \frac{g_{\text{ps}}}{M} \frac{A}{H_{\text{inf}}^2}$$

Axion monodromy inflation (2)

Example

$$M = 3.55 \times 10^{18} \text{GeV}, \quad g_{\text{ps}} = 1.0$$

$$|\xi| = 2.98, \quad H_{\text{inf}} = 6.51 \times 10^{15} \text{GeV}$$

$$n_s = 0.9603$$

$$\longrightarrow \underline{r_{\text{TS}} = 16\epsilon_V = 0.106}$$

This is consistent with BICEP2 result.

Power spectrum of the curvature perturbations

Evolution equation of quantum fluctuations of Y

$$\frac{\partial^2 \delta Y(t, \mathbf{x})}{\partial t^2} + 3H \frac{\partial \delta Y(t, \mathbf{x})}{\partial t} - \frac{\nabla^2 \delta Y(t, \mathbf{x})}{a^2} = \frac{g_{\text{ps}}}{M} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Two-point correlation function in the Fourier space

$$\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle \equiv (2\pi^2/k^3) P_{\mathcal{R}}(k) \delta^{(3)}(\mathbf{k} + \mathbf{k}') \quad \mathcal{R} = - \left(H/\dot{Y} \right) \delta Y$$

$$P_{\mathcal{R}}(k) \simeq \Delta_{\mathcal{R}}^2 \left(\frac{k}{k_*} \right)^{n_s-1} \left(1 + \Delta_{\mathcal{R}}^2 f_S(\xi) \exp(4\pi\xi) \right)$$

: Curvature perturbations

$$\Delta_{\mathcal{R}}^2 = \left(\frac{H_{\text{inf}}}{2\pi} \right)^2 \frac{H_{\text{inf}}^2}{|\dot{Y}|^2}, \quad k_* = 0.002 \text{ Mpc}^{-1}$$

[Barnaby and Peloso, *Phys. Rev. Lett.* **106**, 181301 (2011)]

$$f_S(\xi) \cong 3.0 \times 10^{-5} \xi^{-5.4} \quad \text{for } 2 \leq \xi \leq 3$$

[Barnaby, Namba and Peloso, *JCAP* **1104**, 009 (2011)]

[Meerburg and Pajer, *JCAP* **1302**, 017 (2013)]

Normalization with COBE and Planck data

Slow-roll parameters: $\epsilon_V \equiv \frac{M_{\text{P}}^2}{2} \left(\frac{V'(Y)}{V(Y)} \right)^2$, $\eta_V \equiv M_{\text{P}}^2 \frac{V''(Y)}{V(Y)}$

Spectral index: $n_s \simeq 1 + 6\epsilon_V + 2\eta_V$

With COBE and Planck data, we find

$$f_s(\xi) \exp(4\pi\xi) = \frac{25}{144} \times 10^8$$

$$Y_k = \pm \frac{M_{\text{P}}}{\sqrt{2}\beta} \left[4 \frac{\bar{V}}{M_{\text{P}}^2 m^2 \beta^{-2}} + 1 \pm \sqrt{12 \frac{\bar{V}}{M_{\text{P}}^2 m^2 \beta^{-2}} + 1} \right]^{1/2}$$

$$\beta \equiv \sqrt{\frac{-(n_s - 1)}{8}}$$

Normalization with COBE and Planck data (2)

For $m/H_{\text{inf}} \ll 1$, **we obtain**

$$m \approx \sqrt{6} \xi_k \frac{M}{M_{\text{P}}} X(t_k) H_{\text{inf}}$$

$$Y_k \approx \sqrt{6} M_{\text{P}} \frac{H_{\text{inf}}}{m} = \frac{M_{\text{P}}^2}{\xi_k X(t_k) M}$$

$$\xi_k \approx [Y_k / (6M X(t_k))] (m^2 / H_{\text{inf}}^2)$$

Cf.

$$\dot{Y}(t_{\text{R}}) = \frac{3}{2} \left[-1 + \sqrt{1 + \left(\frac{2m}{3H_{\text{inf}}} \right)^2} \right] H_{\text{inf}} Y_k \exp \left\{ \frac{3}{2} \left[-1 + \sqrt{1 + \left(\frac{2m}{3H_{\text{inf}}} \right)^2} \right] (N - 1) \right\}$$

Tensor-to-scalar ratio (4)

BICEP2 experiment

$$r_{\text{TS}} = 0.20^{+0.07}_{-0.05} \quad (68\% \text{ CL})$$

[Ade *et al.* [BICEP2 Collaboration], PRL (2014)]

- * Inflationary models (Examples)
 - [KB, Cognola, Odintsov, Zerbini, arXiv:1404.4311]
 - [KB, Myrzakulov, Odintsov, Sebastiani, arXiv:1403.6649]
 - [Kobayashi, Seto, PRD (2014); 1404.3102]
 - [Kobayashi, Seto, Yamaguchi, arXiv:1404.5518]
 - [Higaki, Kobayashi, Seto, Yamaguchi, arXiv:1405.0775]
 - [Nakayama, Takahashi, PLB (2014)]
 - [Harigaya, Ibe, Schmitz, Yanagida, PLB (2014)]
 - [Bonvin, Durrer, Maartens, GRG (2014)]

Evolution of $C_+(k, t)$ (3)

- **COBE normalization**

(Amplitude of power spectrum of the curvature perturbations)

$$\Delta_{\mathcal{R}}^2(k) = 2.4 \times 10^{-9} \quad \text{at} \quad k = 0.002 \text{Mpc}^{-1}$$

- **Planck result**

(Spectral index of power spectrum of the curvature perturbations)

$$n_s = 0.9603 \pm 0.0073 \text{ (68\% CL)} \quad \text{at} \quad k = 0.05 \text{Mpc}^{-1}$$

[Ade *et al.* [Planck Collaboration], *Astron. Astrophys.* 571, A22 (2014)]

Cf. $n_s = 0.968 \pm 0.006 \text{ (68\% CL)} \quad \text{at} \quad k = 0.05 \text{Mpc}^{-1}$

[Ade *et al.* [Planck Collaboration], arXiv:1502.02114]

- $\bar{V} = 3H_{\text{inf}}^2 M_{\text{P}}^2$: **Friedmann equation**

X の変動

$$X \equiv \exp(-\lambda\Phi/M_P)$$

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{dU(\Phi)}{d\Phi} = 0$$

$$\longrightarrow \underset{\uparrow}{3H_{\text{inf}}\dot{\Phi}} + dU(\Phi)/d\Phi = 0$$

$$|\ddot{\Phi}/(H_{\text{inf}}\dot{\Phi})| \ll 1$$

$$\rightarrow \text{解: } \Phi = (\lambda\kappa)^{-1} \ln \left[\lambda^2 w H_{\text{inf}} (t - t_R) + \exp(\lambda\kappa\Phi_R) \right]$$

\Rightarrow 定性的に同様の結果が得られる。

Contents

I. Introduction

II. Model

III. Current strength of the magnetic fields

IV. Non-Gaussianity of curvature perturbations

V. Tensor-to-scalar ratio

VI. Conclusions

Origin of cosmic magnetic fields (1)

1. Astrophysical process: **Plasma instability**

(a) **Biermann battery mechanism**

[Biermann, Schlüter, Phys. Rev. (1951)]

[Hanayama *et al.*, ApJ (2005)]

(b) **Weibel instability**

[Weibel, PRL (1959)]

[Fujita, Kato, MNRAS (2005)]

Origin of cosmic magnetic fields (2)

2. Cosmological processes:

- **Phase transitions**

- (i) **Electroweak phase transition (EWPT)**

[Baym, Bödeker, McLerran, PRD (1996)]

- (ii) **Quark-hadron phase transition (QCDPT)**

[Quashnock, Loeb, Spergel, ApJ (1989)]

- **Density perturbations**

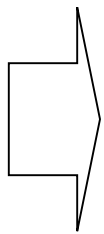
[Ichiki *et al.*, Science (2006)]

[Kobayashi, *et al.*, PRD (2007)]

Inflationary cosmology

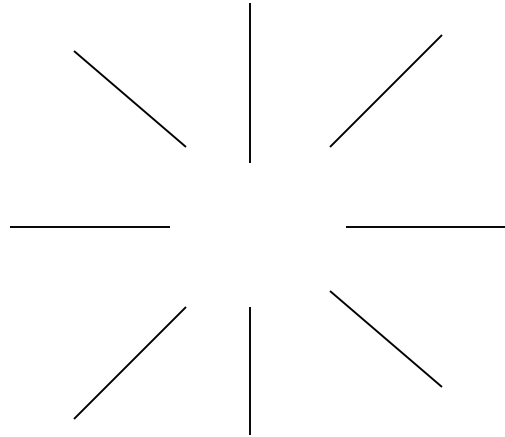
Inflation:

Exponential cosmic acceleration in the early universe

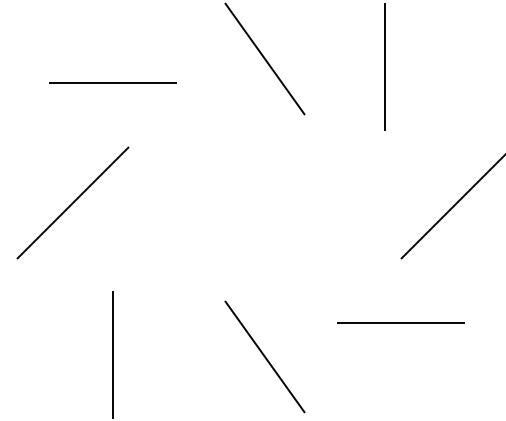


- 1. Homogeneity, Isotropy, Flatness**
- 2. Primordial density perturbations**

Tensor-to-scalar ratio (2)



E-mode polarization



B-mode polarization



Only from the tensor mode

[E. Komatsu, *The astronomical herald*,
The Astronomical Society of Japan, 2003]

IV. Non-Gaussianity of curvature perturbations

Einstein equation

→ Metric perturbation = Matter density perturbation

① Scalar mode (Curvature perturbation)

Temperature perturbation of the cosmic microwave background radiation (CMB): $\frac{\delta T}{T} \simeq 10^{-5}$

(2) Vector mode

③ Tensor mode (Primordial gravitational wave)

V. Tensor-to-scalar ratio

Power spectrum of the curvature perturbation

$$\propto k^{n_s-1} \quad n_s : \text{Spectral index}$$

Tensor-to-scalar ratio

$$r_{\text{TS}} \equiv \frac{\text{Power spectrum of the tensor mode}}{\text{Power spectrum of the scalar mode}}$$

Current strength of the magnetic fields (3)

- **The resultant strength of the magnetic fields satisfies the constraints suggested by the back reaction problem.**

[Demozzi, Mukhanov and Rubinstein, JCAP 0908, 025 (2009)]

Cf. [Kanno, Soda and Watanabe, JCAP 0912, 009 (2009)]

[Suyama and Yokoyama, Phys. Rev. D 86, 023512 (2012)]

[Fujita and Mukohyama, JCAP 1210, 034 (2012)]

[Fujita and Yokoyama, JCAP 1403, 013 (2014)]

* **Observational constraints found from Planck**

$$B < 4.4 \text{ nG on } 1\text{Mpc}$$

[Ade *et al.* [Planck Collaboration], arXiv:1502.01594]

磁場の強度に対する制限

1. CMB

$$B < 5 \times 10^{-9} \text{G}$$

$$L = H_0^{-1}$$

[Barrow, Ferreira,
Silk, PRL (1997)]

2. BBN

$$B < 10^{-6} \text{G}$$

$$h_{70} \equiv h/0.70$$

$$L \sim 1.4 \times 10^{-4} h_{70}^{-1} \text{ Mpc}$$

[Grasso, Rubinstein, PLB (1996)] [Cheng, Olinto, Schramm, Truran, PRD (1996)]

3. ファラデー回転の Rotation measure (RM)

$$B < 6 \times 10^{-10} (n_{e0}/10^{-7} \text{ cm}^{-3})^{-1} \text{ G}$$

n_{e0} : 熱的電子の現在の平均密度 [Vallée, ApJ (1990)]

II B. 磁場の現在での強度

<固有磁場>

$$B_i^{\text{proper}}(t, \boldsymbol{x}) = a^{-1} B_i(t, \boldsymbol{x}) = a^{-2} \epsilon_{ijk} \partial_j A_k(t, \boldsymbol{x})$$

<Fourier 空間での磁場のエネルギー密度>

$$\rho_B(k, t) = \frac{1}{2} \left[|B_+^{\text{proper}}(k, t)|^2 + |B_-^{\text{proper}}(k, t)|^2 \right]$$

$$|B_{\pm}^{\text{proper}}(k, t)|^2 = \frac{1}{a^2} \left(\frac{k}{a} \right)^2 |A_{\pm}(k, t)|^2$$

$$B_{\pm}^{\text{proper}}(k, t) \equiv B_1^{\text{proper}}(k, t) \pm i B_2^{\text{proper}}(k, t)$$

<磁場のエネルギー密度>

$$\rho_B(L, t) = \frac{k^3}{4\pi^2} \left[|B_+^{\text{proper}}(k, t)|^2 + |B_-^{\text{proper}}(k, t)|^2 \right]$$

$$\rho_B(L, t) = \frac{1}{8\pi^2} \left(\frac{k}{a} \right)^4 \underline{\mathcal{I}(k, t)}, \quad \mathcal{I}(k, t) \equiv |C_+(k, t)|^2 + |C_-(k, t)|^2$$

$\rho_B(k, t)$ に
相空間密度:
 $4\pi k^3 / (2\pi)^3$
をかける。

インフレーション期での増幅項

現在の磁場の強度

$$B_{Y_i}^{\text{proper}}(t, \boldsymbol{x}) = \frac{1}{a^2} B_{Y_i}(t, \boldsymbol{x}) = \frac{1}{a^2} \epsilon_{ijk} \partial_j F_k(t, \boldsymbol{x})$$

$$E_{Y_i}^{\text{proper}}(t, \boldsymbol{x}) = a^{-1} E_{Y_i}(t, \boldsymbol{x}) = -a^{-1} \dot{F}_i(t, \boldsymbol{x})$$

$$\rho_{B_Y}(L, t) = \frac{k^3}{4\pi^2} \left[|B_{Y_+}^{\text{proper}}(k, t)|^2 + |B_{Y_-}^{\text{proper}}(k, t)|^2 \right] X$$

$$|B_{Y_{\pm}}^{\text{proper}}(k, t)|^2 = (1/a^2) (k/a)^2 |F_{\pm}(k, t)|^2$$

→ $B(L, t) = \sqrt{2\rho_B(L, t)}$ を用いる。

$$B(L, t_0) = 8.2 \times 10^{18} \exp(-2N) \left(\frac{H_{\text{inf}}}{[\text{GeV}]} \right)^2 \left(\frac{a_R}{a_0} \right)^2 \sqrt{\mathcal{I}(k, t_R)} \quad [\text{G}]$$

$$N = 45 + \ln \left(\frac{L}{[\text{Mpc}]} \right) + \ln \left\{ \frac{[30/(\pi^2 g_R)]^{1/12} \rho_R^{1/4}}{10^{38/3} [\text{GeV}]} \right\} : \begin{array}{l} t_1 \text{ と } t_R \text{ の間} \\ \text{の } e\text{-folds 数} \end{array}$$

$$\rho_R = \frac{\pi^2}{30} g_R T_R^4 : t_R \text{ での輻射のエネルギー密度、} \quad T_R : t_R \text{ での温度}$$

$$\frac{a_R}{a_0} = \left(\frac{g_R}{3.91} \right)^{-1/3} \frac{T_{\gamma 0}}{T_R}$$

$T_{\gamma 0} (= 2.73[\text{K}])$: 現在の温度

$g_R (\approx 100)$: t_R での相対論的粒子の全自由度数